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Entanglement Classification and Machine Learning

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Outline

Introduction

Quantum Particle Workgroup

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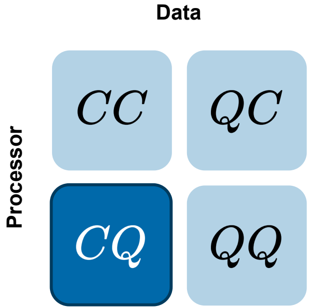
- Quantum Information Theory
- Bound Entanglement and Bell States
- Quantum Machine Learning



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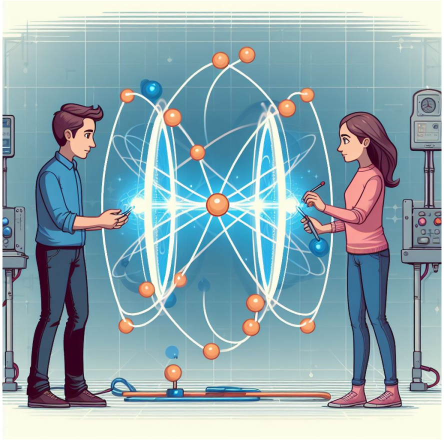
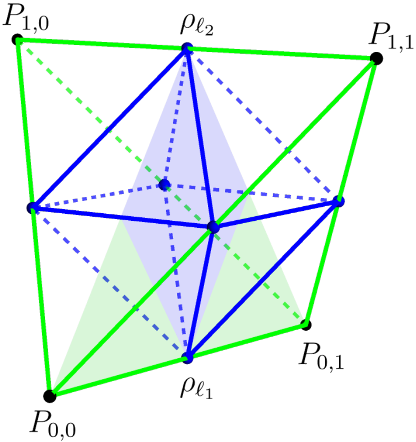
Entanglement and Bell states

- Entangled quantum state:
  - State cannot be described locally.
  - Stronger than classically possible correlations.
- Verifying a state as entangled is a hard problem ("Separability Problem").
- Bell states $|\Omega_{i,j}\rangle, \; P_{i,j} = |\Omega_{i,j}\rangle\langle\Omega_{i,j}|$
  - Maximally entangled states for general dimension d.
  - e.g., $d = 3$: $|\Omega_{0,0}\rangle \propto |0,0\rangle + |1,1\rangle + |2,2\rangle$



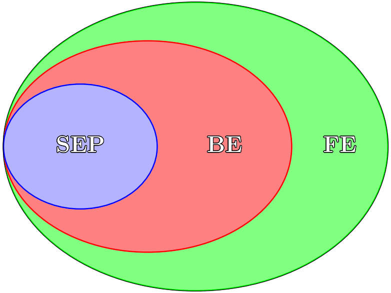
Bell-diagonal states

- Noise affects states:
  - $|\Omega\rangle \to \rho = \sum_{k,l} c_{k,l} P_{k,l}$ "Bell-diagonal state"
  - $c_{k,l} \geq 0, \sum_{k,l} c_{k,l} = 1$, "mixing probabilities"
- Noise affects entanglement:
  - Weak mixing $\to$ "free entangled" ("distillable") state
  - Strong mixing $\to$ "separable" (not entangled) state
  - $d > 2$: "Bound entangled" ("undistillable") states exist.



The separability problem

- Given a quantum state ρ, is it
  - separable (*SEP*)
  - or entangled ($ENT = BE \cup FE$)?
- BE states make the separability problem NP-hard.
- Apply ML-methods for Bell-diagonal states.
- **Focus:**
  - How useful are simple neural networks for the separability problem of Bell-diagonal states?
  - Can we learn something from successful classifiers?



Data and Implementation

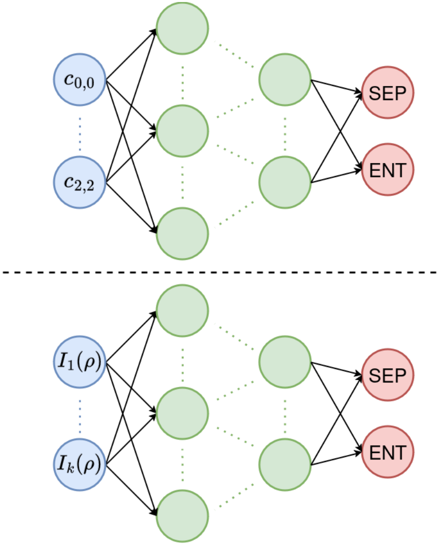
Data

- Uniformly distributed Bell-diagonal qutrit states $\rho = \sum_{k,l=0}^{2} c_{k,l} P_{k,l}$.
- Classified as "SEP" or "ENT" (BE) via analytical and numerical methods.
- Strongly leveraging algebraic and geometric properties of Bell states.
- Related publications:
  - Popp, C., Hiesmayr, B.C. Almost complete solution for the NP-hard separability problem of Bell diagonal qutrits. Sci Rep 12, 12472 (2022).
  - Popp, C., Hiesmayr, B.C. Comparing bound entanglement of bell diagonal pairs of qutrits and ququarts. Sci Rep 13, 2037 (2023)
  - Popp, BellDiagonalQudits: A package for entanglement analyses of mixed maximally entangled qudits. J.Open Source Softw. 8 (2023)

Methods

- NN classifiers:
  - Feedforward neural networks (FNN)
  - Convolutional neural networks (CNN)

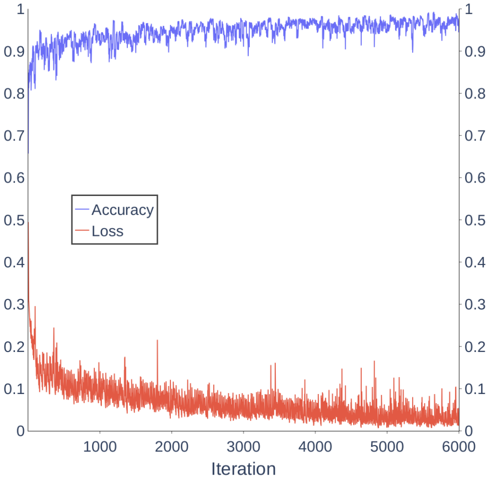
- Compare two feature sets:
  - Mixing probabilities $c_{k,l}$ ("Noise features")
  - Quantum Information features I (e.g., entropies, purity)



Results

Entanglement classification

- Mixing probabilities $c_{k,l}$ as features:
  - Simple FNN (≈ 6000 parameters) with $95\% - 98\%$ accuracy.
  - Simple CNN (≈ 600 parameters) with $\approx 90\%$ accuracy.
- Information quantities I_k as features:
  - Simple FNN (≈ 1000 parameters) with $85\% - 90\%$ accuracy.
  - More complex model (≈ 6000 parameters) does not perform better.

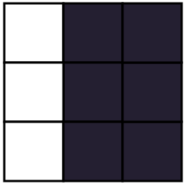


Entanglement structure and CNN

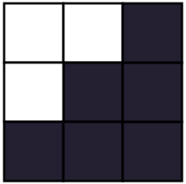
- Algebraic relations of Bell states $\leftrightarrow$ entanglement structure.
- Fact: 3 element mixtures are relevant ("line structure").
- Example:
  - $\rho_s = \frac{1}{3}(P_{0,0} + P_{1,0} + P_{2,0}) \in$ *SEP*.
  - $\rho_e = \frac{1}{3}(P_{0,0} + P_{1,0} + P_{0,1}) \in$ *FE* $\subset$ *ENT*.
- Can CNNs with kernels/filters f capture this structure?

$$\rho \propto \begin{array}{|c|c|c|} \hline c_{0,0} & c_{0,1} & c_{0,2} \\ \hline c_{1,0} & c_{1,1} & c_{1,2} \\ \hline c_{2,0} & c_{2,1} & c_{2,2} \\ \hline \end{array}$$

$\rho_s \propto$



$\rho_e \propto$



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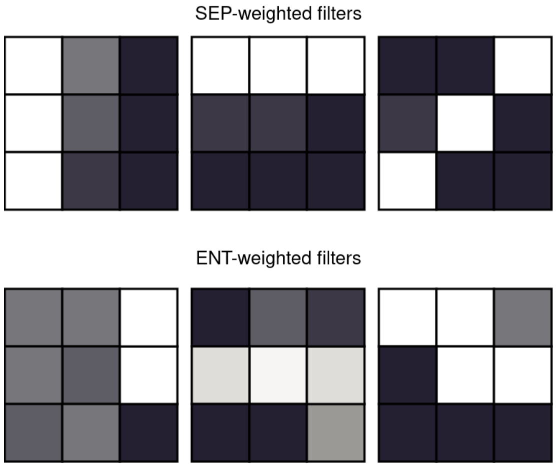
Entanglement structure and CNN

- Apply CNN with 3×3 kernels.
- Normalization of filters $\leftrightarrow$ equivalent representation to states.
- Weights determine relevance of filters for classification of SEP or ENT.
- Visible structures in filters
  $\rightarrow$ "Explainable" relation to entanglement.



Takeaways

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- Simple FNN can classify Bell-diagonal states (NP-hard separability problem) with high accuracy.
  $\rightarrow$ Dominant entanglement structures are not too complex.

- Noise features (mixing probabilities) are more effective than common information quantities features.
  $\rightarrow$ For these information quantities, the required information for classification is not easily accessible.

- Simple CNN models can capture relevant entanglement structures of Bell-diagonal states.
  $\rightarrow$ "Explainable" model provides insight about physics.