Gravitational-Wave Lensing as a Powerful Tool for Cosmology

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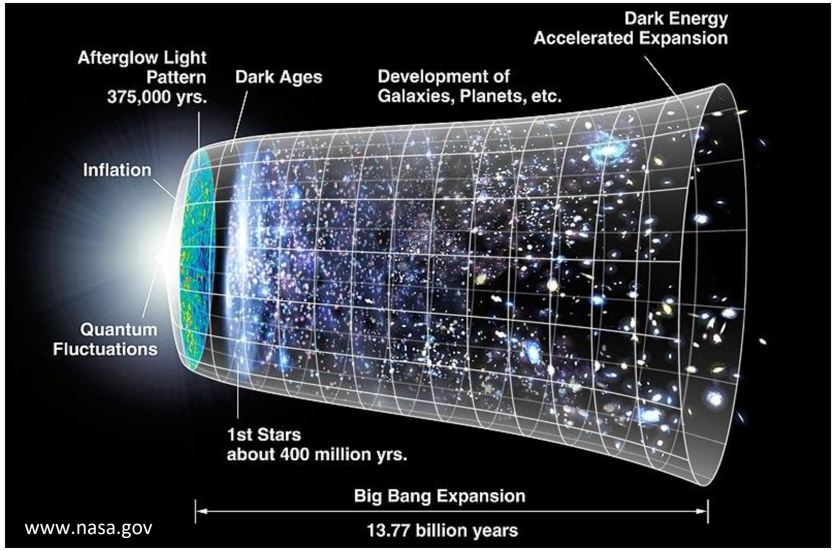
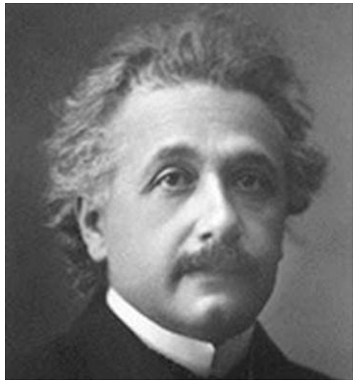
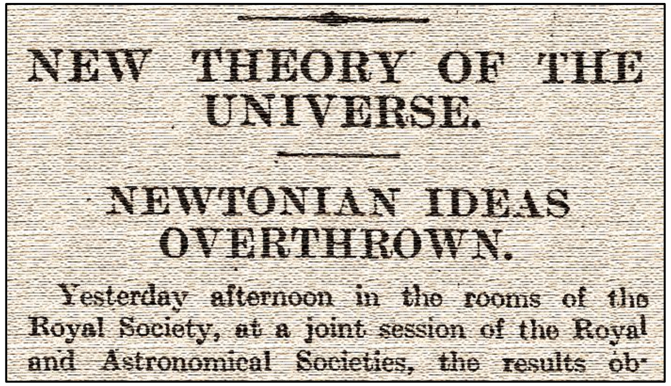


Outline

- GW lensing across different lensing regimes: methodological framework and some computational challenges
- GW lensing and cosmology – the (partial) story so far…and looking ahead…
  - Constraining dark matter profiles with gravitational waves
  - Detecting signatures of microlensing in strongly-lensed GW events
  - Constraining the Hubble expansion with strongly-lensed dark sirens
- Into the 2030s with LISA and 3G detectors:
  - (Multi-messenger) challenges: weak lensing of GW sirens
  - Multi-messenger opportunities: constraining higher dimensions
- Conclusions

With thanks to: **Mick Wright**, **Eungwang Seo**, Zhaofeng Wu, Charlie Mpetha, Justin Janquart, Otto Hannuksela, Xikai Shan, Rachel Gray, Bin Hu, Zhuotao Li, Andy Taylor, Guiseppe Congedo, Max Corman, Celia Rivera, Lok Chan, Lan Nguyen, Tjonnie Li

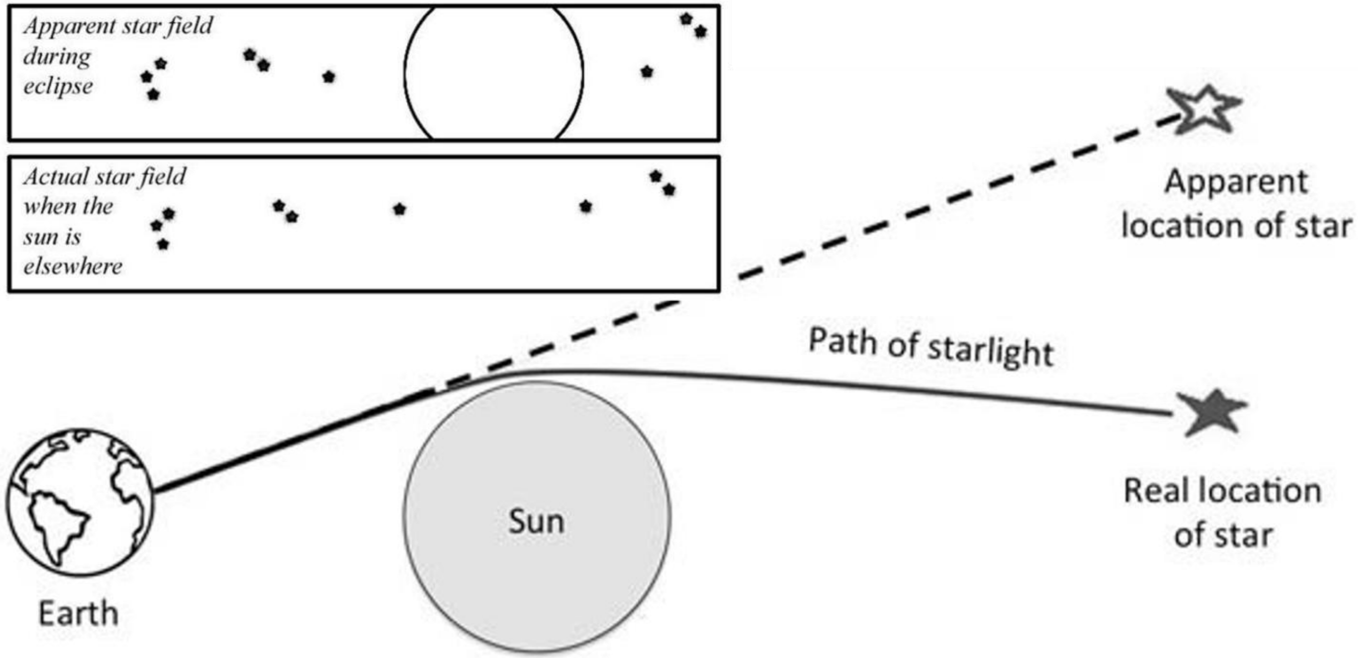


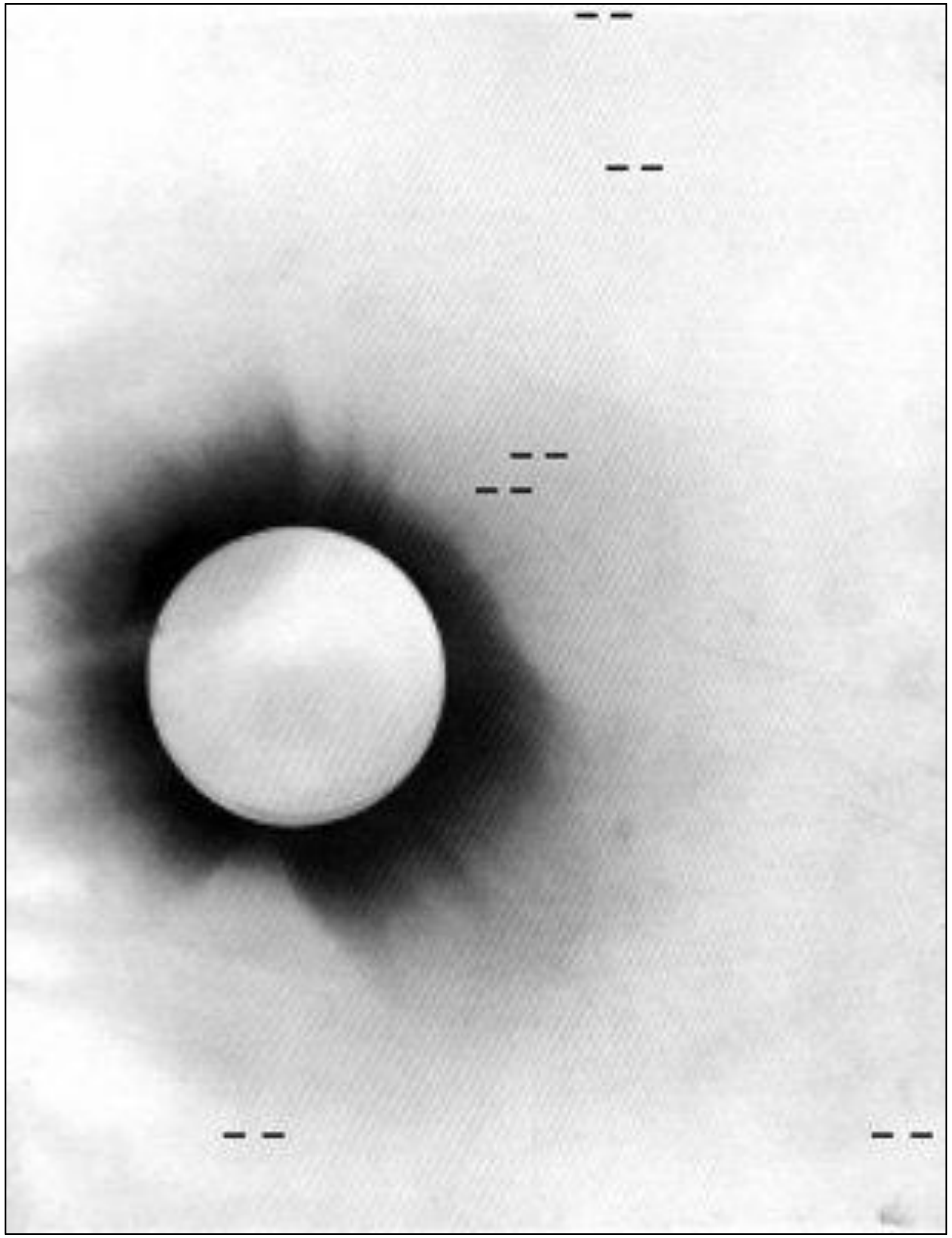


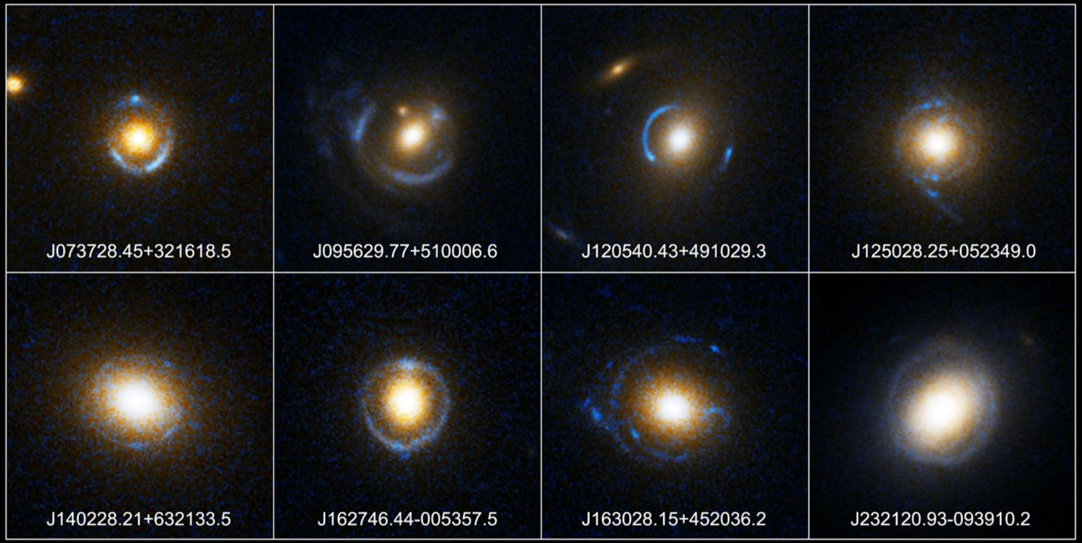
NEW THEORY OF THE UNIVERSE.

NEWTONIAN IDEAS OVERTHROWN.

Yesterday afternoon in the rooms of the Royal Society, at a joint session of the Royal and Astronomical Societies, the results ob-

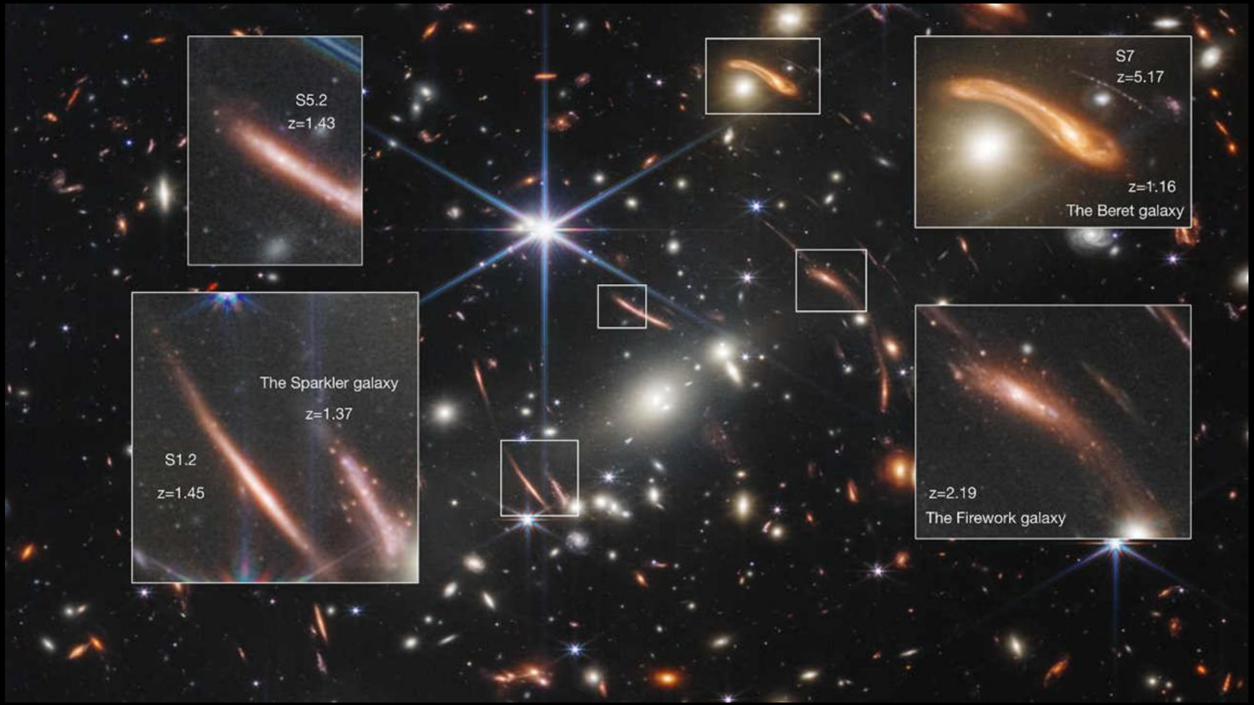


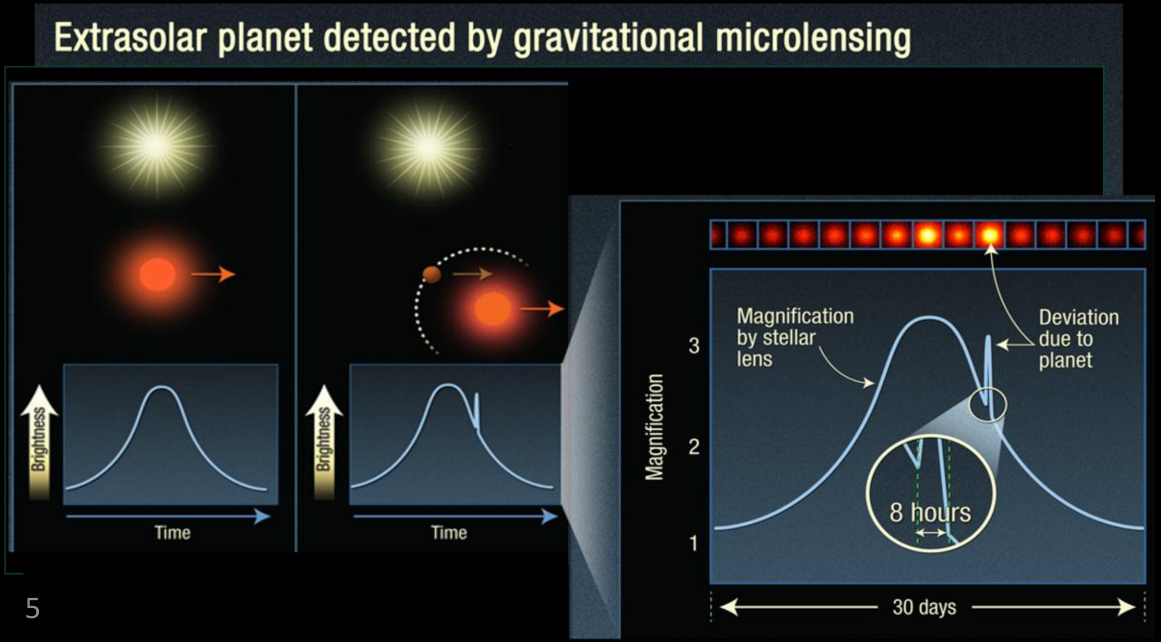


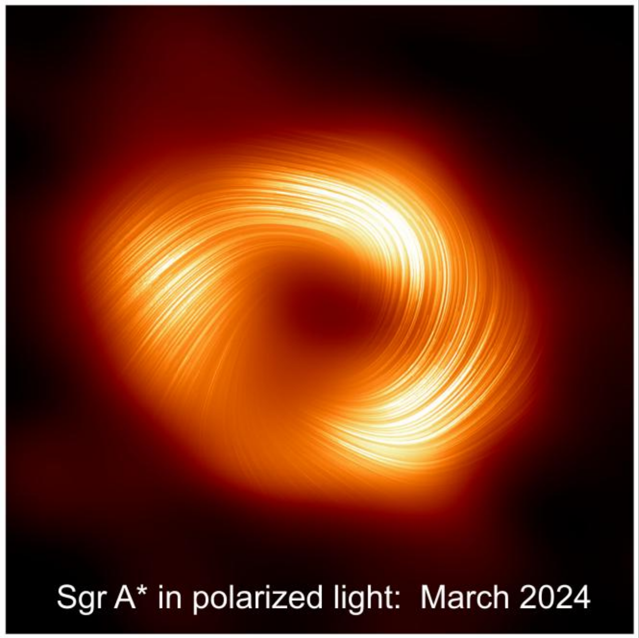


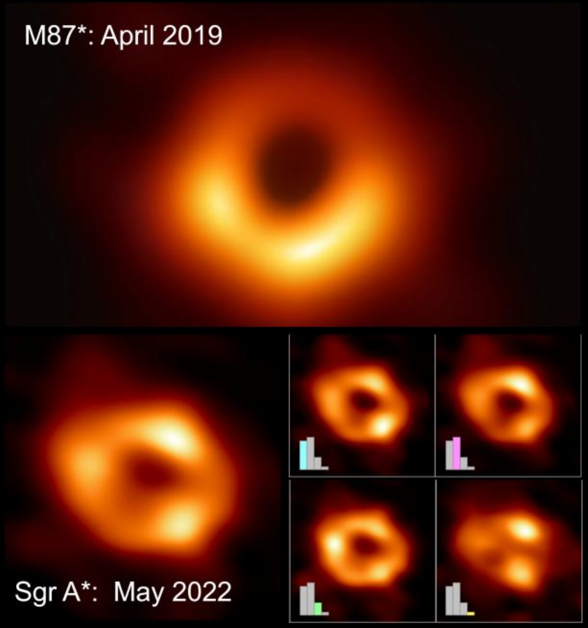
Einstein Ring Gravitational Lenses
Hubble Space Telescope • Advanced Camera for Surveys

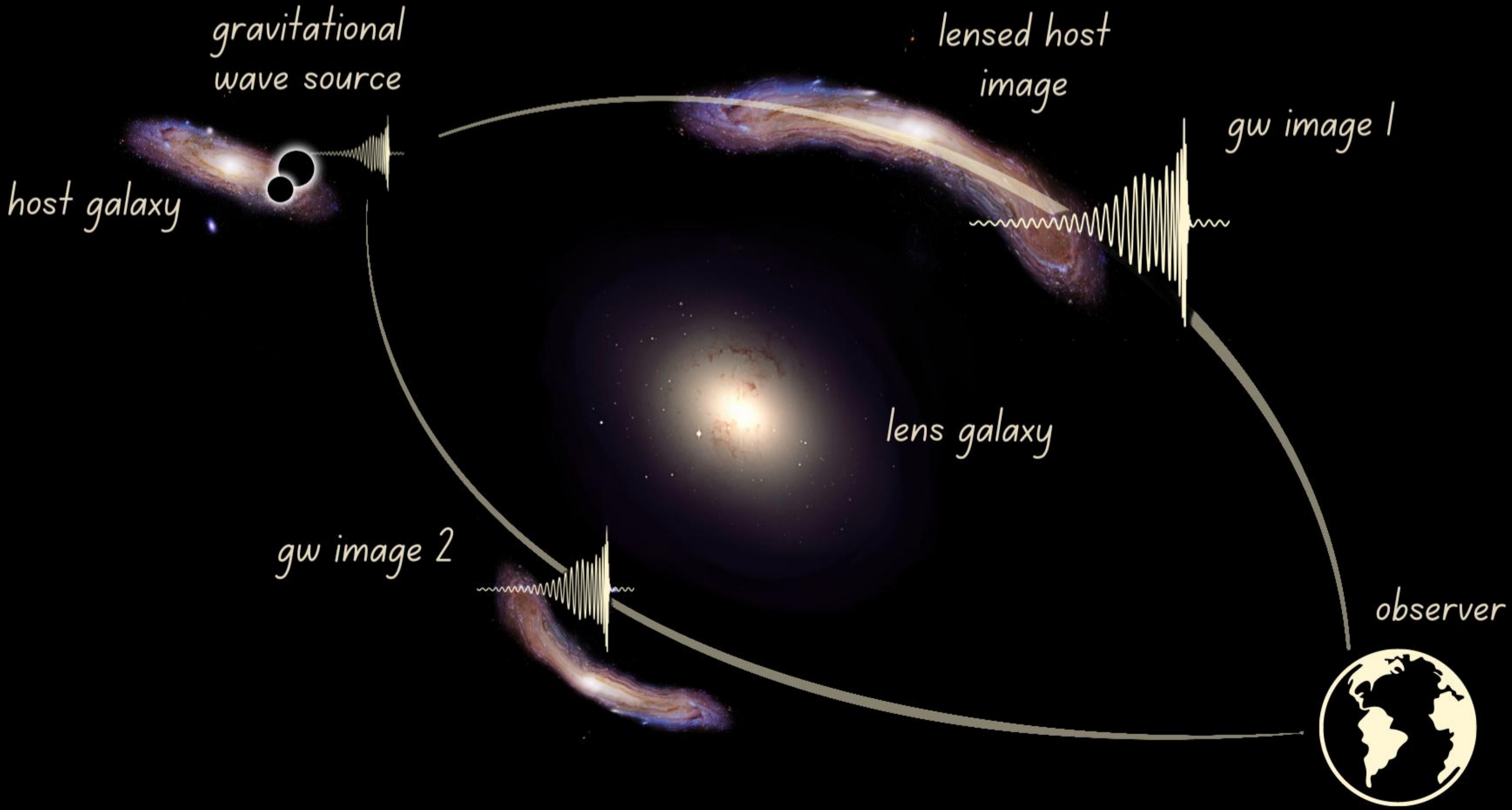
NASA, ESA, A. Bolton (Harvard-Smithsonian CfA), and the SLACS Team STScI-PRC05-32











Credit: Laura E. Uronen, CUHK

Einstein in focus: Gravitational Wave Astronomy and Gravitational Lensing

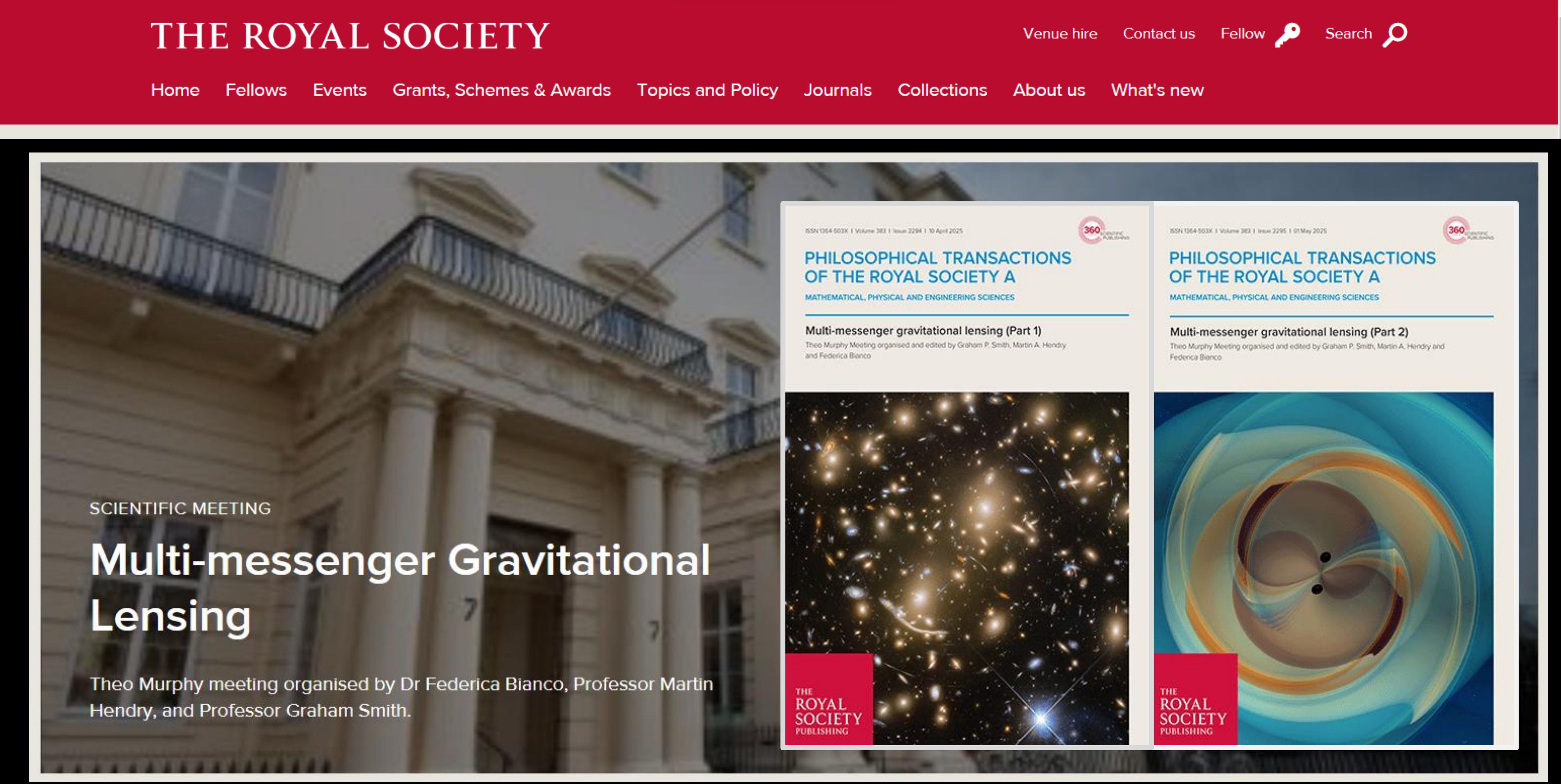
RAS/IOP Specialist Discussion Meeting: October 17, 2008

The meeting will be held in Lecture Theatre of the Royal Astronomical Society, Burlington House, Piccadilly, London W1. Registration is at 9:30, talks and poster session from 9:45 to 15:30, see the program below.

This meeting will bring together the gravitational wave and gravitational lensing communities to discuss issues at the interface of these two fields of study. The main focus of the meeting will be on how both strong and weak gravitational lensing will complicate the interpretation of gravitational waves detected with the next generation of ground-based detectors. Postgraduate and postdoctoral scientists working in both fields are particularly encouraged to attend and to take advantage of the poster session to present their work.

Aims

The nascent field of gravitational wave astronomy offers exciting prospects for astrophysics and cosmology because it opens a new window on the universe, complementary to “traditional” observations in the electromagnetic sector. As the samples of gravitational wave detections grow, an important future priority will be to develop methods of analysis that optimally combine observations of electromagnetic and gravitational radiation.



11 - 12 March 2024

https://royalsocietypublishing.org/rsta/issue/383/2294

https://royalsocietypublishing.org/rsta/issue/383/2295

https://arxiv.org/abs/2503.19973

Multi-messenger gravitational lensing

fundamental science with multiple messengers from explosive transients in distant gravitationally lensed galaxies

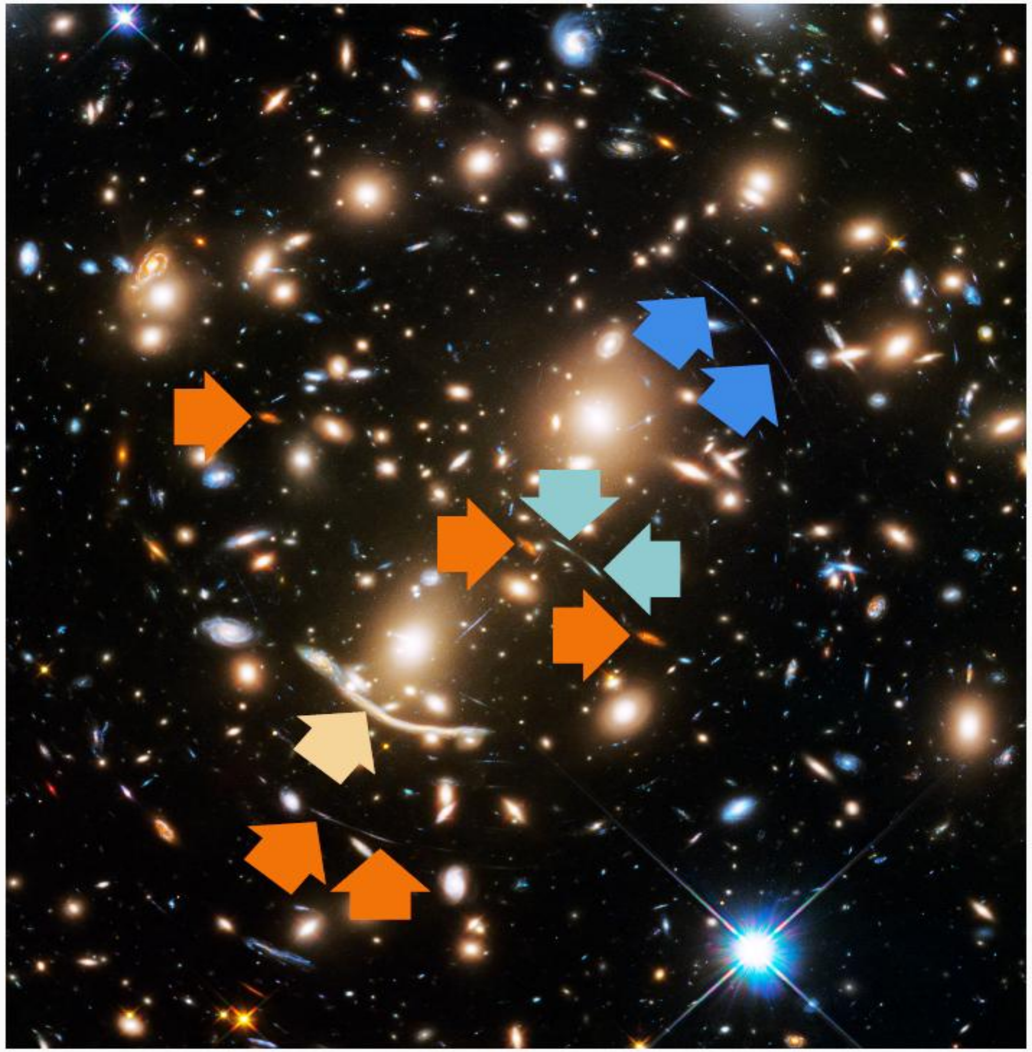


Image credit: STScI

Astrophysics:

- extreme states of matter
- host galaxies at high-z
- chemical enrichment
- compact objects at high-z
- …

Cosmology:

- Hubble tension
- nature of dark matter
- primordial black holes
- …

Fundamental physics:

- testing general relativity
- speed of messengers
- …

Magnification
"gravitational telescope"

Arrival time difference

Image positions

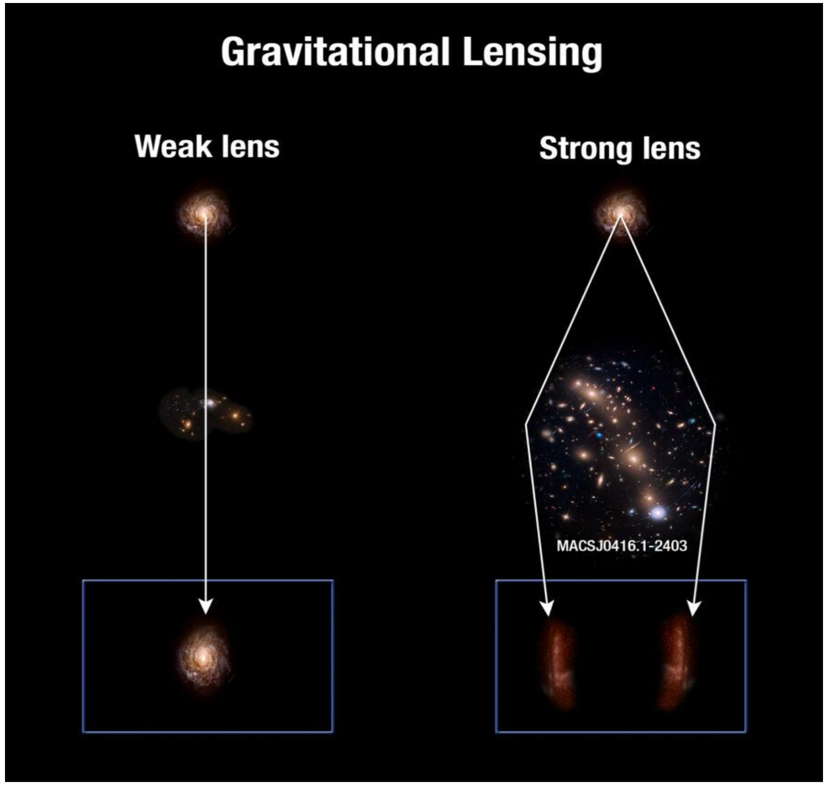
Slide credit: Graham Smith

Gravitational-wave lensing regimes

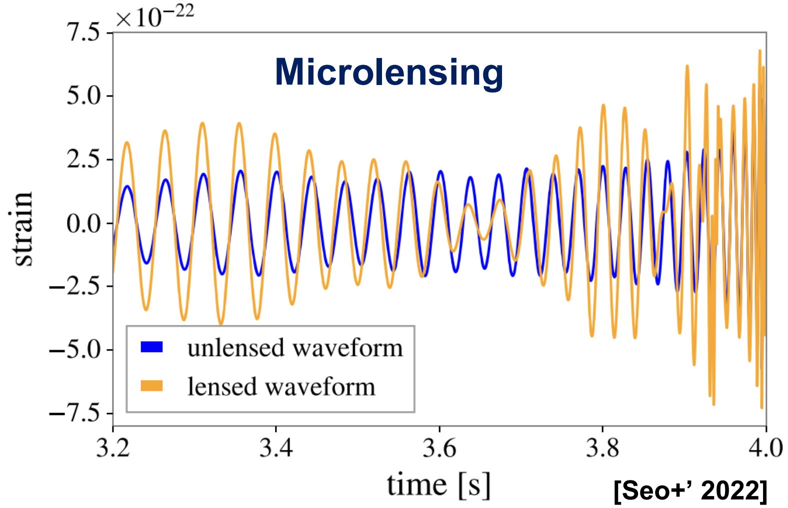
- **Strong lensing**
  - Caused by large-mass objects ($M_l^z \gtrsim 10^8 \mathrm{M}_\odot$)
    (e.g. galaxies and galaxy clusters)
  - Geometrical optics is valid (Lens equation can be solved)
  - Produces multiple signals ("images")
  - Large magnification

- **Weak lensing**
  - Produces no multiple signals
  - Moderate magnification

- **Microlensing**
  - Caused by small-mass objects ($M_l^z < 10^5 \mathrm{M}_\odot$)
    (e.g., stars, compact objects, and star clusters)
  - Dominated by wave optics
  - Beating patterns arise due to short time delays between lensed signals.



Credit: A. Field (STScI)



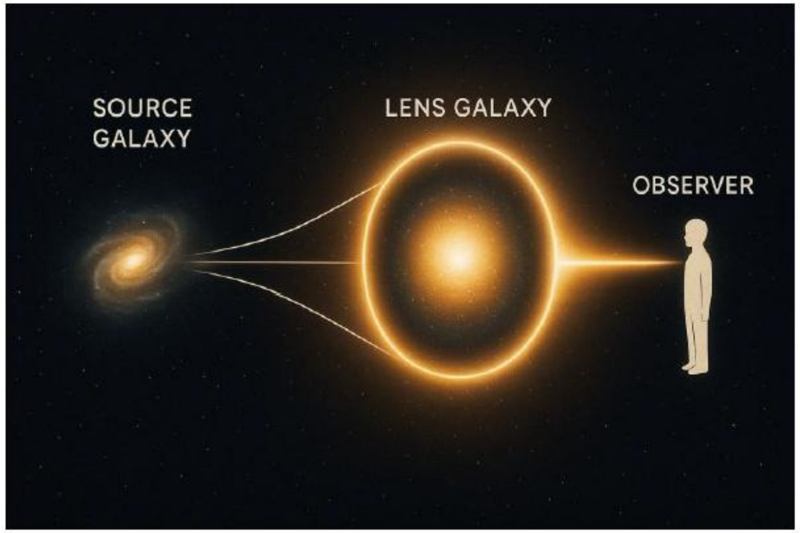
[Seo+' 2022]

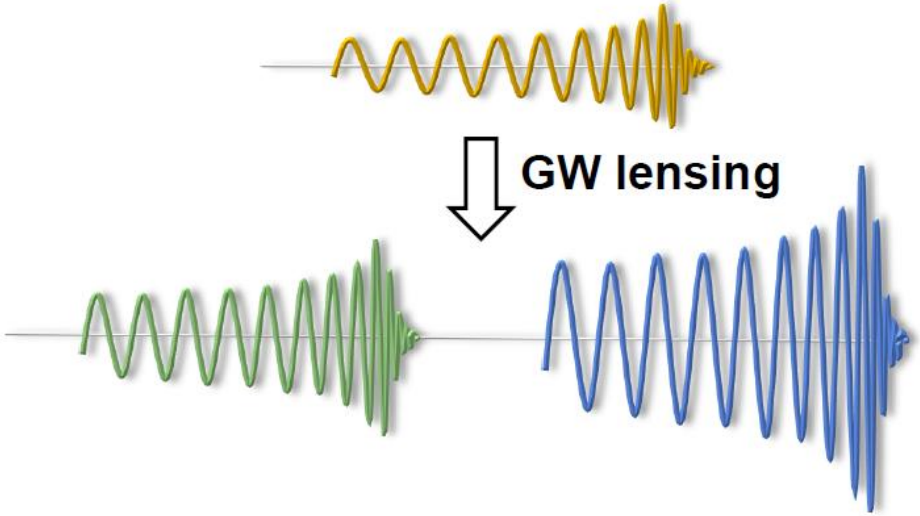
Credit: Eungwang Seo (2026)

Lensing: *Gravitational waves vs Electromagnetic waves*

	Light		GW (strong lensing)	GW (microlensing)
Magnification (μ_k)	**Flux increases** by a factor of μ_k: $\boxed{\hat{\mathcal{F}}_k = \mathcal{F}\mu_k}$	**The apparent magnitude** changes as follows: $\boxed{\hat{m}_k = m - 2.5\log_{10}(\mu_k)}$	**Amplitude increases** by a factor of $\sqrt{\mu_k}$: $\boxed{\hat{A}_k = A\sqrt{\mu_k}}$	**The apparent luminosity distance** changes as follows: $\boxed{\hat{d}_{L,k} = d_L/\sqrt{\mu_k}}$
Optics	**Geometric optics** The lens equation is valid		**Geometric optics** Waveform morphology remains unchanged.	**Wave optics** Waveform morphology can change.

EM lensing

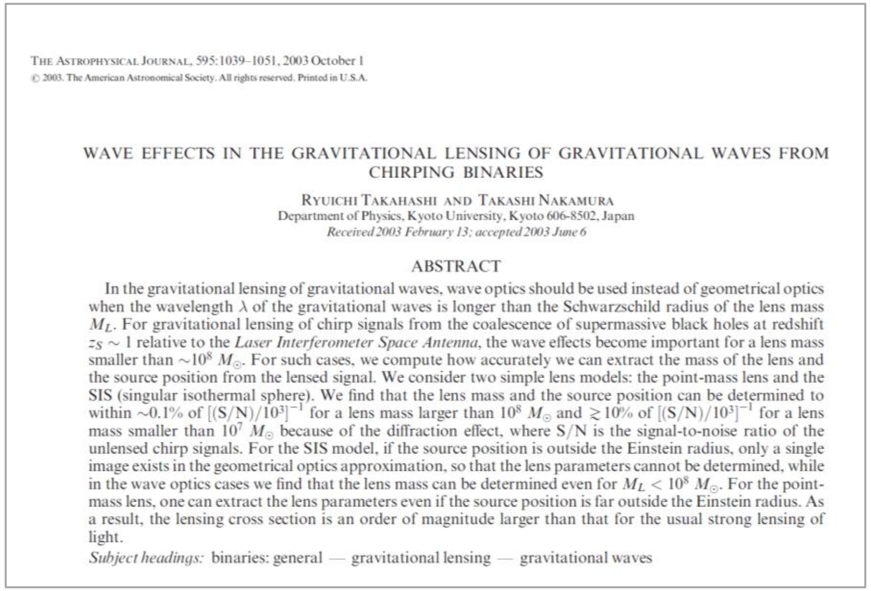




Credit: Eungwang Seo (2026)

Wave optics treatment **essential** for compact, point-like lenses, in both the GW **strong lensing** and **microlensing** cases

Extensive literature on this, over several decades:



THE ASTROPHYSICAL JOURNAL, 595:1039–1051, 2003 October 1

WAVE EFFECTS IN THE GRAVITATIONAL LENSING OF GRAVITATIONAL WAVES FROM CHIRPING BINARIES

RYUICHI TAKAHASHI AND TAKASHI NAKAMURA
Department of Physics, Kyoto University, Kyoto 606-8502, Japan
Received 2003 February 13; accepted 2003 June 6

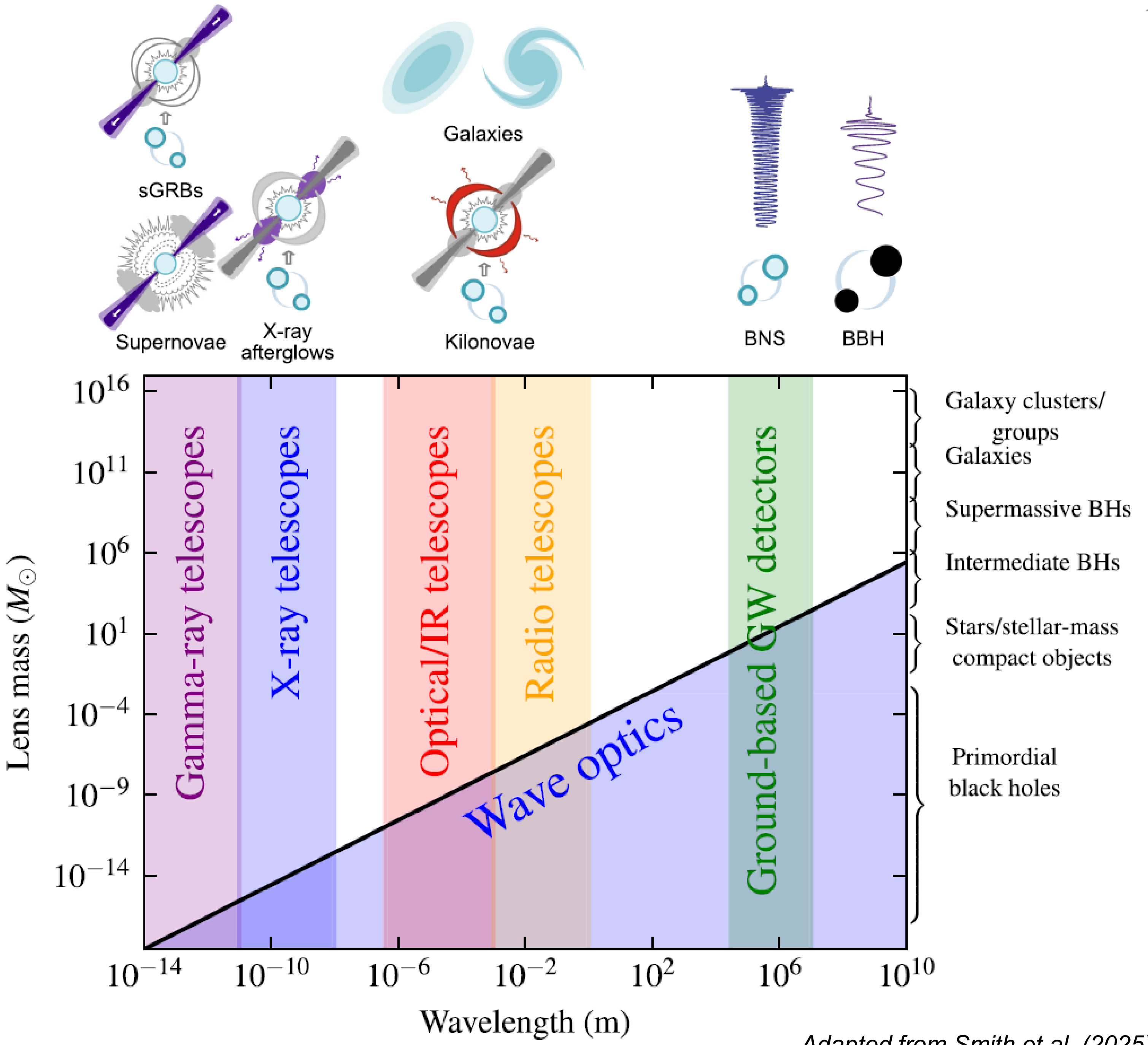
ABSTRACT

In the gravitational lensing of gravitational waves, wave optics should be used instead of geometrical optics when the wavelength λ of the gravitational waves is longer than the Schwarzschild radius of the lens mass M_L. For gravitational lensing of chirp signals from the coalescence of supermassive black holes at redshift $z_S \sim 1$ relative to the *Laser Interferometer Space Antenna*, the wave effects become important for a lens mass smaller than $\sim 10^8\ M_\odot$. For such cases, we compute how accurately we can extract the mass of the lens and the source position from the lensed signal. We consider two simple lens models: the point-mass lens and the SIS (singular isothermal sphere). We find that the lens mass and the source position can be determined to within $\sim 0.1\%$ of $[(S/N)/10^3]^{-1}$ for a lens mass larger than $10^8\ M_\odot$ and $\gtrsim 10\%$ of $[(S/N)/10^3]^{-1}$ for a lens mass smaller than $10^7\ M_\odot$ because of the diffraction effect, where S/N is the signal-to-noise ratio of the unlensed chirp signals. For the SIS model, if the source position is outside the Einstein radius, only a single image exists in the geometrical optics approximation, so that the lens parameters cannot be determined, while in the wave optics cases we find that the lens mass can be determined even for $M_L < 10^8\ M_\odot$. For the point-mass lens, one can extract the lens parameters even if the source position is far outside the Einstein radius. As a result, the lensing cross section is an order of magnitude larger than that for the usual strong lensing of light.

Subject headings: binaries: general — gravitational lensing — gravitational waves

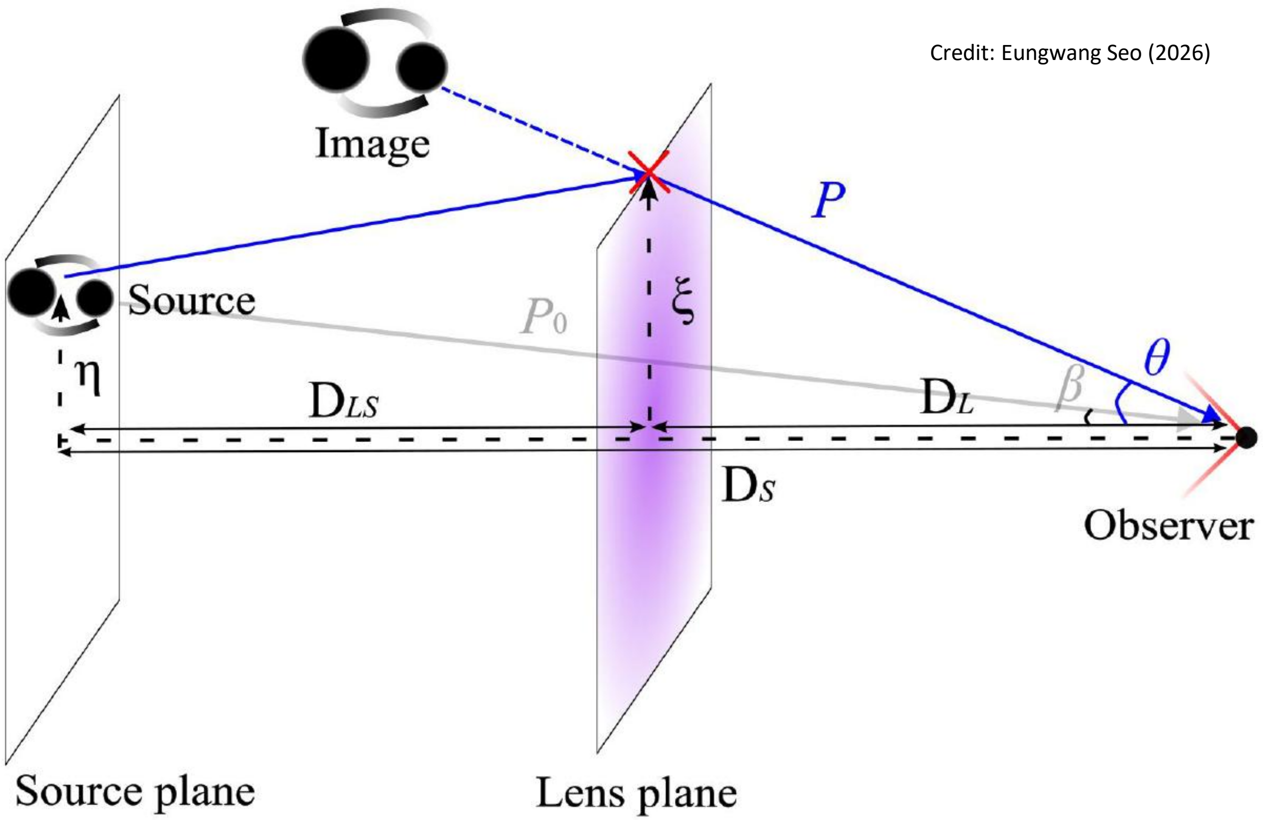
See also Smith et al (2025) for a review.

https://arxiv.org/abs/astro-ph/0305055



Adapted from Smith et al. (2025)

Dimensionless deflections: $\boldsymbol{x} = \frac{\boldsymbol{\xi}}{\xi_0}$, $\boldsymbol{y} = \frac{D_{OL}}{\xi_0 D_{OS}}\boldsymbol{\eta}$



Lensed wave amplitude:

$$\tilde{\phi}_L = \frac{1}{4\pi}\int_S d^2\boldsymbol{\xi}\left[\tilde{\phi}\frac{\partial}{\partial n}\left(\frac{e^{i\omega r}}{r}\right) - \frac{e^{i\omega r}}{r}\frac{\partial}{\partial n}\tilde{\phi}\right]$$

Lensed GW signal: $h^L(f) = h(f) \times F(f)$

where $F(w, \boldsymbol{\eta}) = \frac{\phi^L_{obs}(w, \boldsymbol{\eta})}{\phi_{obs}(w, \boldsymbol{\eta})}$

or, in fully dimensionless arguments

$$\boxed{F(w, \boldsymbol{y}) = \frac{w}{2\pi i}\int d^2x\left[\exp(iwT(\boldsymbol{x}, \boldsymbol{y}))\right]}$$

Dimensionless time delay, from

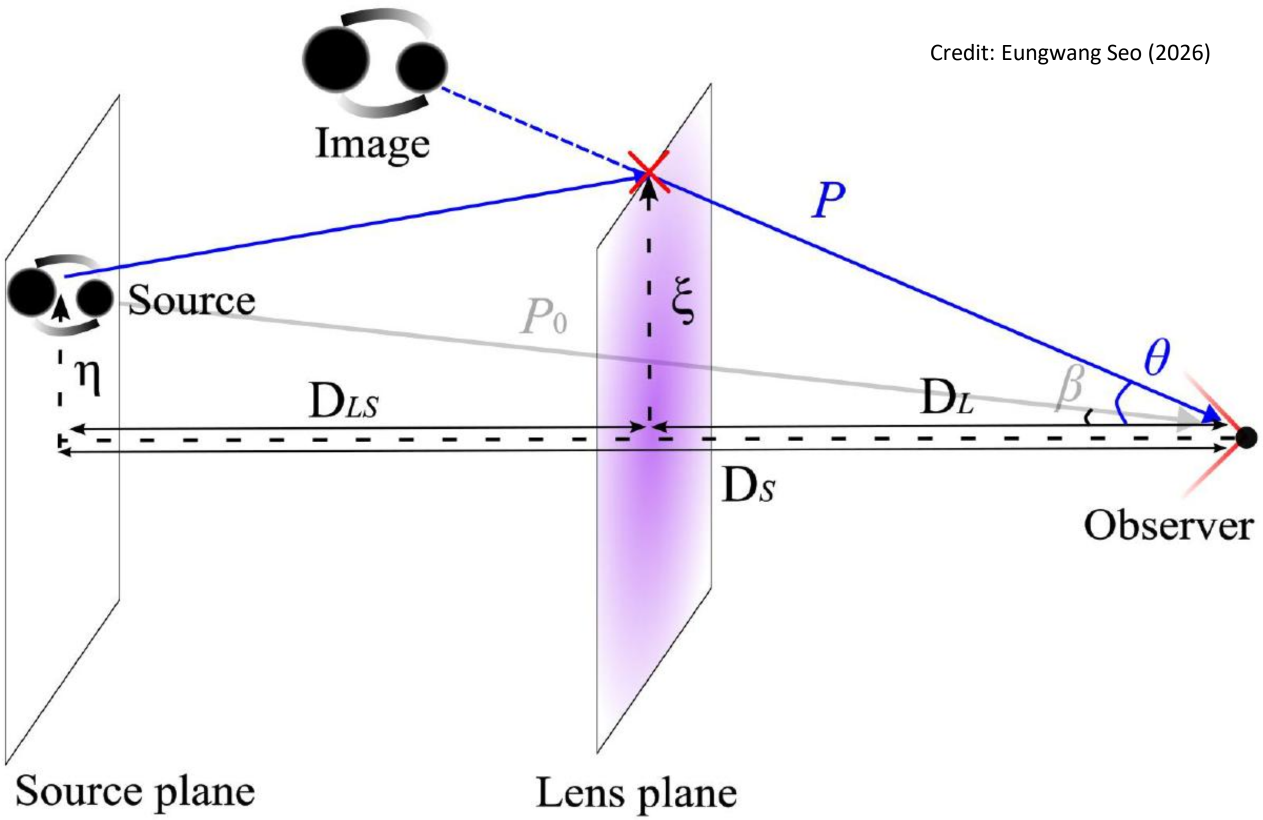
Projected lensing potential

$$\Delta t(\boldsymbol{x}) = \frac{1+z_L}{c}\frac{D_L D_S}{D_{LS}}\left[\frac{1}{2}|\boldsymbol{x}-\boldsymbol{y}|^2 - \psi(\boldsymbol{x})\right]$$
$$= D_{\Delta t} T(\boldsymbol{x}, \boldsymbol{y})$$

with dimensionless frequency $w \equiv \frac{D_S}{D_{LS} D_L}\xi_0^2 (1+z_L)\,\omega.$

Dimensionless deflections: $\boldsymbol{x} = \frac{\boldsymbol{\xi}}{\xi_0}$, $\boldsymbol{y} = \frac{D_{OL}}{\xi_0 D_{OS}} \boldsymbol{\eta}$

Credit: Eungwang Seo (2026)



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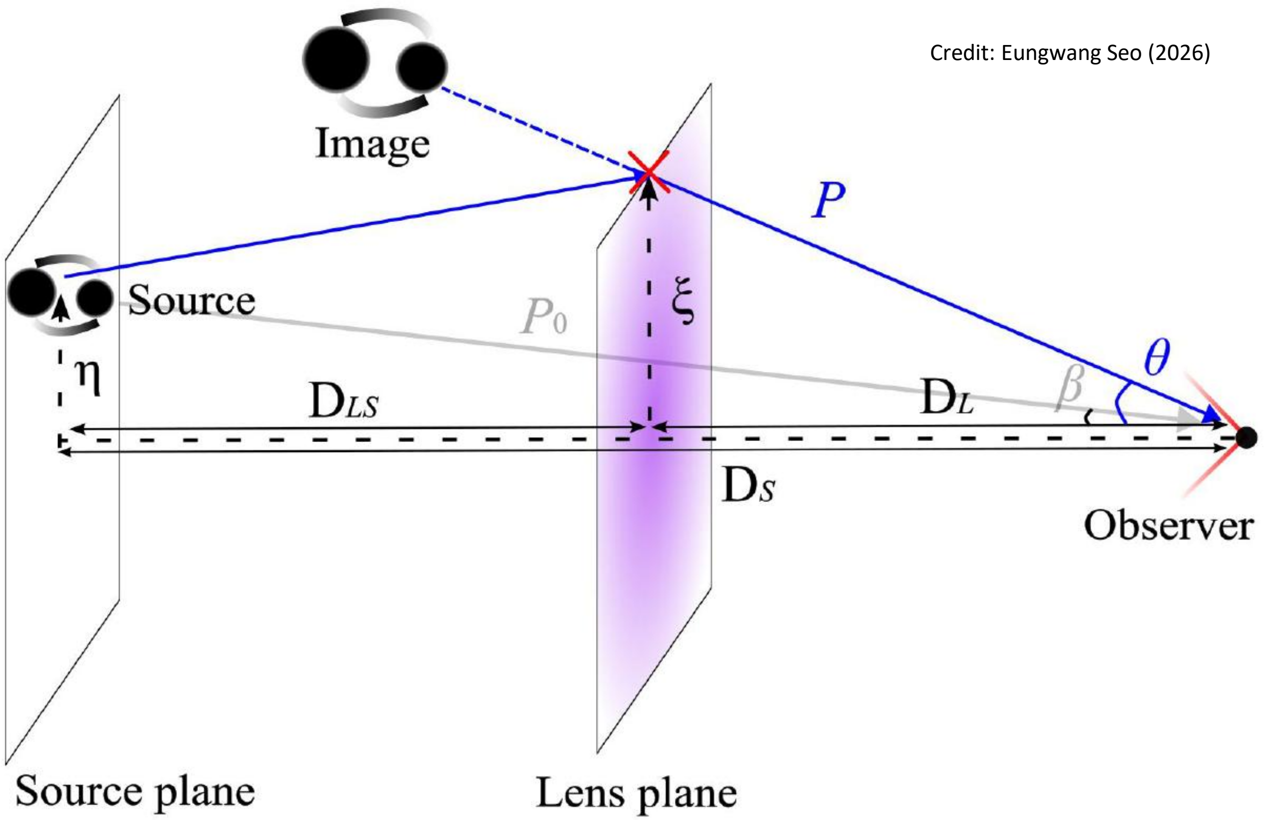
For an axially symmetric lens model:

$$\boxed{F(w,y) = -iwe^{iwy^2/2} \int_0^\infty dx\, x J_0(wxy) \exp\left[iw \left(\frac{1}{2}x^2 - \psi(x) + \phi_m(y) \right) \right]}$$

Minimum arrival time of unlensed signal

Dimensionless deflections: $\boldsymbol{x} = \frac{\boldsymbol{\xi}}{\xi_0} \quad , \quad \boldsymbol{y} = \frac{D_{OL}}{\xi_0 D_{OS}}\boldsymbol{\eta}$



Lensed wave amplitude:

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Lensed GW signal: $h^L(f) = h(f) \times F(f)$

where $$\boxed{F(w,\boldsymbol{y}) = \frac{w}{2\pi i}\int d^2x\left[\exp(iwT(\boldsymbol{x},\boldsymbol{y}))\right]}$$

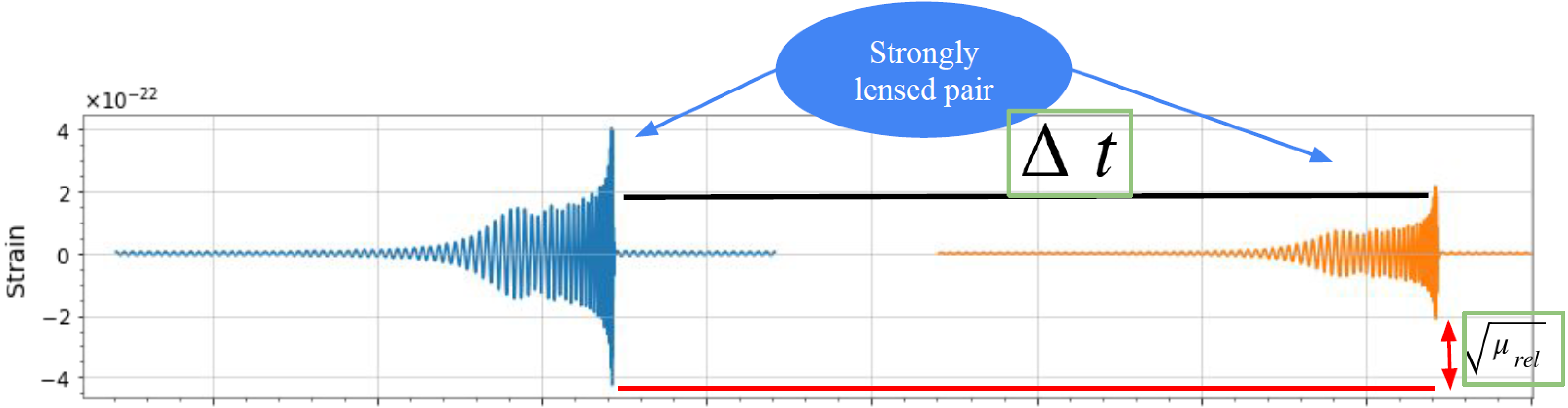
For $w \gg 1$ the phase factor $\exp(iwT(\boldsymbol{x},\boldsymbol{y}))$ becomes highly oscillatory, and contributions from different regions of the lens plane interfere destructively – except at **stationary points** of $T(\boldsymbol{x},\boldsymbol{y})$.

GEOMETRIC OPTICS APPROXIMATION: amplification factor reduces to a discrete sum over the geometric images

$$\boxed{F(w,\boldsymbol{y}) \simeq \sum_j |\mu_j|^{1/2}\exp\left[iwT_j - i\pi n_j\right]}$$

Q: What happens when a GW is ***strongly*** lensed?

A: We would observe multiple (typically N = 2 or 4) repeated events, with same frequency evolution, connected by three lensing parameters



\+ Overall phase shift → Morse factor n_{12}

Credit: Justin Janquart (2021)

Lensing: *Gravitational waves vs Electromagnetic waves*

	Light	GW (strong lensing)	GW (microlensing)
Searching	**Strong lensing & Weak lensing:** Distorted images (rings, arcs, and tails) **Microlensing:** Magnification patterns	**Matched filtering** (comparing templates with data) **Strong lensing:** Similar parameters across multiple signals **Microlensing:** Modelled search with lensed templates	
Observables	**- Deflection angles** **- Image positions** - Relative magnification factors - Time delays (for transient sources)	**- Relative magnification factors** **- Time delays** **- Phase differences** * No image positions due to poor localization	**- Model parameters** Ex) point mass lens, a) redshifted lens mass b) impact parameter

$$F(w, \boldsymbol{y}) \simeq \sum_j |\mu_j|^{1/2} \exp\left[iwT_j - i\pi n_j\right]$$

Amplification factor

Dimensionless time delay (Fermat potential)

Morse phase $n_j = 0, 1/2, 1$ corresponding to Type I, II, III images

Credit: Eungwang Seo (2026)

Lensing: *Gravitational waves vs Electromagnetic waves*

	Light	GW (strong lensing)	GW (microlensing)
Searching	**Strong lensing & Weak lensing:** Distorted images (rings, arcs, and tails) **Microlensing:** Magnification patterns	**Matched filtering** (comparing templates with data) **Strong lensing:** Similar parameters across multiple signals **Microlensing:** Modelled search with lensed templates	
Observables	- **Deflection angles** - **Image positions** - Relative magnification factors - Time delays (for transient sources)	- **Relative magnification factors** - **Time delays** - **Phase differences** * No image positions due to poor localization	- **Model parameters** Ex) point mass lens, a) redshifted lens mass b) impact parameter

$T(\boldsymbol{x}, \boldsymbol{y})$ represents time delay surface:

Hessian: $H_{ij} = \partial^2 T / \partial x_i \partial x_j$

$$H_{ij} = \delta_{ij} - \frac{\partial^2 \psi}{\partial x_i \partial x_j} = \mathcal{A}_{ij}$$

Jacobian of lens equation

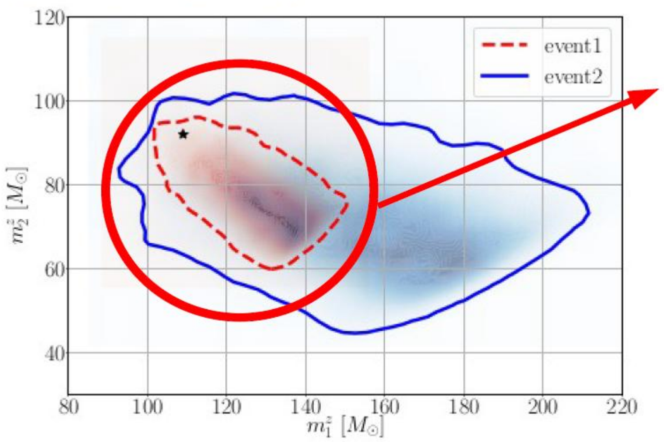
Image Type	Stationary Point	Eigenvalues	det $\mathcal{A}$	tr$\mathcal{A}$	Parity	Arrival Sequence
Type I	Minimum	Both positive	> 0	> 0	Positive	First
Type II	Saddle point	Opposite signs	< 0	–	Negative	Intermediate
Type III	Maximum	Both negative	> 0	< 0	Positive	Last

Credit: Eungwang Seo (2026)

Searching for lensing candidates: the basic idea

- Look for events with consistent posterior distributions
- Compute **posterior odds ratio** of lensed / unlensed hypothesis

Haris et al. https://arxiv.org/pdf/1807.07062



The posteriors are compatible Between lensed images.

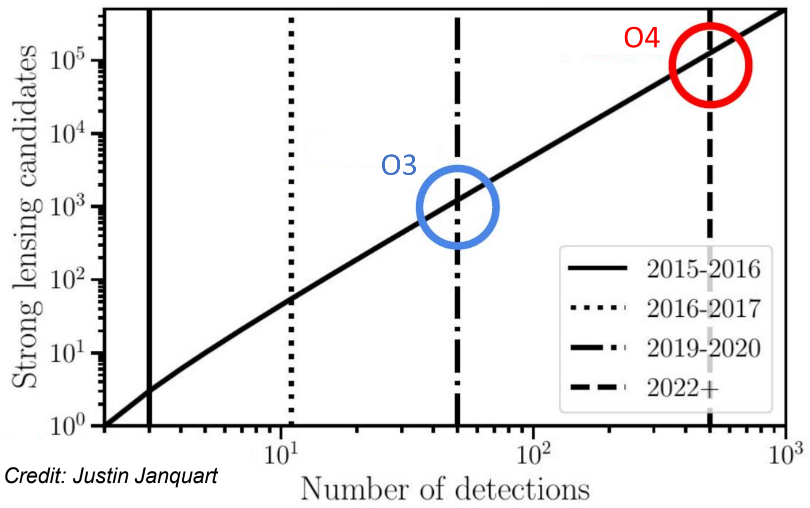
- $\mathcal{H}_L$: The data set $\{d_1, d_2\}$ contain lensed signals from a single binary black hole merger event with parameters $\boldsymbol{\theta}_1 = \boldsymbol{\theta}_2 = \boldsymbol{\theta}$.
- $\mathcal{H}_U$: The data set $\{d_1, d_2\}$ contain signals from two independent binary black hole merger events with parameters $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$.

$$\mathcal{O}^L_U = \frac{P(\mathcal{H}_L|\{d_1, d_2\})}{P(\mathcal{H}_U|\{d_1, d_2\})} = \frac{P(\mathcal{H}_L)}{P(\mathcal{H}_U)} \frac{P(\{d_1, d_2\}|\mathcal{H}_L)}{P(\{d_1, d_2\}|\mathcal{H}_U)} = \mathcal{P}^L_U\, \mathcal{B}^L_U$$

Bayes factor

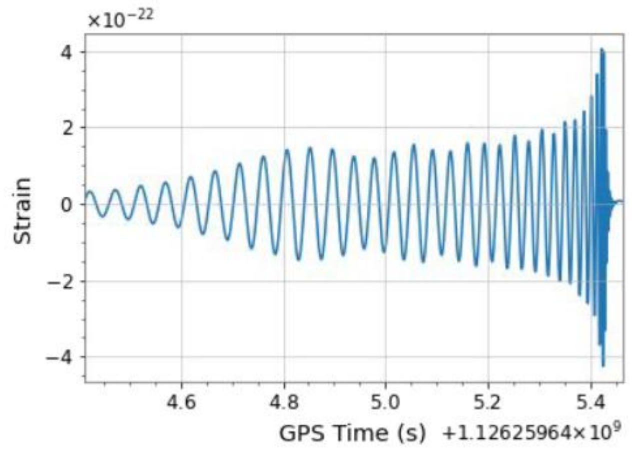
Significant computational challenge!

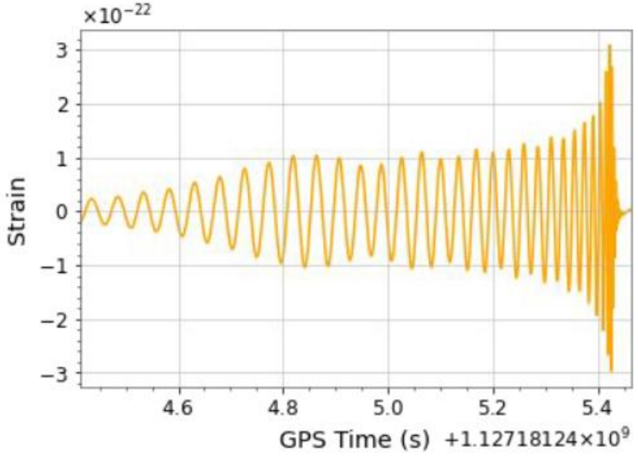
Increasing number of pairs



Credit: Justin Janquart

We need to accelerate the analysis of possible lensed pairs…





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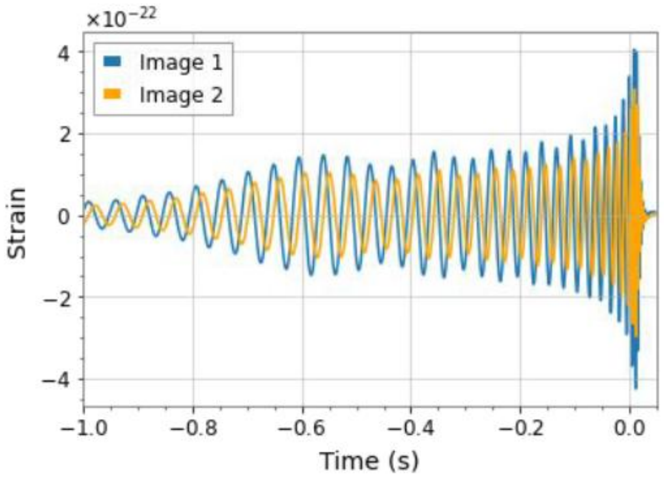


See Janquart et al. https://arxiv.org/abs/2105.04536

GOLUM: Gravitational-wave analysis **O**f **L**ensed and **U**nlensed waveform **M**odels

IDEA: use the posteriors of the first image analysis as priors for the second image

Coherence ratio $$C_U^L = \frac{p(d_1|\mathcal{H}_L)}{p(d_1|\mathcal{H}_U)} \frac{p(d_2|d_1,\mathcal{H}_L)}{p(d_2|\mathcal{H}_U)}$$

To recover the full Bayes factor, we need to account for **selection effects**; see e.g. Lo & Magana-Hernandez (2023)

Adapted from Justin Janquart (2021)

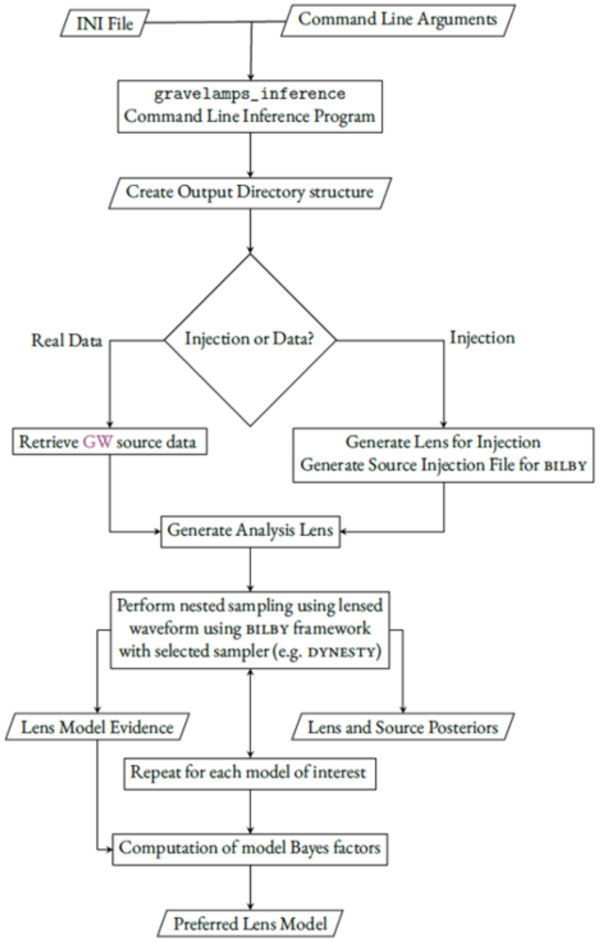
We need to accelerate the analysis of possible lensed pairs…

	Posterior overlap	GOLUM	Full joint PE
Speed	O(CPU minute)	O(CPU hour)	O(CPU month(s))
Precision	low	high	very high
Discriminatory power	weak	good	very good
Extended injection studies	computationally tractable	computationally tractable	computationally expensive

Credit: Justin Janquart (2021)

GRAVELAMPS is designed to be:

- Fast, Efficient, Open and Extendable



…but we also want to be able to compare *different* lens models:

Point mass (incl. microlensing)
SIS, SIE, SPEMD
NFW profiles, others?...

→ **GRAVELAMPS** (Wright & Hendry, 2022)

Gravitational Wave Lensing Mass Profile Model Selection

ApJ, **935**, 68 (2022); https://arxiv.org/abs/2112.07012


The Astrophysical Journal, 935:68 (11pp), 2022 August 20 https://doi.org/10.3847/1538-4357/ac7ec2

OPEN ACCESS

Gravelamps: Gravitational Wave Lensing Mass Profile Model Selection

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Abstract

We present the package GRAVELAMPS, which is designed to analyze gravitationally lensed gravitational wave signals in order to constrain the mass density profile of the lensing object. GRAVELAMPS does this via parameter estimation using the framework of BILBY, which enables estimation of both the lens and the source parameters. The package can be used to study both microlensing and macrolensing cases, where the lensing mass distribution is described by a point-mass and extended-mass density profile, respectively. It allows the user to easily and freely switch between a full wave optics and approximate geometric optics description. The performance of GRAVELAMPS is demonstrated via simulated analysis of both microlensing and macrolensing events, illustrating its capability for both parameter estimation and model selection in the wave optics and hybrid environments. To further demonstrate the utility of the package, the real gravitational-wave event GW170809 was analyzed using GRAVELAMPS; this event was found to yield no strong evidence supporting the lensing hypothesis, consistent with previously published results.

Unified Astronomy Thesaurus concepts: Gravitational wave astronomy (675); Astronomy data analysis (1858); Astronomy software (1855); Gravitational lensing (670); Strong gravitational lensing (1643); Gravitational microlensing (672)

1. Introduction

Credit: Mick Wright (2024)

Example: GW150914-like simulated event, at a **luminosity distance of 410 Mpc**, lensed by an isolated **1000-solar mass** point-mass lens.

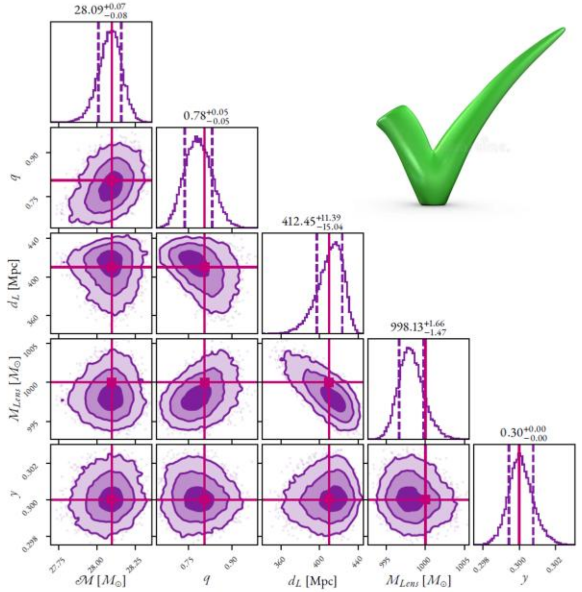
Source Parameter	Value
Primary object mass, $m_1 (M_\odot)$	36
Secondary object mass, $m_2 (M_\odot)$	29
Luminosity distance, d_L (Mpc)	410
Dimensionless primary spin magnitude, a_1	0.4
Dimensionless secondary spin magnitude, a_2	0.3
Primary tilt angle, θ_1	0.5
Secondary tilt angle, θ_2	1.0
Azimuthal angle between component spins, ϕ_{12},	1.7
Azimuthal angle between total binary angular and orbital momenta, ϕ_{jl}	0.3
Angle between total binary angular momentum and line-of-sight, θ_{jn}	0.4
Polarisation angle, ψ	2.659
Phase at reference frequency, ϕ	1.3
Right Ascension, α	1.375
Declination, δ	1.2108
Coalescence Time, t_c	1125259642.413

Y = Model used to fit the data

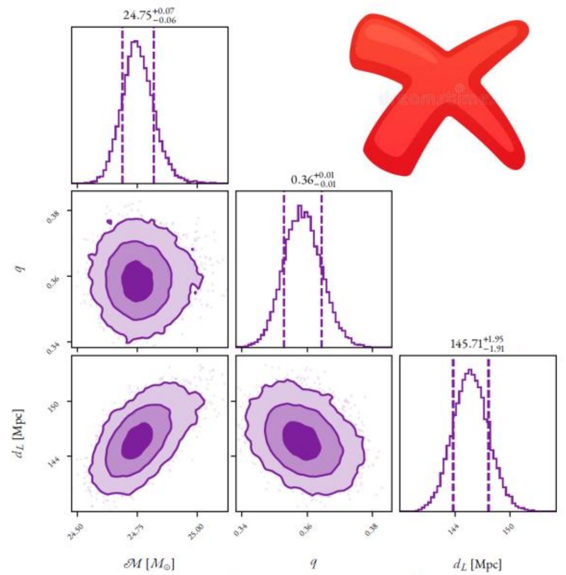
X = Model used to generate data

	Unlensed	Isolated Point Mass	SIS	NFW
Unlensed	0.00	6.06	−0.43	−0.82
Isolated Point Mass	3729.28	0.00	13.25	49.26
SIS	4016.60	49.93	0.00	476.14
NFW	113.80	37.01	27.00	0.00

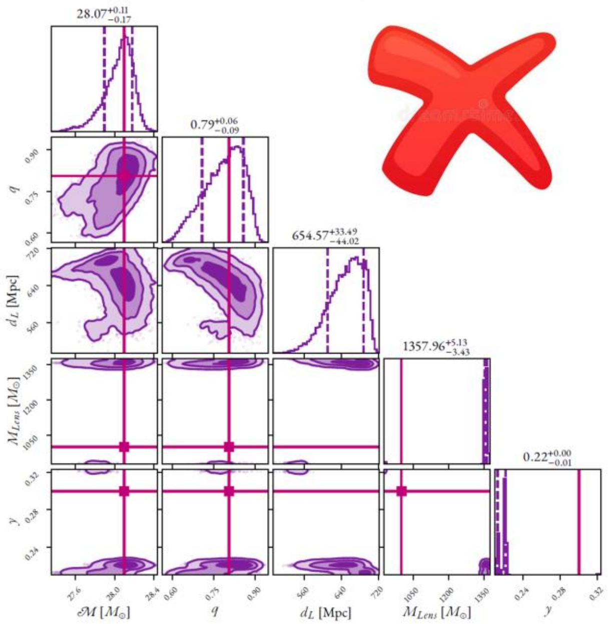
Log_{10} Bayes factors from data generated with model X, fitted with model Y



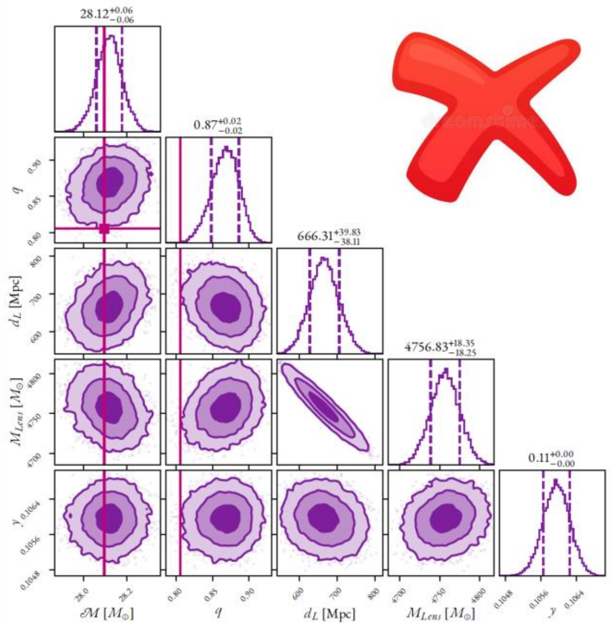
Correctly modelled as a **point lens**



Incorrectly modelled as **unlensed**



Incorrectly modelled as an **SIS lens**



Incorrectly modelled as an **NFW lens**

Credit: Mick Wright (2024)

GRAVELAMPS has now been combined with GOLUM….


THE ASTROPHYSICAL JOURNAL, 959:70 (9pp), 2023 December 20 https://doi.org/10.3847/1538-4357/ad0891

Determination of Lens Mass Density Profile from Strongly Lensed Gravitational-wave Signals

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[2] Department of Physics, Institute for Gravitational and Subatomic Physics (GRASP), Utrecht University, Princetonplein 1, NL-3584 CC Utrecht, The Netherlands
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Abstract

As the interferometers detecting gravitational waves are upgraded, improving their sensitivity, the probability of observing strong lensing increases. Once a detection is made, it will be critical to gain as much information as possible about the lensing object from these observations. In this work, we present a methodology to rapidly perform model selection between differing mass density profiles for strongly lensed gravitational-wave signals, using the results of the fast strong-lensing analysis pipeline GOLUM. We demonstrate the validity of this methodology using some illustrative examples adopting the idealized singular isothermal sphere and point-mass lens models. We take several simulated lensed signals, analyze them with GOLUM, and subject them to our methodology to recover both the model and its parameters. To demonstrate the methodology's stability, we show how the result varies with the number of samples used for a subset of these injections. In addition to the analysis of simulations, we also apply our methodology to the gravitational-wave event pair GW191230–LGW200104, two events with similar frequency evolutions and sky locations, which was analyzed in detail as a potential lensing candidate but ultimately discarded when considering the full population and the uncertain nature of the second event. We find a preference for the singular isothermal sphere model over the point mass, though our posteriors are much wider than for the lensed injections, in line with the expectations for a nonlensed event. The methodology developed in this work is made available as part of the GRAVELAMPS software package.

Unified Astronomy Thesaurus concepts: Gravitational lensing (670); Gravitational waves (678); Strong gravitational lensing (1643)

1. Introduction

In the wake of a rapidly increasing number of gravitational-wave (GW) detections, with 90 confirmed detections as of the latest catalog of events, GWTC-3 (The LIGO Scientific Collaboration et al. 2021), and with the coming years set to deliver significant improvements in the sensitivity of the existing network of ground-based GW detectors, currently comprising Advanced LIGO (Aasi et al. 2015), Advanced Virgo (Acernese et al. 2015), and KAGRA (Somiya 2012; Aso et al. 2013; Akutsu et al. 2021), the opportunities are manifest for the nascent field of GW astronomy to pursue a wide variety of scientific investigations.

One example of a particularly active area of GW research is that of GW lensing (Hannuksela et al. 2019; Abbott et al. 2021; Janquart et al. 2023b; The LIGO Scientific Collaboration et al. 2023). Analogously to the deflection of light by a strong gravitational field (Einstein 1936)—which formed one of the first major tests of Einstein's general relativity (GR; Dyson et al. 1920)—GW signals may be lensed by intervening objects such as stars, black holes, galaxies, and galaxy clusters (Ohanian 1974; Wang et al. 1996; Takahashi & Nakamura 2003; Smith et al. 2017; Mishra et al. 2021).

As of the end of the third observing run of the current ground-based detector network, searches for lensed GW signals have not yet yielded a confirmed detection (Abbott et al. 2021; Janquart et al. 2023b; The LIGO Scientific Collaboration et al. 2023). However, with the increased detection rate of GW events expected during the next observing run, there is a significant probability that one or more of these detections may be identified as *strongly lensed* (Smith et al. 2017; Li et al. 2018; Ng et al. 2018; Smith et al. 2018; Wierda et al. 2021; Xu et al. 2022; Smith et al. 2023). In this scenario the GW signal would be lensed by an intervening galaxy (Dai et al. 2017; Ng et al. 2018; Robertson et al. 2020) or galaxy cluster (Smith et al. 2017, 2018; Robertson et al. 2020), forming multiple distinct "images." These images would be potentially magnified, time-shifted, and overall phase-shifted copies of the original signal that may be detected as repeated events (Dai & Venumadhav 2017; Janquart et al. 2021; Liu et al. 2021; Lo & Magana Hernandez 2021; Janquart et al. 2023a).

Detection of such a lensed event would not only be of enormous scientific interest in itself but would also immediately leverage additional scientific benefit from the information carried by lensed GW signals from compact binary coalescence events—much as the detection of GW signals from such events in the first place has opened the door to completely new ways in which to investigate these phenomena. Examples of the kinds of investigations that could be enabled by lensed GW signals include, but are not limited to, improving the localization of detected GW events (Hannuksela et al. 2020; Wempe et al. 2022), precision cosmography (Sereno et al. 2011; Liao et al. 2017), or tests of the speed of gravity (Fan et al. 2017), or general relativity itself (Ezquiaga & Zumalacárregui 2020; Goyal et al. 2021).

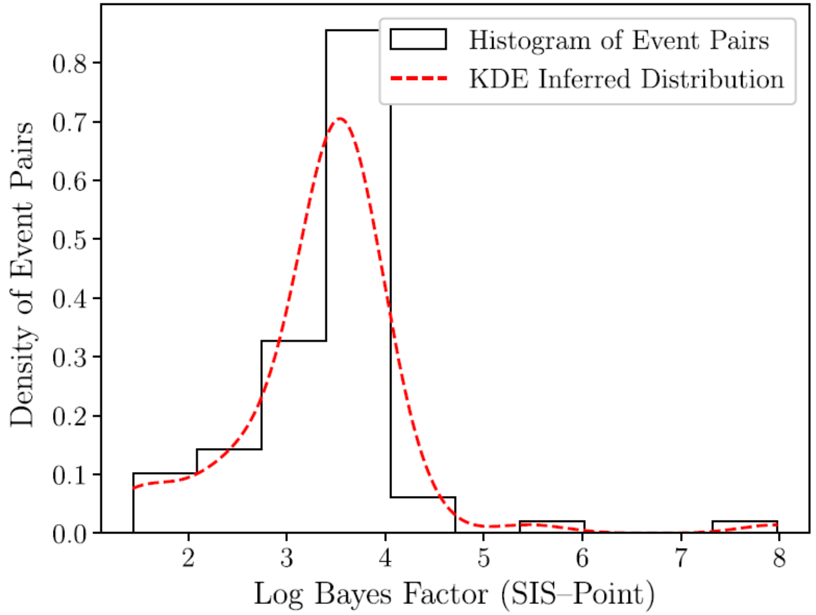
Moreover, investigations of lensed GW systems may also reveal important information about the nature of the lens. For example, it has been demonstrated that the mass-density profile

1

Wright, Janquart & Hendry, ApJ, **959**, 70 (2023)

https://arxiv.org/abs/2307.07852

….allowing rapid Bayesian model comparison of different lens models…



[e.g. injection set of 100 SIS model lensing events]

GRAVELAMPS has now been combined with GOLUM….

THE ASTROPHYSICAL JOURNAL, 959:70 (9pp), 2023 December 20 https://doi.org/10.3847/1538-4357/ad0891

Determination of Lens Mass Density Profile from Strongly Lensed Gravitational-wave Signals

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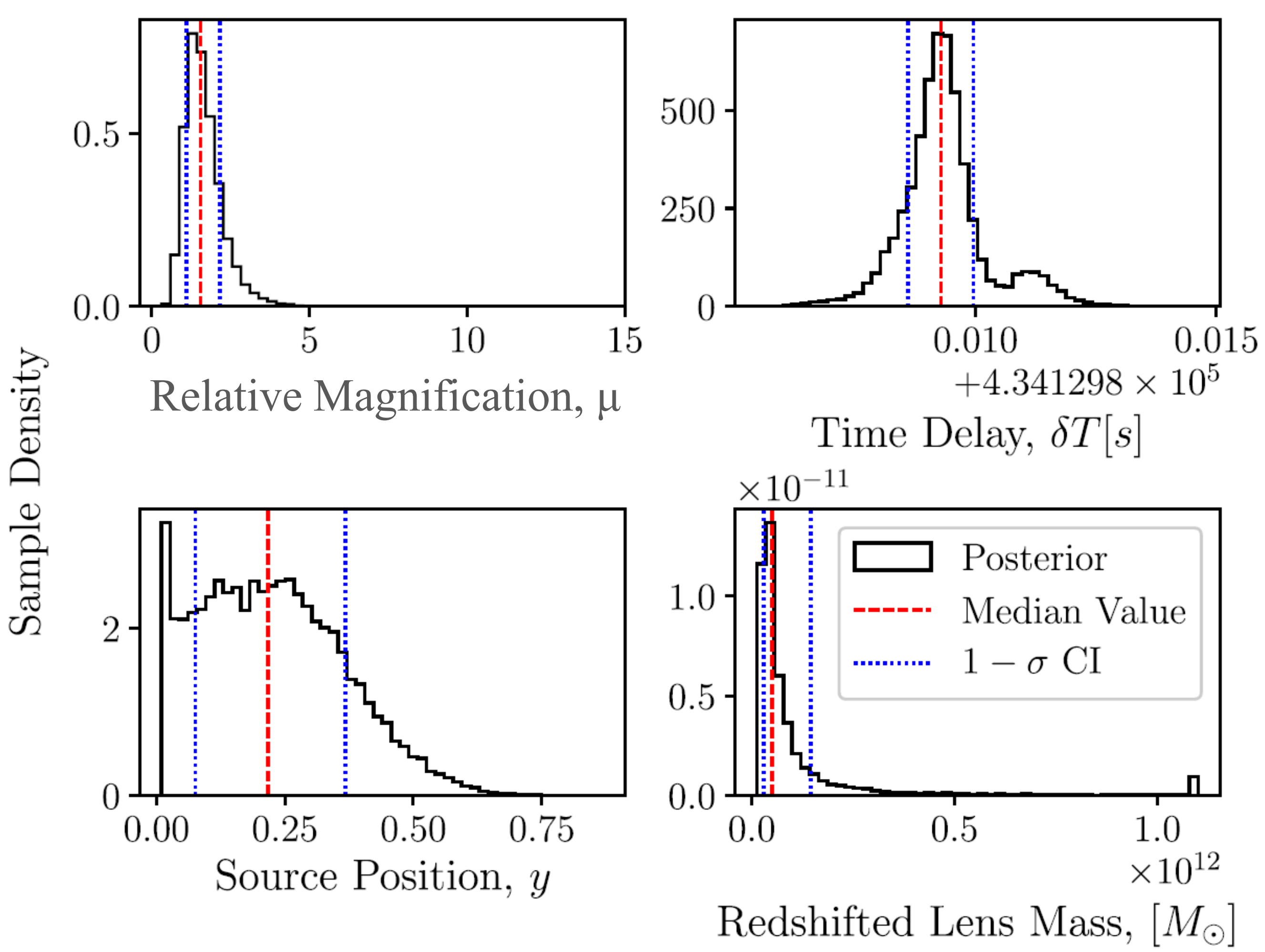
Wright, Janquart & Hendry, **ApJ**, 959, 70 (2023)

https://arxiv.org/abs/2307.07852

….and rapid parameter estimation for lensing candidates

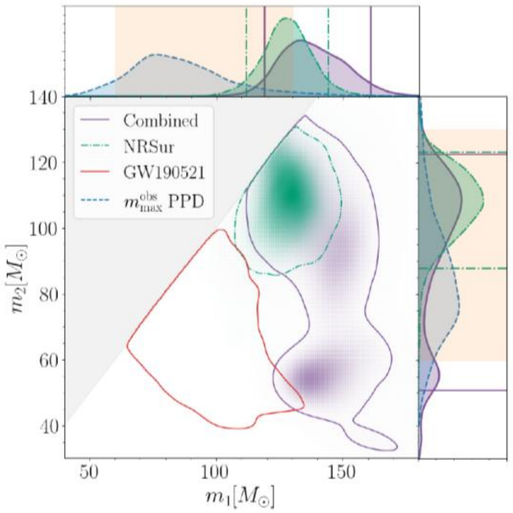


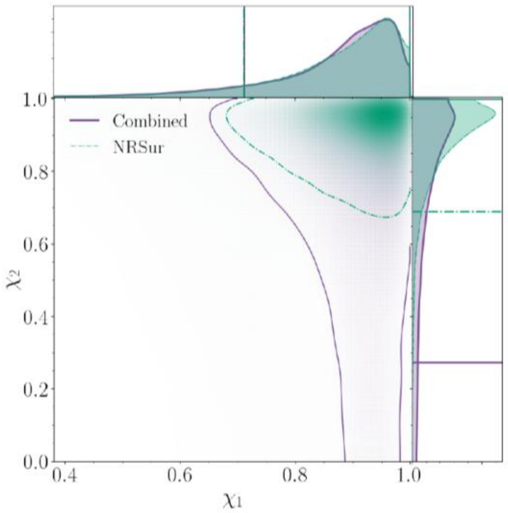
GRAVELAMPS-GOLUM analysis of **GW191230-LGW200104**, the lensing candidate with the highest Bayes factor in the O3 LVK lensing search, https://arxiv.org/abs/2304.08393 .

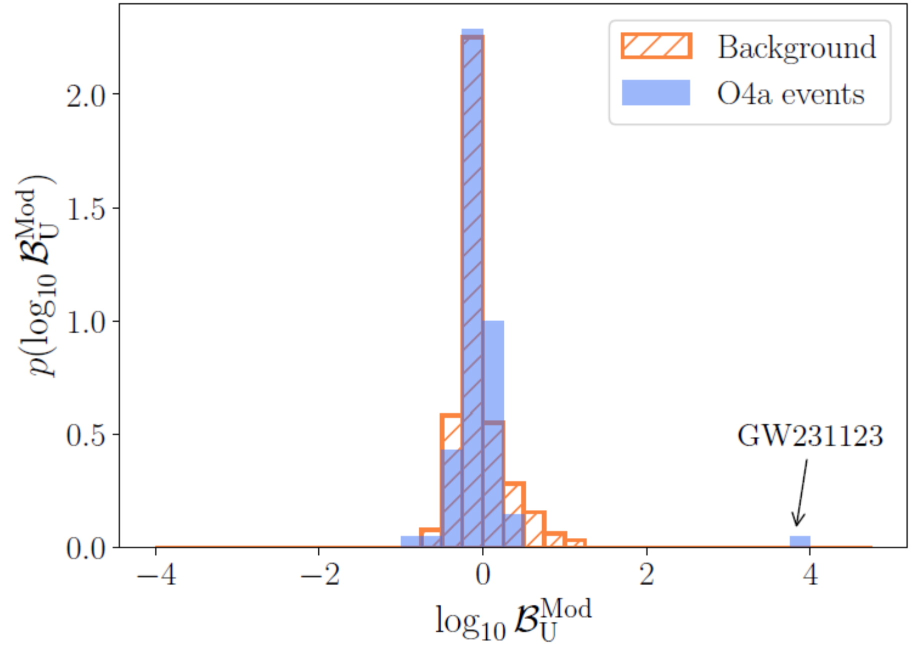
GRAVELAMPS has also been used to search for candidates lensed by point-mass objects, in the latest published **LVK lensing results** (for O4a).... Abac et al (2025): https://arxiv.org/abs/2512.16347

See also Abac et al (2025): https://arxiv.org/abs/2507.08219

Lensing candidate of interest: **GW231123**

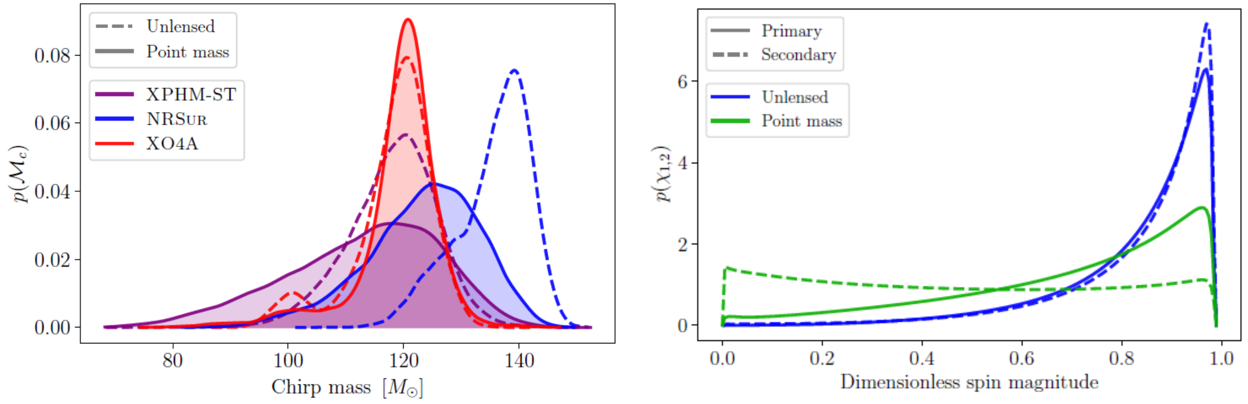






Caveats and issues:

- PE dependence on waveform approximants
- Broader posteriors, but reduced discrepancies
- Only isolated point-mass lens model considered
- Is astrophysical background representative?...



What about composite situations, where both macro- and microlensing is present?

Investigation

- A galactic-scale macrolens splits and distorts a GW into multiple signals, with **additional microlensing by stellar fields causing further distortions**.

- Detecting combined strong lensing (SL) and microlensing (ML) effects requires **computationally expensive templates** incorporating both phenomena.

- Current SLGW search pipelines neglect microlensing, so **we focus on residuals (data – best-fit template) to identify microlensing signatures**, similar to testing GR methodologies (e.g., Abbot et al. (2016))

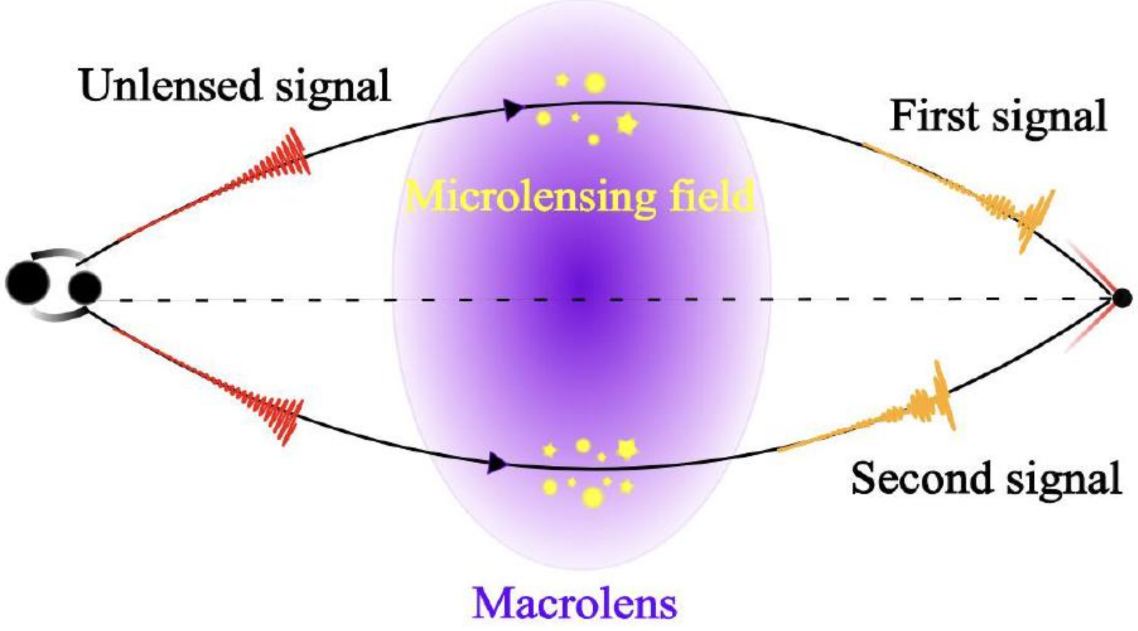


Fig. 1: GW Lensing by microlensing field embedded in a macrolens.

The Astrophysical Journal, 988:159 (12pp), 2025 August 1 https://doi.org/10.3847/1538-4357/ade4bf

Residual Test to Search for Microlensing Signatures in Strongly Lensed Gravitational Wave Signals

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Following Eungwang Seo (2025)

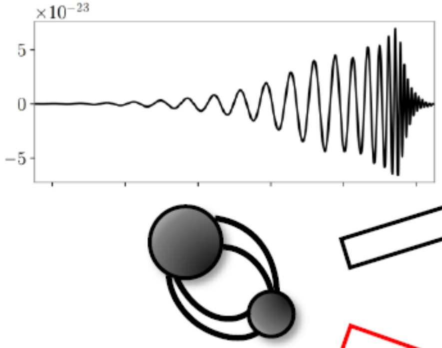
Seo et al., ApJ, **988**, 159 (2025)

https://arxiv.org/abs/2503.02186

Credit: Eungwang Seo (2025)

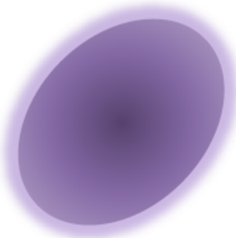
Data generation: Simulating lensed GW signals

A Binary Black hole (BBHs) (a GW signal)

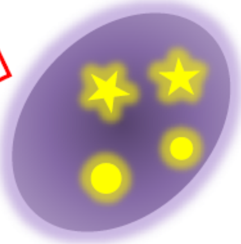


- **Merger rate:** *Xu et al. (2022)*
- **Mass model:** *Power-law + peak (Abbott et al. 2019)*
- **Waveform model:** *IMRPhenomPv2*

<Lensing hypothesis>



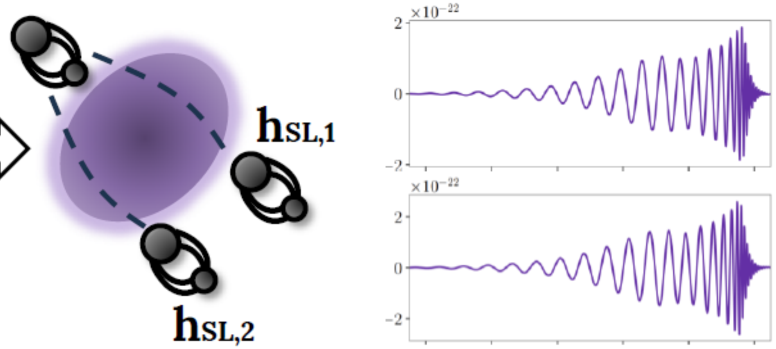
$\mathcal{H}_{\mathrm{SL}}$:
Singular Isothermal Ellipsoid (SIE)

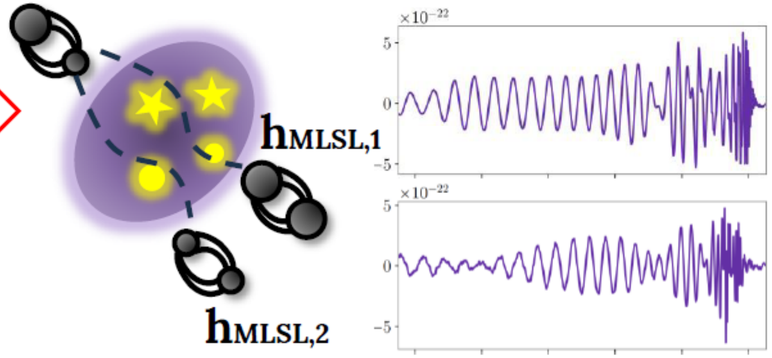


$\mathcal{H}_{\mathrm{MLSL}}$:
SIE + point mass fields (Shan et al. 2023, 2024)

*** The SIE used in $\mathcal{H}_{\mathrm{MLSL}}$ is the same as that used in $\mathcal{H}_{\mathrm{SL}}$**

Resulting lensed BBHs (two lensed GW signals)





SET1: 100 $\times$ ($h_{\mathrm{SL},1}$ + $h_{\mathrm{SL},2}$)
SET2: 100 $\times$ ($h_{\mathrm{MLSL},1}$ + $h_{\mathrm{MLSL},2}$)

We want to compare the results from two SETs.

Credit: Eungwang Seo (2025)

Data analysis: Residual calculation

- We assume all data in i^{th} detector can be written as :

$$\boxed{d^i = h^i_{\text{SL}} + dh^i + n^i}$$

, where ***dh*** **is the microlensing-induced term.**

- For **SET 1** (injected signals are **purely SLGW**), the residuals are computed as follows:

$$\begin{aligned} r^i|\mathcal{H}_{\text{SL}} &\equiv \lim_{dh^i \to 0} \left(d^i - h^{i,\text{maxL}}\right) \\ &= h^i_{\text{SL}} + n^i - h^{i,\text{maxL}} \\ &= \delta h^i_{\text{SL}} + n^i, \end{aligned}$$

- For **SET 2** (injected signals are **MLSLGW**), the residuals are computed as follows:

$$\begin{aligned} r^i|\mathcal{H}_{\text{MLSL}} &\equiv d^i - h^{i,\text{maxL}} \\ &= h^i_{\text{SL}} + dh^i + n^i - h^{i,\text{maxL}} \\ &= \delta h^i_{\text{MLSL}} + n^i, \end{aligned}$$

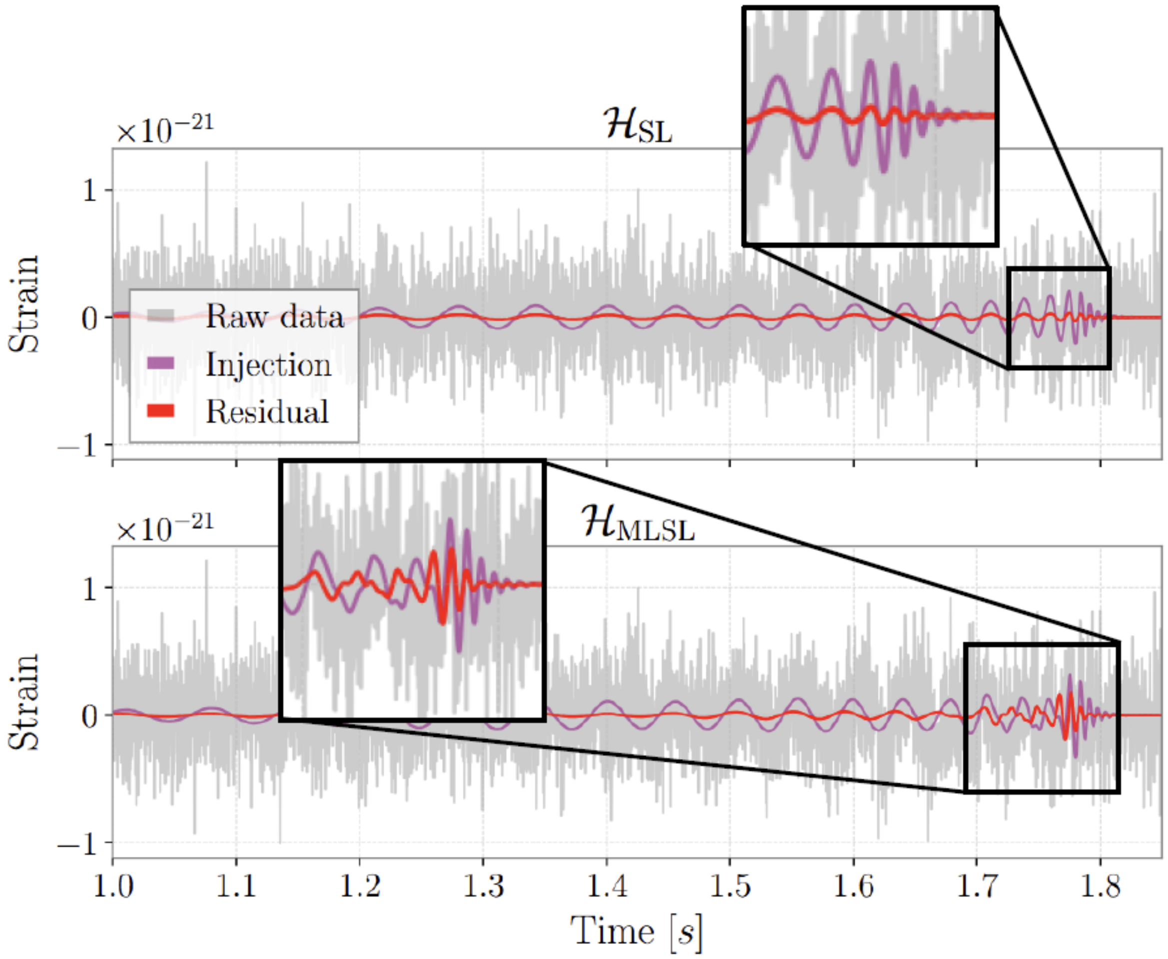


Fig.2: Representative δh_{SL} and δh_{MLSL} at a detector

Credit: Eungwang Seo (2025)

Data analysis: Residual test (Goodness-of-fit test)

Now, we have **200 r_{SL}** and **200 r_{MLSL}**.

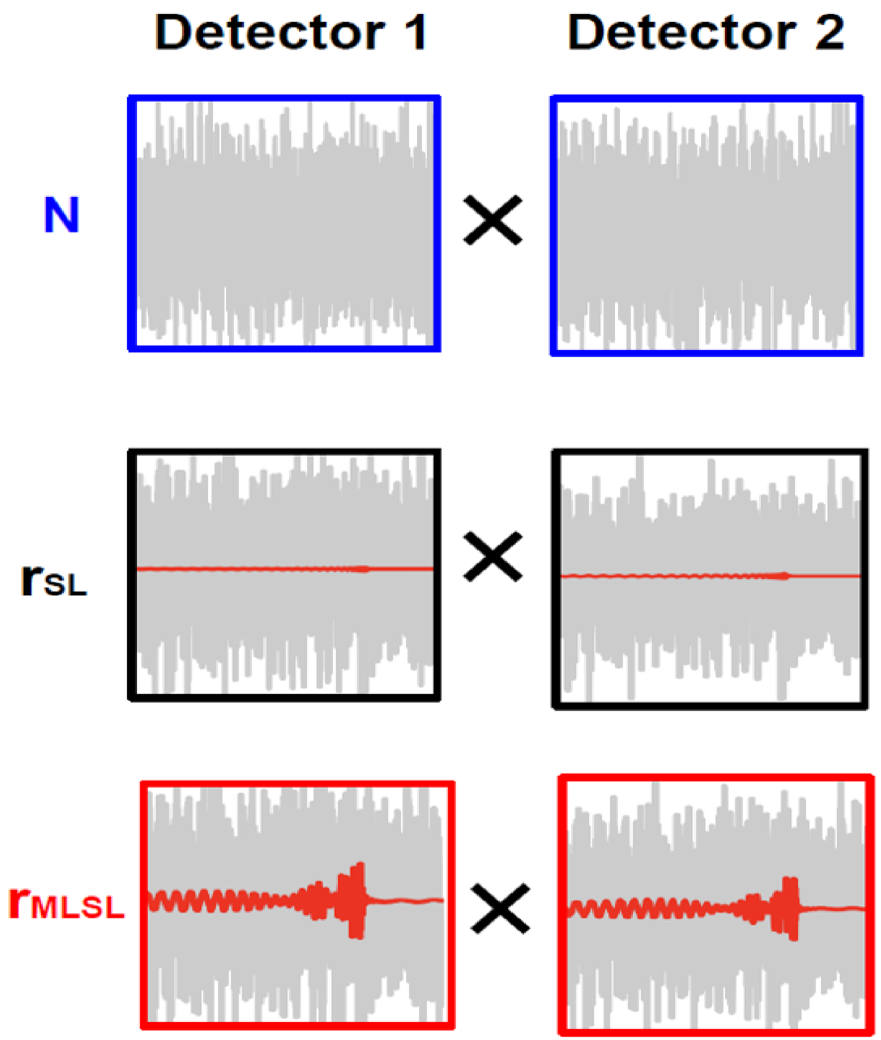
A parameter **Λ**, a function of the **inner product of residuals** in i^{th} and j^{th} detector, for evaluating how significant residuals are:

$$\Lambda_{\mathcal{H}_X} = \exp\left[\frac{1}{2}\langle r^i(t)_{\mathcal{H}_X}, r^j(t+\tau)_{\mathcal{H}_X}\rangle\right]^{*}$$

, where X is either of SL or MLSL and τ is the time delay between the arrival times of a GW at the i^{th} and j^{th} detector.

- In addition to **200 r_{SL}** and **200 r_{MLSL}**, we generated a set of 200 Gaussian noise segments (**N**).
- Expected results:
  1) $\Lambda_{\mathcal{H}_{SL}}$ dist. $\approx$ $\Lambda_{\mathcal{H}_N}$ dist.
  2) $\Lambda_{\mathcal{H}_{MLSL}}$ dist. $\not\approx$ $\Lambda_{\mathcal{H}_N}$ and $\Lambda_{\mathcal{H}_{SL}}$ dist.

$$* \ \langle \mathbf{a}, \mathbf{b}\rangle = 2\int_0^{\infty} \frac{\tilde{a}(f)\tilde{b}^*(f) + \tilde{a}^*(f)\tilde{b}(f)}{S_n(f)} \mathrm{d}f\ ,$$



Adapted from Eungwang Seo (2025)

Result: Detected or Not?

$$\Lambda_{\mathcal{H}_X} = \exp\left[\frac{1}{2}\langle r^i(t)_{\mathcal{H}_X}, r^j(t+\tau)_{\mathcal{H}_X}\rangle\right]$$

Expected results:

1) $\boldsymbol{\Lambda}_{\mathcal{H}_{\mathbf{SL}}}$ dist. $\approx$ $\boldsymbol{\Lambda}_{\mathcal{H}_{\mathbf{N}}}$ dist.

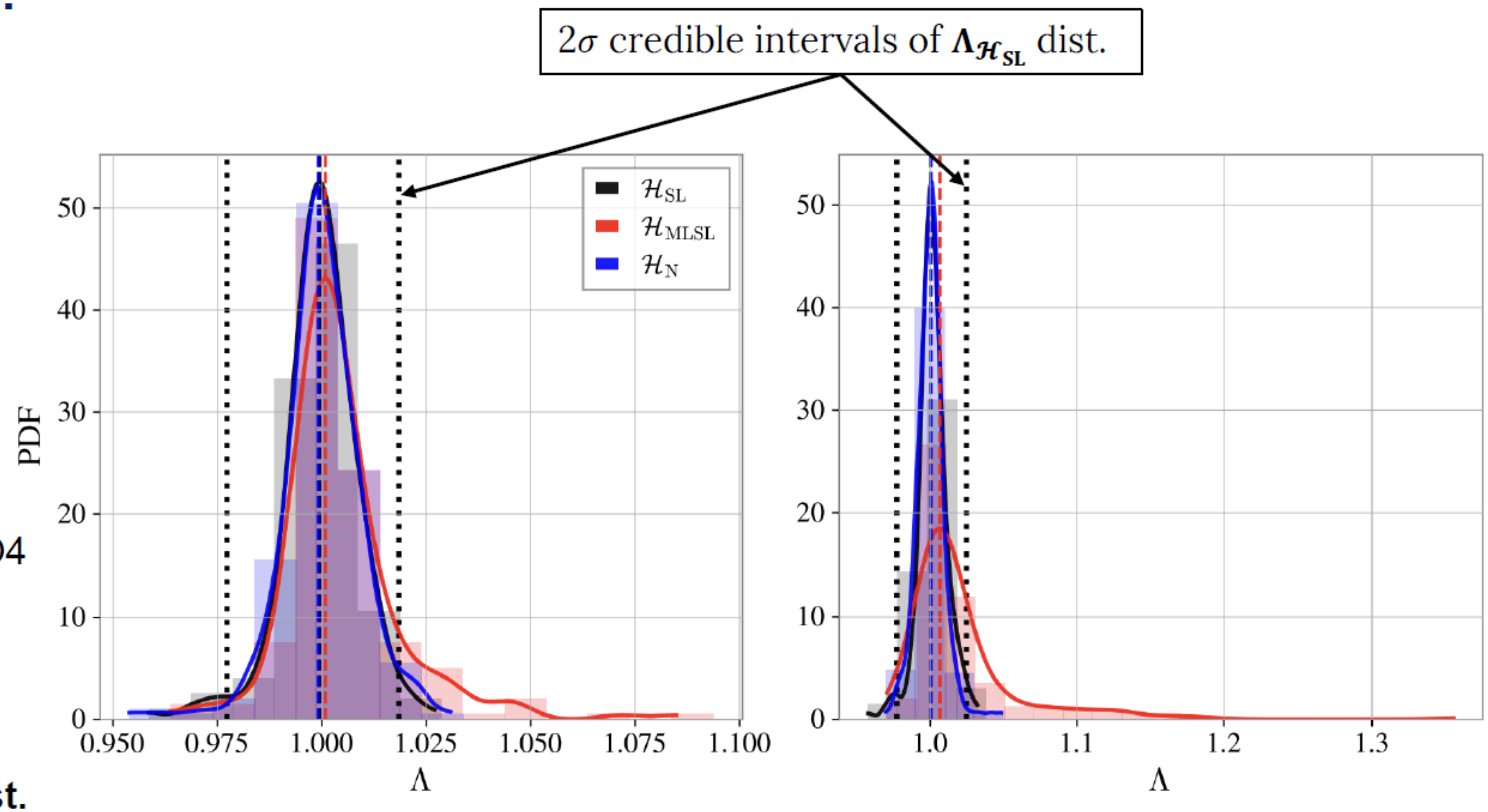
2) $\boldsymbol{\Lambda}_{\mathcal{H}_{\mathbf{MLSL}}}$ dist. $\not\approx$ $\boldsymbol{\Lambda}_{\mathcal{H}_{\mathbf{N}}}$ and $\boldsymbol{\Lambda}_{\mathcal{H}_{\mathbf{SL}}}$ dist.

Results:

- $\boldsymbol{\Lambda}_{\mathcal{H}_{\mathbf{MLSL}}}$ dist. are positively-skewed in both O4 and O5 cases. The outlier events beyond the $\Lambda_{\mathcal{H}_{\mathrm{SL}}}$ distribution should be focused.

*** Detection criterion:**

$\boldsymbol{\Lambda}_{\mathcal{H}_{\mathbf{MLSL}}}$ > upper bound of 2σ C.I. of $\boldsymbol{\Lambda}_{\mathcal{H}_{\mathbf{SL}}}$ dist.



<O4: 22 / 200 (11.0%)**>**

<O5: 43 / 200 (21.5%)**>**

Fig.3: Lambda distributions for Noise, SLGW, and MLSLGW

Credit: Eungwang Seo (2025)

Result: With ‘*discernible*’ microlensed events (1)

With our realistic population of lensed GW events, we would not expect current detectors to be able to identify weaker microlensed events. We therefore check the efficacy of the residual test applied only to microlensed events with **higher mismatch values**, which should be more readily distinguishable with current detectors.

- **Mismatch:**
  dissimilarity between two waveforms

$$\mathrm{MM} \equiv 1 - \mathcal{M}\left(h_{\mathrm{MLSL}}, h_{\mathrm{SL}}\right) = 1 - \max_{\phi_0, t_0} \frac{\langle h_{\mathrm{MLSL}}, h_{\mathrm{SL}} \rangle}{\sqrt{\langle h_{\mathrm{MLSL}}, h_{\mathrm{MLSL}} \rangle \langle h_{\mathrm{SL}}, h_{\mathrm{SL}} \rangle}}.$$

- **Realistic population:**
  Most of events have **MM < 0.03**
  11% (22%) were identified as microlensed in the **O4 (O5)** case.

- **Scenario 2:**
  All detected events have **MM > 0.03**
  28% (52%) were identified as microlensed in the **O4 (O5)** case.

- **Scenario 3:**
  All detected events have **MM > 0.1**
  35% (66%) were identified as microlensed in the **O4 (O5)** case.

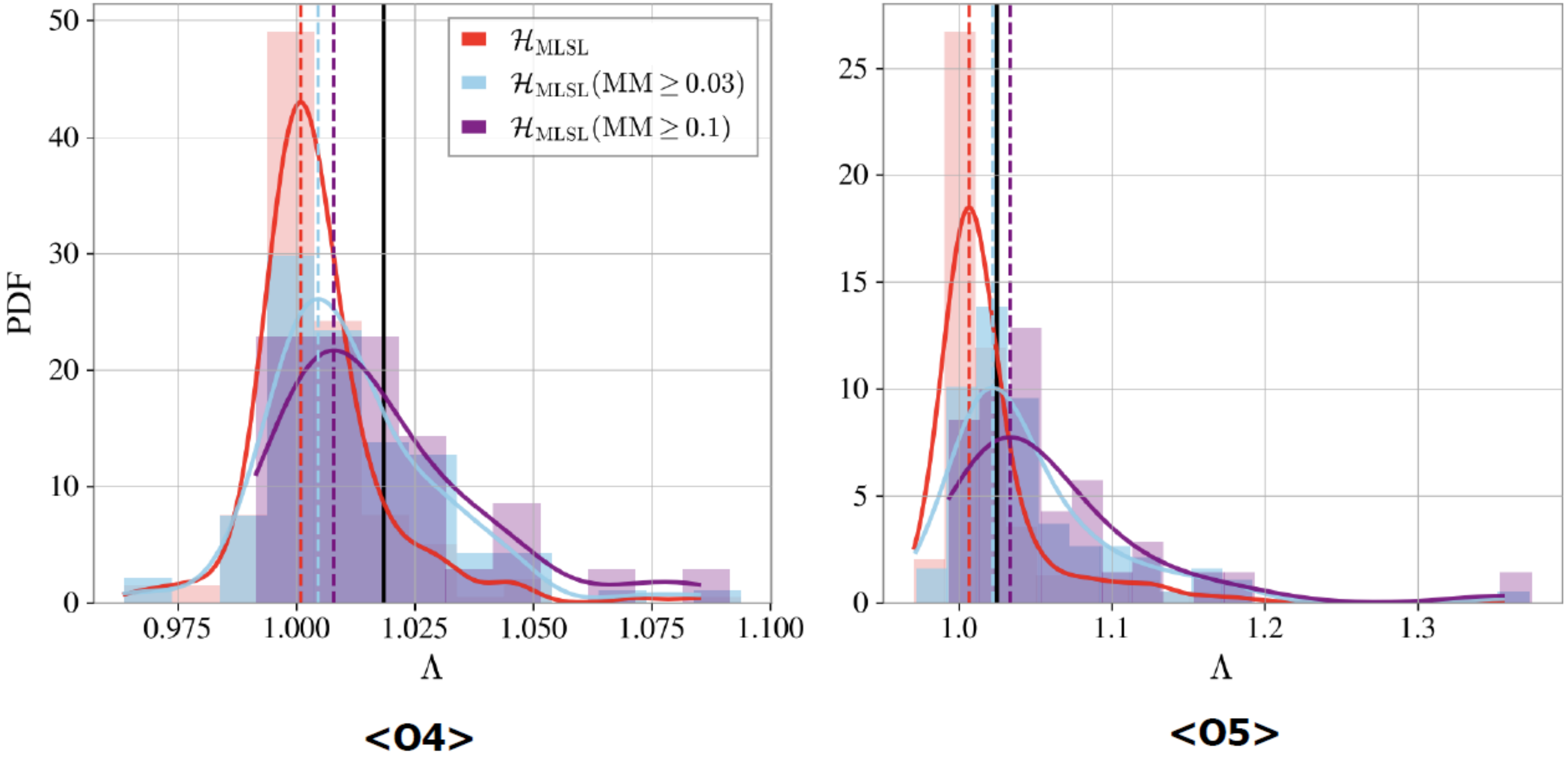


Fig.4: Lambda distributions for realistic pop, MM>0.03, and MM>0.1 events.

Adapted from Eungwang Seo (2025)

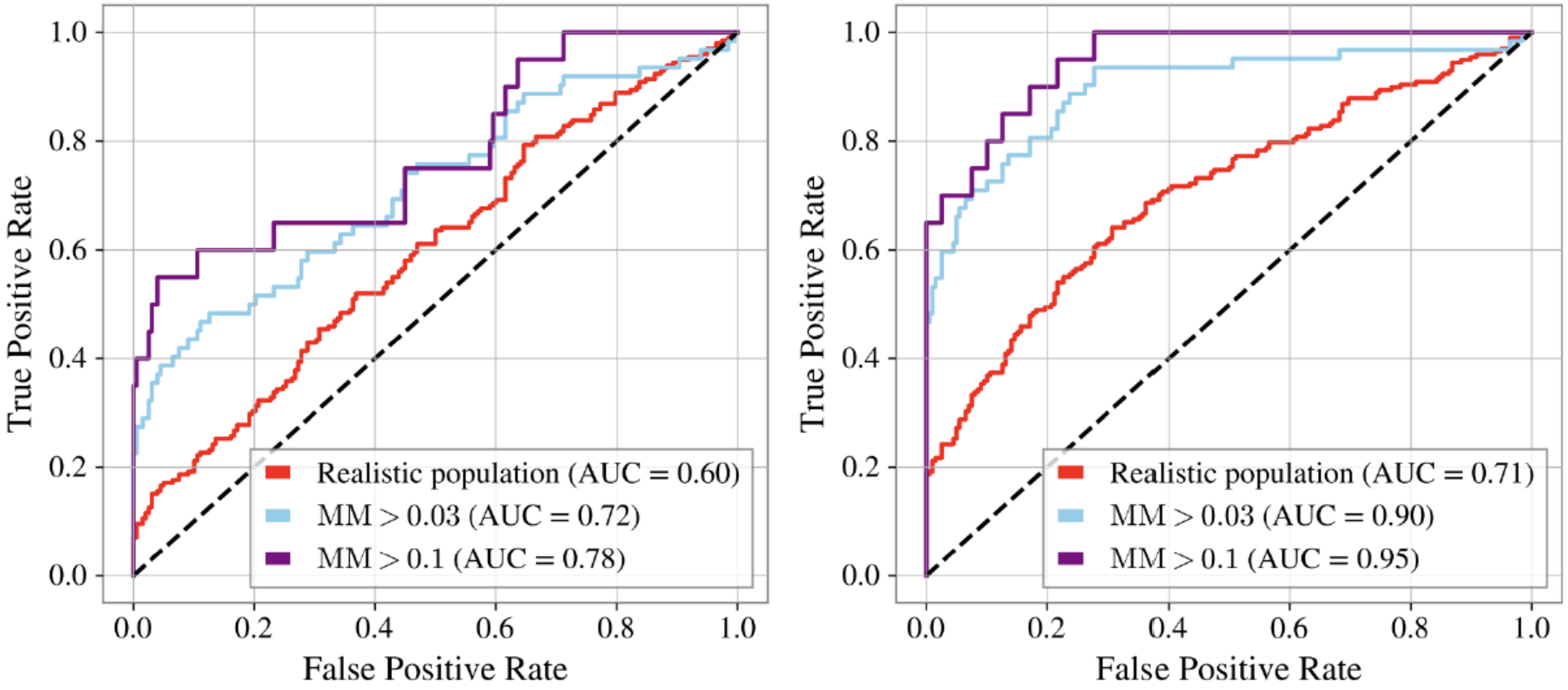
Result: With ‘*discernible*’ microlensed events (2)

We created ROC (Receiver Operating Characteristic) curves showing the performance of our residual test (i.e., classifier).

- **All AUC (Area Under the Curve) values are greater than 0.5**
  → The residual test effectively distinguishes microlensed GW signals.

  * AUC = 1: Perfect classifier
  * AUC = 0.5: Random classifier

- **AUC increases with**
  • higher detector sensitivity
  • larger mismatch values

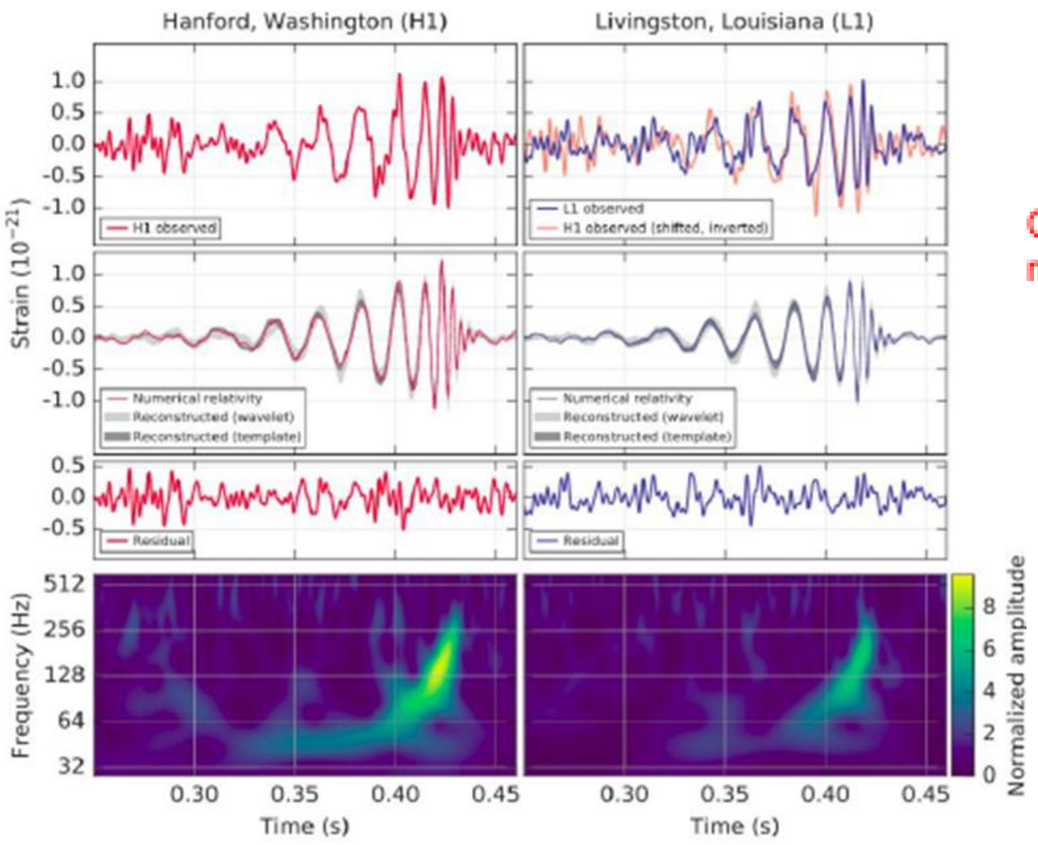


<O4> **< O5>**

Fig.5: ROC curves for realistic pop, MM>0.03, and MM>0.1 events.

Credit: Eungwang Seo (2025)

Cosmology with Standard Sirens



'Chirping' waveform

Chirp mass

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$h \propto \mathcal{M}^{5/3} f^{2/3} D_L^{-1}$$

$$\dot{f} \propto \mathcal{M}^{5/3} f^{11/3}$$

Measure

$$\boxed{h, f, \dot{f} \Rightarrow \mathcal{M}, D_L}$$

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Determining the Hubble constant from gravitational wave observations

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I report here how gravitational wave observations can be used to determine the Hubble constant, H_0. The nearly monochromatic gravitational waves emitted by the decaying orbit of an ultra-compact, two-neutron-star binary system just before the stars coalesce are very likely to be detected by the kilometre-sized interferometric gravitational wave antennas now being designed[1-4]. The signal is easily identified and contains enough information to determine the absolute distance to the binary, independently of any assumptions about the masses of the stars. Ten events out to 100 Mpc may suffice to measure the Hubble constant to 3% accuracy.

The signal from a system of two $1M_\odot$ stars (where $M_\odot$ is the mass of the Sun) will sweep from 100 Hz to 1 kHz in ~3 s. There might be three events per year out to 100 Mpc, and if the detectors achieve their current design sensitivity, such events will be detectable with a signal-to-noise ratio of 30. To determine the distance, the signal has to be observed by a worldwide network of three, and preferably four, detectors. By measuring both the response of the detectors and the delays between the arrival times of the signal at different detectors, the network should be able to locate the source in an error box of ~36 square degrees. There is some chance that the coalescence event will be optically identifiable (I.D. Novikov, personal communication); otherwise, clustering of galaxies provides a statistical method that will still yield H_0 after remarkably few events. Here I give only a brief discussion; full details will be published elsewhere.

Several detectors being developed in the United States and Europe[2-4] will take the form of interferometers with arm lengths 1-4 km, observing bandwidths 10^2-10^4 Hz and r.m.s. noise levels at 100 Hz of $<10^{-24}$ strain $Hz^{-1/2}$. Within 10 years we may expect that there will be four or five such detectors in operation in America and Europe, with typical separations of 6,500 km. [It is possible that bar detectors could contribute to these observations. However, because of their narrow bandwidth, their detection of coalescing binaries requires quite different methods, which have not been studied. I shall therefore concentrate on interferometric detectors.]

Although there are many possible sources of gravitational waves, the most promising for detection by these instruments seems to be the coalescence of binary neutron stars, as will happen to the binary pulsar PSR1913+16 in $\sim 10^8$ yr. The gravitational waves from these sources before coalescence can be predicted very reliably (K. S. Thorne, personal communication). As an orbit decays through the emission of gravitational radiation, its eccentricity is reduced, so we need only consider systems with circular orbits[5]. Consider a binary at a distance $100r_{100}$ Mpc, with total mass $m_T M_\odot$ and reduced mass $\mu M_\odot$, emitting waves at frequency $100f_{100}$ Hz (twice its orbital frequency). The standard 'quadrupole formula' of general relativity[6,7] shows that the waves will have amplitude (r.m.s.-averaged over detector and source orientations)

$$\langle h \rangle = 1 \times 10^{-23} m_T^{2/3} \mu f_{100}^{2/3} r_{100}^{-1} \quad (1)$$

and that their frequency will change on a timescale

$$\tau = f/\dot{f} = 7.8 m_T^{-2/3} \mu^{-1} f_{100}^{-8/3}\ \mathrm{s} \quad (2)$$

Two $1.4M_\odot$ neutron stars will coalesce[1] when $f \approx 10^3$ Hz. By using matched filters to analyse the data[5], the noise can effectively be limited to a bandwidth of $\sim\tau^{-1}$. This will enable the detectors to see binary neutron star sources at 100 Hz at a distance of 100 Mpc, with a mean signal-to-noise ratio (SNR) of >30. An observation will therefore determine τ and h to perhaps 3%. The key to our method is that the stars' masses enter equations (1) and (2) in exactly the same way, so that

$$r_{100} = 7.8 f_{100}^{-2} (\langle h_{23} \rangle \tau)^{-1} \quad (3)$$

where $\langle h_{23} \rangle = \langle h \rangle \times 10^{23}$, independently of the masses of the stars.

This result is not quite so strong as it seems, as equation (1) gives the r.m.s. value of h averaged over orientations, whereas the value of h inferred from the network's observations will depend on the binary system's orientation and position relative to the detectors as well as its distance. However, these can be determined from the observations: as I show below, provided that three or more detectors register the same event, they can determine the location on the sky and the degree of elliptical polarization of the wave. (In general relativity, gravitational waves are transverse and have only two independent polarizations[6-7].) Now, the radiation emitted by the binary along its angular momentum axis is circularly polarized, whereas that in the equatorial plane is linearly polarized. The degree of eliptical polarization therefore determines the inclination of the orbit to the line of sight, which enables us to solve for r_{100} in terms of the observed h. Equation (3) also depends on being able to model the system as two newtonian point masses. As we shall see below, tidal and relativistic corrections are negligible in the range of orbital parameters we require.

Being able to determine r directly from the observations is remarkable in itself, but it is only really useful if the source of the event can be identified. For this an accurate position is required. Because this accuracy is crucial for the determination of the Hubble constant, I will discuss it in some detail.

Each detector has quadrupolar linear polarization, so it is not highly directional; however, the differences in arrival time of a wave at different detectors can be used to triangulate the position. Between any two detectors with separation d, a wave travelling at an angle θ to the line joining the detectors will arrive at the second detector with a delay $\Delta t = d \cos\theta / c$ relative to the first, where c is the speed of light. For $d = 6.5 \times 10^3$ km, we have $|\Delta t| \leq 22$ ms. As the two detectors will generally not have the same polarization, there will be a further effective time delay due to the wave's elliptical polarization. Such a polarization can be regarded as a superposition of the two independent linear polarizations defined by the detectors, with a phase shift between them. This sphase shift means that differently polarized detectors record the wave train with extra time delays of up to one period (+10 ms for a 100-Hz signal). The two independent time delays measured among three detectors and the three measured amplitudes are sufficient to determine the waves' five unknowns: arrival directions (two), amplitudes of the different polarizations (two), and phase lag of the polarizations (one).

The precision with which the source's position and polarization can be measured depends on the two sorts of errors: the accuracy with which the arrival time of the wave at a detector (and hence the time delays) can be determined, and the accuracy with which the amplitude of the detector's response can be measured. In what follows, I will assume that $m_T^{2/3} \mu = 1$ (for example, two stars of $\sim 1.1 M_\odot$) to illustrate the situation. We shall see that the timing accuracy is typically 1% of the maximum timing range (from −22 to +22 ms), and the amplitude error is ~3%. When only three detectors see an event, there are actually two error boxes of size ~10°×10°, which may be too large for our purposes. I will therefore consider events detected in four instruments. The seven data overdetermine the five unknowns, and this redundancy offers us the opportunity to reduce the effective amplitude noise (it also allows a test of Einstein's polarization predictions). In this way, three timing measurements at ±1% and one amplitude measurement with effective error ±3% can be used to locate the source. This suggests that a positional error of ±3° is not unreasonable, giving an error

Schutz, "Determining the Hubble Constant from gravitational wave observation" Nature, **323**, 310 (1986)

$$h_{\rm meas} = \frac{G\boxed{(1+z)\mathcal{M}}/c^2}{D_L(z)} [\pi \boxed{(1+z)\mathcal{M}} f(t)]^{2/3} \mathcal{F}\,(\text{angles}) \cos\Phi\left(m_1, m_2, \vec{S}_1, \vec{S}_2; t\right)$$

Independent route to the Hubble Constant

Bright Sirens

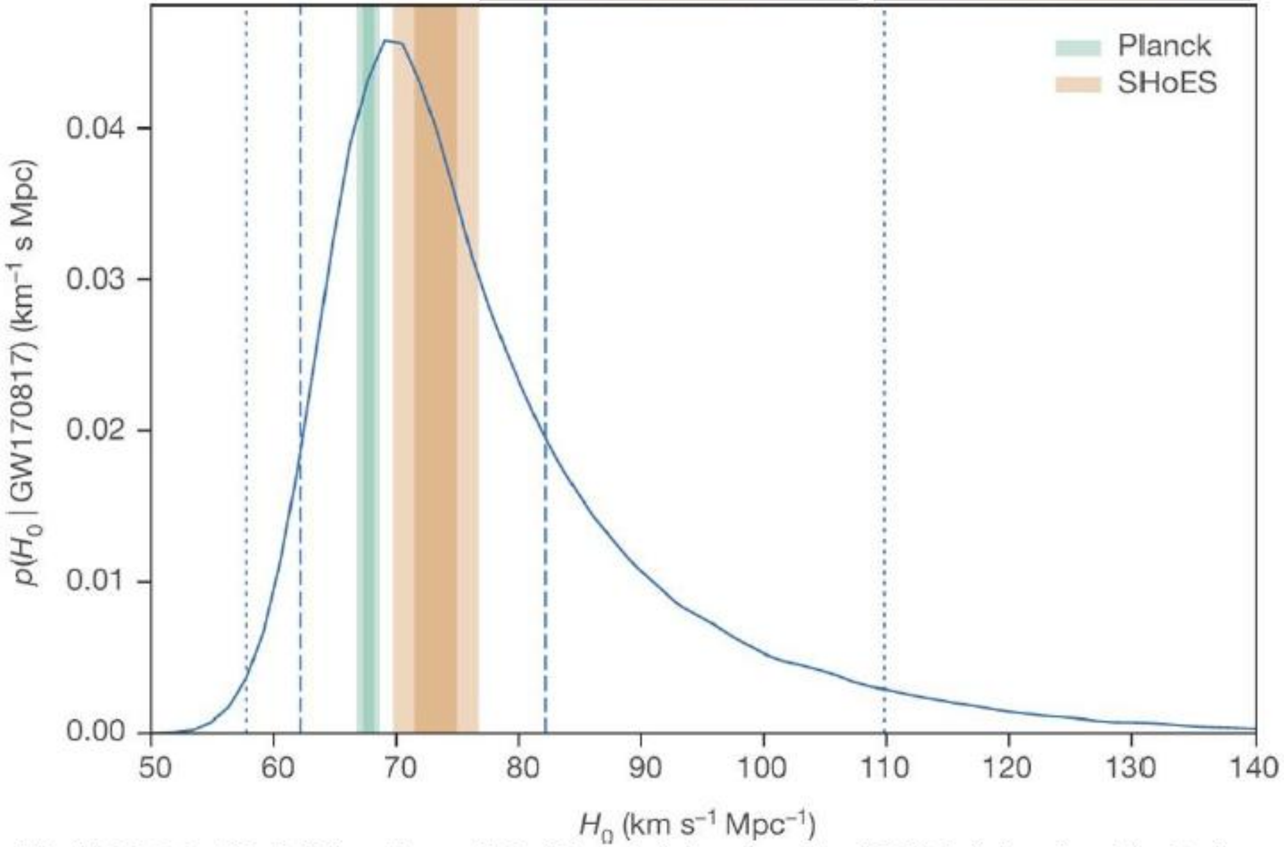
Any GW standard siren with an electromagnetic counterpart.

Observe EM counterpart → identify host galaxy → measure host galaxy's redshift → **measure *H_0***

Complicated by galaxy *peculiar velocities* (see, e.g. Mukherjee+ 2020).

$$\boxed{H_0 = 70.0^{+12.0}_{-8.0}\,\mathrm{km\ s^{-1}\,Mpc^{-1}}}$$

M. Soares-Santos *et al.* 2017 *ApJL* **848** L16



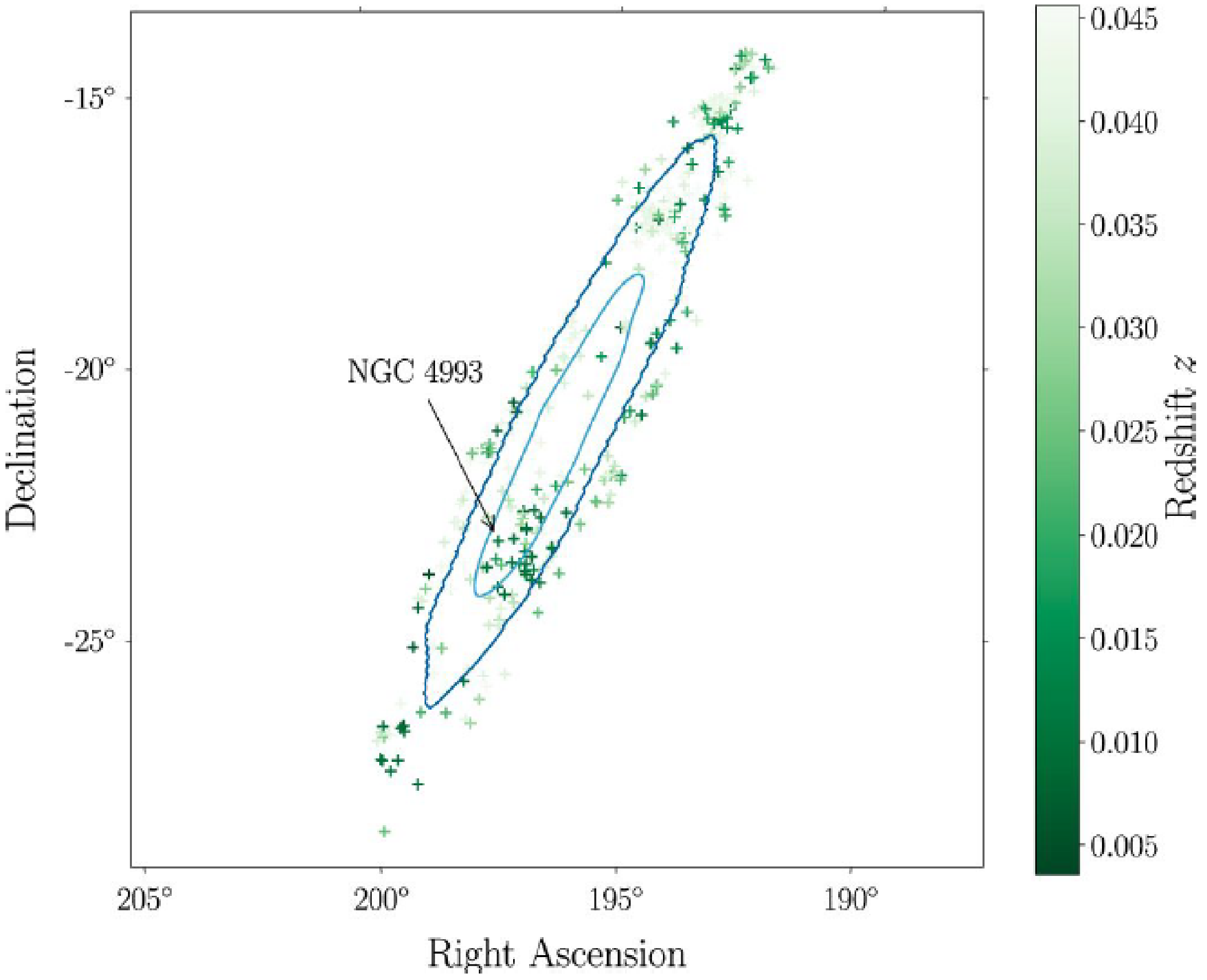
The LIGO Scientific Collaboration and The Virgo Collaboration, The 1M2H Collaboration, The Dark Energy Camera GW-EM Collaboration and the DES Collaboration *et al.* *Nature* **551**, 85–88 (2017).

The dark siren + galaxy catalogue method

Don't know the true host galaxy, so treat all galaxies in the GW localisation as potential hosts and marginalise over them

(see, e.g. Schutz 1986, Del Pozzo 2012, Chen+ 2018, Soares-Santos+ 2019, Gray+ 2020, Finke+ 2021…)

Related method: cross-correlation of large-scale structure distribution with siren information. See e.g. Mukherjee et al. (2024), arXiv:2203.03643,



M. Fishbach *et al.* 2019 *ApJL* **871** L13.

Following Rachel Gray (2023)

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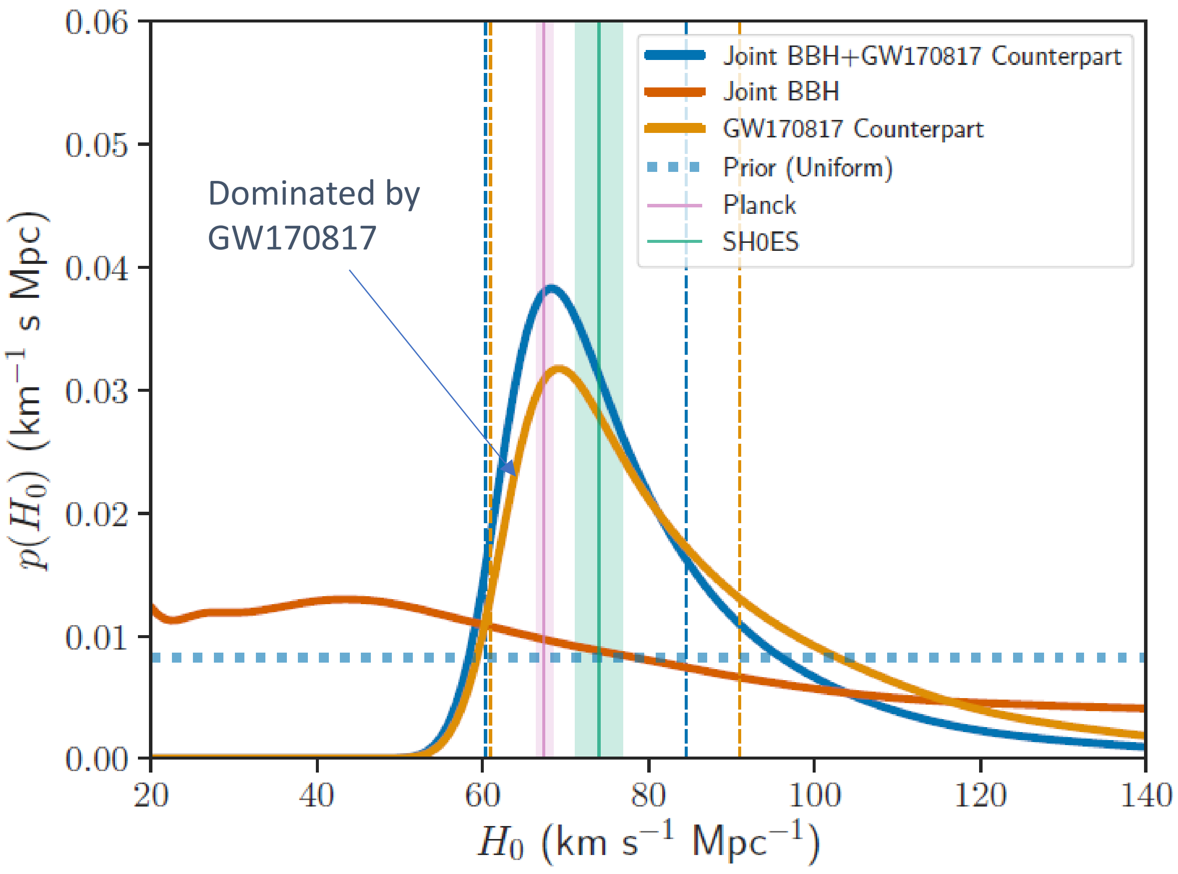


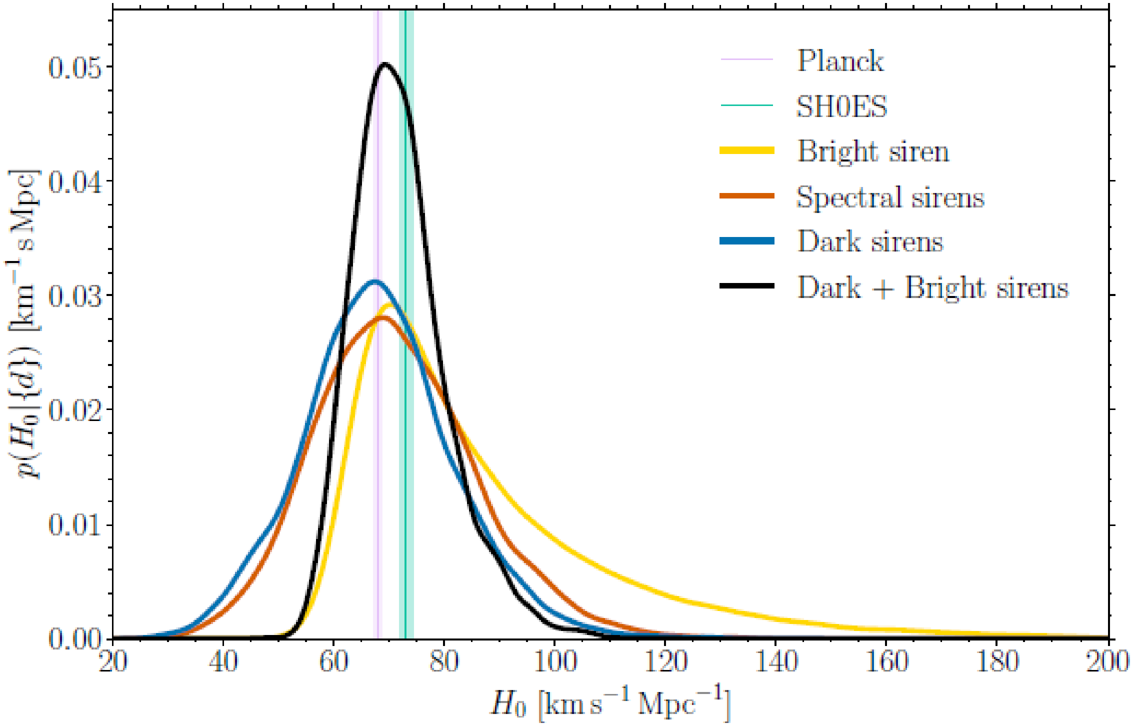
Abbott et al., "A gravitational-wave measurement of the Hubble constant following the second observing run of Advanced LIGO and Virgo", http://arxiv.org/abs/1908.06060

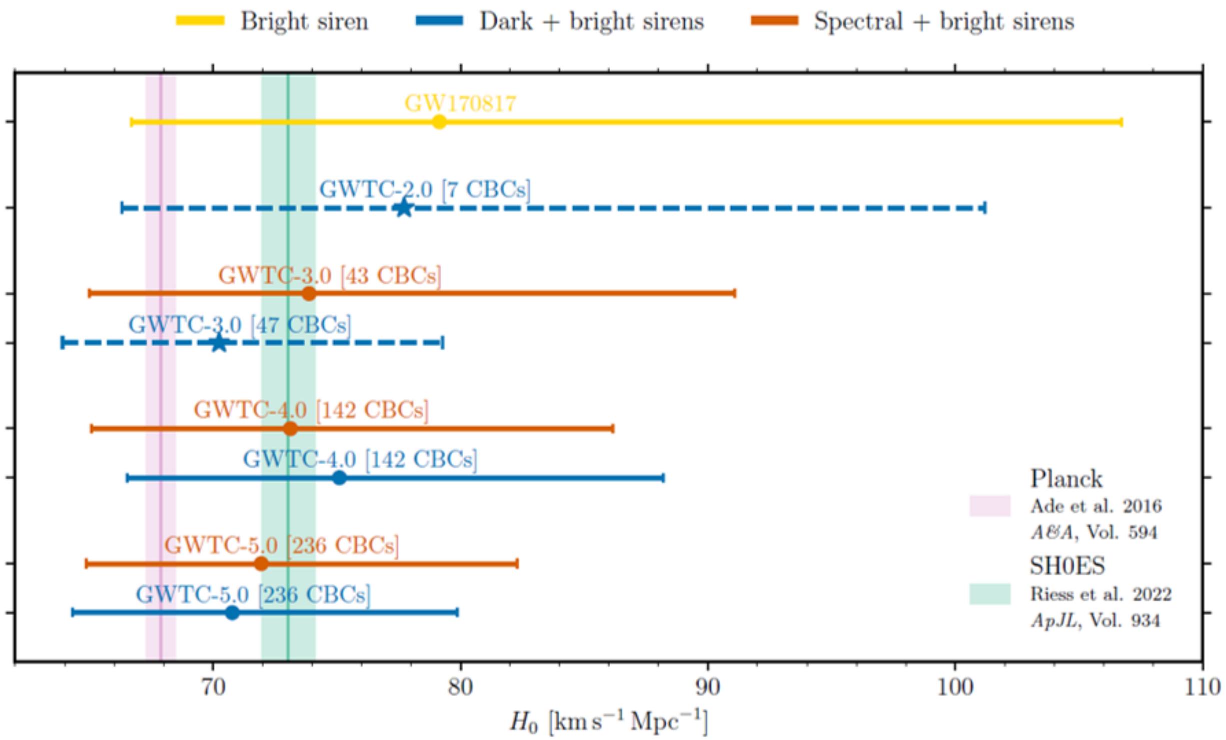
Following Rachel Gray (2023)

We can also use “dark sirens” – no explicit EM counterpart

We ‘marginalise’ over the redshifts of possible host galaxies

From GWTC-5.0





LSC et al., “GWTC-5.0: Constraints on the cosmic expansion rate and modified gravitational-wave propagation”, https://arxiv.org/abs/2605.27227

Impact of strong lensing on Hubble constant measurement?

The Astrophysical Journal Supplement Series, 284:20 (23pp), 2026 May https://doi.org/10.3847/1538-4365/ae579a

The Impact of Strong Lensing on Hubble Constant Measurements with Gravitational-wave Dark Sirens

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Abstract

The disagreement between early and late Universe electromagnetic measurements of the Hubble constant, H_0, known as the Hubble tension, highlights the need for independent and complementary probes. Gravitational-wave events have recently emerged as such a probe for constraining cosmological parameters. H_0 inference using these events relies on sky localization and luminosity distance estimates, both of which can be significantly improved for strongly lensed events with appropriate lens modeling. In this context, we propose utilizing strong lensing of dark sirens, gravitational-wave events without identified electromagnetic counterparts, in combination with strong lensing of galaxies as a novel method for measuring H_0. The constant is inferred from the luminosity distances of these lensed dark sirens and the redshifts of their host galaxies, combining information from individual events to obtain statistically stronger constraints when multiple events are available. We adopt a simulated galaxy catalog, MICECATv2, as the basis for simulating strong lensing of galaxies and to provide the redshift information of host galaxy candidates required to infer H_0. We also examine the impact of galaxy catalog incompleteness on the resulting H_0 inference. Our results demonstrate that using only eight strongly lensed dark sirens, analyzed with a dedicated galaxy–galaxy lensing catalog, can improve the precision of H_0 by roughly 50% compared to 250 unlensed events.

Unified Astronomy Thesaurus concepts: Cosmology (343); Gravitational wave astronomy (675); Hubble constant (758); Strong gravitational lensing (1643); Bayesian statistics (1900)

1. Introduction

In modern cosmology, the Hubble constant, H_0, plays a central role in understanding the evolution of the Universe. Despite its significance, the precise value of H_0 remains under debate due to the persistent discrepancy, the so-called Hubble tension (E. Di Valentino et al. 2021), between early-Universe measurements, such as those inferred from the cosmic microwave background (N. Aghanim et al. 2020) reporting $H_0 = 67.4 \pm 0.5\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, and late-Universe measurements based on the distance ladder using Cepheid variable stars and Type Ia supernovae (A. G. Riess et al. 2022) reporting $H_0 = 72.3 \pm 1.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. This tension has sparked considerable interest in independent and novel methods for determining H_0.

Over the past decade, the era of gravitational-wave (GW) astronomy has flourished with the detection of more than 200 GW signals (A. G. Abac et al. 2025b) by the ground-based LIGO–Virgo–KAGRA (LVK) detector network (J. Aasi et al. 2015; F. Acernese et al. 2015; T. Akutsu et al. 2019). To date, the sources of all confirmed GW events have been consistent with the coalescence of two relativistic compact objects, such as black holes (BHs) or neutron stars (NSs; A. G. Abac et al. 2025b). The inferences drawn from the GW signals have impacted multiple areas of astrophysics: results to date have clarified the statistical distributions of compact-object masses and spins (A. G. Abac et al. 2025c), provided strong evidence for the existence of intermediate-mass BHs (A. G. Abac et al. 2025b), placed constraints on the equation of state for dense matter in NS interiors (B. P. Abbott et al. 2018), and enabled precision tests of general relativity in the highly dynamical, strong-field regime (R. Abbott et al. 2025).

With its current sensitivity, the LVK network is capable of detecting GWs from sources at luminosity distances of a few Gpc (A. G. Abac et al. 2025b), thereby extending GW observations to cosmological scales. Such reach enables GW observations to probe cosmological models and constrain fundamental parameters (J. Zheng et al. 2022; S.-J. Jin et al. 2025). Since the luminosity distance can be directly inferred from the waveform amplitude of compact binary coalescences (CBCs), without reference to local calibrators, these sources are referred to as standard sirens (B. F. Schutz 1986; D. E. Holz & S. A. Hughes 2005). If the redshifts of the sources can be determined separately, GW events provide a measurement of H_0 in a completely new way. In particular, events with identified electromagnetic (EM) counterparts, so-called bright sirens, allow for a direct determination of the source redshift (B. P. Abbott et al. 2017; G. P. Smith et al. 2025). In contrast, the redshift must be inferred statistically for dark sirens, for which EM counterparts are absent (M. Soares-Santos et al. 2019; B. P. Abbott et al. 2021a; S. Mukherjee et al. 2021; R. Gray et al. 2023; W. Ballard et al. 2023; C. R. Bom et al. 2024). Several statistical methodologies have been developed.

One method exploits characteristic features in the mass distributions of NSs and BHs to partially break the degeneracy between the intrinsic source mass and cosmological redshift, which we refer to as the spectral siren method (J. M. Ezquiaga & D. E. Holz 2022). Cross-correlation techniques instead

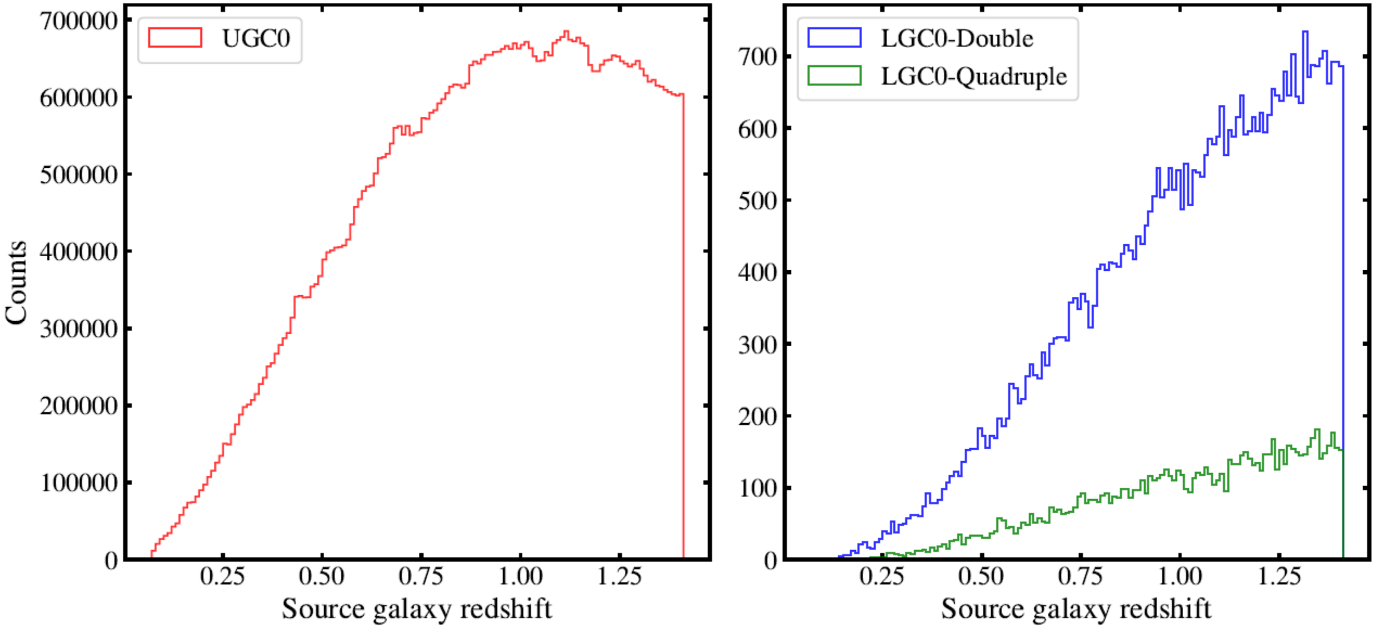
Seo et al., ApJS, **284**, 20 (2026): https://arxiv.org/abs/2603.01321

Two case studies:

1. Single dark siren lensed by double- or quadruple-lensed system
2. Multiple lensed dark sirens + galaxy catalog

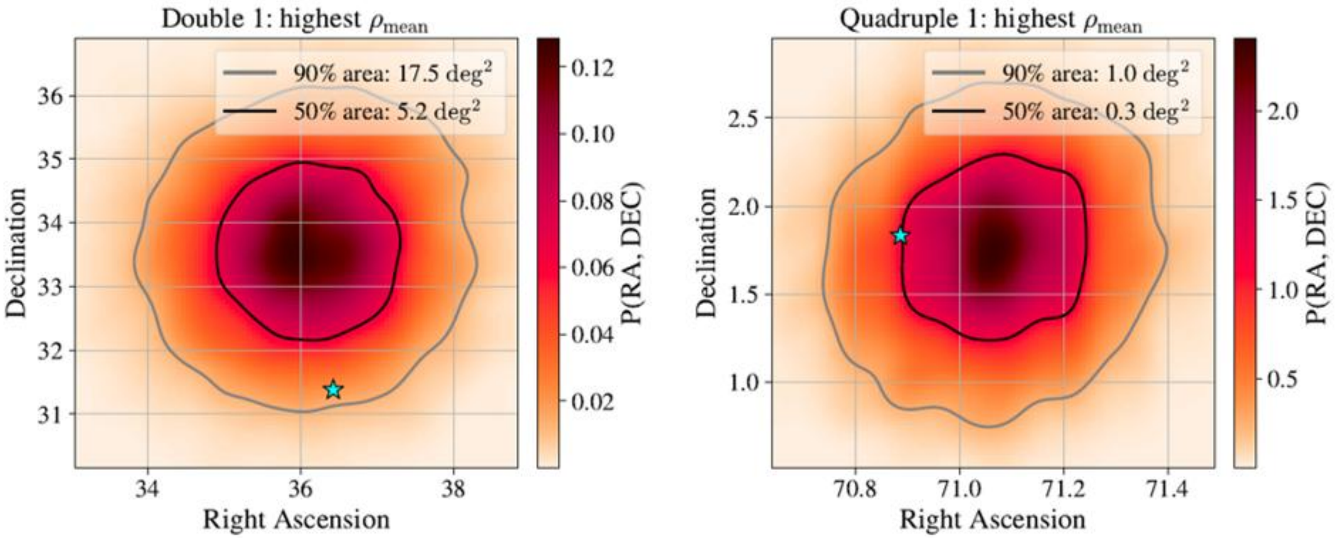
   → extend **GWCOSMO** pipeline to include lensed events

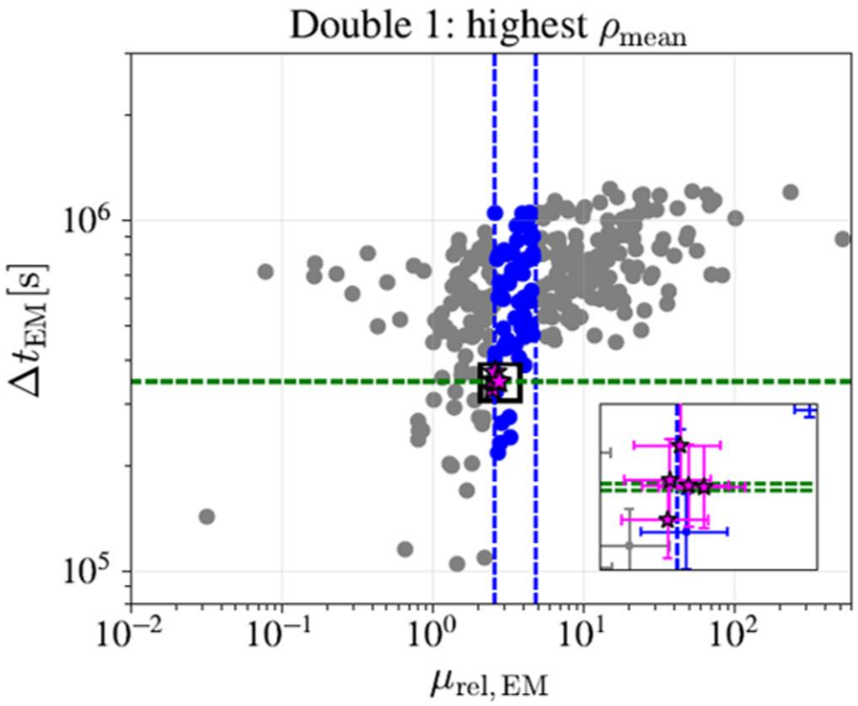
- Generate mock unlensed and lensed catalogs, derived from **MICECATv2** catalog (Fosalba et al. 2014), assuming SIE lenses



Single dark siren lensed by double- or quadruple-lensed system

- Infer **GW source parameters** and **lensing observables** from GW observations:
  - relative magnifications, time delays, Morse factors
- Identify **candidate lensed galaxies** within the GW sky localisation area.
- Use observed **EM properties** of these candidate galaxies to narrow down the possible GW hosts.
- Convert GW lensing observables into (model-dependent) **lens parameters** [using "Rejection Sampling" method (Seo et al. 2024)]
  - for SIE model: velocity dispersion, axis ratio, elliptical angle, redshifted lens mass
- Compare lens model parameters with EM properties of potential host galaxies to **further narrow down GW host** and improve inference of **luminosity distance**.





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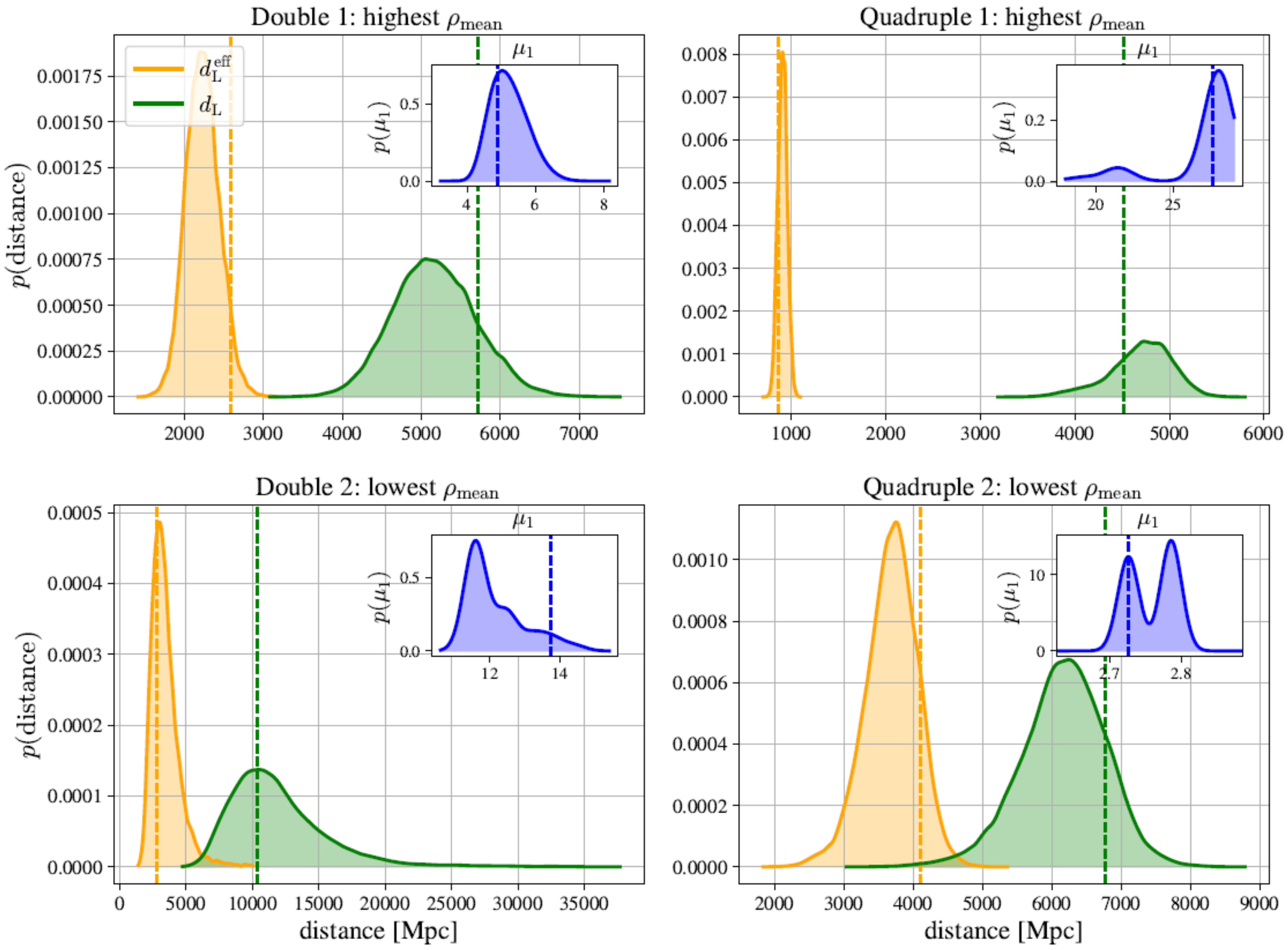

*Joint inference of lensing magnifications and 'true' luminosity distance = **de-lensing***

Seo et al., ApJS, **284**, 20 (2026): https://arxiv.org/abs/2603.01321

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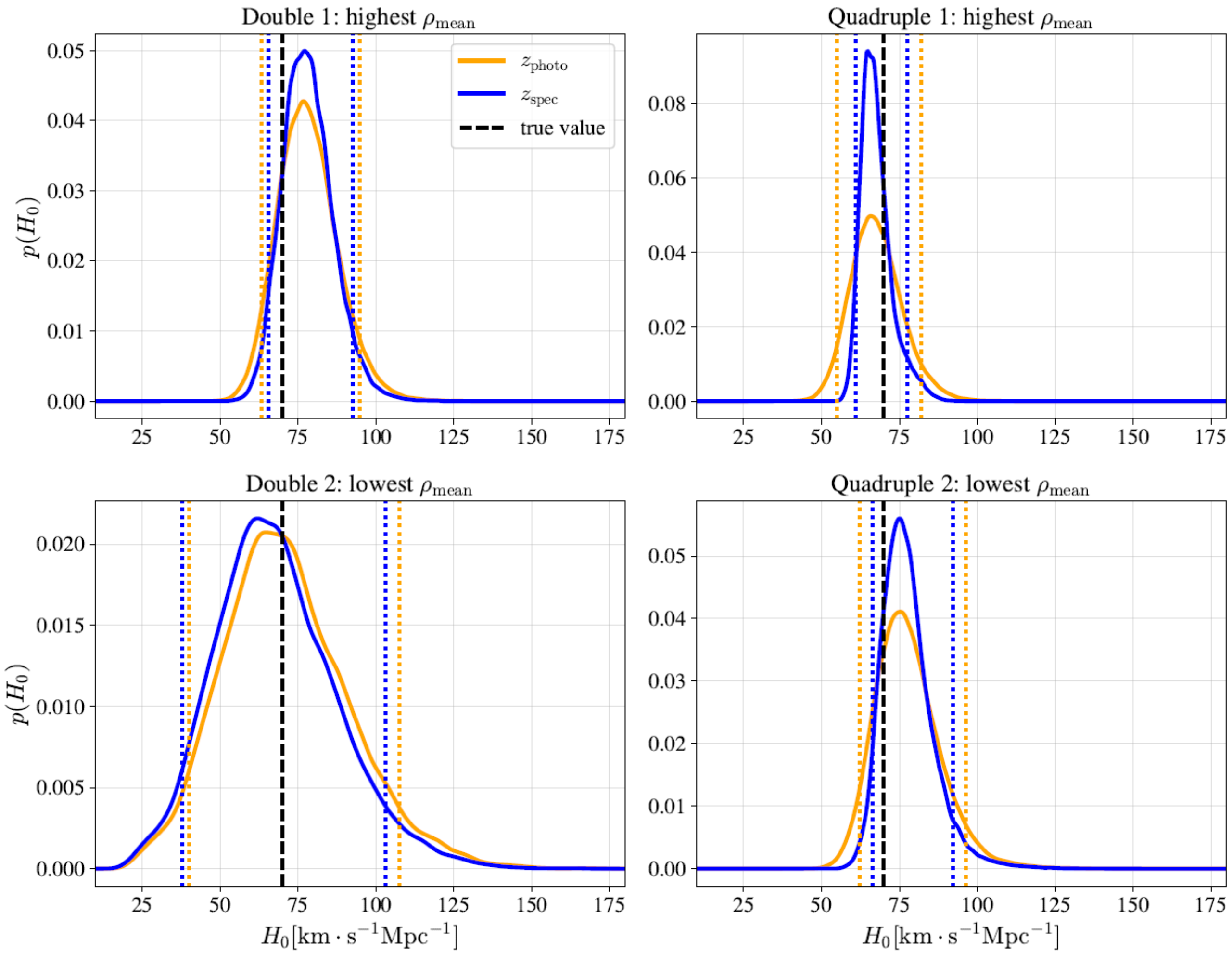


System	H_0 (km s^{-1} Mpc^{-1})	δ_{H_0}
DOUBLE 1	$77.7^{+14.6}_{-11.8}$	0.34
DOUBLE 2	$67.4^{+48.4}_{-33.6}$	1.22
QUADRUPLE 1	$66.3^{+7.2}_{-5.7}$	0.19
QUADRUPLE 2	$76.5^{+16.3}_{-10.0}$	0.34

$$\delta_{H_0} \equiv \frac{90\%\ \text{C.I.}}{H_{0,\rm med}}$$

Seo et al., ApJS, **284**, 20 (2026): https://arxiv.org/abs/2603.01321

Multiple lensed dark sirens

- Adapt **GWCOSMO** analysis pipeline (Gray et al. 2020) to include lensed GW sources.
- Create **magnitude-limited catalog** (approx. 50% completeness)

Basic formalism:

$$p(H_0|\{x_{\mathrm{lgw}}\}, \{D_{\mathrm{lgw}}\}) \propto p(H_0)\, p(N_{\mathrm{lens,det}}|H_0) \times \prod_{i=1}^{N_{\mathrm{lens,det}}} \mathcal{L}(x_{\mathrm{lgw},i}|D_{\mathrm{lgw},i}, H_0),$$

$$\mathcal{L}(x_{\mathrm{lgw}}|D_{\mathrm{lgw}}, H_0) = \frac{p(x_{\mathrm{lgw}}|H_0)}{p(D_{\mathrm{lgw}}|H_0)}$$

Likelihood of observing data x_{lgw} given i^{th} lensed detection and given value of H_0

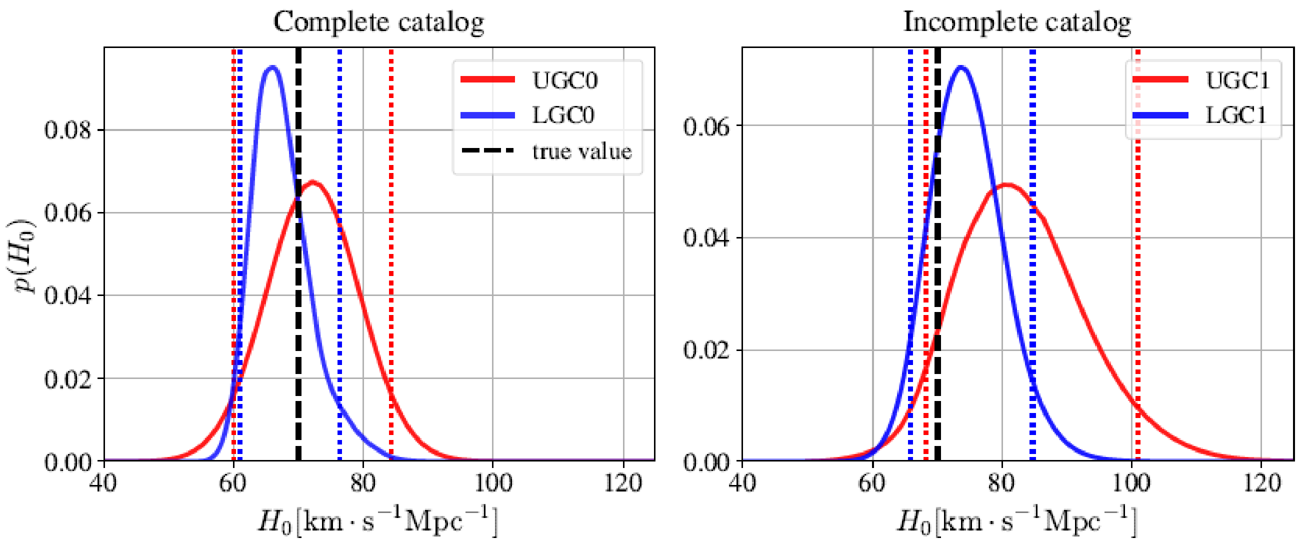
where

$$p(D_{\mathrm{lgw}}|H_0) = \int p(D_{\mathrm{lgw}}|x_{\mathrm{lgw}}, H_0) \times p(x_{\mathrm{lgw}}|H_0)\, dx_{\mathrm{lgw}}$$

probability of detecting lensed GW event, given data x_{lgw} and given value of H_0

Comparisons:

- **250** unlensed dark sirens vs
  6 doubly-lensed, **2** quadruply-lensed systems
- Complete vs incomplete catalogs



Median Values and 90% Credible Intervals of Recovered H_0 Posteriors and Their Relative C.I.s for Each Catalog Case

Catalog	Completeness	H_0 (km s^{-1} Mpc^{-1})	δ_{H_0}
UGC0	100	$72.12^{+12.12}_{-12.12}$	0.34
UGC1	50	$82.73^{+18.18}_{-14.55}$	0.40
LGC0	100	$66.85^{+9.49}_{-5.77}$	0.23
LGC1	34	$74.29^{+10.45}_{-8.53}$	0.26

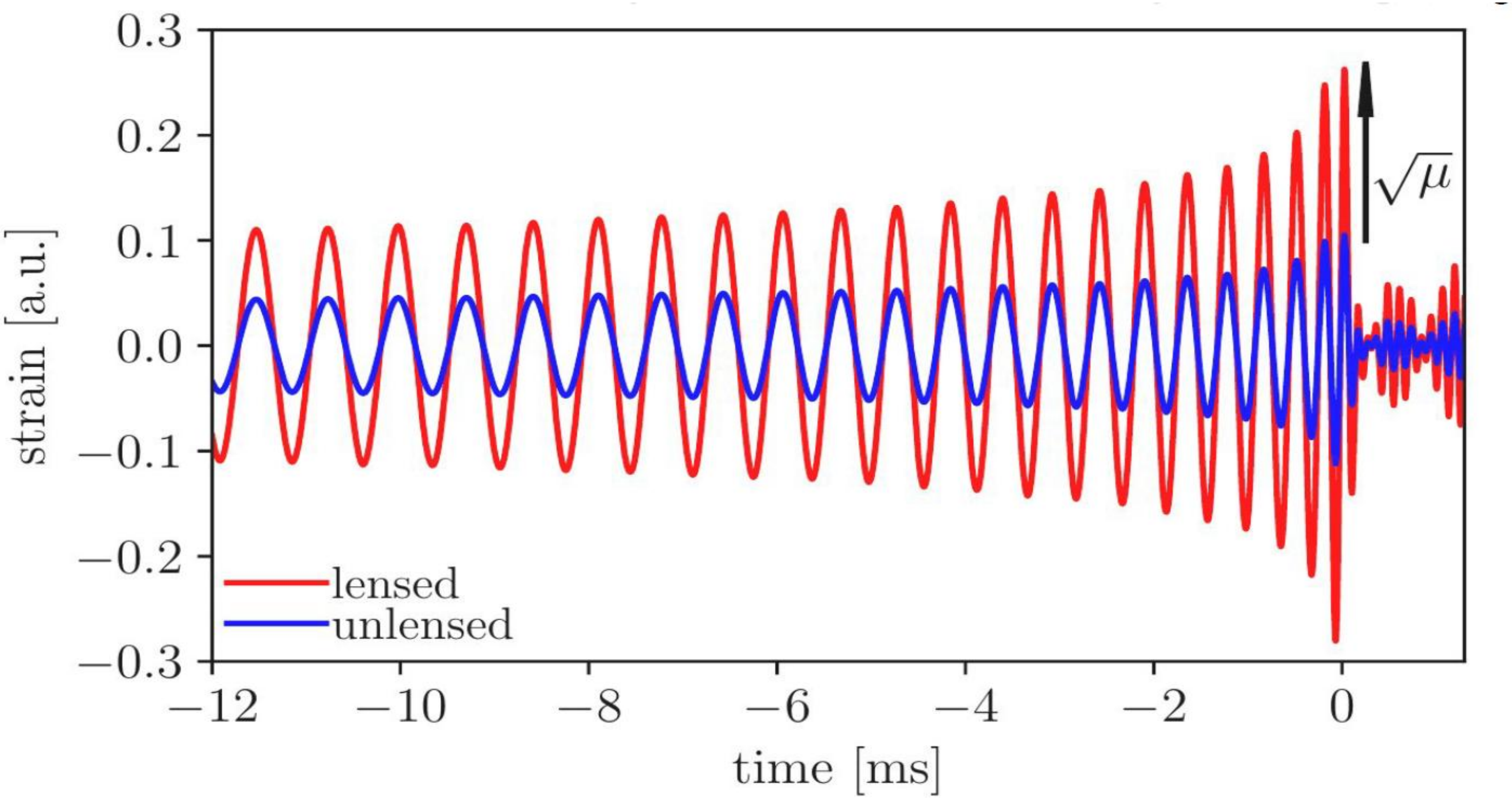
Significant improvement in H_0 precision

Seo et al., ApJS, **284**, 20 (2026): https://arxiv.org/abs/2603.01321

Q: What happens when a GW is *weakly* lensed?

A: We observe a single magnified waveform → biased luminosity distance

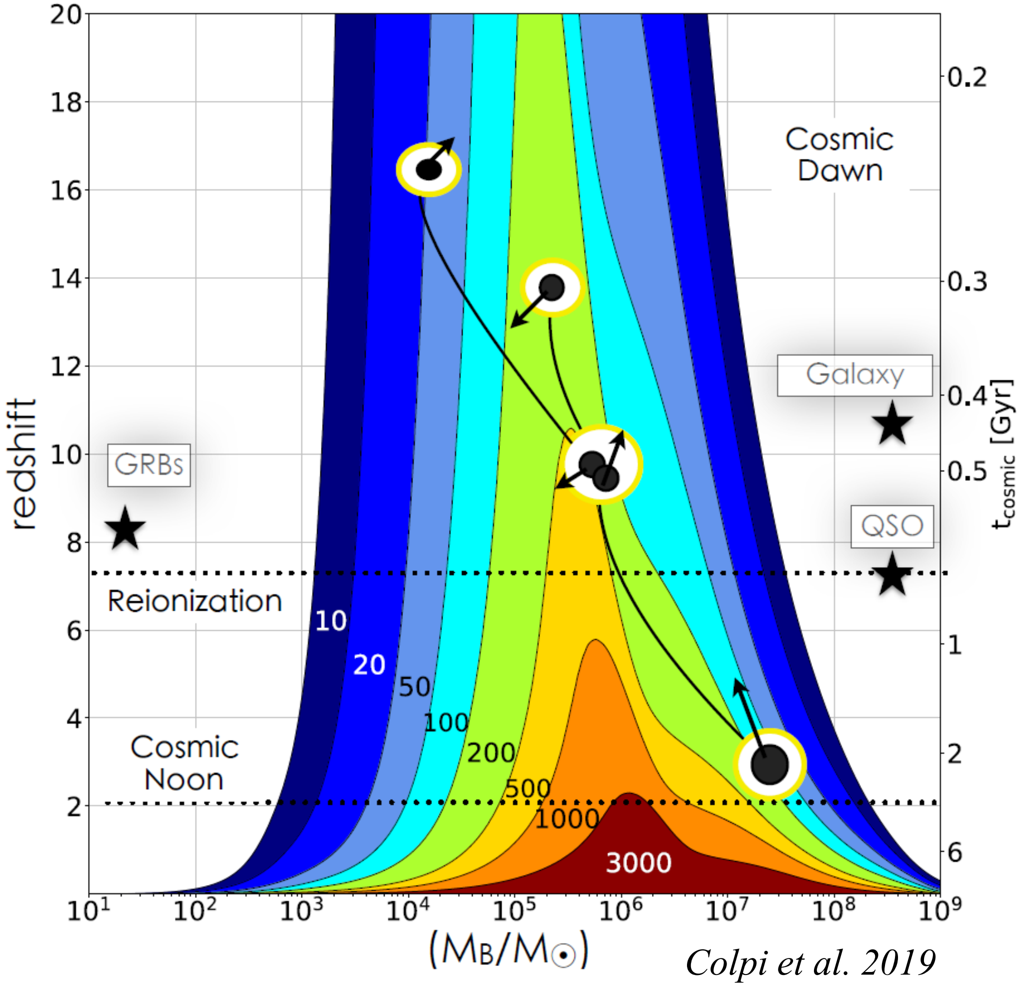
- For BBH mergers magnification is not detectable in waveform
- [De-lensing may be possible – see later]
- For BNS mergers degeneracy can be broken using tidal deformability (Pang et al. 2020)



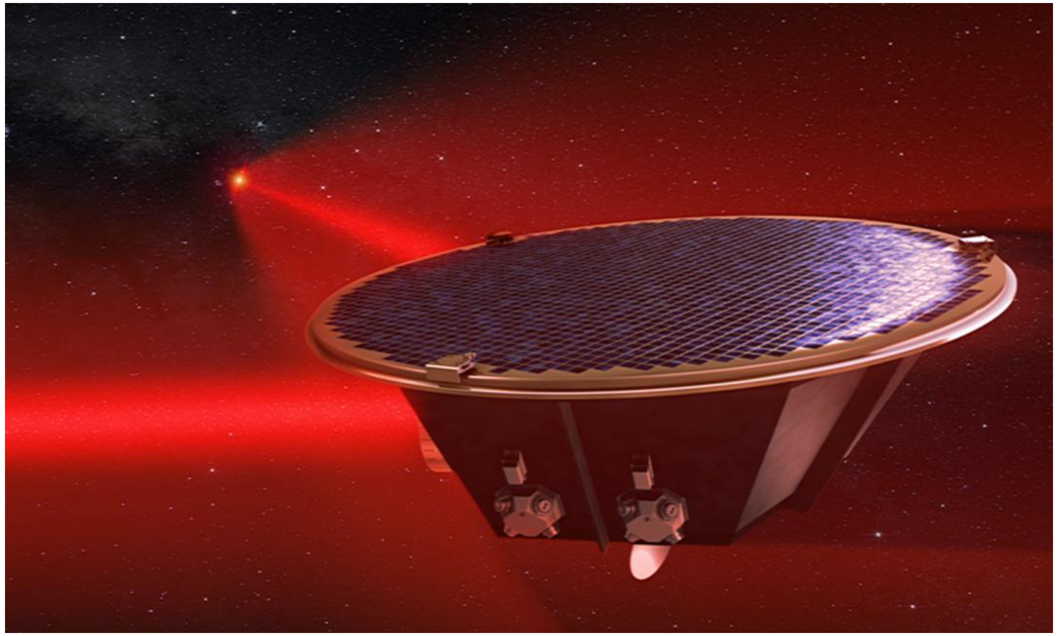
Credit: Justin Janquart (2021)

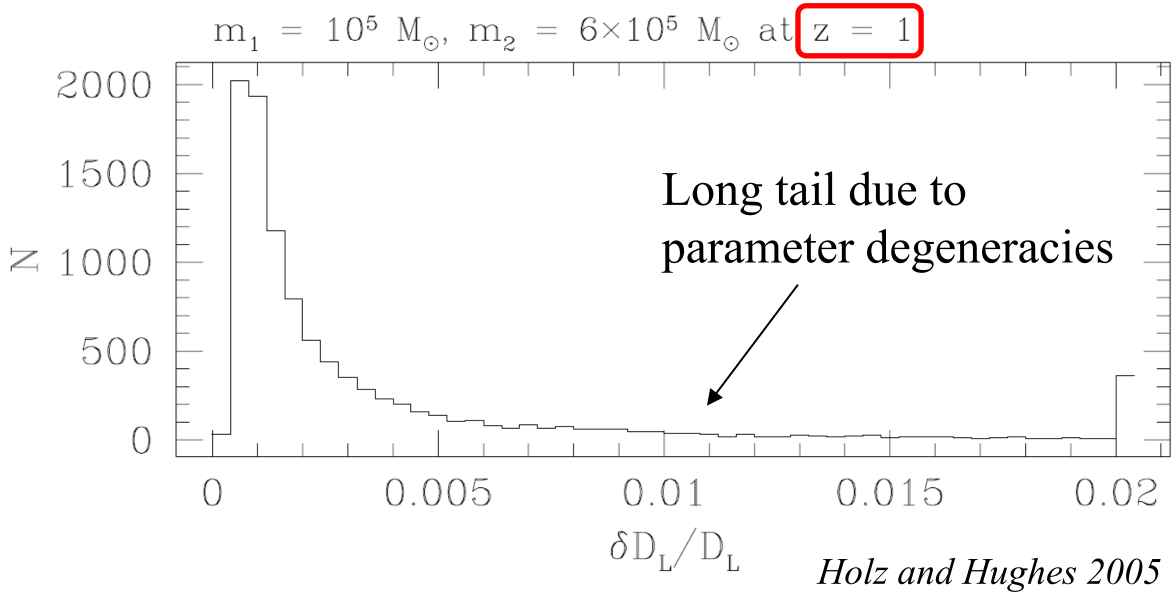
LISA will 'see' very high-SNR massive black hole binary mergers to $z > 20$

- Exquisite tests of GR from waveforms
- Standard siren Hubble diagram to high redshift



Colpi et al. 2019

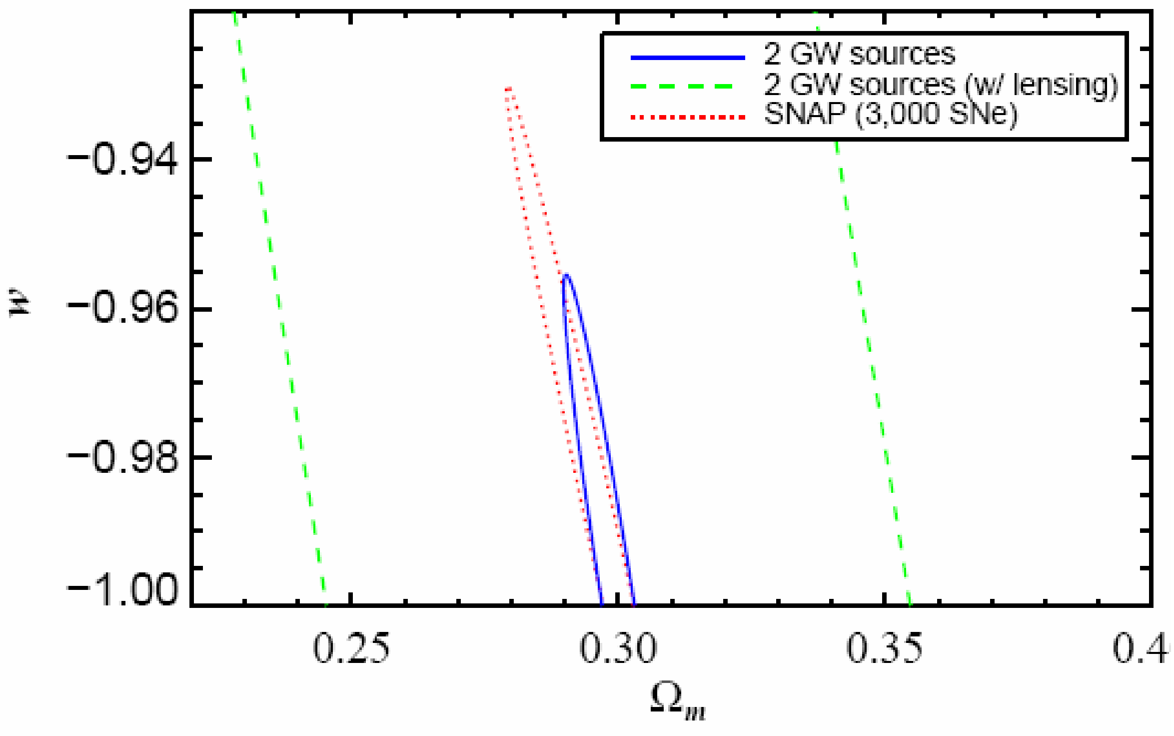




Holz and Hughes 2005

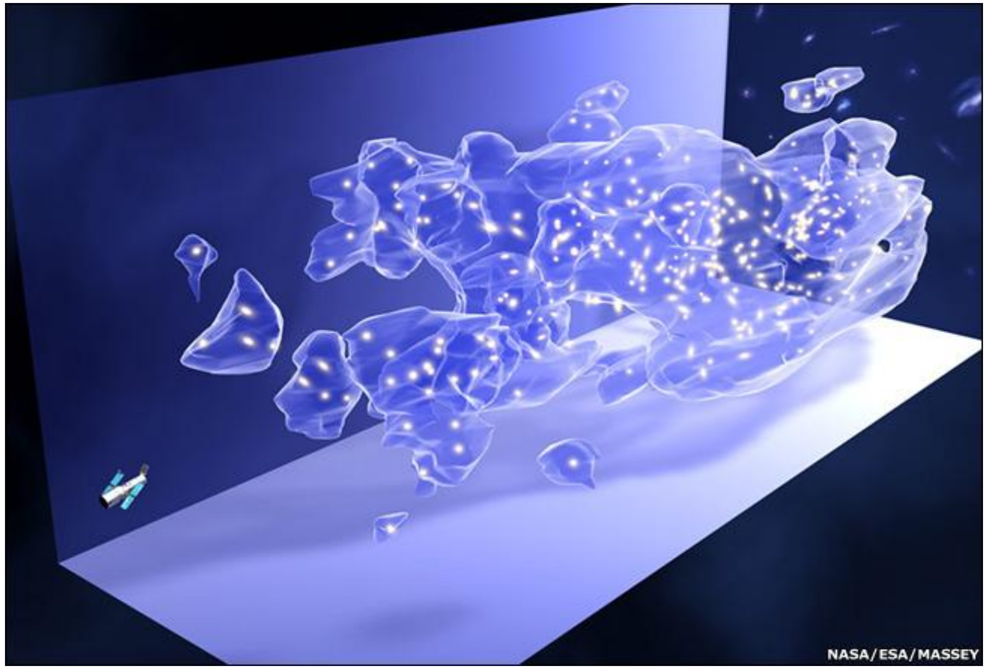
Weak lensing is a significant issue (opportunity?) for doing cosmology with sirens

Adapted from Holz & Hughes (2005)



GW sources will be (de-)magnified by WL due to LSS.

Same effect as for SN. However, WL has much greater impact for e.g. SMBH sirens, because of their much smaller *intrinsic* scatter.



- Observed siren brightness increased by magnification μ
- Could we correct individual sirens by inferring μ on small angular scales from EM data?

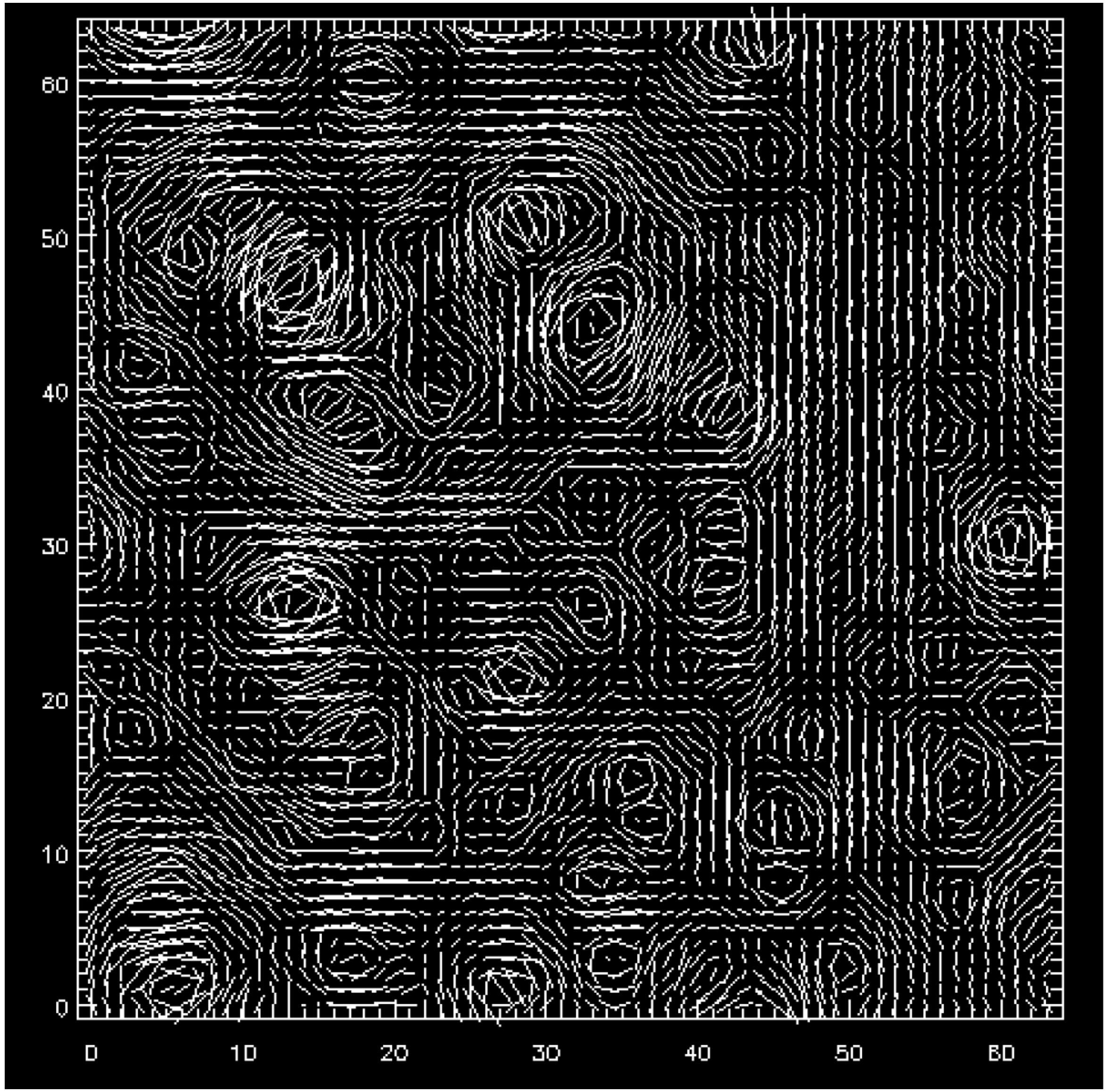
De-lensing

Correcting for weak lensing?...

- Observed siren brightness increased by magnification μ
- What μ can we expect?
- We can measure σ_μ from shear maps derived from simulations
- Typically $\sigma_\mu \sim 0.1$ at $z = 2$ which means ~5% error in D_L
- Could we correct individual sirens by mapping μ on small angular scales?

  [Similar to de-lensing of SLGW events in Seo et al. (2026)]

Multimessenger challenge?!



Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

https://arxiv.org/abs/0907.3635

Mon. Not. R. Astron. Soc. **404**, 858–866 (2010) doi:10.1111/j.1365-2966.2010.16317.x

Delensing gravitational wave standard sirens with shear and flexion maps

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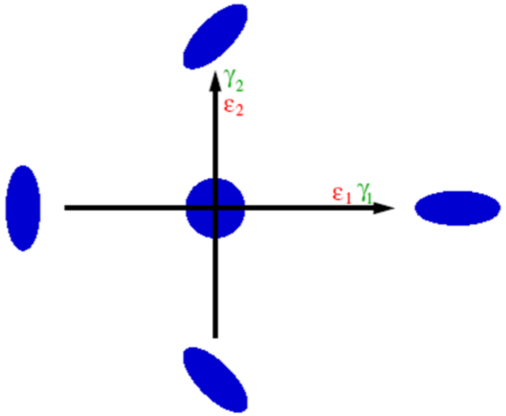


$$\Psi_{ij} \equiv \frac{\partial(\delta\theta_i)}{\partial\theta_j} \equiv \begin{pmatrix} \kappa+\gamma_1 & \gamma_2 \\ \gamma_2 & \kappa-\gamma_1 \end{pmatrix}$$

Convergence

Shear

Following Refregier 2003

Weak lensing limit:

$$\mu \simeq 1 + 2\kappa$$

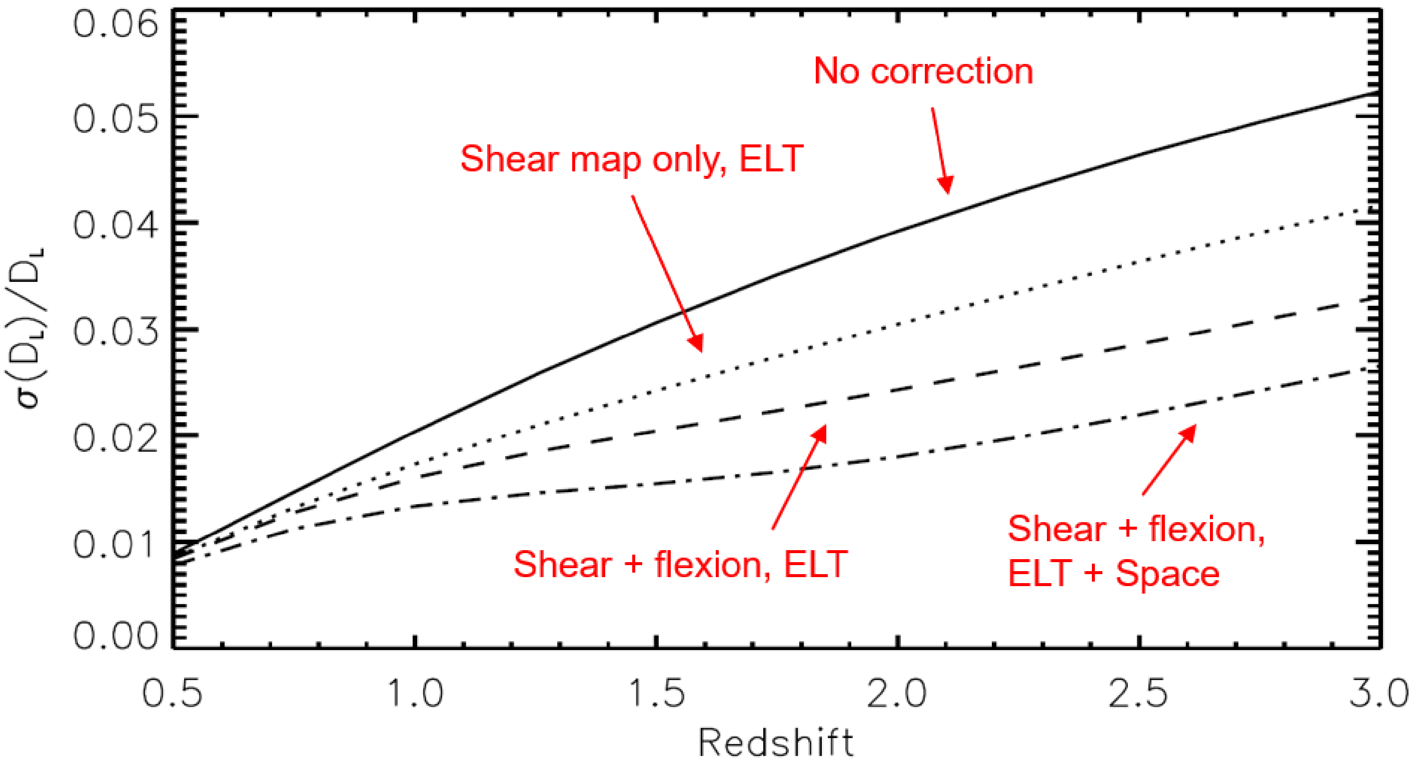
Magnification

Convergence

- Estimate local, smoothed shear
- Convert to estimated convergence (e.g. Kaiser & Squires 1993)

$$\tilde{\kappa}(\boldsymbol{l}) = \frac{l_1^2 - l_2^2}{l^2}\tilde{\gamma}_1 + 2\frac{l_1 l_2}{l^2}\tilde{\gamma}_2$$

- Compute magnification, and correct the siren's estimated distance



Similar results in Hilbert et al. (2011) *Also, see e.g. Rowe et al (2013)…*

Zhu et al., MNRAS, **522**, 4059 (2023):
https://arxiv.org/abs/2211.15160

- Re-visits Shapiro et al. calculations
- “End to end” simulation:
  - realistic, current DM simulations
    https://slics.roe.ac.uk/
  - self-consistent galaxy + siren catalogues
  - realistic errors on galaxy shape measurements, reconstructed shear
  - 3-D galaxy redshift information
  - updated best-case ground- and space-based observations

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY
MNRAS **522**, 4059–4077 (2023)
Advance Access publication 2023 May 5
https://doi.org/10.1093/mnras/stad1194

Reducing the impact of weak-lensing errors on gravitational-wave standard sirens

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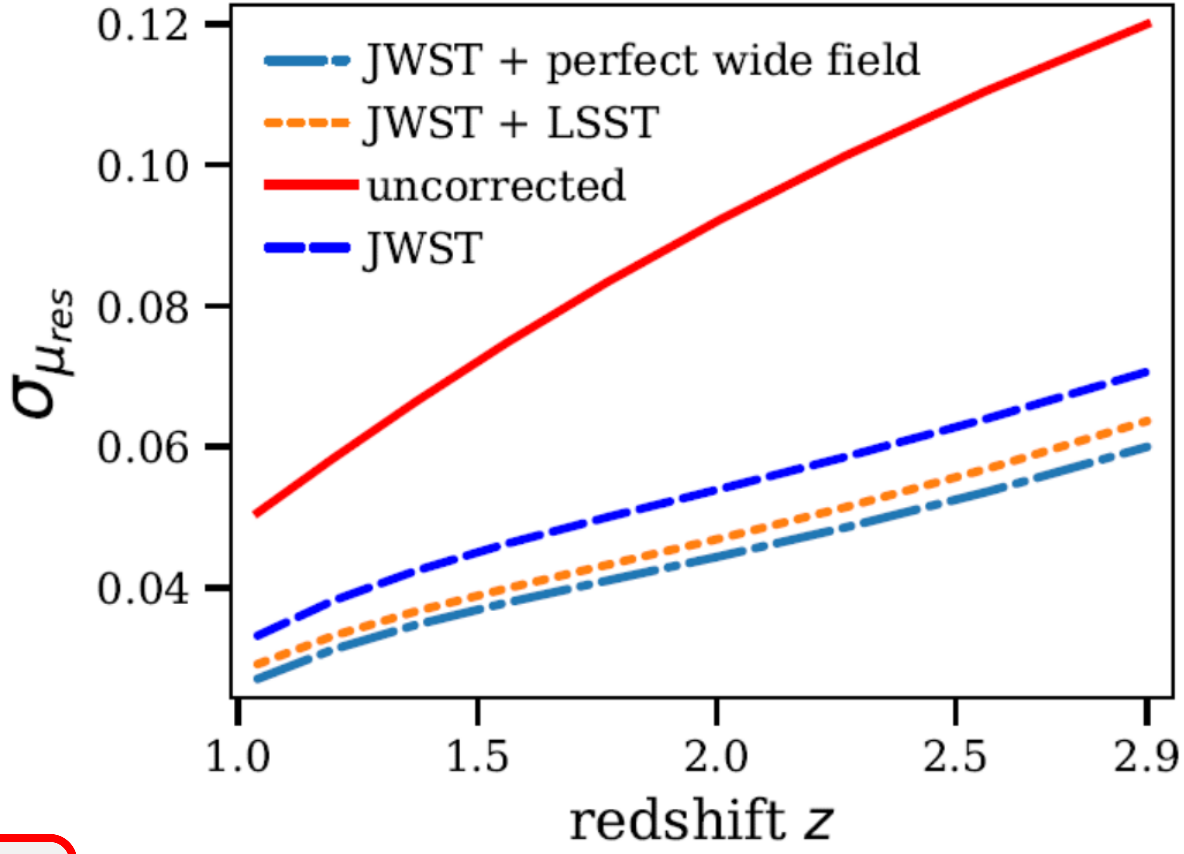
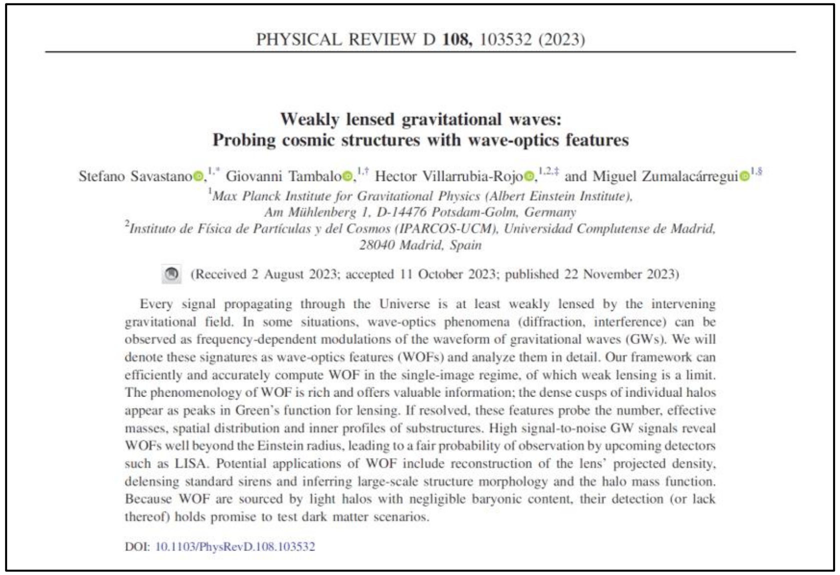


Similar improvement to earlier work, but only with ultra-deep field observations for <u>every</u> siren

[Aside #1: Is de-lensing possible *without* EM observations?...]



PHYSICAL REVIEW D **108**, 103532 (2023)

Weakly lensed gravitational waves: Probing cosmic structures with wave-optics features

Stefano Savastano,[1,*] Giovanni Tambalo,[1,†] Hector Villarrubia-Rojo,[1,2,‡] and Miguel Zumalacárregui[1,§]
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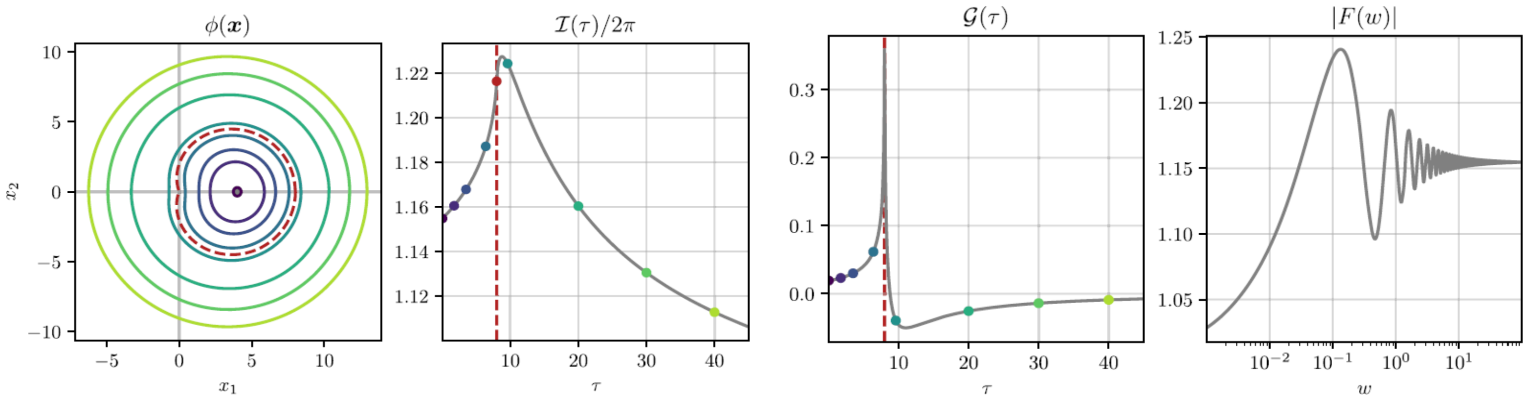
(Received 2 August 2023; accepted 11 October 2023; published 22 November 2023)

Every signal propagating through the Universe is at least weakly lensed by the intervening gravitational field. In some situations, wave-optics phenomena (diffraction, interference) can be observed as frequency-dependent modulations of the waveform of gravitational waves (GWs). We will denote these signatures as wave-optics features (WOFs) and analyze them in detail. Our framework can efficiently and accurately compute WOF in the single-image regime, of which weak lensing is a limit. The phenomenology of WOF is rich and offers valuable information; the dense cusps of individual halos appear as peaks in Green's function for lensing. If resolved, these features probe the number, effective masses, spatial distribution and inner profiles of substructures. High signal-to-noise GW signals reveal WOFs well beyond the Einstein radius, leading to a fair probability of observation by upcoming detectors such as LISA. Potential applications of WOF include reconstruction of the lens' projected density, delensing standard sirens and inferring large-scale structure morphology and the halo mass function. Because WOF are sourced by light halos with negligible baryonic content, their detection (or lack thereof) holds promise to test dark matter scenarios.

DOI: 10.1103/PhysRevD.108.103532

Savastano et al. Phys Rev D, **108**, 103532 (2023)
https://arxiv.org/abs/2306.05282

- Wave-optics features **not likely** to be imprinted on LISA SLGW signals, as geometric optics generally applies.
- However, WOF ***can*** be observed in the WL region for LISA, appearing as frequency-dependent waveform modulations.
- Can express lensed signal in the **time-domain** $\mathcal{I}(\tau)$ as a 1-D integral over contours of equal **time delay** (Fermat potential)
- Compute Green's function $G(\tau) \equiv \frac{1}{2\pi}\frac{\mathrm{d}}{\mathrm{d}\tau}\mathcal{I}(\tau)$.
- Amplification factor $F(w)$ is the inverse F.T. of $G(\tau)$



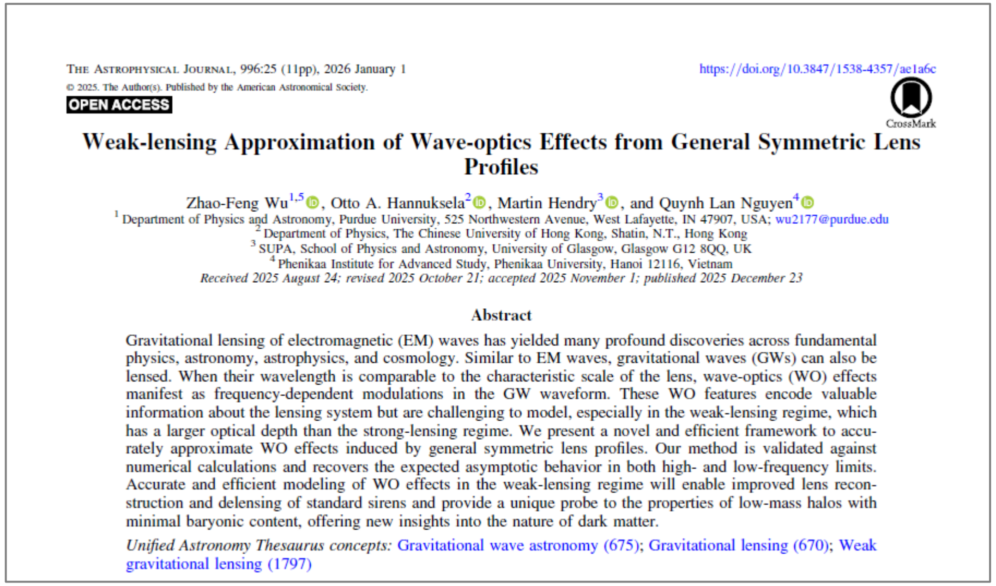
Example: SIS model, with $y = 3$

Scaling relations between lensing magnification and amplification factor

e.g. **SIS**: $\mu - 1 \propto w_0^{1/2}$

$\mu - 1 \propto |F(w_0)|.$

[Aside #2: Simplified calculation of WO amplification factors]



THE ASTROPHYSICAL JOURNAL, 996:25 (11pp), 2026 January 1 https://doi.org/10.3847/1538-4357/ae1a6c

Weak-lensing Approximation of Wave-optics Effects from General Symmetric Lens Profiles

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Abstract

Gravitational lensing of electromagnetic (EM) waves has yielded many profound discoveries across fundamental physics, astronomy, astrophysics, and cosmology. Similar to EM waves, gravitational waves (GWs) can also be lensed. When their wavelength is comparable to the characteristic scale of the lens, wave-optics (WO) effects manifest as frequency-dependent modulations in the GW waveform. These WO features encode valuable information about the lensing system but are challenging to model, especially in the weak-lensing regime, which has a larger optical depth than the strong-lensing regime. We present a novel and efficient framework to accurately approximate WO effects induced by general symmetric lens profiles. Our method is validated against numerical calculations and recovers the expected asymptotic behavior in both high- and low-frequency limits. Accurate and efficient modeling of WO effects in the weak-lensing regime will enable improved lens reconstruction and delensing of standard sirens and provide a unique probe to the properties of low-mass halos with minimal baryonic content, offering new insights into the nature of dark matter.

Unified Astronomy Thesaurus concepts: Gravitational wave astronomy (675); Gravitational lensing (670); Weak gravitational lensing (1797)

Wu et al. ApJ, **996**, 25 (2026)
https://arxiv.org/abs/2508.17486

For an axially symmetric lens model:

$$\boxed{F(w,y) = -iwe^{iwy^2/2}\int_0^\infty dx\; xJ_0(wxy)\exp\left[iw\left(\frac{1}{2}x^2 - \psi(x) + \phi_m(y)\right)\right]}$$

Analytic solutions exist for e.g. SIS model but expressions remain complicated and unwieldy

Extends to e.g. NFW and general axisymmetric case

Example

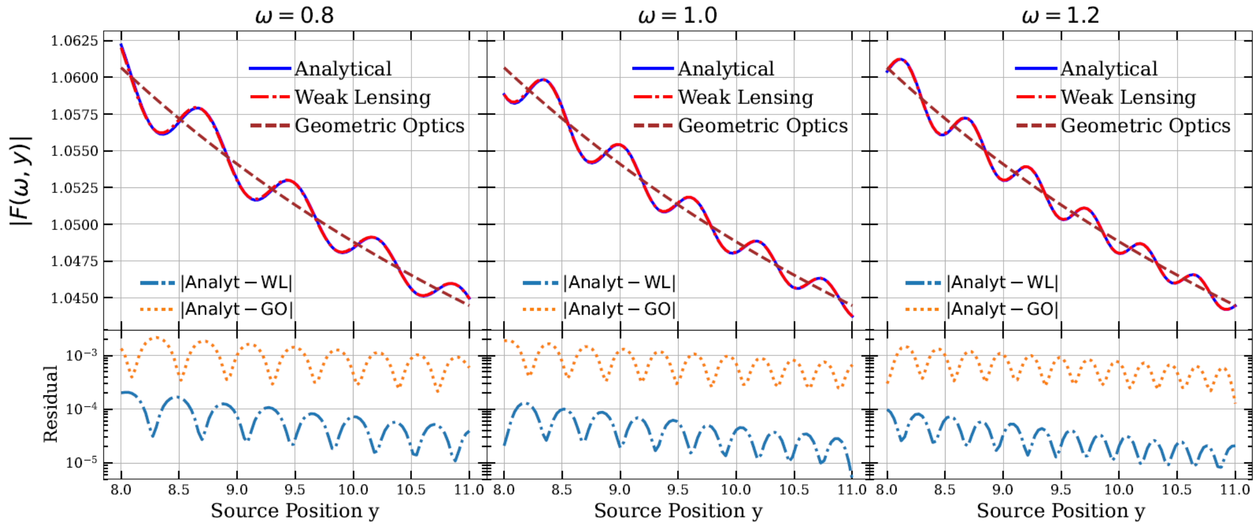
Full SIS analytic solution, valid in SL and WL case:

$$F(\omega, y) = e^{\frac{i}{2}\omega(y^2-2\phi_m(y))}\sum_{n=0}^{\infty}\frac{\Gamma\left(1+\frac{n}{2}\right)}{n!}(-i2\omega)^{n/2} \times {}_1F_1\left(1+\frac{n}{2}, 1; -\frac{i}{2}\omega y^2\right)$$

WL approximation, with $\omega y^2/2 \gg 1$

Can use asymptotic expansion of conf. hypergeometric function to simplify to:

$$F(\omega, y) = \sqrt{1+\frac{1}{y}}\left(1+\frac{i}{8\omega y^3}\right) + \frac{1}{\omega y^3}e^{i\omega\left(\frac{1}{2}y^2+y+1/2\right)} + \frac{1}{(\omega y^2)}\mathcal{O}(y^{-3}).$$



Fully self-consistent treatment?...

Mpetha et al., Phys Rev D, **110**, 023502 (2024):

https://arxiv.org/abs/2402.19476

Lensed distance: $d'_L = \dfrac{d_L}{\sqrt{\mu}}$

Distance uncertainty:

$$\sigma^2_{d_L} = \sigma^2_{\mathrm{GW}} + \sigma^2_{\mathrm{WL}} + \left(\frac{\partial d_L}{\partial z}\right)^2 (\sigma^2_z + \sigma^2_{v_{\mathrm{pec}}})$$

$$\frac{\sigma_{\mathrm{WL}}(z)}{d_L(z)} = \frac{1}{2}\frac{\sigma_\mu(z)}{\langle \mu(z) \rangle} = \frac{1}{2}\sigma_\mu(z)$$

But $\sigma_\mu(z)$ depends on cosmological parameters

\+ details of simulation, ray tracing etc

What if we mis-characterise $\sigma_\mu(z)$?

→ **Biases inference of parameters**

PHYSICAL REVIEW D **110,** 023502 (2024)

Impact of weak lensing on bright standard siren analyses

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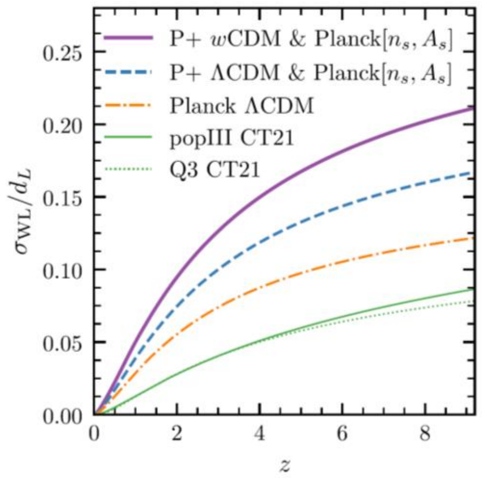
(Received 11 March 2024; accepted 6 June 2024; published 2 July 2024)

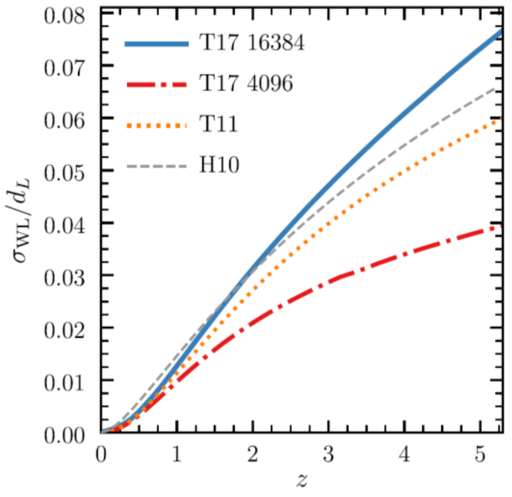
Gravitational waves from binary mergers at cosmological distances will experience weak lensing by large scale structure. This causes a (de)magnification, μ, of the wave amplitude, and a completely degenerate modification to the inferred luminosity distance d_L. The customary method to address this is to increase the uncertainty on d_L according to the dispersion of the magnification distribution at the source redshift, $\sigma_\mu(z)$. But this term is dependent on the cosmological parameters that are being constrained by gravitational wave "standard sirens," such as the Hubble parameter H_0, and the matter density fraction Ω_{m}. The dispersion $\sigma_\mu(z)$ is also sensitive to the resolution of the simulation used for its calculation. Tension in the measured value of H_0 from independent datasets, and the present use of weak-lensing fitting functions calibrated using outdated cosmological simulations, suggest $\sigma_\mu(z)$ could be underestimated. This motivates an investigation into the consequences of mischaracterizing $\sigma_\mu(z)$. We consider two classes of standard siren, supermassive black hole binary and binary neutron star mergers. Underestimating H_0 and Ω_{m} when calculating $\sigma_\mu(z)$ increases the probability of finding a residual lensing bias on these parameters greater than 1σ by 1.5–3 times. Underestimating $\sigma_\mu(z)$ by using low resolution/small sky-area simulations can also significantly increase the probability of biased results; the probability of a $1\sigma(2\sigma)$ bias in H_0 and Ω_{m} found from binary neutron star mergers is 54% (19%) in this case. For neutron star mergers, the mean bias on H_0 caused by magnification selection effects is $\Delta H_0 = -0.1\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. The spread around this mean bias—determined by assumptions on $\sigma_\mu(z)$—is $\Delta H_0 = \pm 0.25\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, comparable to the forecasted uncertainty. These effects do not impact merging neutron stars' utility for addressing the H_0 tension, but left uncorrected they limit their use for precision cosmology. For supermassive black hole binaries, the spread of possible biases on H_0 is significant, $\Delta H_0 = \pm 5\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, but $\mathcal{O}(200)$ observations are needed to reduce the variance below the bias. To achieve accurate subpercent level precision on cosmological parameters using standard sirens, first much improved knowledge on the form of the magnification distribution and its dependence on cosmology is needed.

DOI: 10.1103/PhysRevD.110.023502

$\frac{\sigma_{\rm WL}(z)}{d_L(z)}$ varies for different simulated cosmologies…



…as well as different simulation resolutions



…and explored biases from mis-characterisation

Example:

True cosmology:

P+ ΛCDM & Planck[n_s, A_s]

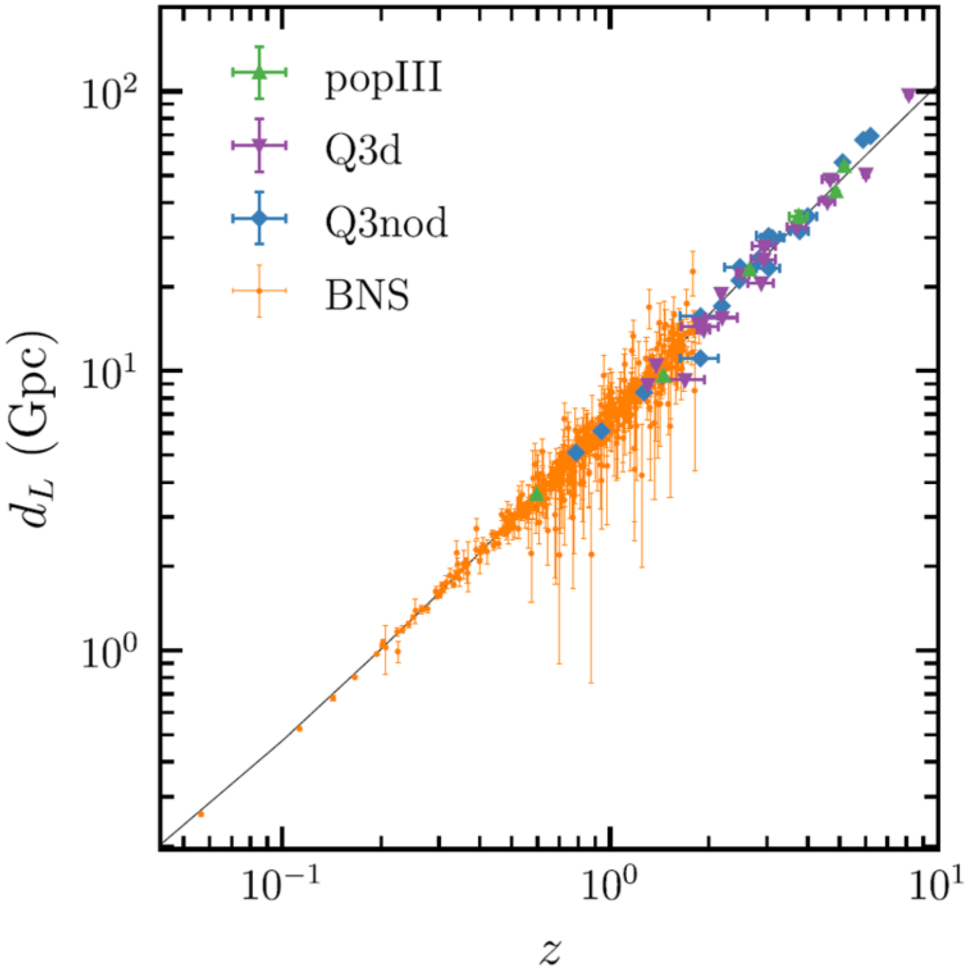
BUT

Correct WL errors using:

Planck ΛCDM

We simulated realistic populations of bright (BNS) and dark sirens….

1. For 3G detectors
2. For LISA (with different BH seeds)



Example of biases…

True cosmology

	Pantheon+ ΛCDM		
Population	$P(b(h) \geq \pm x_h)$	$P(b(\Omega_m) \geq \pm x_{\Omega_m})$	
popIII	0.55	0.56	$x_h = 0.02,\ x_{\Omega_m} = 0.05.$
Q3	0.66	0.64	
Q3nod	0.44	0.53	
BNS	0.32	0.24	$x_h = 0.0025,\ x_{\Omega_m} = 0.015$

True cosmology:

P+ ΛCDM & Planck$[n_s, A_s]$

BUT

Correct WL errors using:

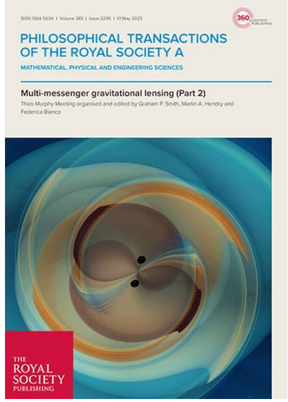
Planck ΛCDM

Take-aways:

- Mischaracterising $\sigma_\mu(z)$ can lead to quite **significant bias**, particularly for LISA
- We need to improve our modelling of $\sigma_\mu(z)$ over the next decade!

Tests of GR and non-GR models?...

See e.g. **Smith et al (2025):** https://arxiv.org/abs/2503.19973
"Multi-messenger gravitational lensing"



Keitel (2025): https://doi.org/10.1098/rsta.2024.0128
"False positives for gravitational lensing: the gravitational-wave perspective"

See also **Wright, Janquart & Johnson-McDaniel (2025)**
"Effect of deviations from General Relativity on Searches for Gravitational-wave Microlensing And Type II Strong Lensing"


The Astrophysical Journal, 981:133 (33pp), 2025 March 10 https://doi.org/10.3847/1538-4357/ad9d3e

Effect of Deviations from General Relativity on Searches for Gravitational-wave Microlensing and Type II Strong Lensing

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Received 2024 March 26; revised 2024 December 4; accepted 2024 December 9; published 2025 March 4

Abstract
As the gravitational-wave (GW) detector network is upgraded and the sensitivity of the detectors improves, novel scientific avenues open for exploration. For example, tests of general relativity (GR) will become more accurate as smaller deviations can be probed. Additionally, the detection of lensed GWs becomes more likely. However, these new avenues could also interact with each other, and a GW event presenting deviations from GR could be mistaken for a lensed one. Here, we explore how phenomenological deviations from GR or binaries of exotic compact objects could impact those lensing searches focusing on a single event. We consider strong lensing, millilensing, and microlensing, and find that certain phenomenological deviations from GR may be mistaken for all of these types of lensing. Therefore, our study shows that future candidate lensing events would need to be carefully examined to avoid a false claim of lensing where instead a deviation from GR has been seen.
Unified Astronomy Thesaurus concepts: Gravitational lensing (670); Gravitational microlensing (672); Gravitational waves (678); General relativity (641)

ApJ **981**, 133 (2025): https://arxiv.org/abs/2403.08957

See also Bahamonde **et al** (2023)

Rep. Prog. Phys., **86**, 026901


IOP Publishing Reports on Progress in Physics
Rep. Prog. Phys. 86 (2023) 026901 (207pp) https://doi.org/10.1088/1361-6633/ac9cef

Review

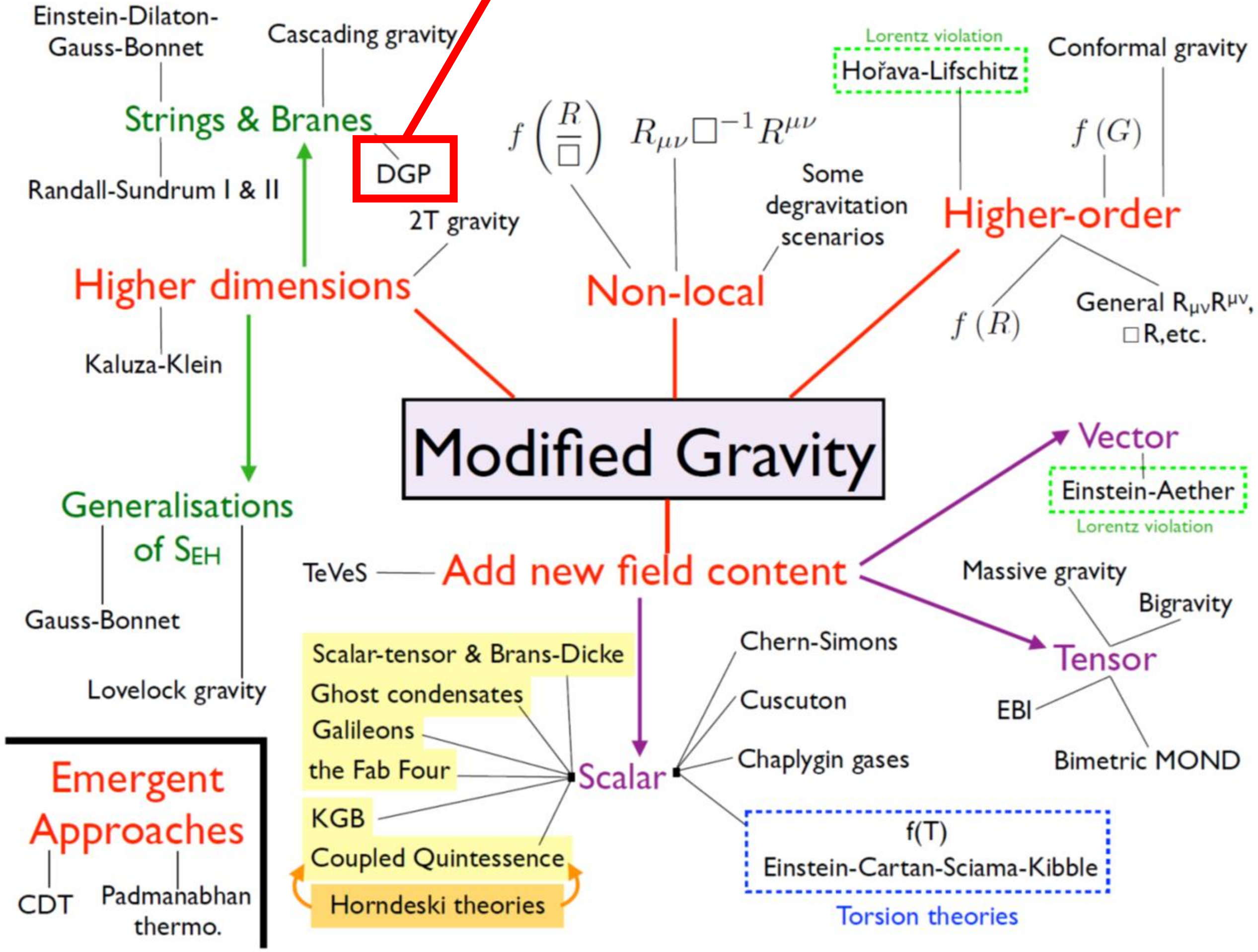
Teleparallel gravity: from theory to cosmology

Sebastian Bahamonde[1,2], Konstantinos F Dialektopoulos[3,4,5], Celia Escamilla-Rivera[6], Gabriel Farrugia[7,8], Viktor Gakis[7,8,9], Martin Hendry[10], Manuel Hohmann[1], Jackson Levi Said[7,8,*], Jurgen Mifsud[7,8] and Eleonora Di Valentino[11]

https://arxiv.org/abs/2106.13793

Explore GW constraints on extra dimensions in Dvali, Gabadadze and Porrati Braneworld models:

Corman et al. (2021, 2022); Nagarajan et al. (in prep.)

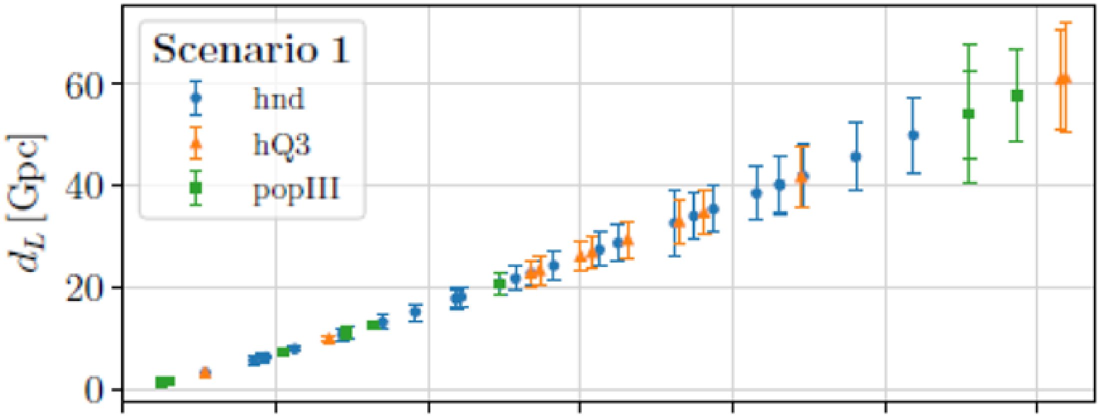


Credit: Bull et al. arxiv: 1512. 05356

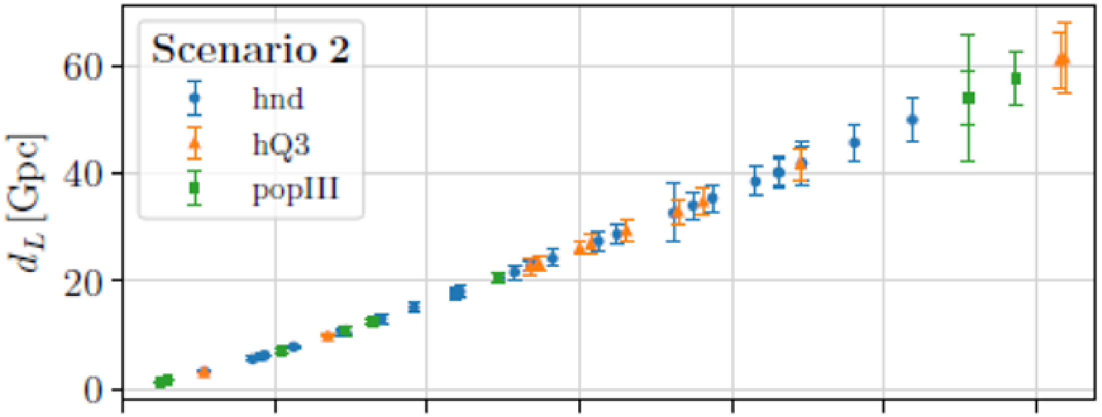
Corman et al. (2021), constraints on D from simulated **LISA catalogs.** arxiv.org/2004.04009

Based on catalogs in Tamanini et al. JCAP (2016); arXiv:1601.07112
Belgacem et al., JCAP (2019); arXiv:1906.01593

“Realistic” errors on d_L



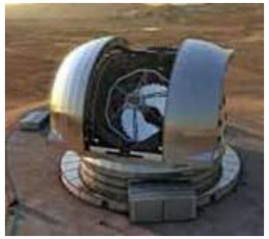
“Optimistic” errors on d_L



1. ‘Realistic’ errors: photometric redshift measurements were assumed to have a relative 1-σ error of $\Delta z_{\text{photo}} = 0.05(1+z)$ while spectroscopic measurements have a relative 1-σ error of $\Delta z_{\text{spect}} = 0.01(1+z)^2$.

2. ‘Optimistic’ errors: photometric redshift measurements were assumed to have a relative 1-σ error of $\Delta z_{\text{photo}} = 0.03(1+z)$ while spectroscopic measurements have a fixed relative 1-σ error of Δ_spect $= 0.01$ and it is assumed that a ‘delensing’ procedure can be applied that is capable of reducing by 50% the luminosity distance error due to weak lensing.

LSST/VRO



EELT

SKA

Corman et al. (2022) Phys. Rev. D **105**, 064061


PHYSICAL REVIEW D **105**, 064061 (2022)

Constraining cosmological extra dimensions with gravitational wave standard sirens: From theory to current and future multimessenger observations

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(Received 14 November 2021; accepted 22 February 2022; published 29 March 2022)

The propagation of gravitational waves (GWs) at cosmological distances offers a new way to test the gravitational interaction at the largest scales. Many modified theories of gravity, usually introduced to explain the observed acceleration of the Universe, can be probed in an alternative and complementary manner with respect to standard electromagnetic (EM) observations. In this paper we consider a homogeneous and isotropic cosmology with extra spatial dimensions at large scales, which represents a simple phenomenological prototype for extradimensional modified gravity cosmological models. By assuming that gravity propagates through the higher-dimensional spacetime, while photons are constrained to the usual four dimensions of general relativity, we derive from first principles the relation between the luminosity distance measured by GW detectors and the one inferred by EM observations. We then use this relation to constrain the number of cosmological extra dimensions with the binary neutron star event GW170817 and the binary black hole merger GW190521. We further provide forecasts for the Laser Interferometer Space Antenna (*LISA*) by simulating multimessenger observations of massive black hole binary (MBHB) mergers. This paper extends and updates previous analyses which crucially neglected an additional redshift dependency in the GW-EM luminosity distance relation which affects results obtained from multimessenger GW events at high redshift, in particular constraints expected from *LISA* MBHBs.

DOI: 10.1103/PhysRevD.105.064061

I. INTRODUCTION

The first observation of gravitational waves (GWs) from binary black hole coalescences [1], the first observation of a neutron star binary coalescence [2], and the identification of an explicit electromagnetic (EM) counterpart [3] have opened a new era of GW and multimessenger astronomy. In the near future, with advanced LIGO and advanced Virgo reaching their design sensitivity [4–6], and KAGRA[1] and LIGO-India[2] joining the global network of second-generation ground-based detectors, we expect GW detections to take place on an almost daily basis. Furthermore, in the 2030s the space-based interferometer *LISA*[3] and third-generation[4] ground-based interferometers, such as the Einstein Telescope[5] and Cosmic Explorer,[6] will be capable of detecting large numbers of coalescing compact binaries at cosmological redshifts, $z \gg 1$. These developments will open up excellent opportunities for constraining cosmological parameters and testing cosmological models beyond

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[1]Kamioka Gravitational Wave Detector, https://gwcenter.icrr.u-tokyo.ac.jp/en/ [7].
[2]Laser Interferometer Gravitational-Wave Observatory, https://www.ligo-india.in, (https://dcc.ligo.org/LIGO-M1100296/public).
[3]Laser Interferometer Space Antenna, https://lisa.nasa.gov.
[4]https://gwic.ligo.org/3Gsubcomm/.
[5]https://www.einsteintelescope.nl.
[6]https://cosmicexplorer.org.

2470-0010/2022/105(6)/064061(15) 064061-1

- (Re-)derive from first principles the relationship between the luminosity distances measured by GW detectors and inferred from EM observations

  We identify an extra (1+z) term previously neglected

$$\boxed{d_L^{\rm GW} = d_L^{\rm EM}\left[1+\left(\frac{d_L^{\rm EM}}{R_c(1+z)}\right)^n\right]^{(D-4)/(2n)}}$$

- Constrain the number of cosmological dimensions using GW + EM observations of GW170817 and GW190521*

- Forecast the constraints achievable with LISA, using simulated multi-messenger observations of SMBH mergers.

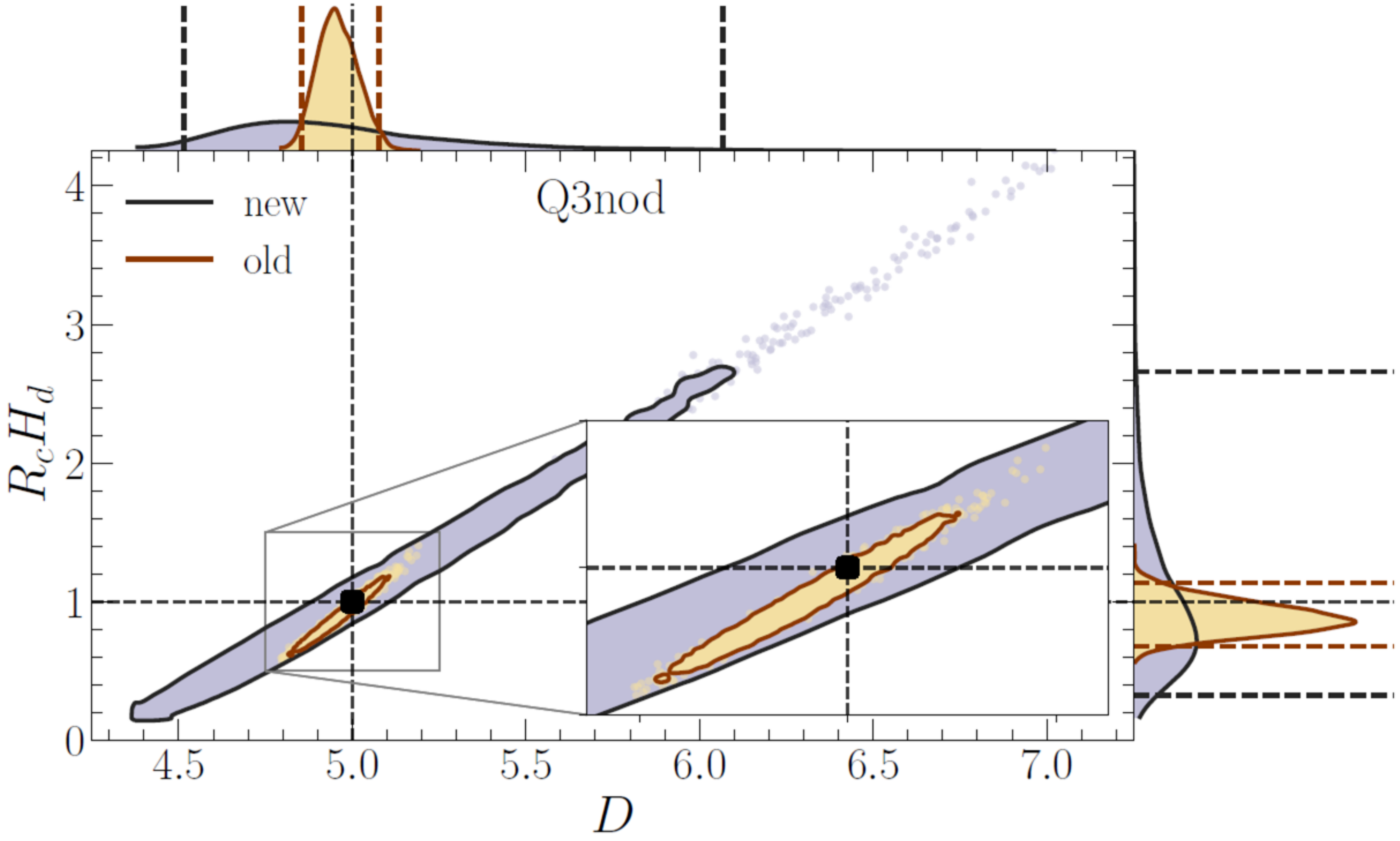
** assuming putative EM counterpart: AGN flare reported in Graham et al (2020)*

For LISA the extra (1+z) term has a significant impact:

- Systematic shift in median value of D away from true value
- Increased spread in the posterior distribution compared with previous analysis
- Similar shift / spread seen for all three merger models
- Inference of screening scale also displays increased bias and larger variance

However…

Bayes factors **still sufficient** to clearly distinguish between ΛCDM (D = 4) and extra-dimension models



Model	D		$R_c H_d$		$\ln B$	
	old	new	old	new	old	new
popIII	$5.01^{+0.18}_{-0.14}$	$5.83^{+1.17}_{-1.04}$	$1.03^{+0.38}_{-0.28}$	$2.17^{+1.84}_{-1.54}$	1074^{+627}_{-672}	141^{+126}_{-101}
HQ3	$5.02^{+0.14}_{-0.13}$	$5.47^{+1.34}_{-0.75}$	$1.05^{+0.30}_{-0.25}$	$1.69^{+2.07}_{-1.11}$	1568^{+722}_{-868}	171^{+125}_{-81}
HND	$4.99^{+0.12}_{-0.10}$	$5.26^{+1.23}_{-0.49}$	$1.01^{+0.25}_{-0.19}$	$1.35^{+1.87}_{-0.70}$	2748^{+629}_{-1217}	321^{+152}_{-145}

Constraining Cosmological Extra Dimensions in Third-Generation Gravitational-wave Detectors

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I. INTRODUCTION

Gravitational-waves offer a perspective into cosmological inference that is entirely independent of the cosmic distance ladder [1–5]. It is possible to probe cosmological parameters using the luminosity distance, assuming an accurate knowledge of the source redshift. It was proposed in [?], that gravitational wave (GW) sources could be used to measure the luminosity distance (d_L) without the need to rely on the cosmological distance ladder, but would require knowledge about the redshift (z) via electromagnetic (EM) observations. This information can be obtained from the EM counterparts of Binary Neutron Star (BNS) or Neutron Star Black Hole (NSBH) mergers, if they exist, but is typically obtained from redshift measurement of the estimated host galaxy. Given the low fraction of BNS and NSBH mergers that would produce an observable short gamma-ray burst at current detector sensitivities [6], there have been studies that have focused on alternate means of source redshift determination in the absence of an EM counterpart [7, 8]. It is evident from these studies that redshift determination is the current bottleneck for the inference of cosmological parameters from GW sources. With third generation (3G) gravitational-wave detectors, however, estimates suggest a large enough observation of BNS mergers with EM counterparts for a sub-percent constraint on cosmological parameters [9].

One of the primary questions that we hope to probe with the planned third generation detectors (LISA [10], TianQin [11], Einstein Telescope [12] and Cosmic Explorer [13]) and the proposed Deci-Hertz detectors (DECIGO [14] and Big Bang Observer [15]) is to test extensions to the Λ-Cold-Dark-Matter model (ΛCDM) cosmological model [16]. Although ΛCDM has had significant observational success, it hosts a multitude of theoretical issues: the cosmological constant problem [17], the small scale problems of ΛCDM [18], issues with cold dark matter [19] and the inadequate testability of inflationary cosmology [20, 21]. The proposals addressing these problems are numerous, some of which include - adding symmetry [22], back-reaction mechanisms [23] and modifications to general relativity [24]. In this work, we are specifically interested in the modifications to general relativity that include the addition of higher dimensions of space that allow for the leakage of gravity into extra dimensions at cosmological distance scales [25–27].

Investigating this class of solutions is particularly interesting since the existence of large higher dimensions, at least as large as the wavelength of the considered gravitational waves [28], would hint at solutions to the hierarchy problem [29, 30]. Moreover, extrapolating the big bang back in time to when the size of the extra dimensions were comparable to the spatial dimensions would provide insight into the evolution of the universe and have profound impact on cosmology [31].

Estimating the existence or number of extra spatial dimensions has been performed in various ways [32–34] but gravitational-waves provide a unique avenue for applying constraints on large extra dimensions. The detection of a binary neutron star merger, GW170817 [35], and its electromagnetic counterpart, GRB170817A [36], made it possible to explore this avenue [28]. No conclusive evidence was found to suggest the existence of extra dimensions from this observation, but given the expected degree of leakage of gravity and the distance to the source, this result conforms to expectations. This can be mitigated when we detect sources with large enough redshift that allow for sufficient leakage. Since LISA is expected to probe the universe up to a redshift of ≈8, [37] conducted a study that constrains the number of extra dimensions, D, using a simulated gravitational-wave siren catalogue at LISA sensitivity. It was found that LISA massive black hole binary (MBHB) observations can be used to provide unequivocal evidence of integer extra dimensions if they existed. However, both [28] and [37] did not include the effects of redshift in their estimates, which would likely broaden the posterior on D. This was later included in [38] to produce an updated estimate of

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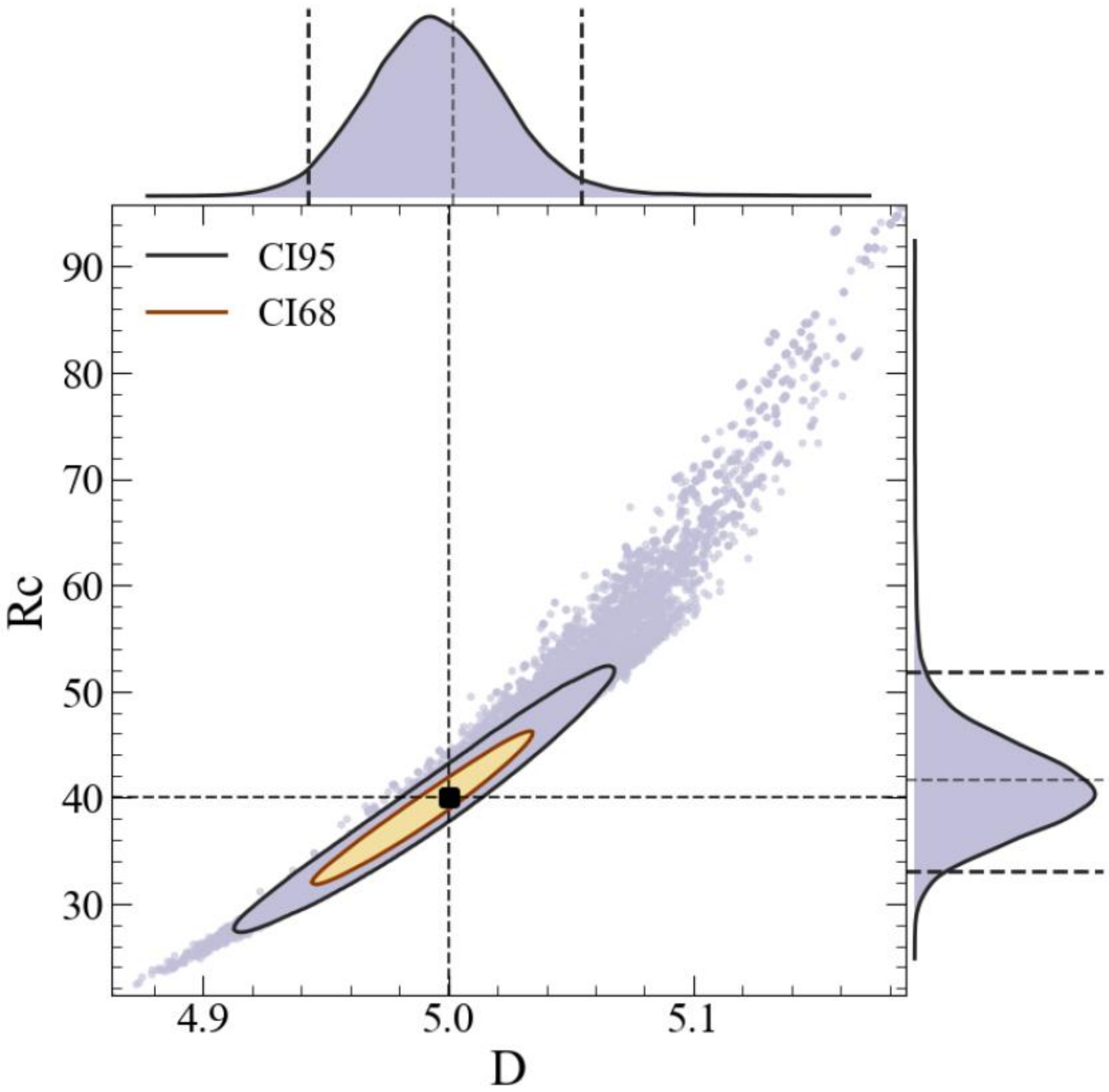

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Conclusions

- GW lensing is a **relatively young but rapidly maturing** field – exciting times ahead!
- Ground-based **SLGW searches well established**, working to address important **computational and statistical** issues (although challenges remain with modelling/understanding of false positives and astrophysical backgrounds).
- Exciting possibilities for **GW microlensing** (and **WO effects**) with computational tools under development to enable **model selection** and **multiple lensing**.
- **Weak lensing** likely to impact on future GW cosmology studies, particularly with **LISA**. **De-lensing** possible, but significant (multi-messenger) challenge.
- Better **"noise" modelling** still needed!