Models of Distribution Amplitudes and GPDs

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January 24^{th}, 2019

Part 1:
Models of Distribution Amplitudes
and consequences

DVMP and DA

- Deep meson production: a key channel for GPDs extractions
- "Golden objective": combined DVCS & DVMP extraction
- Possibility to do this at NLO ?

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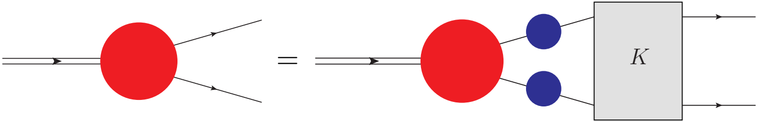
Pragmatic approach

Gather as much information from any side, and make "reasonable" assumptions

- Dyson-Schwinger techniques:
  - Multiple meson can be addressed
    - Pion DA (leading and sub-leading twist)
    - ρ-meson DA (leading twist)
    - J/Ψ DA (leading twist)
  - Many Mellin moments can be computed
  - Interaction is approximated
  - Computation of the gauge link remains to be addressed (although claimed to be small in the case of DA)
- Lattice QCD computations
  - Precise computations of the first non-trivial Mellin Moment
  - x dependence with LaMET but with wide errors
- Possible parametrisations combining information from the two ?

- Two-body bound states obey their own equation called the Bethe-Sapeter equation:



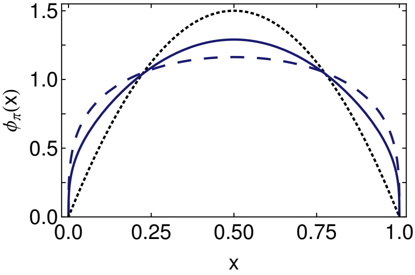
- It is needed to approximate K consistently the quark propagator used to fulfil QCD symmetries (especially the Axial-Vector Ward-Takahashi Identities)
- The DA is given by projecting the Bethe-Salpeter wave function:

$$\varphi(x) \propto \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \delta\left(x - \frac{k \cdot n}{P \cdot n}\right) \chi(k, P)$$

- Advantages and Drawbacks

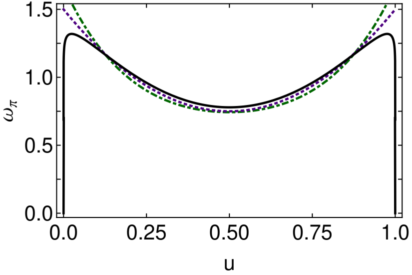
Pion DA in BSE framework

Leading twist (MOM 2GeV)



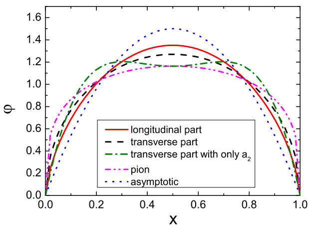
L. Chang *et. al*, PRL 110 (2013) no.13, 132001

Higher twist (MOM 2GeV)



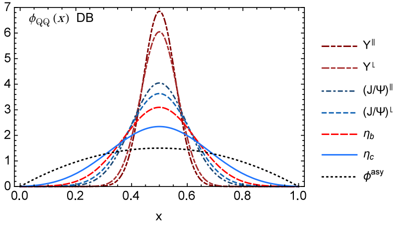
C. Shi *et. al*, Phys.Rev. D92 (2015) 014035

MOM 2 GeV



F. Gao *et al.*, Phys.Rev. D90 (2014) no.1, 014011

MOM 2 GeV



M. Ding *et al.*,Phys.Lett. B753 (2016) 330-335

Lattice QCD Moments for the pion

- Lattice can compute local operators related to $\langle \xi^m \rangle$:

$$\langle \xi^m \rangle(\mu) = \int \mathrm{d}x(2x-1)^m \varphi(x,\mu)$$

- Calculation possible for $m = 2$, beyond operator mixing makes it difficult:

$$\langle \xi^2 \rangle^{\overline{\mathrm{MS}}}(\mu = 2\mathrm{GeV}) = 0.2361(41)(39)(?)$$

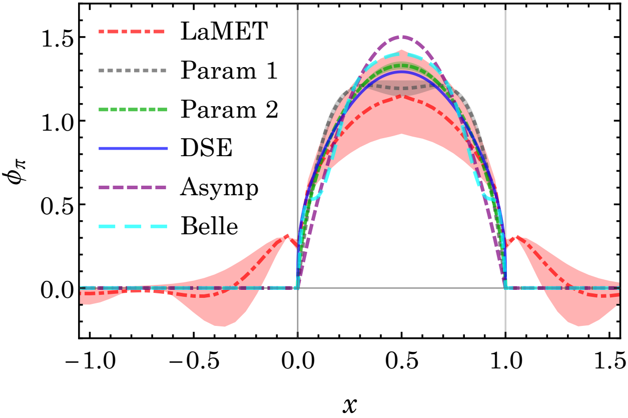
V. Braun *et al.*, Phys.Rev. D92 (2015) no.1, 014504

- Preliminary results shown at ECT* workshop last September for $\langle \xi^2 \rangle$ at *physical point* **and** in the continuum limit, but with renormalisation to be finalised:

$$\langle \xi^2 \rangle^{\overline{\mathrm{MS}}}(\mu = 2\mathrm{GeV}) = 0.2399(64)$$

G. Bali, talk at "Mapping PDF and PDA", ECT*, September 10-14, 2018

LaMET for the pion



Jian-Hui Zhang *et al.*, Phys.Rev. D95 (2017) no.9, 094514

Parametrisations

Assumptions

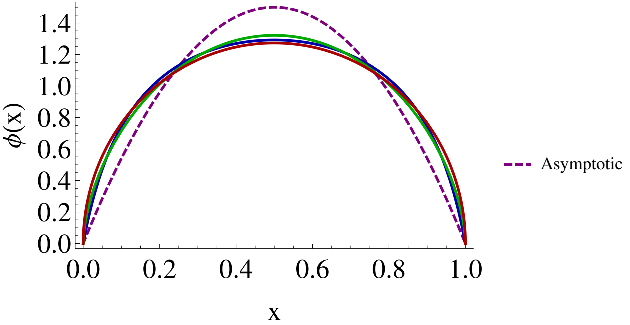
1) DSE + LaMET $\to$ broad unimodal DA is favoured over bimodal one
2) Lattice computation of $\langle \xi^2 \rangle$ is reliable

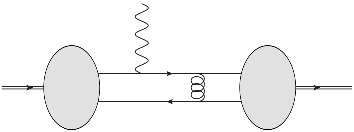
- Asymptotic DA
$$\varphi_{As}(x) = 6x(1-x)$$
- Logarithmic DA (one parameter κ fitted on lattice data)
$$\varphi_{\ln}(x) \propto 1 - \frac{\ln\left[1+\kappa x(1-x)\right]}{\kappa x(1-x)}$$
- Power DA (one parameter ν fitted on lattice data)
$$\varphi_{\nu}(x) \propto x^{\nu}(1-x)^{\nu}$$
- Square root DA
$$\varphi_{1/2}(x) \propto \sqrt{x(1-x)}$$

$n = -1$ Mellin Moment

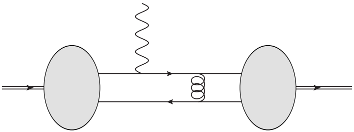
$$\langle x^{-1} \rangle = \int_0^1 \mathrm{d}x \frac{\varphi(x)}{1-x}$$

	$\varphi_{As}(x)$	$\varphi_{\ln}(x)$	$\varphi_{\nu}(x)$	$\varphi_{1/2}(x)$
$\langle x^{-1} \rangle$	3	3.41	3.66	4
$\frac{\langle x^{-1} \rangle}{\langle x^{-1} \rangle_{As}}$	1	1.14	1.22	1.33





$$Q^2F(Q^2) = \mathcal{N}\int [\mathrm{d}x_i][\mathrm{d}y_i]\varphi(x,\zeta_x^2)T(x,y,Q^2,\zeta_x^2,\zeta_y^2)\varphi(y,\zeta_y^2)$$



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- LO Kernel and NLO kernels are known
- $T_0 \propto \frac{\alpha_S(\mu_R^2)}{(1-x)(1-y)}$
- $T_1 \propto \frac{\alpha_s^2(\mu_R^2)}{(1-x)(1-y)}(f_{UV}(\mu_R^2) + f_{IR}(\zeta^2) + f_{finite})$

R Field *et al.*, NPB 186 429 (1981)
F. Dittes and A. Radyushkin, YF 34 529 (1981)
B. Melic *et al.*, PRD 60 074004 (1999)

Pion FF

- The UV scale dependent term behaves like:

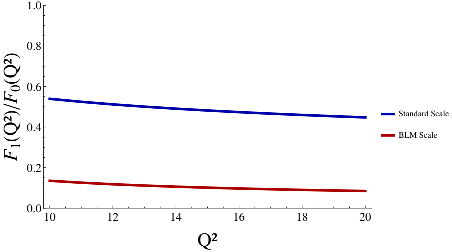
$$f_{UV}(\mu_R^2) \propto \beta_0 \left(5/3 - \ln((1-x)(1-y)) + \ln\left(\frac{\mu_R^2}{Q^2}\right)\right)$$

- Here I take two examples:
  - the standard choice of $\zeta_x^2 = \zeta_y^2 = \mu^2 = Q^2/4$
  - the regularised BLM-PMC scale $\zeta_x^2 = \zeta_y^2 = \mu^2 = e^{-5/3}Q^2/4$

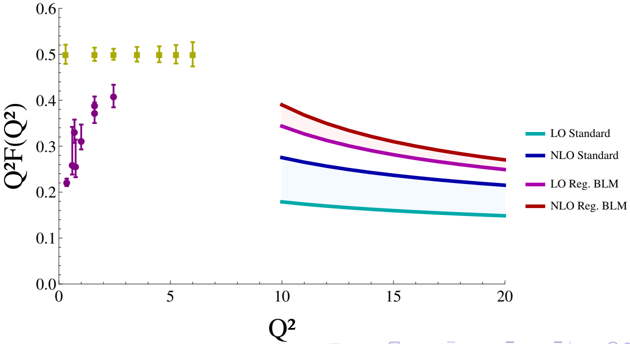
S. Brodsky *et al.*, PRD 28 228 (1983)
S. Brodsky and L. Di Giustino, PRD 86 085026 (2011)

- Take $\varphi_{\ln}$ for our calculations

Pion FF



1.0
0.8
0.6
0.4
0.2
0.0
$F_1(Q^2)/F_0(Q^2)$
10
12
14
16
18
20
Q²
Standard Scale
BLM Scale



0.6
0.5
0.4
0.3
0.2
0.1
0.0
Q²F(Q²)
0
5
10
15
20
Q²
LO Standard
NLO Standard
LO Reg. BLM
NLO Reg. BLM

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S. Brodsky *et al.*, PRD 28 228 (1983)
S. Brodsky and L. Di Giustino, PRD 86 085026 (2011)

- Take $\varphi_{\ln}$ for our calculations
- BLM scale reduces significantly the impact of the NLO corrections and increase dramatically the LO one.

Part 2:
Modeling Generalised Parton Distributions

Properties

- Polynomiality Property:

$$\int_{-1}^{1} \mathrm{d}x \; x^m H^q(x,\xi,t) = \sum_{j=0}^{\left[\frac{m}{2}\right]} \xi^{2j} C^q_{2j}(t) + mod(m,2)\xi^{m+1} C^q_{m+1}(t)$$

Lorentz Covariance

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

$$\left|H^q(x,\xi,t) - \frac{\xi^2}{1-\xi^2}E^q(x,\xi,t)\right| \leq \sqrt{\frac{q\left(\frac{x+\xi}{1+\xi}\right)q\left(\frac{x-\xi}{1-\xi}\right)}{1-\xi^2}}$$

A. Radysuhkin, Phys. Rev. **D59**, 014030 (1999)
B. Pire *et al.*, Eur. Phys. J. **C8**, 103 (1999)
M. Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)
P.V. Pobilitsa, Phys. Rev. **D65**, 114015 (2002)

Positivity of Hilbert space norm

Properties

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

$$x \in [-1; 1]$$

M. Diehl and T. Gousset, Phys. Lett. **B428**, 359 (1998)

Relativistic quantum mechanics

Properties

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

Relativistic quantum mechanics

- Soft pion theorem (pion GPDs only)

M.V. Polyakov, Nucl. Phys. **B555**, 231 (1999)
CM *et al.*, Phys. Lett. **B741**, 190 (2015)

Axial-Vector WTI

- Polynomiality Property:

  Lorentz Covariance

- Positivity property:

  Positivity of Hilbert space norm

- Support property:

  Relativistic quantum mechanics

- Soft pion theorem (pion GPDs only)

  Axial-Vector WTI

No model (so far) fulfils all the constraints a priori

People emphasise either:

- Polynomiality through Double Distribution or conformal moments modeling,
- Positivity through LFWFs approaches

Double Distributions for spin 0

- Definition in terms of matrix element for $z^2 = 0$:

$$
\begin{aligned}
\langle P + \frac{\Delta}{2} | \bar{q}\left(-\frac{z}{2}\right) \gamma_\mu q\left(\frac{z}{2}\right) | P - \frac{\Delta}{2} \rangle \;=\; & 2P_\mu \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \, e^{-i\beta(P \cdot z) + i\alpha \frac{(\Delta \cdot z)}{2}} F^q(\beta, \alpha, t) \\
& - \Delta_\mu \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \, e^{-i\beta(P \cdot z) + i\alpha \frac{(\Delta \cdot z)}{2}} G^q(\beta, \alpha, t) \\
& + \text{higher twist terms.}
\end{aligned}
$$

D. Müller *et al.*, Fortsch. Phy. 42 101 (1994)
A. Radyushkin, Phys. Rev. **D56**, 5524 (1997)

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A. Radyushkin, Phys. Rev. **D56**, 5524 (1997)

- Simple relation to GPDs:

$$H(x,\xi,t) = \int_\Omega \mathrm{d}\beta\mathrm{d}\alpha\,\delta(x-\beta-\alpha\xi)\left[F(\beta,\alpha,t) + \xi G(\beta,\alpha,t)\right]$$

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- Automatically fulfil the polynomiality property
- But positivity is not fulfilled a priori

LFWFs

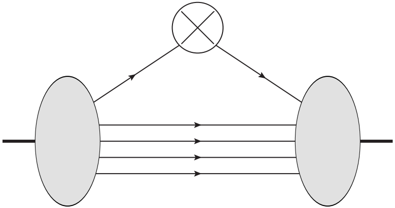
- Lightfront quantization allows to expand hadrons on a Fock basis

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \dots$$

$$|P,N\rangle \propto \sum_\beta \Psi_\beta^{qqq}|qqq\rangle + \sum_\beta \Psi_\beta^{qqq,q\bar{q}}|qqq,q\bar{q}\rangle + \dots$$

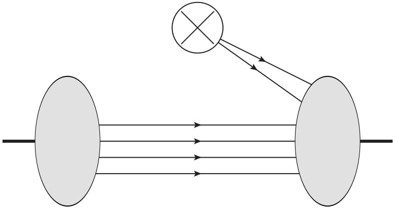
- Lightfront quantization allows to expand hadrons on a Fock basis

DGLAP: $|x| > |\xi|$



- Same N LFWFs
- Truncation unambiguous

ERBL: $|x| < |\xi|$



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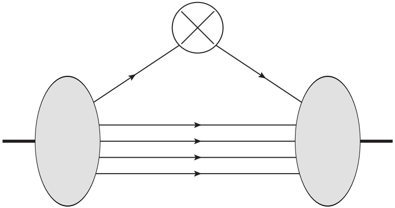

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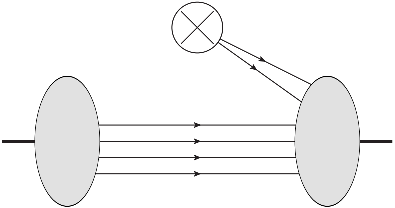

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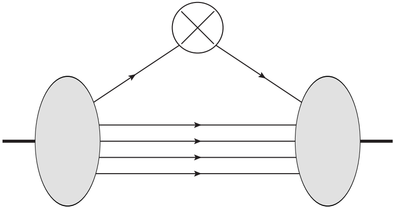

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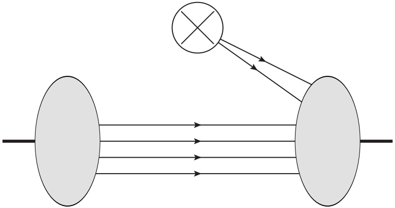

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LFWFs formalism has the positivity property inbuilt but polynomiality is lost by truncating both in DGLAP and ERBL sectors.

Is there a solution to get all the good properties?

- A specific algebraic parametrisation of LFWF was introduced by D. Mueller and D. Hwang

D. Müller and D. Hwang PLB 660 (2008) 350-359

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Question

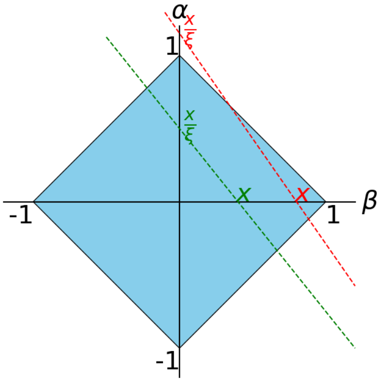
How general is the procedure? Can it be done for any LFWFs?

Chapter 4:
The Inverse Radon Transform

N.Chouika, CM, H. Moutarde, J. Rodriguez-Quintero,
EPJC 77 (2017) no.12, 906

Intuitive picture

$$H(x,\xi) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \delta(x - \beta - \alpha\xi) \left[F(\beta,\alpha) + \xi G(\beta,\alpha)\right]$$



- DGLAP (red) and ERBL (green) lines cut $\beta = 0$ outside or inside the square
- Every point $(\beta \neq 0, \alpha)$ contributes **both** to DGLAP and ERBL regions
- For every point $(\beta \neq 0, \alpha)$ we can draw an infinite number of DGLAP lines.

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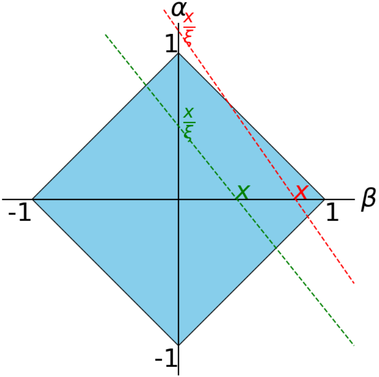

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- For every point $(\beta \neq 0, \alpha)$ we can draw an infinite number of DGLAP lines.

Is it possible to recover the DDs from the DGLAP region only?

- We can define a D-term such that:

$$\int_{-1}^{1} \mathrm{d}x\, x^m \left(H(x,\xi) - D(x/\xi)\right) = \sum_{i \text{ even}}^{m} (2\xi)^i C_{m,i},$$

yielding the **Ludwig-Helgason** consistency conditions.

- From **Hertle theorem** (1983), we know that $H - D$ is in the range of the Radon transform and that:

$$H(x,\xi) = D(x/\xi) + \int_{\Omega} \mathrm{d}\beta \mathrm{d}\alpha \delta(x - \beta - \alpha\xi) F_D(\beta,\alpha)$$

This allows us to identify the DD F_D with the Radon transform of $H - D$. This has been first noticed by O. Teryaev (PLB510 2001 125).

- It should be possible to use the **limited** Radon inverse transform to obtain the DD and thus the ERBL part.

Radon Transform and GPDs

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NB: This is equivalent to fixing the DD to the Polyakov-Weiss scheme. The same argument can be done in other schemes, but the D-term remains ambiguous.

Uniqueness of the Extension

- Since DD are compactly supported, we can use the **Boman and Todd-Quinto theorem** which tells us

$$H(x,\xi) = 0 \quad \text{for} \quad (x,\xi) \in \text{DGLAP} \Rightarrow F_D(\beta,\alpha) = 0 \quad \text{for all} \quad (\beta \neq 0, \alpha) \in \Omega$$

Boman and Todd-Quinto, Duke Math. J. 55, 943 (1987)

insuring the uniqueness of the extension up to D-term like terms.

- The DGLAP region almost completely characterises the entire GPD.

New modeling strategy

- Compute the DGLAP region through overlap of LFWFs
  $\Rightarrow$ **fulfilment of the positivity property**
- Extension to the ERBL region using the Radon inverse transform
  $\Rightarrow$ **fulfilment of the polynomiality property**

Chapter 5:
An example on the pion

N.Chouika, CM, H. Moutarde, J. Rodriguez-Quintero,
PLB 780 (2018) 287-293

An algebraic model for the Pion BSWF

- Consider the Euclidean Bethe-Salpeter Wave Function based on the Nakanishi representation:

$$\Psi(k,P) = S(k-P/2)\Gamma(k,P)S(k+P/2)$$

$$S(k) = \frac{i\gamma\cdot k + M}{k^2+M^2} \qquad \Gamma(k,P) = i\mathcal{N}\gamma_5 \int_{-1}^{1} \frac{\mathrm{d}z(1-z^2)M^2}{\left[\left(k-\frac{1-z}{2}P\right)^2 + M^2\right]}$$

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- We can compute from it the 2-body LFWFs:

$$\begin{aligned}
\Phi_{\uparrow\downarrow}(x,\mathbf{k}_\perp) &= 8\sqrt{15}\,\pi \frac{M^3}{\left(\mathbf{k}_\perp^2+M^2\right)^2}(1-x)\,x \\
\Phi_{\uparrow\uparrow}(x,\mathbf{k}_\perp) &= -8i\sqrt{15}\,\pi \frac{M^2}{\left(\mathbf{k}_\perp^2+M^2\right)^2}(1-x)\,x
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Nakanishi Representation

The present model is very simple, but the Nakanishi formalism is general, and can be straigthforwardly apply to more complicated models.

Algebraic Results

$$H^u_{\pi^+}(x,\xi,t)|_{\xi\le x} = \frac{15}{2}\,\frac{(1-x)^2(x^2-\xi^2)}{(1-\xi^2)^2}\,\frac{1}{(1+\zeta)^2}\left(3+\frac{1-2\zeta}{1+\zeta}\,\frac{\mathrm{arctanh}\left(\sqrt{\frac{\zeta}{1+\zeta}}\right)}{\sqrt{\frac{\zeta}{1+\zeta}}}\right),$$

$$\zeta = \frac{-t}{4M^2}\,\frac{(1-x)^2}{1-\xi^2}\,,$$

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Few comments

- Simple LFWFs yield quite complicated GPDs in the DGLAP region;
- Yet algebraic results can be obtained both for the DD and the GPD in the ERBL region;
- Provide us with a benchmarck for numerical approaches

Numerical Inversion

- For more complicated LFWFs, algebraic inversion is not possible, we need to develop a **systematic** numerical method to handle it.

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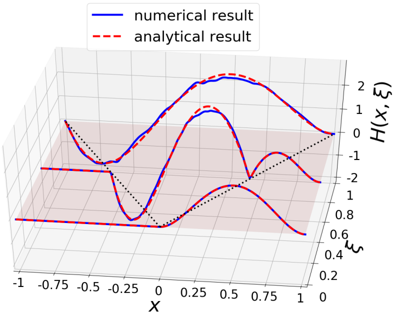
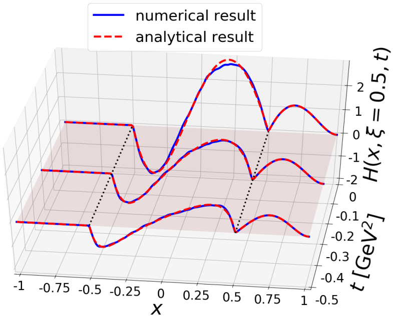
- For more complicated LFWFs, algebraic inversion is not possible, we need to develop a **systematic** numerical method to handle it.
- Difficulty: The limited inverse Radon transform is a severely ill-posed problem in the sens of Hadamard.
- Using finite element analysis we obtained:

$H(x, \xi, 0)$



$H(x, 0.5, t)$



Modeling through LFWF

- We have now a generic technique to model GPDs from LFWFs guaranteeing both positivity and polynomiality.
- PARTONS allows us to go from LFWFs up to the DVCS observables.
- The question of building a model based on effective LFWFs can be addressed
- Advantages:
  - Fulfil all properties by construction
  - Bridges with other hadron physics communities (?)
- Drawbacks: hole at $x = \xi$ kinematics due to LFWF being *zero* at their edges.
  - filled by evolution (?)
  - effective LFWFs not vanishing at the edges (?)
  - specific $x = \xi$ physics (?)

Conclusion

Perspectives and questions

Modeling DA

- Progresses have been done on the theory side
- Theoretical calculations seem to favour the broad unimodal DA
- Is it compatible with phenomenology (?)
- Would help extraction of GPDs from DVMP

Modeling GPDs

- New modeling approach based on LFWFs
- Fulfil all theoretical constraints
- Is it compatible with phenomenology (?)
- Use for global GPD fit and beyond (?)

Thank you for your attention

Back up slides

$$
\begin{aligned}
H(x,\xi,t) &= (1-x)\int_\Omega \mathrm{d}\beta\mathrm{d}\alpha\delta(x-\beta-\alpha\xi)h_P(\beta,\alpha,t)\\
h_P(\beta,\alpha,t) &= \frac{15}{2}\theta(\beta)\left[1+\frac{-t}{4M^2}\left((1-\beta)^2-\alpha^2\right)\right]^{-3}\\
&\times\left[1-3(\alpha^2-\beta^2)-2\beta+\frac{-t}{4M^2}\left(1-(\alpha^2-\beta^2)^2-4\beta(1-\beta)\right)\right],
\end{aligned}
$$

From the algebraic DD we can deduce the GPD in ERBL region

$$
H(x,\xi,0)\big|_{|x|\leq\xi} = \frac{15}{2}\frac{(1-x)(\xi^2-x^2)}{\xi^3(1+\xi)^2}\left(x+2x\xi+\xi^2\right) ,
$$

- Use of a P_1 (planar by pieces) basis
- We have to trade of precision and noise:
  In ill-posed inverse problem, small errors coming from our discretisations can trigger significant increases in the numerical noise.