Relational observables and quantum reference frames

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Outline of the talk

1. AQFT
   - Algebraic QFT...
   - ... and beyond
2. QRFs
   - QRFs and measurement
   - QRFs from quantum fields

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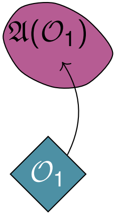

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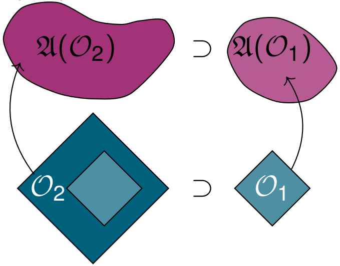
- The physical notion of subsystems is realized by the condition of isotony, i.e.: $\mathcal{O}_1 \subset \mathcal{O}_2 \Rightarrow \mathfrak{A}(\mathcal{O}_1) \subset \mathfrak{A}(\mathcal{O}_2)$. We obtain a net of algebras.



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One can also ask for further, physically motivated properties: causality and time-slice axiom.

- **Causality**: If $\mathcal{O}_1, \mathcal{O}_2 \subset \mathbb{M}$ are spacelike separated (no causal curve joining them)

$$\text{then} \quad [\mathfrak{A}(\mathcal{O}_1), \mathfrak{A}(\mathcal{O}_2)] = \{0\},$$

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where $[.,.]$ is the commutator in the sense of $\mathfrak{A}(\mathcal{O}_3)$, where $\mathcal{O}_3$ contains $\mathcal{O}_1$ and $\mathcal{O}_2$.
- **Time-slice axiom:** If $\mathcal{N}$ is a neighborhood of a Cauchy-surface in $\mathcal{O}$, then $\mathfrak{A}(\mathcal{N}) = \mathfrak{A}(\mathcal{O})$.

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- One can also assign algebras of observables to globally hyperbolic spacetimes in a covariant way [Brunetti-Fredenhagen-Verch **CMP** 2003].
- For constructing physical models we often use perturbation theory. More on that here: *Perturbative algebraic quantum field theory. An introduction for mathematicians*, KR, Springer 2016.

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- And now we make it all quantum!

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- One can think of the choice of dynamical local coordinates X^μ as the choice of observer (or reference frame). See also [Donelly-Freidel 2016], earlier works by [Dittrich, Giesel, Rovelli, Thiemann, Bergmann-Komar, Brown-Kuchar. . .].

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- Fix a background Γ_0 such that the map

$$X_{\Gamma_0} : x \mapsto (X^0_{\Gamma_0}, \dots, X^3_{\Gamma_0})$$

is injective.

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- Under construction: phrase this in the language of operational QRFs.

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- See also papers by Fröb et. al. [JCAP 2017, CQG 2018].

The (p)AQFT perspective on quantum observables

Perturbative algebraic quantum field theory (pAQFT) is a framework that allows to build interacting QFT models on curved spacetimes using deformation quantization and formal power series.

From classical to quantum

pAQFT is a machinery to turn functionals of classical field configurations (classical observables) into quantum observables.

The choice of diffeomorphism invariant objects is made on the classical level.

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- Dynamics: we use a modification of the Lagrangian formalism (fully covariant).

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- Note: In the dynamical coordinate approach, f depends on the dynamical coordinates!

Classical Dynamics

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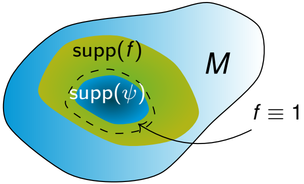

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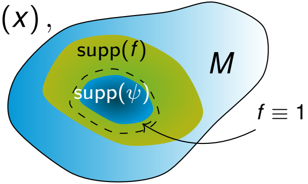
$$\langle dL(\varphi_0), \psi \rangle \doteq \int \frac{\delta L(f)}{\delta \varphi(x)}(\varphi_0)\psi(x)\,,$$

where $\varphi_0 \in \mathcal{E}$, $\psi \in \mathcal{E}_c$ and $f \equiv 1$ on $\operatorname{supp}\psi$.



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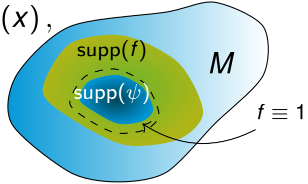

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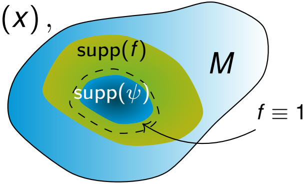

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- It is a differential equation (i.e. local) and we assume that it is normally hyperbolic (essentially a wave equation).

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- We add to $\frac{i}{2}\Delta$ a symmetric distribution H with $P_x H(x,y) = 0 = P_y H(x,y)$, so that the resulting $W = \frac{i}{2}\Delta + H$ is of positive type and has a particularly nice singluarity structure (it ends up being the 2-point function of a Hadamard state for the quantized theory).

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- We introduce the Feynman propagator $\Delta^{\mathrm{F}} = \frac{i}{2}(\Delta^{\mathrm{R}} + \Delta^{\mathrm{A}}) + H$. This is much more singular than W! this is the source of divergences in QFT.

Time-ordered product

- Denote $D_{ij} = \left\langle \hbar\Delta^{\mathrm{F}}, \frac{\delta^2}{\delta\varphi_i\delta\varphi_j} \right\rangle$ and for $F_1, \dots, F_n \in \mathcal{F}_{\mathrm{loc}}$ with pariwise disjoint supports define their time-ordered product by

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- One of the crucial axioms is Causal factorisation property:

$$\mathcal{T}_n(V_1, \ldots, V_n) = \mathcal{T}_k(V_1, \ldots, V_k) \star \mathcal{T}_{n-k}(V_{k+1}, \ldots, V_n)\,,$$

if $\mathrm{supp}\, V_{k+1}, \ldots, \mathrm{supp}\, V_n$ not later than $\mathrm{supp}\, V_1, \ldots, \mathrm{supp}\, V_k$.

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- Relational observables quantised! To do: connection to QRFs.

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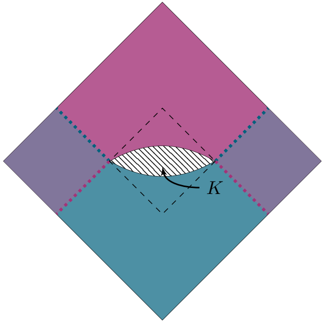

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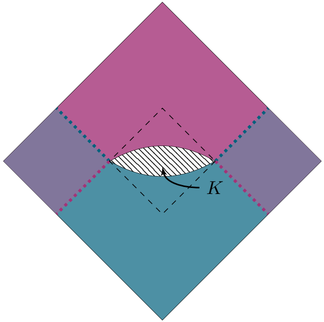

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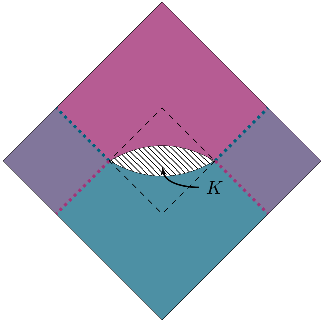

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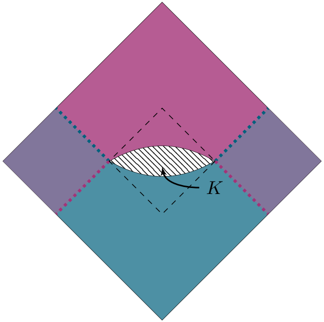

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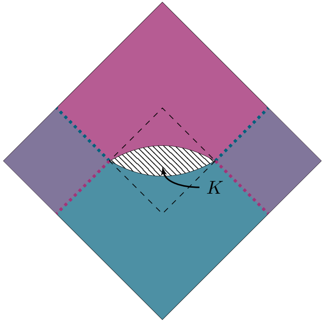

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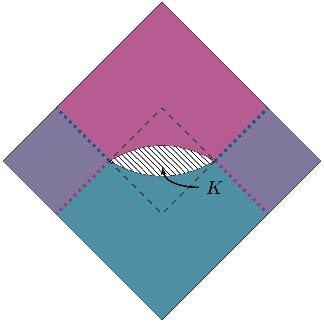

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- Such scheme is interpreted as measurement of A.

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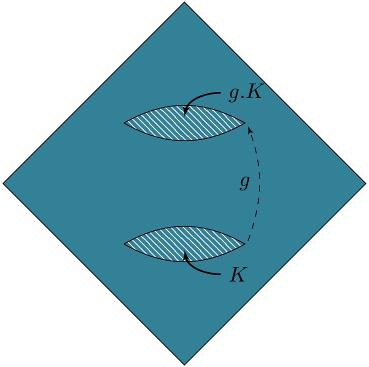

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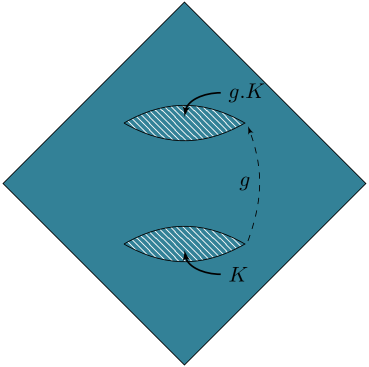
- A transformation of the measurement scheme leads to covariant transformation of induced system observable $A \mapsto \alpha(g)A$.



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- Approaches to QRFs perspectival, perspective neutral, operational, ...

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A QRF is called sharp if E is a PVM (and unsharp otherwise). It is compactly stabilised if H is compact.

Example: particle on a line

A QRF $(\lambda, P, \mathcal{H})$ for translation group $\mathbb{R}$ (with $\Sigma = \mathbb{R}$)

$$\mathcal{H} = L^2(\mathbb{R}), \qquad (\lambda(y)\psi)(x) = \psi(x - y), \qquad (P(X)\psi)(x) = \chi_X(x)\psi(x).$$

Here P the spectral (projection valued) measure of position operator $\mathbf{x}$ and $\lambda(y) = \exp(-i\mathbf{p}y)$ left translation generated by momentum. Formally, we thus have

$$\lambda(y)P(X)\lambda(y)^* = P(X + y) \iff [\mathbf{x}, \mathbf{p}] = i.$$

Perimeter Institute 26-30 October 2026

Relativity Reframed: Quantum Reference Frames and Gravity

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Quantum reference frames (QRFs) have emerged as a powerful and rapidly developing framework in fundamental physics, providing a systematic approach to formulating theories without fixed classical backgrounds. By making explicit the relational structure of physical laws, QRFs supply concrete tools to define subsystems, observables, and quantum information in diffeomorphism-invariant and dynamically fluctuating spacetimes. Their applications span quantum gravity, algebraic and curved-spacetime quantum field theory, quantum information, cosmology, and phenomenology, refining our understanding of locality, causality, symmetry, and measurement beyond semiclassical regimes. In gravitational settings, QRFs clarify relational observables, edge modes, soft degrees of freedom, and entanglement in quantum spacetime, and shed new light on the role of observers in dynamical geometries.

Current and future research

Examples of QRFs in (p)AQFT:

- Another QM system, e.g. a single particle, harmonic oscillator, etc. [Fewster-Janssen-Loveridge-KR-Waldron, CMP 2024], generalizing [Chandrasekaran-Longo-Penington-Witten 2023]. Similar methods apply also to QFT on Schwarzschild spacetime exterior or the Rindler wedge.

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- Dynamical coordinate fields [Brunetti-Fredenhagen-KR, CMP 2016].

EM on spacetimes with corners

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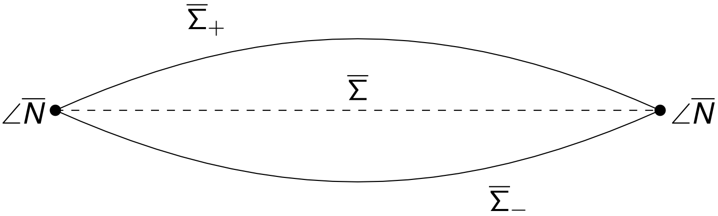
Finite Cauchy lens compact spacetime $\overline{N}$ with boundary $\partial\overline{N} = \overline{\Sigma}^+ \cup \overline{\Sigma}^-$ and (smooth) corner $\angle\overline{N} = \overline{\Sigma}^+ \cap \overline{\Sigma}^- = \partial\overline{\Sigma}^\pm$.



QRFs for EM corner symmetries

- Given external current J, fields are 1-forms $A \in \Omega^1(\overline{N})$, with action

$$S(A) = -\int_{\overline{N}} \frac{1}{2} \mathrm{d}A \wedge \star \mathrm{d}A + A \wedge \star J.$$

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- We want a QRF for this group!

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Classical observables given by smearing of initial data on $\overline{\Sigma} \subset \overline{N}$.

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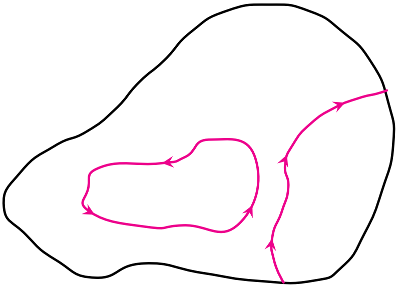

Smearing the vector potential $\mathbf{A}$ at $\overline{\Sigma}$ with co-closed $F \in \Omega^1_{\mathrm{d}^*}(\overline{\Sigma})$

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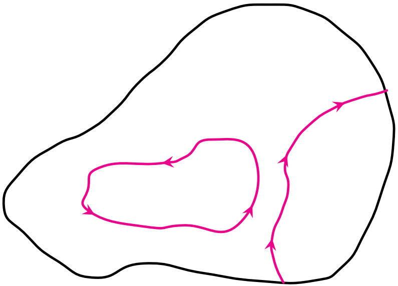

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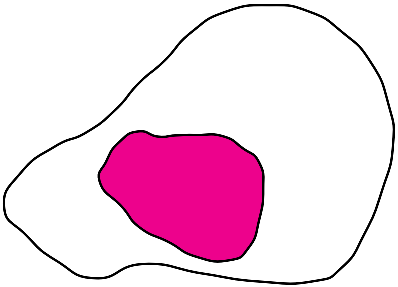

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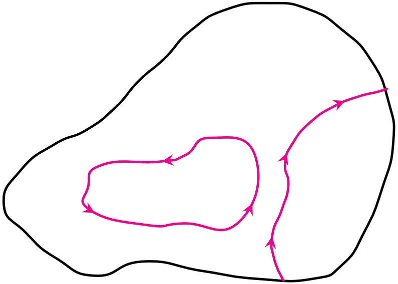

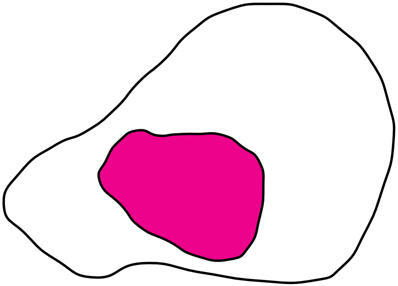

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These 'equal-time' observables satisfy $\{\mathbf{A}(F), \mathbf{E}(H)\} = \langle F, H \rangle_{\overline{\Sigma}} \mathbb{1}$.

QRFs for EM corner symmetries

- The algebra of semi-local observables is the Weyl C^*-algebra of the symplectic space of observables, i.e. $\mathfrak{A}_{\mathrm{sloc}}(\overline{N}) \cong \mathrm{Weyl}(\mathrm{Sol}^{J}_{\mathscr{G}}(\overline{N}), \sigma)$.

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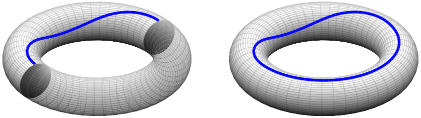
- Auxiliary surface fields $\varphi \in \Omega^0(\partial\overline{\Sigma})$ play the role of QRF.
- Given a QRF and a "bare" observable a, there is a map $¥$ that assigns to it the relativised (dressed) observables $¥(a)$.

Relativised observables



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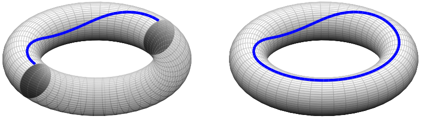

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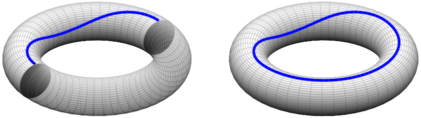

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- Another interpretation: gluing two pieces of a cut loop.
- Loops in partition 2 act as a QRF for loops in partition 1 and we construct the gluing map:

$$\yen : \mathfrak{A}(\overline{N_1}) \to \pi_1(\mathfrak{A}_{\mathrm{sloc}}(\overline{N}_1))'' \otimes \pi_2(\mathfrak{A}_{\mathrm{sloc}}(\overline{N}_2))'',$$

injective C^*-algebra homeomorphism.



Thank you very much for your attention!

(some pictures were generated using pixlr)