



THE 12TH CONFERENCE OF
THE POLISH SOCIETY ON
RELATIVITY



2-7 August 2026
Pałac Będlewo, Poland

DESI: Is Dark Energy evolving in time?

Marek Biesiada

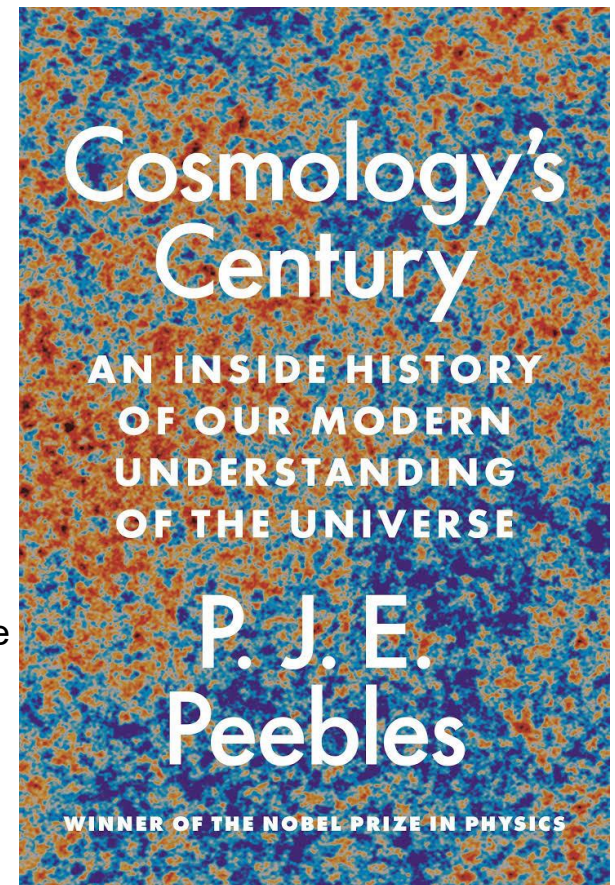
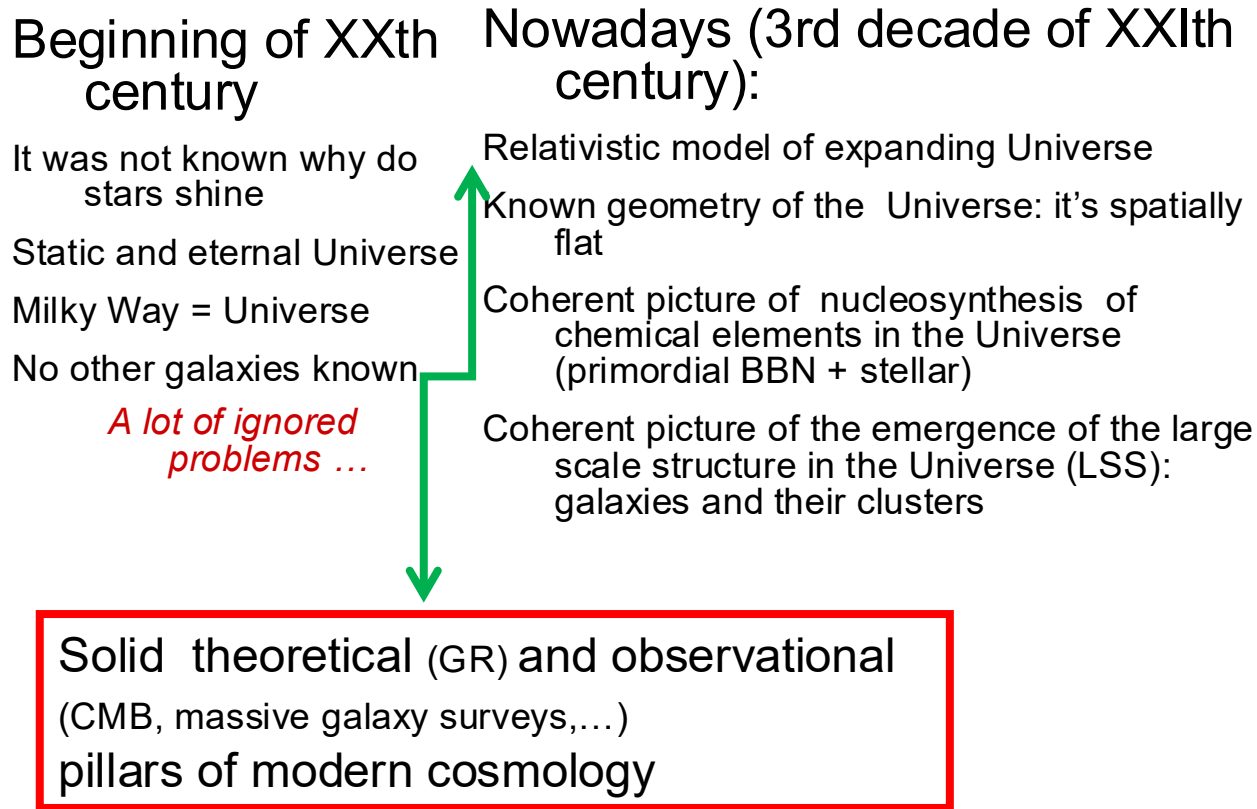
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ŚWIERK

Introduction

Golden Age of Cosmology



Modern cosmology – elevated to the precision science,
(Λ CDM model in agreement with most of the data)
but some TENSIONS emerged ...

A. Sandage

Physics Today **23**, 2, 34 (1970)

„quest for two numbers”

THE DYNAMICAL PARAMETERS OF THE UNIVERSE: H_0 , q_0 , Ω_0 , Λ , and K

Allan Sandage

Mount Wilson and Las Campanas Observatories of the Carnegie Institution of Washington, Pasadena, CA, USA.

G. A. Tammann*)

Astronomisches Institut der Universität Basel, Binningen, Switzerland and European Southern Observatory, Garching bei München, FRG.

1984

Accelerated expansion of the Universe



Nobel Prize in Physics 2011

What is causing it?

We don't know



But a tentative answer is already with us - the cosmological constant Λ

General Relativity (1915):

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}$$

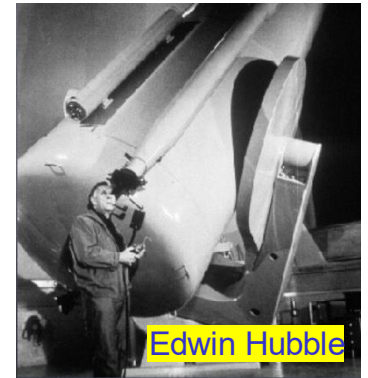
geometry ↗ *matter*

Cosmological constant
– in order to make the Universe static

$$G_{\mu\nu} + \cancel{\Lambda g_{\mu\nu}} = -8\pi G T_{\mu\nu}$$



Albert Einstein



Edwin Hubble

Einstein's „biggest blunder“

Revives in the 60-ties of XXth century
Yakov Zeldovich, Stephen Weinberg

Currently the Λ CDM model
is the standard cosmological model

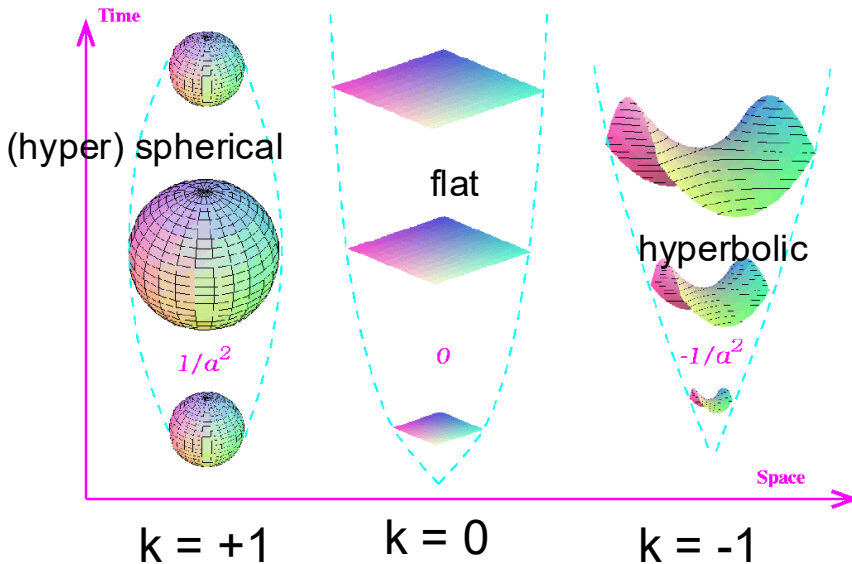
$$G_{\mu\nu} = -8\pi G (T_{\mu\nu} + T_{\mu\nu}^{vac})$$

$$T_{\mu\nu}^{vac} = \rho_{vac} g_{\mu\nu} = \frac{\Lambda}{8\pi G} g_{\mu\nu}$$

Λ CDM model
is very simple,
fits well to all the data, ... BUT
is burdened with lots of conceptual problems!

Modern cosmology in brief

$$G_{\mu\nu} = -8\pi G T_{\mu\nu}$$



Geometry of the Universe
homogeneous, isotropic
Friedman-Lemaitre-Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

cosmological redshift

$$z = \frac{1 - a(t)}{a(t)}$$

all spatial distances
(including wavelength of light)
are scaled by $a(t)$
which changes in time

$$a(t) = \frac{1}{1 + z}$$

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu - p g^{\mu\nu}$$

$$\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho$$

$$H \equiv \frac{\dot{a}}{a} \quad \text{expansion rate (Hubble function)}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$q \equiv -\frac{\ddot{a}}{a} \frac{1}{H^2} \quad \text{deceleration parameter as soon as}$$

$$\rho + 3p > 0$$

Friedman equation

$$H(t)^2 = \Omega_i H_0^2 - \frac{k}{a(t)^2}$$

$$\Omega_i = \frac{\rho_i}{\rho_{cr}} = \rho_i \frac{8\pi G}{3H_0^2}$$

Friedman equation – clue to cosmological model testing

$$H(z)^2 = H_0^2 \left[\Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_k(1+z)^2 + \Omega_{DE,0}(1+z)^{3(1+w_0+w_a)} \exp\left(-\frac{3w_a z}{1+z}\right) \right]$$

↑ CDM + baryonic matter
 ↑ radiation
 ↑ Curvature parameter
 ↑ DE equation of state

DE equation of state – we don't know it

$$p_{DE} = w_{DE}(z)\rho_{DE}$$

$$\{w_{DE}(z) = -1\} \equiv \Lambda$$

$$\Omega_{\Lambda} = \text{const.}$$

Assuming that it could be changing in the history of the Universe – first order Taylor expansion

$$w(a) = w_0 + w_a(1 - a)$$

$$w(z) = w_0 + w_a \frac{z}{1+z}$$

Chevallier – Polarski – Linder
(CPL) parametrization

challenge for observers

reconstruct history of the expansion of the Universe

Hubble
constant

$$H(z) = H_0 h(z)$$

c/H_0 distance scale

$1/H_0$ time scale

3 types of distances:

- *comoving distance*— not measurable !

$$D_C(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{h(z')}$$

$\Omega_i \quad \{i = m, r, k, DE\}$

- *luminosity distance* $D_L(z) = (1+z)D_C(z) = (1+z) \frac{c}{H_0} \int_0^z \frac{dz'}{h(z')}$

- *angular diameter distance* $D_A(z) = \frac{1}{1+z} D_C(z) = \frac{1}{1+z} \frac{c}{H_0} \int_0^z \frac{dz'}{h(z')}$

Standard candles – objects of known (or standardizable)

luminosity L ; we measure the flux F , and calculate the distance

$$L = 4\pi D_L(z) F$$

Standard rulers – objects of known sizes D ; we measure

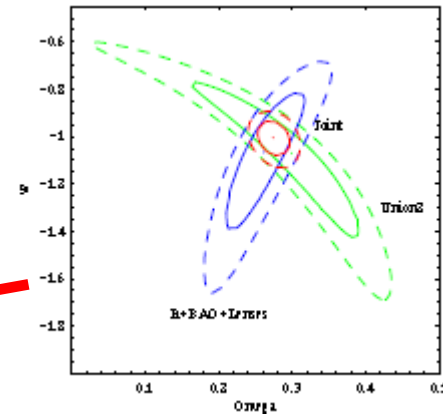
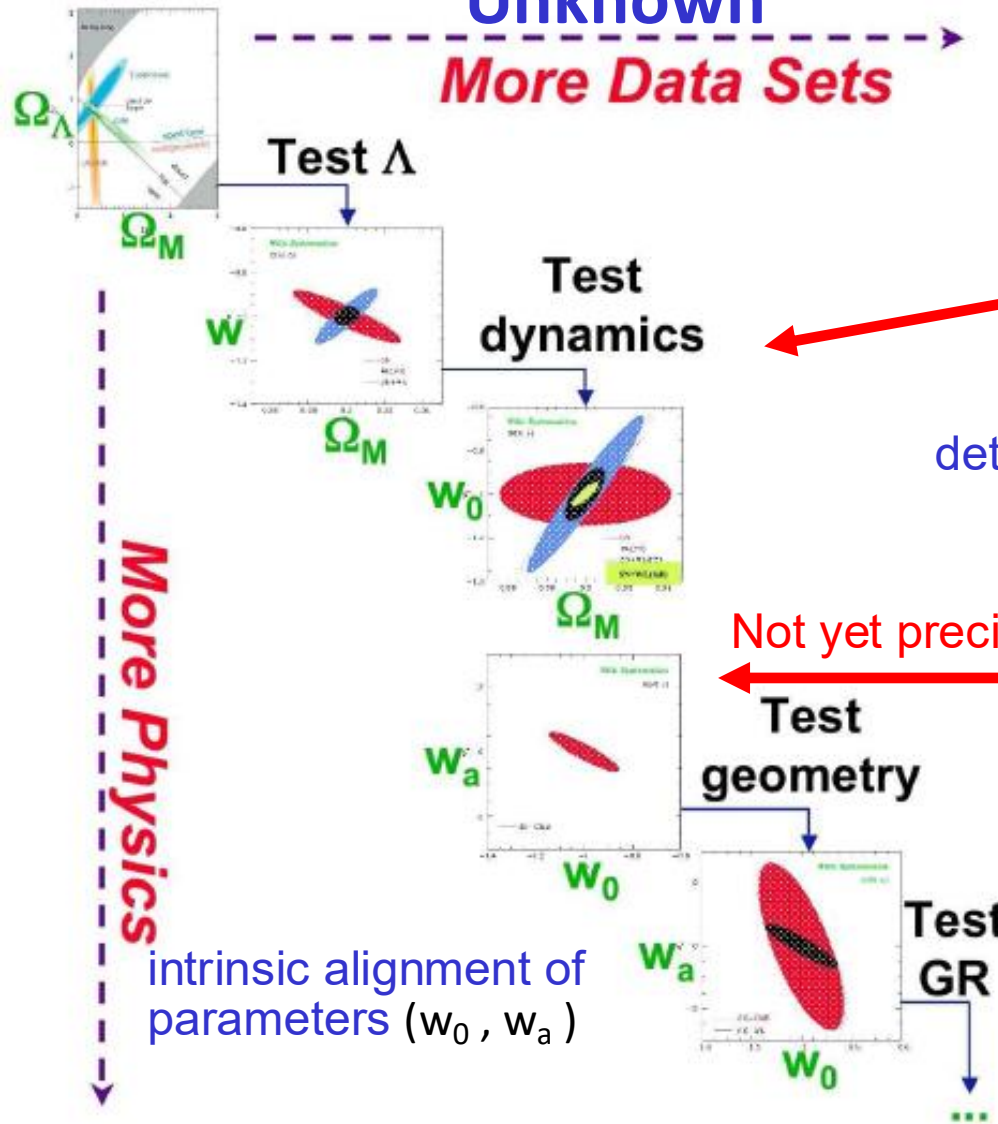
angular diameter θ and calculate the distance

$$D = D_A(z) \theta$$

Standard sirens – coalescing compact binaries

D_L is directly measurable (but z – unknown!)

Modern cosmology: Incremental Exploration of the Unknown

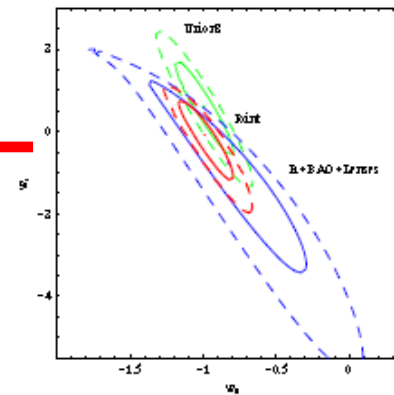


MB,
B.Malec,
A.Piórkowska
2011

determine DE equation of state

$$p_X = w\rho_X$$

determine if it
changes in time



$$w(z) = w_0 + w_a \frac{z}{1+z}$$

We need **complementary probes** –
to break degeneracies
and for cross-checks

Linder (astro-ph/0511197)

DESI Opens Access to the Largest 3D Map of the Universe Yet

DESI Opens Access to the Largest 3D Map of the Universe Yet

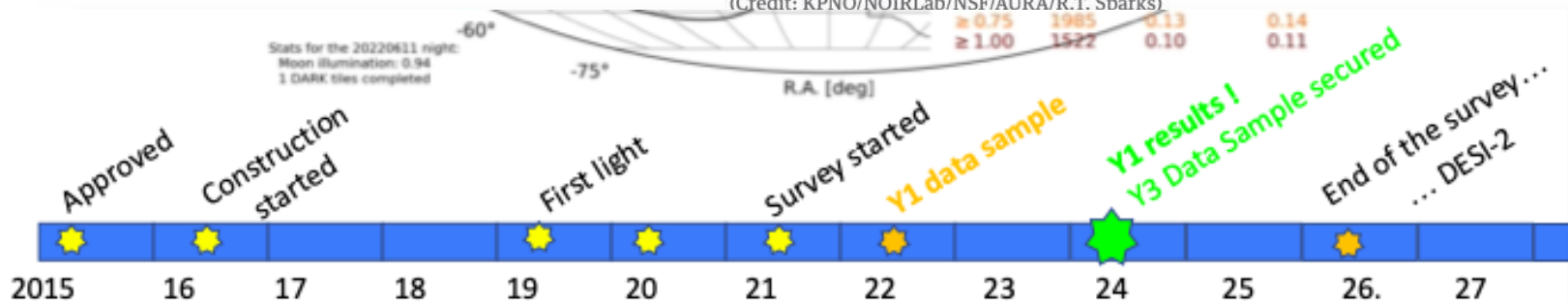
By Elizabeth Ball and Lauren Biron

March 19, 2025

A new data release from the Dark Energy Spectroscopic Instrument is now available for researchers to explore. The collection contains information on 18.7 million galaxies, quasars, and stars – the largest dataset of its kind ever shared.



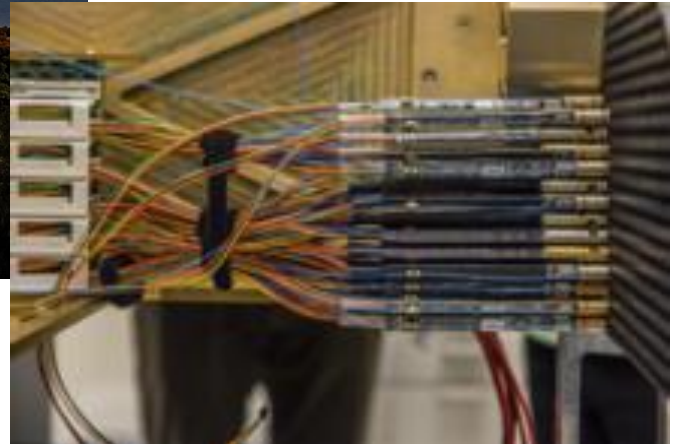
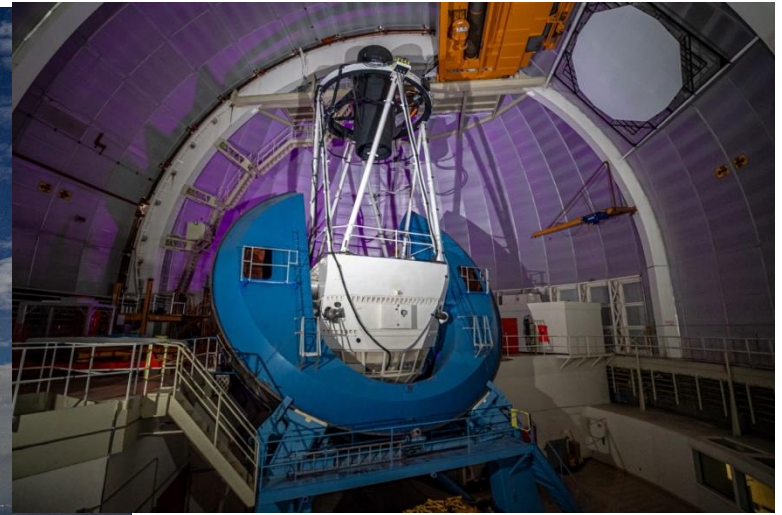
DESI observes the sky from the Mayall Telescope, shown here beneath the Milky Way.
(Credit: KPNO/NOIRLab/NSF/AURA/R.T. Sparks)



Dark Energy Spectroscopic Instrument DESI



Kitt Peak Observatory, AZ
Mayal 4m telescope



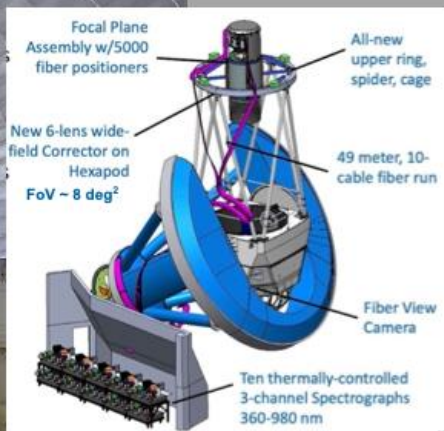
Credits: <https://www.desi.lbl.gov/photos/>



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Mayall 4m telescope @ Kitt Peak (AZ)



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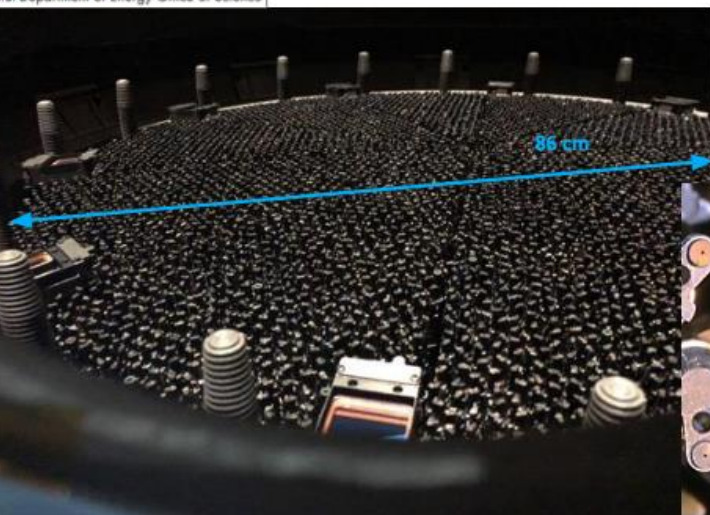
...and Spectra



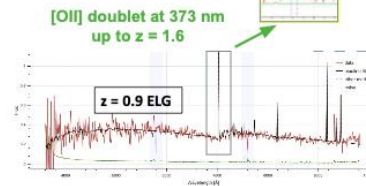
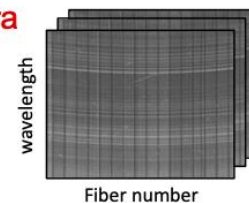
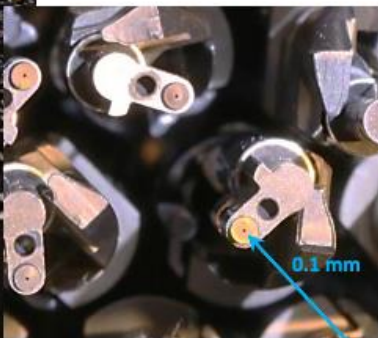
**DARK ENERGY
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INSTRUMENT**

U.S. Department of Energy Office of Science

Focal plane – 5000 robotic positioners



- Exposure time (dark) : 1000s
- Configuration of the whole focal plane ~140s
- CCD Readout
- Go to next pointing



Credit: E. Burtin Moriond 2024



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SPECTROSCOPIC
INSTRUMENT

U.S. Department of Energy Office of Science

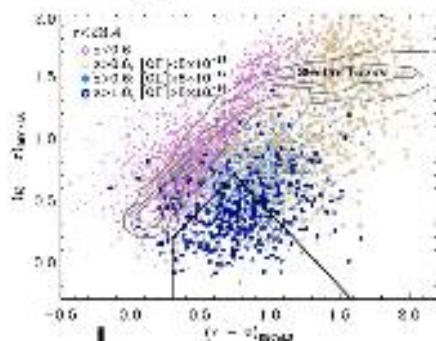
INR

Multi-object Spectroscopy

Imaging Surveys (2014-2019)

+WISE IR sat

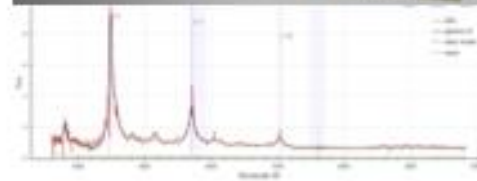
Target Selection



Observation...



...of 5000 objects
every ~20mins...

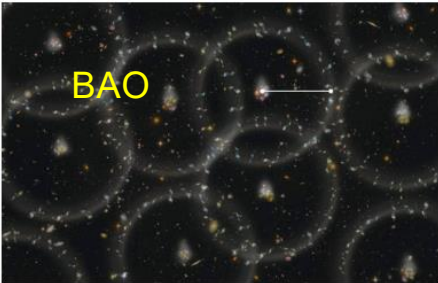
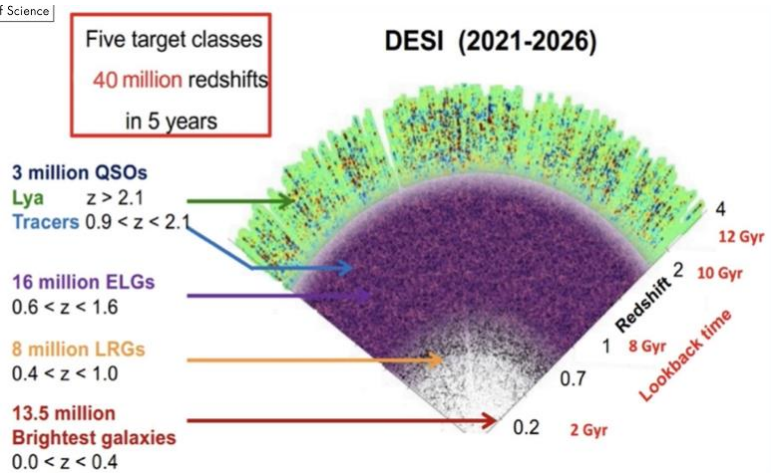


...and measure their redshift

Credit: E. Burtin Moriond 2024

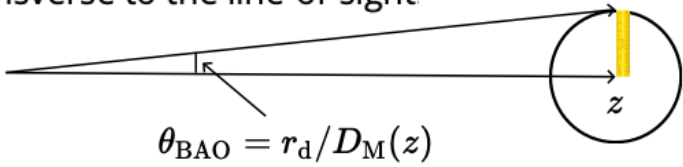
Dark Energy Spectroscopic Instrument DESI

Credit: E. Burtin Moriond 2024

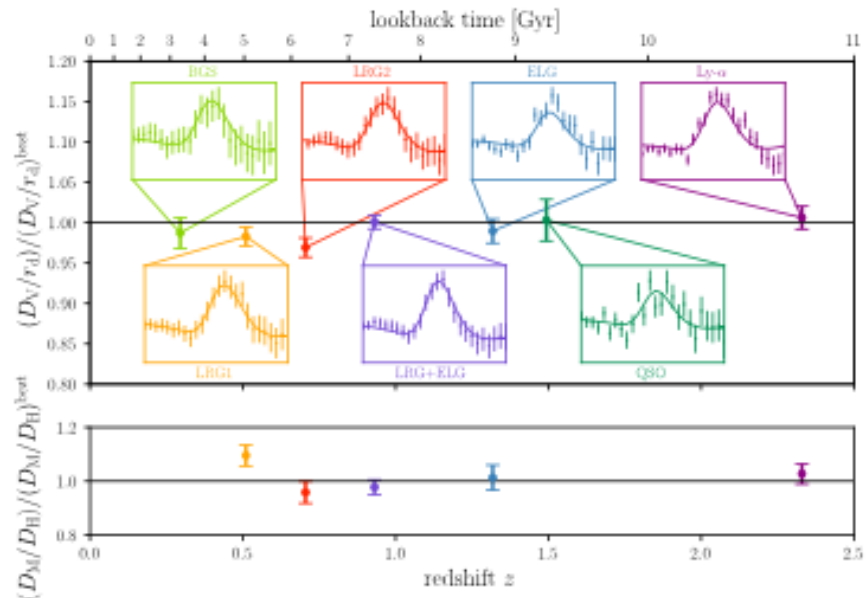
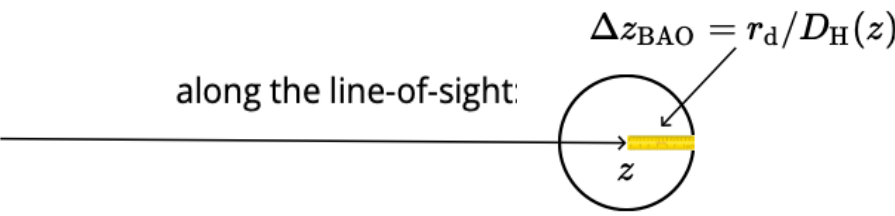


BAO measures ratios of distances over the sound horizon scale at the drag epoch ["standard ruler"] r_d

transverse to the line-of-sight



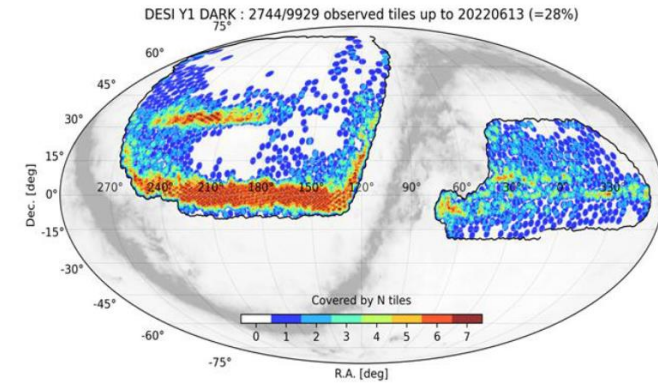
along the line-of-sight:



- transverse to the line-of-sight: $D_M(z)/r_d$
- along the line-of-sight: $D_H(z)/r_d = c/(H(z)r_d)$
- isotropic average: $D_V(z)/r_d = (zD_M^2(z)D_H(z))^{1/3}/r_d$



The DESI DR1 sample (DESI 2024)



Hee-Jong Seo

6M objects
Biggest data set.
Catalog-level blinded analysis

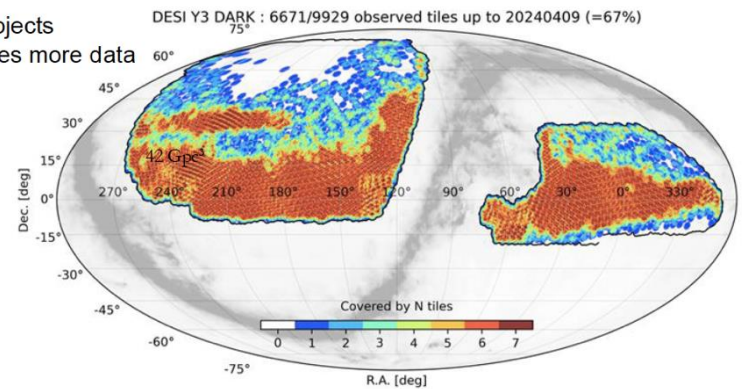
Aggregate distance precision with Year 1 data:
0.49%

Cf. All BAOs over two decades:
0.60%

16

The DESI DR2 sample (2025)

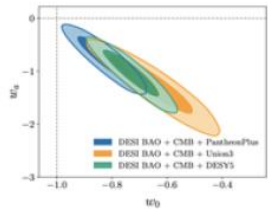
14M objects
2.4 times more data



Hee-Jong Seo

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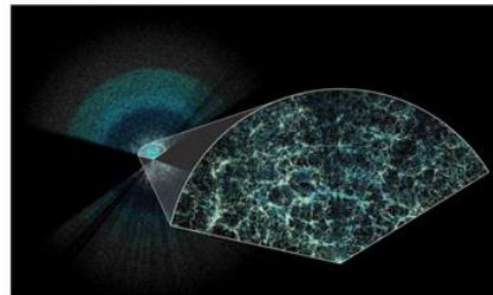
DESI 2024 results



arXiv:2404.03002,
2404.03001, 2404.03002

New 3D cosmic map raises quest over future of universe, scientist

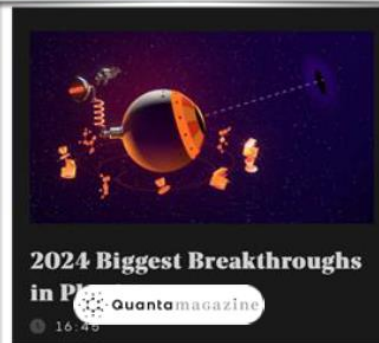
Researchers say findings from map with three times more galaxies than previous efforts could challenge standard idea of dark energy



The New York Times

A 'Tantalizing 'Hint' That Astronomers Got Dark Energy All Wrong

Scientists may have discovered a major flaw in their understanding of that mysterious cosmic force. That could be good news for the fate of the universe.



Credit: H.-J. Seo
Phenomenology 2025 Symposium

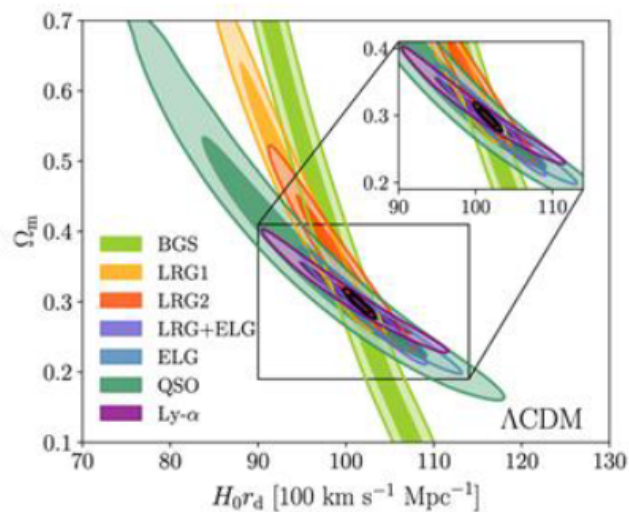
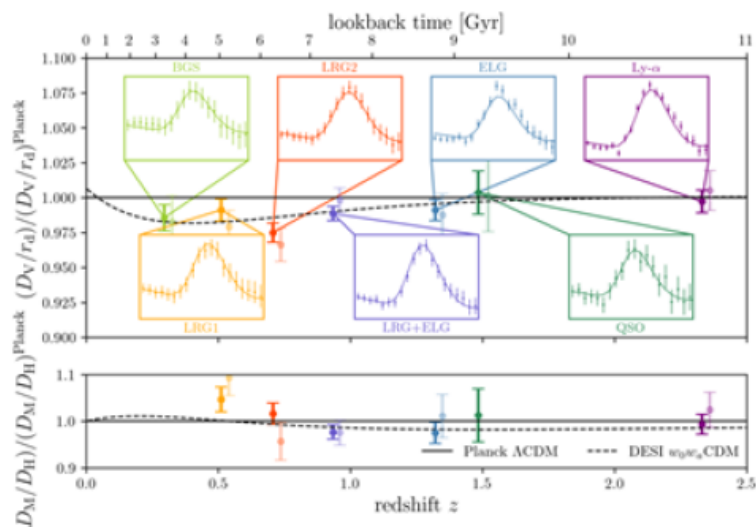
1
7

DESI 2025 BAO

BAO data: $\Delta\theta$ and $\Delta z \longrightarrow D_M / r_d$ and D_H / r_d

$$D_V = (z D_M(z)^2 D_H(z))^{1/3}$$

Ω_M and $H_0 r_d$



Aggregated precision on BAO distance scale:

0.28% (vs. 0.49% for DR1)

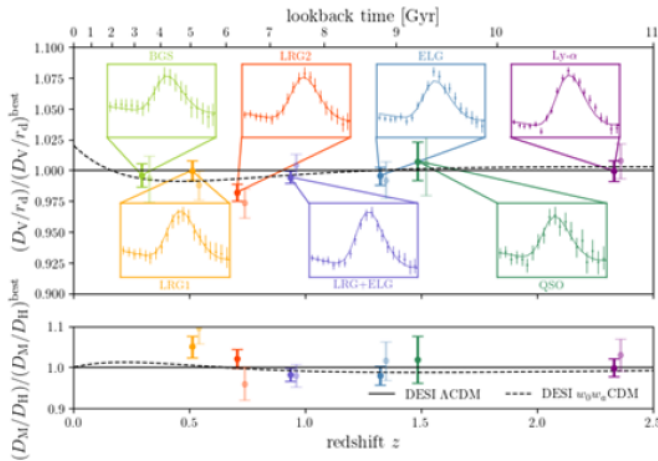
Agreement & complementarity
between tracers

Hee-Jong Seo

2

Dark Energy with constant w

Credit: H.-J. Seo
Phenomenology 2025 Symposium



DESI data are comparable with $w=-1$ (Λ CDM) when w is assumed constant.

But we observe mild tensions between Λ CDM fits of different expansion probes (e.g., CMB and SNe).

Hee-Jong Seo

2
7

Dark Energy Equation of State

Constant EoS parameter w

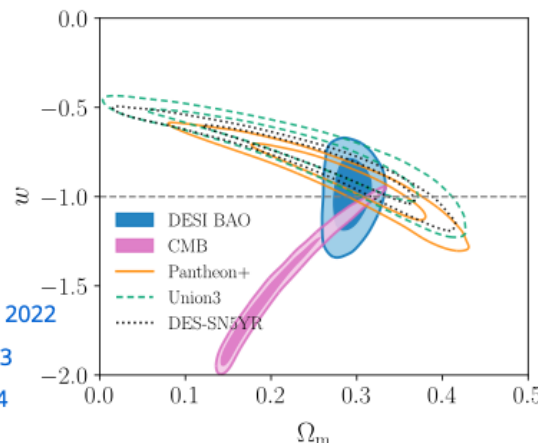
$$\Omega_m = 0.293 \pm 0.015 \quad (5.1\%)$$

$$w = -0.99^{+0.15}_{-0.13} \quad (15\%)$$

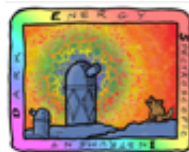
DESI

SNe:

- **Pantheon+** Brout, Scolnic, Popovic et al., 2022
- **Union3** Rubin, Aldering, Betoule et al. 2023
- **DES-SN5YR** DES Collaboration et al. 2024



Credit: A.de Mattia Moriond 2024



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Credit: A.de Mattia Moriond 2024

Dark Energy Equation of State

DR1

Combining all DESI + CMB + SN

$$w_0 = -0.827 \pm 0.063 \quad w_a = -0.75^{+0.29}_{-0.25}$$

DESI + CMB + Pantheon+ $\Rightarrow 2.5\sigma$

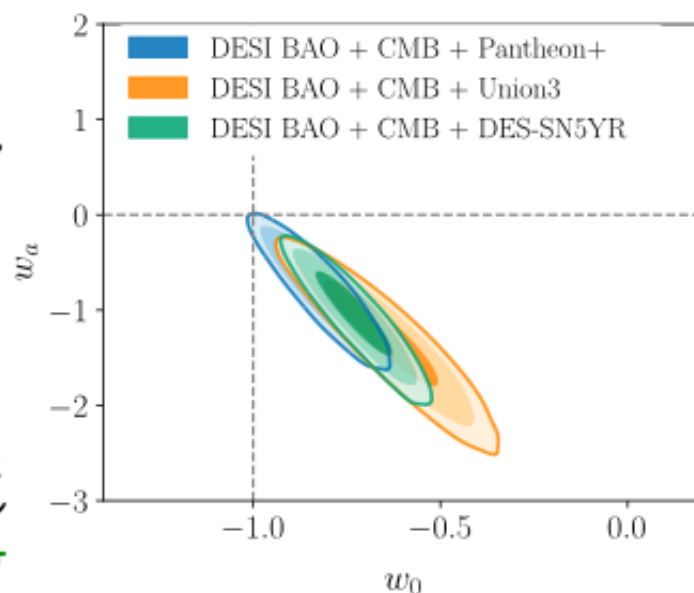
$$w_0 = -0.64 \pm 0.11 \quad w_a = -1.27^{+0.40}_{-0.34}$$

DESI + CMB + Union3 $\Rightarrow 3.5\sigma$

$$w_0 = -0.727 \pm 0.067 \quad w_a = -1.05^{+0.31}_{-0.27}$$

DESI + CMB + DES-SN5YR $\Rightarrow 3.9\sigma$

$w_0 > -1, w_a < 0$ favored, level varying on the SN dataset



Essentially confidence contours shrink
and become detached from $(w_0 = -1, w_a = 0)$



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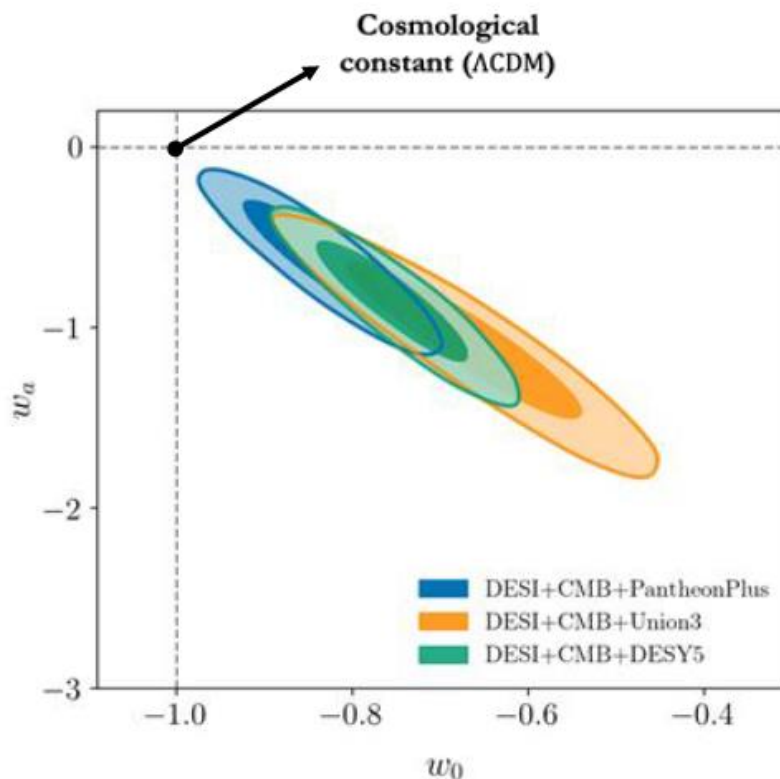
Dynamical Dark Energy: DR2

Varying EoS

$$w(a) = w_0 + (1 - a)w_a$$

- Significance of rejection of Λ CDM:
 - DESI+CMB+Pantheon+: 2.8σ
 - DESI+CMB+Union3 : 3.8σ
 - DESI+CMB+DESY5: 4.2σ

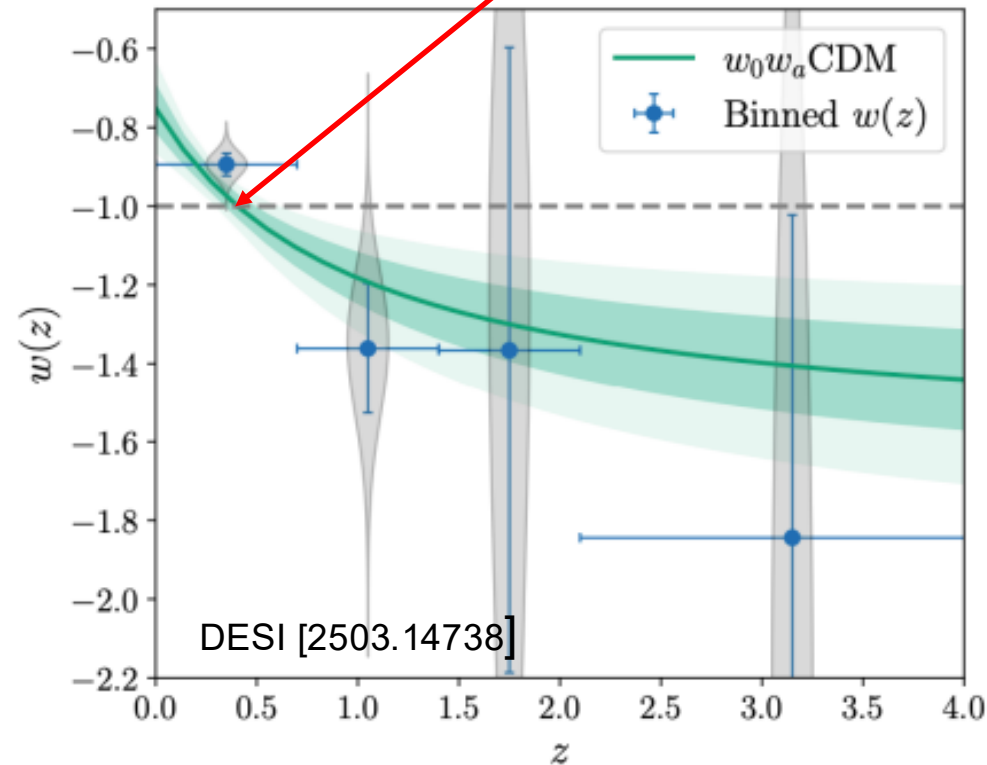
Pantheon+, Union3, DESY5:
SNIa data



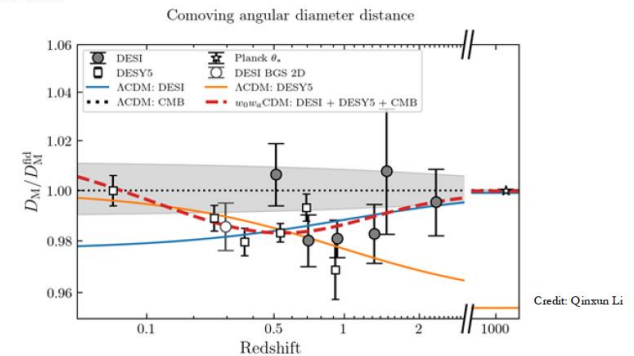
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Data prefer phantom crossing

Phenomenologically $w(z)$ is not tracing scalar field DE



Dynamic DE reconciles different probes



But, except for the H_0 tension...

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Inspired by data-suggested phantom crossing

THE ASTROPHYSICAL JOURNAL LETTERS, 1001:L21 (8pp), 2026 April 10

<https://doi.org/10.3847/2041-8213/ac5a36>

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Metastability in Emergent Dark Energy: A New Framework Confronting Cosmological Observations

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Assumed e.o.s.

$$w(z) = -1 - \Delta \tanh\left(\log_{10}\left(\frac{1+z}{1+z_t}\right)\right).$$

CAMB Boltzmann solver



Confront with
CMB, BAO, SNIa

DE density

$$\rho_{\text{de}}(z) = \rho_{\text{de},0} \left[\frac{\cosh\left(\log_{10}\left(\frac{1+z}{1+z_t}\right)\right)}{\cosh\left(\log_{10}\left(\frac{1}{1+z_t}\right)\right)} \right]^{-3\Delta \ln 10}$$

linear perturbations

$$\delta'_{\text{de}} = -(1 + w_{\text{de}})(\theta_{\text{de}} - 3\phi') - 3\mathcal{H}\delta_{\text{de}}(c_{\text{s,de}}^2 - w_{\text{de}}) - 9(1 + w_{\text{de}})(c_{\text{s,de}}^2 - c_{\text{a,de}}^2)\mathcal{H}^2 \frac{\theta_{\text{de}}}{k^2},$$

$$\theta'_{\text{de}} = -(1 - 3c_{\text{s,de}}^2)\mathcal{H}\theta_{\text{de}} + \frac{c_{\text{s,de}}^2}{1 + w_{\text{de}}}k^2\delta_{\text{de}} + k^2\psi.$$

MEDE fitted to the data - Results

Table 1
Constraints on Cosmological Parameters for the Flat MEDE Model

Model Data	MEDE Model		
	CMB+DESI	CMB+DESI+DES-Dovekie	CMB+DESI+PantheonPlus
z_t	$0.500^{+0.130}_{-0.110}$	$0.418^{+0.070}_{-0.085}$	$0.425^{+0.084}_{-0.120}$
Δ	>1.12	1.03 ± 0.32	$0.87^{+0.29}_{-0.35}$
$\Omega_b h^2$	0.02226 ± 0.00012	0.02226 ± 0.00012	0.02226 ± 0.00013
$\Omega_c h^2$	0.11895 ± 0.00078	0.11902 ± 0.00077	0.11890 ± 0.00081
$\log(10^{10} A_s)$	3.046 ± 0.012	3.045 ± 0.012	3.046 ± 0.012
n_s	0.9662 ± 0.0036	0.9661 ± 0.0035	0.9664 ± 0.0035
τ_{reio}	0.0564 ± 0.0067	0.0561 ± 0.0062	0.0567 ± 0.0066
H_0 (km s $^{-1}$ Mpc $^{-1}$)	$66.30^{+1.20}_{-1.40}$	67.43 ± 0.56	67.56 ± 0.58
Ω_m	0.3230 ± 0.0120	0.3122 ± 0.0055	0.3108 ± 0.0056
σ_8	0.8020 ± 0.0120	0.8116 ± 0.0073	0.8120 ± 0.0079
r_{drag}	147.51 ± 0.21	147.49 ± 0.20	147.51 ± 0.20
χ^2_{min}	11006.35	12632.67	12406.45
$\Delta\chi^2_{\text{MEDE-}\Lambda\text{CDM}}$	-1.65	-8.43	-3.94
$\Delta\chi^2_{\text{MEDE-CPL}}$	4.22	4.53	3.39
$\Delta\text{DIC}_{\text{MEDE-}\Lambda\text{CDM}}$	-6.53	-11.36	-9.29
$\Delta\text{DIC}_{\text{MEDE-CPL}}$	1.71	1.32	0.74

Note. Results are shown for the CMB+DESI, CMB+DESI+DES-Dovekie, and CMB+DESI+PantheonPlus datasets, and we also include the minimum χ^2 , $\Delta\chi^2$, and ΔDIC values relative to the ΛCDM and CPL models.

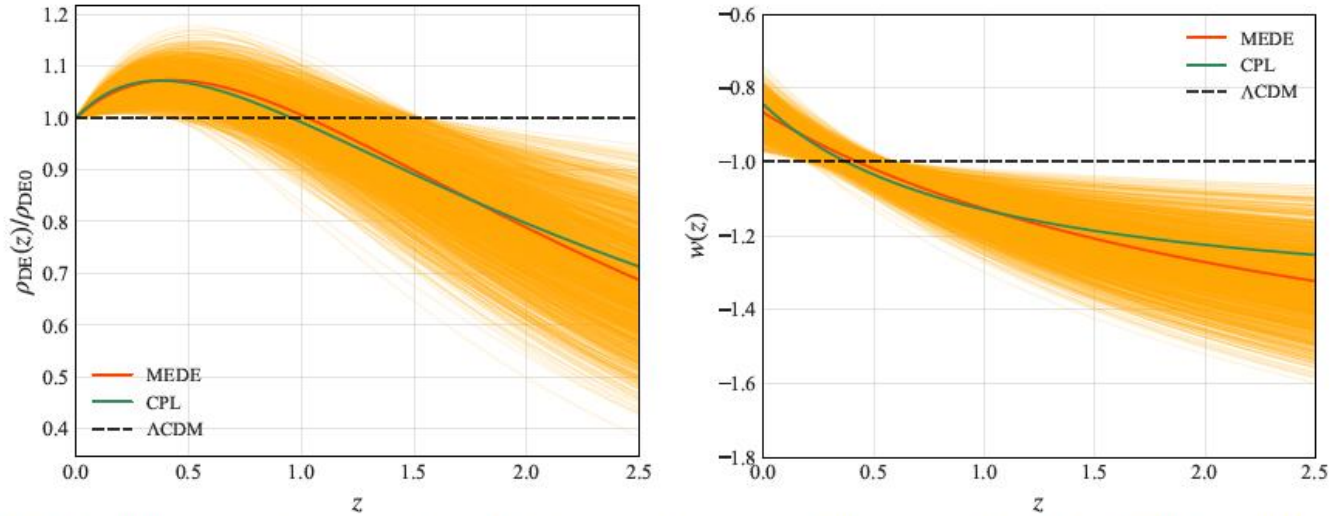


Figure 3. Redshift evolution of the dark-energy sector. Left: the dark-energy density $\rho_{DE}(z)/\rho_{DE0}$. Right: the equation of state $w_{DE}(z)$. The red solid line represents the best-fit MEDE model (CMB+DESI+PantheonPlus). The light-orange region shows the 2σ confidence region derived from MCMC chains, illustrating the posterior distribution of parameters. The green solid line shows the CPL parameterization ((CMB+DESI+PantheonPlus best-fit), and the black dashed line represents Λ CDM. The MEDE model exhibits a smooth transition around $z_t \sim 0.42$, crossing the phantom divide $w = -1$ from phantom-like ($w < -1$) at high redshifts to quintessence-like ($w > -1$) at low redshifts.

correspondence between CPL and MEDE:

$$w_0 = -1 + \Delta \tanh[\log_{10}(1 + z_t)], \quad (8)$$

$$w_a = -\frac{1}{\ln 10} \Delta \operatorname{sech}^2[\log_{10}(1 + z_t)]. \quad (9)$$

Substituting the central values of Δ and z_t fitted to the (CMB+DESI+DES-Dovekie) data (see Table 1), one obtains $w_0 = -0.845^{+0.076}_{-0.067}$, $w_a = -0.44^{+0.13}_{-0.13}$, which is within 1σ of the DESI (M. Abdul Karim et al. 2025) best fit $w_0 = -0.838 \pm 0.055$, $w_a = -0.62^{+0.22}_{-0.19}$. This explains the similarity in performance between CPL and MEDE in the redshift range probed by current data: they can be viewed as two sides of the same coin, both conveying the message that an evolving equation of state should be taken seriously as an alternative to the Λ CDM model.

Next step ... (will appear in arxiv soon)

Scalar-Quintom Reconstruction of a DESI-Motivated Phantom-Divide Transition

Tonghua Liu,¹ Xiaolei Li,² Marek Biesiada,³ Tian-Nuo Li,⁴ Arman Shafieloo,^{5,6} and Xin Zhang^{4,7,8}

Take MEDE

$$w_{\text{de}}(z) = -1 - \Delta \tanh \left[\ln \left(\frac{1+z}{1+z_t} \right) \right]$$

Auxiliary variable $u(z) = \ln \left(\frac{1+z}{1+z_t} \right)$

Suppose that MEDE is derived from the scalar-quintom field theory

$$\begin{aligned} \text{scalar quintom} \quad & \rho_q = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} \dot{\psi}^2 + V, \quad p_q = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} \dot{\psi}^2 - V, \\ \text{kinetic imbalance} \quad & K(z) = -\frac{\Delta}{2} \tanh u(z) \rho_{\text{de}}(z), \\ & V(z) = \left[1 + \frac{\Delta}{2} \tanh u(z) \right] \rho_{\text{de}}(z). \end{aligned}$$

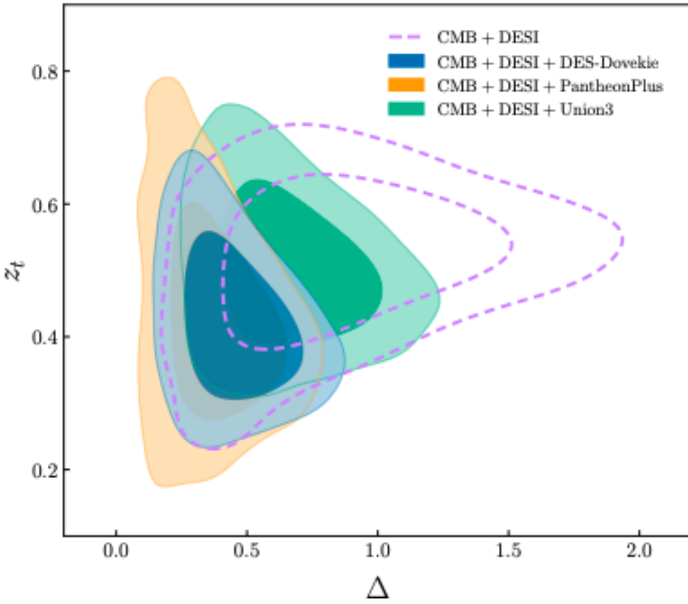
$$S_q = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\partial\phi)^2 + \frac{1}{2} (\partial\psi)^2 - V(\phi, \psi) \right] + S_m + S_r.$$

Consistency test:

- Fit MEDE parameters to the data CMB+BAO+SN Ia
- Take best fit MEDE values to fix reconstructed scalar-quintom fields
- Sample fields around MEDE trajectory and evolve

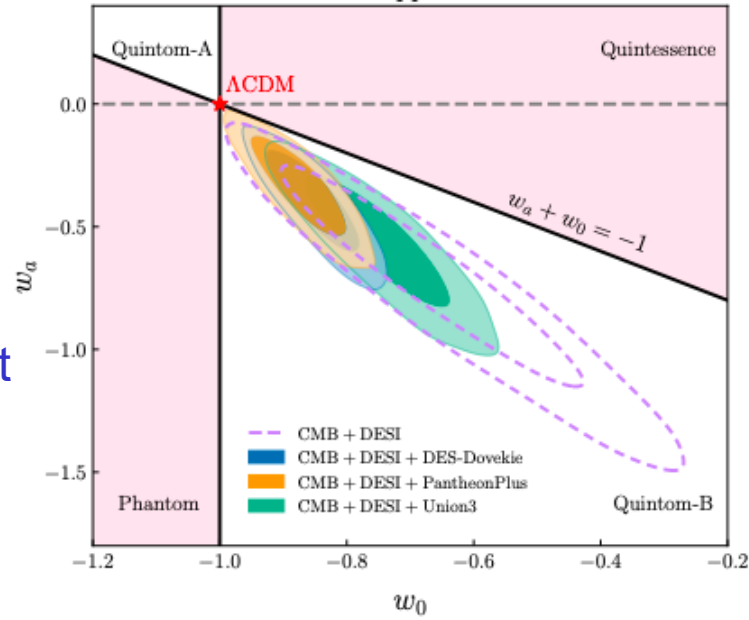
$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} + V_{,\phi} &= 0, \\ \ddot{\psi} + 3H\dot{\psi} - V_{,\psi} &= 0, \\ 3M_{\text{Pl}}^2 H^2 &= \rho_m + \rho_r + \rho_q, \\ \dot{H} &= -\frac{\rho_m + 4\rho_r/3 + \dot{\phi}^2 - \dot{\psi}^2}{2M_{\text{Pl}}^2}. \end{aligned}$$

MEDE

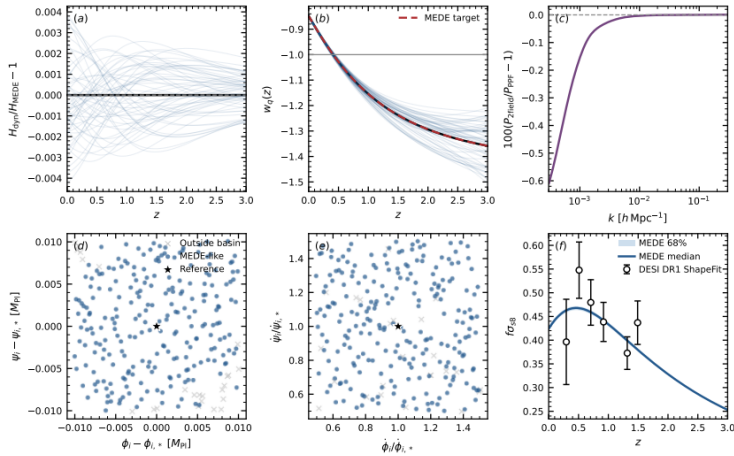


Fit

MEDE mapped to CPL



Conclusion.— Present distance data place a phantom-divide transition near $z \simeq 0.4\text{--}0.5$, although CPL fits the same data equally well. The principal result is therefore dynamical rather than statistical. Once a representative potential is fixed, the coupled field and Friedmann equations reproduce the expansion; the crossing persists across a finite range of late-time initial conditions; and the field perturbations remain regular through $w = -1$.



Test

Is Dark Energy evolving in time?

the DESI data for the first time gave a strong support for the time evolving cosmic equation of state

Is this a “funeral” of Λ CDM ?

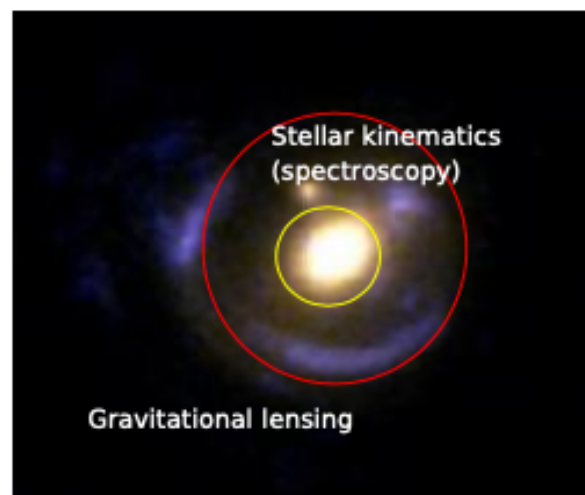
Not really – remember Yakov Zeldovich saying

“ cosmological constant is like a jinn from the bottle – once released, it’s not easy to put it back there “

But we should try – more alternative probes are needed



Strong lensing combined with stellar kinematics



1. The simplest lens model $\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$
Singular Isothermal Sphere (SIS)
 $\theta_E = 4\pi \left(\frac{\sigma_v}{c} \right)^2 \frac{D_{ls}}{D_s}$

2. Spherically symmetric power-law mass density profile $\rho(r) \sim r^{-\gamma}$

stellar dynamics (spherically symmetric Jeans equation):
 mass projected inside the aperture radius
 scaled to the Einstein radius

Strong lensing: mass inside the Einstein radius

$$M_{lens} = \frac{c^2}{4G} \frac{D_l D_s}{D_{ls}} \theta_E^2 \quad + \quad M_{dyn} = \frac{\pi}{G} \sigma_{ap}^2 D_l \theta_E \left(\frac{\theta_E}{\theta_{ap}} \right)^{2-\gamma} f(\gamma)$$

$$\theta_E = 4\pi \frac{\sigma_{ap}^2}{c^2} \frac{D_{ls}}{D_s} \left(\frac{\theta_E}{\theta_{ap}} \right)^{2-\gamma} f(\gamma)$$

3. More general spherically symmetric power-law model

$$\rho(r) = \rho_0 \left(\frac{r}{r_0} \right)^{-\alpha} \quad \text{total mass}$$

$$\nu(r) = \nu_0 \left(\frac{r}{r_0} \right)^{-\delta} \quad \text{luminous matter}$$

$$\beta(r) = 1 - \sigma_t^2 / \sigma_r^2 \quad \text{stellar anisotropy}$$

$$\theta_E = 2\sqrt{\pi} \frac{\sigma_{ap}^2}{c^2} \frac{D_{ls}}{D_s} \left(\frac{\theta_E}{\theta_{ap}} \right)^{2-\alpha} (\xi - 2\beta) \lambda(\alpha)$$

Strong lensing cosmography – e.o.s. of the Universe

PHYSICAL REVIEW D 73, 023006 (2006)

Strong lensing systems as a probe of dark energy in the universe

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Department of Astrophysics and Cosmology, University of Silesia, Uniwersytecka 4, 40-007 Katowice, Poland
(Received 3 November 2005; published 23 January 2006)

Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

JCAP03(2012)016

Constraints on cosmological models from strong gravitational lensing systems

Shuo Cao,^a Yu Pan,^{a,b} Marek Biesiada,^c Włodzimierz Godłowski^d
and Zong-Hong Zhu^{a,1}

10 cluster lenses
70 galaxy lenses
from SLACS

Cosmological model	Best-fitting parameters ($n = 80$)	Best-fitting parameters ($n = 46$)
Λ CDM	$\Omega_m = 0.20^{+0.07}_{-0.07}$	$\Omega_m = 0.26^{+0.11}_{-0.10}$
w CDM	$w = -1.02^{+0.26}_{-0.26}$	$w = -1.15^{+0.34}_{-0.35}$
CPL	$w_0 = 0.60 \pm 1.76$ $w_a = -7.37 \pm 8.05$	$w_0 = -0.24 \pm 2.42$ $w_a = -6.35 \pm 9.75$

THE ASTROPHYSICAL JOURNAL, 806:185 (12pp), 2015 June 20
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COSMOLOGY WITH STRONG-LENSING SYSTEMS

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118 lenses from
SLACS sample

Monthly Notices

ROYAL ASTRONOMICAL SOCIETY

Mon. Not. R. Astron. Soc. **406**, 1055–1059 (2010)

doi:10.1111/j.1365-2966.2010.16725.x

Cosmic equation of state from strong gravitational lensing systems

Marek Biesiada,^{*} Aleksandra Piórkowska^{*} and Beata Malec^{*}

Department of Astrophysics and Cosmology, Institute of Physics, University of Silesia, Uniwersytecka 4, 40-007 Katowice, Poland

Accepted 2010 March 22. Received 2010 March 18; in original form 2010 February 5

20 galaxy lenses
from SLACS + LSD

WMAP7+BAO+H0

$$\Omega_m = 0.272$$

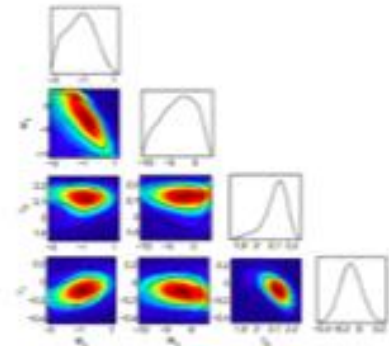
$$w = -1.10 \pm 0.14$$

$$w_0 = -0.93 \pm 0.13$$

$$w_a = -0.41 \pm 0.71$$

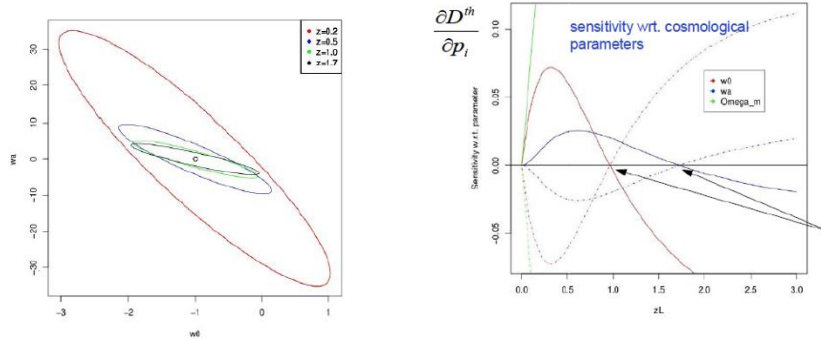
Komatsu et al. 2011

doi:10.1088/0004-637X/806/2/185



Cosmological applications of strong lensing systems

- SL systems may have different degeneracies than SN Ia



A. Piórkowska, MB et al. APP B 2013

THE ASTROPHYSICAL JOURNAL SUPPLEMENT SERIES, 285:4 (15pp), 2026 July
© 2026. The Author(s). Published by the American Astronomical Society.

OPEN ACCESS

<https://doi.org/10.3847/1538-4365/aef>

Hierarchical Cosmological Constraints through a Strong-lensing Distance Ratio

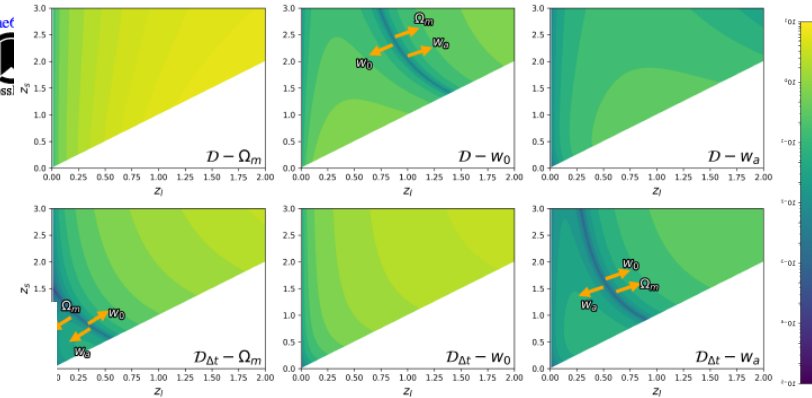
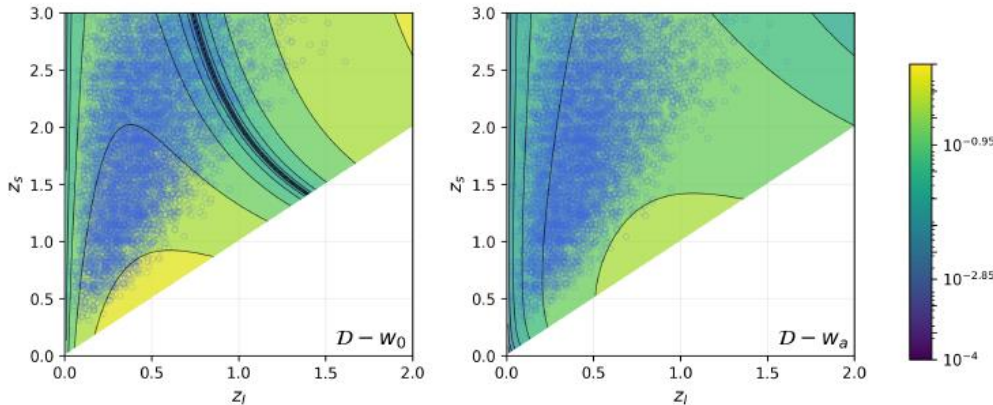
Shuaibo Geng¹, Shuo Cao^{2,3}, Marek Biesiada¹, Xinyue Jiang^{1,2,3}, Yalong Nan^{2,3}, and Chenfa Zheng^{2,3}

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Received 2026 March 3; revised 2026 April 22; accepted 2026 April 29; published 2026 June 16



Goal: select the (LSST, Euclid)
gold sample
best for the purpose

More cosmological applications of strong lensing systems

- Measurements of cosmic curvature

Distance sum rule – valid in any FLRW metric

$$d_{ls} = \sqrt{1 + \Omega_k d_l^2} d_s - \sqrt{1 + \Omega_k d_s^2} d_l$$

$$\Omega_k(z_l, z_s) = \frac{d_l^4 + d_s^4 + d_{ls}^4 - 2d_l^2 d_s^2 - 2d_l^2 d_{ls}^2 - 2d_s^2 d_{ls}^2}{4d_l^2 d_s^2 d_{ls}^2}$$

This is a function of two redshifts, but within the FLRW metric it should be just a single number !

J..Qi, MB et al. MNRAS 2019
S. Cao, MB et al. SciRep 2019
T. Liu MB et al. MNRAS 2020
J. Qi, MB et al. MNRAS 2021
S.Cao, MB et al. ApJ 2022

- Tests of GR – γ PPN parameter and modified gravity

THE ASTROPHYSICAL JOURNAL, 835:92 (7pp), 2017 January 20

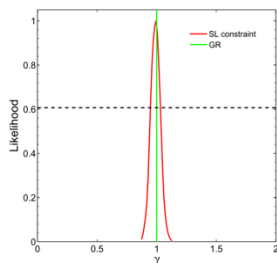


Figure 3. Normalized posterior likelihood of the PPN γ parameter obtained with rigid priors on the nuisance parameters (α, β, δ) .

$$\gamma = 0.995^{+0.037}_{-0.047}$$

S. Cao, MB et al. ApJ 2017
Y.Lian MB et al. MNRAS 2021



$$\frac{\Delta\gamma}{\gamma} \sim 10^{-3} - 10^{-4}$$

Conclusions

History of modern cosmology witnessed numerous tensions.

Age of the Universe vs. globular clusters ...

matter budget issues ...

Modern tensions are embedded in subtle issues of calibrations and systematics

Their persistent significance may be telling us something, e.g. that Λ CDM model or even FLRW framework are too simplified.

We need further development of alternative cosmological probes (strong lensing, GW standard sirens, lensing of transient events, AGNs, compact radio-sources, etc.)

Supplementary slides – Hubble tension

Tensions within Λ CDM

- H_0 tension** – persistent significant disagreement between early and late-time cosmological probes

$$H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$H_0 = 73.17 \pm 0.86 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

- S_8 tension**

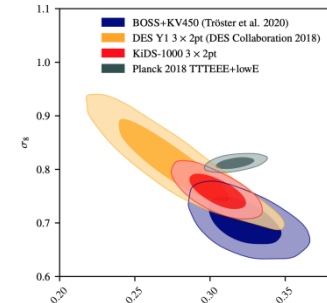
$S_8 \equiv \sigma_8 \sqrt{\Omega_m/0.3}$ parameter.

CMB

$$S_8 = 0.834 \pm 0.016$$

DES3Y

$$S_8 = 0.772 \pm 0.017$$



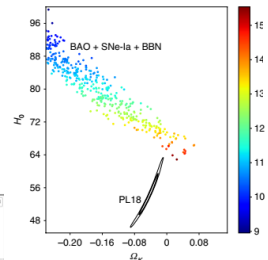
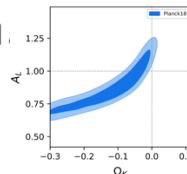
Credit: C. Heymans et al. 2020
arXiv:2007.15632

- Curvature parameter Ω_k**



Planck evidence for a closed Universe and possible crisis for cosmology

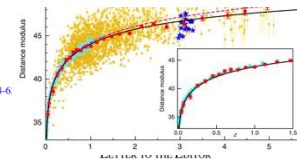
Eleonora Di Valentino¹, Alessandro Melchiorri^{2*} and Joseph Silk^{3,4,5}



Cosmological constraints from the Hubble diagram of quasars at high redshifts

G. Risaliti^{1,2*} and E. Lusso³

A&A 628, L4 (2019)
<https://doi.org/10.1051/0004-6369/201935504>
© ESO 2019



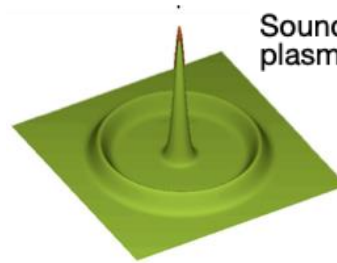
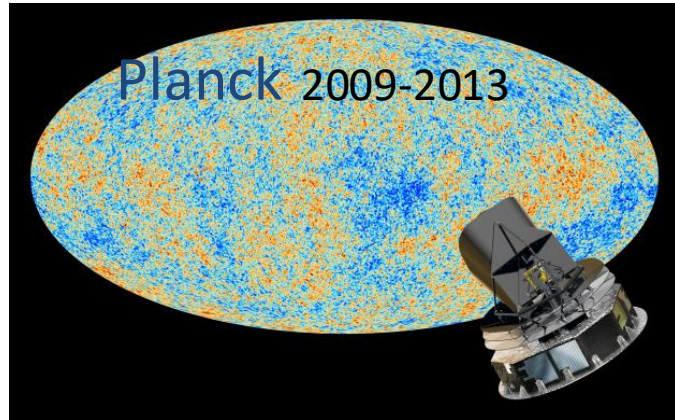
Astronomy
Astrophysics

Tension with the flat Λ CDM model from a high-redshift Hubble diagram of supernovae, quasars, and gamma-ray bursts

E. Lusso^{1,2}, E. Piedipalumbo^{3,4}, G. Risaliti^{1,2}, M. Paolillo^{3,4,5}, S. Bisogni², E. Nardini², and L. Amati^{5,6}

- Validity of Λ CDM itself** – DESI results gave evidence of evolving DE

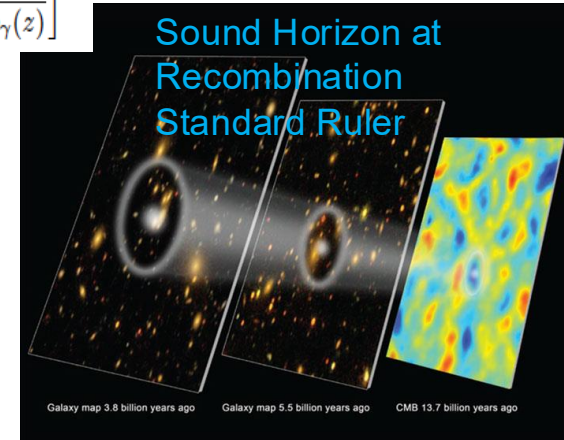
Early time probes CMB, BAO



Sound waves in primordial plasma:

standard ruler – sound horizon

$$r_s = \int_{z_*}^{\infty} \frac{c_s(z)}{H(z)} dz$$

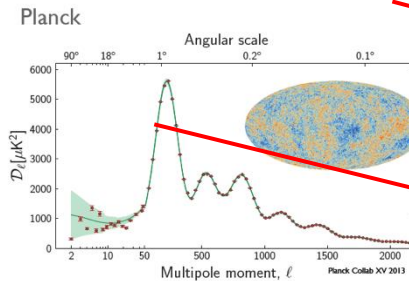


A&A 641, A6 (2020)
https://doi.org/10.1051/0004-6361/201833910
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Planck 2018 results

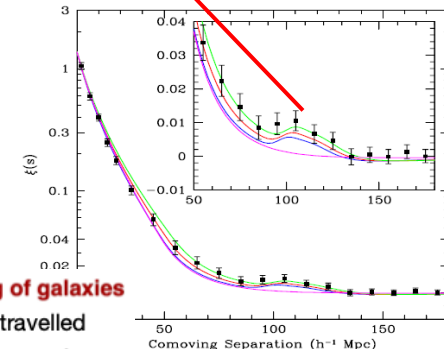
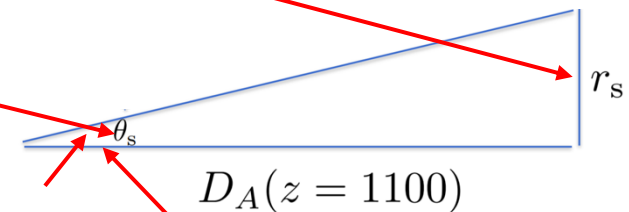
Astronomy
Astrophysics
Special issue

Planck 2018 results
VI. Cosmological parameters



calculated

measured



⇒ a residual spherical peak in clustering of galaxies

size of feature = distance sound wave travelled

= sound horizon scale at recomb.

≈ 150 Mpc

$r_s = 147.6$ Mpc from Planck and BBN
values of ρ_b , ρ_γ and ρ_{CDM}

Measuring H_0 with CMB

Steps within Λ CDM: See e.g. Knox & Millea's *Hubble Hunter's Guide*, PRD 101 (2020) 043533

- Infer ω_b from even/odd peak height modulation
- Infer ω_m from “potential envelope” effect (early ISW effect)
- Calculate $r_s^* \sim \int_{z_*}^{\infty} dz c_s(z, \omega_b) / \sqrt{\omega_m(1+z)^3 + \omega_r(1+z)^4}$
- Measure $\theta_s \sim \pi / \Delta \ell$ from peak spacing
- With r_s^* and θ_s known, infer $D_A^* = r_s^* / \theta_s$

$$\theta_s = \frac{r_s}{D_A(z)} = \frac{H_0 r_s}{\int_0^z \frac{dz}{E(z)}}$$

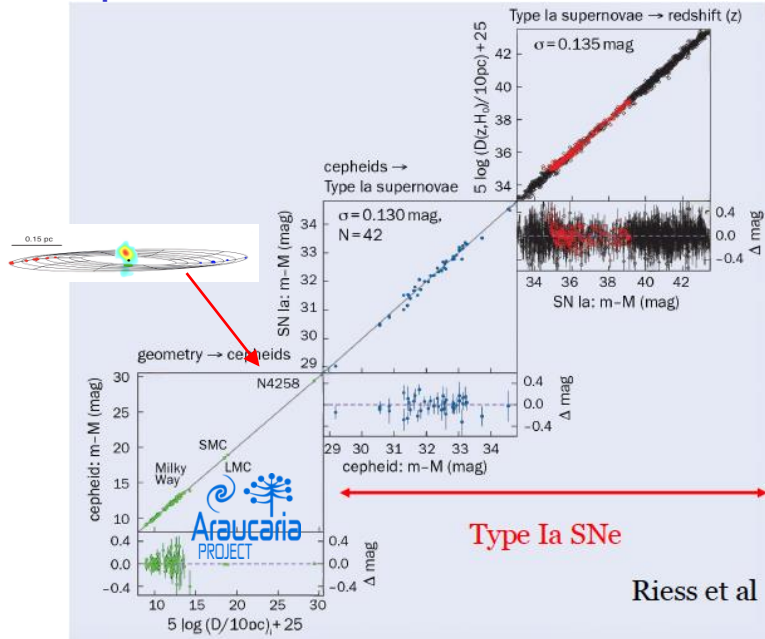
Here Λ CDM is assumed

$r_s = 147.6$ Mpc from Planck and BBN
values of ρ_b , ρ_v and ρ_{CDM}

$$E(z) = [\Omega_{0m}(1+z)^3 + (1 - \Omega_{0m})]^{\frac{1}{2}}$$

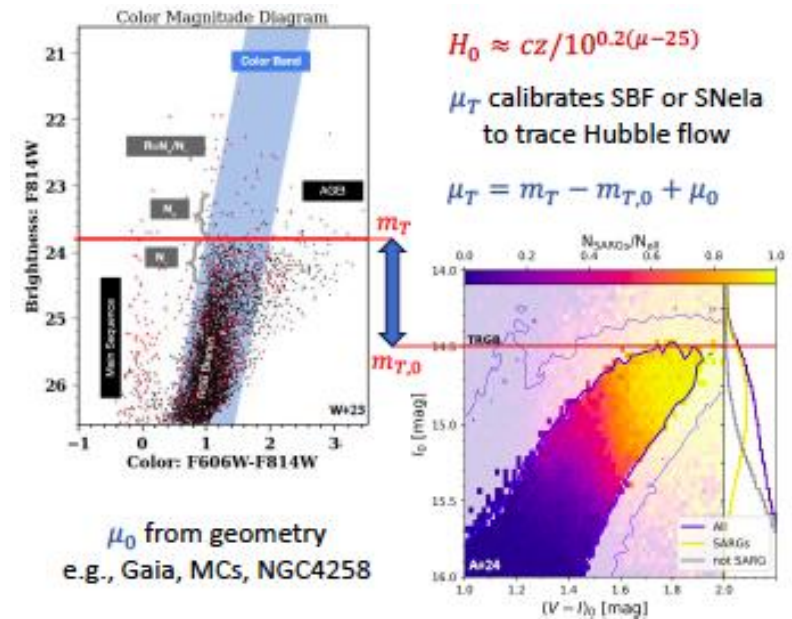
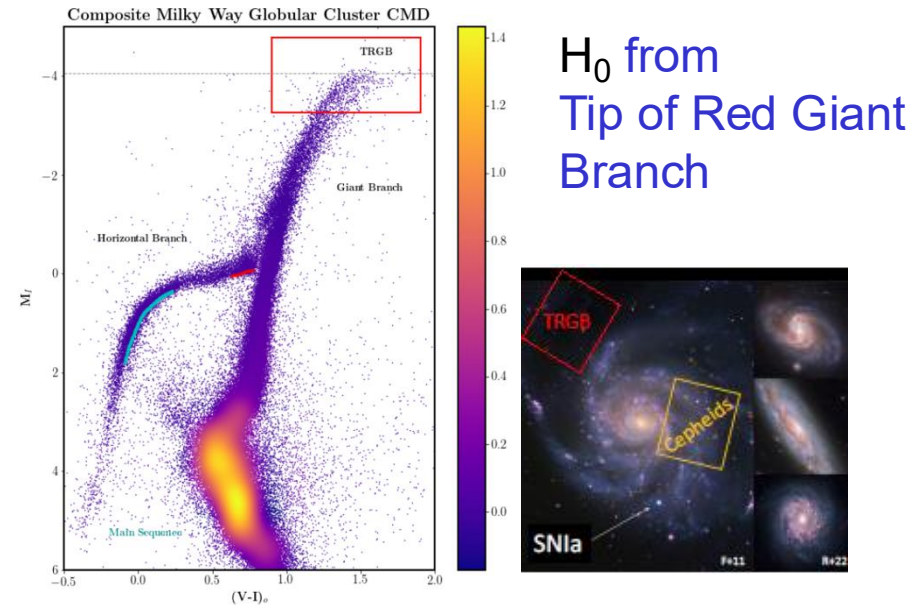
Degeneracy between r_s and H_0 and $E(z)$.

H₀ from 3-rung distance ladder Cepheids



Systematics:

- multiple photometric systems 1-2%
- crowding
- metallicity
- Cepheids in binaries



$$\hat{H}_0 = 70.39 \pm 1.22 \text{ (stat)} \pm 1.33 \text{ (sys)} \pm 0.70 \text{ } (\sigma_{\text{SN}})$$

Measuring H_0 with standard candles: late time calibrators

Fit for H_0 using SNIa Standard Candles, ($0.02 < z < 0.1$)

fit with approximate formula ($0.01 < z < 0.1$)

$$d_L = c(1+z) \int_0^z \frac{dz'}{H(z')}$$

$$D_L(z, q_0) = cz \left[1 + \frac{1}{2}(1 - q_0)z \right]$$

$$m_{th}(z) = M + 5 \log_{10} \left[\frac{d_L(z)}{Mpc} \right] + 25$$

$$D_L(z) = \frac{H_0 d_L(z)}{c}$$

$$m_{th}(z) = M + 5 \log_{10}(D_L(z)) + 5 \log_{10} \left(\frac{c/H_0}{1 Mpc} \right) + 25$$

Degeneracy between M (measured at $z < 0.01$) and H_0
(fit at $z > 0.01$). No $E(z) = H(z)/H_0$ dependence.

measure

measure locally ($z < 0.01$, 40Mpc) using
relative distance indicators (e.g. Cepheids)

Fit (assume M is the
same in the Hubble
flow ($z > 0.01$))

Fit for $H(z)$ and cosmological parameters (Ω_{0m}) $z_{\max} \sim 2$.

Parametrize

$H(z)$:
$$H(z)^2 = H_0^2 [\Omega_{0m}(1+z)^3 + (1 - \Omega_{0m})]$$

$$m_{th}(\Omega_{0m}, \mathcal{M}) = 5 \log_{10} D_L(z; \Omega_{0m}) + \mathcal{M}(M, H_0)$$

$$D_L(z, \Omega_{0m}) = c(1+z) \int_0^z \frac{dz'}{[\Omega_{0m}(1+z')^3 + (1 - \Omega_{0m})]^{1/2}}$$

Minimize:

$$\chi^2(\mathcal{M}, \Omega_{0m}) = \sum_i \left[\frac{m_{obs,i} - m_{th}(z_i; \Omega_{0m}, \mathcal{M})}{\sigma_i^2} \right]$$

$$\mathcal{M} = M + 5 \log \frac{c/H_0}{Mpc} + 25$$

Issues in the SH0ES Analysis for H_0

SN Ia calibration parameters:

absolute magnitude M_B

color and stretch parameters (α, β)

Cepheid calibration parameters:

b_W (period-luminosity),

Z_W (metallicity-luminosity),

M_W (Cepheid zero-point amplitude),

R_W (Cepheid color-luminosity)

best fit values of these parameters

are not consistent among different subgroups of
SN Ia+Cepheid data

E. Mortsell, A. Goobar, et al. ApJ 2022

R. Wojtak, J. Hjorth, MNRAS 2022

CMB with Planck
 Balkenhol et al. (2021), Planck 2018+SPT+ACT : 67.49 ± 0.53
 Pogosian et al. (2020), eBOSS+Planck $\Omega_m H^2$: 69.6 ± 1.8
 Aghanim et al. (2020), Planck 2018: 67.27 ± 0.60
 Aghanim et al. (2020), Planck 2018+CMB lensing: 67.36 ± 0.54
 Ade et al. (2016), Planck 2015, $H_0 = 67.27 \pm 0.66$

CMB without Planck
 Dutcher et al. (2021), SPT: 68.8 ± 1.5
 Aiola et al. (2020), ACT: 67.9 ± 1.5
 Aiola et al. (2020), WMAP9+ACT: 67.6 ± 1.1
 Zhang, Huang (2019), WMAP9+BAO: $68.36^{+0.53}_{-0.51}$
 Hinshaw et al. (2013), WMAP9: 70.0 ± 2.2

No CMB, with BBN
 D'Amico et al. (2020), BOSS DR12+BBN: 68.5 ± 2.2
 Colas et al. (2020), BOSS DR12+BBN: 68.7 ± 1.5
 Philcox et al. (2020), P_1 +BAO+BBN: 68.6 ± 1.1
 Ivanov et al. (2020), BOSS+BBN: 67.9 ± 1.1
 Alam et al. (2020), BOSS+eBOSS+BBN: 67.35 ± 0.97

$P_1(k)$ + CMB lensing
 Philcox et al. (2020), $P_1(k)$ +CMB lensing: $70.6^{+2.3}_{-2.9}$

Cepheids – SNIa
 Riess et al. (2020), R20: 73.2 ± 1.3
 Breuval et al. (2020): 72.8 ± 2.7
 Riess et al. (2019), R19: 74.0 ± 1.4
 Camarena, Marra (2019): 75.4 ± 1.7
 Burns et al. (2018): 73.2 ± 2.3
 Dhawan, Jha, Leibundgut (2017), NIR: 72.8 ± 3.1
 Follin, Knox (2017): 73.3 ± 1.7
 Feeney, Mortlock, Dalmasso (2017): 73.2 ± 1.8
 Riess et al. (2016), R16: 73.2 ± 1.7
 Cardona, Kunz, Pettorino (2016), HPS: 73.8 ± 2.1
 Freedman et al. (2012): 74.3 ± 2.1

TRGB – SNIa
 Soltis, Casertano, Riess (2020): 72.1 ± 2.0
 Freedman et al. (2020): 69.6 ± 1.9
 Reid, Pesce, Riess (2019), SH0ES: 71.1 ± 1.9
 Freedman et al. (2019): 69.8 ± 1.9
 Yuan et al. (2019): 72.4 ± 2.0
 Jang, Lee (2017): 71.2 ± 2.5

Miras – SNIa
 Huang et al. (2019): 73.3 ± 4.0

Masers
 Pesce et al. (2020): 73.9 ± 3.0

Tully – Fisher Relation (TFR)
 Kourkchi et al. (2020): 76.0 ± 2.6
 Schombert, McGaugh, Lelli (2020): 75.1 ± 2.8

Surface Brightness Fluctuations
 Blakeslee et al. (2021) IR-SBF w/ HST: 73.3 ± 2.5
 Khetan et al. (2020) w/ LMC DEB: 71.1 ± 4.1

SNII
 de Jaeger et al. (2020): $75.8^{+5.3}_{-4.9}$

HII galaxies
 Fernández Arenas et al. (2018): 71.0 ± 3.5

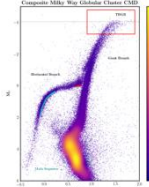
Lensing related, mass model – dependent
 Denzel et al. (2021): $71.8^{+3.9}_{-3.8}$
 Birrer et al. (2020), TDCOSMO+SLACS: $67.4^{+5.1}_{-4.1}$, TDCOSMO: $74.5^{+1.7}_{-1.7}$
 Yang, Birrer, Hu (2020): $H_0 = 73.65^{+1.7}_{-1.7}$
 Millon et al. (2020), TDCOSMO: 74.2 ± 1.6
 Baxter et al. (2020): 73.5 ± 5.3
 Qi et al. (2020): $73.6^{+1.8}_{-1.8}$
 Liao et al. (2020): $72.8^{+1.6}_{-1.6}$
 Liao et al. (2019): 72.2 ± 2.1
 Shajib et al. (2019), STRIDES: $74.2^{+2.2}_{-2.2}$
 Wong et al. (2019), H0LICOW 2019: $73.3^{+1.9}_{-1.9}$
 Birrer et al. (2018), H0LICOW 2018: $72.5^{+2.1}_{-2.1}$
 Bonvin et al. (2016), H0LICOW 2016: $71.9^{+2.0}_{-2.0}$

Optimistic average
 Di Valentino (2021): 72.94 ± 0.75
Ultra – conservative, no Cepheids, no lensing
 Di Valentino (2021): 72.7 ± 1.1

GW related
 Gayathri et al. (2020), GW190521+GW170817: $73.4^{+6.9}_{-6.9}$
 Mukherjee et al. (2020), GW170817+ZTF: $67.6^{+14.4}_{-14.4}$
 Mukherjee et al. (2019), GW170817+VLBI: $68.3^{+4.2}_{-4.2}$
 Abbott et al. (2017), GW170817: $70.0^{+12.0}_{-12.0}$

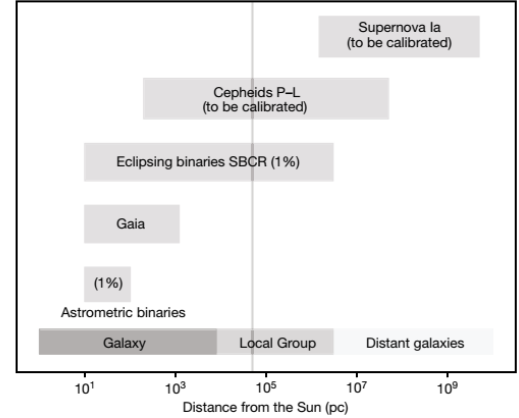
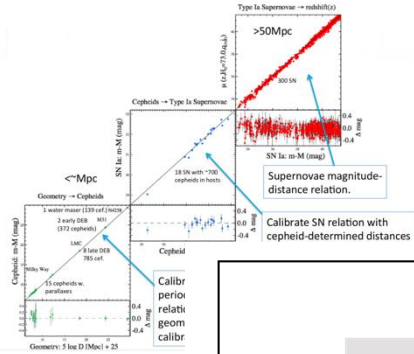
H_0
 [km s⁻¹ Mpc⁻¹]

Indirect
 Direct



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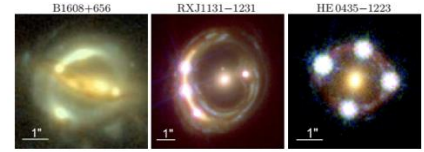
3-rung distance ladder Adapted from Adam Riess and Silvia Galli



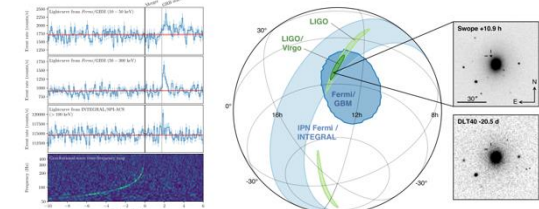
A distance to the Large Magellanic Cloud that is precise to one per cent

G. Pietrzyński^{1,2*}, D. Graczyk^{1,2,3}, A. Gajda^{1,2,3}, W. Gieren¹, I. B. Thompson⁴, B. Pilecki¹, P. Karczmarek², M. Górski², K. Suchomska², M. Taormina¹, B. Zgierski¹, P. Wiegierski¹, Z. Kozłowski^{1,4}, P. Konorski², S. Villanova², N. Nardetto², P. Kervella¹, F. Bresolin⁵, R. P. Kudritzki^{6,7,1}, J. Storm⁸, R. Smolec¹ & W. Narloch¹

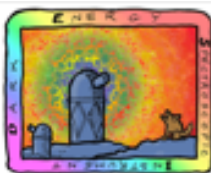
Alternative probes



Strong lensing
 time delays



GW170817



DARK ENERGY
SPECTROSCOPIC
INSTRUMENT

U.S. Department of Energy Office of Science

Hubble constant

$$H_0 = (68.53 \pm 0.80) \text{ km s}^{-1} \text{ Mpc}^{-1}$$

DESI + BBN

$$H_0 = (68.52 \pm 0.62) \text{ km s}^{-1} \text{ Mpc}^{-1}$$

DESI + θ_* + BBN

- Consistency with SDSS
- In agreement with CMB
- In 3.7σ tension with SH0ES

CMB (no lensing)

CMB

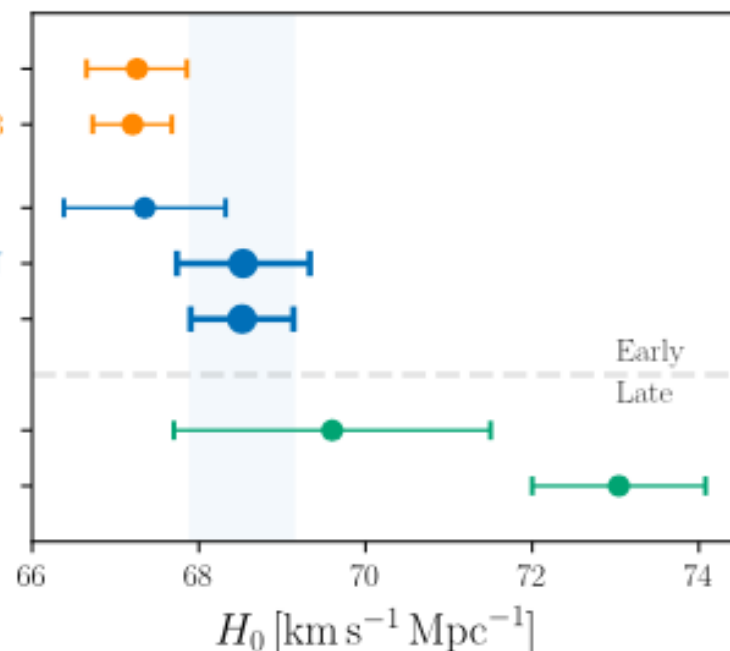
SDSS: BAO+BBN

DESI: BAO + BBN

DESI: BAO + θ_* + BBN

CCHP: TRGB

SH0ES: Cepheids





Newest Measurements of Hubble Constant from DESI 2024 Baryon Acoustic Oscillation Observations

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Chengsheng Mu^{1,2}, and Dadian Cheng^{1,2}

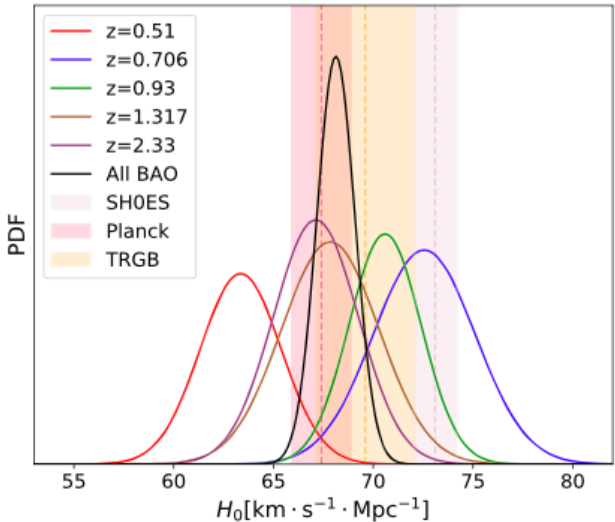
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Abstract

In this Letter, we use the latest results from the Dark Energy Spectroscopic Instrument (DESI) survey to measure the Hubble constant. Baryon acoustic oscillation (BAO) observations released by the DESI survey, allow us to determine H_0 from the first principles. Our method is purely data-driven and relies on unanchored luminosity distances reconstructed from Type Ia supernovae (SN Ia) data and $H(z)$ reconstruction from cosmic chronometers. Thus, it circumvents calibrations related to the value of the sound horizon size at the baryon drag epoch or intrinsic luminosity of SN Ia. We find $H_0 = 68.4^{+1.0}_{-0.8} \text{ km s}^{-1} \text{ Mpc}^{-1}$ at a 68% confidence level, which provides the Hubble constant at an accuracy of 1.3% with minimal assumptions. Our assessments of this fundamental cosmological quantity using the BAO data spanning the redshift range $z = 0.51\text{--}2.33$ agree very well with Planck's results and TRGB results within 1σ . This result is still in a 4.3σ tension with the results of the Supernova H0 for the Equation of State.

$$\frac{d_L(z)}{(1+z)^2 d_A(z)} = 1.$$

Etherington relation



$$H_0 = \frac{1}{(1+z)^2} \frac{H_0 d_L(z)}{H(z) d_A(z)} H(z).$$

$$H_0 = 68.4^{+1.0}_{-0.8} \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Table 1
Value of the Hubble Constant at Different Redshifts Corresponding to BAO Data Points from DESI

z	H_0 ($\text{km s}^{-1} \text{ Mpc}^{-1}$)
Total	$68.4^{+1.0}_{-0.8}$
0.51	$63.4^{+2.3}_{-2.1}$
0.706	$72.4^{+2.8}_{-2.5}$
0.93	70.8 ± 1.8
1.317	$67.9^{+2.9}_{-2.4}$
2.33	67.5 ± 2.3

Note. The joint posterior H_0 value is denoted as total.

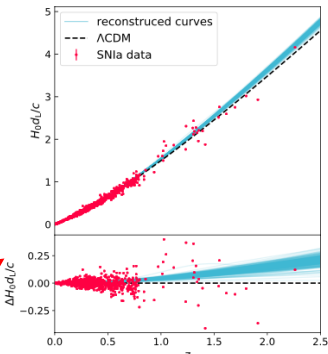


Figure 1. Reconstructed $H_0(z)$ from SN Ia data using Gaussian regression. The black dashed line shows the best-fitted ACDM model. The residuals between reconstructed $H_0(z)$ and the best-fit ACDM are also shown in the bottom panel.

SN Ia Pantheon+

Cosmic Chronometers

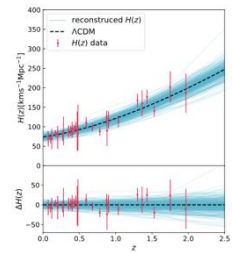


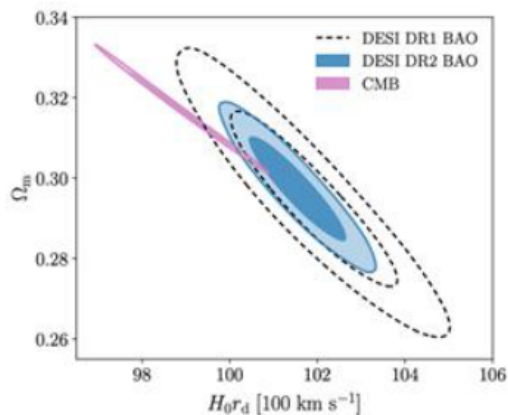
Figure 2. Reconstructed $H(z)$ from CC data using Gaussian regression. The black dashed line shows the best-fit ACDM model. The residuals between reconstructed $H(z)$ and the best-fit ACDM are also shown in the bottom panel.

$$\frac{D_M}{D_H} = \frac{(1+z)d_A(z)}{cH(z)^{-1}} = \frac{1+z}{c} H(z) d_A(z).$$

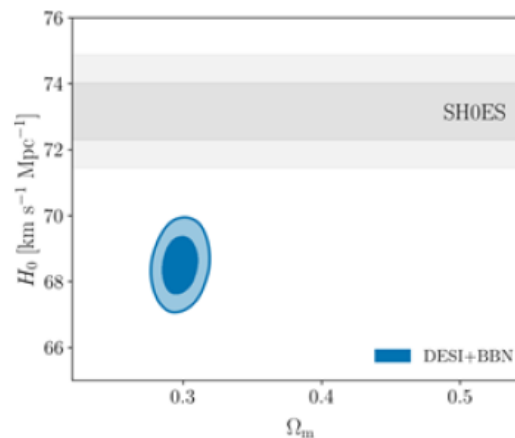
DESI 2024
5 BAO data points

$$r_d H_0 \rightarrow H_0$$

DESI 2025: [arXiv:2503.14738](https://arxiv.org/abs/2503.14738)



r_d from BBN



$$H_0 = (68.51 \pm 0.58) \text{ km s}^{-1} \text{ Mpc}^{-1} \quad (\text{DESI+BBN})$$

- Still in agreement with **CMB** (1.9-
2.3sigma discrepancy)

In Λ CDM, the tension between the **DESI+BBN** and SH0ES H_0 (Breuval++2024) now stands at 4.5σ , independent of the CMB.

Conclusions

History of modern cosmology witnessed numerous tensions.

Age of the Universe vs. globular clusters ...
matter budget issues ...

Modern tensions are embedded in subtle issues of calibrations and systematics

Their persistent significance may be telling us something, e.g. that Λ CDM model or even FLRW framework are too simplified.

We need further development of alternative cosmological probes (strong lensing, GW standard sirens, lensing of transient events, AGNs, compact radio-sources, etc.)

