Global analysis of DVCS data

Paweł Sznajder National Centre for Nuclear Research, Warsaw



Prospects for extraction of GPDs from global fits of current nad future data Warsaw, January 22 - 25, 2019

- Introduction
- PARTONS project
- Global analysis of DVCS
- Summary

Deeply Virtual Compton Scattering (DVCS)



factorization for $|t|/Q^2 \ll 1$

Chiral-even GPDs: (helicity of parton conserved)

$H^{q,g}(x,\xi,t)$	$E^{q,g}(x,\xi,t)$	for sum over parton helicities
$\widetilde{H}^{q,g}(x,\xi,t)$	$\widetilde{E}^{q,g}(x,\xi,t)$	for difference over parton helicities
nucleon helicity conserved	nucleon helicity changed	

Nucleon tomography

$$q(x, \mathbf{b}_{\perp}) = \int \frac{\mathrm{d}^2 \mathbf{\Delta}}{4\pi^2} e^{-i\mathbf{b}_{\perp} \cdot \mathbf{\Delta}} H^q(x, 0, t = -\mathbf{\Delta}^2)$$



- Study of long. polarization with GPD H
- Study of distortion in transv. polarized nucleon with GPD E

Impact parameter \mathbf{b}_{\perp} defined w.r.t. center of momentum, such as $\sum x \mathbf{b}_{\perp} = 0$



Energy momentum tensor in terms of form factors:

$$\langle p', s' | \hat{T}^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \left[\frac{P^{\mu}P^{\nu}}{M} A(t) + \frac{\Delta^{\mu}\Delta^{\nu} - \eta^{\mu\nu}\Delta^{2}}{M} C(t) + M\eta^{\mu\nu} \bar{C}(t) + \frac{P^{\mu}i\sigma^{\nu\lambda}\Delta_{\lambda}}{4M} \left[A(t) + B(t) + D(t) \right] + \frac{P^{\nu}i\sigma^{\mu\lambda}\Delta_{\lambda}}{4M} \left[A(t) + B(t) - D(t) \right] u(p, s)$$

Access to total angular momentum and forces acting on quarks

$$A^{q}(0) + B^{q}(0) = \int_{-1}^{1} x \left[H^{q}(x,\xi,0) + E^{q}(x,\xi,0) \right] = 2J^{q}$$



Paweł Sznajder / Prospects for extraction of GPDs / Jan 24, 2019

H. Moutarde, P. S., J. Wagner "*Border and skewness functions from a leading order fit to DVCS data*" arXiv:1807.07620 [hep-ph]

Goal: global extraction of Compton Form Factors (CFFs) from DVCS data using LO/LT formalism

Analysis done within PARTONS project

imaginary part

$$Im\mathcal{G}(\xi,t) = \pi G^{(+)}(\xi,\xi,t) = \pi \sum_{q} e_q^2 G^{q(+)}(\xi,\xi,t)$$

 $G^{q(+)}(x,\xi,t) = G^{q}(x,\xi,t) \mp G^{q}(-x,\xi,t)$ $G^{q(+)}(\xi,\xi,t) = G^{q_{\text{val}}}(\xi,\xi,t) + 2G^{q_{\text{sea}}}(\xi,\xi,t)$

"-" for
$$G \in \{H, E\}$$

"+" for $G \in \{\widetilde{H}, \widetilde{E}\}$

real part

$$Re\mathcal{G}(\xi,t) = P.V. \int_0^1 G^{(+)}(x,\xi,t) \left(\frac{1}{\xi-x} \mp \frac{1}{\xi+x}\right) dx$$
$$Re\mathcal{G}(\xi,t) = P.V. \int_0^1 G^{(+)}(x,x,t) \left(\frac{1}{\xi-x} \mp \frac{1}{\xi+x}\right) dx + C_G(t)$$
$$C_H(t) = -C_E(t) \qquad C_{\widetilde{H}}(t) = C_{\widetilde{E}}(t) = 0$$

Relation between subtraction constant and D-term:

$$C_{G}^{q}(t) = 2 \int_{-1}^{1} \frac{D^{q}(z,t)}{1-z} dz \equiv 4D^{q}(t)$$

 $z = \frac{x}{\xi}$

where

Decomposition into Gegenbauer polynomials:

$$D^{q}(z,t) = (1-z^{2}) \sum_{i=0}^{\infty} d_{i}^{q}(t) C_{2i+1}^{3/2}(z)$$

Connection to EMT FF:

$$D^{q}(t) = \sum_{\substack{i=1\\\text{odd}}}^{\infty} d_{i}^{q}(t)$$

$$d_1^q(t) = 5C^q(t)$$

Comparing CFFs evaluated with two methods

$$C_G^q(t) = \int_0^1 \left(G^{q(+)}(x,\xi,t) - G^{q(+)}(x,x,t) \right) \left(\frac{1}{\xi - x} - \frac{1}{\xi + x} \right) dx$$

for $\xi = 0$

$$C_G^q(t) = 2\int_0^1 \left(G^{q(+)}(x, x, t) - G^{q(+)}(x, 0, t) \right) \frac{1}{x} dx$$

divergent integral!

but

$$C_{G,j}^{q}(t) = 2 \int_{0}^{1} \left(G^{q(+)}(x, x, t) - G^{q(+)}(x, 0, t) \right) x^{j} dx$$

well defined for odd positive j

Subtraction constant as analytic continuation of Mellin moments to j = -1

$$C_{G}^{q}(t) = C_{G,-1}^{q}(t) = 2 \int_{(0)}^{1} \left(G^{q(+)}(x,x,t) - G^{q(+)}(x,0,t) \right) \frac{1}{x} dx$$

Analytic regularization prescription

$$\int_{(0)}^{1} \frac{f(x)}{x^{a+1}} = \int_{0}^{1} \frac{f(x) - f(0) - xf'(0) - \dots}{x^{a+1}} + f(0) \int_{(0)}^{1} \frac{dx}{x^{a+1}} + f'(0) \int_{(0)}^{1} \frac{dx}{x^{a}} + \dots = \int_{0}^{1} \frac{f(x) - f(0) - xf'(0) - \dots}{x^{a+1}} - \frac{f(0)}{a} - \frac{f'(0)}{a - 1} + \dots$$

applicable if f(x) analytic and not-vanishing at x = 0

 $G^q(x,0,t) = \mathrm{pdf}_G^q(x) \; \exp(f_G^q(x)t)$

$$f_G^q(x) = A_G^q \log(1/x) + B_G^q (1-x)^2 + C_G^q (1-x)x$$

- modify "classical" log(1/x) term by $B_{G^q}(1-x)^2$ in low-x and by $C_{G^q}(1-x)x$ in high-x regions
- polynomials found in analysis of EFF data → good description of data
- allows to use the analytic regularization prescription
- finite proton size at $x \rightarrow 1$

$$G^{q}(x, x, t) = G^{q}(x, 0, t) g^{q}_{G}(x, x, t)$$

$$g_G^q(x, x, t) = \frac{a_G^q}{(1 - x^2)^2} \left(1 + t(1 - x)(b_G^q + c_G^q \log(1 + x))\right)$$



- at $x \rightarrow 1$ reproduce power behaviour predicted for GPDs in Phys. Rev. D69, 051501 (2004)
- t-dependence similar to DD-models with (1-x) to avoid any t-dep. at x = 1



Ansatz for H and \widetilde{H}

"trouble" with analytic regularization

$$\int_{(0)}^{1} \frac{f(x)}{x^{a+1}} = \int_{0}^{1} \frac{f(x) - f(0) - xf'(0) - \dots}{x^{a+1}} - \frac{f(0)}{a} - \frac{f'(0)}{a-1} + \dots$$

where in our case

$$a = \delta + A_G^q t$$

$$q(x) \sim x^{-\delta}$$

$$C_G^q(x, 0, t) = C_G^q(x, 0, t) (x^q(x, t) - 1)$$

$$f(x) = \frac{G^q(x, x, t) - G^q(x, 0, t)}{x^{-a}} = \frac{G^q(x, 0, t) \left(g^q_G(x, t) - 1\right)}{x^{-a}}$$

compensating terms singular for $t \equiv t_0^{\infty} = -\delta/A_G^q$ and $t \equiv t_1^{\infty} = (1 - \delta)/A_G^q$ unless f(0) = 0 at t_0^{∞} and f'(0) = 0 at t_1^{∞} , condition provided by:

$$b_{G}^{q} = \frac{A_{G}^{q}(a_{G}^{q}-1)}{a_{G}^{q}\delta} \qquad c_{G}^{q} = \frac{(a_{G}^{q}-1)}{p_{0}\left(\delta-1\right)a_{G}^{q}\delta}\left(p_{0}\left(2B_{G}^{q}-C_{G}^{q}\right)\left(\delta-1\right) + A_{G}^{q}p_{0}\left(\delta-1-\alpha\right) + A_{G}^{q}p_{1}\right)$$

where δ, α, p_0, p_1 are PDF parameterization parameters

for GPD E

$$\begin{split} E^{q_{\text{val}}}(x,0,t) &= e^{q_{\text{val}}}(x) \exp(f_E^{q_{\text{val}}}(x)t) \\ e^{q_{\text{val}}}(x) &= \kappa_q N_{q_{\text{val}}} x^{-\alpha_{q_{\text{val}}}}(1-x)^{\beta_{q_{\text{val}}}}(1+\gamma_{q_{\text{val}}}\sqrt{x}) \\ E^{q_{\text{val}}}(x,x,t) &= E^{q_{\text{val}}}(x,0,t) g_E^{q_{\text{val}}}(x,t) \\ g_E^{q_{\text{val}}}(x,t) &= \frac{a_E^{q_{\text{val}}}}{(1-x^2)^3} \\ \text{from Phys. Rev. D69, 051501 (2004)} \\ \text{for GPD }\tilde{\mathsf{E}} \end{split}$$

$$\widetilde{\mathcal{E}}(\xi, t) = N_{\widetilde{E}}\widetilde{\mathcal{E}}_{\mathrm{GK}}(\xi, t)$$

CFF from GK GPD model

$$H^q(x,0,0) \equiv q(x)$$

$$\widetilde{H}^q(x,0,0) \equiv \Delta q(x)$$

Ansatz:

$$pdf_G(x, Q^2) = x^{-g(\delta_p, \delta_q, Q^2)} (1 - x)^{\alpha} \sum_{i=0}^4 g(p_i, q_i, Q^2) x^i$$
$$g(p, q, Q^2) = p + q \log \frac{Q^2}{Q_0^2}$$

13 parameters:

$$\delta_p, \delta_q, lpha, p_i, q_i$$
 where $i=0,1,\ldots,4$

constrained by NNPDF3.0 and NNPDFpol11 sets (per each flavor and each PDF replica)



Free parameters for valance quarks and GPDs H and E constrained by EFF data

$$\int_{-1}^{1} H^{q}(x,\xi,t) \equiv F_{1}^{q}(t)$$
$$\int_{-1}^{1} E^{q}(x,\xi,t) \equiv F_{2}^{q}(t)$$

From Dirac and Pauli partonic FFs to Sachs nucleon FFs

$$\begin{split} F_{i}^{p} &= e_{u}F_{i}^{u} + e_{d}F_{i}^{d} & i = 1,2 \\ F_{i}^{n} &= e_{u}F_{i}^{d} + e_{d}F_{i}^{u} \\ G_{M}^{i} &= F_{1}^{i} + F_{2}^{i} & i = p,n \\ G_{E}^{i} &= F_{1}^{i} + \frac{t}{4m^{2}}F_{2}^{i} \end{split}$$

Observables

$$G_{M,N}^{i}(t) = \frac{G_{M}^{i}(t)}{\mu_{i}G_{D}(t)} \qquad i = p, n$$

$$R^{i}(t) = \frac{\mu_{i}G_{E}^{i}(t)}{G_{M}^{i}(t)}$$

$$r_{nE}^2 = 6 \frac{dG_E^n(t)}{dt} \Big|_{t=0}$$

for the selection of observables and experimental data we follow *Eur. Phys.J.* C73 (2013) 4, 2397



Performance:

$$\chi^2/\mathrm{ndf} = 129.6/(178 - 9) \approx 0.77$$

Replication of experimental data to estimate corresponding uncertainties:

$$v_i \pm \Delta_i^{\text{tot}} \xrightarrow{\text{replica } j} \operatorname{rnd}_j(v_i, \Delta_i^{\text{tot}}) \pm \Delta_i^{\text{tot}}$$

 $\Delta_i^{\text{tot}} = \sqrt{(\Delta_i^{\text{stat}})^2 + (\Delta_i^{\text{sys}})^2}$

Fitted values:

Parameter	Mean	Data unc.	Unpol. PDF unc.
$A^{u_{\mathrm{val}}}_{H/E}$	0.99	0.01	0.08
$B_{H}^{\hat{u}_{\mathrm{val}}'}$	-0.50	0.02	0.14
$A_{H/E}^{d_{\mathrm{val}}}$	0.70	0.02	0.08
$B_{H}^{d_{\mathrm{val}}^{'}}$	0.47	0.07	0.24
α	0.69	0.01	0.03
$B_E^{u_{ m val}}$	-0.69	0.04	0.18
$C_E^{\widetilde{u}_{ ext{val}}}$	-0.92	0.04	0.09
$B_E^{d_{\mathrm{val}}}$	-0.54	0.06	0.20
$C_E^{\overrightarrow{d}_{ ext{val}}}$	-0.73	0.06	0.22

Paweł Sznajder / Prospects for extraction of GPDs / Jan 24, 2019

DVCS data

All DVCS proton data used in the fit, except:

- HERA data
- Hall A cross sections

Kinematic cuts:

$$Q^2 > 1.5 \text{ GeV}^2$$
$$-t/Q^2 < 0.25$$



No.	Collab.	Year	Observable		Kinematic dependence	No. of points used / all
1	HERMES	2001	A_{LU}^+		ϕ	10 / 10
2		2006	$A_C^{\cos i\phi}$	i = 1	t	4 / 4
3		2008	$A_C^{\cos i\phi}$	i = 0, 1	x_{Bj}	18 / 24
			$A_{UT, DVCS}^{\sin(\phi - \phi_S) \cos i\phi}$	i = 0		
			$A_{UT}^{\sin(\phi-\phi_S)\cos i\phi}$	i = 0, 1		
			$A_{UT,I}^{\cos(\phi-\phi_S)\sin i\phi}$	i = 1		
4		2009	$A_{LU,I}^{\sin i\phi}$	i = 1, 2	x_{Bj}	35 / 42
			$A_{LU,\mathrm{DVCS}}^{\sin i\phi}$	i = 1		
			$A_C^{\cos i\phi}$	i = 0, 1, 2, 3		
5		2010	$A_{UL}^{+,\sin i\phi}$	i = 1, 2, 3	x_{Bj}	18 / 24
			$A_{LL}^{+,\cos i\phi}$	i = 0, 1, 2		
6		2011	$A_{LT, \text{DVCS}}^{\cos(\phi - \phi_S)\cos i\phi}$	i = 0, 1	x_{Bj}	24 / 32
			$A_{LT, \text{DVCS}}^{\sin(\phi - \phi_S) \sin i\phi}$	i = 1		
			$A_{LT I}^{\cos(\phi - \phi_S)\cos i\phi}$	i = 0, 1, 2		
			$A_{LT,I}^{\sin(\phi-\phi_S)\sin i\phi}$	i = 1, 2		
7		2012	$A_{LU,I}^{\sin i\phi}$	i = 1, 2	x_{Bj}	35 / 42
			$A_{LU,\mathrm{DVCS}}^{\sin i\phi}$	i = 1		
			$A_C^{\cos i\phi}$	i = 0, 1, 2, 3		
8	CLAS	2001	$A_{LU}^{-,\sin i\phi}$	i = 1, 2		0 / 2
9		2006	$A_{UL}^{-,\sin i\phi}$	i = 1, 2		2 / 2
10		2008	A_{LU}^-		ϕ	283 / 737
11		2009	A_{LU}^-		ϕ	22 / 33
12		2015	$A_{LU}^-, A_{UL}^-, A_{LL}^-$		ϕ	311 / 497
13		2015	$d^4\sigma^{UU}$		ϕ	1333 / 1933
14	Hall A	2015	$\Delta d^4 \sigma^{LU}$		ϕ	228 / 228
15	001 (D 1 00	2017	$\Delta d^4 \sigma^{LU}$		ϕ	276 / 358
16	COMPASS	2018	b			1 / 1

v

DVCS data

Performance:

 $\chi^2/\text{ndf} = 2346.3/(2600 - 13) \approx 0.91$

No.	Collab.	Year	χ^2	n	χ^2/n
1	HERMES	2001	9.8	10	0.98
2		2006	2.9	4	0.72
3		2008	24.2	18	1.35
4		2009	40.1	35	1.15
5		2010	40.3	18	2.24
6		2011	14.5	24	0.60
$\overline{7}$		2012	25.4	35	0.73
8	CLAS	2001		0	
9		2006	0.9	2	0.47
10		2008	371.1	283	1.31
11		2009	36.4	22	1.66
12		2015	351.4	311	1.13
13		2015	937.9	1333	0.70
14	Hall A	2015	220.2	228	0.97
15		2017	258.8	276	0.94
16	COMPASS	2018	10.7	1	10.67

Fitted values:

	-						
Parameter	Mean	Data unc.	Unpol. PDF unc.	Pol. PDF unc.	EFF unc.		
$a_H^{q_{\mathrm{val}}}$	0.81	0.04	0.17	0.02	< 0.01		
$a_{H}^{\widehat{q}_{ ext{sea}}}$	0.99	0.01	0.02	< 0.01	< 0.01		
$a^{\hat{q}}_{\widetilde{H}}$	1.03	0.04	0.30	0.24	0.01		
$N_{\widetilde{E}}$	-0.46	0.10	0.09	0.06	0.01		
$A_H^{q_{ m sea}}$	2.56	0.23	0.30	0.09	0.03		
$B_H^{\widetilde{q}_{ ext{sea}}}$	-5	at the limit					
$C_{H}^{\overline{q}_{ ext{sea}}}$	34	27	49	14	3		
$A^{u_{\mathrm{val}}}_{\widetilde{H}}$	0.77	0.12	0.30	0.23	0.07		
$B^{t\!t_{\mathrm{val}}}_{\widetilde{\iota\iota}}$	-0.02	0.26	0.75	0.24	0.15		
$C_{\widetilde{H}}^{\widetilde{H}_{\mathrm{val}}}$	-0.92	0.07	0.44	0.24	0.04		
$A^{d_{\mathrm{val}}}_{\widetilde{H}}$	0.64	0.24	0.30	0.28	0.05		
$B^{\overline{d}_{\mathrm{val}}}_{\widetilde{H}}$	-1.19	0.45	0.91	0.98	0.22		
$C_{\widetilde{H}}^{d_{\mathrm{val}}}$	-0.55	0.24	0.26	0.27	0.10		

Replication of experimental data to estimate corresponding uncertainties:

$$v_i \pm \Delta_i^{\text{tot}} \xrightarrow{\text{replica } j} \left(\operatorname{rnd}_j(v_i, \Delta_i^{\text{tot}}) \pm \Delta_i^{\text{tot}} \right) \times \operatorname{rnd}_j(1, \Delta_i^{\text{norm}}) \quad \Delta_i^{\text{tot}} = \sqrt{\left(\Delta_i^{\text{stat}}\right)^2 + \left(\Delta_i^{\text{sys}}\right)^2}$$

CLAS data:





Phys. Rev. Lett. 115(21), 212003 (2015) $x_{Bj} = 0.244, t = -0.15 \text{ GeV}^2, Q^2 = 1.79 \text{ GeV}^2$

Phys. Rev. D91(5), 052014 (2015) $x_{Bj} = 0.257, t = -0.23 \text{ GeV}^2, Q^2 = 2.02 \text{ GeV}^2$ HERMES data:





JHEP 06, 066 (2008) t = -0.12 GeV², $Q^2 = 2.5 \text{ GeV}^2$

Results

this analysisGK model ----VGG model

Hall A data:



Phys. Rev. C92(5), 055202 (2015) $x_{Bj} = 0.392, t = -0.233 \text{ GeV}^2, Q^2 = 2.054 \text{ GeV}^2$ COMPASS and HERA:





arXiv: hep-ex/1802.02739 $Q^2 = 1.8 \text{ GeV}^2$

this analysisGK model ----VGG model

Compton Form Factors:



 $t = -0.3 \text{ GeV}^2, Q^2 = 2 \text{ GeV}^2$

this analysis GK model ----VGG model

Compton Form Factors:



 $t = -0.3 \text{ GeV}^2, Q^2 = 2 \text{ GeV}^2$





Paweł Sznajder / Prospects for extraction of GPDs / Jan 24, 2019

Subtraction constant:





t = 0

Nucleon tomography:













Neural network analysis

- Independent parameterisation of CFFs $\mathcal{H}, \mathcal{E}, \tilde{\mathcal{H}}, \tilde{\mathcal{E}}$
- Neural networks with genetic algorithm training and early stopping regularization
- Enclosure test to prove that the method works
- Careful estimation of uncertainties
- Interpretation of results in terms of GPDs
- All existing DVCS data (from JLab to HERA)
- Complementary to the work presented today



Fits to DVCS data

- New parameterizations of border and skewness function proposed
 - \rightarrow basic properties of GPD as building blocks
 - \rightarrow small number of parameters
 - \rightarrow encoded access to nucleon tomography and subtraction constant
- Successful to fit EFF and DVCS data
 - \rightarrow replica method for a careful propagation of uncertainties

What next?

- Neural network parameterization of CFFs
- Include other channels and more observables
- Include new and already existing theory developments
- Make consistent analysis of all those ingredients → PARTONS

Inequalities:

$$\begin{aligned} |\Delta q(x, \mathbf{b}_{\perp})| &\leq q(x, \mathbf{b}_{\perp}) \\ \frac{\mathbf{b}_{\perp}^2}{m^2} \left(\frac{\partial}{\partial \mathbf{b}_{\perp}^2} e(x, \mathbf{b}_{\perp}) \right)^2 &\leq (q(x, \mathbf{b}_{\perp}) + \Delta q(x, \mathbf{b}_{\perp})) \times (q(x, \mathbf{b}_{\perp}) - \Delta q(x, \mathbf{b}_{\perp})) \end{aligned}$$

to avoid violation of the positivity in the impact parameter space



Paweł Sznajder / Prospects for extraction of GPDs / Jan 24, 2019