Flavor dependence of GPDs in the Large-N_c limit

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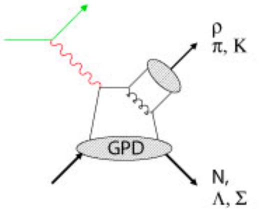
Outline

- Chiral-odd GPDs
- Bag Model
- Chiral-odd GPDs in Bag Model
- Large-N_c expansion in Bag Model
- Phenomenological implications

Chiral-odd GPDs

- Accessible through exclusive meson production processes



- There are four chiral-odd GPDs $H_T, \tilde{H}_T, E_T, \tilde{E}_T$ at leading twist

$$
\begin{aligned}
&\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar{\psi}(-\tfrac{1}{2}z)\, i\sigma^{+i}\, \psi(\tfrac{1}{2}z)\, |p,\lambda\rangle \Big|_{z^+=0,\, \mathbf{z}_T=0} \\
&= \frac{1}{2P^+}\bar{u}(p',\lambda') \left[H_T^q\, i\sigma^{+i} + \tilde{H}_T^q \frac{P^+\Delta^i - \Delta^+P^i}{m^2} \right. \\
&\qquad \left. + E_T^q \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^q \frac{\gamma^+P^i - P^+\gamma^i}{m} \right] u(p,\lambda).
\end{aligned}
$$

where $i = 1, 2$ is the transversity index [Diehl '03]

Properties of chiral-odd GPDs

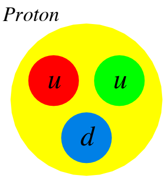
- In the forward limit $\Delta \to 0$, H_T reduces to transversity PDF; $H_T(x,0,0) \to h_1(x)$
- It follows from the time reversal invariance that the GPDs $H_T, \tilde{H}_T, E_T$ are invariant under the transformation $\xi \to -\xi$. Whereas $\tilde{E}_T$ is subject to sign change, i.e.

$$\begin{aligned} GPD(x,\xi,t) =& GPD(x,-\xi,t) \qquad \text{for } GPD = H_T, \tilde{H}_T, E_T \\ GPD(x,\xi,t) =& -GPD(x,-\xi,t) \quad \text{for } GPD = \tilde{E}_T \end{aligned}$$

- Polynomiality! In particular, first moments of the chiral-odd GPDs are the nucleon's tensor form factors, i.e.

$$\begin{aligned} \int_{-1}^{1} \left\{ H_T, \tilde{H}_T, E_T \right\}(x,\xi,t)dx =& H_T(t), \tilde{H}_T(t), E_T(t) \\ \int_{-1}^{1} \left\{ \tilde{E}_T \right\}(x,\xi,t)dx =& 0 \end{aligned}$$

Bag Model



- Quarks are constrained inside of a finite size "bag"
- Quarks are free inside the bag (Asymptotic freedom), however are subject to sharp boundary conditions on the surface to implement the confinement.
- The Lagrangian density of the system for massless quarks is given by

$$\mathcal{L} = \left(i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - B\right)\Theta_V - \frac{1}{2}\bar{\psi}\psi\delta_S$$

where Θ_V is the volume inside the bag and δ_S is a δ-function at the bag surface. The constant B is the energy needed to create the perturbative vacuum inside the bag.

Bag Model

- Quarks inside the bag satisfy the Dirac equation
- Equations of motion of the system further asserts that

$$\eta_\mu j^\mu = \eta_\mu \bar{\psi}\gamma^\mu\psi = 0 \text{ (conservation of current)}$$
$$\partial_\mu T^{\mu\nu} = 0 \text{ (conservation of EMT)}$$

- Bag model has been used to obtain the first estimations for chiral-even GPDs [Ji, Melnitchouk, Song '97]
- We use the Bag model to evaluate the off-forward matrix elements in the Breit frame

$$p^\mu = (\bar{m}, -\vec{\Delta}/2) \quad \text{and} \quad p'^\mu = (\bar{m}, \vec{\Delta}/2)$$

where $\bar{m}^2 = P^2$.

Chiral-odd GPDs in Bag Model

- The momentum space wave function in the bag is given by

$$\varphi(\vec{k}) = \sqrt{4\pi} N R^3 \begin{pmatrix} t_0(k)\chi_m \\ \vec{\sigma}\cdot\hat{k} \quad t_1(k)\chi_m \end{pmatrix}$$

where N is the normalization constant, R is the bag radius and

$$t_0(k) = \frac{j_0(w_0)\cos(kR) - j_0(kR)\cos(w_0)}{w_0^2 - \vec{k}^2 R^2}$$

$$t_1(k) = \frac{j_0(kR) j_1(w_0) kR - j_0(w_0) j_1(kR) w_0}{w_0^2 - \vec{k}^2 R^2}$$

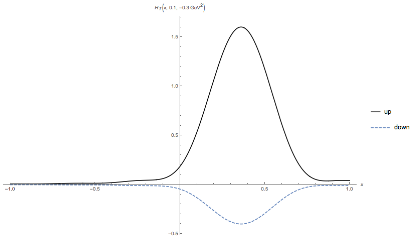
- Use this wave function to evaluate the correlators on the left hand side;

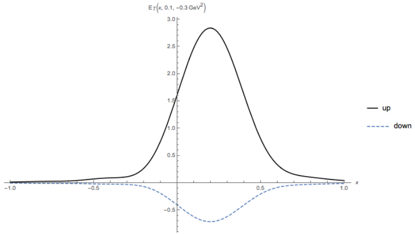
$$\varphi^\dagger(k') S(\Lambda_{-\vec{v}}) \gamma^0 \Gamma S(\Lambda_{\vec{v}}) \varphi(k)$$

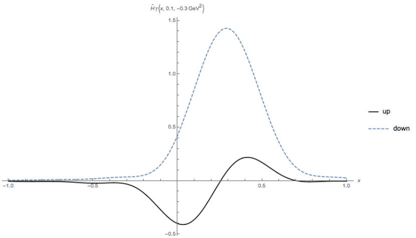
where $\Gamma = i\sigma^{+i}$ and $S(\Lambda_{\vec{v}})$ is the Lorentz boost transformation
- We have 2 equation (for $i = 1, 2$) and 4 unknowns; project on different spin components to obtain 4 equations with 4 unknowns

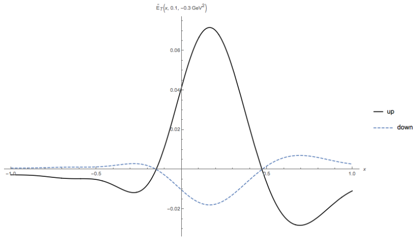
Chiral-odd GPDs in Bag Model

- Chiral-odd GPDs in Bag Model at $\xi = 0.1, t = -0.3GeV^2$









Large-N_c expansion

- Usually once we can not solve a problem analytically, we tend to use perturbation theory; anharmonic oscillator in QM, ϕ^4 theory in QFT, ect.
- In QCD, however, the coupling constant g is high at low energies. Hence is not a good expansion parameter.
- The only known expansion parameter valid in all regions in QCD is $1/N_c$ obtained by generalizing the color gauge group $SU(3) \to SU(N_c)$ [t'Hooft '74]
- As $N_c \to \infty$, QCD simplifies significantly and can be approached nonperturbatively; with an expansion parameter $1/N_c$

Large-N_c expansion

- In this picture, baryons appear as solitons in the background of weakly interacting mesons [Witten '79]
- Large-N_c results can be checked in various ways: Diagrammatic techniques, chiral soliton model, Large-N_c quark model
- Large-N_c expansion connects QCD with Skyrme model and quark model [Manohar '84 - Dashen, Manohar '93]
- This is due to QCD has the same spin-flavor symmetry with Skyrme model and quark model in the Large-N_c limit
- We use Bag Model as a tool to investigate model independent (N_c-scaling) results of GPDs in the Large-N_c framework

Large-N_c expansion in Bag Model

- In Large-N_c limit, the nucleon mass scales with N_c while retaining a stable size

$$M_N \sim N_c$$
$$R \sim N_c^0$$

Hence the nucleon becomes denser and denser.

- On the other hand

$$\Delta^0 \sim N_c^{-1} \quad \text{and} \quad \vec{\Delta} \sim N_c^0$$
$$x \sim N_c^{-1}$$
$$\xi \sim N_c^{-1}$$
$$t \sim N_c^0$$

Large-N_c expansion in Bag Model

- In the Large-N_c framework, a GPD G is asymptotically equivalent to a product
$$G(x,\xi,t) \sim N_c^k \times F(N_c x, N_c \xi, t)$$
where $k \in \mathbb{Z}_+$ and F is the limiting function which arise in the limit of $N_c \to \infty$.
- Here k depends on the GPD in question and the function F depends on the dynamical model used
- The leading flavor combinations of chiral-even GPDs in $1/N_c$ expansion is found to be [Goeke,Polyakov,Vanderhaeghen '01]

$$H^{u+d} \sim N_c^2, \quad E^{u-d} \sim N_c^3$$
$$\tilde{H}^{u-d} \sim N_c^2, \quad \tilde{E}^{u-d} \sim N_c^4.$$

Large-N_c expansion in Bag Model

- By using Bag Model, we obtain the following scaling properties of chiral-odd GPDs

$$
\begin{aligned}
H_T^q &\sim N_c^2 \\
E_T^q &\sim N_c^4 \\
\tilde{H}_T^q &\sim N_c^4 \\
\tilde{E}_T^q &\sim N_c^3.
\end{aligned}
\qquad (1)
$$

- Here we note that among chiral-odd GPDs there is a special linear combination, $\bar{E}_T^q = E_T^q + 2\tilde{H}_T^q$, which shows a cancellation of leading order scalings in the Large-N_c expansion

$$\bar{E}^q \sim N_c^3.$$

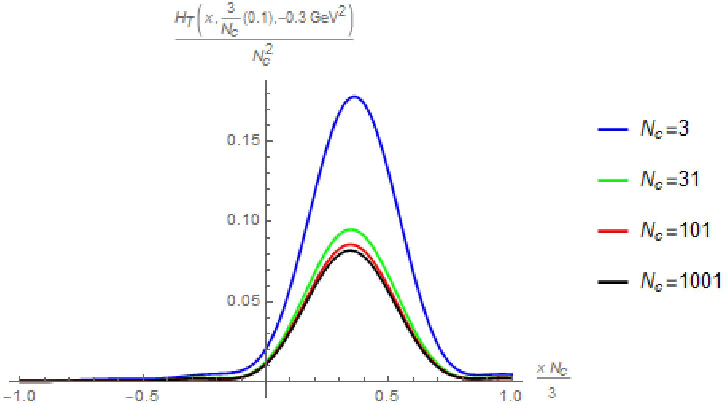


Figure: N_c-scaling of the chiral-odd GPD H_T^u as a function of $\frac{xN_c}{3}$ fixed at $\xi = 0.1 \times \frac{3}{N_c}$ and $t = -0.3 GeV^2$.

Large-N_c expansion in Bag Model

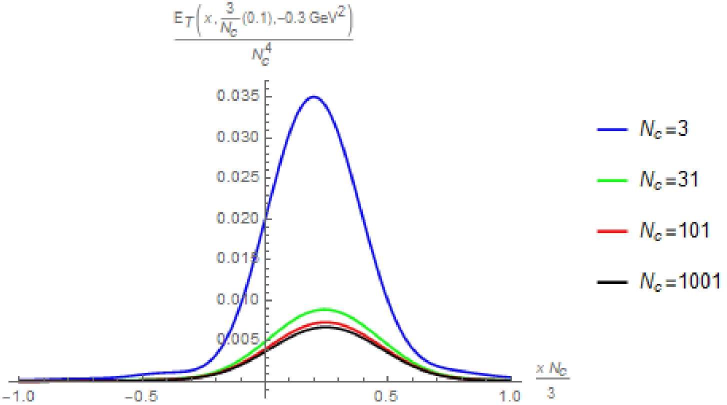


Figure: N_c-scaling of the chiral-odd GPD E_T^u as a function of $\frac{xN_c}{3}$ fixed at $\xi = 0.1 \times \frac{3}{N_c}$ and $t = -0.3GeV^2$.

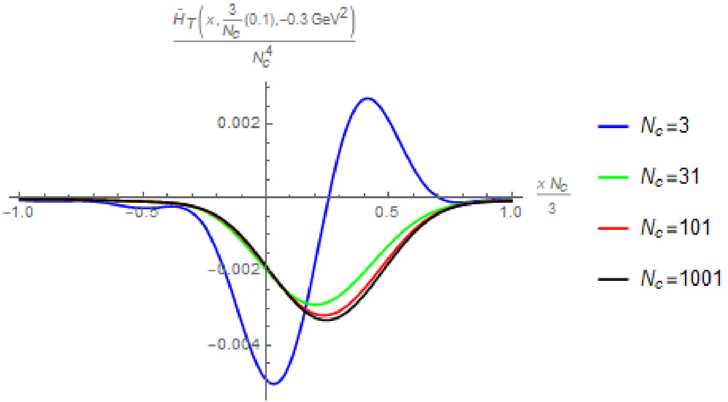


Figure: N_c-scaling of the chiral-odd GPD $\tilde{H}_T^u$ as a function of $\frac{xN_c}{3}$ fixed at $\xi = 0.1 \times \frac{3}{N_c}$ and $t = -0.3GeV^2$.

Large-N_c expansion in Bag Model

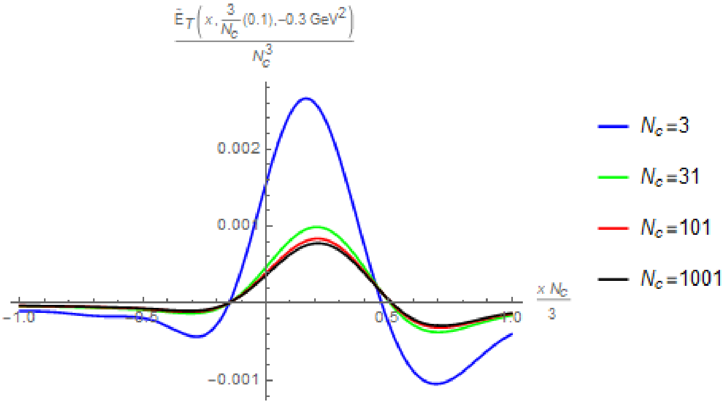


Figure: N_c-scaling of the chiral-odd GPD $\tilde{E}_T^u$ as a function of $\frac{xN_c}{3}$ fixed at $\xi = 0.1 \times \frac{3}{N_c}$ and $t = -0.3 GeV^2$.

Large-N_c expansion in Bag Model

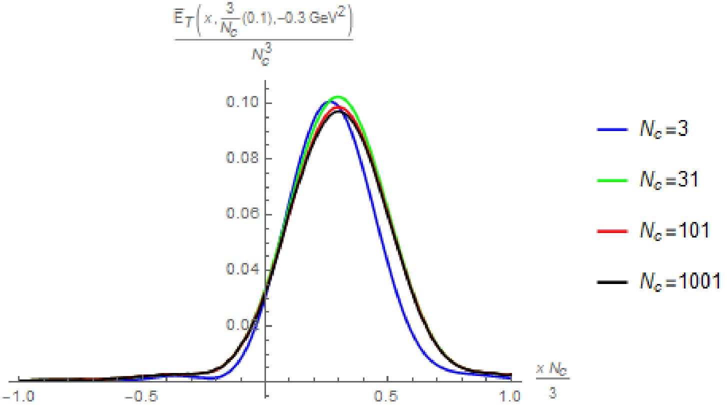


Figure: N_c-scaling of the chiral-odd GPD $\bar{E}_T^u$ as a function of $\frac{xN_c}{3}$ fixed at $\xi = 0.1 \times \frac{3}{N_c}$ and $t = -0.3GeV^2$.

Large-N_c expansion in Bag Model

- The partonic helicity amplitudes are defined by [Diehl '03]

$$A_{\lambda'\mu',\lambda\mu} = P^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{z}{2}) \mathcal{O}_{\mu'\mu} \psi(\frac{z}{2}) | p, \lambda \rangle \Big|_{z^+=0, \vec{z}_T=0}$$

where μ',μ are the light-front helicities of the final,initial quark.

- The relation between chiral-odd GPDs and helicity-flipping amplitudes are given as follows

$$A_{++,+-} = \epsilon \frac{\sqrt{t_0 - t}}{2m} \left(\tilde{H}_T^q + (1-\xi) \frac{E_T^q + \tilde{E}_T^q}{2} \right)$$

$$A_{-+,--} = \epsilon \frac{\sqrt{t_0 - t}}{2m} \left(\tilde{H}_T^q + (1+\xi) \frac{E_T^q - \tilde{E}_T^q}{2} \right)$$

$$A_{++,--} = \sqrt{1-\xi^2} \left(H_T^q + \frac{t_0 - t}{4m^2} \tilde{H}_T^q - \frac{\xi^2}{1-\xi^2} E_T^q + \frac{\xi}{1-\xi^2} \tilde{E}_T^q \right)$$

$$A_{-+,+-} = -\sqrt{1-\xi^2} \frac{t_0 - t}{4m^2} \tilde{H}_T^q$$

Large-N_c expansion in Bag Model

- With the covariant normalization of the nucleon states, the partonic helicity amplitudes scale by N_c^2
- Recall that

$$
\begin{aligned}
A_{++,+-} &= \epsilon\frac{\sqrt{t_0-t}}{2m}\left(\tilde{H}_T^q + (1-\xi)\frac{E_T^q+\tilde{E}_T^q}{2}\right)\\
A_{-+,--} &= \epsilon\frac{\sqrt{t_0-t}}{2m}\left(\tilde{H}_T^q + (1+\xi)\frac{E_T^q-\tilde{E}_T^q}{2}\right)\\
A_{++,--} &= \sqrt{1-\xi^2}\left(H_T^q + \frac{t_0-t}{4m^2}\tilde{H}_T^q - \frac{\xi^2}{1-\xi^2}E_T^q + \frac{\xi}{1-\xi^2}\tilde{E}_T^q\right)\\
A_{-+,+-} &= -\sqrt{1-\xi^2}\frac{t_0-t}{4m^2}\tilde{H}_T^q
\end{aligned}
$$

- In the large-N_c limit, we confirm that this is indeed the case with

$$
\begin{aligned}
\tilde{H}_T^q \sim N_c^4, &\quad E_T^q \sim N_c^4\\
H_T^q \sim N_c^2, &\quad \tilde{E}_T^q \sim N_c^3.
\end{aligned}
$$

Large-N_c expansion in Bag Model

- On the other hand, dominant isospin combinations of chiral-odd GPDs in the Large-N_c limit appear as

$$\begin{aligned} H_T^{u-d}(x,\xi,t) &\sim N_c^2 \\ E_T^{u+d}(x,\xi,t) &\sim N_c^4 \\ \tilde{H}_T^{u+d}(x,\xi,t) &\sim N_c^4 \\ \tilde{E}_T^{u-d}(x,\xi,t) &\sim N_c^3 \\ \bar{E}_T^{u+d}(x,\xi,t) &\sim N_c^3. \end{aligned} \tag{2}$$

- Whereas, opposite isospin combinations are suppressed by order one
- The N_c scaling behaviors of the isospin combinations of chiral-odd GPDs: $\bar{E}_T, H_T$ and $\tilde{E}_T$ were discussed by [Schweitzer, Weiss '16] using a solitonic field with known symmetry properties. The results are confirmed in the bag model

Phenomenological implications

- What are the phenomenological implications of our findings?
- Since we have Large-N_c relations among flavor-singlet and flavor-nonsinglet components of GPDs, this order among them predicts the relative sign of flavor decomposed GPDs
- For instance, dominance of flavor-nonsinglet $(u - d)$ component of the GPD H_T in the Large-N_c limit implies a sign difference in the flavor decomposition of H_T
- Similarly for $\bar{E}_T$, flavor-singlet $(u + d)$ component is dominant in the Large-N_c limit. This implies that the flavor decomposition is expected to have the same sign

Phenomenological implications

- Preliminary π^0, η electroproduction data at JLab confirms our predictions for H_T and $\bar{E}_T$

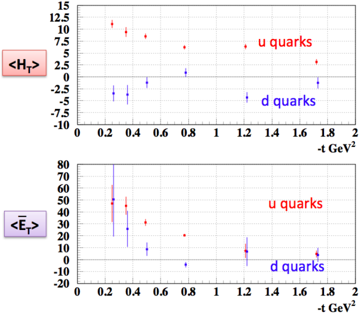


Figure: Preliminary [Kubarovsky '15], talk given at EMP and Short Range Hadron Structure. $Q^2 = 1.8 GeV^2$

where $< ... >$ denotes a convolution of the associated GPDs with a subprocess as introduced in *GK* model.

Conclusions

- Chiral-odd GPDs at leading twist are evaluated in the MIT Bag Model
- In the Large-N_c limit, scaling properties of GPDs and their isospin combinations were analyzed
- Phenomenological results supports the Large-N_c expectations.