Bifurcations in Ratra-Peebles quintessence models and their extensions

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Cosmology with scalar field

Scalar field approach:

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- In order to maintain covariance of the action, **requires implementation of the potential** $U(\phi)$, affecting the scalar field ϕ.

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$$S = S_g + S_\phi,$$

where

$$S_g = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}R \quad \text{(Einstein-Hilbert action)},$$

$$S_\phi = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[\underbrace{\varepsilon\nabla^\alpha\phi\nabla_\alpha\phi}_{\text{kinetic energy}}\ \underbrace{+2U(\phi)}_{\text{potential energy}}\ \underbrace{+\varepsilon\xi R\phi^2}_{R-\phi\text{ coupling}}\right],$$

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  - $\kappa^2 = 8\pi G$, $c = 1$ and the signature is $(-+++)$.

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- We use the **Ratra-Peebles potential**

$$U(\phi) = \frac{M^{n+4}}{\phi^n},$$

where n is a dimensionless parameter and $M > 0$ is a dimensional constant.

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- In new variables, after variation of action w.r.t. $g^{\mu\nu}$ and ϕ, we obtain the **dynamical system**

$$u' = \left[-\frac{1}{2}u^2(2+n) - \frac{3}{2}u(1+4\xi n) + \frac{1}{2}\varepsilon n v^2 - 3\xi(1+n)\right]\left[\frac{1}{3}\varepsilon v^2 - 2\xi(1-6\xi)\right]^2 +$$
$$+ (6\xi + u)\left[\frac{1}{3}\varepsilon v^2 - 2\xi(1-6\xi)\right] \cdot$$
$$\cdot \left(u^2\left[1-\xi(2-n)\right] + 4\xi u(2+3\xi n) - \frac{1}{2}\varepsilon v^2(1+2\xi n) + 3\xi\left[1+2\xi(1+n)\right]\right),$$
$$v' = -uv\left[\frac{1}{3}\varepsilon v^2 - 2\xi(1-6\xi)\right]^2,$$

where $f' = \frac{\mathrm{d}f}{\mathrm{d}\ln a} = H^{-1}\dot{f}$.

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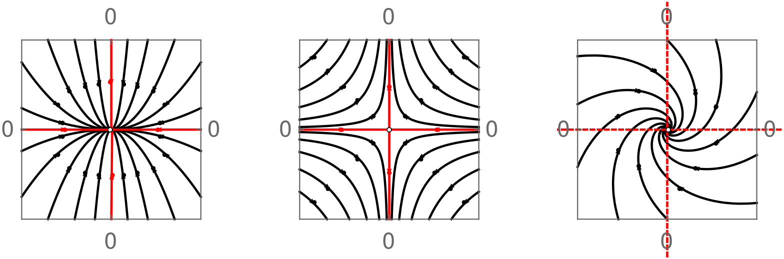

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- **Definition:** The appearance of topologically nonequivalent phase portrait under variation of parameters is called a **bifurcation**.

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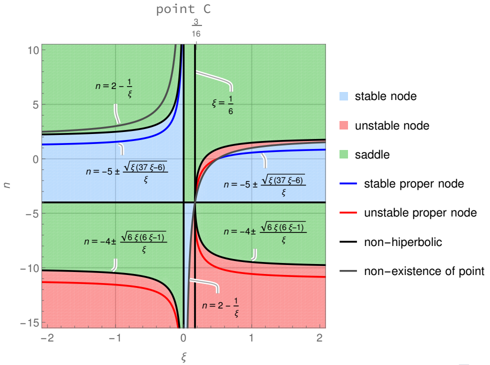

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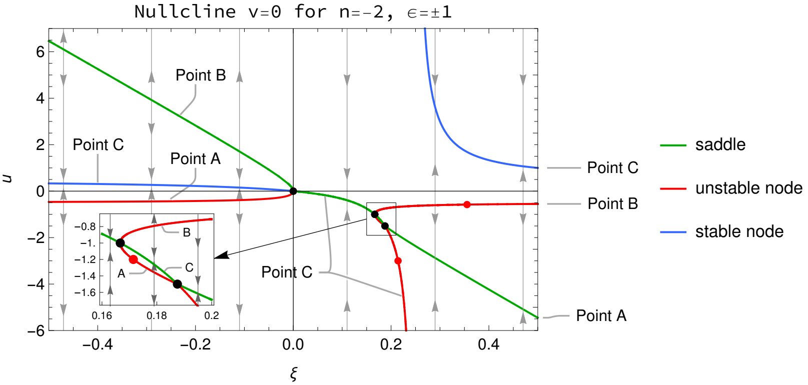
Table: Sets of parameters for which the universe undergoes the evolution starting from the de Sitter state and finishing in the de Sitter state.

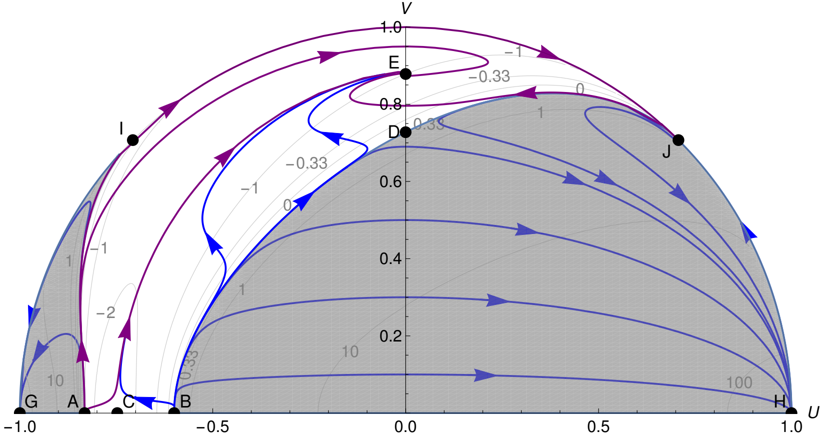
no.	ξ	n	ε	starting point	final point
1. Generic de Sitter–de Sitter evolution					
(a)	$\frac{3}{16}$	$(0, +\infty)$	$+1$	unstable node A	stable focus E
(b)	$\frac{3}{16}$	$(-2, 0)$	-1	unstable node A	stable focus E
(c)	$\left(\frac{3}{16}, \frac{1}{4}\right)$	-2	-1	unstable node C	stable focus E
2. Non-generic de Sitter–de Sitter evolution					
(d)	$(-\infty, 0]$	-2	$+1$	saddle E	stable node C
(e)	$\frac{3}{16}$	$\left(-3\frac{5}{9}, -2\right)$	-1	saddle A	stable focus E
(f)	$\frac{3}{16}$	$\left[-4, -3\frac{5}{9}\right)$	-1	saddle A	saddle E
(g)	$\left[0, \frac{3}{16}\right)$	-2	-1	saddle C	stable node/focus E
(h)	$\left(\frac{1}{3}, +\infty\right)$	-2	-1	saddle E	stable node C

‘Collisions’ of equilibria for generic dS–dS scenarios

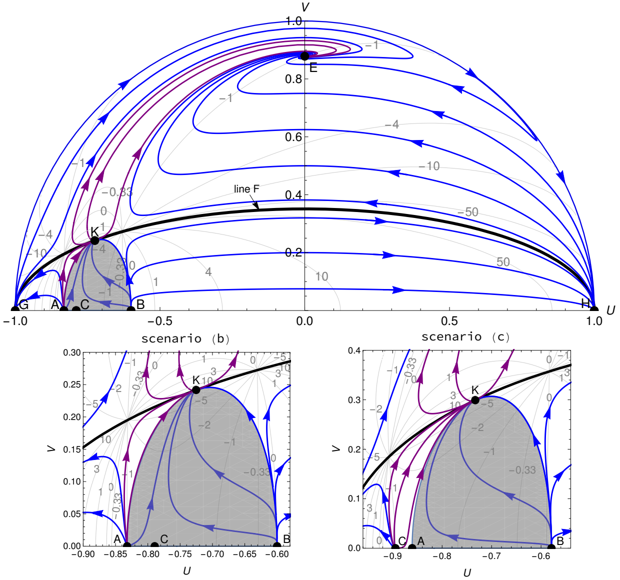
- Again, bifurcation diagrams (showing ‘collisions’ of equilibria) indicated that one phase portrait is fully representative for each of scenarios (a)–(c).



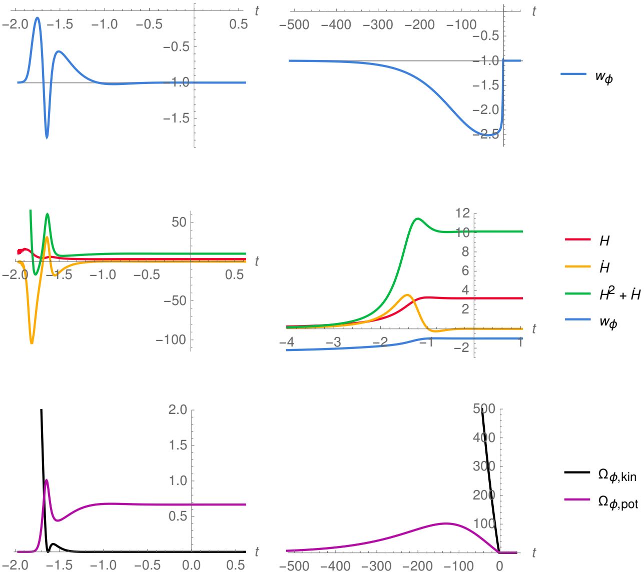
Phase portrait for scenario (a)



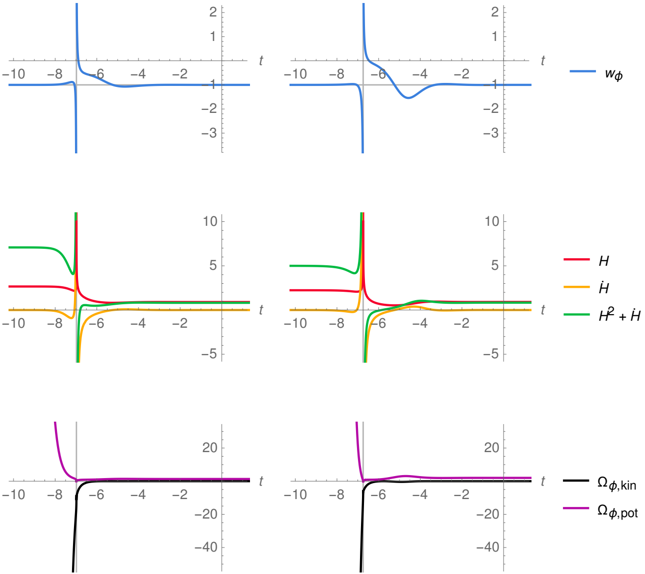
Phase portrait for scenarios (b) and (c)



Evolution of physical quantities for case (a)



Evolution of physical quantities for cases (b) and (c)



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- *THANK YOU FOR YOUR ATTENTION!*