Utilizing Gaussian Boson Sampling for hyperspectral data reduction

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Semptember 11, 2026, WMLQ2026, NCBJ, Warsaw



Quantum Cosmos Lab
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We participated in three ESA co-funded projects:

- Quantum machine learning for analyzing multi- and hyperspectral satellite images (2022-2024).

  Collaboration: KP Labs
- Quantum Advantage for Earth Observation - QA4EO (2023).

  Collaboration: DLR, VTT, Syderal Poland, CSC, ETOS
- Gaussian Boson Sampling for Earth Observation - GBS4EO (2025-2026).

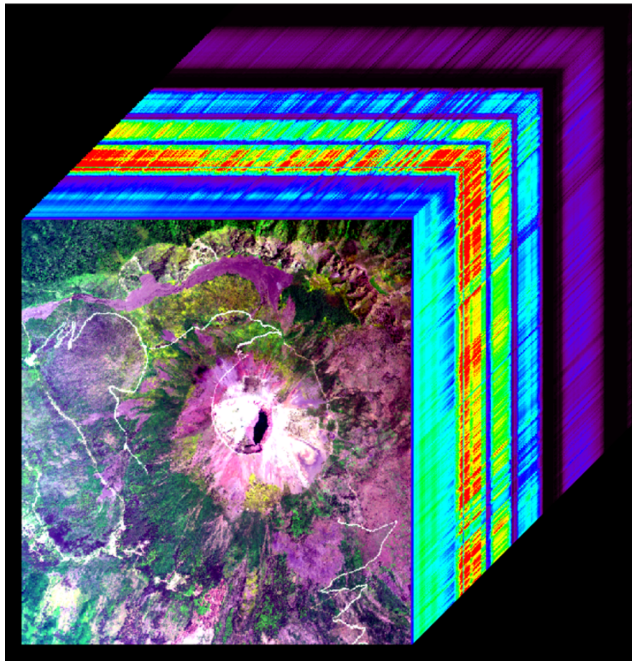
  Collaboration: KP Labs, IT4I

Selected previous results

- A. Miroszewski, et. al, Detecting Clouds in Multispectral Satellite Images Using Quantum-Kernel Support Vector Machines, in IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing, vol. 16, pp. 7601-7613 (2023)
- A. Miroszewski, et. al, Quo Vadis, Quantum Machine Learning?: Quantum kernel methods meet Earth observation, in IEEE Geoscience and Remote Sensing Magazine, vol. 14, no. 1, pp. 306-334 (2026).
- A. Delilbasic et al., Large-Scale Quantum Kernels for Hyperspectral Data Classification, in IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing, doi: 10.1109/JSTARS.2026.3727251 (2026).

The problem

Feature/band selection for the multi/hyper-spectral data.



Credit: JPL, NASA

The task is to reduce the number of bands in the original dataset while preserving its essential information. This reduction produces a more compact representation of the data, making it computationally lighter and easier to analyze.

Turning the bands into a graph

By introducing an appropriate distance between spectral bands:

$$D(B_i, B_j),$$

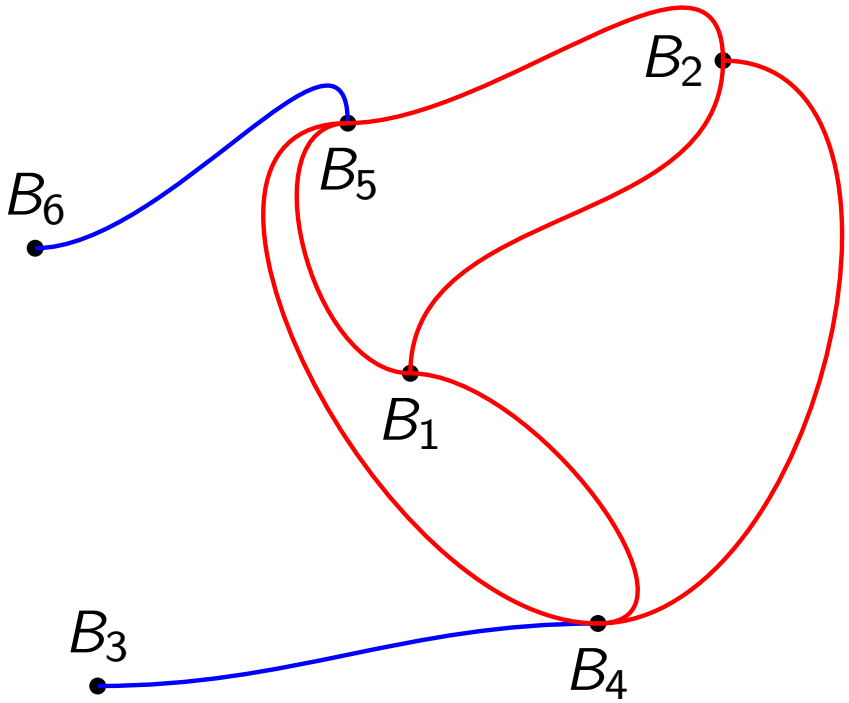
it is possible to quantify the relationship between two bands in terms of their informational content and local variability.

Based on this measure, the set of bands can be represented as a graph, where each vertex corresponds to a spectral band.

In particular, an edge between bands B_i and B_j is defined according to the threshold D_0:

- $D(B_i, B_j) < D_0$ – an edge is created,
- $D(B_i, B_j) \geqslant D_0$ – no edge is created.

Let us consider an example with 6 bands $\{B_1, B_2, B_3, B_4, B_5, B_6\}$, which leads to the following graph:



The bands $\{B_1, B_2, B_4, B_5\}$ form a clique which can be reduced into a singe node.

Then, the original six bands are reduced to: $\{B_1, B_3, B_6\}$

Finding a clique can be related to the computation of the hafnian of the corresponding adjacency matrix.

The hafnian of a $2m \times 2m$ symmetric matrix A, is defined as:

$$\mathrm{haf}(A) = \sum_{M \in \mathrm{PMP}(2m)} \prod_{\{i,j\} \in M} A_{ij},$$

where $PMP(2m)$ is the set of all perfect matchings of $\{1, ..., 2m\}$.

A perfect matching of a set with an even number of elements is a partition of the elements into disjoint pairs, so that every element appears in exactly one pair. E.g. for 4×4 matrix:

$$\mathrm{haf}(A_4) = a_{12}a_{34} + a_{13}a_{24} + a_{14}a_{23}.$$

Finding the perfect matching and in conseqnece the hafnian is #P-Hard (Sharp-P-Hard). Expected to be harder than NP-hard.

For the exemplary graph, the adjacency matrix is:

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

The hafnias for the graph and subgraphs are the following:

- $\mathrm{haf}(A) = A_{12}A_{34}A_{56} = 1$
- In total we have 15 4-node subgraphs. The maximal hafnian is: $\mathrm{haf}(A_{\{1,2,4,5\}}) = A_{12}A_{45} + A_{14}A_{25} + A_{15}A_{24} = 3$.
- The remaining 4-node hafnians are equal to 0 or 1.
- The 2-node hafnians are equal $\mathrm{haf}(A_2) = A_{ij} = \{0, 1\}$.

Gaussian Boson Sampling (GBS)

Gaussian Boson Sampling (GBS) [Hamilton et al., PRL **119**, 170501 (2017)] is a photonic process in which the outcome probability is given by:

$$P(n_1, n_2, ..., n_m) = \frac{1}{\sqrt{\det(Q)}} \frac{|\text{haf}\,(B_S)|^2}{n_1! \cdots n_m!},$$

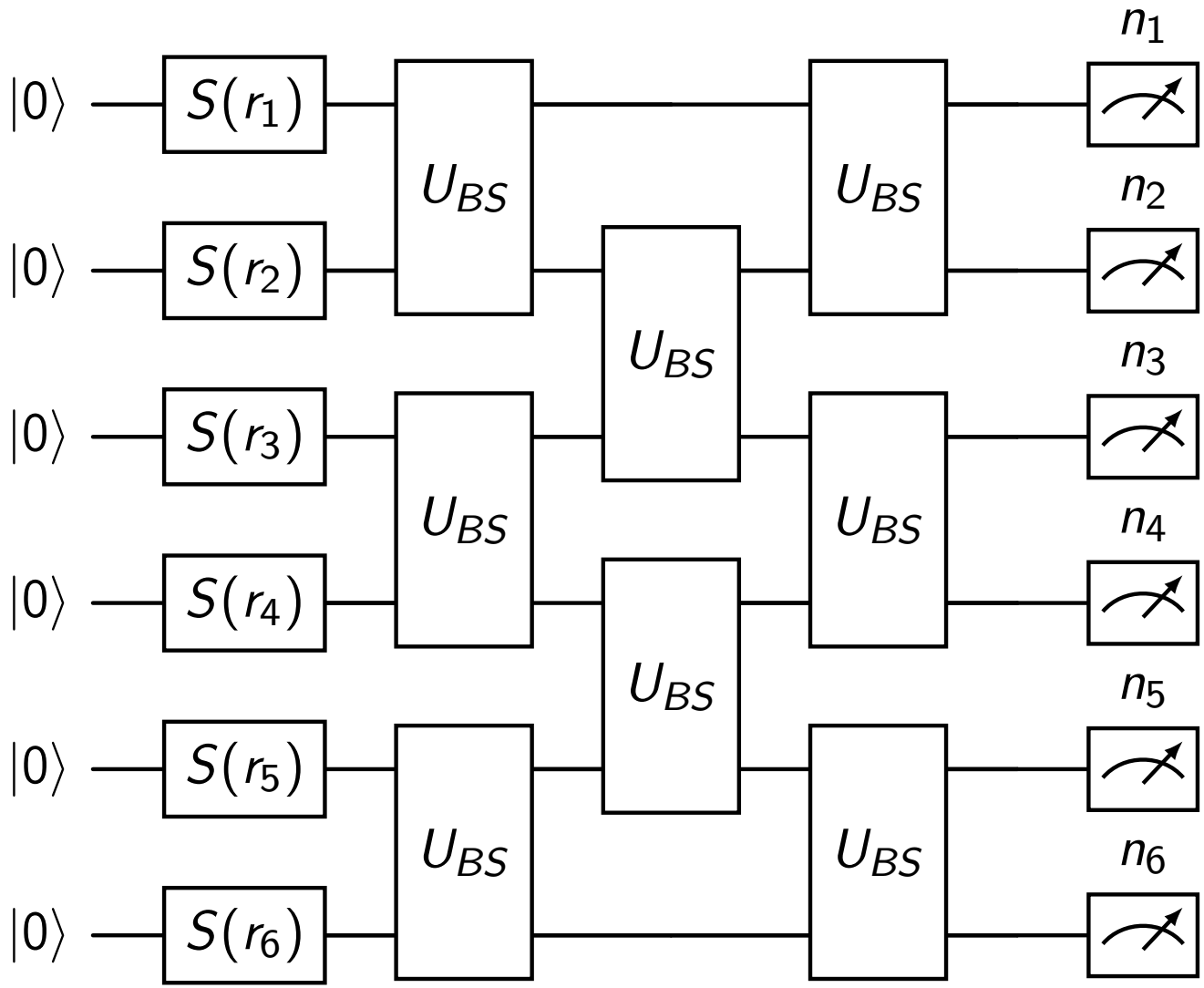
where n_i corresponds to the number of photon counts in the m photonic channels. Here, B_S is a submatrix of a symmetric matrix:

$$B = U\,\text{diag}(\tanh r_1, \ldots, \tanh r_m) U^T.$$

where U is the unitary transformation of the photonic circuit and $\tanh r_i$ parametrize the input squeezing of the optical modes.

Given the adjacency matrix A, a corresponding matrix B is constructed and encoded into the GBS circuit.

The most probable outputs are associated with cliques.



Sampled are the outcomes with the photon counts:
$(n_1, n_2, n_3, n_4, n_5, n_6)$.

Mapping GBS into the qubit-based architectures

Can we run it on a standard qubit-based quantum computer?

The optical GBS circuit can be approximately mapped onto a qubit-based architecture by truncating infinite-dimensional Fock space:

$$\mathcal{H} = \text{span}\{|0\rangle, |1\rangle, |2\rangle, ...\},$$

where $|n\rangle$ denotes the n-photon occupation state, to the finite-dimensionl space:

$$\mathcal{H} = \text{span}\{|0\rangle, |1\rangle, |2\rangle, \ldots, |N-1\rangle\}.$$

We developed a quantum compiler that automatically translates photonic circuits (Blackbird format) into qubit circuits (OpenQASM format) operating in the truncated Fock-space encoding: `https://github.com/Quantum-Cosmos-Lab/gbs-compiler`.

The current version works for $N = 2$ (fermionic case) and $N = 4$.

Truncated Hilbert space

The truncation of the Fock space to N dimensions is introduced by the cut-off annihilation and creation operators:

$$\hat{b} = \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \sqrt{2} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \sqrt{3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \sqrt{N-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix},$$

$$\hat{b}^\dagger = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \sqrt{N-1} & 0 \end{pmatrix}.$$

The operators $\hat{b}, \hat{b}^\dagger$ are adjoint to each other and satisfy the following algebra:

$$[\hat{b}, \hat{b}^\dagger] = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -(N-1) \end{pmatrix} = \hat{\mathbb{I}} - N \left|N\right\rangle \left\langle N\right|,$$

which differs from the standard infinite-dimensional CCR algebra. The resulting occupation number operator $\hat{N}_b := \hat{b}^\dagger \hat{b}$ is symmetric, and the standard eigenvalue equation holds:

$$\hat{N}_b \left|n\right\rangle = n \left|N\right\rangle.$$

Importantly, for $n = N - 1$, the action of $\hat{b}^\dagger$ is terminated:

$$\hat{b}^\dagger \left|N-1\right\rangle = 0.$$

Mapping Fock states into qubits

To represent photonic Fock states within the framework of qubit-based quantum circuits, we adopt a binary encoding convention that assigns each photon-number degree of freedom to a register of qubits.

For the $N = 4$, case the first four Fock states can be encoded using two qubits:

$$
\begin{aligned}
|0\rangle &\rightarrow |00\rangle\,, \\
|1\rangle &\rightarrow |01\rangle\,, \\
|2\rangle &\rightarrow |10\rangle\,, \\
|3\rangle &\rightarrow |11\rangle\,.
\end{aligned}
$$

In case when $N = 2^n$, with $n \in \mathbb{N}$, the number of Fock states fits the number of required qubits. E.g. $2 = 2^1$, $4 = 2^2$, $8 = 2^3$,... .

Truncated operators

To implement the GBS circuits the following operators are needed:

- Phase shift operator:

$$\hat{U}(\theta) := \exp\left(-i\theta\hat{N}\right)$$

- Squeezing operator:

$$\hat{S}(z) = \exp\left[(z^*\hat{b}^2 - z(\hat{b}^\dagger)^2)/2\right],$$

where the complex squeezing parameter is represented as $z = re^{i\phi}$.

- Action of beam splitter:

$$\hat{U}_{BS} = \exp\left[\theta\left(e^{i\phi}\hat{b}_1^\dagger \otimes \hat{b}_2 - e^{-i\phi}\hat{b}_1 \otimes \hat{b}_2^\dagger\right)\right].$$

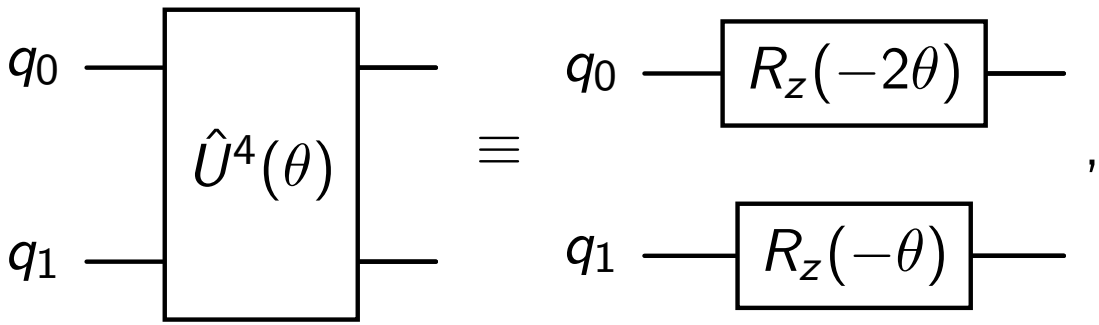
The case of symmetric beam splitter is obtained for $\theta = \frac{\pi}{4}$ and $\phi = 0$.

Phase shift operator

For the case under consideration, we find that the phase shift operator is:

$$\hat{U}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-i\theta} & 0 & 0 \\ 0 & 0 & e^{-i2\theta} & 0 \\ 0 & 0 & 0 & e^{-i3\theta} \end{pmatrix} = \exp(-i\theta\hat{N}).$$

The gate decomposition is:



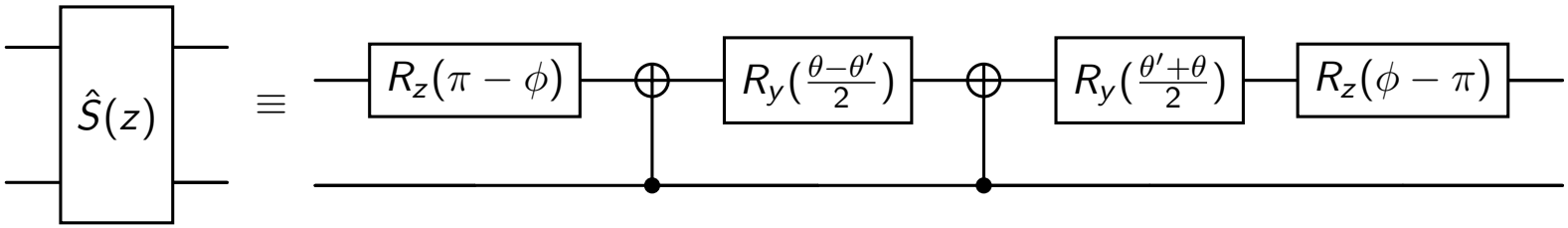
with a simple generalization to higher N cases.

Squeezing operator

The truncated squeezing operator can be expressed as follows:

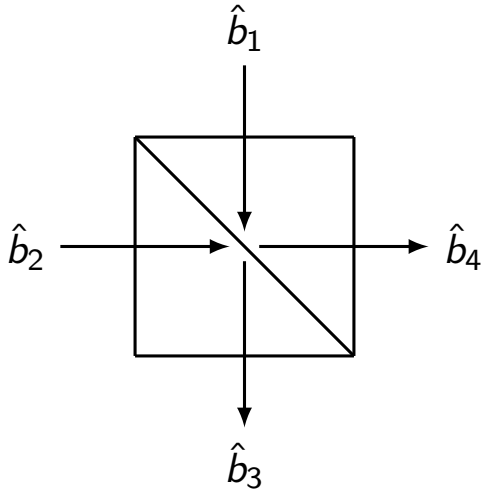
$$\hat{S}(z) = \begin{pmatrix} \cos\left(\frac{r}{\sqrt{2}}\right) & 0 & e^{-i\phi}\sin\left(\frac{r}{\sqrt{2}}\right) & 0 \\ 0 & \cos\left(\sqrt{\frac{3}{2}}r\right) & 0 & e^{-i\phi}\sin\left(\sqrt{\frac{3}{2}}r\right) \\ -e^{i\phi}\sin\left(\frac{r}{\sqrt{2}}\right) & 0 & \cos\left(\frac{r}{\sqrt{2}}\right) & 0 \\ 0 & -e^{i\phi}\sin\left(\sqrt{\frac{3}{2}}r\right) & 0 & \cos\left(\sqrt{\frac{3}{2}}r\right) \end{pmatrix}.$$

The gate decomposition is:



where $\theta = \sqrt{2}r$ and $\theta' = \sqrt{6}r$.

Beam splitter



The transformation operator for the symmetric beam splitter can now be expressed as:

$$\hat{U}_{BS} = \exp\left[\theta\left(e^{i\phi}\hat{b}_1^\dagger \otimes \hat{b}_2 - e^{-i\phi}\hat{b}_1 \otimes \hat{b}_2^\dagger\right)\right],$$

where the phase ϕ dependence can be extracted as follows:

$$\hat{U}_{BS}(\theta,\phi) = (\hat{U}(\phi)^\dagger \otimes \hat{\mathbb{I}})\hat{U}_{BS}(\theta,0)(\hat{U}(\phi) \otimes \hat{\mathbb{I}}) = e^{\hat{A}}.$$

Then, the $\hat{A}$ operator can be expressed as follows:

$$\hat{A} = \theta \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & -\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sqrt{6} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & -\sqrt{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{6} & 0 & 0 & 0 & 0 & 0 & -\sqrt{6} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{6} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}.$$

From here:

$$U_{BS}(\theta,0)=\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \cos(\theta) & 0 & 0 & -\sin(\theta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \cos^2(\theta) & 0 & 0 & -\frac{\sin(2\theta)}{\sqrt{2}} & 0 & 0 & \sin^2(\theta) & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \cos^3\theta & 0 & 0 & -\sqrt{3}\sin\theta\cos^2\theta & 0 & 0 & \sqrt{3}\cos\theta\sin^2\theta & 0 & 0 & -\sin^3\theta & 0 & 0 & 0\\
0 & \sin(\theta) & 0 & 0 & \cos(\theta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \frac{\sin(2\theta)}{\sqrt{2}} & 0 & 0 & \cos(2\theta) & 0 & 0 & -\frac{\sin(2\theta)}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \sqrt{3}\sin\theta\cos^2\theta & 0 & 0 & \frac{\cos(\theta)+3\cos(3\theta)}{4} & 0 & 0 & \frac{\sin(\theta)-3\sin(3\theta)}{4} & 0 & 0 & \sqrt{3}\cos\theta\sin^2\theta & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1+\cos\left(2\sqrt{3}\theta\right)}{2} & 0 & 0 & -\frac{\sqrt{2}\sin\left(2\sqrt{3}\theta\right)}{2} & 0 & 0 & \sin^2\left(\sqrt{3}\theta\right) & 0 & 0\\
0 & 0 & \sin^2(\theta) & 0 & 0 & \frac{\sin(2\theta)}{\sqrt{2}} & 0 & 0 & \cos^2(\theta) & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \sqrt{3}\cos\theta\sin^2\theta & 0 & 0 & \frac{-\sin(\theta)+3\sin(3\theta)}{4} & 0 & 0 & \frac{\cos(\theta)+3\cos(3\theta)}{4} & 0 & 0 & -\sqrt{3}\sin\theta\cos^2\theta & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sin\left(2\sqrt{3}\theta\right)}{\sqrt{2}} & 0 & 0 & \cos\left(2\sqrt{3}\theta\right) & 0 & 0 & -\frac{\sin\left(2\sqrt{3}\theta\right)}{\sqrt{2}} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos(3\theta) & 0 & 0 & -\sin(3\theta) & 0\\
0 & 0 & 0 & \sin^3\theta & 0 & 0 & \sqrt{3}\cos\theta\sin^2\theta & 0 & 0 & \sqrt{3}\sin\theta\cos^2\theta & 0 & 0 & \cos^3\theta & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin^2\left(\sqrt{3}\theta\right) & 0 & 0 & \frac{\sin\left(2\sqrt{3}\theta\right)}{\sqrt{2}} & 0 & 0 & \cos^2\left(\sqrt{3}\theta\right) & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin(3\theta) & 0 & 0 & \cos(3\theta) & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}.$$

The transformation $\hat{U}_{BS}(\theta,0)=e^{\hat{A}}$ can be decomposed according to the total number of photons involved, i.e., $n=n_1+n_2$. Then the transformation of the basis states can be written as the following direct sum:

$$U_{BS}^{N=4}(\theta,0)=\bigoplus_{i=0}^{6}U_i.$$

- $n = 0$ (no photons) - basis $(|0,0\rangle)$
- $n = 1$ (1 photon) - basis $(|0,1\rangle, |1,0\rangle)$
- $n = 2$ (2 photons) - basis $(|0,2\rangle, |1,1\rangle, |2,0\rangle)$

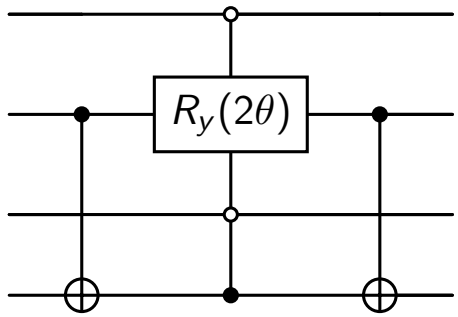
$$U_2 = \begin{pmatrix} \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \end{pmatrix}.$$

So that $U_2 |1,1\rangle = \frac{1}{\sqrt{2}}(|0,2\rangle - |2,0\rangle)$, which leads to the Hong–Ou–Mandel effect!

- $n = 3$ (3 photons) - basis $(|0,3\rangle, |1,2\rangle, |2,1\rangle, |3,0\rangle)$
- $n = 4$ (4 photons) - basis $(|1,3\rangle, |2,2\rangle, |3,1\rangle)$
- $n = 5$ (5 photons) - basis $(|2,3\rangle, |3,2\rangle)$
- $n = 6$ (6 photons) - basis $(|3,3\rangle)$

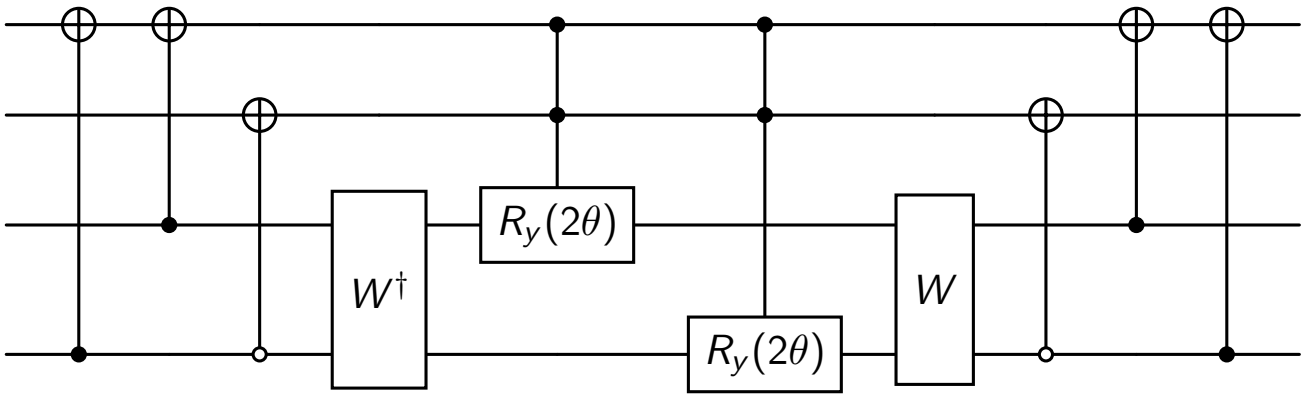
- **n=0** - We do nothing.
- **n=1**

$U_1 \equiv$



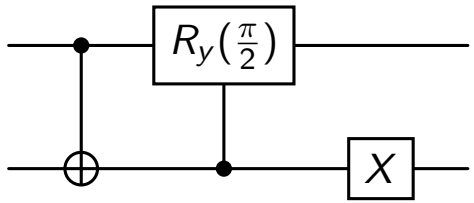
- **n=2**

$U_2 \equiv$

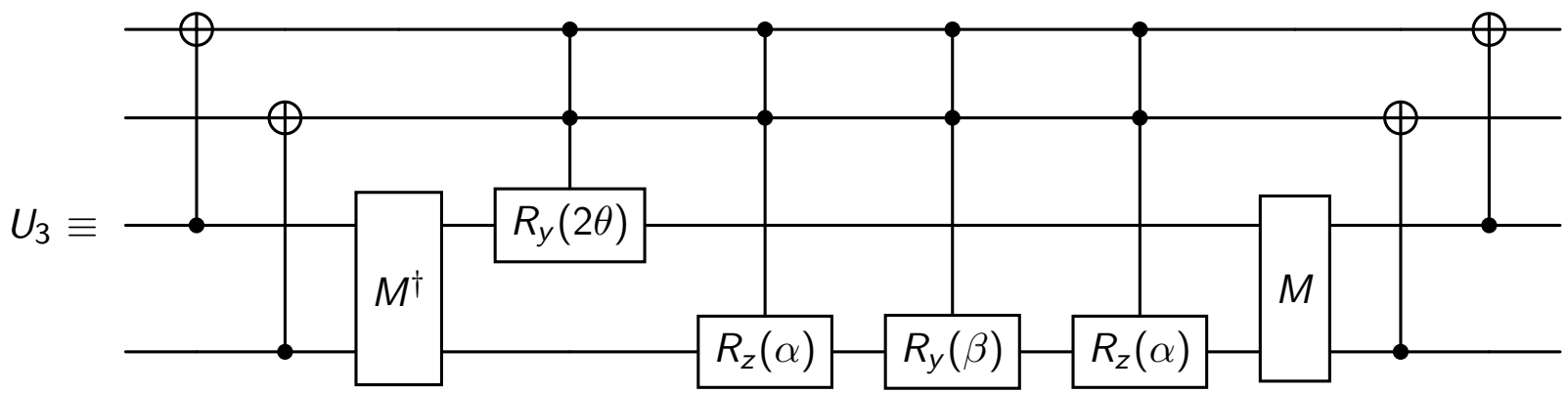


where

$W \equiv$



- **n=3**

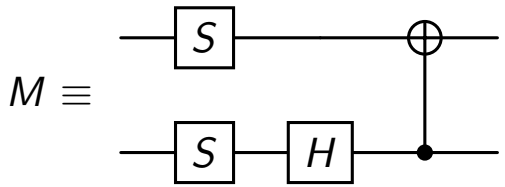


where the angles involved can be expressed as follows:

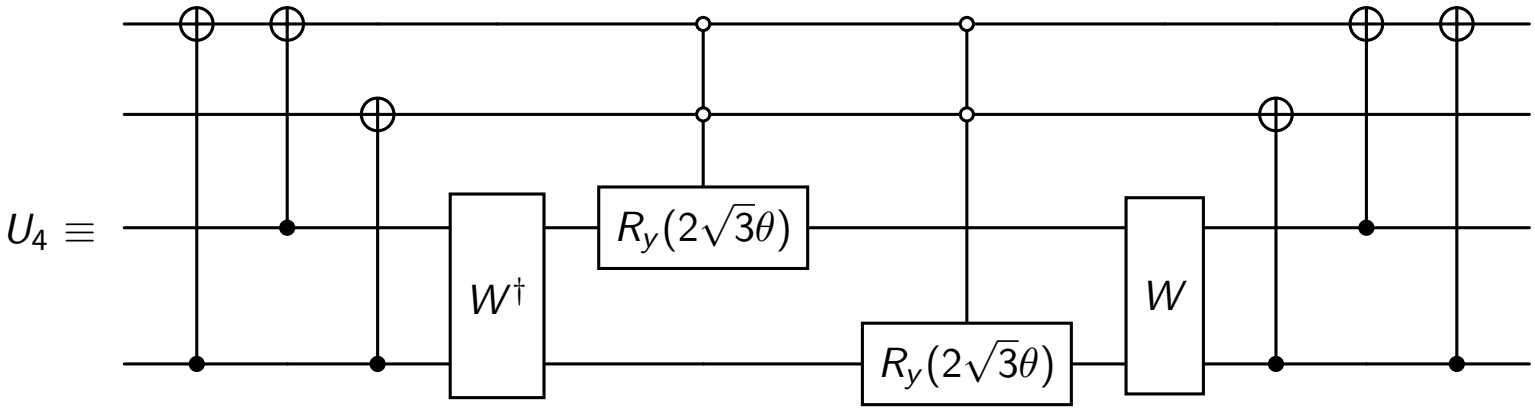
$$\alpha(\theta) = -\operatorname{arc\,tg}\left(\frac{\sqrt{3}\sin 2\theta}{2\cos 2\theta}\right),$$

$$\beta(\theta) = 2\operatorname{arc\,sin}\left(\frac{1}{2}\sin 2\theta\right),$$

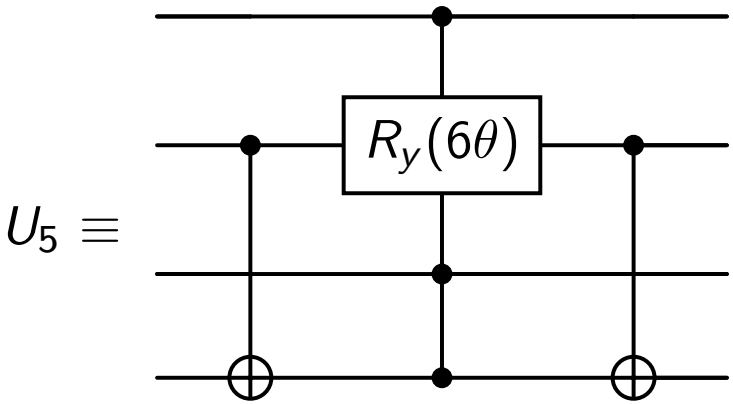
and where:



- **n=4**



- **n=5**



- **n=6** - We do nothing.

Summary

- The band selection problem can be formulated as a clique-finding problem.
- Clique detection is related to computation of the hafnian, which is a #P-hard problem.
- Cliques can be identified through the most probable outcomes of the Gaussian Boson Sampling (GBS) process.
- The number of spectral bands (graph vertices) corresponds directly to the number of optical modes (channels).
- Near-term prospects for optical implementations:
  - China: Jiuzhang 4.0 - 1024 squeezed states.
  - Canada: Borealis (Xanadu) - 216 squeezed states.
  - Europe: large-scale photonic GBS platforms are needed!
- Mapping GBS circuits onto qubit-based architectures is possible. However, it requires substantial quantum resources.