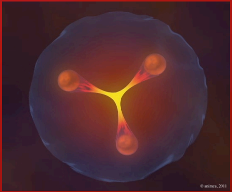
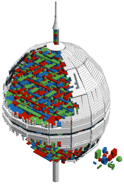
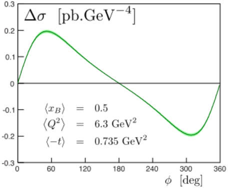
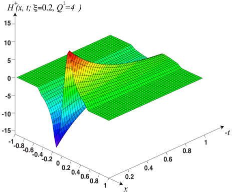
DE LA RECHERCHE À L'INDUSTRIE

Introduction to discussion: phenomenology of GPDs









Hands-on workshop on GPDs | Hervé MOUTARDE

Jan. 24, 2019

Big questions.

What are our priorities as a community?

- What is a "good" GPD measurement?
  - What is the most discriminating channel?

    Photon or mesons?
  - What is the most discriminating observable?

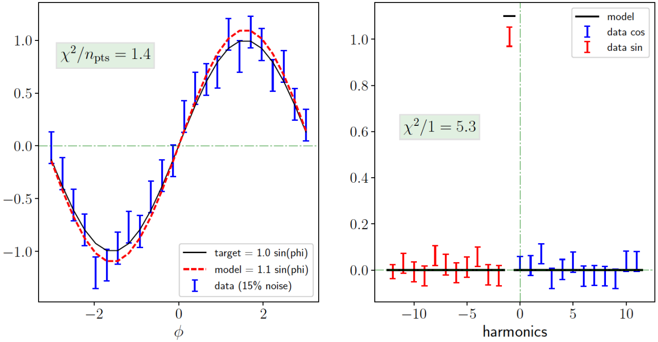
    (Weighted) Fourier harmonics, cross sections, asymmetries?
  - What is the most discriminating kinematics?

    BH vs DVCS, etc.
- What tools are needed for the optimization of experimental setups?
  - Event generation?

    Computing speed, generic tools, interfaces, etc.
  - Radiative corrections?

    What is needed: 4-momenta, cross sections?
- How can we achieve (fast) a global multi-channel analysis?

Case #2: Sine toy model



- ϕ-space view can be misleading

Observables.

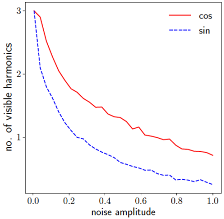
ϕ-dependent observables or Fourier harmonics?

How many harmonics are there?

- Case #3: Target function is

$$f(\phi) = 0.42 - 0.28\cos(\phi) + 0.08\cos(2\phi) + 0.02\cos(3\phi) - 0.13\sin(\phi) - 0.03\sin(2\phi) + 0.006\sin(3\phi)$$

  and is used to generate data with variable noise
- Fitting Fourier series with increasing number of harmonics until $\chi^2/\mathrm{d.o.f.}$ w.r.t. target function starts to deteriorate



But we don't know target function in real life

Propagation of uncertainties to harmonics

- Consider three types of uncertainty:
  1. uncorrelated point-to-point uncertainty (absolute size ϵ)
  2. correlated normalization uncertainty (relative size ϵ)
  3. correlated modulated (ϕ-dependent) uncertainty (e.g., relative size $\epsilon \cos(\phi)$)
- Uncorrelated uncertainty: $\Delta c_k = \sqrt{2/N}\,\epsilon$
- Normalization uncertainty: $\Delta c_k / c_k = \epsilon$
- Correlated modulated uncertainty: more complicated, but for hierarchical case $c_0 \gg c_1 \gg \cdots$ one obtains

$$\frac{\Delta c_0}{c_0} = \frac{c_1}{2c_0}\epsilon, \qquad \frac{\Delta c_1}{c_1} = \frac{c_0}{c_1}\epsilon$$

i.e. we have enhancement of uncertainty for subleading harmonics!

$$(c_0 + c_1 \cos\phi + \cdots) \times (1 + \epsilon\cos\phi) = c_0\left(1 + \frac{c_1}{2c_0}\epsilon\right) + c_1\left(1 + \frac{c_0}{c_1}\epsilon\right)\cos\phi$$

If one measures cross-sections, one can also perform normal Fourier analysis, or it may be favorable to work with specially weighted Fourier integral measure [27]

$$d\phi \;\to\; dw \equiv \frac{2\pi \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)}{\int_{-\pi}^{\pi} d\phi\, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} d\phi \,, \qquad (136)$$

thus cancelling strongly oscillating factors $1/(\mathcal{P}_1(\phi)\mathcal{P}_2(\phi))$ in Bethe-Heitler and interference terms, eqs. (119-120). Series of such *weighted* harmonic terms, *e.g.*

$$\sigma^{\sin n\phi, w} \equiv \frac{1}{\pi}\int_{-\pi}^{\pi} dw\; \sin n\phi\, \sigma(\phi) \,, \qquad (137)$$

converges then faster with increasing n than normal Fourier series.

Kumerički *et al.*, Eur. Phys. J. **A52**, 157 (2016)

What is then a good DVCS measurement?

- I would define a good DVCS measurement as:
  - A measurement maximizing the gain about GPD information
  - For the lesser experimental cost.

- *Problem: How to measure the gain in GPD-information from a measurement*:
  => I am **very glad PARTONS** is available!!! (Just need money for Post-doc/Student now)
  => Until then, from my past experience, I learnt a set of rules (to be discussed):
  - Aim at kinematics not covered yet, or poorly constrained. (go Jlab12, go COMPASS, go EIC!!!)

  - **Not too much Bethe-Heitler** contribution for unpolarized cross sections.
  (Where too much Bethe-Heitler, can you tell much without the unpolarized cross sections?)

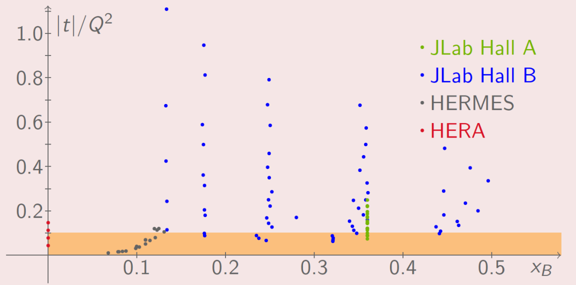
  - Since we mostly work on **CFF** for the moment, try to cover a **complete phi-acceptance**.
  (Sometimes you have to make a choice between statistics and acceptance.)
  (What about when you work at GPD-level?)
  (With multi-channel analysis?)

- These questions are of tremendous importance, at least for Jefferson Lab, for which we have flexibility on the experimental configuration:
  - For CLAS12, Torus polarity change the statistics and acceptance. (Phi vs photon electroproduction)
  - Still for CLAS12, trade-off between luminosity and detector proximity of the beamline.

Optimized measurements?
Discriminating power and uncertainty propagation.

Kinematics of existing DVCS measurements.
Looking for the Bjorken regime.

What is large Q^2 ?



- World data cover **complementary kinematic regions**.
- Q^2 is **not so large** for most of the data.
- **Higher twists**, finite-t and target mass corrections ?

Dataset waiting for a yet-to-come theoretical framework...

Algebraic Inversion

$$H(x,\xi,t) = (1-x)\int_{\Omega} \mathrm{d}\beta \mathrm{d}\alpha \delta(x-\beta-\alpha\xi) h_P(\beta,\alpha,t)$$

$$h_P(\beta,\alpha,t) = \frac{15}{2}\theta(\beta)\left[1+\frac{-t}{4M^2}\left((1-\beta)^2-\alpha^2\right)\right]^{-3} \times\left[1-3(\alpha^2-\beta^2)-2\beta+\frac{-t}{4M^2}\left(1-(\alpha^2-\beta^2)^2-4\beta(1-\beta)\right)\right],$$

From the algebraic DD we can deduce the GPD in ERBL region

$$H(x,\xi,0)|_{|x|\leq\xi} = \frac{15}{2}\frac{(1-x)(\xi^2-x^2)}{\xi^3(1+\xi)^2}\left(x+2x\xi+\xi^2\right),$$

x-space or conformal space?

Practical implementation.

$$q(x,\eta,\Delta^2) = \frac{1}{2i}\oint_{(0)}^{(\infty)} dj\, \frac{1}{\sin(\pi j)} p_j(x,\eta)\, m_j(\eta,\Delta^2)\,. \quad (45)$$

Here we included a factor $1/\sin(\pi j)$, which has the residue $\mathrm{Res}_{j=n} 1/\sin(\pi j) = (-1)^n/\pi$ for $n = 0, 1, 2, \cdots$. Thus, if no other singularities are present inside the integration contour, the residue theorem leads to the conformal partial wave expansion (41). The main difficulty is to find an appropriate analytic continuation[10] of both functions $p_j(x,\eta)$ and $m_j(\eta,\Delta^2)$ with respect to the conformal spin $n+2$.

The analytic continuation of the polynomials $m_n(\eta,\Delta^2)$ is denoted as $m_j(\eta,\Delta^2)$. These functions will be also analytic in η, however, might have branch points at $\eta = 0$, $\eta = \pm 1$, and $\eta = \infty$. It would be desirable to have an integral representation that makes this property transparent and might allow the continuation from $\eta \geq 1$ to $\eta \leq 1$ or even to negative values. Moreover, we will also require that the moments $m_j(\eta,\Delta^2)$ are bounded at large j. It turned out that with these requirements the analytic or numerical calculation of moments from a given GPD is a rather intricate task. We have to admit that this mathematical problem is also not solved here for any conceivable GPD. In the following we give, however, some recipes to evaluate the conformal moments for complex conformal spin in the region $|\eta| \leq 1$.

Müller and Schäfer, Nucl. Phys. **B739**, 1 (2006)

- Reminder: PDFs fits have been performed by **more groups** for a **longer time**.
- **Encouraging results** have been obtained in the last five years in fitting DVCS data.
- In progress: inclusion of DVMP data in fits.
- Today it is not clear that existing strategies will be able to handle **very precise** data on a **large kinematic domain**.
- **All approaches should be explored**, each with its own advantages and drawbacks.
- Global fits seem **unavoidable** at some point (direct GPD fit? Two-step fit, CFF, then GPDs? Extrapolations?).
- Experimental 3D imaging is far more complicated than PDF or charge radius fitting, but **possible in principle**.

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