On the Coincidence Problem in Cosmology

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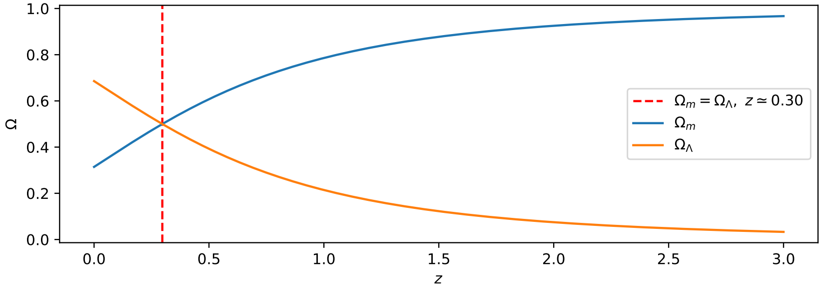
The ‘weak’ coincidence problem

- The value of Λ is such that Ω_Λ begins to dominate over Ω_m only recently
- The onset of cosmic acceleration ($z \sim 0.6$) coincides with the epoch when nonlinear large-scale structure becomes important

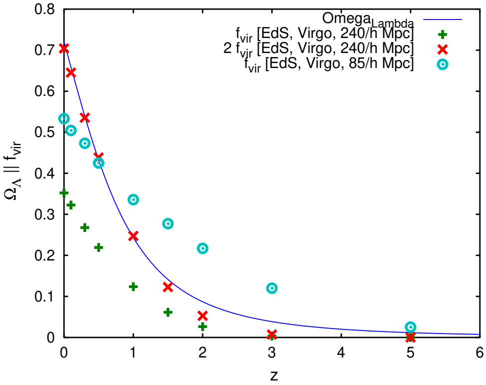


The ‘strong’ coincidence problem

- Ω_Λ appears to track some measures of late-time cosmic inhomogeneity

- There is no unique measure of inhomogeneity

- $f_{vir} = \frac{\text{particles in structures}}{\text{number of all particles}}$



Roukema, JO, Buchert; JCAP (2013)

AvERA code, Racz et al. MNRAS (2017)

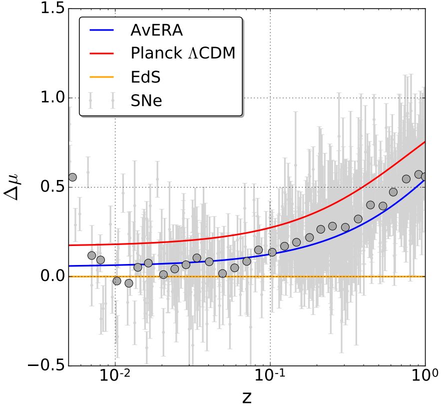
Standard approach vs AvERA separate universe evolution

$$\left\langle \begin{array}{c} \Omega_{m,1} \\ \Omega_{m,2} \\ \dots \\ \Omega_{m,N} \end{array} \right\rangle \Rightarrow \boxed{\text{Friedmann eq.}} \Rightarrow \; \Delta V \; \Rightarrow a(t+\delta t)$$

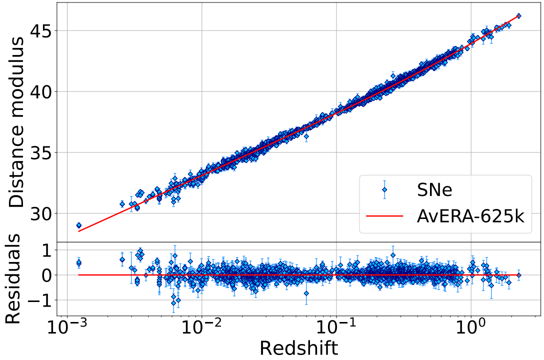
$$\begin{array}{c} \Omega_{m,1} \\ \Omega_{m,2} \\ \dots \\ \Omega_{m,N} \end{array} \Rightarrow \boxed{\text{Friedmann eq.}} \Rightarrow \left\langle \begin{array}{c} \Delta V_1 \\ \Delta V_2 \\ \dots \\ \Delta V_N \end{array} \right\rangle \Rightarrow a(t+\delta t)$$

AvERA vs ΛCDM model

Local FLRW evolution can mimic a dark-energy-like distance-redshift relation.



Rácz et al, MNRAS (2017)



Pataki et al, A&A (2025)

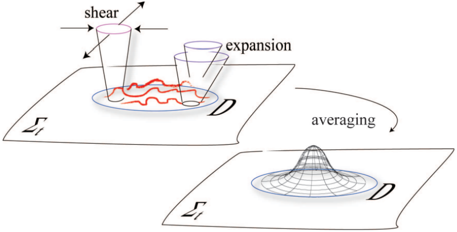
Buchert averaging

- spatial averaging of scalar parts of Einstein's equations

$$\langle \mathcal{A} \rangle_{\mathcal{D}} = \frac{1}{V_{\mathcal{D}}} \int_{\mathcal{D}} \mathcal{A} \, \mathrm{d}\mu \ , \quad \langle \theta \rangle_{\mathcal{D}} = \frac{\partial_t V_{\mathcal{D}}}{V_{\mathcal{D}}} \ .$$

- averaging and time evolution do not commute

$$\partial_t \langle \mathcal{A} \rangle_{\mathcal{D}} - \langle \partial_t \mathcal{A} \rangle_{\mathcal{D}} = \langle \mathcal{A}\theta \rangle_{\mathcal{D}} - \langle \mathcal{A} \rangle_{\mathcal{D}} \langle \theta \rangle_{\mathcal{D}} \ .$$



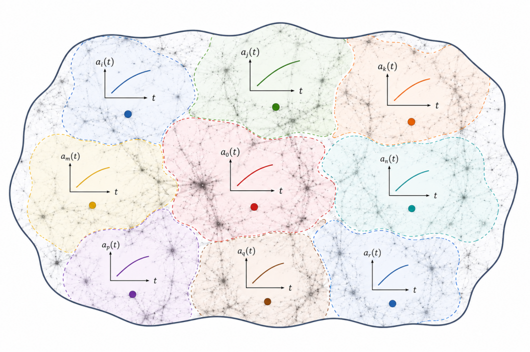
Buchert, Carfora, 2008

- the averaged acceleration equation is sourced by the backreaction term

$$3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} + 4\pi G \langle \rho \rangle_{\mathcal{D}} = \mathcal{Q}_{\mathcal{D}} \ , \ \mathcal{Q}_{\mathcal{D}} = \frac{2}{3}\left(\langle \theta^2 \rangle_{\mathcal{D}} - \langle \theta \rangle_{\mathcal{D}}^2\right) - 2\langle \sigma^2 \rangle_{\mathcal{D}} \ .$$

Multiscale partitioning

- divide the hypersurface into smaller regions
- each region is described by a scale factor that solves the Buchert equations
- for scalar-valued functions, we have $\langle f \rangle_{\mathcal{D}} = \sum_\ell \lambda_\ell \langle f \rangle_{\mathcal{F}_\ell}$



The global backreaction term reads

$$\mathcal{Q}_{\mathcal{D}} = \sum_\ell \lambda_\ell \mathcal{Q}_\ell + 6 \sum_{\ell<k} \lambda_\ell \lambda_k (H_\ell - H_k)^2 \ , \ \lambda_i = \frac{V_i}{V_{\mathcal{D}}}$$

Buchert-Ehlers theorem

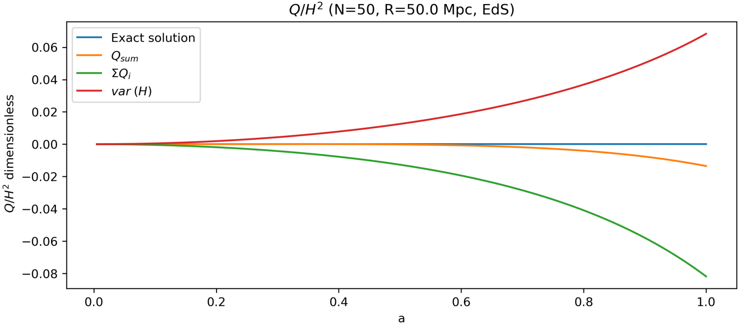
- in the Newtonian setup, the backreaction can be written as a divergence
- a vanishing boundary term or spherical symmetry gives $\mathcal{Q}_{\mathcal{D}} = 0$
- AvERA treats each region as a local matter-only FLRW domain, hence

$$\mathcal{Q}_{\mathcal{D}} = \xcancel{\sum_{\ell} \lambda_\ell \mathcal{Q}_\ell} + 6 \sum_{\ell<k} \lambda_\ell \lambda_k (H_\ell - H_k)^2 \ .$$

- there is no directly equivalent theorem in General Relativity

Numerical check

- initial conditions generated on the $\mathbb{T}^3$ hypersurface
- partitioning into 50Mpc domains (comoving)
- evolution is approximated by the *Relativistic Zel'dovich Approximation*



Buchert, Ostrowski, Bovon, in prep.

Timescape cosmology

- **Cosmological Equivalence Principle** (Wiltshire, PRD (2008))

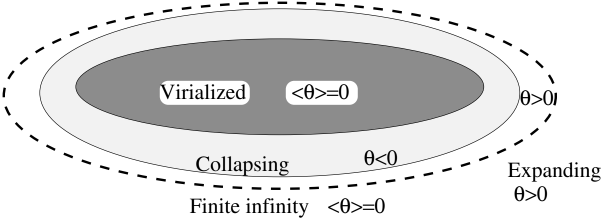
  At any event, always and everywhere, it is possible to choose a suitably defined spacetime neighbourhood, the cosmological inertial frame, in which average motions (timelike and null) can be described by geodesics in a geometry which is Minkowski up to some time-dependent conformal transformation

$$\mathrm{d}s^2_{\mathrm{CIF}} = a^2_{\mathrm{CIF}}(\eta)\left(-\mathrm{d}\eta^2 + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right) \ ,$$

- *A suitably defined region here refers to one which is smaller than the scalar curvature scale within underdense voids, or alternatively is the finite infinity scale for systems containing overdensities*

Two-scale solution, finite infinity

- we have different definitions of dust particles in voids and in walls
- two-scale partitioning: union of voids $\mathcal{E}$, union of finite infinities (walls) $\mathcal{M}$
- quasi-local scale factors solve Friedmann equations $\rightarrow \mathcal{Q}_{\mathcal{E}} = \mathcal{Q}_{\mathcal{M}} = 0$.
- evolution of statistical geometry is driven by $6\lambda_{\mathcal{M}}\lambda_{\mathcal{E}}(H_{\mathcal{E}} - H_{\mathcal{M}})^2$.



Wiltshire, 2016

Backreaction scale: a two-region estimate

Two-scale partitioning

$$Q_{\mathcal{D}} = 6\lambda_M(1-\lambda_M)(H_M - H_E)^2$$

For finite-infinity / wall regions:

$H_M \simeq 0$ and $H_{\mathcal{D}} \simeq (1-\lambda_M)H_E$.

Therefore

$$\frac{Q_{\mathcal{D}}}{H_{\mathcal{D}}^2} = \frac{6\lambda_M}{1-\lambda_M}.$$

Require a Λ-sized effect

$$Q_{\mathcal{D}} \sim \Lambda = 3\Omega_\Lambda H_{\mathcal{D}}^2$$

Thus

$$\lambda_M \sim \frac{\Omega_\Lambda}{2+\Omega_\Lambda}.$$

For $\Omega_\Lambda \simeq 0.685$,

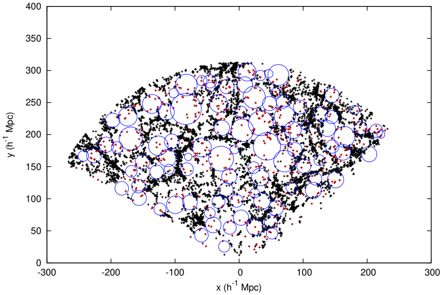
$$\boxed{\lambda_M \simeq 0.25, \qquad \lambda_E \simeq 0.75.}$$

Λ-sized effect requires a void-dominated volume, together with a non-negligible wall/finite-infinity fraction

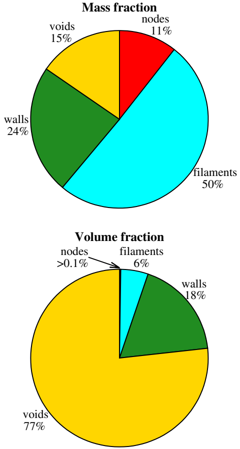
Observed and simulated void volume fractions

$f_{void} \sim 0.62$ (Pan et al.)

$f_{void} \sim 0.77$ (Cautun et al.)



Pan et al., MNRAS (2012)



Cautun et al., MNRAS (2014)

Conclusions

- The variance of quasi-local Hubble parameters appears to track Ω_Λ as long as the partitioning scale is larger than roughly the virialization scale
- This term arises naturally in several inhomogeneous cosmological models: Timescape, AvERA (based on the separate universe conjecture) and others
- Newtonian simulations have to be treated with caution - Buchert-Ehlers theorem
- In a simple two-scale picture, the Hubble variance is not constant in time: it is negligible in the homogeneous limit, grows during void-wall differentiation, and declines once voids dominate the volume.