Origin of Irreversibility

From Quantum Cosmology

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Direction of time

Paul Cézanne, *Nature morte au crâne* (Barnes Foundation, Pennsylvania)

Arrows of time

Almost all the fundamental laws of Nature are time-symmetric; but we observe classes of phenomena that distinguish a direction of time:

- Radiation (advanced versus retarded)
- Thermodynamics (increase of entropy)
- Quantum theory (measurement process)
- Gravity (expansion of the Universe;
  formation of structure; black holes)
- …

Master arrow of time?

Radiation

Fields that interact with local sources are usually desribed by retarded solutions:

$$\begin{aligned} A^\mu &= \text{source term plus boundary term} \\ &= A^\mu_{\mathrm{ret}} + A^\mu_{\mathrm{in}} \\ &= A^\mu_{\mathrm{av}} + A^\mu_{\mathrm{out}} \end{aligned}$$

$$\boxed{\text{Why is } A^\mu_{\mathrm{in}} \approx 0?}$$

$\Longrightarrow$ Cosmology

Einstein und Ritz (1909):

" . . . Ritz considers the restriction to the form of the retarded potentials as one of the roots of the Second Law, while Einstein believes that the irreversibility is exclusively based on reasons of probability. "

Thermodynamics

Second Law of thermodynamics: Increase of entropy

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \underbrace{\left(\frac{\mathrm{d}S}{\mathrm{d}t}\right)_{\mathrm{ext}}}_{\mathrm{d}S_{\mathrm{ext}}=\delta Q/T} + \underbrace{\left(\frac{\mathrm{d}S}{\mathrm{d}t}\right)_{\mathrm{int}}}_{\geq 0}$$

Statistical foundation?

- Loschmidt's *Umkehreinwand*
- Zermelo's *Wiederkehreinwand*

Need special boundary condition!

Entropy increase by coarse graining

Transformation of relevant information about macroscopic differences into irrelevant information (pure microscopic differences without controllable macroscopic consequences)

classic example: Gibbs's ink drop analogy

$$\boxed{S = -k_{\mathrm{B}} \mathrm{tr}\,(\rho \ln \rho)}$$

of special interest: entanglement entropy

Quantum theory

The Schrödinger equation is time reversal invariant, but the measurement process distinguishes a direction of time, either by

- a dynamical collapse of the wave function, or
- a branching of the Everett paths.

Increasing entanglement leads to
Decoherence: irreversible emergence of classical properties through interaction with the environment

Arrow of time from cosmology

Where does the Sun come from?

$\Downarrow$

Gravitational instability of dust clouds

$\Downarrow$

Cosmology

Ludwig Boltzmann (1898):

That in Nature the transition from a probable to an improbable state does not happen equally often as the opposite transition, should be sufficiently explained by the assumption of a very improbable initial state of the whole Universe surrounding us . . . .

Past hypothesis

Sir Arthur Eddington (1931):

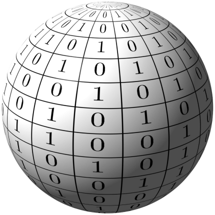
Accordingly, we sweep anti-chance out of the laws of physics—out of the differential equations. Naturally, therefore, it reappears in the boundary conditions ...

We are thus driven to admit anti-chance; and apparently the best thing we can do with it is to sweep it up into a heap at the beginning of time, as I have already described.

Entropy of black holes

If black holes have a temperature, they also have an entropy:

$$S_{\mathrm{BH}} = k_{\mathrm{B}} \frac{Ac^3}{4\hbar G} \equiv k_{\mathrm{B}} \frac{A}{4l_{\mathrm{P}}^2} \overset{\text{Schwarzschild}}{\approx} 1.07 \times 10^{77} k_{\mathrm{B}} \left(\frac{M}{M_{\odot}}\right)^2$$



Quantum gravity?

Entropy of the Universe

- The non-gravitational entropy is dominated by the photons of the Cosmic Microwave background (CMB): it contributes about $2 \times 10^{89}\, k_{\mathrm{B}}$.
- It is exceeded by the entropy of the Galactic Black Hole, which has a Bekenstein–Hawking entropy of about $7 \times 10^{90}\, k_{\mathrm{B}}$; all supermassive black holes together in the observable Universe together yield an (estimated) entropy of about $3.1 \times 10^{104}\, k_{\mathrm{B}}$.
- If the observed acceleration of our Universe is caused by a cosmological constant Λ, the entropy in the far future would be dominated by the entropy of the cosmological event horizon ("Gibbons–Hawking entropy"),

$$S_{\mathrm{GH}} = \frac{3\pi}{\Lambda l_{\mathrm{P}}^2}\, k_{\mathrm{B}} \approx 2.9 \times 10^{122}\, k_{\mathrm{B}}.$$

How special is the Universe?

Penrose (1981):

Entropy of the observed part of the Universe (up to the cosmic particle horizon) is maximal if all its mass is in one black hole; the naive probability for our Universe would then be

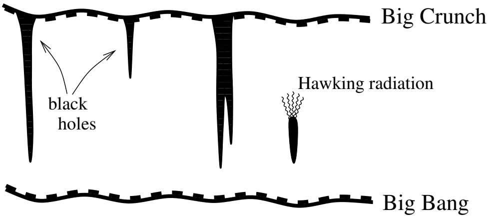
(updated version from C.K. arXiv:0910.5836)

$$\frac{\exp\left(\frac{S}{k_{\mathrm{B}}}\right)}{\exp\left(\frac{S_{\max}}{k_{\mathrm{B}}}\right)} \sim \frac{\exp\left(3.1\times 10^{104}\right)}{\exp\left(1.8\times 10^{121}\right)} \approx \exp\left(-1.8\times 10^{121}\right)$$

(Gibbons–Hawking entropy connected with Λ is neglected here.)

Weyl curvature hypothesis

HYPOTHESIS (CLASSICAL)*:* The Weyl curvature vanishes at all past singularities, as the singularity is approached from future directions. *(This condition can be weakened by only demanding that the Weyl tensor be finite, rather than diverging)*



(Penrose 1979)

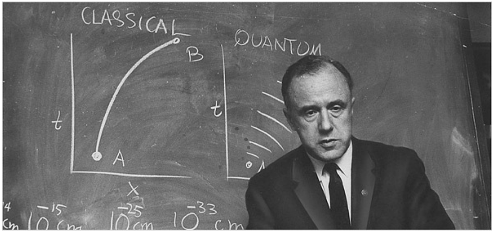
Quantum cosmology

Gell-Mann and Hartle 1990:
Quantum mechanics is best and most fundamentally understood in the framework of quantum cosmology.

John Bell 1990:
A serious formulation [of quantum theory] will not exclude the big world outside the laboratory.

- A universally valid quantum theory must be applied to the Universe as a whole as the only closed quantum system in the strict sense;
- need quantum theory of gravity, since gravity dominates on large scales

Quantum geometrodynamics



(a) John Archibald Wheeler

(b) Bryce DeWitt

Canonical quantization of general relativity leads to

$$\hat{\mathcal{H}}_\perp \Psi \equiv \left(-16\pi G \hbar^2 G_{abcd} \frac{\delta^2}{\delta h_{ab} \delta h_{cd}} - (16\pi G)^{-1} \sqrt{h} \big({}^{(3)}R - 2\Lambda \big) + \sqrt{h} \hat{\rho} \right) \Psi = 0$$

Wheeler–DeWitt equation

$$\hat{\mathcal{H}}^a \Psi \equiv -2 D_b \frac{\hbar}{\mathrm{i}} \frac{\delta \Psi}{\delta h_{ab}} + \sqrt{h} \hat{j}^a \Psi = 0$$

quantum diffeomorphism (momentum) constraint

Comparison between geometrodynamics and particle dynamics		
underlying notion	geometrodynamics	particle dynamics
dynamical object	space	particle
configuration	h_{ij} 3-geometry	x, t events
classical description	$h_{ij}(t)$ 4-geometry	$x(t)$ trajectory
dynamical arena	superspace	spacetime
wave function(al)	$\Psi[h_{ij}]$	$\psi(x, t)$

C.K. and P. Peter, *Universe* 8 (2022) 36

Time in quantum gravity

- External time t has vanished from the formalism.
- This holds also for loop quantum gravity and probably for string theory;
- it holds for *any* theory that has no absolute time at the classical level.
- The Wheeler–DeWitt equation has the structure of a wave equation any may therefore allow the introduction of an intrinsic time.
- The Hilbert-space structure in quantum mechanics is connected with the probability interpretation, in particular with probability conservation *in time* t; what happens with this structure in a timeless situation?

Closed Friedmann–Lemaître universe with scale factor a, containing a homogeneous massive scalar field ϕ (two-dimensional *minisuperspace*)

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}\Omega_3^2$$

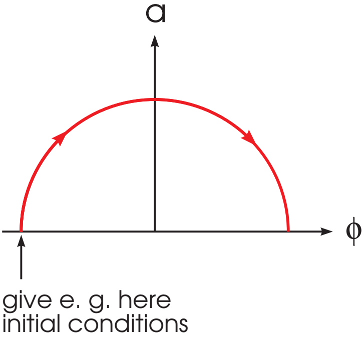
The Wheeler–DeWitt equation reads (with units $2G/3\pi = 1$)

$$\frac{1}{2}\left(\frac{\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{\hbar^2}{a^3}\frac{\partial^2}{\partial\phi^2} - a + \frac{\Lambda a^3}{3} + m^2 a^3 \phi^2\right)\psi(a,\phi) = 0$$

Factor ordering chosen in order to achieve covariance in minisuperspace

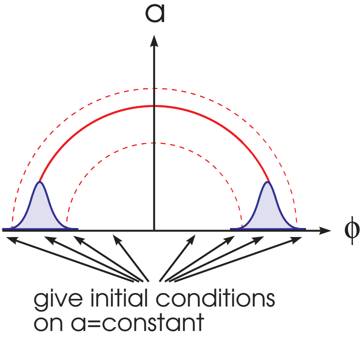
Determinism in classical and quantum theory

Classical theory



Recollapsing part is deterministic successor of expanding part

Quantum theory



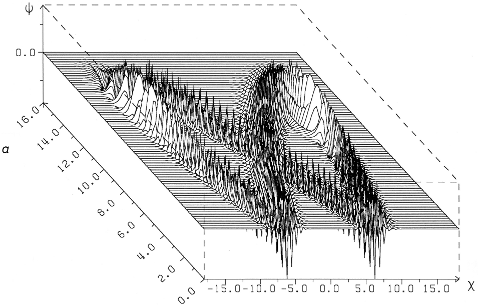
‘Recollapsing’ wave packet must be present ‘initially’

No intrinsic difference between ‘big bang’ and ‘big crunch’!

Example

Indefinite Oscillator

$$\hat{H}\psi(a,\chi) \equiv (-H_a + H_\chi)\psi \equiv \left(\frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \chi^2} - a^2 + \chi^2\right)\psi = 0$$



C.K., *Nucl. Phys. B* **341**, 273 (1990)

COMPLEXITY, ENTROPY AND THE PHYSICS OF INFORMATION

SANTA FE INSTITUTE WORKSHOP

April 16-21, 1990

Supported by: The Santa Fe Institute
Air Force Office of Scientific Research

Tentative Schedule

All events take place at the Santa Fe Institute, 1120 Canyon Road unless otherwise noted

Monday, April 16, 1990

3:00 p.m.		Opening Session
	George Cowan	Welcome & Introduction to SFI
5:00 p.m.		Reception
6:00 p.m.		Dinner at El Farol Restaurant, 808 Canyon Road

Tuesday, April 17, 1990

8:30 a.m.	Coffee	
9:00 a.m.	J.A. Wheeler	Towards "It From Bit"
	W.H. Zurek	Measurements, Demons, and the Second Law
	G. Nicolis	Markovian Coarse-Graining, Information and Complexity in Unstable Dynamical Systems
12:00 p.m.	LUNCH	
2:30 p.m.	M. Gell-Mann**	Quantum Mechanics in the Light of Quantum Cosmology: Progress Report
	J.B. Hartle	Generalized Quantum Mechanics
	A. Peres	Neumark's Theorem and Quantum Inseparability

Wednesday, April 18, 1989

8:30 a.m.	Coffee	
9:00 a.m.	E.T. Jaynes	The Second Law as Physical Fact and as Human Inference
	C.M. Caves	New Quantum Optical Tests of Complementarity and Observation in Quantum Mechanics
	R. Landauer	Title to be Announced
12:30 p.m.	LUNCH	
2:30 p.m.	Charles Bennett	Quantum Cryptography
	R. Ingarden	Information Dynamics in Classical and Quantum Systems
	T.M. Cover	Topics in Information Theory, Complexity & Physics***

Thursday, April 19, 1990

8:30 a.m.	Coffee	
9:00 a.m.	B. DeWitt	Deutsch's Proof that the Mathematical Formalism of Quantum Mechanics Already Implies the Probability Interpretation
	W.G. Unruh	Title to be announced
	A.J. Leggett	The Quantum Preparation Problem***
12:30 a.m.	LUNCH	
2:30 p.m.	L. Schulman	Quantum Measurements and the Arrow of Time
	C. Kiefer	The Emergence of Classical Properties in Quantum Gravity***
	W.K. Wootters	The Information Content of a Random Wavefunction

Friday, April 20, 1990

8:30 a.m.	Coffee	
9:00 a.m.	S.A. Kauffman*	Title to be Announced
	A.J. Albrecht*	The Role of the Observer in Quantum Cosmology
	W.A. Miller	Is There an Analogue of Mach's Principle in Information Theory?
12:30 p.m.	LUNCH	
2:30 p.m.	J.J. Halliwell	A Topic in Quantum Cosmology
	S. Lloyd	Logical Friction
	B. Schumacher	How Much Does Information Weigh?

Saturday, April 21, 1990

8:30 a.m.	Coffee
9:00 a.m.	Summary?

* tentative speaker
** speaker stuck in Nepal without helicopter
*** tentative title

Quantum Weyl curvature hypothesis

HYPOTHESIS (QUANTUM)*:* The quantum states for the Weyl scalars describing tensor modes assume the form of adiabatic vacuum states in a (quasi-) de Sitter space, as the region of small-enough scale factors is approached from future directions.

$$
\begin{aligned}
\Psi_4 &:= -\frac{1}{8c^2}\left(\ddot{h}_+ - \mathrm{i}\ddot{h}_\times\right), \\
\Psi_0 &:= -\frac{1}{2c^2}\left(\ddot{h}_+ + \mathrm{i}\ddot{h}_\times\right).
\end{aligned}
$$

C.K., *AVS Quantum Science* **4**, 015607 (2022).

Decoherence in quantum cosmology

In quantum cosmology, arbitrary superpositions of the gravitational field and matter states can occur. How can we understand the emergence of an (approximate) classical Universe?

- *System*: global degrees of freedom (scale factor, inflaton field, . . .)
- *Environment*: small density fluctuations, gravitational waves, . . .

(Zeh 1986, C.K. 1987)

Example: scale factor a of a de Sitter universe ($a \propto \mathrm{e}^{H_{\mathrm{I}}t}$) ('system') experiences decoherence by gravitons ('environment') according to

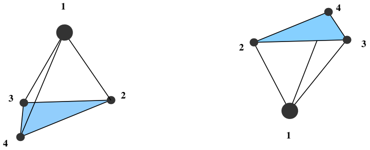
$$\rho_0(a,a') \to \rho_0(a,a') \exp\left(-CH_{\mathrm{I}}^3 a(a-a')^2\right)\ ,\ C>0$$

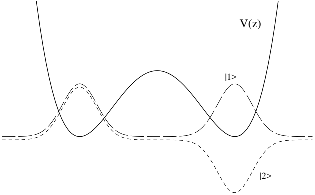
The Universe assumes classical properties at the beginning of inflation

(Barvinsky, Kamenshchik, C.K. 1999)

Time from symmetry breaking

Analogy from molecular physics: emergence of chirality





dynamical origin: decoherence through scattering by light or air molecules

Quantum cosmology: decoherence between $\exp(\mathrm{i}S_0/G\hbar)$- and $\exp(-\mathrm{i}S_0/G\hbar)$-components of the wave function through interaction with e.g. weak gravitational waves

Example for a decoherence factor:
$\exp\left(-\frac{\pi m H_0^2 a^3}{128\hbar}\right) \sim \exp\left(-10^{43}\right)$ (C.K. 1992)

Interpretation of quantum cosmology

Almost all approaches to quantum gravity preserve the linear structure of quantum theory and thus the strict validity of the superposition principle.

Bryce S. DeWitt 1967:

Everett's view of the world is a very natural one to adopt in the quantum theory of gravity, where one is accustomed to speak without embarassment of the 'wave function of the universe.' It is possible that Everett's view is not only natural but essential.

The Everett interpretation together with decoherence suffices to explain the main conceptual features

Origin of irreversibility

Fundamental asymmetry with respect to “intrinsic time” ($\alpha := \ln(a/a_0)$) :

$$\hat{H}\Psi = \left(\frac{\partial^2}{\partial\alpha^2} + \sum_n \left[-\frac{\partial^2}{\partial x_n^2} + \underbrace{V_n(\alpha, x_n)}_{\to 0 \text{ for } \alpha \to -\infty} \right] \right) \Psi = 0.$$

This is compatible with a simple boundary condition:

$$\Psi \stackrel{\alpha \to -\infty}{\longrightarrow} \psi_0(\alpha) \prod_n \psi_n(\alpha, x_n),$$

where the wave functions $\psi_n(\alpha, x_n)$ denote the adiabatic ground states of scalar and tensor modes (the scalar modes are described by the “Mukhanov–Sasaki variables”).

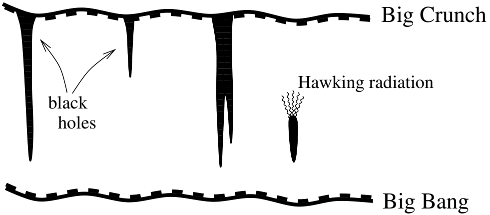
This simple boundary condition corresponds to an initial condition of low entanglement entropy (after the "environmental degrees of freedom" are integrated out). It is a quantum manifestation of the past hypothesis.

The entropy $-k_{\rm B}{\rm Tr}\rho\ln\rho$ then increases with increasing α, since entanglement with other degrees of freedom increases. This defines a direction towards increasing size of the Universe, which in the semiclassical limit coincides with increasing semiclassical (WKB) time t.

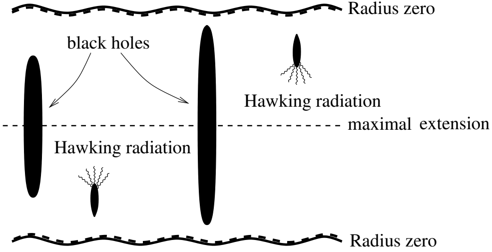
Is the expansion of the Universe a tautology?

See L. Chataigner, C. Kiefer, M. Tyugi, J. Phys. Conf. Ser. **2883** (2024) 012008 and the references therein

Arrow of time in a recollapsing quantum universe



(Penrose 1979)



(C.K. and Zeh 1995)

Emergence of probabilities?

For solutions of

$$\hat{H}|\Psi\rangle = 0,$$

one can define *branch states*

$$|\mathcal{B}_i\rangle := \hat{P}_i|\Psi\rangle.$$

Semiclassical worlds are represented by branches that satisfy an *approximate decoherence condition*

$$(\mathcal{B}_i|\mathcal{B}_j) = (\Psi|\hat{P}_i\hat{P}_j\Psi) \approx \delta_{ij}(\Psi|\hat{P}_i\Psi).$$

One can show that $(\Psi|\hat{P}_i\Psi)$ gives the "percentage" of the universal state that corresponds to a given branch; it can be identified with the "probability" that observers will find themselves in that branch.

See L. Chataigner, C. Kiefer, M. Tyugi, J. Phys. Conf. Ser. **3017** (2025) 012007 and references therein

Kwantowa kosmologia – to tam, gdzie fizyka rodzi się z metafizyki.

(Michał Heller)