# Corrections to dijet production cross section in Deep Inelastic Scattering within the CGC formalism.

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# Outline

- Quantum Chromodynamics
  - Structure of the theory
  - Main properties
  - Perturbative calculations and their limitation

- Factorization in hadron physics
  - Deep Inelastic Scattering
  - Collinear factorization
  - Parton Distribution Functions
  - TMD factorization and TMD PDF

- Color Glass Condensate
  - Small-x physics
  - Saturation momentum scale
  - Color Glass Condensate as a effective field theory
  - Dynamics of CGC

- General outline of our work

# What is the idea

- **Understanding the structure of hadrons via different processes**

Hadrons, even if they at first appear as somewhat simple composite objects, in reality are extremely complicated and cannot be easily described by first principles. This leads us to consider various effective theories and a mass of approximations.

- **How the effective theories work and what are their limitations**

Theories and approximations work only in specific regimes – sometimes in very different ones. To really use them, as a main idea behind the physics or only as a useful calculational scheme, we have to acquire a great knowledge about their regime of work

- **How to make a connection between different kinematical regimes**

The different schemes live in different scales, but they describe basically the same physical systems. It would be ideal to find a connection between them in order to have as unified picture as possible

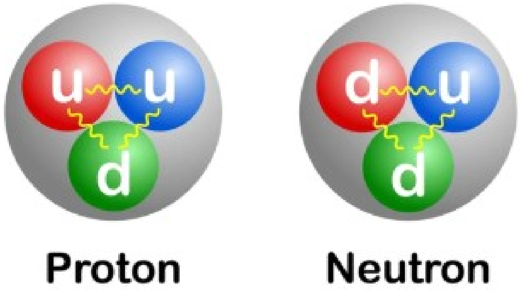
**My goal is to broaden the connection by considering less strict approximations in perturbative calculations of effective theories within so-called Quantum Chromodynamics**

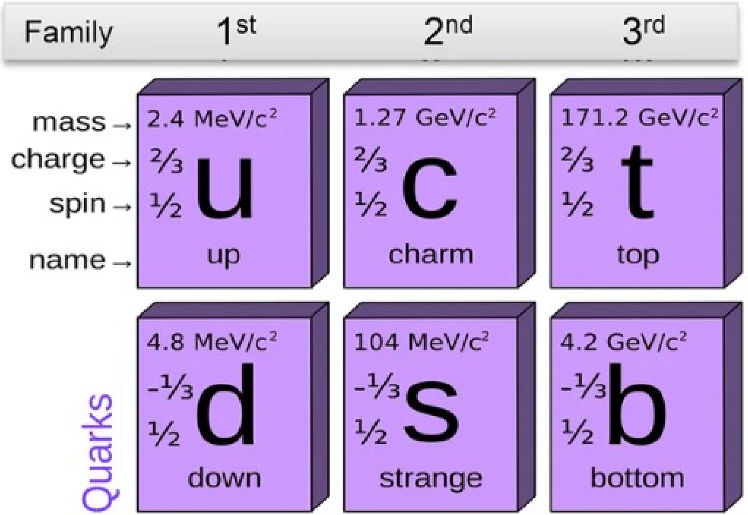
# Basics of Quantum Chromodynamics

- The main theoretical framework regarding the structure of hadrons we currently have is known as **Quantum Chromodynamics** (QCD in short).

- QCD describes the dynamics of quarks and gluons via the Strong interaction.
    - Quarks are the only particles of matter that interact strongly. Thus, they form a separate family within the Standard Model.
    - Gluons are massless carriers of the Strong force.

There are six recognized types (or flavors) of quarks. However, the major part of all hadronic matter consists of up (u) and down (d) quarks.



# Basics of Quantum Chromodynamics

- In Electrodynamics, we deal with charged particles interacting by emission or absorption of photons. Similarly, in QCD, quarks possess a new type of charge (called **color**) that permits emission and absorption of gluons

- However, in contrast to Electrodynamics:
    - There are three types of the color charge (creatively called red, green and blue)
    - Gluons **are color-charged**

Especially the second point carries a striking difference between these two interactions. Photons do not have an electric charge, and therefore they do not "see" each other during any process.

**Gluons can interact with themselves.** Due to this, the structure of QCD interactions is significantly richer.

# QCD – formal structure

- Formally, QCD is a Quantum Field Theory with a local SU(3) gauge symmetry described be the following Lagrangian:

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi - \frac{1}{4}F^{a}_{\mu\nu}F^{\mu\nu}_{a}$$

- This term describes quarks and their interaction with gluons
- Quarks are fermions with spin ½ and a fractional electric charge.

$$D_{\mu} = \partial_{\mu} - igA^{a}_{\mu}t^{a}$$

QCD coupling constant

Gluon vector field

Gell-Mann matrices

- F stands for the gluon field strength tensor. It is similar to its electrodynamic counterpart, but involves an additional term:

$$F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} + gf^{abc}A^{b}_{\mu}A^{c}_{\nu}$$

Structure constants

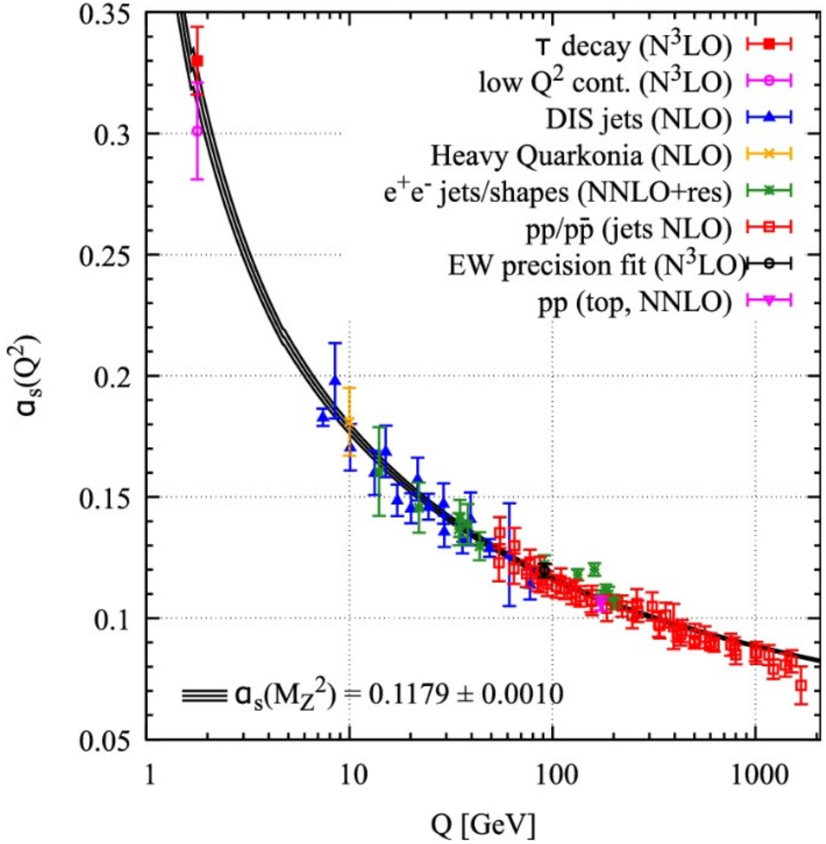
# Asymptotic Freedom and Confinement

One of the most important aspects of QCD: Its coupling (denoted by g or $\alpha_s$) **decreases with increasing energy scale:**

$$\alpha_s(k^2) \approx \frac{1}{\beta_0 \ln(k^2/\Lambda^2)}$$

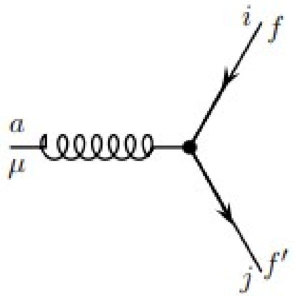
Gives rise to two specific phenomena:

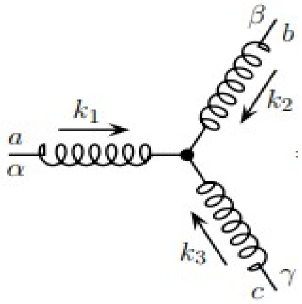
- **Asymptotic Freedom –** At high energies (or, equivalently, small distances r ~ 1/k), quark are essentially free.
- **Confinement –** all partons have to be confined within a composite structure – hadrons. There are no free quarks/gluons in nature.

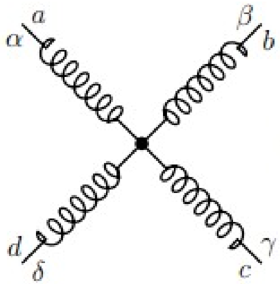


# Perturbative calculations in QCD

It is possible to deduce three vertices from the QCD Lagrangian:




$$-ig\gamma^{\mu}t^{a}$$

$$-igf^{abc}(g^{\alpha\beta}(k_1 - k_2)^{\gamma} + perm.)$$

$$-ig^2 f^{abe} f^{cbe}(g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma}) + perm.$$

Together with expressions for gluon and quark propagators, they form a basis for, technically, all perturbative calculations within QCD.

# Perturbative QCD - limitations

The main problem with perturbative calculations in QCD:

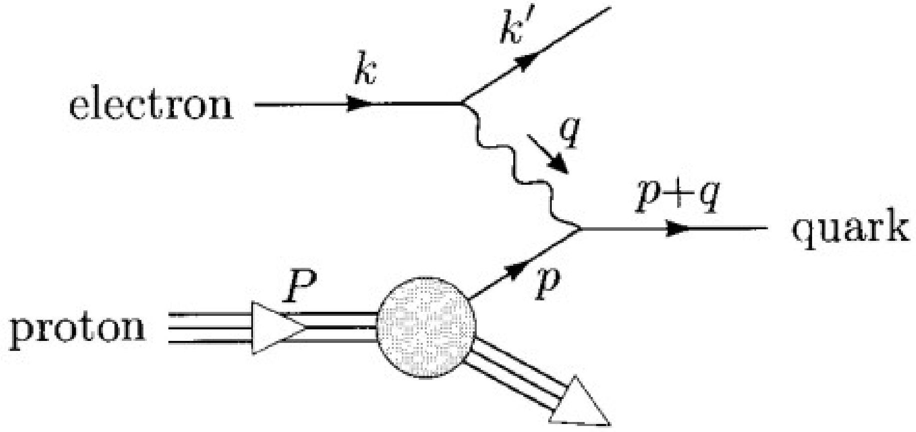
- Due to the coupling constant $\alpha_s$ scaling with energy, the interactions become strong and break the perturbative series below a certain characteristic energy scale ~ 1 GeV.

- It is possible to use perturbative expansion in high-energy scattering processes (asymptotic freedom). However, even in that case it is not so simple:
    - All scatterings involve hadrons. The hadronic wave-functions is not known, and its internal dynamics encompasses also soft scales. As such, a hadron is inherently a non-perturbative object.
    - Even if the scattering process regards a hard scale during collision, the total cross section for it would still be non-perturbative.

<u>Fortunately, there are ways to proceed with perturbative calculations in specific regimes</u>

# Deep Inelastic Scattering

- One of the well-established scattering processes – **Deep Inelastic Scattering**
- Involves a fermion (electron) scattering off of a hadron (proton) by a virtual photon exchange




- Low momentum transfer – scattering off a hadron directly
- High momentum transfer – the photon “penetrates” the hadron involved, scattering of a parton inside
    - Can shatter the hadron completely

Kinematics:

$q^\mu = k^\mu - k'^\mu$ - photon momentum (spacelike)

$Q^2 = -q^2$ - momentum exchange squared

$x = \frac{Q^2}{2P \cdot q}$ - Bjorken x variable, related to the longitudinal momentum fraction of the struck parton: p ~ xP

$y = \frac{q \cdot P}{k \cdot P}$ - Inelasticity (fraction of the lepton’s energy loss)

- Both lie in the range of (0, 1)

# Deep Inelastic Scattering

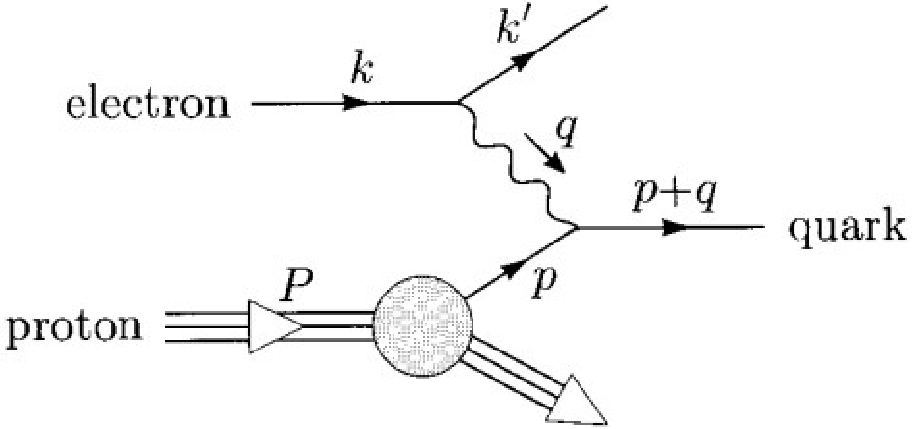
- DIS is **inclusive** (we are interested only in the scattered lepton)
- In high-energy limit, the DIS cross section may be written quite generally as:

$$E' \frac{d\sigma}{d\Pi} \simeq \frac{2\alpha_s^2}{sQ^4} L^{\mu\nu} W_{\mu\nu}$$

s – square of the center-of-mass energy

Hadronic tensor

Leptonic tensor




$$y = \frac{Q^2}{xs}$$

- Relation between CoM energy and inelasticity y

Can be expanded (for unpolarized scattering, neglecting both lepton and hadron masses) in terms of our kinematical variables:

$$\frac{d\sigma}{dydx} = \frac{4\pi\alpha_s^2 s}{Q^4} \left\{ (1-y) F_2(x,Q^2) + xy F_1(x,Q^2) \right\}$$

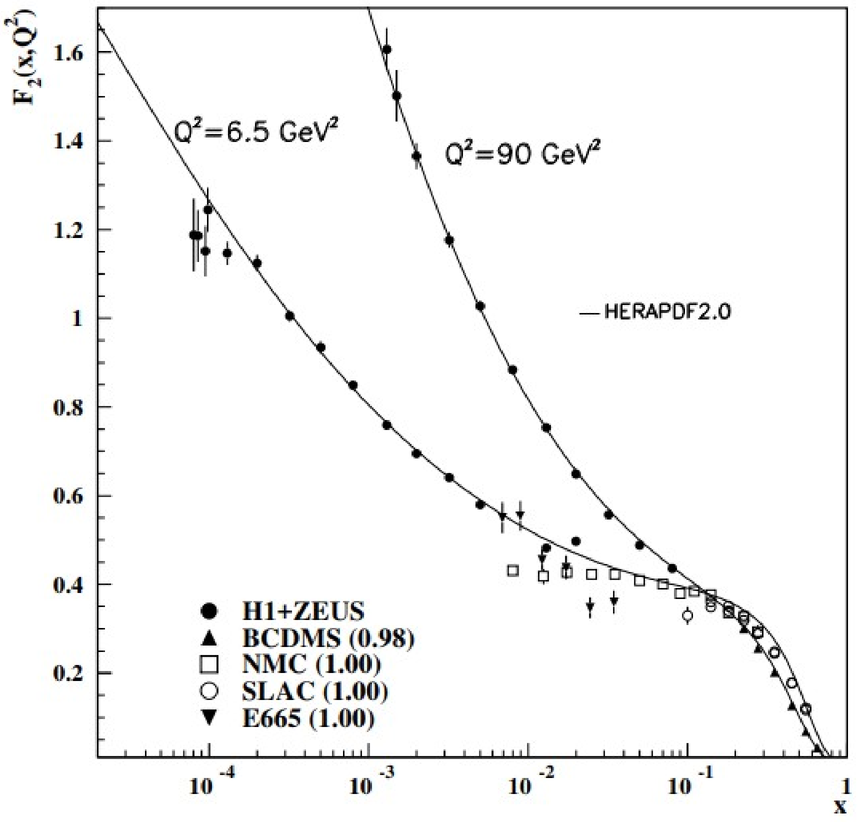
# Structure Functions

$F_1(x, Q^2)$ and $F(x, Q^2)$ are called **Structure Functions** of the hadron. They depend on both the momentum exchange (resolution) and the longitudinal momentum fraction carried by partons.

- Non-perturbative. Need some external experimental or numerical data
- Polarized cross-section have other types of structure functions as well

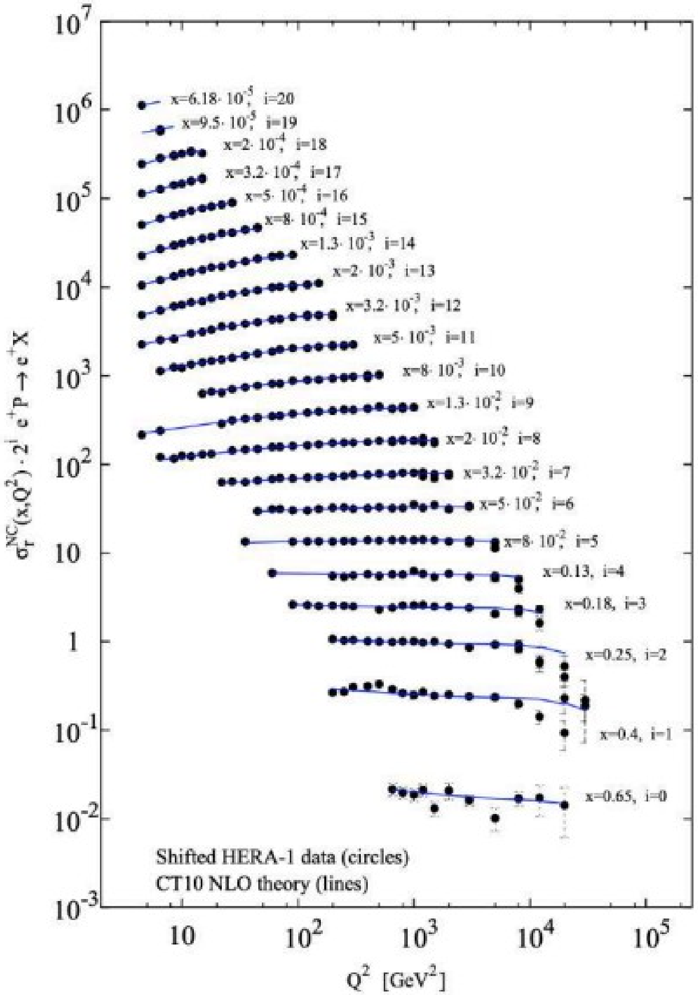
Worth noticing – for large values of x (>0.1), structure function are similar for very different $Q^2$

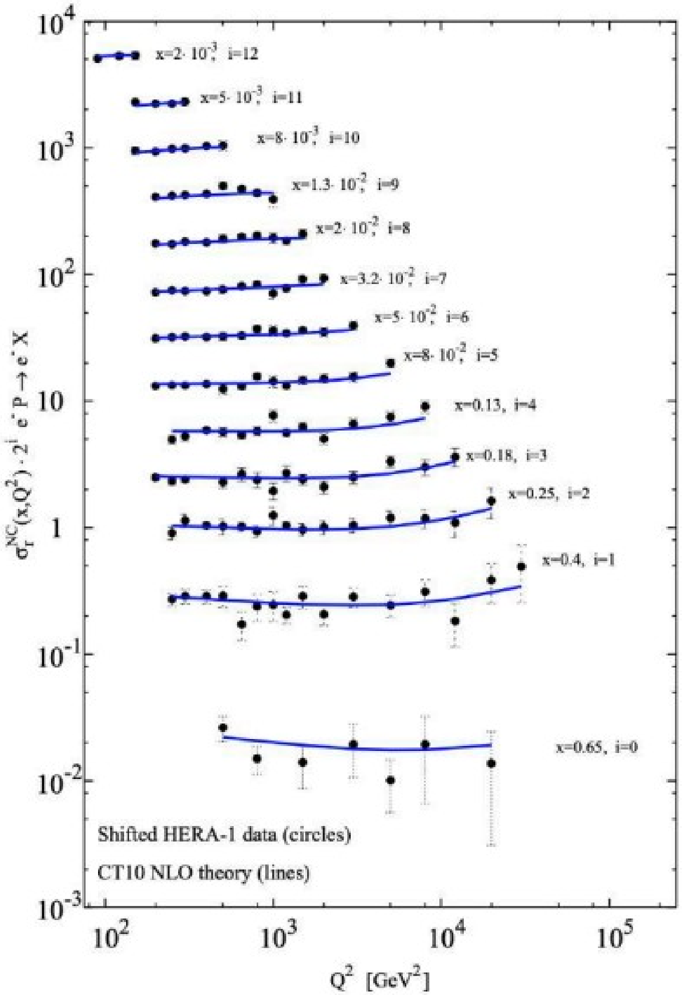
**- Bjorken scaling**




Example of determination of one of the structure functions

HERA DIS data:



# Collinear Factorization

- In high-energy scattering, hadrons can be view as a free collection of quarks and gluons (Feynman's **parton model**). Time scale of the probing is much shorter (~ 1/Q) than the characteristic time scale of processes inside (~ $1/\Lambda_{QCD}$)
  - Parton model is equivalent to leading order in perturbative expansion. For more – need QCD corrections

- All partons move collinearly with the parent nucleon – transverse momenta are negligible

These conditions allow us to utilize an extremely convenient way of describing the DIS cross sections: **Collinear Factorization.** In this formalism, the whole cross-section can be split:

(Holds when Q >> $\Lambda_{QCD}$)

$$d\sigma_{DIS} = \sum_i \int_0^1 dx\, f_i(x, Q^2) \hat{\sigma_{p_i}} = \sum_i f_i(x, Q^2) \otimes C_{p_i}$$

**Parton Distribution Function (PDF)**

**Partonic cross section**

**Hard factor** (related to the partonic cross section)

Connection between Structure Functions and PDFs:

$$F_1(x, Q^2) = \frac{1}{2} \sum_i e_i^2 f_i(x, Q^2) + \mathcal{O}(\alpha_s),$$

$$F_2(x, Q^2) = \sum_i e_i^2 x f_i(x, Q^2) + \mathcal{O}(\alpha_s).$$

$$F_2(x, Q^2) = 2x F_1(x, Q^2) + \mathcal{O}(\alpha_s)$$

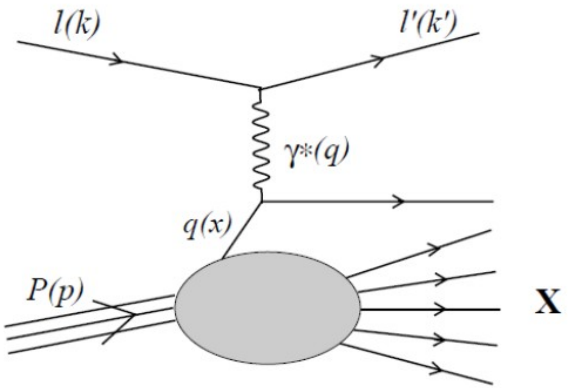
- Callan – Gross relation

# Parton Distribution Functions

$$d\sigma_{DIS} = \sum_i \int_0^1 dx\, f_i(x, Q^2)\hat{\sigma_{p_i}} = \sum_i f_i(x, Q^2) \otimes C_{p_i}$$

- Hard factors are **process-dependent** and can be calculated perturbatively order by order

- PDFs are non-perturbative and need additional external input (experimental, lattice). However, they are **universal for a given hadron – describe structure**

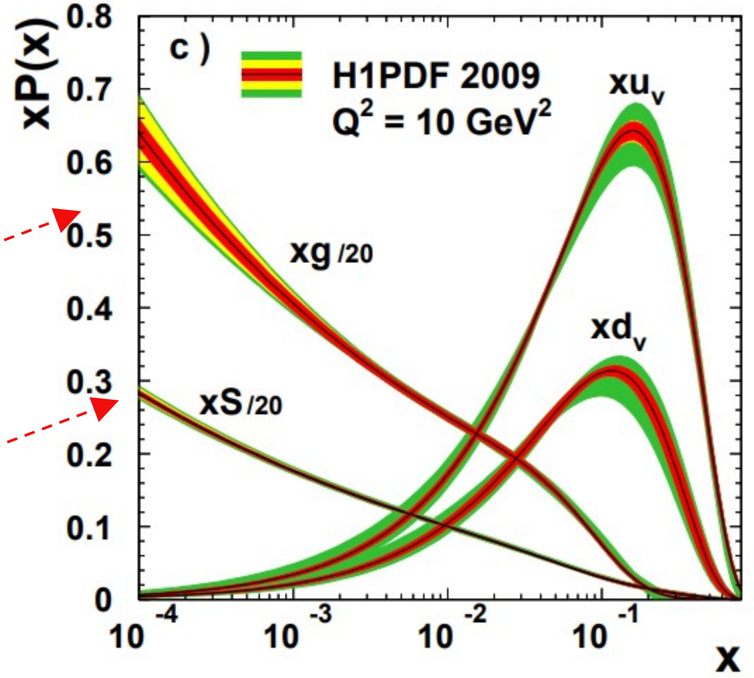



Collinear factorization formula: we can divide whole DIS cross-section into perturbative part regarding lepton scattering off of a given parton and into non-perturbative, but **universal** distributions.

# Parton Distribution Functions

- Universality: Once we know the hadron's PDF, all we have to do is to calculate the hard factors up to a desired order, provided all kinematical criteria for factorization are met.

Due to that, there was a high effort to find the properties of the parton distributions.

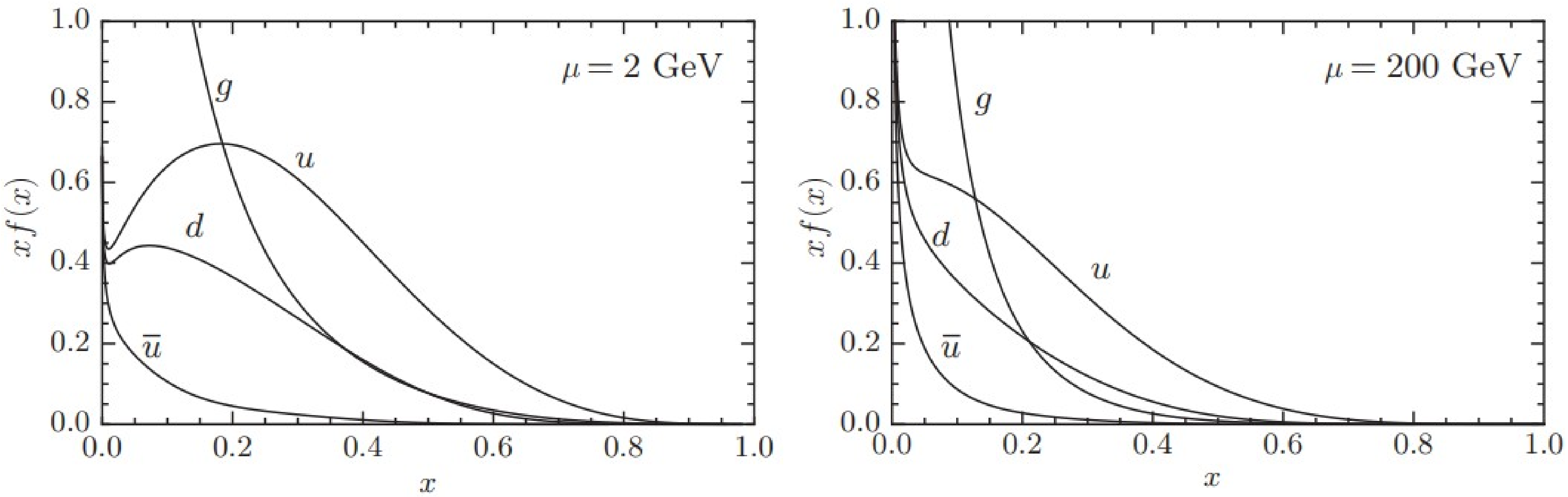



Gluons dominate at small x – important later

Sea quarks

Valence quarks visible

# Parton Distribution Functions




Various PDFs with two additional scales. The properties here are similar to the previous plot:

- Valence quarks are clearly visible at higher values of x
- For small enough x, gluons dominate – that picture is universal for hadrons

The literature focus mostly on proton's PDFs. However, determination of other can also be found.

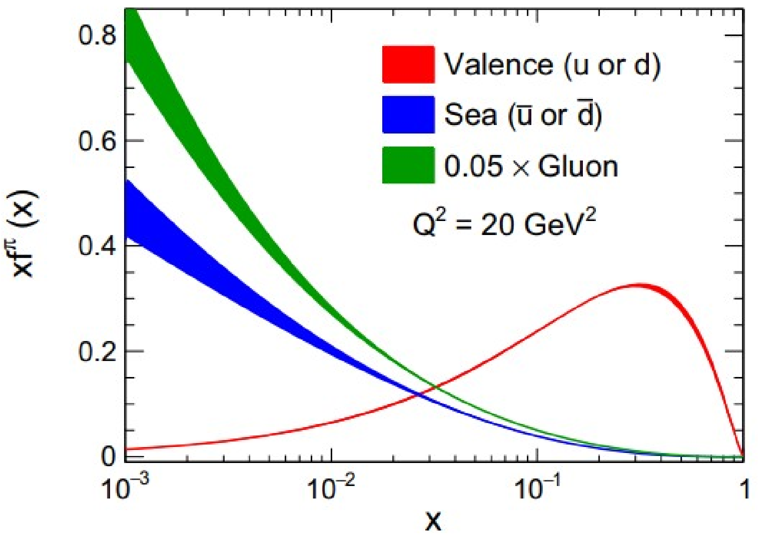



**Fig. 6** The valence quark, sea quark and gluon distributions of the pion from this work. The error bands show the $3\sigma$ uncertainties of the quantities

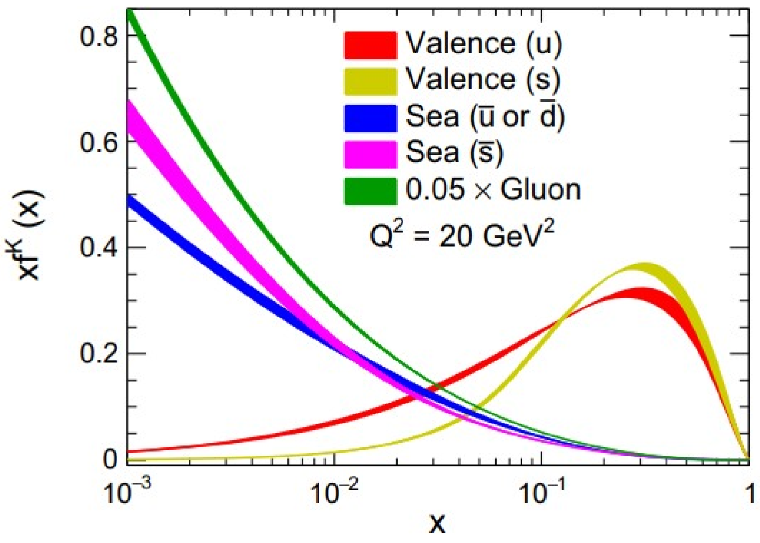



**Fig. 7** The valence quark, sea quark and gluon distributions of the kaon from this work. The error bands show the $3\sigma$ uncertainties of the quantities

Determination of PDFs in the case of pions and kaons. As mentioned, the gluon sea behavior appears to be universally dominant at small-x scales.

*[Han, C., Xie, G., Wang, R. et al. An analysis of parton distribution functions of the pion and the kaon with the maximum entropy input. Eur. Phys. J. C 81, 302 (2021)]*

# Operator form of PDFs

It is possible (and frequently useful) to rewrite the parton densities as a (Fourier transformed) matrix element of a bilocal operator

$$f_i(x) = \int \frac{d\omega^-}{2\pi} e^{-ixP^+\omega^-} \langle P|\bar{\psi}_i(0,\omega^-,0_\perp)\Gamma\psi_i(0)|P\rangle_c$$

Important note: to get gauge-invariant expression, we need to connect these fields with a gauge link – Wilson line. It is a path-ordered exponential of the gluon field U ~ exp(ig·∫A)

Quark fields at two different spacetime points.

Placeholder for any Dirac matrix that may appear here – polarized and unpolarized cases.

We work in **lightcone coordinates:**

$$x^\mu = (t,x,y,z) \rightarrow (x^+, x^-, \mathbf{x}_\perp)$$

Where: $x^\pm = \frac{z \pm t}{\sqrt{2}}$ $\qquad x \cdot y = x^+y^- + x^-y^+ - \mathbf{x} \cdot \mathbf{y}$

Lightcone coordinates behave nicely under Lorentz boosts.

- For large Q scales, only $p^+ = xP^+$ parton momentum is important.

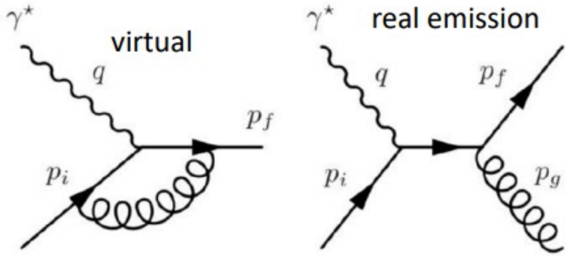
The Fourier conjugate of $p^+$ is $\omega^-$ (and vice versa). This is why only that particular component appears in the operational definition of PDF – the rest is suppressed in highly boosted frames.

# Evolution of PDFs – DGLAP equation

- Parton model – only first order approximation to general QCD scheme
- The picture is more complicated in next orders – need resummation of loops and taking care of various divergences

PDFs are non-perturbative, but their evolution with momentum scale can be perturbatively achieved. Not going into details, it is possible to derive a Renormalization Group Equation (RGE) for their evolution.

It is known as the DGLAP (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi) equation:




Next-to-leading order (NLO) contributions involving gluons

$$\mu \frac{d}{d\mu}\begin{pmatrix} f_i(x,\mu) \\ f_g(x,\mu) \end{pmatrix} = \sum_j \frac{\alpha_s}{\pi} \int_x^1 \frac{d\xi}{\xi} \begin{pmatrix} P_{q_i q_j}(\frac{x}{\xi}) & P_{q_i g}(\frac{x}{\xi}) \\ P_{g q_j}(\frac{x}{\xi}) & P_{gg}(\frac{x}{\xi}) \end{pmatrix} \begin{pmatrix} f_j(\xi,\mu) \\ f_g(\xi,\mu) \end{pmatrix}$$

Careful: μ instead of $Q^2$, as the PDFs are evolving with momentum scale here. May end at the Q value after evolution.

DGLAP kernel

Splitting functions
- perturbative

$$P_{qq}(z) = C_F\left[\frac{1+z^2}{[1-z]_+} + \frac{3}{2}\delta(1-z)\right] \qquad P_{gq}(z) = C_F\left[\frac{z^2+(1-z)^2}{z}\right]$$

$$P_{qg}(z) = T_F\left[z^2+(1-z)^2\right] \qquad P_{gg}(z) = 2C_A\left[\frac{z}{[1-z]_+} + \frac{1-z}{z} + z(1-z)\right] + \frac{\beta_0}{2}\delta(1-z)$$

Splitting functions at LO (leading order). Extremely complicated even at NLO

# Transverse Momentum Dependent PDF

- In highly boosted frames the longitudinal momentum $p^+$ dominates, while $p^-$ is always much smaller.
- However, transverse momentum may be relevant in a detailed description: $\Lambda^2_{QCD} \lesssim \mathbf{q}^2 \ll Q^2$

To go beyond the collinear regime, we need a new factorization - **Transverse Momentum Dependent (TMD) factorization:**

$$d\sigma \sim \sum_i \int_0^1 dx\, f_i(x, Q^2) \cdot \hat{\sigma}_i(Q^2) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$$

$$d\sigma \sim \sum_i \int_0^1 dx \int d^2\mathbf{k}\, f_i(x, \mathbf{k}_\perp, Q^2) \cdot \hat{\sigma}_i(\mathbf{k}_\perp, Q^2) + \mathcal{O}\left(\frac{\mathbf{q}^2}{Q^2}\right) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$$

Often useful to work in Fourier-transform:

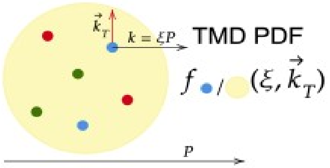
$$\tilde{f}(x, \mathbf{b}_\perp) = \int d^2\mathbf{k} e^{-i\mathbf{k}_\perp \cdot \mathbf{b}_\perp} \cdot f(x, \mathbf{k}_\perp)$$

Impact parameter

Interpretation of the TMD factorization is similar to its collinear counterpart!

# Transverse Momentum Dependent PDF

$$d\sigma \sim \sum_i \int_0^1 dx \int d^2\mathbf{k}\, f_i(x,\mathbf{k}_\perp,Q^2)\cdot\hat{\sigma}_i(\mathbf{k}_\perp,Q^2) + \mathcal{O}\left(\frac{\mathbf{k}^2}{Q^2}\right) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$$




**Transverse Momentum Dependent Parton Distribution Function (TMD PDF)**
- direct upgrade of the collinear PDF (still non-perturbative, universal)

- Many TMDs – depend on the correlation between parton's and the nucleon's spin in polarized scatterings
  - More than PDFs

- Gives more information on the hadron's 3D structure

**Leading Quark TMDPDFs** — Nucleon Spin, Quark Spin

| Nucleon Polarization | Quark Polarization: Un-Polarized (U) | Quark Polarization: Longitudinally Polarized (L) | Quark Polarization: Transversely Polarized (T) |
|---|---|---|---|
| U | $f_1$ = Unpolarized | | $h_1^\perp$ = Boer-Mulders |
| L | | $g_1$ = Helicity | $h_{1L}^\perp$ = Worm-gear |
| T | $f_{1T}^\perp$ = Sivers | $g_{1T}^\perp$ = Worm-gear | $h_1$ = Transversity<br>$h_{1T}^\perp$ = Pretzelosity |

# TMD - Operation definition

Similar to unpolarized PDF operational definition:

$$\tilde{f}_i(x, \mathbf{b}_\perp) = \int \frac{db^-}{2\pi} e^{-ixP^+b^-} \langle P|\bar{\psi}_i(0, b^-, \mathbf{b}_\perp)\frac{\gamma^+}{2} U(b^\mu)\psi_i(0)|P\rangle_c$$

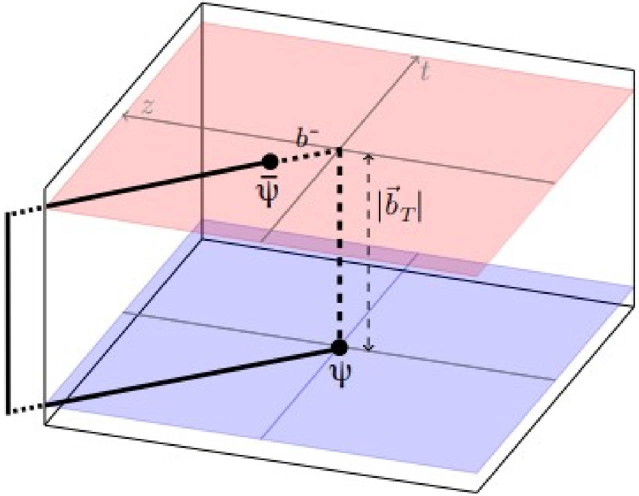
Unpolarized TMD

- This time the bilocal operator is defined at two different transverse points.
- The Wilson Line structure here is more complicated – it spans, firstly, to infinity:

$$U(b^\mu) = \lim_{Y^+\to\infty} U([Y^+, \mathbf{b}_\perp, 0^-] \to [b^+, \mathbf{b}_\perp, 0^-]) \cdot U([Y^+, \mathbf{0}_\perp, 0^-] \to [Y^+, \mathbf{b}_\perp, 0^-]) \cdot U([0^+, \mathbf{0}_\perp, 0^-] \to [Y^+, \mathbf{0}_\perp, 0^-])$$

Structure needed for gauge invariance

$$U(a, b) = P\exp\left[-ig\int_a^b dsn \cdot A^a(a + sb)t_a\right]$$



# TMD - Operation definition

We can construct other TMD functions using the operation formalism. For example:

$$\tilde{f}_i^{[\Gamma]}(x, \mathbf{b}_\perp) = \int \frac{db^-}{2\pi} e^{-ixP^+b^-} \langle P, S|\bar{\psi}_i(0, b^-, \mathbf{b}_\perp)\frac{\Gamma}{2}U(b^\mu)\psi_i(0)|P, S\rangle_c$$

Generalization of TMDs – include polarization

$$\sim \Phi^{\mu\nu[\Gamma]}(b_1^+, b_2^+, \mathbf{b}_\perp) = \langle P, S|\bar{\psi}(b_1^+, \mathbf{b}_\perp)\mathcal{F}^{\mu\nu}(b_2^+, \mathbf{b}_\perp)\frac{\Gamma}{2}\psi(0)|P, S\rangle$$

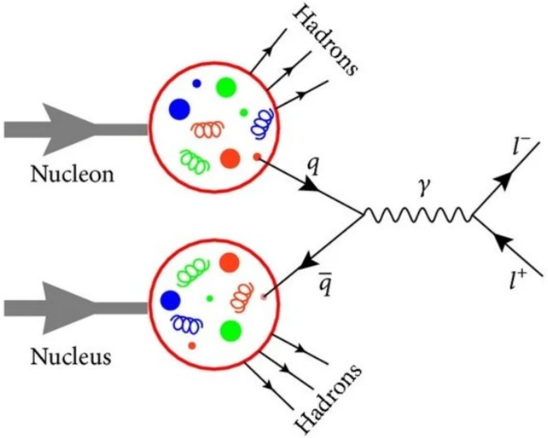
Three-point TMD – more complicated and still not understood very well

- Always need to be careful about the Wilson line structure in the case of TMDs – type of the operator, type of the process

# Other processes - examples

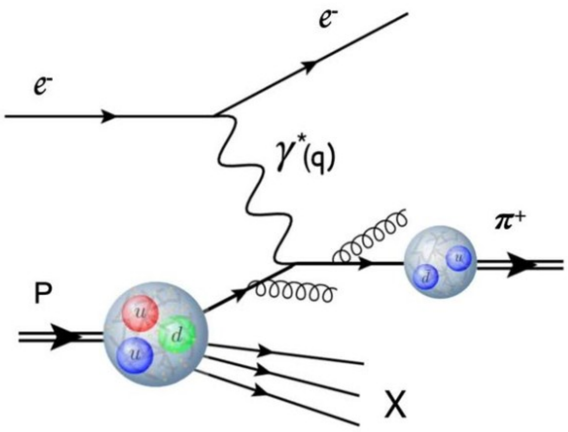
So far – only DIS has been considered. Other processes involve factorization into universal parton densities. As an example:

- **Drell-Yan process: $H_a + H_b \rightarrow l^- + l^+ + X$**




$$d\sigma_{D-Y} \sim f_{i/H_a}(x_a, \mathbf{k}_\perp, Q) f_{j/H_b}(x_b, \mathbf{q}_\perp - \mathbf{k}_\perp, Q)$$

- **Semi-Inclusive DIS (SIDIS): $e^- + P \rightarrow e^- + H + X$**




$$d\sigma_{SIDIS} \sim f_{i/P}(x_a, \mathbf{k}_\perp) D_{j/H}(z_h, \mathbf{p}_\perp)$$

Fragmentation function: Probability density of creating a hadron from a quark of type j

z – ratio of energy of the hadron to the energy of the photon

- related to the fraction of parent parton's momentum

# Different scales

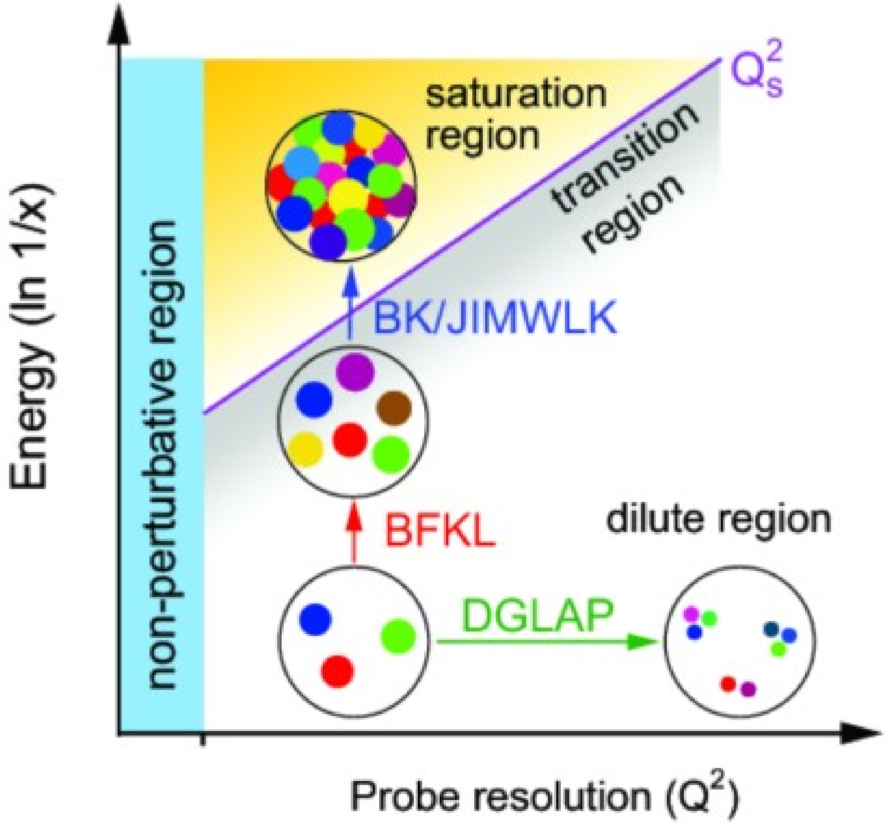
So far we focused on a high Q scale:

- Time of the probing is of order ~1/Q << $1/\Lambda_{QCD}$ – at high values of the momentum exchange the machinery of PDF/TMD factorization is at hand
- The transverse "size" of the partons is of the same order ~ $1/Q^2$.

This means that even if we can see more partons, their size is decreasing and **the system is dilute**.

Two different regimes:

- Bjorken limit: $Q^2 \to \infty, x = const$
- Regge-Gribov limit: $x \to 0, Q = const$

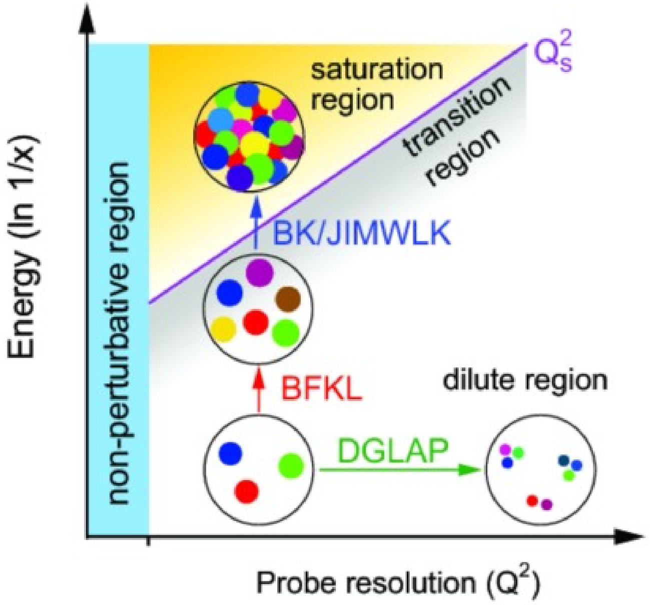


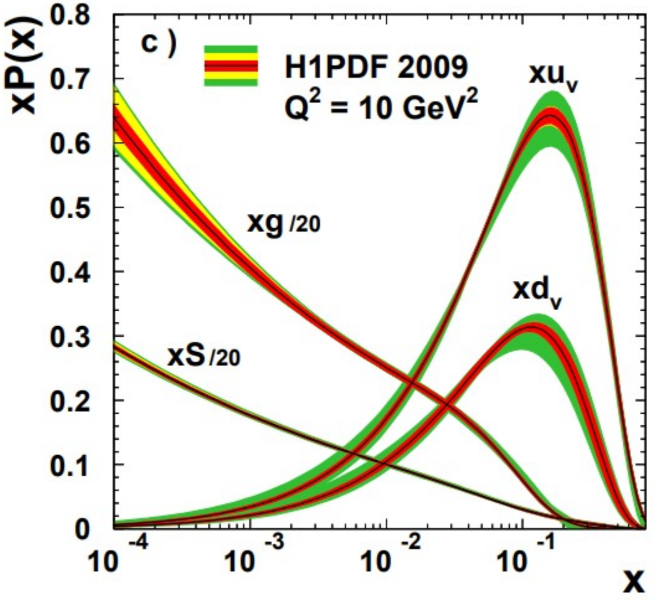
# Small-x physics

Regge-Gribov limit: $x \to 0, Q = const$

- From PDF determination mentioned before – gluon density grows rapidly (+ sea quarks) at small-x scales. Small-x physics is mostly equivalent to working with dynamics of a gluon condensate
- We should have a mechanism that allows a creation of such number of gluons, but it cannot grow indefinitely

High gluon densities – transverse momenta are important



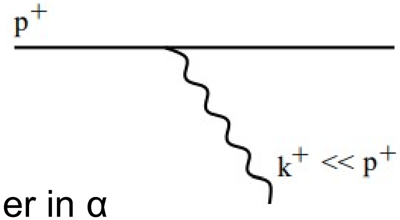



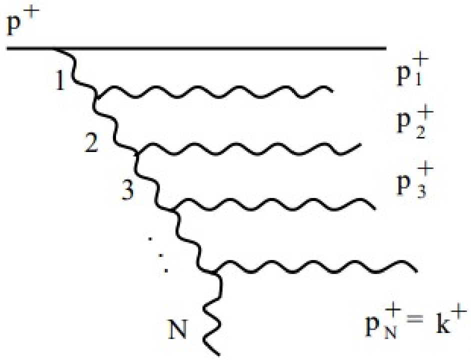
# Small-x physics – gluon cascade

General picture: at x << 1, a parton can emit a gluon with lower momenta with probability:

$$dP \sim \frac{\alpha_s C_R}{\pi^2} \frac{d^2\mathbf{k}_\perp}{\mathbf{k}^2} \frac{dx}{x}$$

To first order in α




Every such gluon can then radiate other "softer" gluons as this process continue:

$$\alpha_s^n \int_x^1 \frac{\mathrm{d}x_n}{x_n} \int_{x_n}^1 \frac{\mathrm{d}x_{n-1}}{x_{n-1}} \cdots \int_{x_2}^1 \frac{\mathrm{d}x_1}{x_1} = \frac{1}{n!}\left(\alpha_s \ln\frac{1}{x}\right)^n$$

Suppression in $\alpha^n$, but at small enough x the whole value increases

Rapidity evolution of (unintegrated) gluon distribution, including QCD corrections to a desired order, is governed by so-called **BFKL** (Balitsky-Fadin-Kuraev-Lipatov) equation:

$$\frac{\partial \phi(x, \mathbf{k}_\perp)}{\partial Y} \sim \mathcal{K}_{g\to gg} \otimes \phi(x, \mathbf{k}_\perp)$$

Resummation of all such cascades of n gluons:

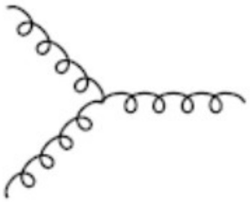
$$N_g \sim \frac{1}{\mathbf{k}_\perp^2} e^{\omega \alpha_s Y}$$

$$Y = \ln\frac{1}{x}$$ - Rapidity

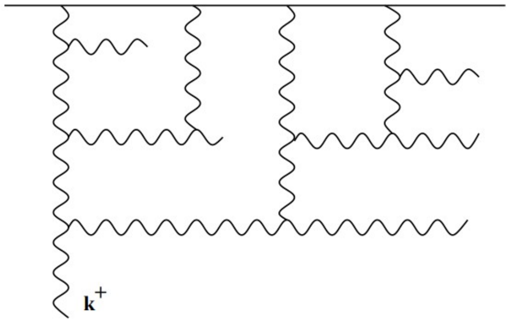
# Small-x physics – gluon recombination

- Gluon number density cannot grow without limit – violation of unitarity, overlapping of gluons
- Gluons at high enough densities start to merge – **gluon recombination**

Probability of gluon recombination depends on the **square** of the gluon density. The recombination effect then stops the indefinite gluon growth.



$$\sim \alpha_s^2 \rho^2 \frac{1}{Q^2}$$




To accommodate for the recombination, BFKL has to be modified:

$$\frac{\partial \phi(x, \mathbf{k}_\perp)}{\partial Y} \sim \mathcal{K}_{g\to gg} \otimes \phi(x, \mathbf{k}_\perp)$$

$$\frac{\partial \phi(x, \mathbf{k}_\perp)}{\partial Y} \sim \mathcal{K}_{g\to gg} \otimes \phi(x, \mathbf{k}_\perp) - \phi(x, \mathbf{k}_\perp)^2$$

Non-linear modification

Those non-linear effects start to be important if:

$$n \sim \mathcal{O}(\alpha_s^{-1})$$

This is known as the **BK** (Balitsky-Kovchegov) equation.

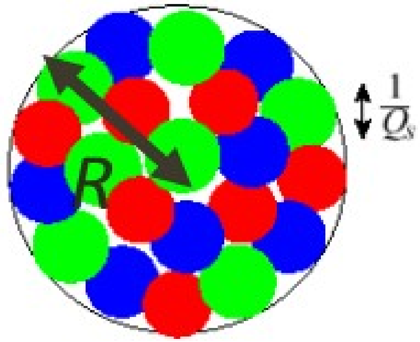
# Saturation scale

- The number density of gluons stops growing due to the recombination. The momentum scale at which this happens is called **Saturation Scale $Q_s$(x).**

- Gluons saturate when their number fill the whole transverse area of the hadron, and their mutual interaction cannot be further neglected – $n \sim \mathcal{O}(\alpha_s^{-1})$

- Saturation scale is large (>> $\Lambda_{QCD}$) – gluons weakly coupled

- Typical "Transverse scale" of a gluon: ~ 1/$Q_s$




$$\frac{\alpha_s}{Q^2} xG(x, \mathbf{k}_\perp) = \pi R^2$$

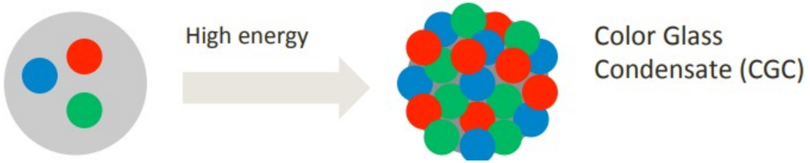
Saturation scale is also dependent on x: $Q_s^2 \sim \left(\frac{1}{x}\right)^\lambda$ $\lambda \sim 0.3$ (behavior known at NLO)

May be different for nuclei ~ **A**[1\3]

# Color Glass Condensate

The most prominent effective field theory at the saturation scale is **Color Glass Condensate (CGC)**.

- Named that because:
  - Gluons are color-charged
  - The number density of gluons is large and saturated




<u>Main idea behind CGC:</u> Fast moving gluons and slow moving gluons (x << 1) are separated due to time dilation.

From the perspective of slow gluons, the fast moving ones are “frozen” in time – act like **semi-classical, background color sources**

The color sources are, during a given, randomly distributed
How do we treat the partons:

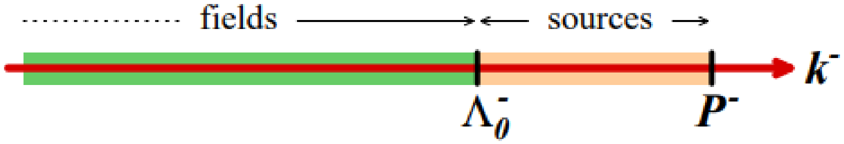
Color charge density

- Fast gluons as color currents:

$$J_a^\mu = \delta^{\mu+}\rho_a$$

- Slow gluons as gauge fields $A^\mu$

$$J^\mu A_\mu$$



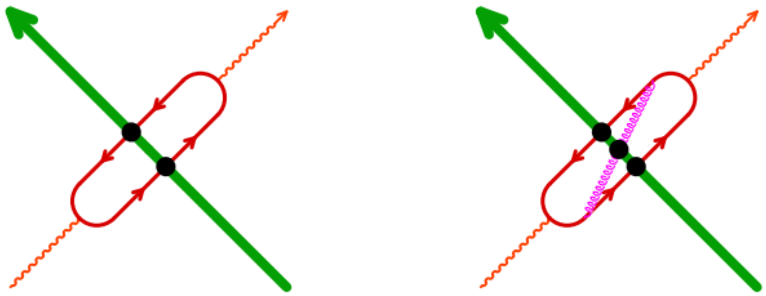
# Color Glass Condensate

Usage of CGC formalism in collisions (DIS) – at small x, the virtual photon disintegrate to a quark-antiquark pair. In such situation, the DIS cross section may be schematically written as:

$$\sigma_{DIS} = \int_0^1 dz \int d^2\mathbf{r}_\perp |\psi|^2 \sigma_{dipole}(x, \mathbf{r}_\perp)$$

LO

QED part due to the photon-quark vertex



$$\sigma_{\text{dipole}}^{\text{LO}}(x, \boldsymbol{r}_\perp) = 2 \int d^2\mathbf{b} \int [D\rho] W_{\Lambda_0^-}[\rho]\, \boldsymbol{T}_{\text{LO}}(\boldsymbol{b} + \frac{\boldsymbol{r}_\perp}{2}, \boldsymbol{b} - \frac{\boldsymbol{r}_\perp}{2})$$

$$\boldsymbol{T}_{\text{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = 1 - \frac{1}{N_c} \text{tr}\left(U(\boldsymbol{x}_\perp) U^\dagger(\boldsymbol{y}_\perp)\right)$$

Forward scattering amplitude

LO: Target provides only the static background source

# Outline of our work

Most of the calculations in CGC treat the target as a dense, background gluon field and the projectile as color-charge density

- In the case of DIS, we have a photon splitting into quark-antiquark pair – need for a quark propagator

Most works are based on so-called **eikonal approximation –** taking into account only terms with leading powers of energy. This translates to:

- Localization of the background field in the longitudinal direction $x^+ = 0$
- Only the leading component of the background field is taken ($A^-$)
- Background field is independent of $x^-$ component

$$\mathcal{A}_a^\mu(x^-, x^+, \mathbf{x}) \approx \delta^{\mu-}\delta(x^+)\mathcal{A}_a^-(\mathbf{x})$$

Feynman diagram for quark propagator within CGC formalism (gives a complicated expression).
Having the propagator, we can use the machinery of field theory to determine the cross section.

Issue: Eikonal approximation may not be enough

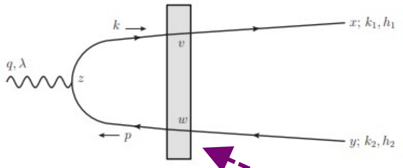
- New colliders (EIC) do not reach high enough energies
- Point of contact with TMD formalism

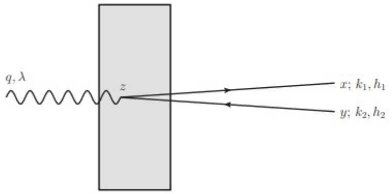

*T. Altinoluk, G. Beuf, A. Czajka, A Tymowska, Phys.Rev.D 104 (2021) 1, 014019*

Assumptions of **next-to-eikonal approximation** (Neik) – taking into account subleading powers of energy:

- The target is not point-localized and has a finite longitudinal width: $\left[-\frac{L^+}{2}, \frac{L^+}{2}\right]$
- The transverse component of the background gluon field is not neglected

Quark propagator – <u>calculated fully to Neik order</u>





Quark propagator inside the background field

Subsequently, cross section for photon → q$\bar{\text{q}}$ – <u>calculated to Neik order</u> within so called back-to-back limit

Back-to-back (correlation) limit: quark-antiquark leave the target as hadronic jets with almost opposite transverse momenta: $|\mathbf{k}| \ll |\mathbf{P}|$

$$\frac{\mathbf{P}^2}{W^2} \sim \frac{\mathbf{k}^2}{W^2}$$

Next-to-eikonal terms (subleading in energy)

$$\frac{|\mathbf{k}|^2}{|\mathbf{P}|^2}$$

Next-to-leading power (subleading in correlation limit)

$$\frac{d\sigma_{\gamma_L^* \to q_1 \bar{q}_2}}{d\text{P.S.}} = \frac{d\sigma_{\gamma_L^* \to q_1 \bar{q}_2}}{d\text{P.S.}}\Bigg|_{\text{Gen. Eik}} + \frac{d\sigma_{\gamma_L^* \to q_1 \bar{q}_2}}{d\text{P.S.}}\Bigg|_{\text{NEik corr.}} + O(\text{NNEik})$$

*T. Altinoluk, G. Beuf, A. Czajka, A Tymowska, Phys.Rev.D 107 (2023) 7, 074016*

# Outline of our work

Cross section can be further separated after calculation:

$$\left.\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\right|_{\text{Eik+NEik}} = \left.\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\right|_{\mathcal{F}^{\perp-}\mathcal{F}^{\perp-}} + \left.\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\right|_{\mathcal{F}^{\perp-}\mathcal{F}^{\perp-}\mathcal{F}^{\perp-}} + \left.\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\right|_{\mathcal{F}^{+-}\mathcal{F}^{\perp-}}$$

The subscript refers to the number of gluon insertions in the formula

Schematically:

$$\left.\frac{d\sigma}{dP.S.}\right|_{\mathcal{F}^{\perp+}\mathcal{F}^{\perp+}} \sim g^2 \int dz_1 dz_2 \langle F_j^-(z_2^+, \mathbf{b}_2) F_i^-(z_1^+, \mathbf{b}_1)\rangle \cdot \Pi^{ij}(\mathbf{P}, \mathbf{k}, \{z\})$$

Some horrible coefficients depending on momenta and positions

$$\left.\frac{d\sigma}{dP.S.}\right|_{\mathcal{F}^{\perp+}\mathcal{F}^{\perp+}\mathcal{F}^{\perp+}} \sim g^3 \int dz_1 dz_2 dz_3 \langle F_k^-(z_3^+, \mathbf{b}_2) F_j^-(z_2^+, \mathbf{b}_2) F_i^-(z_1^+, \mathbf{b}_1)\rangle \cdot \Pi^{ijk}(\mathbf{P}, \{z\})$$

Average over color → changes to an operator

**We want to make a contact point with TMD formalism – need to view the average as an correlator inside a hadron of a specific operator**

$$\langle \mathcal{O} \rangle = \lim_{P'_{tar}\to P_{tar}} \frac{\langle P'_{tar}|\hat{\mathcal{O}}|P_{tar}\rangle}{\langle P'_{tar}|P_{tar}\rangle}$$

# Outline of our work

Thus, n-point insertion would become a sum of n-point transverse-dependent operators – **n-point TMD**

$$\left.\frac{d\sigma}{dP.S.}\right|_{\mathcal{F}^{\perp}+\mathcal{F}^{\perp}+\mathcal{F}^{\perp}+} \sim g^3 \int dz_1 dz_2 dz_3 \langle F_k^-(z_3^+, \mathbf{b}_2) F_j^-(z_2^+, \mathbf{b}_2) F_i^-(z_1^+, \mathbf{b}_1)\rangle \cdot \Pi^{ijk}(\mathbf{P}, \{z\})$$

Example → change to 3-point TMD with correlation among two different transverse point

Another issue → There are indications that on Next-to-eikonal level the classical background approximation **cannot be fully trusted**

- **Time ordering of the gluon insertions becomes important at the amplitude level**. We have to re-derive the results to account for it
- Cannot assume the ordering from the start – we integrate over whole phase-space in most expressions. Easiest to do by separating the phase space into disjoint pieces

Time ordering procedure alters the momenta coefficients and, in principle, should be closer to TMD formalism (especially for three-point functions!)

# Summary

- Quantum Chromodynamics achieved great success in describing the strong interaction, but is problematic in use due to non-perturbative scales and lack of simple asymptotic states.
- In processes involving hadrons at high energy the physics can be separated into universal non-perturbative parton densities and perturbative process-dependent hard factors.
- In first approximation, only longitudinal momenta of partons are important. In more detailed prescription, transverse momenta are also important. Knowledge of transverse-dependent parton densities leads to “3D picture” of a hadron.
- Gluon number density (= dense hadron) dominates at small longitudinal momentum fraction – leads to saturation scale and invent of Color Glass Condensate
- Cross sections of processes in CGC are calculated up to next-to-eikonal approximation
- Need to be careful about classical approximations in CGC - time ordering of operators when going into more dilute regimes