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UNIVERSITÉ PARIS-SACLAY Mechanical properties of proton based on mulipole model

> Arkadiusz P. Trawiński in collaboration with Cédric Lorcé and Hervé Moutarde [arXiv:1810.09837]

CPHT/École polytechnique and IRFU/CEA Université Paris-Saclay

Warsaw, 22 Jan 2019



Energy momentum tensor

Mulipole model for proton

The EMT in the BF

The EMT in the EF

Stability conditions

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Energy momentum tensor

Mechanical properties Connection with GPDs DVCS process

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In classical continuum mechanics the energy-momentum tensor

$$T^{\mu
u}(\mathbf{x},t) = egin{pmatrix} rac{T^{00}}{T^{10}} & T^{01} & T^{02} & T^{03} \ \hline T^{10} & T^{11} & T^{12} & T^{13} \ T^{20} & T^{21} & T^{22} & T^{23} \ T^{30} & T^{31} & T^{32} & T^{33} \ \end{pmatrix} \,,$$

where:

 T^{00} - energy density, T^{ii} - (not summed) represents pressure, T^{ij} - $i \neq j$ represent shear stress, T^{0i} - momentum flux, T^{i0} - mass (energy) flux. Classically $T^{\mu\nu} = T^{\nu\mu}$.



In QCD $\hat{T}^{\mu\nu}_{\text{QCD}}$ is an operator. However its expectation value on state is interpreted in the same way:

$$\mathcal{T}^{\mu\nu}(x) = \operatorname{Tr}[\hat{T}^{\mu\nu}_{\text{QCD}}(x)\,\rho(\vec{0}\,,\vec{P}\,)]\,,$$

where the Wigner distribution ρ on the proton state with average momentum \vec{P} and position \vec{X} is

$$\rho_{R,P} = \int \frac{\mathrm{d}P^2}{2\pi} \int \frac{\mathrm{d}^4 \Delta}{(2\pi)^4} 2\pi \,\delta(2P \cdot \Delta) \,2\pi \,\delta(P^2 + \frac{\Delta^2}{4} - M^2) \\ \left|P - \frac{\Delta}{2}\right\rangle \langle P + \frac{\Delta}{2} \left| e^{-i\Delta \cdot R} \right|.$$

E. P. Wigner, Phys.Rev.40 (1932) 749

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Then the expectation value of the EMT reads

$$\mathcal{T}^{\mu
u}(x) = \int rac{d^3ec{\Delta}}{(2\pi)^3} \, e^{i\Delta\cdot x} \left<\!\!\left< T^{\mu
u}_{a}(0) \right>\!\!\right>,$$

where for unpolarized proton state we get

$$\langle\!\langle T^{\mu\nu}_{a}(0)
angle\!\rangle = rac{1}{2} \sum_{s=\uparrow,\downarrow} rac{\langle p',s|\hat{T}^{\mu\nu}_{\mathsf{QCD}}(0)|p,s
angle}{\sqrt{2p'_{0}2p_{0}}}\,,$$

where
$$\Delta = p' - p$$
, $P = \frac{1}{2}(p' + p)$ and $t = \Delta^2$.

M. V. Polyakov, Phys.Lett.B555 (2003) 57 C. Lorcé, L. Mantovani, B. Pasquini, Phys.Lett.B776 (2018) 38

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Matrix elements of the general local asymmetric energy-momentum tensor for a spin-1/2 target read

$$\begin{split} \left\langle p',s' \right| \hat{T}_{\text{QCD}}^{\mu\nu}(0) | p,s \rangle &= \\ &= \bar{u}(p',s') \Biggl\{ \frac{P^{\mu}P^{\nu}}{M} \mathcal{A}(t) + \frac{\Delta^{\mu}\Delta^{\nu} - \eta^{\mu\nu}\Delta^{2}}{M} \mathcal{C}(t) + M\eta^{\mu\nu}\bar{\mathcal{C}}(t) \\ &+ \frac{P^{\mu}i\sigma^{\nu\lambda}\Delta_{\lambda}}{4M} \Bigl[\mathcal{A}(t) + \mathcal{B}(t) + \mathcal{D}(t) \Bigr] \\ &+ \frac{P^{\nu}i\sigma^{\mu\lambda}\Delta_{\lambda}}{4M} \Bigl[\mathcal{A}(t) + \mathcal{B}(t) - \mathcal{D}(t) \Bigr] \Biggr\} u(p,s) \,. \end{split}$$

X.-D. Ji, Phys.Rev.Lett.78 (1997) 610 B.L.G. Bakker, E. Leader, T.L. Trueman, Phys.Rev.D70 (2004) 114001

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The study of the EMT became especially important after obtaining by Ji a relation between the EMT and GPDs

$$\int_{-1}^{1} dx \, x \, H(x,\xi,t) = A(t) + 4\xi^2 C(t) \,,$$
$$\int_{-1}^{1} dx \, x \, E(x,\xi,t) = B(t) - 4\xi^2 C(t) \,,$$
$$\int_{-1}^{1} dx \, \tilde{H}(x,0,t) = -D(t) \,.$$

Beside this, $\overline{C}(t)$ can be related to the scalar form factor.

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X.-D. Ji, Phys.Rev.Lett.78 (1997) 610 C. Lorcé, L. Mantovani, B. Pasquini, Phys.Lett.B776 (2018) 38

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X.-D. Ji, Phys.Rev.D55 (1997) 7114 H. Moutarde, P. Sznajder, J. Wagner, Eur.Phys.J.C78 (2018) 890

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The study of the EMT is important because:

- $T^{\mu\nu}$ is a fundamental quantity, which allows to access for example a spin decomposition.
- DVCS gives a way to experimentally measure $T^{\mu\nu}$, *e.g.* JLab.
- lts form factors have a clear interpretation as spatial densities $(\vec{\Delta} \text{ is related to } \vec{r}).$
- EMT form factors and GPDs constrain each other.

E. Leader, C. Lorcé, Phys.Rept.541 (2014) 163



Energy momentum tensor

Mulipole model for proton

General EMT for unpolarized proton Parameters of the mulipole model Why dipole is not sufficient? Justification of parameters

The EMT in the BF

The EMT in the EF

Stability conditions

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The unpolarized amplitude $\langle\!\langle T^{\mu\nu}_a(0) \rangle\!\rangle =$

$$= \frac{P^{2} + MP^{0}}{P^{0}\mathcal{N}} \left\{ \frac{P^{\mu}P^{\nu}}{M} A_{a}(t) + \frac{\Delta^{\mu}\Delta^{\nu} - \eta^{\mu\nu}\Delta^{2}}{M} C_{a}(t) + M\eta^{\mu\nu}\bar{C}_{a}(t) \right\} \\ + \frac{\Delta^{2}}{4P^{0}\mathcal{N}} \left\{ \left[\frac{2P^{\mu}P^{\nu}}{M} + P^{\{\mu}\eta^{\nu\}0} \right] \frac{A_{a}(t) + B_{a}(t)}{2} + P^{[\mu}\eta^{\nu]0} \frac{D_{a}(t)}{2} \right\} \\ - \frac{\Delta^{0}}{4P^{0}\mathcal{N}} \left\{ P^{\{\mu}\Delta^{\nu\}} \frac{A_{a}(t) + B_{a}(t)}{2} + P^{[\mu}\Delta^{\nu]} \frac{D_{a}(t)}{2} \right\}.$$

where $\mathcal{N} = \sqrt{p'^0 + M} \sqrt{p^0 + M}$ and label *a* refers to contribution from quarks or gluons.

We adopt a simple multipole Ansatz for the GFFs

$$F_a(t) = \frac{F_a(0)}{\left(1 - t/\Lambda_{F_a}^2\right)^{n_F}},$$

which is supported by Goeke, *et.al.* calculations for |t| < 1 GeV².

We adopt a standard dipole Ansatz (i.e. $n_F = 2$) for A_a , \bar{C}_a and D_a , but for B_a and C_a we choose a tripole Ansatz (i.e. $n_F = 3$) in order for the energy and pressure distributions to be realistic.

K. Goeke, J. Grabis, J. Ossmann, M.V. Polyakov, P. Schweitzer, A. Silva, D. Urbano, Phys.Rev.D75 (2007) 094021

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Parameters for the multipole model of the GFFs, in the $\overline{\text{MS}}$ scheme with renormalization scale $\mu = 2$ GeV.

| Fa | n _F | $F_q(0)$ | Λ_{F_q} [GeV] | $F_G(0)$ | Λ_{F_G} [GeV] |
|-------------|----------------|----------|-----------------------|----------|-----------------------|
| Aa | 2 | 0.55 | 0.91 | 0.45 | 0.91 |
| Ba | 3 | -0.07 | 8. 0 | 0.07 | 8.0 |
| Ca | 3 | -0.32 | 8. 0 | -0.56 | 8.0 |
| $ar{C}_{a}$ | 2 | -0.11 | 0.91 | 0.11 | 0.91 |
| D_a | 2 | -0.33 | 1.74 | - | - |



- $A_q(0)$ is taken from the MMHT2014 analysis [EPJC75:204].
- ► B_q(0) is suggested by the AdS/QCD correspondence [EPJC76:4], [EPJA53:237] and agrees in magnitude with Lattice QCD [PRD91:014505], [arXiv:1810.04626].
- ► C_q(0) = d₁^q(0)/5 and d₁^q(0) is obtained in a dispersive analysis of DVCS [PLB739:133] which is close to a recent experimental extraction reported in [Nature557:396].
- $\bar{C}_q(0)$ is given by a phenomenological estimate [EPJC78:120] and is supported by a Lattice calculation [PRL121:212001].
- $D_q(0) = -G_A^q(0)$ is obtained from a leading twist NNLO analysis by the HERMES collaboration [PRD75:012007]

- Λ_{A_q} and Λ_{C̄_q} are motivated by results obtained in the chiral quark-soliton model [PRD75:094021],[JHEP09:156].
- We estimated the tripole mass Λ_{Cq} by multiplying the reported dipole mass in the chiral quark-soliton model with √(3/2) so as to leave the quantity d(Cq(t))/dt | t=0 unchanged.
 No Λ_{Bq} has been reported, so we simply choose Λ_{Bq} = Λ_{Cq}.
 Λ_{Dq} = Λ_{GA}^q is taken from a Lattice estimate [PRD96:054507].

• $A_G(0)$, $B_G(0)$ and $\overline{C}_G(0)$ are determined by the sum rules:

$$\sum_{a=q,G} A_a(0) = 1, \qquad \sum_{a=q,G} B_a(0) = 0, \qquad \sum_{a=q,G} \bar{C}_a(t) = 0.$$

- ► As suggested in [PRD98:074003], we use the simple relation $C_G(0) = \frac{16}{3n_f} C_q(0)$ with $n_f = 3$.
- Since we lack information about the gluon GFFs, we simply set $\Lambda_{F_G} = \Lambda_{F_q}$ for $F = A, B, C, \overline{C}$.



Energy momentum tensor

Mulipole model for proton

The EMT in the BF Figures of ϵ , m, p_r , p_t , p, sProton radius The EMT in the FL

The EMT in the EF

Stability conditions

In the Breit frame (BF) defined by ${m P}={f 0}$ the EMT reduces to

$$egin{aligned} &\langle\!\langle T^{\mu
u}_{a}(0)
angle
ight
angle_{\mathsf{BF}} &= M \Big\{ \eta^{\mu 0} \eta^{
u 0} \left[\mathcal{A}_{a}(t) + rac{t}{4M^{2}} \, \mathcal{B}_{a}(t)
ight] \ &+ \eta^{\mu
u} \left[ar{\mathcal{C}}_{a}(t) - rac{t}{M^{2}} \, \mathcal{C}_{a}(t)
ight] + rac{\Delta^{\mu} \Delta^{
u}}{M^{2}} \, \mathcal{C}_{a}(t) \Big\} \,, \end{aligned}$$

which (after the Fourier transform) has the same structure as the EMT of an anisotropic-spherically symmetric compact star

$$\Theta^{\mu\nu}(\mathbf{r}) = [\varepsilon(r) + p_t(r)] u^{\mu}u^{\nu} - p_t(r)\eta^{\mu\nu} + [p_r(r) - p_t(r)] \chi^{\mu}\chi^{\nu},$$

for $u^{\mu} = \eta^{\mu 0}$, where u^{μ} and $\chi^{\mu} = x^{\mu}/r$ are unit timelike and spacelike four-vectors orthogonal to each other $(r = |\mathbf{r}|)$.

S.S. Bayin, Astrophys.J.303 (1986) 101





The tensor can alternatively be written as

$$\Theta^{\mu\nu}(\mathbf{r}) = [\varepsilon(r) + p(r)] u^{\mu}u^{\nu} - p(r)\eta^{\mu\nu} + s(r)\left(\chi^{\mu}\chi^{\nu} - \frac{1}{3}h^{\mu\nu}\right)$$

with $h^{\mu\nu} = u^{\mu}u^{\nu} - \eta^{\mu\nu}$, where isotropic pressure p(r) and pressure anisotropy s(r) are related to and pressures as follows

$$p(r) = rac{p_r(r) + 2 p_t(r)}{3}, \qquad s(r) = p_r(r) - p_t(r).$$

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Definitions

$$\begin{split} \varepsilon_{a}(r) &= M \int \frac{d^{3} \mathbf{\Delta}}{(2\pi)^{3}} e^{-i\mathbf{\Delta}\cdot\mathbf{r}} \left\{ A_{a}(t) + \bar{C}_{a}(t) + \frac{t}{4M^{2}} \left[B_{a}(t) - 4C_{a}(t) \right] \right\} \\ p_{r,a}(r) &= M \int \frac{d^{3} \mathbf{\Delta}}{(2\pi)^{3}} e^{-i\mathbf{\Delta}\cdot\mathbf{r}} \left\{ -\bar{C}_{a}(t) - \frac{4}{r^{2}} \frac{t^{-1/2}}{M^{2}} \frac{d}{dt} \left(t^{3/2} C_{a}(t) \right) \right\} \\ p_{t,a}(r) &= M \int \frac{d^{3} \mathbf{\Delta}}{(2\pi)^{3}} e^{-i\mathbf{\Delta}\cdot\mathbf{r}} \left\{ -\bar{C}_{a}(t) + \frac{4}{r^{2}} \frac{t^{-1/2}}{M^{2}} \frac{d}{dt} \left[t \frac{d}{dt} \left(t^{3/2} C_{a}(t) \right) \right] \right\} \\ p_{a}(r) &= M \int \frac{d^{3} \mathbf{\Delta}}{(2\pi)^{3}} e^{-i\mathbf{\Delta}\cdot\mathbf{r}} \left\{ -\bar{C}_{a}(t) + \frac{2}{3} \frac{t}{M^{2}} C_{a}(t) \right\} \\ s_{a}(r) &= M \int \frac{d^{3} \mathbf{\Delta}}{(2\pi)^{3}} e^{-i\mathbf{\Delta}\cdot\mathbf{r}} \left\{ -\frac{4}{r^{2}} \frac{t^{-1/2}}{M^{2}} \frac{d^{2}}{dt^{2}} \left(t^{5/2} C_{a}(t) \right) \right\} \end{split}$$

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The energy density is always positive, thus it allows to define the corresponding average squared mass radius as

$$R_M^2 = \frac{1}{M} \int d^3 \mathbf{r} \, r^2 \, \varepsilon(\mathbf{r}) = 6 \left[\frac{dA(t)}{dt} \Big|_{t=0} - \frac{1}{M^2} \, C(0) \right]$$

▶ *R_M* = 0.905 fm

- *R_e* = 0.879 fm [J.Phys.Chem.Ref.Data44:031203]
- $R_{\mu} = 0.841$ fm [Nature466:213], [Science353:669]

Also knowing the distribution of energy density, it is also easy to derive the standard mass function

$$m(r) = 4\pi \int_0^r \mathrm{d}r' \, r'^2 \, \varepsilon(r') \, .$$







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In the forward limit (FL) defined by $\pmb{\Delta}=0$ the EMT reduces to

$$\langle\!\langle T^{\mu\nu}_{a}(0) \rangle\!\rangle \big|_{\mathsf{FL}} = rac{P^{\mu}P^{
u}}{E_P} A_{a}(0) + rac{M^2}{E_P} \eta^{\mu
u} \bar{C}_{a}(0) \,,$$

where $E_P = \sqrt{M^2 + P^2}$, which has the same structure as the EMT of perfect fluid

$$\theta^{\mu\nu}(\mathbf{r}) = (\bar{\varepsilon} + \bar{\rho}) \, u^{\mu} u^{\nu} - \bar{\rho} \, \eta^{\mu\nu} \, ,$$

where $u^{\mu} = P^{\mu}/M$. This suggests that the following combinations

$$ar{arepsilon}_{a}(oldsymbol{r}) = ig[A_{a}(0) + ar{C}_{a}(0)ig] \; rac{M^{2}}{E_{P}} rac{ heta(r < R_{M})}{V} \, , \ ar{p}_{a}(oldsymbol{r}) = -ar{C}_{a}(0) \; rac{M^{2}}{E_{P}} \; rac{ heta(r < R_{M})}{V} \, .$$

C. Eckart, Phys.Rev. 58 (1940) 919

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The matrix element of the EMT in the FL can be decompose into two ways (for ${m P}={f 0})$

$$\begin{split} \left\langle T_{a}^{\mu\nu}(0) \right\rangle \Big|_{\mathsf{FL}} &= 2 \, A_{a}(0) \, M^{2} \, \eta^{\mu 0} \eta^{\nu 0} + 2 \, \bar{C}_{a}(0) \, M^{2} \, \eta^{\mu \nu} \\ &= 2 \, A_{a}(0) \, M^{2} \Big[\eta^{\mu 0} \eta^{\nu 0} - \frac{1}{4} \eta^{\mu \nu} \Big] \\ &+ \left[A_{a}(0) + 4 \, \bar{C}_{a}(0) \right] \frac{1}{2} \eta^{\mu \nu} \, M^{2} \, , \end{split}$$

where $\left[\eta^{\mu 0}\eta^{\nu 0} - \frac{1}{4}\eta^{\mu \nu}\right]$ - traceless part. Ji's coefficients $a_a = A_a$ and $b_a = A_a + 4\bar{C}_a$.

X.D. Ji, Phys.Rev.Lett.74 (1995) 1071
X.D. Ji, Phys.Rev.D52 (1995) 271
C. Lorcé, Eur.Phys.J.C78 (2018) 120

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Energy momentum tensor

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The spin denisty Projection relations Figures of ρ , σ_r , σ_t , σ , Π

Stability conditions

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In the elastic frame (EF) defined by $\boldsymbol{P} \cdot \boldsymbol{\Delta} = 0$ the EMT reduces to

$$\begin{split} \left\langle \left\langle T_{a}^{\mu\nu}(0) \right\rangle \right\rangle \Big|_{\mathsf{EF}} &= \\ &= \left[1 - \frac{\boldsymbol{P}^{2}}{N^{2}} \right] \left\{ \frac{P^{\mu}P^{\nu}}{M} A_{a}(t) + \frac{\Delta_{\perp}^{\mu}\Delta_{\perp}^{\nu} + \eta^{\mu\nu}\Delta_{\perp}^{2}}{M} C_{a}(t) + M\eta^{\mu\nu}\bar{C}_{a}(t) \right\} \\ &\quad - \frac{\Delta_{\perp}^{2}}{4N^{2}} \left\{ \left[\frac{2P^{\mu}P^{\nu}}{M} + P^{\{\mu}\eta^{\nu\}0} \right] \frac{A_{a}(t) + B_{a}(t)}{2} + P^{[\mu}\eta^{\nu]0} \frac{D_{a}(t)}{2} \right\} \\ &\text{where } P^{0} = \sqrt{\boldsymbol{P}^{2} + \frac{\Delta_{\perp}^{2}}{4} + M^{2}}, \ N^{2} = P^{0}(P^{0} + M) \text{ and} \\ \Delta_{\perp}^{0} = \Delta_{\perp}^{\parallel} = 0. \end{split}$$

C. Lorcé, L. Mantovani, B. Pasquini, Phys.Lett.B776 (2018) 38

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Interpretation of D(t)

The EMT in the EF is not symmetric,

$$\langle\!\langle T^{[\mu\nu]}(0) \rangle\!\rangle \big|_{\mathsf{EF}} = rac{\mathbf{\Delta}_{\perp}^2}{4N^2} D(t) \, \eta^{0\mu} P^{\nu} = -i \Delta_{\lambda} \left\langle S^{\lambda\mu\nu} \right\rangle \,,$$

where the spin tensor reads

$$\left\langle S^{\lambda\mu\nu}\right\rangle = -i \, \frac{G_A(t)}{4N^2} \epsilon^{\lambda\mu\nu\sigma} \epsilon_{\sigma 0\alpha\beta} P^{\alpha} \Delta^{\beta} \, .$$

Thus $D(t) = -G_A(t)$ and

$$egin{aligned} m{S}(m{x}_{ot}) &= -i \, \int rac{\mathrm{d}^2 m{\Delta}_{ot}}{(2\pi)^2} e^{-im{\Delta}_{ot}\cdotm{x}_{ot}} rac{D(t)}{4N^2} \left(m{P} imesm{\Delta}_{ot}
ight)
ight|_{\mathsf{EF}} \, , \ &= \left(m{P} imesm{\partial}_{ot}
ight) \int rac{\mathrm{d}^2m{\Delta}_{ot}}{(2\pi)^2} e^{-im{\Delta}_{ot}\cdotm{x}_{ot}} rac{D(t)}{4N^2} \end{aligned}$$

C. Lorcé, L. Mantovani, B. Pasquini, Phys.Lett.B776 (2018) 38 C. Lorcé, Eur.Phys.J.C78 (2018) 785

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Spin density $m{S}(m{b}_{ot})$



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The 2D Fourier transform of the EMT in EF has structure of an anisotropic axially symmetric compact star in two dimensions

$$t^{\alpha\beta}(\boldsymbol{b}_{\perp}; \boldsymbol{P}_{z}) = [\gamma^{2}\rho(\boldsymbol{b}, \boldsymbol{P}_{z}) + \sigma_{t}(\boldsymbol{b}, \boldsymbol{P}_{z})] \mathbf{v}^{\alpha} \mathbf{v}^{\beta} - \sigma_{t}(\boldsymbol{b}, \boldsymbol{P}_{z}) \eta^{\alpha\beta} + [\sigma_{r}(\boldsymbol{b}, \boldsymbol{P}_{z}) - \sigma_{t}(\boldsymbol{b}, \boldsymbol{P}_{z})] \chi^{\alpha} \chi^{\beta},$$

where $v^{lpha}=(1, {f 0}_{\perp})$ and $\chi^{lpha}=(0, {m b}_{\perp}/b).$

The condition $\mathbf{P} \cdot \mathbf{\Delta} = 0$ for $\mathbf{P} = (0_{\perp}, P_z)$ means that $\Delta_z = 0$. The condition $\Delta_z = 0$ is equivalent to integration over z.

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Projection



Figure:
$$\rho_q(b,0) = \int \mathrm{d}z \,\varepsilon_q(r)$$

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Integrating the EMT in BF over z allows us to express 2D energy density and pressures as follows

$$\begin{split} \rho_{a}(b,0) &= \int dz \, \varepsilon_{a}(r) \,, \\ \sigma_{r,a}(b,0) &= \int dz \, \frac{b^{2} p_{r,a}(r) + z^{2} p_{t,a}(r)}{r^{2}} \,, \\ \sigma_{t,a}(b,0) &= \int dz \, p_{t,a}(r) \,, \\ \sigma_{a}(b,0) &= \int dz \left[p_{a}(r) + \frac{b^{2} - 2z^{2}}{6r^{2}} \, s_{a}(r) \right] \,, \\ \Pi_{a}(b,0) &= \int dz \, \frac{b^{2}}{r^{2}} \, s_{a}(r) \,, \end{split}$$

with
$$r = \sqrt{b^2 + z^2}$$

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Energy density ρ



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Figure: $\rho_a(b, P_z) \approx M A_a(b)$, others properties are the same.

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- Energy momentum tensor
- Mulipole model for proton
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- Stability conditions

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For a stable system, it is expected that

- 1. $\varepsilon(0) < \infty$, $p(0) < \infty$ and s(0) = 0;
- 2. $\varepsilon(r) > 0$ and $p_r(r) > 0$;

3.
$$\frac{\mathrm{d}\varepsilon(r)}{\mathrm{d}r} < 0$$
 and $\frac{\mathrm{d}p_r(r)}{\mathrm{d}r} < 0$.

The fulfilled these requirements potentially gives new constrains on GPDs.



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- The formulas for energy and pressures densities are model independent.
- Simple multipole model provides reasonable results.
- Stability condition gives new constrains on GPDs.

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UNIVERSITÉ PARIS-SACLAY proton based on mulipole model

> Arkadiusz P. Trawiński in collaboration with Cédric Lorcé and Hervé Moutarde [arXiv:1810.09837]

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Warsaw, 22 Jan 2019