GPD myths and prospects

Dieter Müller

- Motivation
- Basics of generalized parton distributions (GPDs)
- Status of theory
- Challenges in phenomenology
- The future
- Conclusions

some past work in collaboration with

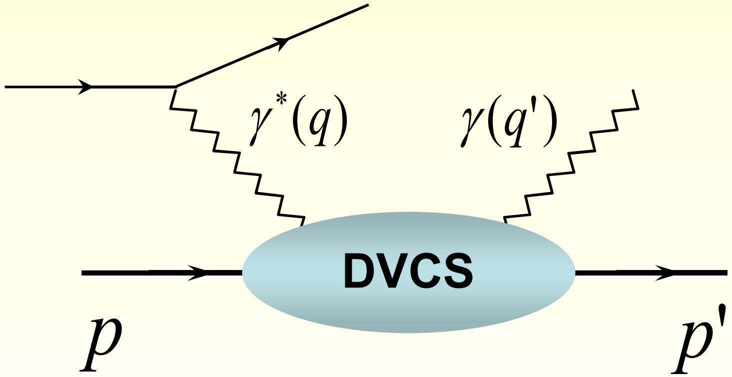
A. Belitsky, Y. Ji; V. Braun, A. Manashov, B. Pirnay;
M. Polyakov, K. Semenov-Tian-Shansky; O.Teryaev; D.S. Hwang;
K. Passek-Kumerički (KP-K), G. Duplancic; T. Lautenschlager, A.Schäfer;
K. Kumerički (KK), E. Aschenauer, S. Firzo, M. Murray; … … …

Motivation

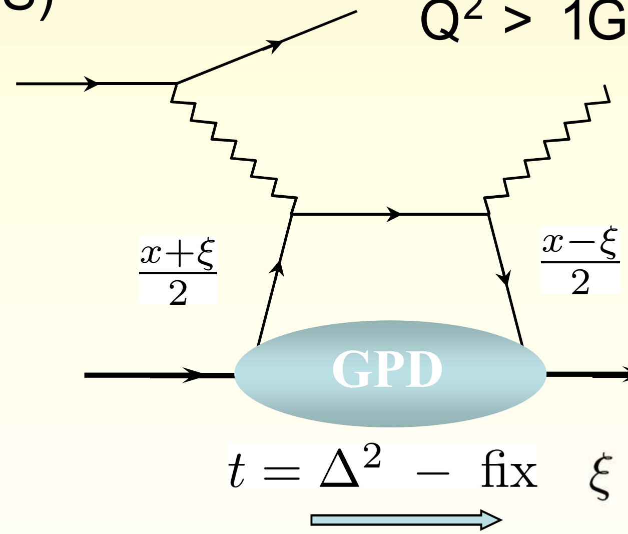
[DM, Robaschik, Geyer, Dittes, Hořejši (91/94)]

GPDs and their crossed counterparts generalized distribution amplitudes GDAs embed like parton distribution functions (PDFs) non-perturbative physics

e.g., deeply virtual Compton scattering (DVCS)



$Q^2 > 1\mathrm{GeV}^2$

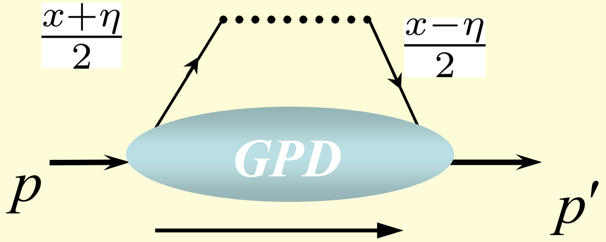


$t = \Delta^2$ – fix $\quad \xi \approx \frac{x_B}{2-x_B}$

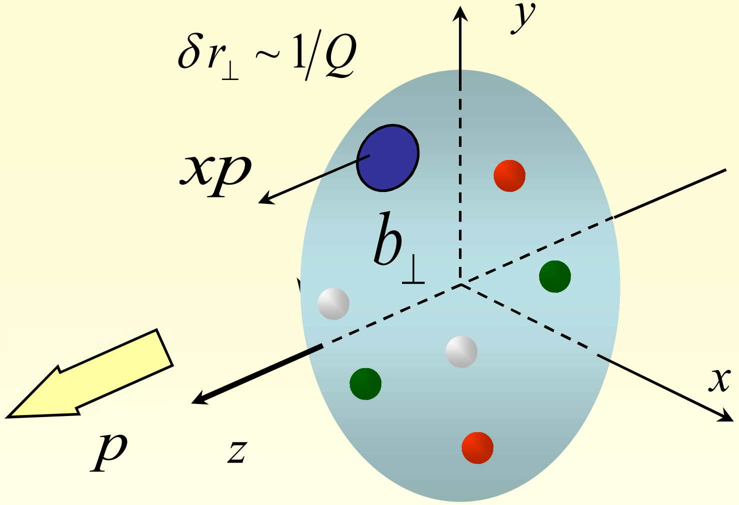
$$\mathcal{F}(x_B, \mathcal{Q}^2, t) = \int_{-1}^{1} dx\, C(x, \xi, \alpha_s(\mu), \mathcal{Q}/\mu) F(x, \xi, t, \mu) + O(\frac{1}{\mathcal{Q}^2})$$

CFF	**hard scattering part**	**GPD**	**higher twist**
Compton form factor observable	perturbation theory (our conventions/microscope)	universal (conventional)	depends on approximation

Partonic interpretation of GPDs



$$\Delta_\perp = p'_\perp - p_\perp$$



GPDs simultaneously carry information on longitudinal and transverse distribution of partons in a proton

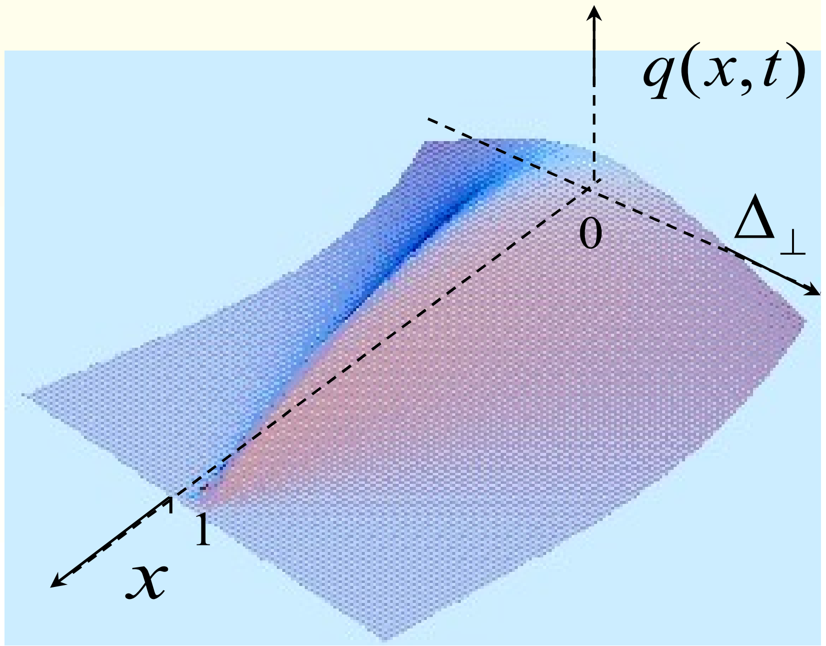
for η=0 they have a probabilistic interpretation (infinite momentum frame) **[Burkhardt (00)]**

$$\langle b_\perp^2 \rangle(x) = 4\frac{d}{dt}\ln H(x,0,t)\Big|_{t=0}$$

GPDs contain also information on partonic angular momentum **[X. Ji (96)]**

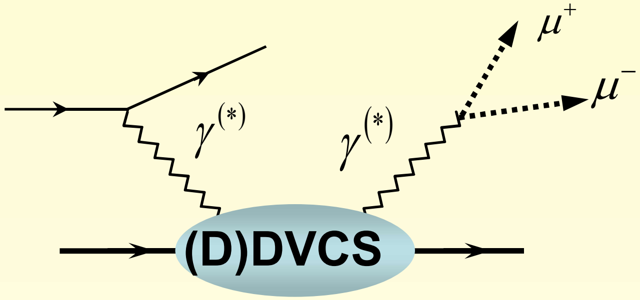
$$\frac{1}{2} = \sum_{a=q,G} J_a^z$$

$$J_a^z = \lim_{t\to 0}\frac{1}{2}\int_{-1}^{1} dx\, x\,(H_a + E_a)(x,\eta,t)$$



Other hard exclusive channels

- deeply virtual Compton scattering processes & crossed process



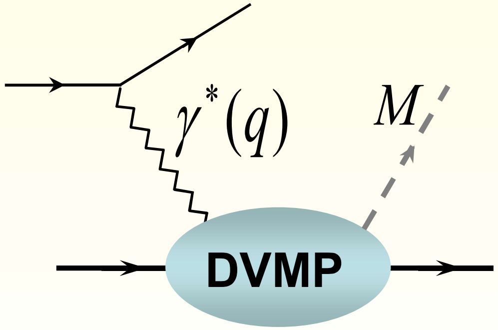
$$ep \to ep\gamma$$
$$en \to en\gamma$$
$$eA \to eA\gamma$$
$$\gamma p \to p\mu^+\mu^-$$

[DM et. al (91/94)
Radyushkin (96)
Ji (96)
Diehl, Pire, Teryaev (99)]

$$e^-e^+ \to e^-e^+h^+h^- \quad h \in \{\rho, \cdots\}$$

- deeply virtual meson production (DVMP) & crossed processes (exclusive Drell-Yan)

[Collins, Frankfurt, Strikman (96)]



$$ep \to epM \qquad M \quad \in \{\pi, \rho, \phi, \omega, J/\psi, \Upsilon \cdots\}$$
$$ep \to enM^+ \quad M^+ \in \{\pi^+, \rho^+, \cdots\}$$
$$pp \to ppJ/\psi$$
$$pA \to pAJ/\psi$$

- large data set arose from systematic measurements (started at ~ 2000) at DESY (H1, ZEUS, HERMES), JLAB (CLAS, HALL A, HALL C), CERN (COMPASS, ALICE)
- planned dedicated GPD programs at COMPASS II and JLAB@12GeV
- essential part of physics program for proposed machines

Field theoretical GPD definition

- GPDs (GDAs) are defined as non-perturbative matrix elements of **renormalized** **light-ray** operators:

[DM, Robaschik, Geyer, Dittes, Hoŕejśi (91/94)]

$$\boxed{F(x,\eta,t,\mu^2) = \int_{-\infty}^{\infty} \frac{d\kappa}{2\pi}\, e^{i\kappa\, x\, n\cdot P} \langle P_2 | \mathcal{R} : \phi(-\kappa n)[(-\kappa n),(\kappa n)]\phi(\kappa n) : | P_1 \rangle,\ n^2 = 0}$$

momentum fraction *x* , skewness $\eta = \frac{n\cdot\Delta}{n\cdot P}$ $\Delta = P_2 - P_1$ $P = P_1 + P_2$ $\Delta^2 \equiv t$

- their evolution is perturbatively calculable (evolution equations, like for PDFs)

$$\boxed{\mu^2 \frac{d}{d\mu^2} F(x,\eta,t,\mu^2) = \int_{-1}^{1} \frac{dy}{2\eta} V\left(\frac{x}{\eta}, \frac{y}{\eta} \middle| \alpha_s(\mu^2)\right) F(y,\eta,t,\mu^2)}$$

- for a nucleon target we have four chiral even twist-two GPDs:

$$\bar\psi_i \gamma_+ \psi_i \quad \Rightarrow \quad {}^{i}q^{V} = \bar U(P_2,S_2)\gamma_+ U(P_1,S_1) H_i + \bar U(P_2,S_2)\frac{i\sigma_{+\nu}\Delta^\nu}{2M} U(P_1,S_1) E_i$$

$$\bar\psi_i \gamma_+ \gamma_5 \psi_i \quad \Rightarrow \quad {}^{i}q^{A} = \bar U(P_2,S_2)\gamma_+ \gamma_5 U(P_1,S_1) \widetilde H_i + \bar U(P_2,S_2)\frac{\gamma_5}{2M} U(P_1,S_1) \widetilde E_i$$

shorthands (Ji`s convention):

$$F = \{H, E, \widetilde H, \widetilde E\}$$ *& CFFs/TFFs:* $$\mathcal{F} = \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$$

GPD properties (from definition)

- polynomiality arises from Poincaré covariance

$$\int_{-1}^{1} dx\, x^n F(x,\eta,t) = \text{polynom of order } n \text{ or } n+1 \text{ in } \eta$$

- satisfied within double distribution representation **[DM et. al (91/94) Radyushkin (96)]**

  (*Radon transform*, inverse transform was rederived once more in 2000)

$$F(x,\eta,t) = (1-x)^p \int_0^1 dy \int_{-1+y}^{1-y} dz\, \delta(x-y-z\eta) f(y,z,t),\ p=\{0,1\}$$

- lowest moment: partonic form factor – related to observables
- first moment: expectation value of energy-momentum tensor **[Ji (96)]**
- generalized form factors are measured in lattice simulations
- reduction to parton distribution functions (PDFs)

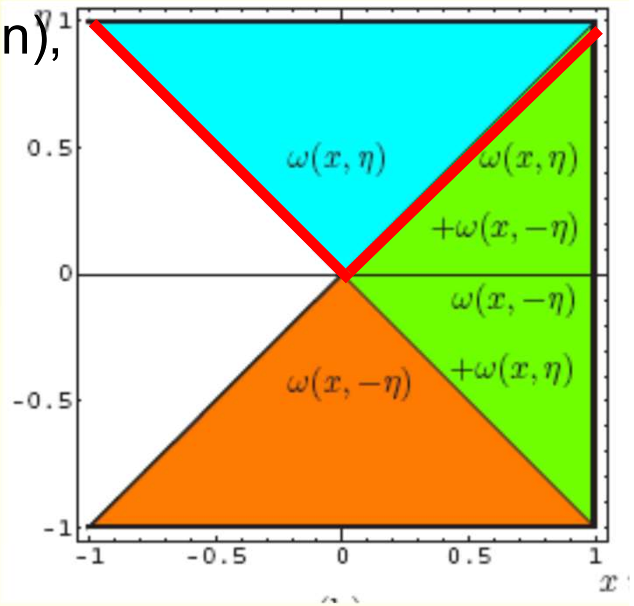
$$q(x) = \lim_{\Delta\to 0} H(x,\eta,t), \quad \Delta q(x) = \lim_{\Delta\to 0} \widetilde{H}(x,\eta,t)$$

- positivity constraints are automatically satisfied in the LFWF overlap representation

[Pobylitsa (00,02)]

A partonic duality interpretation

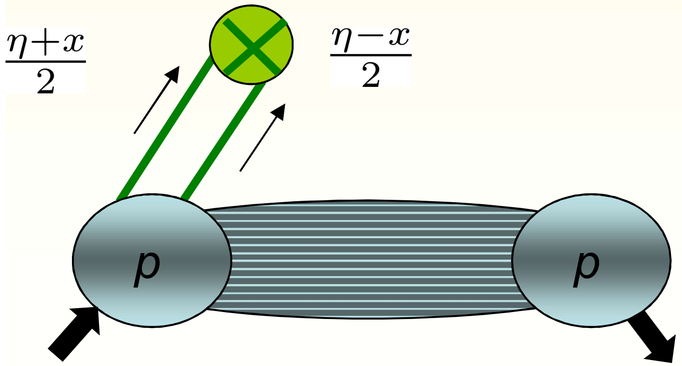
GPD reads explicitly (double distribution representation), e.g. for quark GPD (anti-quark *x* → -*x*):



$$F(x,\eta,t) = \theta(-\eta \le x \le 1)\,\omega(x,\eta,t) + \theta(\eta \le x \le 1)\,\omega(x,-\eta,t)$$

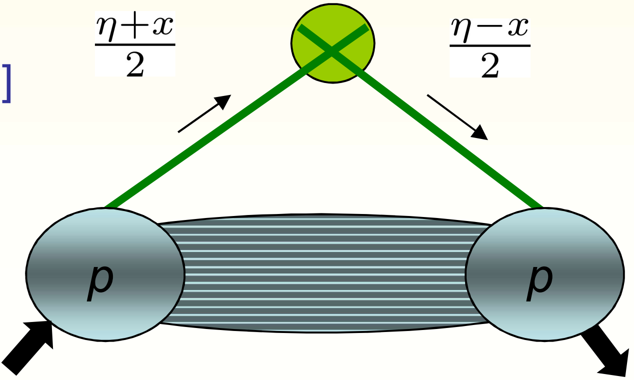
$$\omega\,(x,\eta,t) = \frac{(1-x)^p}{\eta}\int_0^{\frac{x+\eta}{1+\eta}} dy\, f\,(y,(x-y)/\eta,t)$$

dual interpretation on partonic level:



support extension is unique [DM et al. 91/94]

ambiguous (*D*-term)
[DM, A. Schäfer (05);
KMP-K (07);
Hwang DM (07)
DM (17)]



central region - η < x < η

mesonic exchange in *t*-channel

outer region η < x

partonic exchange in *s*-channel

- Observables are given in terms of CFFs and TFFs

$$\Im\mathrm{m}\,\mathcal{F}^{\mathrm{A}}(x_{\mathrm{B}},t,\mathcal{Q}^2) \overset{\mathrm{Tw-2}}{=} \int_{\xi}^{1}\frac{dx}{x}\,{}^{\sigma}t\left(\frac{\xi}{x}\middle|\alpha_s(\mu),\frac{\mathcal{Q}^2}{\mu^2}\right)F^{\mathrm{A}}(x,\xi,t,\mu^2)\,.$$

$$\Im\mathrm{m}\,\mathcal{F}^{\mathrm{A}}_{\mathrm{M}}(x_{\mathrm{B}},t,\mathcal{Q}^2) \overset{\mathrm{Tw-2}}{=} \frac{\pi\,C_F f_{\mathrm{M}}}{N_c\mathcal{Q}}\int_{\xi}^{1}\frac{dx}{x}\,\varphi_{\mathrm{M}}(v,\mu^2)\overset{v}{\otimes}{}^{\sigma}t\left(\frac{\xi}{x},v\middle|\alpha_s(\mu),\frac{\mathcal{Q}^2}{\mu^2}\right)F^{\mathrm{A}}(x,\xi,t,\mu^2)\,.$$

$$\Im\mathrm{m}\,\mathcal{F}^{\mathrm{A}}_{(\mathrm{M})}(x_{\mathrm{B}},t,\mathcal{Q}^2) \propto F^{\mathrm{A}}(\xi,\xi,t,\mu^2)+O(\alpha_s)$$

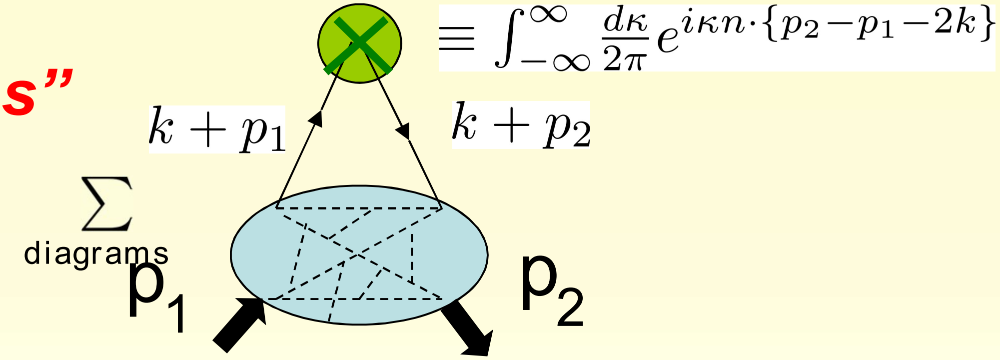
$$\Re\mathrm{e}\,\mathcal{F}(x_{\mathrm{B}},t,\mathcal{Q}^2) \overset{\mathrm{Tw-2}}{=} \mathcal{P}\int_{0}^{1}\frac{dx}{\xi^2-x^2}\begin{Bmatrix}2x\\2\xi\end{Bmatrix}\frac{1}{\pi}\Im\mathrm{m}\,\mathcal{F}\left(\frac{2x}{1+x},t,\mathcal{Q}^2\right)+\mathcal{C}_{\mathcal{F}}(t,\mathcal{Q}^2)\ \text{ for }\ \sigma(\mathcal{F})=\begin{cases}+1\\-1\end{cases},$$

GPD myths

- measurement of quark angular momentum
- GPD tomography (probabilistic interpretation)
- measurement of pressure inside the proton

only a model dependent access is possible

GPD representations



``light-ray spectral functions”

diagrammatic α-representation

DM, Robaschik, Geyer,
Dittes, Hořejśi (88 91/94)

called *double distributions*

A. Radyushkin (96)

light cone wave function overlap

(Hamiltonian approach in light-cone quantization)

Diehl, Feldmann,
Jakob, Kroll (98,00)

Diehl, Brodsky,
Hwang (00)

SL(2,R) (conformal) expansion

(resummed series of local operators, rather similar to Mellin transform of PDFs)

Radyushkin (97);
Belitsky, Geyer, DM, Schäfer (97);
Shuvaev (99,02); Noritzsch (00)
Polyakov (02,07)
DM, Schäfer (05); Kirch et. al (05)

➡ each representation has its own ***advantages***, however, they are ***equivalent*** **(clearly spelled out in [Hwang, DM07])**

SL(2,R) representations for GPDs

- support is a consequence of Poincaré invariance (polynomiality)

$$F_n(\eta, t, \mu^2) = \int_{-1}^{1} dx\, \eta^n C_n^{\frac{3}{2}}\left(\frac{x}{\eta}\right) F(x, \eta, t, \mu^2)$$

- conformal moments evolve autonomous (LO and beyond in a special scheme)

$$\mu \frac{d}{d\mu} F_n(\eta, t, \mu^2) = -\frac{\alpha_s(\mu)}{2\pi} \gamma_n^{(0)} F_n(\eta, t, \mu^2) \;\Rightarrow\; F_n(\eta, t, \mathcal{Q}^2) = E_n(\mathcal{Q}^2, \mathcal{Q}_0^2) F_n(\eta, t, \mathcal{Q}_0^2)$$

- GPDs are now given as a series of generalized functions:

$$F(x, \eta, t) = \sum_{n=0}^{\infty} (-1)^n p_n(x, \eta) F_n(\eta, t),\; p_n(x, \eta) \propto \theta(|x| \leq \eta) \frac{\eta^2 - x^2}{\eta^{n+3}} C_n^{\frac{3}{2}}\left(\frac{-x}{\eta}\right)$$

- ***Mellin-Barnes integral*** based on Sommerfeld-Watson transform **[DM, Schäfer (05)]**

$$F(x, \eta, t, \mathcal{Q}^2) = \frac{i}{2} \int_{c-i\infty}^{c+i\infty} dj\, \frac{p_j(x, \eta)}{\sin(\pi j)} F_j(\eta, t, \mathcal{Q}^2)$$

- this technique is utilized in the ***existing*** GPD fitting procedure

$$\mathcal{F}(\xi, t, \mathcal{Q}^2) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj\, \frac{2^{j+1}\Gamma(5/2+j)}{\xi^{j+1}\Gamma(3/2)\Gamma(3+j)} \left[i - \frac{\cos(\pi j) \mp 1}{\sin(\pi j)}\right] F_j(\xi, t, \mathcal{Q}^2)$$

Advantages of the Mellin-Barnes integral

✓ another possibility to parameterize GPDs [analog to Shuvaev`s suggestion] (basic properties are implemented, essential for flexible fitting routines)

✓ (LO) solution of the evolution equation is trivial implemented

$$F(x,\eta,t,\mathcal{Q}^2) = \frac{i}{2}\int_{c-i\infty}^{c+i\infty} dj\, \frac{p_j(x,\eta)}{\sin(\pi j)} \exp\left\{-\frac{\gamma_j^{(0)}}{2}\int_{\mathcal{Q}_0^2}^{\mathcal{Q}^2}\frac{d\sigma}{\sigma}\frac{\alpha_s(\sigma)}{2\pi}\right\} F_j(\eta,t,\mathcal{Q}_0^2)$$

✓ fast and robust numerical evaluation

✓ simple representation of amplitudes

$$\mathcal{F}(x_B,t,\mathcal{Q}^2) = \int_{-1}^{1} dx \left[\frac{1}{\xi - x - i\epsilon} \mp \frac{1}{\xi + x - i\epsilon}\right] F(x,\xi,t,\mathcal{Q}^2)$$

$$\mathcal{F} = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\, \xi^{-j-1}\frac{2^{j+1}\Gamma(5/2+j)}{\Gamma(3/2)\Gamma(3+j)}\left(i - \frac{\cos(\pi j)\mp 1}{\sin(\pi j)}\right) F_j(\xi,t,\mathcal{Q}^2)$$

✓ MS factorization conventions can be implemented at NLO

✓ CS factorization conventions enable us to explore NNLO corrections

What is `dual' parameterization ?

- *t*-channel scattering angle and skewness parameter are related: $\cos\theta \approx -1/\eta$
- labeling the conformal moments by the *t*-channel ***angular momentum*** *J* (conjugated variable to θ or in some sense to η)

$$F_j(\eta,t) = \eta^{j+1}\sum_{J=J^{\min}}^{j+1} f_{j,J}(t)\, d_J(1/\eta)$$

[Polyakov (99) Ji, Lebed (00) Diehl (03), KMP-K (07),…]

partial wave amplitudes depending on j and J

reduced Wigner rotation matrices

- ➢ primary `quantum numbers' are *j+2* and the difference *2v= j+1-J*
- ➢ in ``dual'' parameterization *j+2* is replaced by conjugate momentum fraction *y*

$$F(x,\eta,t)=\sum_{\nu=0}^{\infty}\int_0^1 dy\, K_{2\nu}(x,\eta|y)\, y^{2\nu} Q_{2\nu}(y,t)$$

[Polyakov, Shuvaev (02)]

- GPD model building in terms of $f_{j,j+1-2v}(t)$ or $Q_v(y,t)$ (one-to-one to DDs)

`dual' parameterization ***[Guzey, Teckentrup (06)]*** effectively took *v=0* ***[Polyakov (07)]***

- deeper insights ***[Polyakov, Semenov-Tian-Shansky (07,08) and DM (15)]***

A flexible GPD model

- take three effective SO(3) partial waves

$$\begin{aligned} F_j(\eta,t) &= \hat{d}_j(\eta) f_j^{j+1}(t) + \eta^2 \hat{d}_{j-2}(\eta) f_j^{j-1}(t) + \eta^4 \hat{d}_{j-4}(\eta) f_j^{j-3}(t), \quad j \geq 4 \\ f_j^{j-2\nu}(\eta,t) &= s_\nu f_j^{j+1}(\eta,t)\,, \quad \nu = 1,2,\cdots \end{aligned}$$

- rewrite Mellin-Barnes integral

$$\begin{aligned} \mathcal{F} \;=\; & \frac{1}{2i} \sum_{\nu=0}^{2} \int_{c-i\infty}^{c+i\infty} dj\, \xi^{-j-1} \frac{2^{j+1+2\nu}\Gamma(5/2+j+2\nu)}{\Gamma(3/2)\Gamma(3+j+2\nu)} \left(i - \frac{\cos(\pi j) \mp 1}{\sin(\pi j)} \right) \\ & \times s_\nu E_{j+2\nu}(\mathcal{Q}^2) f_j^{j+1}(t) \hat{d}^{j+1}(\xi) + \text{two `subtraction terms'} \quad s_0 = 1 \end{aligned}$$

NOTE:

- first partial wave amplitude is fixed by PDFs (if they exist) and FFs
- “Regge poles” should be in the angular momentum *J*-plane (not in the *j*-plane)

$$H(x,x,t=0,\mathcal{Q}^2) \overset{x\to 0}{=} \sum_{\nu=0}^{2} s_\nu \frac{2^{\alpha+2\nu}\Gamma(3/2+\alpha+2\nu)}{\Gamma(3/2)\Gamma(2+\alpha+2\nu)} q(x,\mathcal{Q}^2)$$

- a *J*-pole is associated with a series of spurious poles in the *j*-plane

LFWF overlap representation

Diehl et al. (00)

parton diagonal overlap representations for outer region $(x \geq \eta)$

$$H(x,\eta,t)+\frac{t_{\min}}{4M^2}E(x,\eta,t)=\sum_{n,s_1}\iint d^2\mathbf{k}_\perp \sum\!\!\!\!\!\!\int_{(n-1)} \psi^{*\Rightarrow}_{(n)}(X'_i,\mathbf{k}'_{\perp i},s_i)\,\psi^{\Rightarrow}_{(n)}(X_i,\mathbf{k}_{\perp i},s_i)$$

$$\frac{|\bar{\boldsymbol{\Delta}}_\perp|\,e^{-i\varphi}}{2M}E(x,\eta,t)=\sum_{n,s_1}\iint d^2\mathbf{k}_\perp \sum\!\!\!\!\!\!\int_{(n-1)} \psi^{*\Rightarrow}_{(n)}(X'_i,\mathbf{k}'_{\perp i},s_i)\,\psi^{\Leftarrow}_{(n)}(X_i,\mathbf{k}_{\perp i},s_i)$$

where

$$X'_1=\frac{x-\eta}{1-\eta}\,,\qquad \mathbf{k}'_{\perp 1}=\mathbf{k}_{\perp 1}-\frac{1-x}{1-\eta}\boldsymbol{\Delta}_\perp\qquad \text{for the struck parton,}$$

$$X'_i=\frac{1+\eta}{1-\eta}X_i\,,\qquad \mathbf{k}'_{\perp i}=\mathbf{k}_{\perp i}+\frac{1+\eta}{1-\eta}X_i\,\boldsymbol{\Delta}_\perp\qquad \text{for the spectator } i\in\{2,\cdots,n\}\,,$$

How to restore the full GPD (or DD)? ***new result*** **[DM (17)]**

$$h(y,z)=\sum_n\frac{1}{2}\int_0^\infty d\lambda\,\lambda\,e^{-\lambda}\varphi_n(y^{\mathrm{i}}\lambda)\varphi_n^*(y^{\mathrm{f}}\lambda)$$

$$\varphi_n(\lambda)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}dr\,e^{\lambda r}\phi_n\!\left(\frac{r-1}{r}\right)$$

$$h(y,z)=\sum_n\frac{-1}{2(2\pi)^2}\int_{-\infty}^{\infty}d\tau^{\mathrm{i}}\int_{-\infty}^{\infty}d\tau^{\mathrm{f}}\,\frac{\phi_n(\frac{\tau^{\mathrm{i}}}{\tau^{\mathrm{i}}+i})\phi_n^*(\frac{\tau^{\mathrm{f}}}{\tau^{\mathrm{f}}+i})}{(y^{\mathrm{i}}\tau^{\mathrm{i}}+y^{\mathrm{f}}\tau^{\mathrm{f}}-iy)^2}$$

$$H(x,\eta)=\frac{1}{1-x}\sum_n\frac{1}{2\pi i}\int_{-\infty}^{\infty}d\tau\,\frac{\phi_n(\frac{\tau}{\tau+i})\phi_n^*(\frac{(\eta-x)\tau}{(\eta-x)\tau-i(x+\eta)})}{\tau-i\frac{x+\eta}{1-x}}$$

Status of theory

[Belitsky, DM (97);
Mankiewicz et. al (97);
Ji,Osborne (97/98);
Pire, Szymanowski, Wagner (11)

✓ ***twist-two*** DVCS coefficients at ***next-to-leading*** order

✓ twist-two DVMP coefficients at ***next-to-leading*** order

time-like$^{\text{new}}$ **DM, Pire, Szymanowski, Wagner (11)]**

[Belitsky, DM (01);
Ivanov, Szymanowski,Krasnikov (04)]

new

NLO effects are well understood generically
large-ξ: logarithmical enhancement
valence region: weak evolution implies moderate effects
small-ξ: model dependence

DM, T. Lautschlager,
K. Passek-Kumericki.
A. Schaefer (13);
G. Duplancic, DM,
K. Passek-Kumericki (16),

✓ anomalous dimensions & evolution kernels at ***next-to-leading*** order

evolution effects can be called moderate, except for H/E at small- ξ
NLO analyses have to include NLO evolution

[Belitsky, DM (98)
+ Freund (01)]

✓ DVCS gluon transversity at ***next-to-leading*** order **[Belitsky, DM (00)]**

✓ ***next-to-next-to-leading*** order for DVCS in a specific subtraction scheme

[DM (06);
KMP-K,
Schaefer (06)]

NLO → NNLO corrections can be called moderate w.r.t. LO → NLO

✓ ***twist-three*** including quark-gluon-quark correlation at LO

[Anikin,Teryaev, Pire (00);
Polyakov et. al (00),
Belitsky, DM (00); Kivel et. al,
Weiss, Radyushkin (00)]

✓ partially, ***twist-three*** sector at ***next-to-leading*** order

[Kivel, Mankiewicz (03)]

? \`target mass corrections' (not understood to that time) **[Belitsky, DM (01)]**

new

✓ **kinematical twist-four** corrections at LO for DVCS **[Braun, Manashov (11)]**

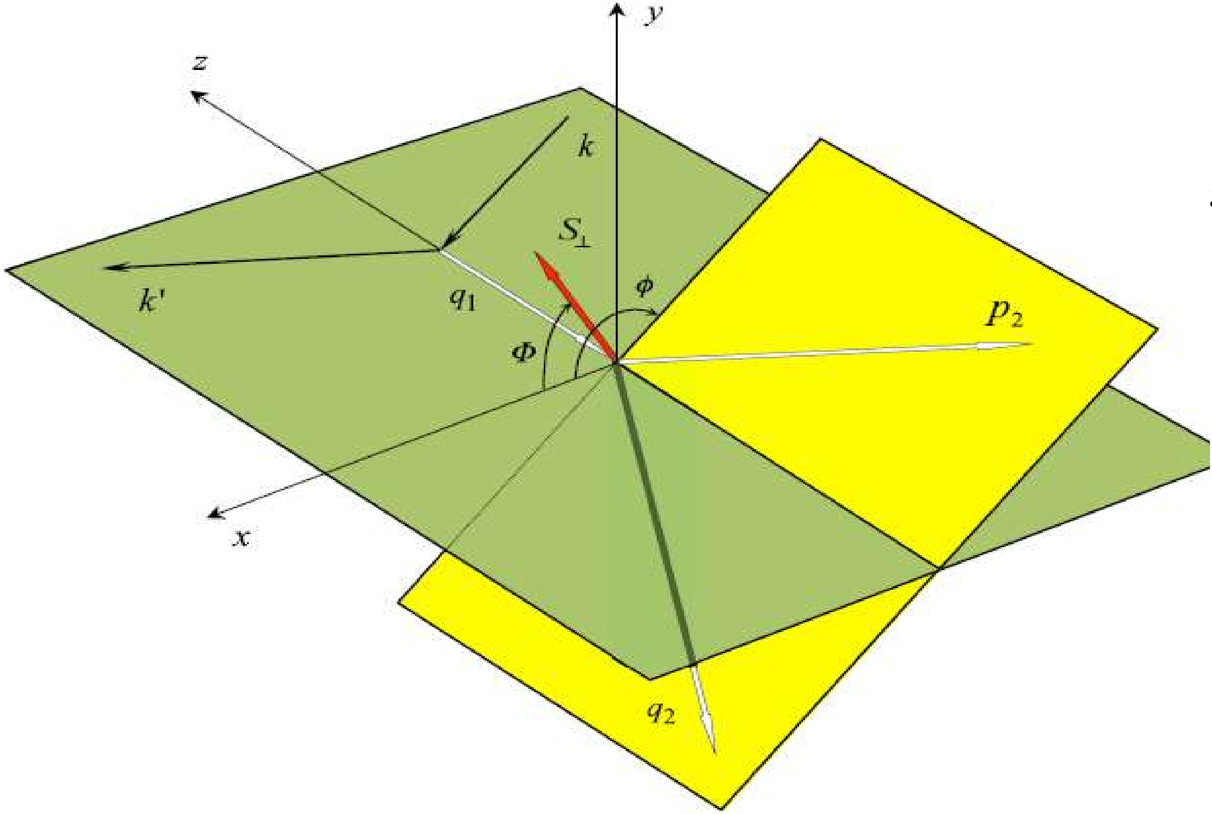
leptoproduction of photons

$$e^{\pm}(k)N(p_1) \to e^{\pm}(k')N(p_2)\gamma(q_2)$$

measured by ***H1, ZEUS, HERMES, CLAS, HALL A*** collaborations,

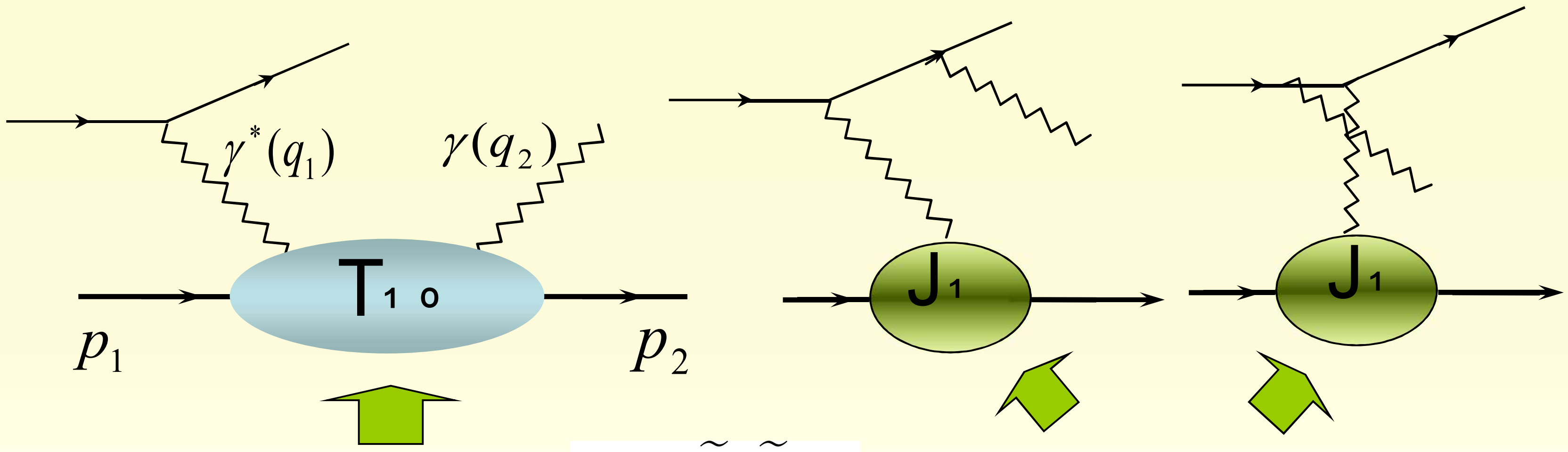
planed at ***COMPASS, JLAB@12GeV***

$$\frac{d\sigma}{dx_{\rm Bj}dydtd\phi d\varphi} = \frac{\alpha^3 x_{\rm Bj} y}{16\,\pi^2\,\mathcal{Q}^2}\left(1+\frac{4M^2x_{Bj}^2}{\mathcal{Q}^2}\right)^{-1/2}\left|\frac{\mathcal{T}}{e^3}\right|^2$$



$$\begin{aligned} x_{\rm Bj} &= \frac{\mathcal{Q}^2}{2p_1\cdot q_1} \approx \frac{2\xi}{1+\xi}, \\ y &= \frac{p_1\cdot q_1}{p_1\cdot k}, \\ t &\equiv \Delta^2 \ (\text{fixed},\ \text{small}), \\ \mathcal{Q}^2 &= -q_1^2 \ \ (> 1\text{GeV}^2), \end{aligned}$$

interference of *DVCS* and *Bethe-Heitler* processes



12 Compton form factors $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}, \cdots$
(helicity amplitudes)

elastic form factors F_1, F_2

$$|\mathcal{T}_{\rm BH}|^2 = \frac{e^6(1+\epsilon^2)^{-2}}{x_{\rm Bj}^2 y^2 t\, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\rm BH} + \sum_{n=1}^{2} c_n^{\rm BH} \cos(n\phi) \right\},$$

exactly known
(LO, QED)

$$|\mathcal{T}_{\rm DVCS}|^2 = \frac{e^6}{y^2 \mathcal{Q}^2} \left\{ c_0^{\rm DVCS} + \sum_{n=1}^{2} \left[c_n^{\rm DVCS} \cos(n\phi) + s_n^{\rm DVCS} \sin(n\phi) \right] \right\},$$

harmonics
⇕ **1:1**
helicity ampl.

$$\mathcal{I} = \frac{\pm e^6}{x_{\rm Bj} y^3 t \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\}.$$

harmonics
⇕ **1:1**
helicity a~~mpl~~.

all harmonics are given by twist-2 and -3 GPDs:

**[Diehl et. al (97)
Belitsky, DM, Kirchner (01)]**

$$\begin{Bmatrix} c_1 \\ s_1 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta}{\mathcal{Q}}\ \text{tw-2(GPDs)} + O(1/\mathcal{Q}^3), \qquad c_0^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2}\ \text{tw-2(GPDs)} + O(1/\mathcal{Q}^4),$$

$$\begin{Bmatrix} c_2 \\ s_2 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2}\ \text{tw-3(GPDs)} + O(1/\mathcal{Q}^4), \qquad \begin{Bmatrix} c_3 \\ s_3 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta\alpha_s}{\mathcal{Q}} (\text{tw-2})^{\text{T}} + O(1/\mathcal{Q}^3),$$

$$c_0^{\text{CS}} \propto\ (\text{tw-2})^2, \qquad \begin{Bmatrix} c_1 \\ s_1 \end{Bmatrix}^{\text{CS}} \propto \frac{\Delta}{Q}\ (\text{tw-2})\ (\text{tw-3}), \qquad \begin{Bmatrix} c_2 \\ s_2 \end{Bmatrix}^{\text{CS}} \propto \alpha_s (\text{tw-2})(\text{tw-2})^{\text{GT}}$$

e.g., *n=1* odd harmonic is approximately given by \`CFF' combination

$$\begin{Bmatrix} c^{\mathcal{I}}_{1,\text{unp}} \\ s^{\mathcal{I}}_{1,\text{unp}} \end{Bmatrix} = 8K \begin{Bmatrix} -(2-2y+y^2) \\ \lambda y(2-y) \end{Bmatrix} \begin{Bmatrix} \Re\text{e} \\ \Im\text{m} \end{Bmatrix} \mathcal{C}^{\mathcal{I}}_{\text{unp}}(\mathcal{F})\ , \mathcal{C}^{\mathcal{I}}_{\text{unp}} = F_1\mathcal{H} + \frac{x_{\text{B}}}{2-x_{\text{B}}}(F_1+F_2)\widetilde{\mathcal{H}} - \frac{\Delta^2}{4M^2}F_2\mathcal{E}$$

relations among **harmonics** and (helicity dependent) **CFFs** are not more based on a 1/*Q* expansion:

**[Belitsky, DM (10) --
Belitsky, DM, Ji (12)]**

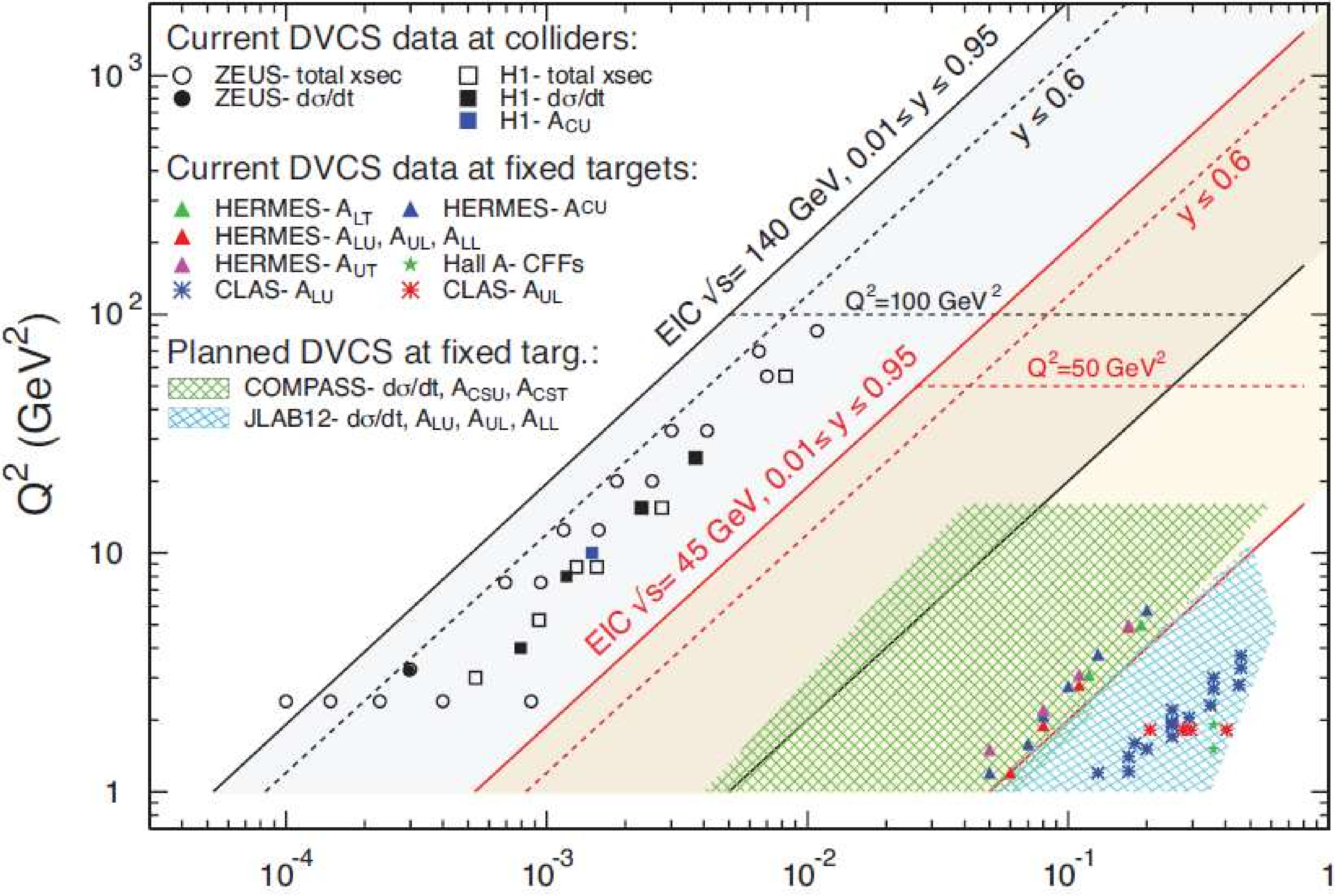
$$s^{\mathcal{I}}_{1,\text{unp}} = \frac{8\widetilde{K}\lambda\sqrt{1-y-\frac{y^2\gamma^2}{4}}(2-y)y}{Q(1+\gamma^2)} \Im\text{m}\Big\{ \mathcal{C}^{\mathcal{I}}_{\text{unp}}\Big(\Big[1 - \frac{\varkappa}{2Q^2}\frac{Q^2+t}{\sqrt{1+\gamma^2}}\Big]\mathcal{F}_{++} + \Big[1 - \frac{2+\varkappa}{2Q^2}\frac{Q^2+t}{\sqrt{1+\gamma^2}}\Big]\mathcal{F}_{-+} + \frac{(Q^2+t)\varkappa_0}{Q^2\sqrt{1+\gamma^2}}\mathcal{F}_{0+}\Big)$$

$$+ \frac{-t(Q^2+t)}{\sqrt{1+\gamma^2}Q^4}\Delta\mathcal{C}^{\mathcal{I}}_{\text{unp}}\left(\mathcal{F}_{-+} + \frac{\varkappa}{2}[\mathcal{F}_{++}+\mathcal{F}_{-+}] - \varkappa_0\mathcal{F}_{0+}\right)\Big\}, \tag{70}$$

new improved *C* coefficients ensure the cancellation of kinematical singularities

relations among CFFs and GPDs are always based on a *1/Q* expansion

DVCS world data set



Strategies to analyze DVCS data

GPD model approach:

(ad hoc) modeling: **VGG** code [**Goeke et. al (01)** based on **Radyuskin's** DDA]
BKM model [**Belitsky, Kirchner, DM (01)** based on RDDA]
`aligned jet' model [**Freund, McDermott, Strikman (02)**]
Kroll/Goloskokov (05, 07, 13) based on RDDA
`dual' model [**Polyakov,Shuvaev 02;Guzey,Teckentrup 06; Polyakov 07**]
" -- " [**KMP-K (07)** in MBs-representation]
polynomials **[Belitski et al. (98), Liuti et. al (07), Moutarde (09)]**

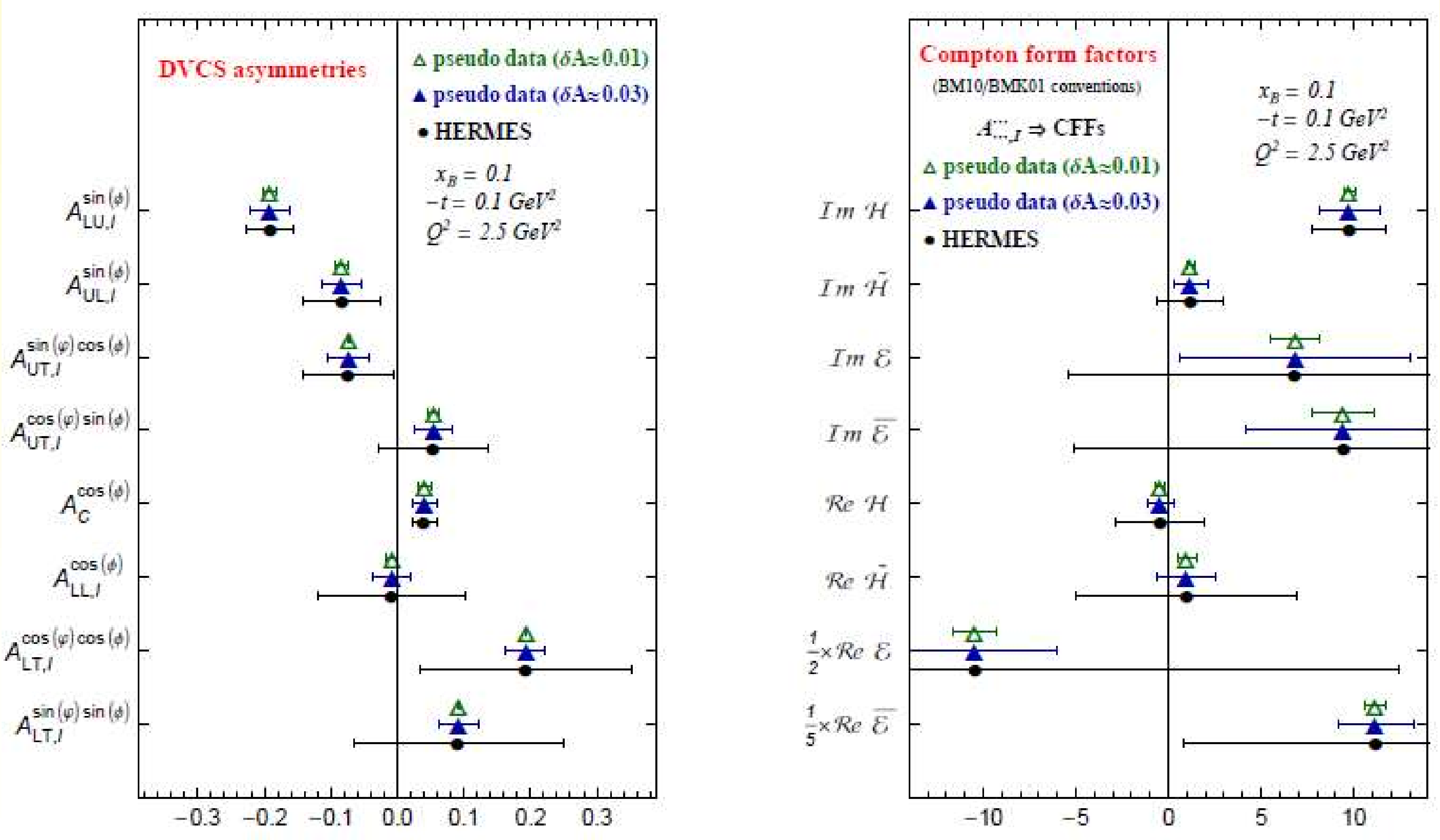
dynamical models: not used **[Radyuskin et.al (02); Tiburzi et.al (04); Hwang, DM (07,14)]…**

flexible models: can be set up in any representation
(for fits) **KMP-K (07/08)** for H1/ZEUS in MBs-representation

extracting CFFs (real and imaginary part)/GPDs from data:

i. CFF extraction with formulae **[BMK (01), HALL-A (06)]**
fits **[Guidal, Moutarde (08...)]**
neural networks **[KM, Schaefer (11)]**
ii. `dispersion integral' fits **[KMP-K (08), KM (08...)]**
iii. flexible GPD modeling **[KM (08...)]**

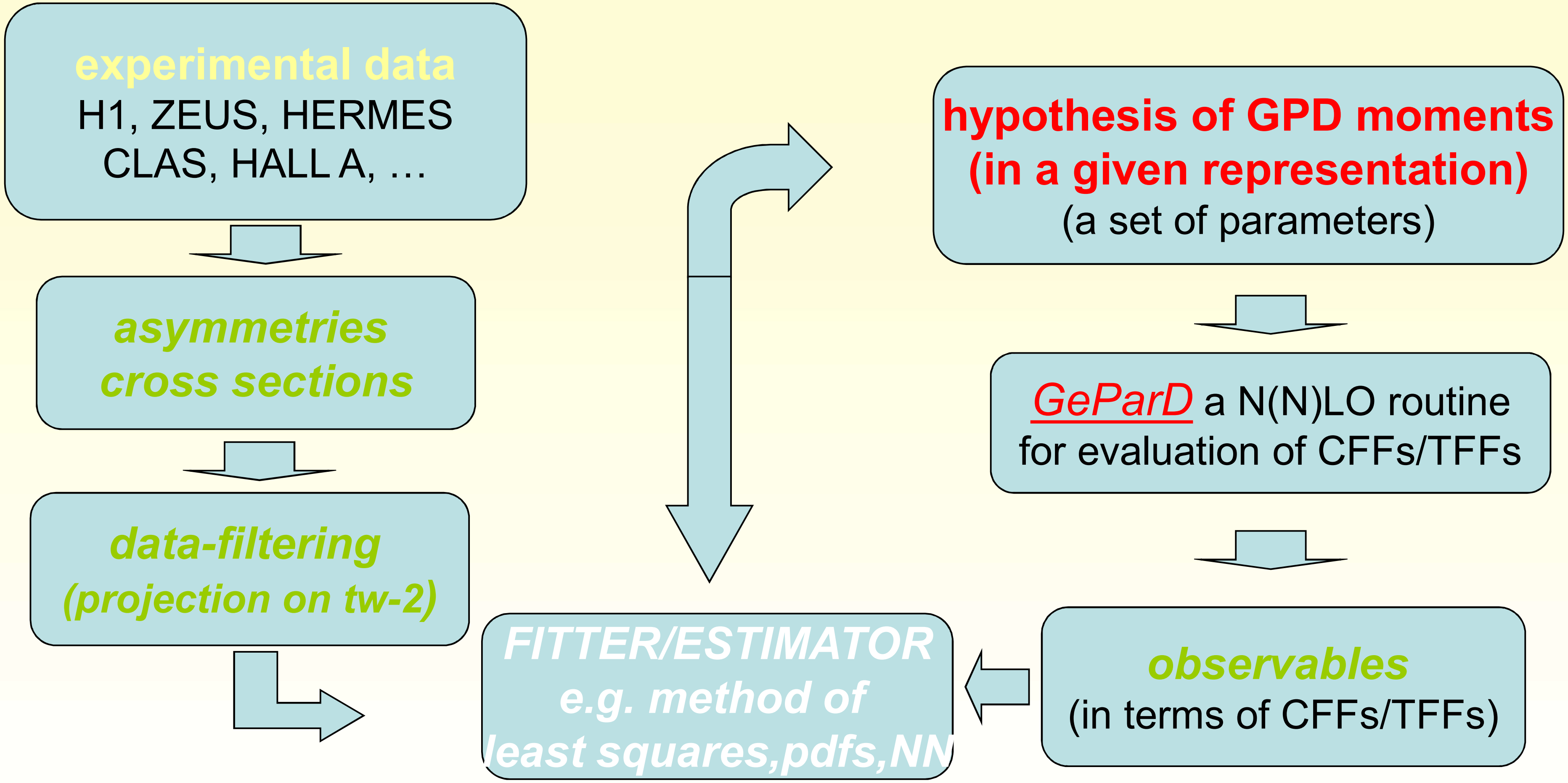
- a complete measurement allows in principle to pin down all CFFs **KK, DM, Murray (13)**
- missing information in incomplete measurements can be filled with noise
  (Guidal`s philosophy: use noise together with hypotheses and model constraints, our results are compatible)



- larger statistics:
  some CFF *E* constraint might have been obtained by HERMES

Flexible GPD model fits

[K. Kumerički, DM (08)]



experimental data
H1, ZEUS, HERMES
CLAS, HALL A, …

asymmetries
cross sections

data-filtering
(projection on tw-2)

FITTER/ESTIMATOR
e.g. method of
least squares,pdfs,NN

hypothesis of GPD moments
(in a given representation)
(a set of parameters)

GeParD a N(N)LO routine
for evaluation of CFFs/TFFs

observables
(in terms of CFFs/TFFs)

Requirements on software developments:

- development should not require much man power
- most flexible structure for processing of information
- robust and fast numerics

KM fits to DVCS

- a hybrid model: three effective SO(3) PWs for sea quarks/gluons
  dispersion relations for valence quarks
  still *E* GPD is neglected (only D-term)
  still $\hat{E}$ GPD only flexible pion pole contribution
- asking for GPD *H* and `*D-term*' *($\hat{H}$ is considered as effective d.o.f.)*

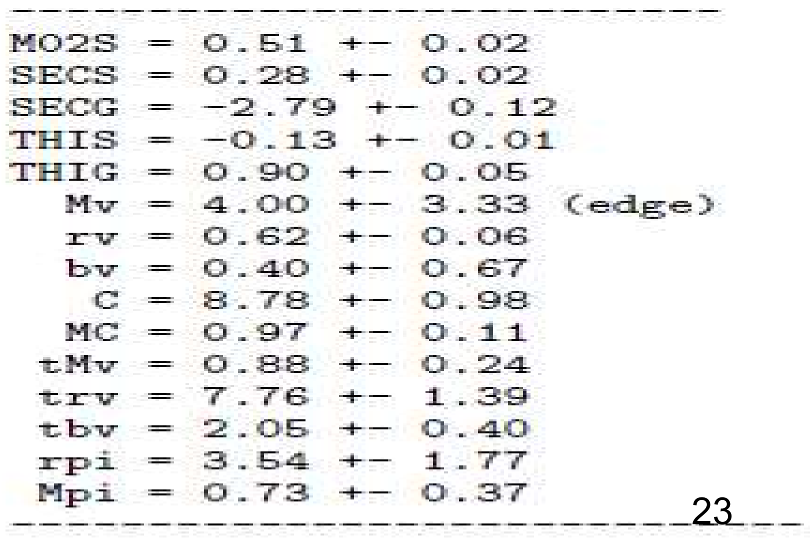
leading order, including evolution for sea quarks/ gluons
quark twist-two dominance hypothesis within CFF convention **[BM10]**

- data selection (taking moments of azimuthal angle harmonics)

KM10a: neglecting HALL-A data
KM10b: forming ratios of moments
KM10: original HALL-A data
neglecting large –*t* BSA CLAS data

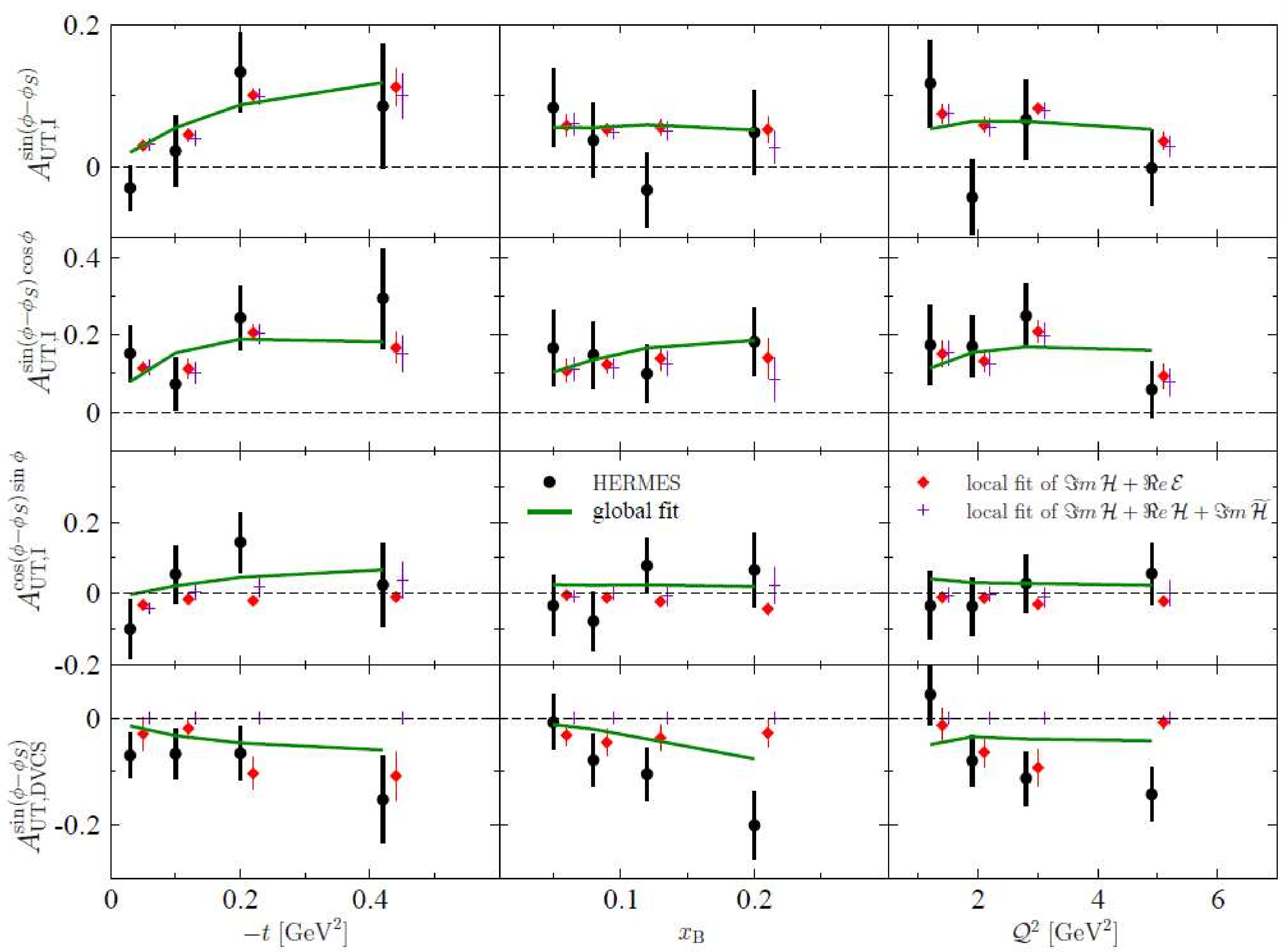
15 parameter fit, e.g.,
including all HALL-A data

175 data points
χ^2/d.o.f. =132/165



```
------------------------------
MO2S = 0.51 +- 0.02
SECS = 0.28 +- 0.02
SECG = -2.79 +- 0.12
THIS = -0.13 +- 0.01
THIG = 0.90 +- 0.05
  Mv = 4.00 +- 3.33 (edge)
  rv = 0.62 +- 0.06
  bv = 0.40 +- 0.67
   C = 8.78 +- 0.98
  MC = 0.97 +- 0.11
 tMv = 0.88 +- 0.24
 trv = 7.76 +- 1.39
 tbv = 2.05 +- 0.40
 rpi = 3.54 +- 1.77
 Mpi = 0.73 +- 0.37
------------------------------
```

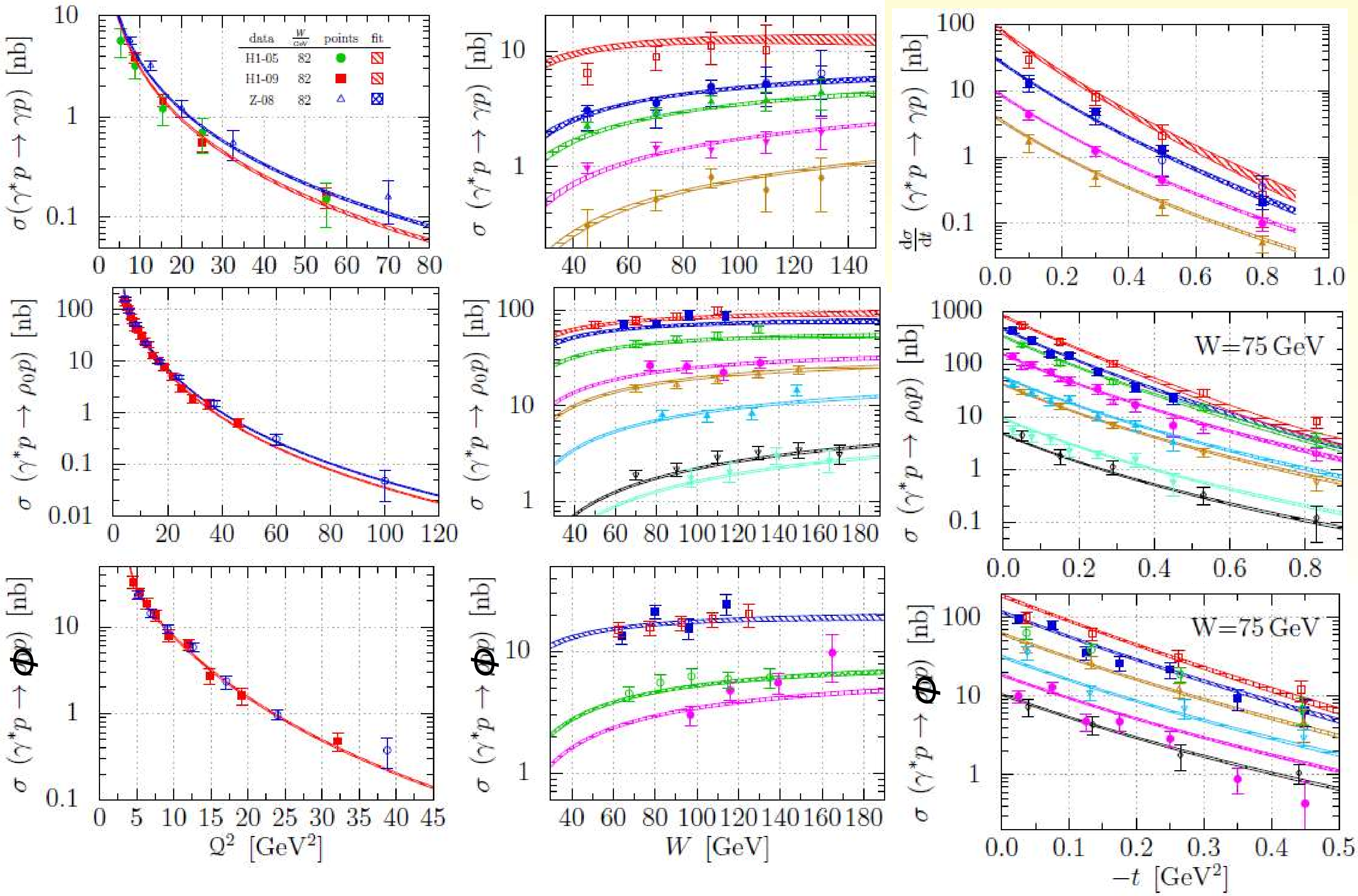
- KMM12/15 includes polarized target DVCS data
  (global fit to most of DVCS data , e.g., χ^2/*d.o.f* $\approx$ *1.6*
  e.g., transverse polarized HERMES asymmetries looks as)



DIS+DVCS+DVMP phenomenology at small-x_B (H1,ZEUS)

works somehow without DIS at LO **[T. Lautenschlager, DM, A. Schäfer (13)]**

works at NLO ($Q^2 > 4\ GeV^2$), done with Bayes theorem (probability distribution function)



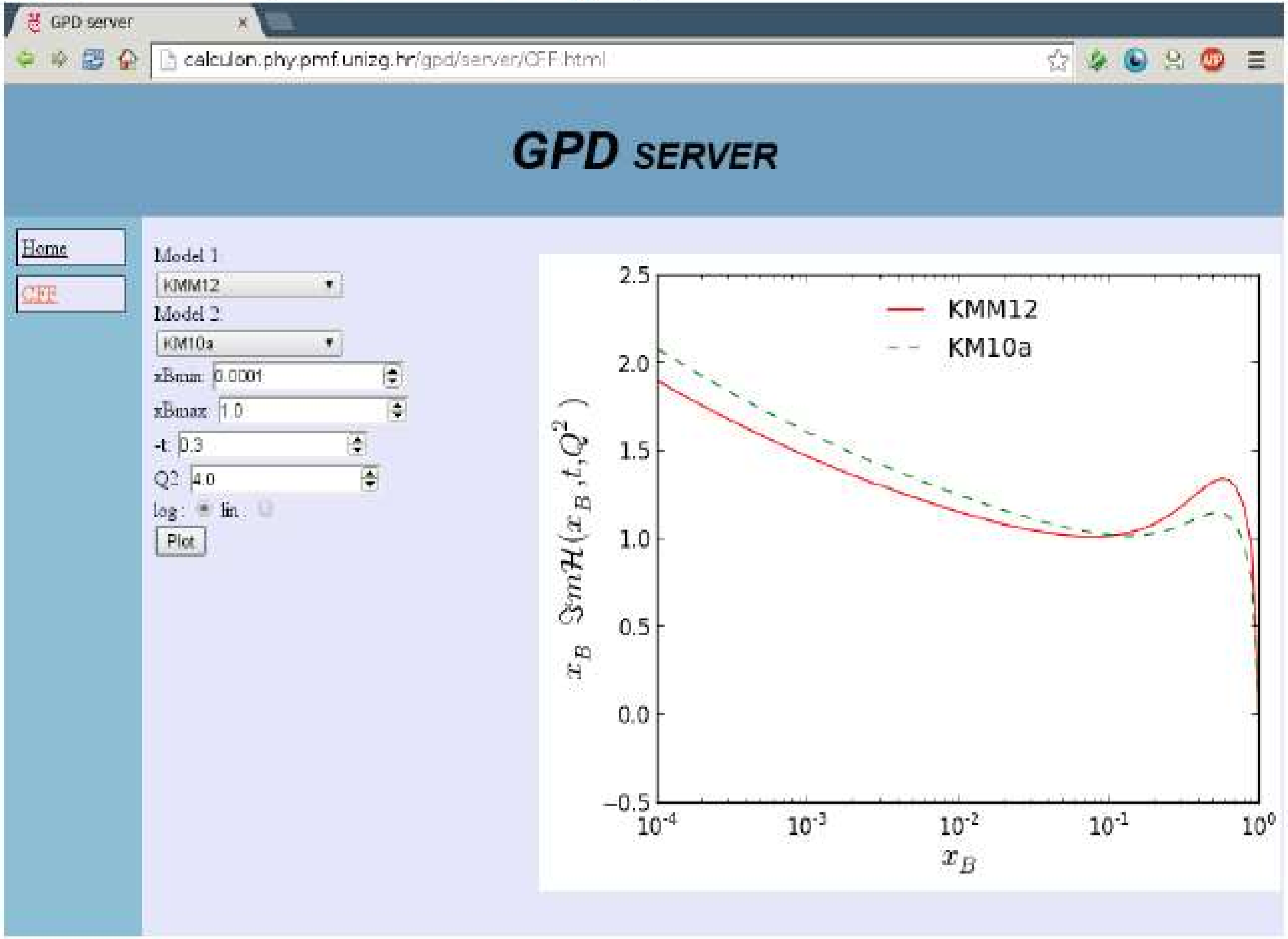
KM models are available at WWW

- http://calculon.phy.hr/gpd/ — binary code for cross sections

```
% xs.exe

  xs.exe  ModelID  Charge  Polarization  Ee  Ep  xB  Q2  t  phi

returns cross section (in nb) for scattering of lepton of energy Ee
on unpolarized proton of energy Ep. Charge=-1 is for electron.

ModelID is one of
   0 debug, always returns 42,
   1 KM09a - arXiv:0904.0458 fit without Hall A,
   2 KM09b - arXiv:0904.0458 fit with Hall A,
   3 KM10  - preliminary hybrid fit with LO sea evolution, from Trento presentation,
   4 KM10a - preliminary hybrid fit with LO sea evolution, without Hall A data
   5 KM10b - preliminary hybrid fit with LO sea evolution, with Hall A data

xB Q2 t phi  -- usual kinematics (phi is in Trento convention)

% xs.exe 1 -1 1 27.6 0.938  0.111 3. -0.3 0

  0.18584386497251
```

GPD page and server

- Durham-like CFF/GPD server page



GPD SERVER

Model 1: KMM12

Model 2: KM10a

xBmin: 0.0001

xBmax: 1.0

-t: 0.3

Q2: 4.0

log: ● lin: ○

- Do we need "Les Houches Accord" CFF/GPD interface?

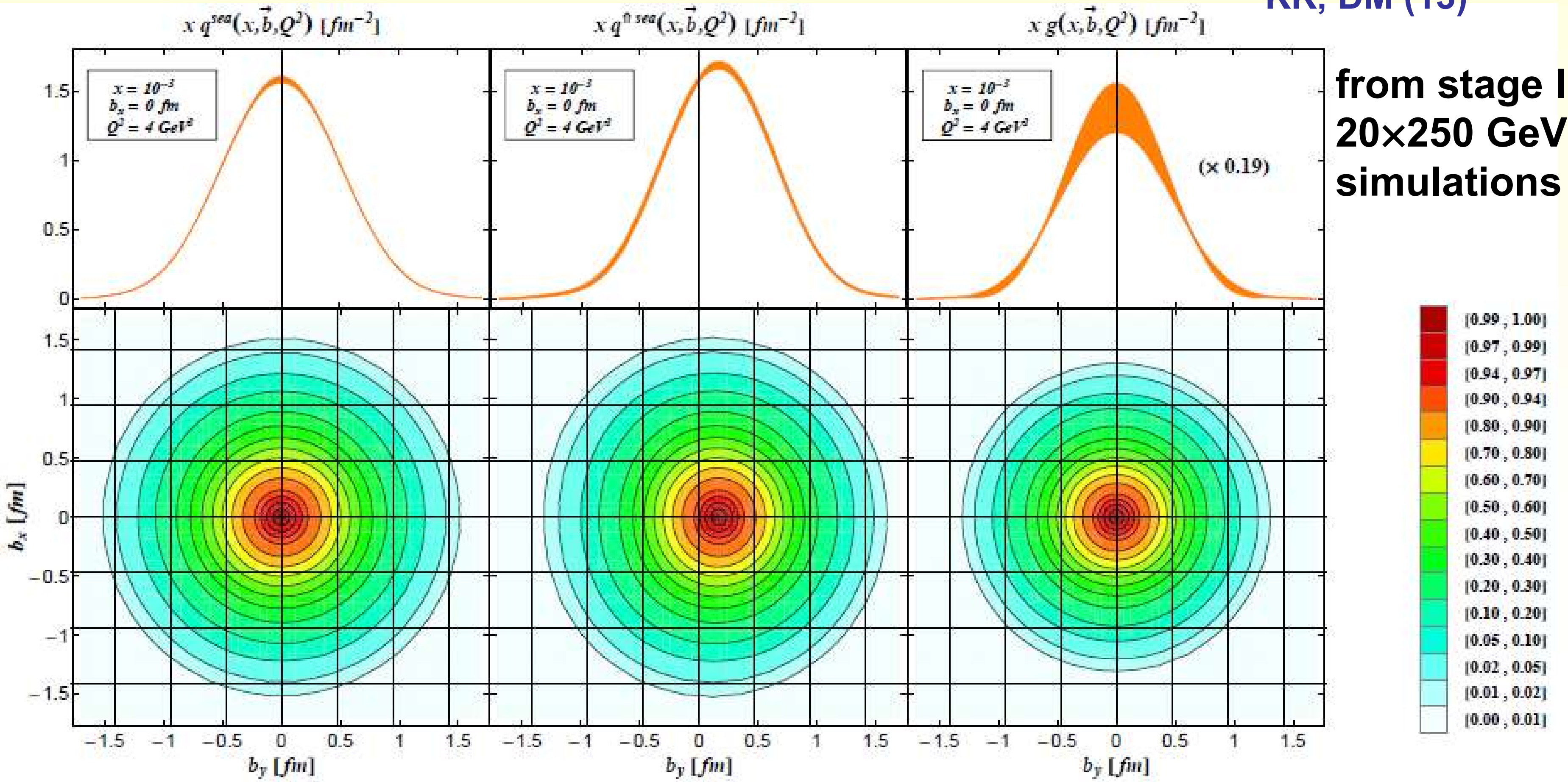
The Future

- ✓ COMPASS II
- ✓ JLAB@12 GeV
- ? ENC@GSI
- ? LHeC@CERN
- ? **EIC@BNL** or EIC@JLAB

J-PARC

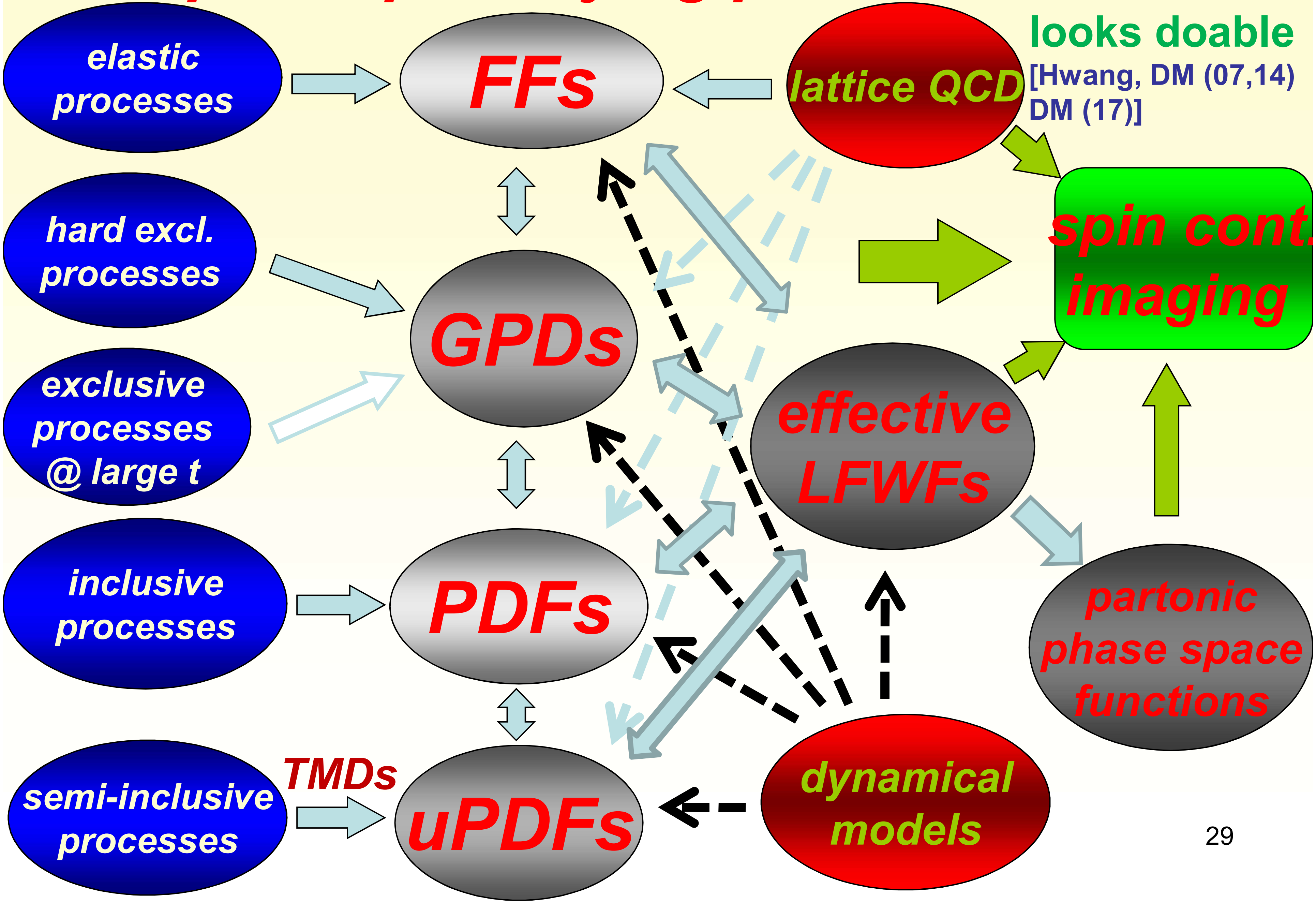
PANDA@GSI

Aschenauer, Firzo
KK, DM (13)



from stage II $20{\times}250\ GeV^2$ simulations

Prospect: quantifying partonic content



[Hwang, DM (07,14)
DM (17)]

Summary

GPDs are intricate and (thus) a promising tool

- ➢ to reveal the transverse distribution of partons (to some extend done at small x_B)
- ➢ to address the spin content of the nucleon (not possible at present in pheno.)
- ➢ providing a bridge to LFWFs & non-perturbative methods (e.g., lattice)
- ➢ CFFs have their own interest, bridging low and high virtuality regimes

first decade of hard exclusive leptoproduction measurements

- DVCS data are describable by means of GPDs, first new qualitative insights
- DVCS and DVMP data are describable in global NLO fits at small x
- moving on: to NLO, kinematical twist, full GPD models, DIS+DVCS+DVMP+...
- theory & software development is needed to address phenomenological goals
- covering the kinematical region between HERA (COMPASS) experiments within a high luminosity machine and dedicated detectors is needed to quantify exclusive and inclusive QCD phenomena: handle on GPD E & 3D