Parton distribution functions from Lattice QCD

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in collaboration with:

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Karl Jansen (DESY Zeuthen)
Aurora Scapellato (Univ. of Cyprus, Univ. of Wuppertal)
Fernanda Steffens (Univ. of Bonn)

Extended ~~European~~ Twisted Mass Collaboration
new logo soon!

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Outline of the talk

1. Lattice QCD
   - Short intro
2. Parton distribution functions
   - Approaches
   - Quasi-PDFs
   - Procedure
   - Results
3. Prospects – GPDs
4. Summary

Results on PDFs based on:

- **C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Reconstruction of light-cone parton distribution functions from lattice QCD simulations at the physical point", Phys. Rev. Lett. 121 (2018) 112001**
- **C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Transversity parton distribution functions from lattice QCD", Phys. Rev. D (Rapid Communications), in press, arXiv: 1807.00232 [hep-lat]**
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, F. Steffens, "A complete non-perturbative renormalization prescription for quasi-PDFs", Nucl. Phys. B923 (2017) 394-415 (invited Frontiers Article)

Review:

- K. Cichy, M. Constantinou, "A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results", arXiv: 1811.07248 [hep-lat]

Lattice QCD

QCD and the need for the lattice

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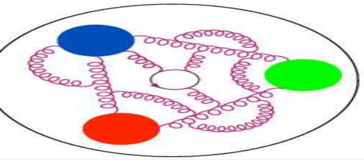
  Hence, it is replaced (analytical continuation) by a real valued exponential e^{-S}.

  Formally, one then evaluates a thermodynamic expectation value with respect to the Boltzmann factor e^{-S}.



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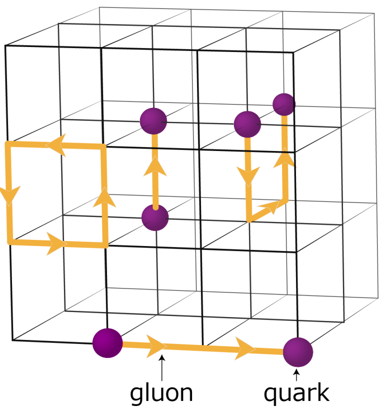

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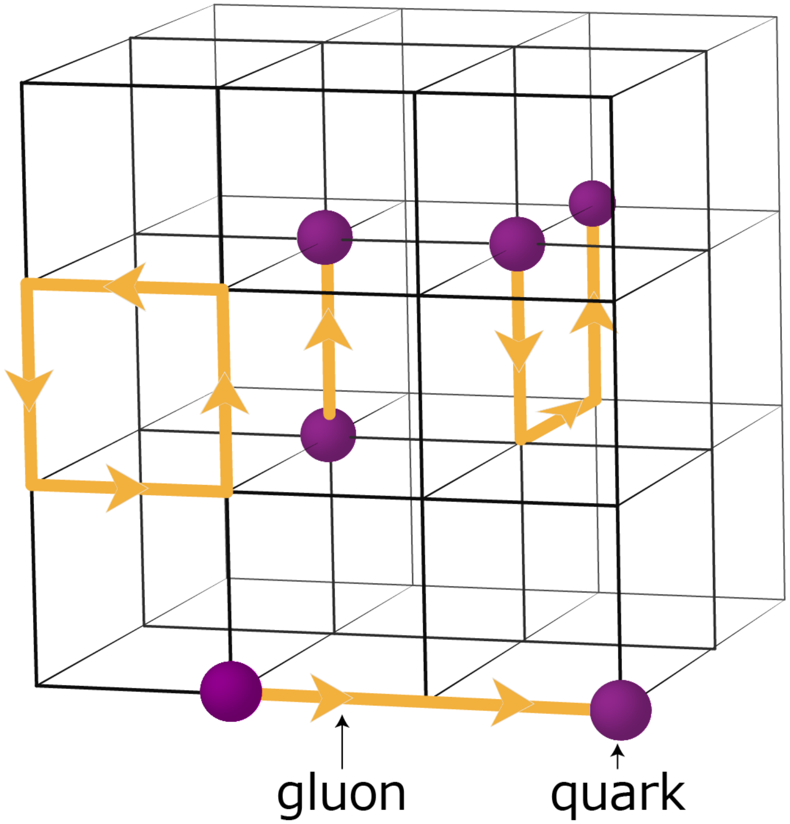
- **2nd step:** regularize the theory → finite space-time lattice (discretization of the theory).



Source: JICFuS, Tsukuba

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  - ⋆ quark fields on lattice sites,
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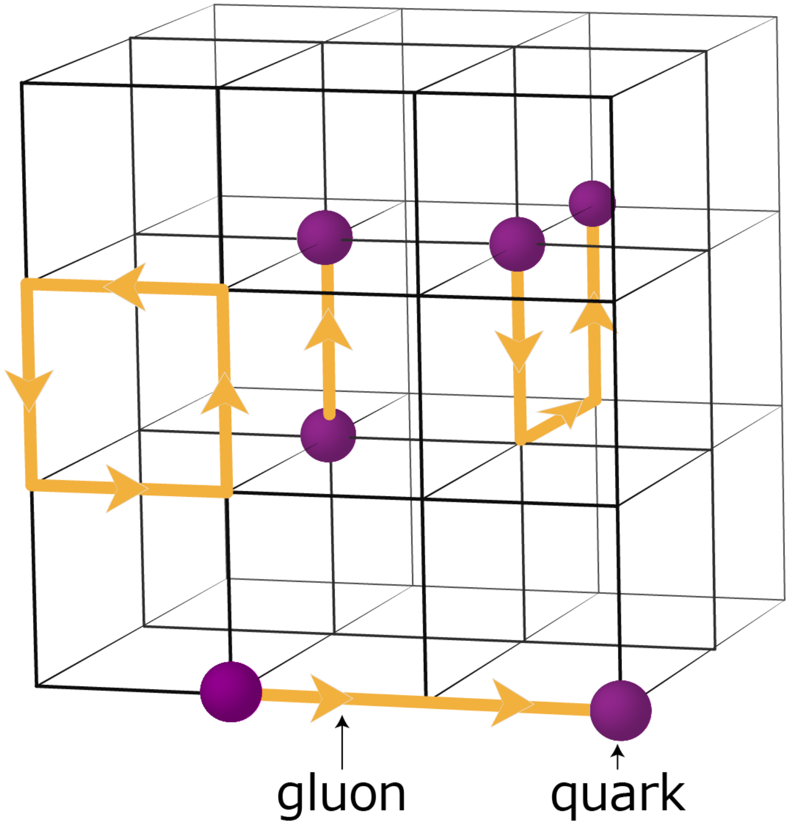

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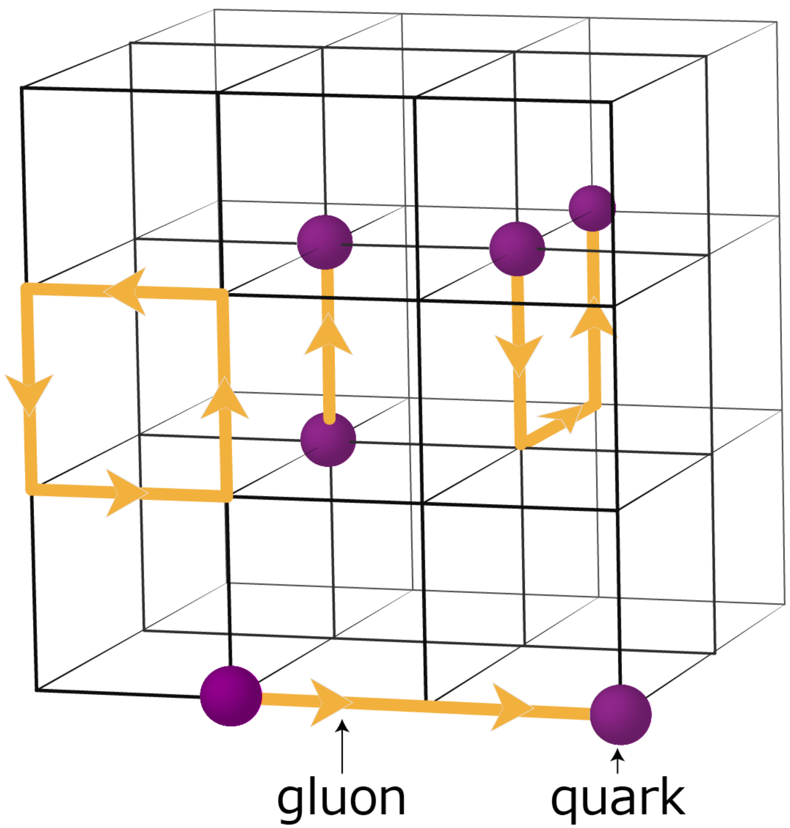
Lattice formulation



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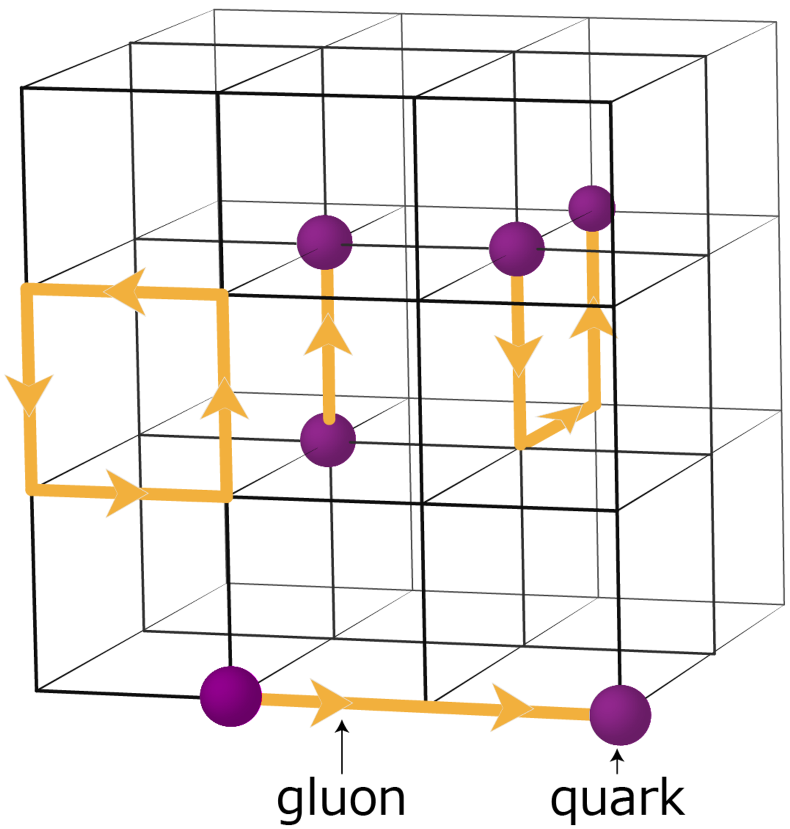

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- Remove the regulator:
  - ⋆ continuum limit $a \to 0$,
  - ⋆ infinite volume limit $L \to \infty$.

Discretization of the action

- gluonic part – "easy" – gauge action constructed from Wilson loops of size 1x1 (plaquettes) and 1x2 (rectangles):

$$S_G[U] = \frac{\beta}{3} \sum_x \Big(b_0 \sum_{\mu,\nu=1} \mathrm{Re\,Tr}\big(1 - P^{1\times1}_{x;\mu,\nu}\big) + b_1 \sum_{\mu\neq\nu} \mathrm{Re\,Tr}\big(1 - P^{1\times2}_{x;\mu,\nu}\big)\Big),$$

where $\beta = 6/g_0^2$, g_0 is the bare coupling and the b_0, b_1 parameters are normalized according to: $b_0 = 1 - 8b_1$.

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$\xrightarrow{a\to 0}$ $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \times \mathcal{O}(a^n)$

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This work by the Extended Twisted Mass Collaboration* uses TM fermions with a clover term.

* formerly known as the European Twisted Mass Collaboration

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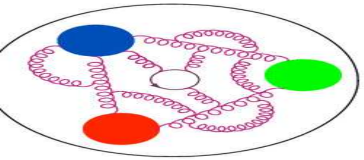

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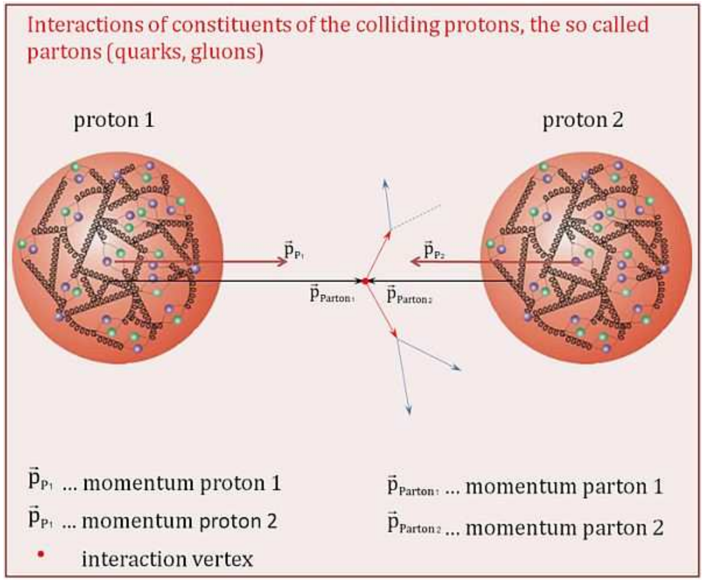
- The power of the lattice approach: **the possibility to control ALL conceivable systematic effects**.

Parton distribution functions (PDFs)

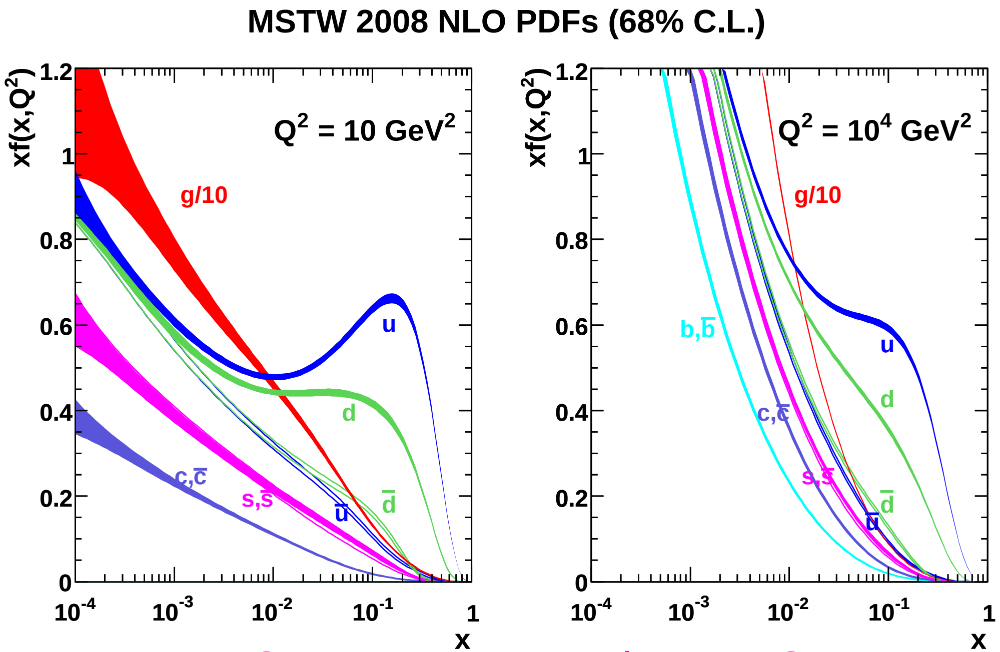
PDFs

- Hadrons are complicated systems with properties resulting from the strong dynamics of quarks and gluons inside them.
- This dynamics is characterized in terms of, among others, parton distribution functions (PDFs).
- PDFs are essential in making predictions for collider experiments.

$$\sigma_{AB} = \sum_{a,b=q,g} \sigma_{ab} \otimes f_{a|A}(x_1, Q^2) \otimes f_{b|B}(x_2, Q^2)$$



Source: LHC, CERN



MSTW2008, Eur. Phys. J. C63, 189

PDFs – why is it difficult on the lattice?

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$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle,$$

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- Accessible on the lattice – moments of the distributions, but:
  - ⋆ higher derivatives noisy,
  - ⋆ operator mixing.

- The common feature of all the approaches is that they rely to some extent on the factorization framework:

$$Q(x,\mu_R) = \int_{-1}^{1} \frac{dy}{y} C\left(\frac{x}{y},\mu_F,\mu_R\right) q(y,\mu_F),$$

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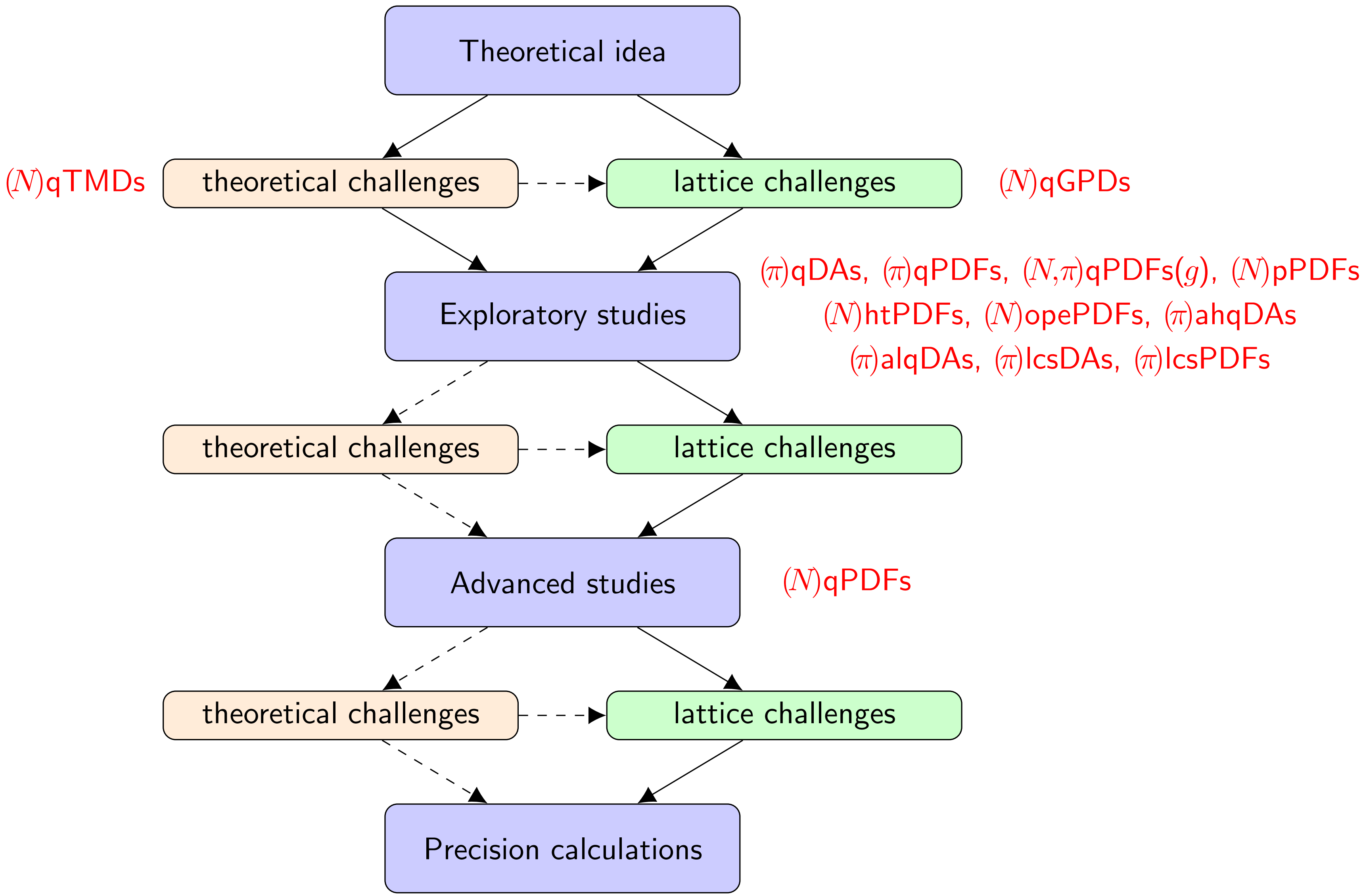
- Matrix elements: $\langle N|\bar{\psi}(z)\Gamma F(z)\Gamma'\psi(0)|N\rangle$ with different choices of Γ,Γ' Dirac structures and objects $F(z)$.
  - ⋆ **hadronic tensor** – K.-F. Liu, S.-J. Dong, 1993
  - ⋆ **auxiliary scalar quark** – U. Aglietti et al., 1998
  - ⋆ **auxiliary heavy quark** – W. Detmold, C.-J. D. Lin, 2005
  - ⋆ **auxiliary light quark** – V. Braun, D. Müller, 2007
  - ⋆ **quasi-distributions** – X. Ji, 2013
  - ⋆ **"good lattice cross sections"** – Y.-Q. Ma, J.-W. Qiu, 2014, 2017
  - ⋆ **pseudo-distributions** – A. Radyushkin, 2017
  - ⋆ **"OPE without OPE"** – QCDSF, 2017

Approaches to light-cone PDFs



A guide to light-cone PDFs from lattice QCD: an overview of approaches, techniques and results

Krzysztof Cichy[1], Martha Constantinou[2] [a]

[1] *Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*

[2] *Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA*

- **97 pages, arXiv:1811.07248, accepted for publication in Special Issue of Adv. in High Energy Physics *Transverse Momentum Dependent Observables from Low to High Energy: Factorization, Evolution, and Global Analyses***
- discusses in detail quasi-distributions:
  nucleon: **non-singlet quark qPDFs**, qGPDs, qTMDs, singlet qPDFs, gluon qPDFs; pion: qPDFs, qDAs
- reviews also other approaches:
  hadronic tensor, auxiliary scalar quark, auxiliary heavy quark, auxiliary light quark, pseudo-distributions, “OPE without OPE”, lattice cross sections

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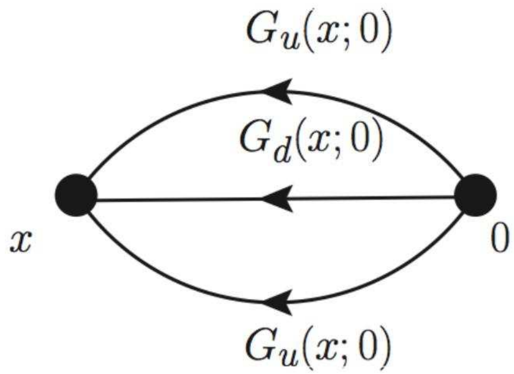

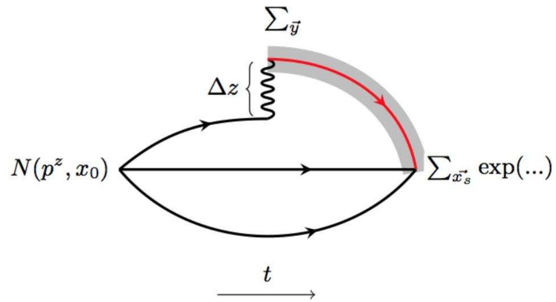

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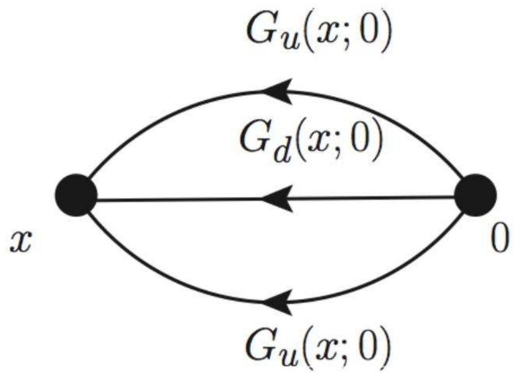
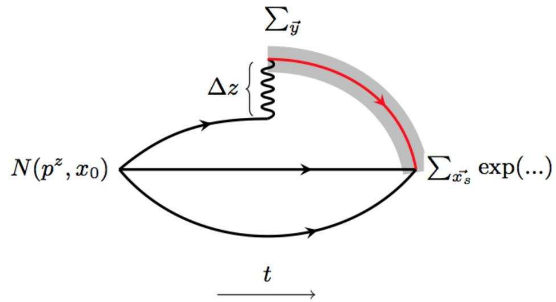
- Differs from light-front PDFs by $\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_3^2},\frac{m_N^2}{P_3^2}\right)$.





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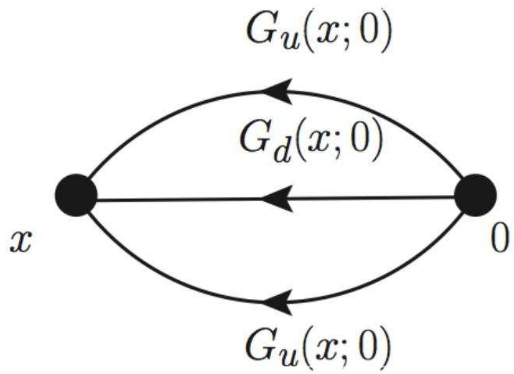
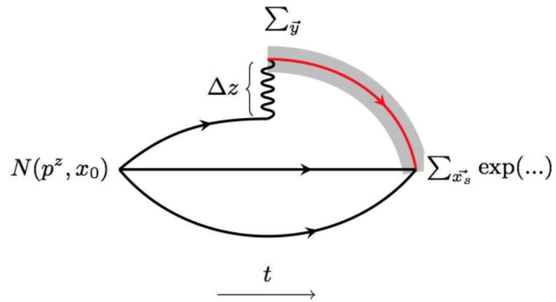
$$\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi} e^{ixP_3 z} \langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle.$$

- z – distance in any *spatial* direction z,
- P_3 – momentum boost in this direction.
- e.g. $\Gamma=\gamma_0,\ \gamma_3$ – unpolarized, $\Gamma=\gamma_5\gamma_3$ – helicity, $\Gamma=\sigma_{31},\ \sigma_{32}$ – transversity
- Theoretically very appealing and intuitive!
- Differs from light-front PDFs by $\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_3^2},\frac{m_N^2}{P_3^2}\right)$.
- The highly non-trivial aspect:
  how to relate $\tilde{q}(x,\mu^2,P_3)$ to the light-front PDF $q(x,\mu^2)$ (infinite momentum frame)
  $\Rightarrow$ **Large Momentum Effective Theory (LaMET)**





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Non-perturbative renormalization scheme: RI'-MOM.

G. Martinelli et al., Nucl. Phys. B445 (1995) 81

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Explicit formulae for 1-loop perturbative matching:

- transverse momentum cutoff scheme to $\overline{\mathrm{MS}}$ matching X. Xiong et al., PRD **90** (2014) 014051
- $\overline{\mathrm{MS}}$ to $\overline{\mathrm{MS}}$ matching W. Wang, S. Zhao, R. Zhu, arXiv:1708.02458 [hep-ph]
- RI to $\overline{\mathrm{MS}}$ matching I.W. Stewart, Y. Zhao, arXiv:1709.04933 [hep-ph]
- treatment of the UV log divergence in wave function corrections T. Izubuchi et al., arXiv:1801.03917 [hep-ph], C. Alexandrou et al., arXiv:1803.02685, 1807.00232 [hep-lat]

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6. Relate quasi-PDFs to light-cone PDFs via a matching procedure.
7. Apply target mass corrections to eliminate residual m_N/P_3 effects.

Lattice setup

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 2.1$
- gauge field configurations generated by ETMC

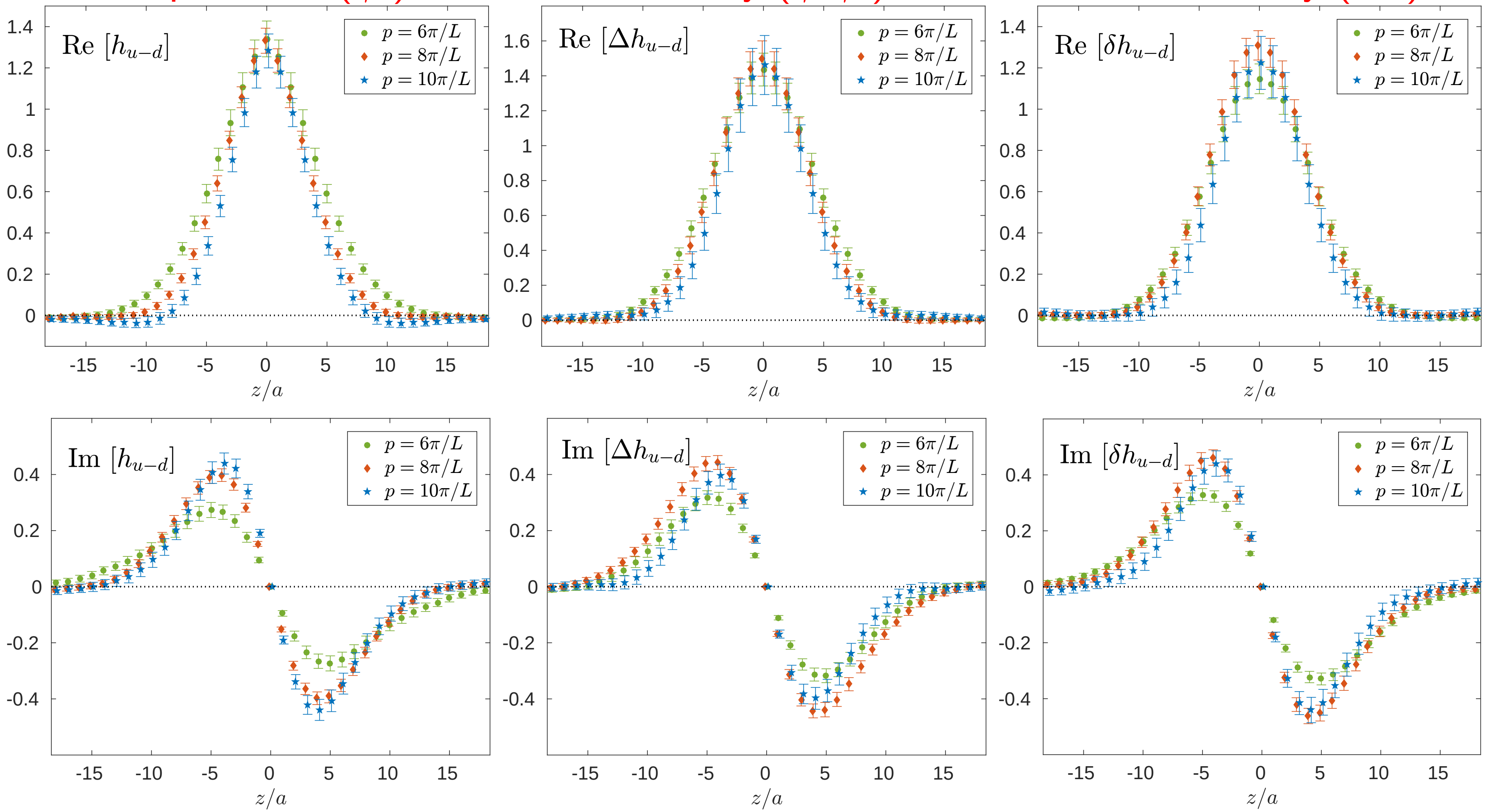
β=2.10,	$c_{\rm SW}$=1.57751, a=0.0938(3)(2) fm
$48^3 \times 96$	$a\mu = 0.0009$ $m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV $m_\pi L = 2.98(1)$

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001
C. Alexandrou et al., Phys. Rev. D98 (2018) 091503 (Rapid Communications)

Results

Bare matrix elements at $t_s = 12a$

unpolarized (γ_0) — helicity $(\gamma_5\gamma_3)$ — transversity (σ_{3i})

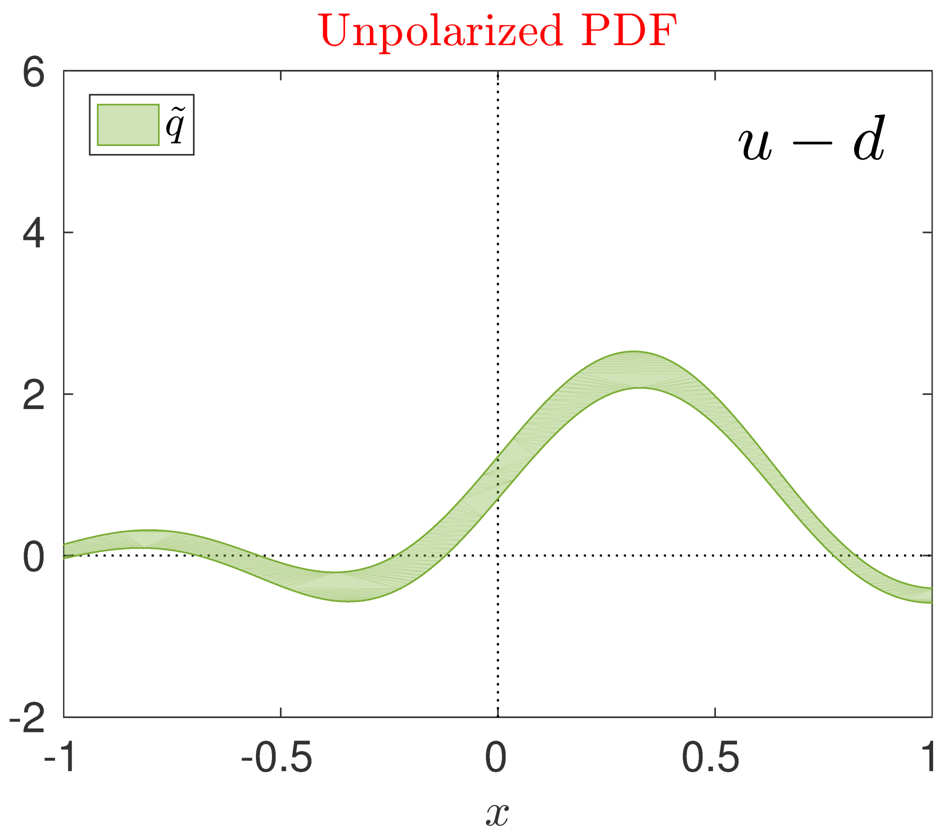


C. Alexandrou et al.: Phys. Rev. Lett. 121 (2018) 112001 and 1807.00232 [hep-lat]

STATISTICS:

unpolarized	helicity	transversity
$P_3 = \frac{6\pi}{L}$ – 4800 meas.	$P_3 = \frac{6\pi}{L}$ – 6240 meas.	$P_3 = \frac{6\pi}{L}$ – 9600 meas.
$P_3 = \frac{8\pi}{L}$ – 38250 meas.	$P_3 = \frac{8\pi}{L}$ – 38250 meas.	$P_3 = \frac{8\pi}{L}$ – 38250 meas.
$P_3 = \frac{10\pi}{L}$ – 72990 meas.	$P_3 = \frac{10\pi}{L}$ – 72990 meas.	$P_3 = \frac{10\pi}{L}$ – 72990 meas.

Nucleon momentum $\frac{10\pi}{48}$



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Quasi-PDFs

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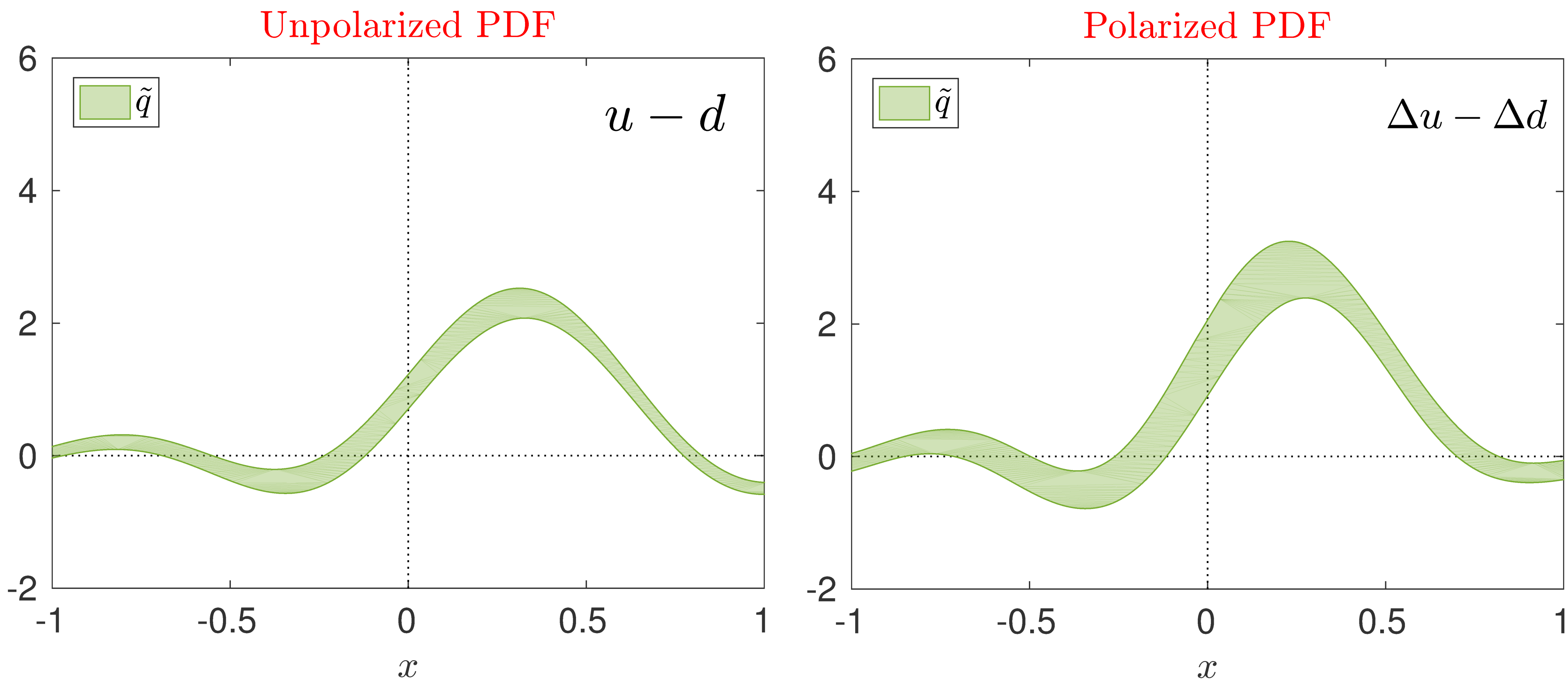

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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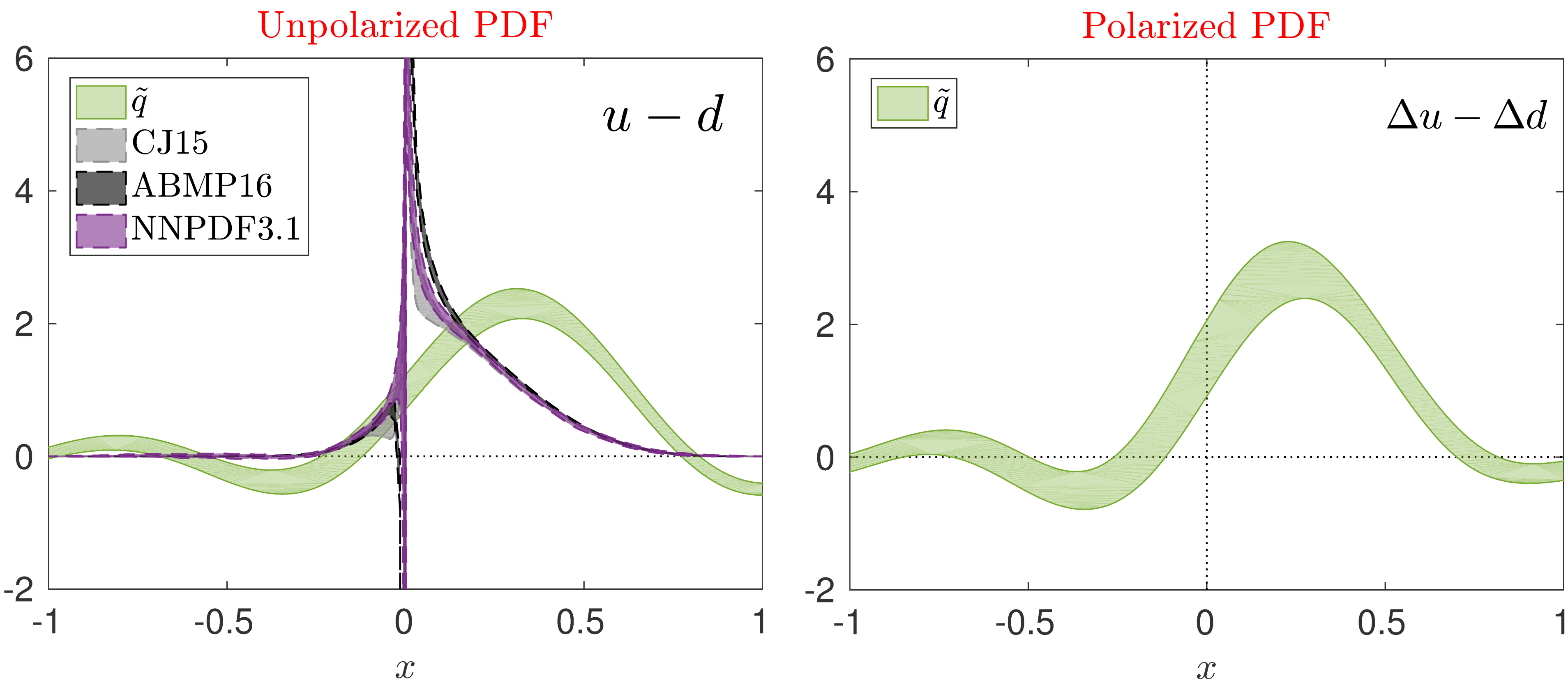

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Quasi-PDFs + pheno

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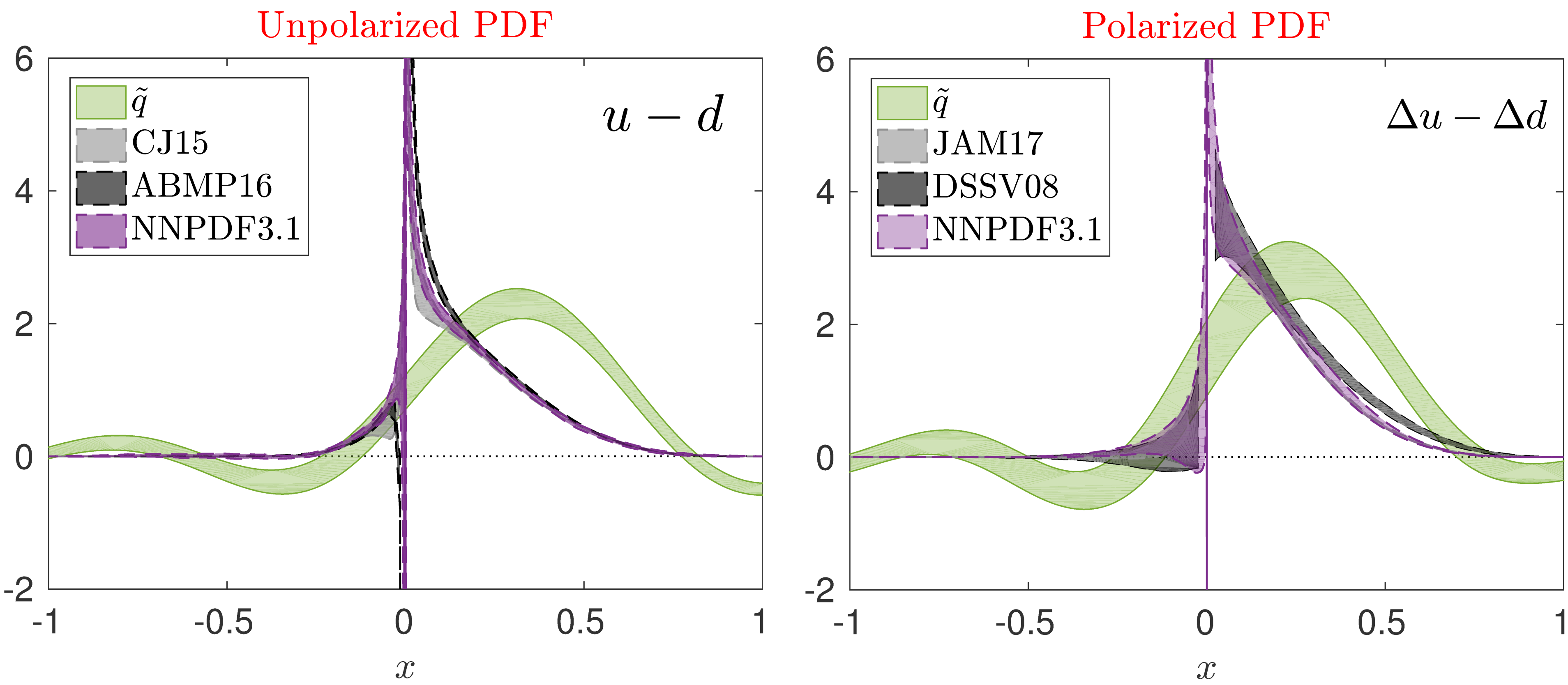

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

The matching formula can be expressed as:

$$q(x,\mu)=\int_{-\infty}^{\infty}\frac{d\xi}{|\xi|}\,C\left(\xi,\frac{\mu}{xP_3}\right)\tilde{q}\left(\frac{x}{\xi},\mu,P_3\right)$$

Matching to light-front PDFs

The matching formula can be expressed as:

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C – matching kernel:

$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_+ & \xi > 1, \\ \left[\frac{1+\xi^2}{1-\xi} \ln\frac{x^2P_3^2}{\xi^2\mu^2} (4\xi(1-\xi)) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_+ & 0 < \xi < 1, \\ \left[-\frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0, \end{cases}$$

[T. Izubuchi et al., arXiv:1801.03917 [hep-ph], C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001]

$\iota=0$ for γ_0 and $\iota=1$ for $\gamma_3/\gamma_5\gamma_3$.

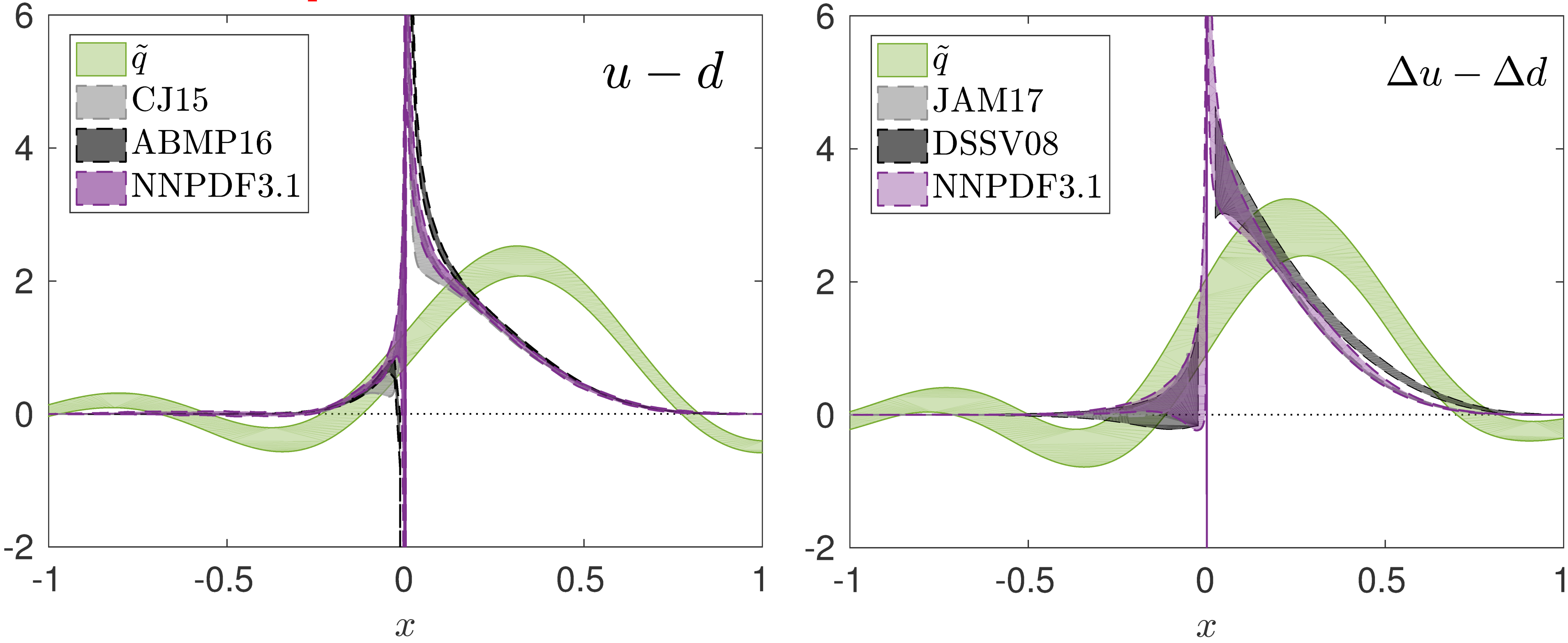
Plus prescription at $\xi=1$:

$$\int \frac{d\xi}{|\xi|} \left[C\left(\xi, \frac{\xi\mu}{xP_3}\right)\right]_+ \tilde{q}\left(\frac{x}{\xi}\right) = \int \frac{d\xi}{|\xi|} C\left(\xi, \frac{\xi\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}\right) - \tilde{q}(x) \int d\xi\, C\left(\xi, \frac{\mu}{xP_3}\right).$$

Matched PDFs

Nucleon momentum $\frac{10\pi}{48}$

Unpolarized PDF — Polarized PDF



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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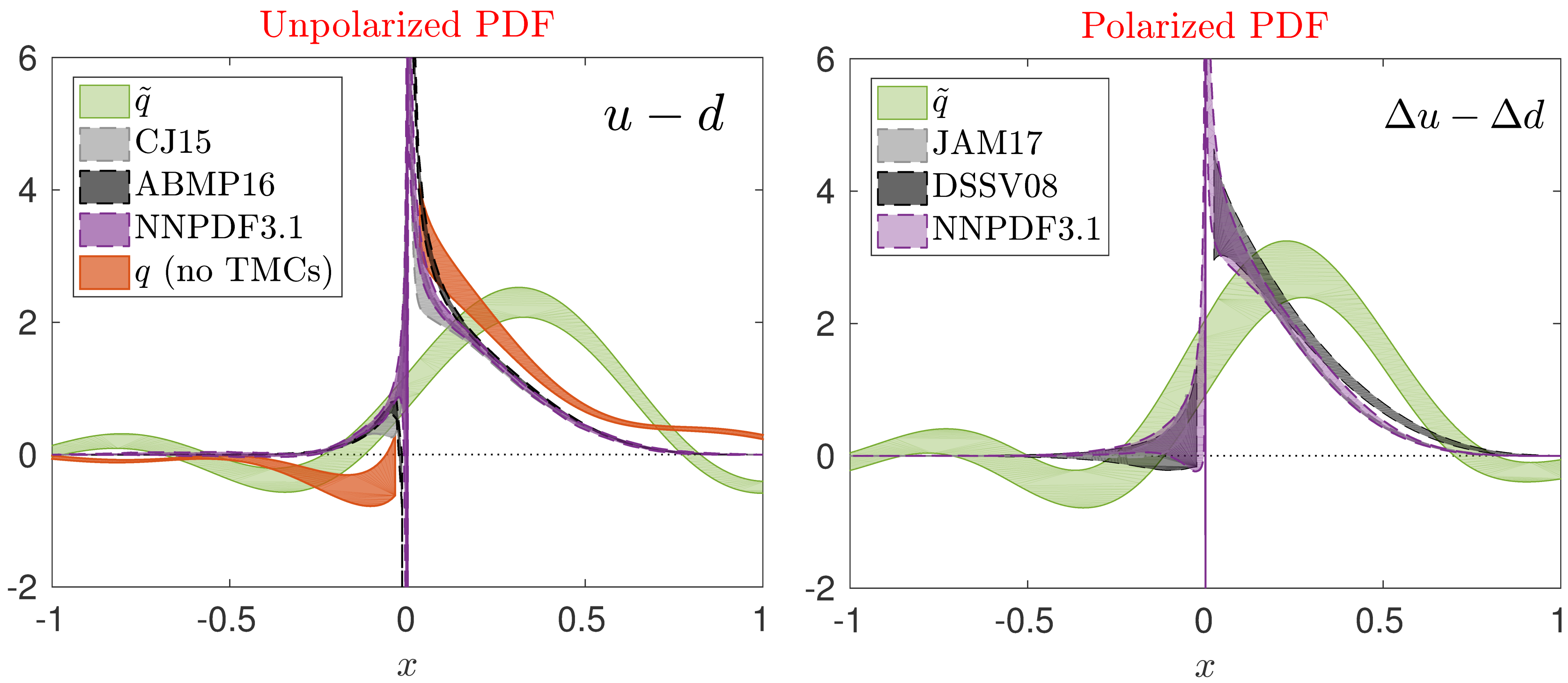

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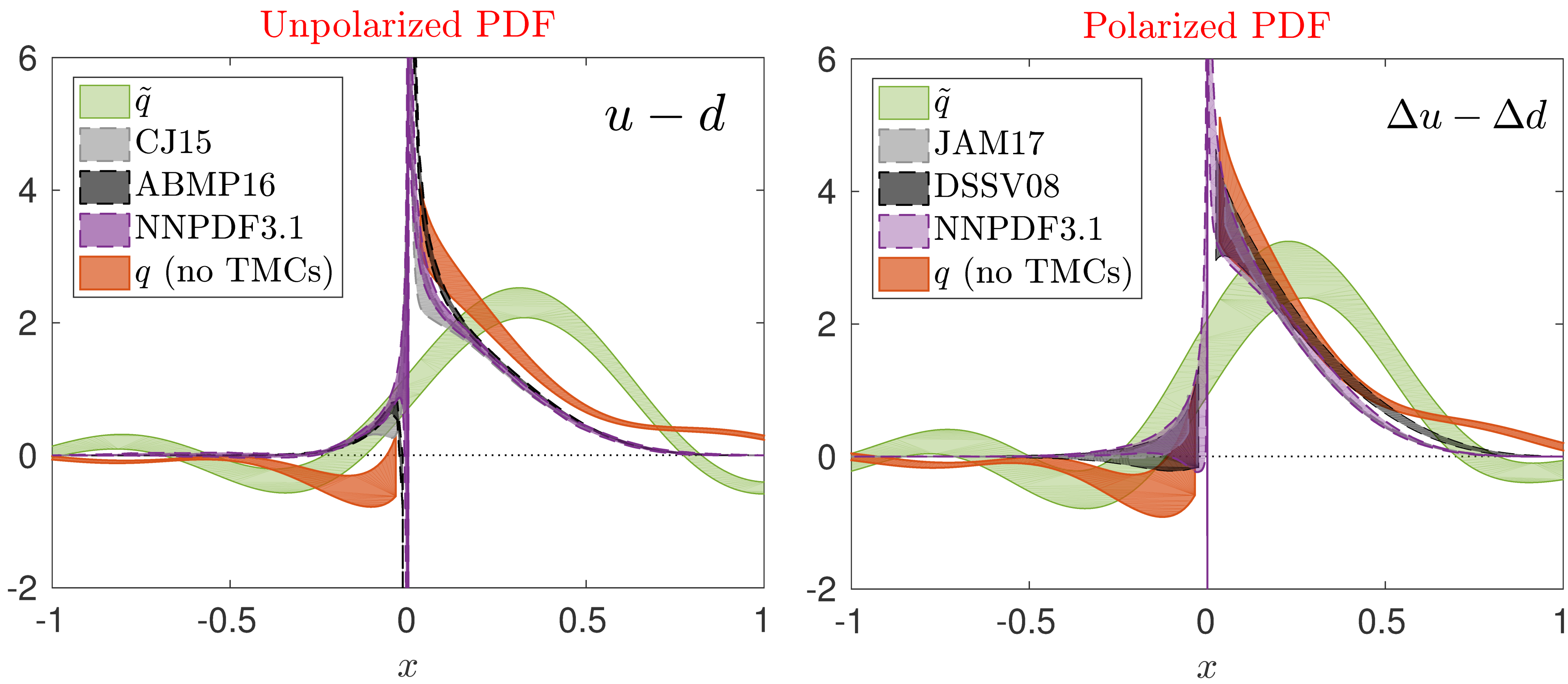

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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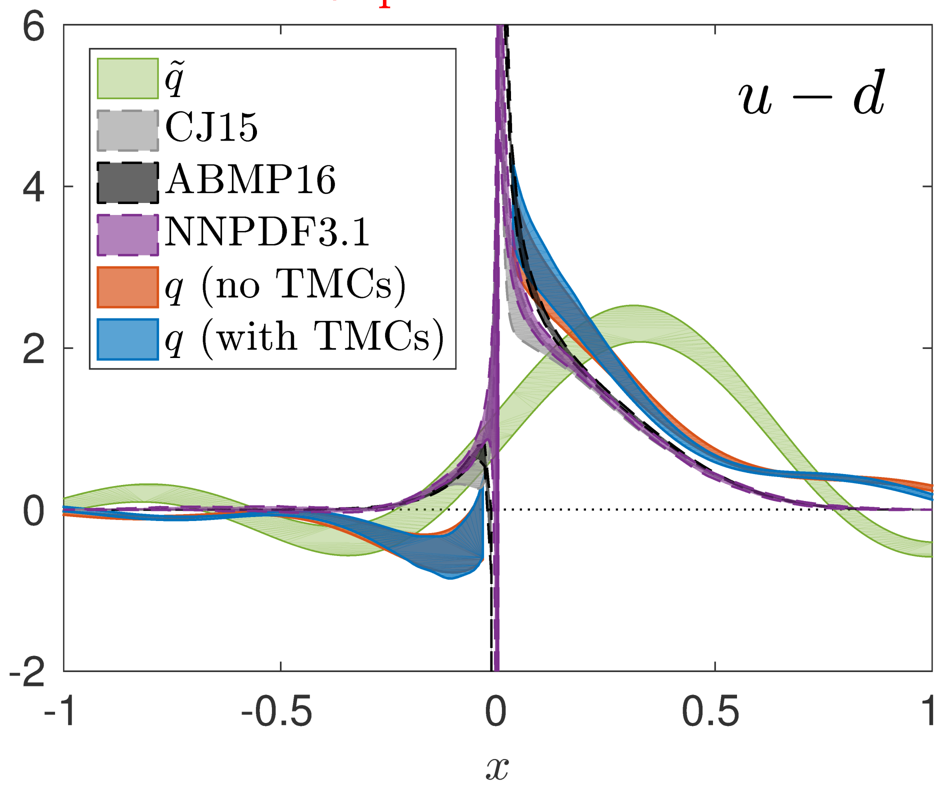
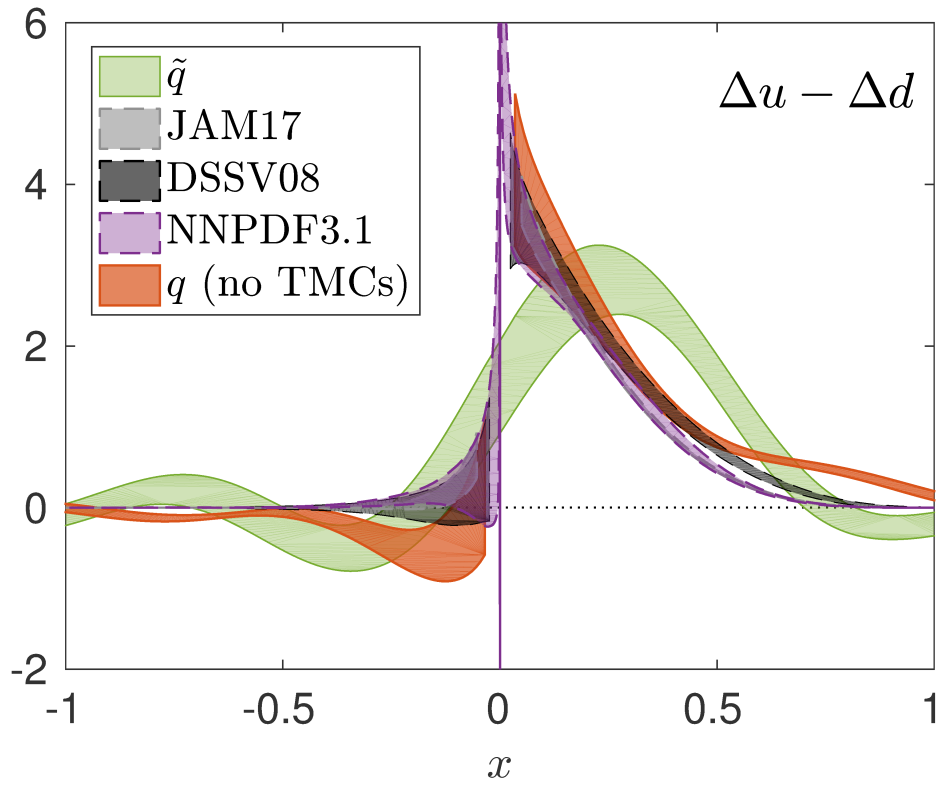
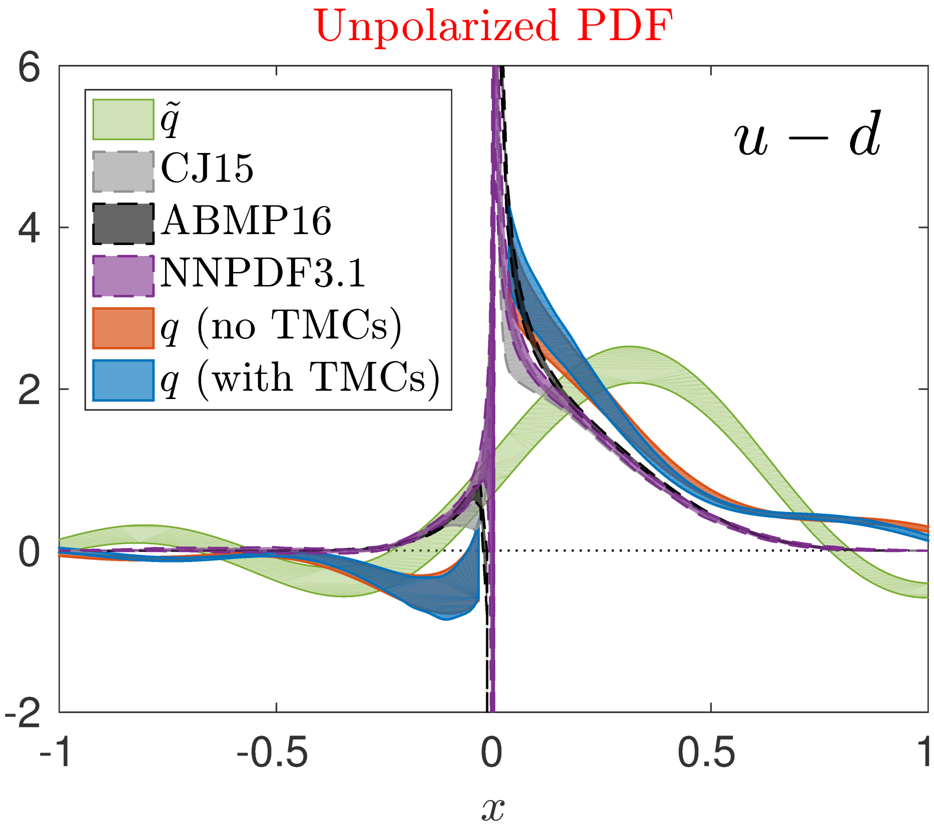


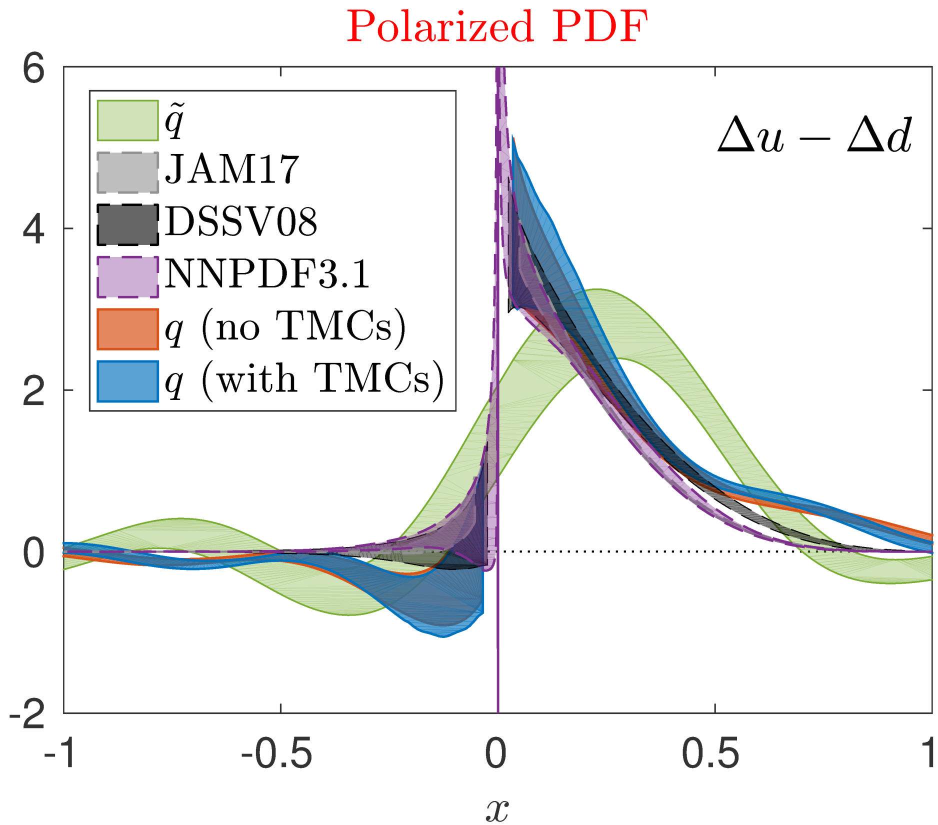


C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Matched PDF + TMCs

Nucleon momentum $\frac{10\pi}{48}$

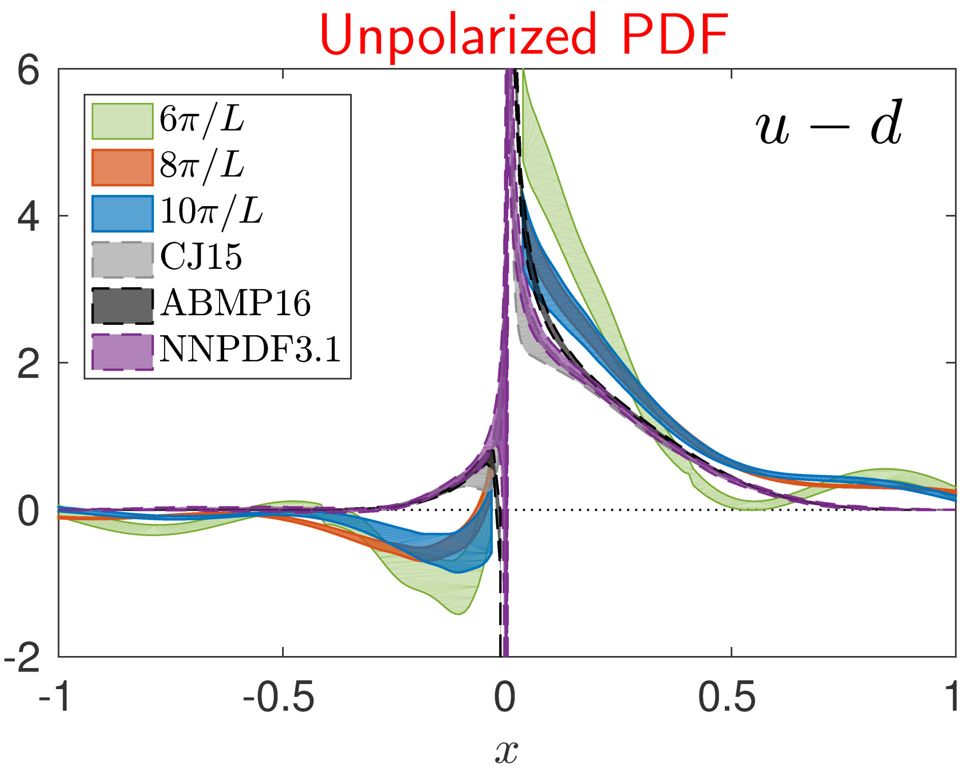




C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Momentum dependence of final PDF

Nucleon momenta $\frac{\{6,8,10\}\pi}{48}$



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Momentum dependence of final PDF

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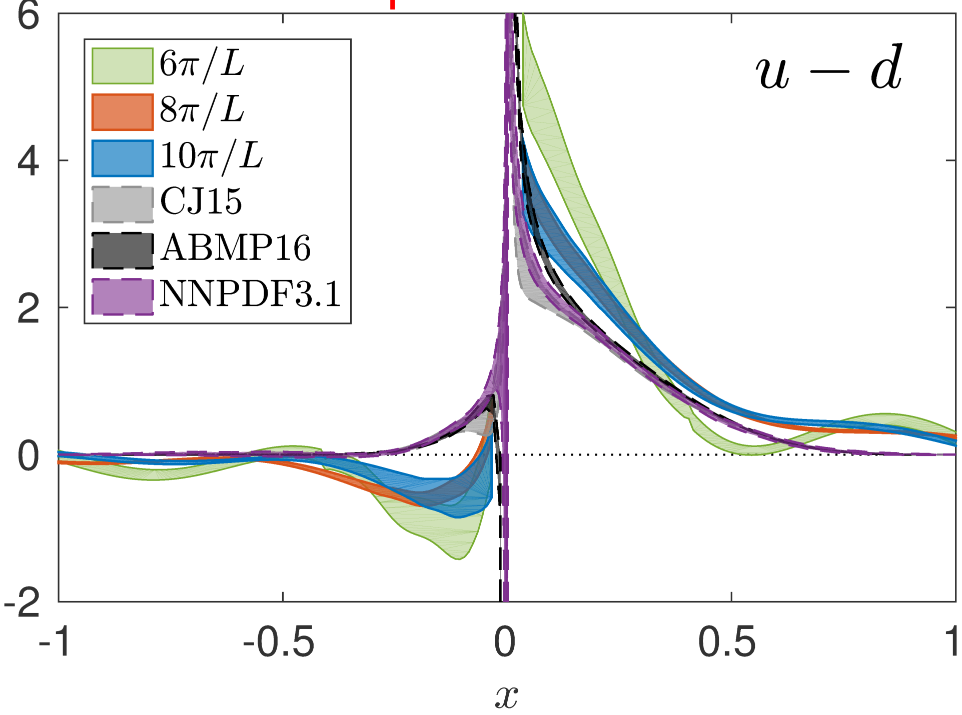
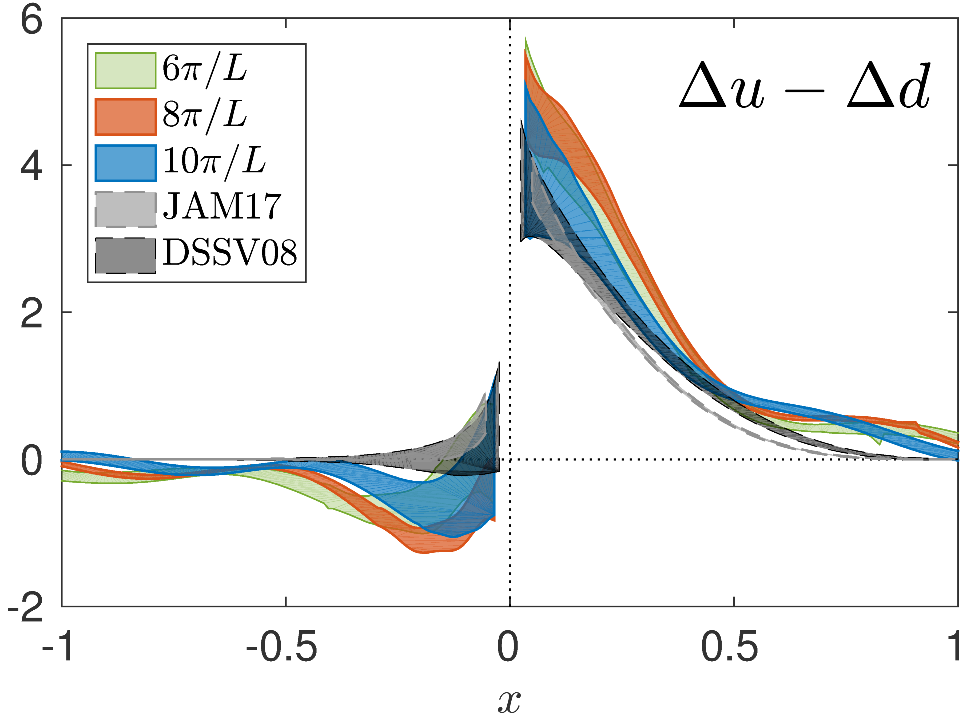
Unpolarized PDF

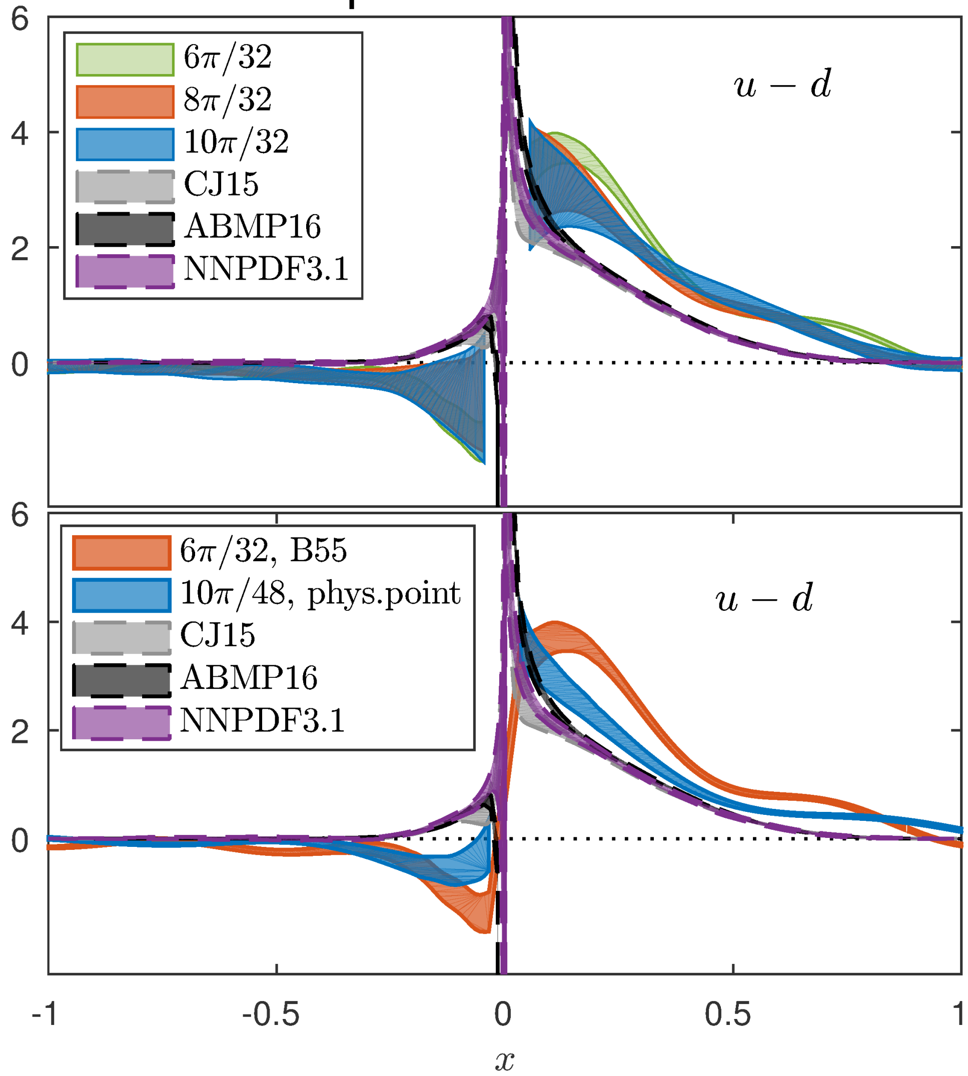


Polarized PDF



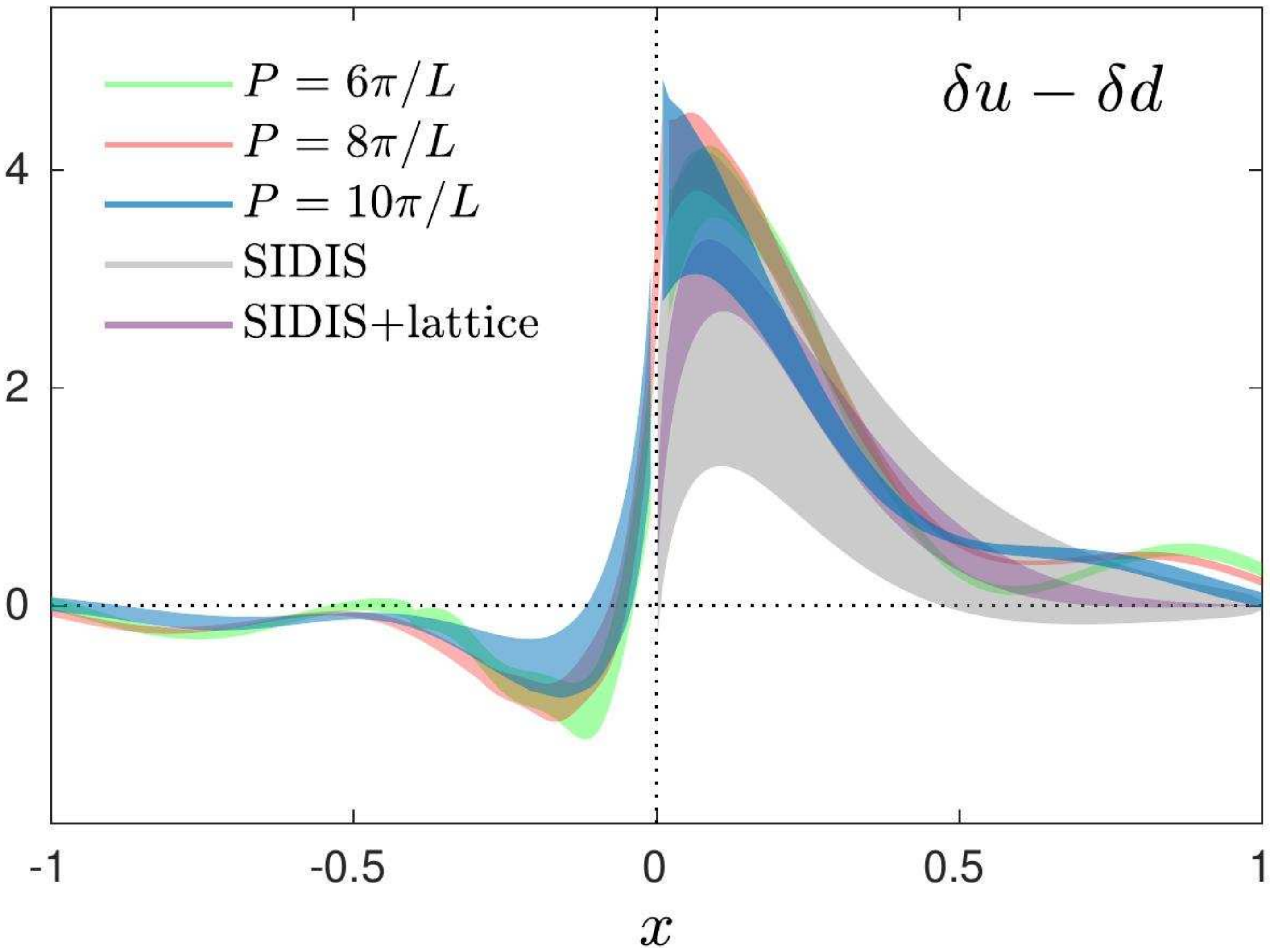
C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Physical vs. non-physical pion mass – 135 vs. 375 MeV
unpolarized PDF



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

Transversity PDF



C. Alexandrou et al., Phys. Rev. D98 (2018) 091503 (Rapid Communications)

Systematics

Different systematic effects still need to be addressed:

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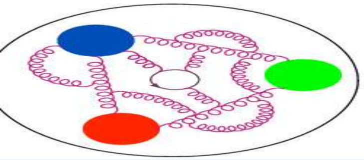

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Biggest challenge:
Reach large momenta at large source-sink separations

Generalized parton distributions

X. Ji et al., *One-loop matching for generalized parton distributions*, Phys. Rev. D92 (2015) 014039

X. Xiong, J.-H. Zhang, *One-loop matching for transversity generalized parton distribution*, Phys. Rev. D92 (2015) 054037

$$
\begin{aligned}
\mathcal{F}_q(x,\xi,t,p^z) &= \int \frac{dz}{4\pi} e^{-ixp^z z} \langle p''|\bar{\psi}(-\frac{z}{2})\gamma^z L(-\frac{z}{2},\frac{z}{2})\psi(\frac{z}{2})|p'\rangle \\
&= \frac{1}{2p^z}\left[\mathcal{H}(x,\xi,t,p^z)\bar{u}(p'')\gamma^z u(p') + \mathcal{E}(x,\xi,t,p^z)\bar{u}(p'')\frac{i\sigma^{z\nu}\Delta_\nu}{2m}u(p')\right], \\
\tilde{\mathcal{F}}_q(x,\xi,t,p^z) &= \int \frac{dz}{4\pi} e^{-ixp^z z} \langle p''|\bar{\psi}(-\frac{z}{2})\gamma^z\gamma^5 L(-\frac{z}{2},\frac{z}{2})\psi(\frac{z}{2})|p'\rangle \\
&= \frac{1}{2p^z}\left[\tilde{\mathcal{H}}(x,\xi,t,p^z)\bar{u}(p'')\gamma^z\gamma^5 u(p') + \tilde{\mathcal{E}}(x,\xi,t,p^z)\bar{u}(p'')\frac{\gamma^5\Delta^z}{2m}u(p')\right], \\
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Generalized parton distributions

X. Ji et al., *One-loop matching for generalized parton distributions*, Phys. Rev. D92 (2015) 014039
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X. Ji et al., *One-loop matching for generalized parton distributions*, Phys. Rev. D92 (2015) 014039

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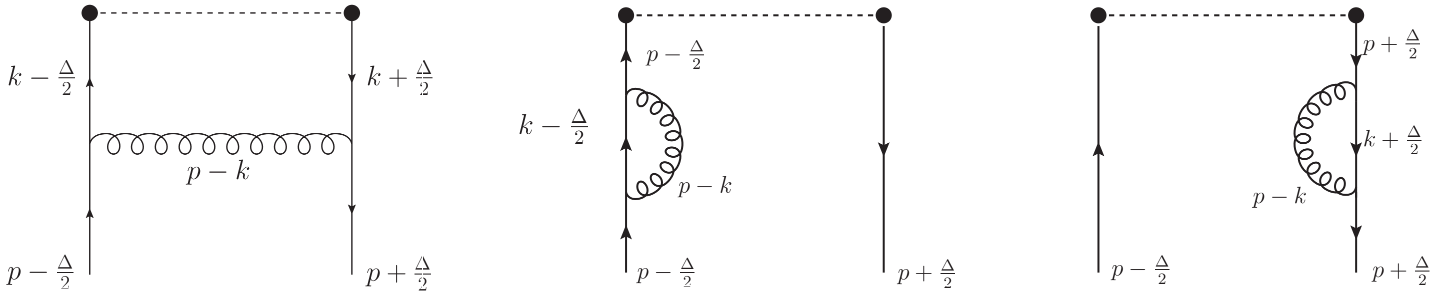

Matching of quasi-GPDs

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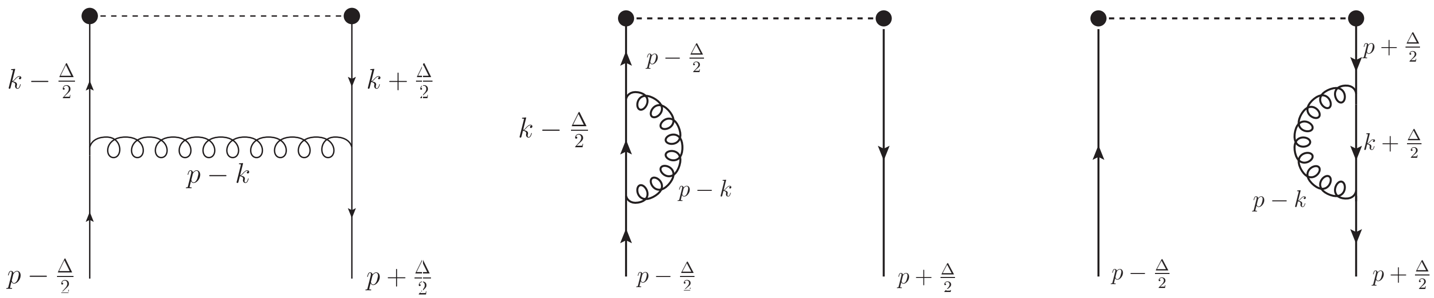

- Matching derived in transverse momentum cutoff scheme.

Matching of quasi-GPDs

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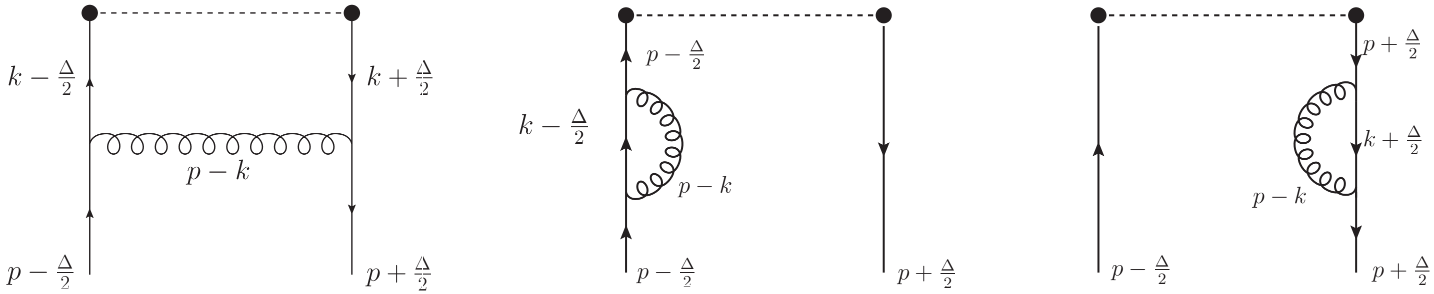

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- For $\xi{=}t{=}0$, the $\mathcal{H}(x,0,0)$, $\tilde{\mathcal{H}}(x,0,0)$ and $\mathcal{H}_T(x,0,0)$ quasi-functions become the standard quasi-PDFs.

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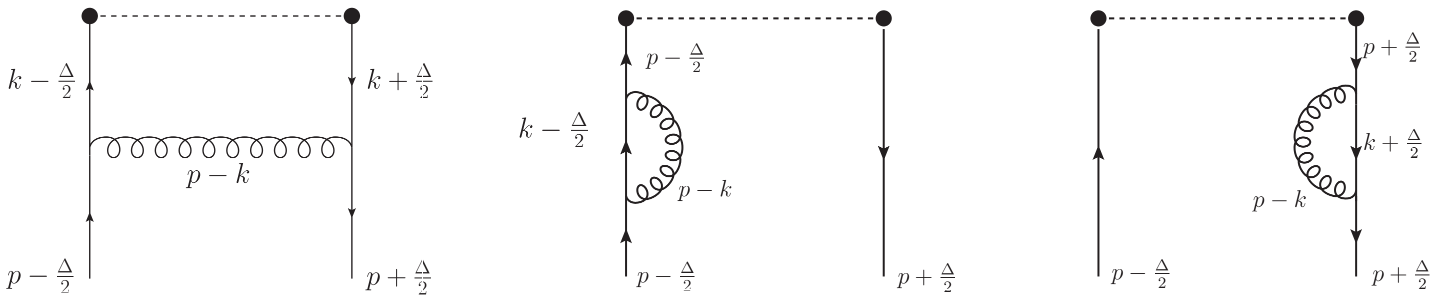

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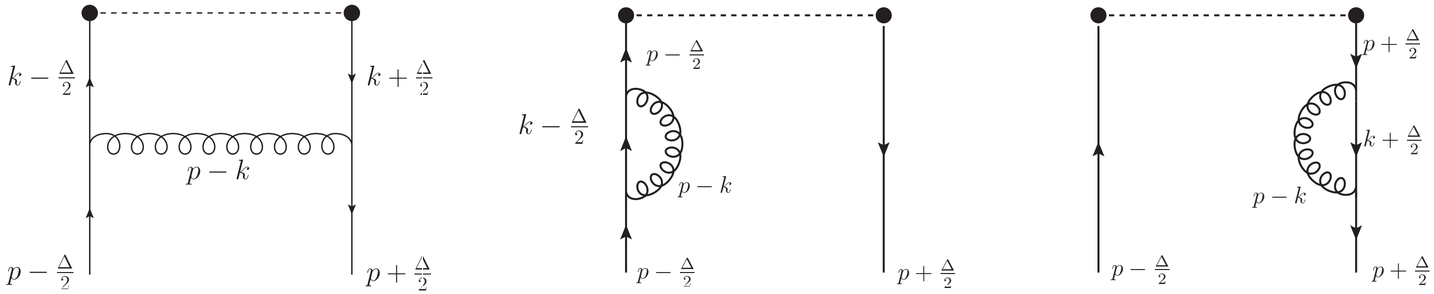

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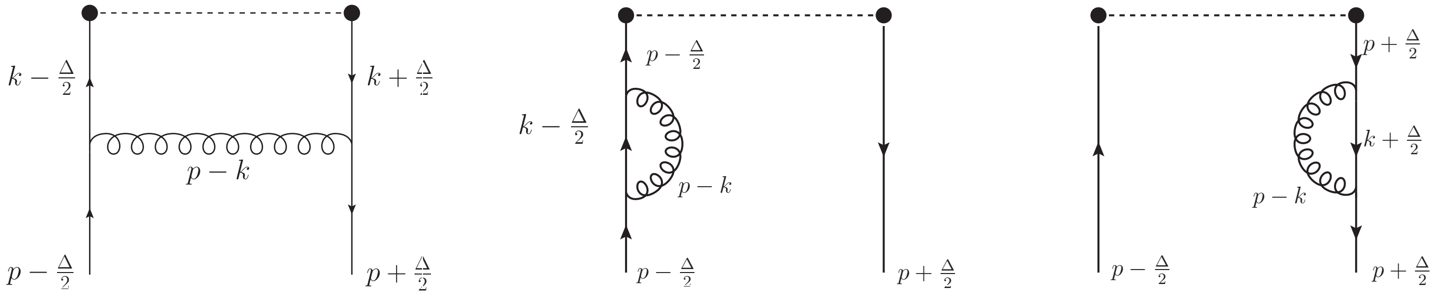

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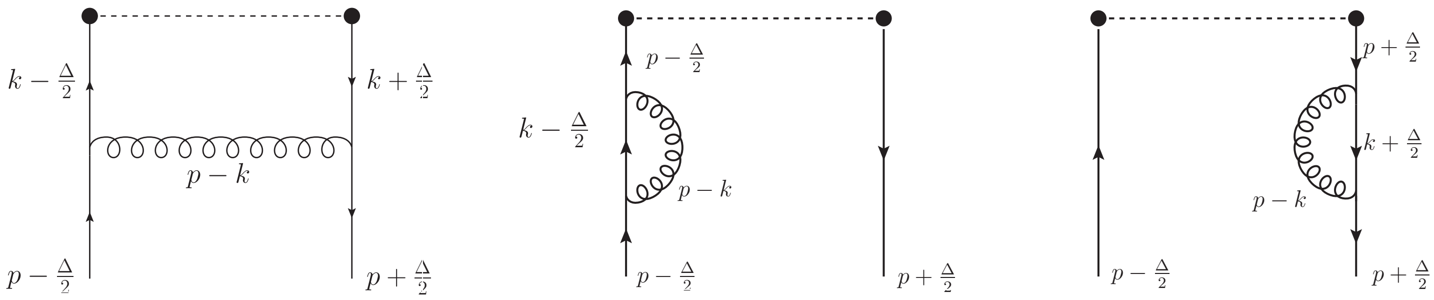

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- For transversity: x-integrals of H_T, $\tilde{H}_T$, E_T, $\tilde{E}_T$ give the generalized tensor form factors $G_T(t)$, $\tilde{A}_{T10}(t)$, $B_{T10}(t)$, $\tilde{B}_{T10}(t) = 0$.

Matching of quasi-GPDs

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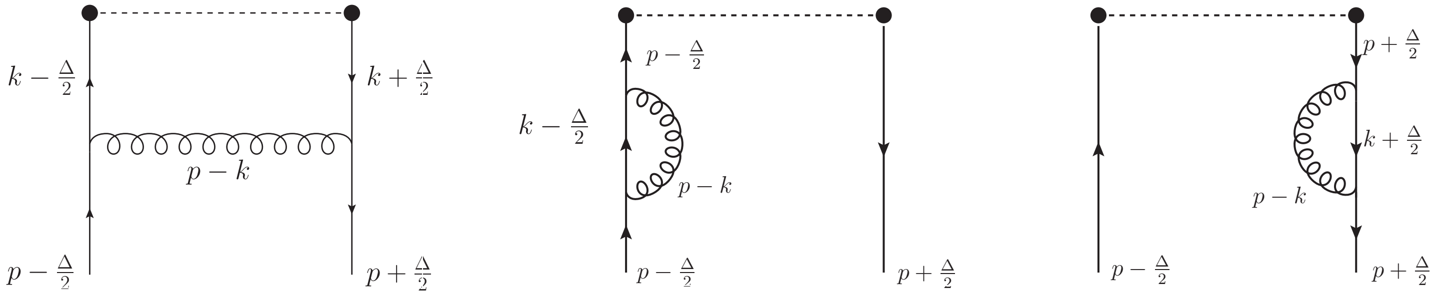

- For quasi-PDFs: 3 intervals in x – physical $-1 < x < 1$ and 2 non-physical $-1 > x$, $x > 1$.

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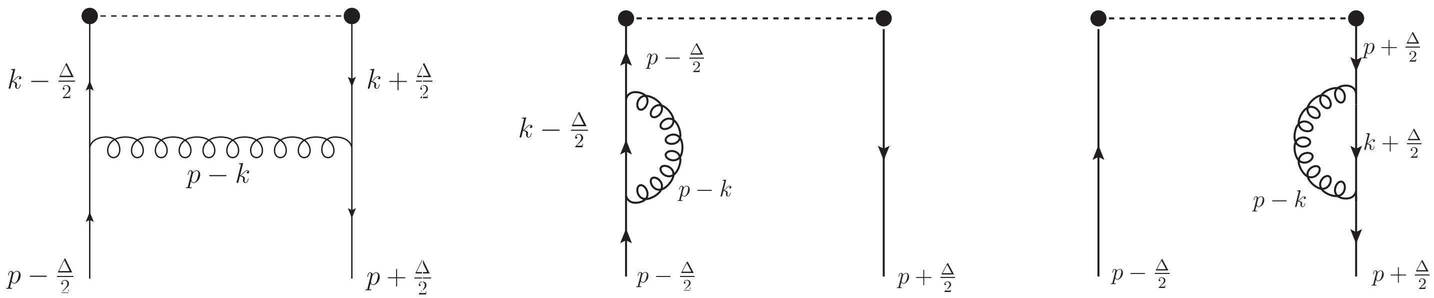

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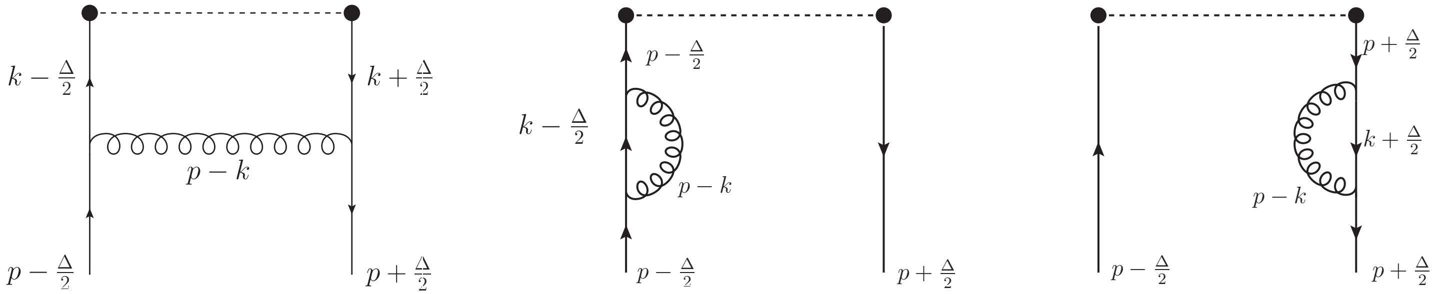

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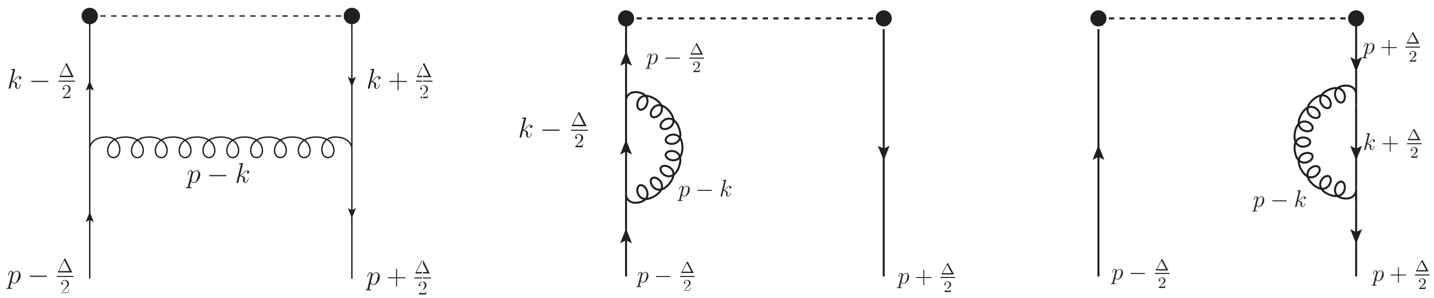

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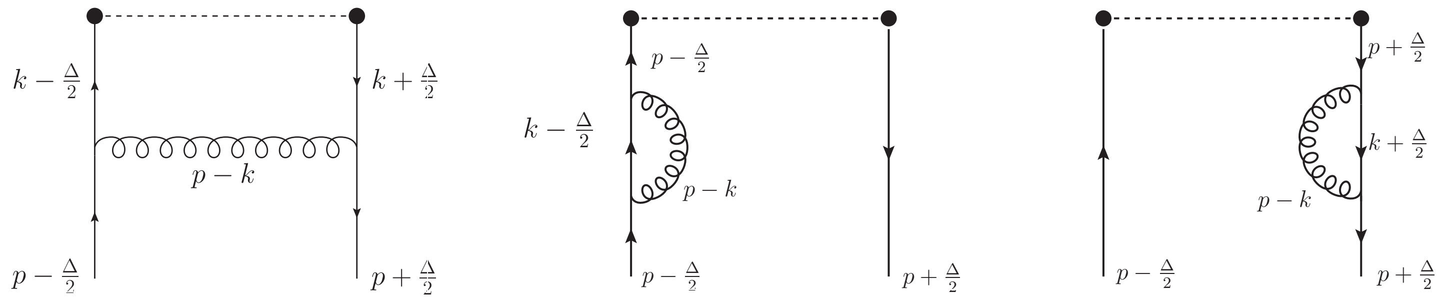

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- The fourth transversity quasi-GPD, $\tilde{\mathcal{H}}_T$, is power-suppressed by the hadron momentum and omitted at leading power accuracy.

Prospects for GPDs

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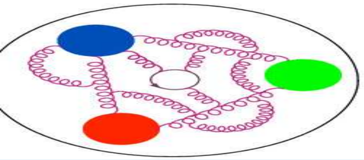
So: **stay tuned!**



Conclusions and prospects

C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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Thank you for your attention!