From theory to automation: Decoupling renormalization for BSM Higgs decays

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Overview

How to read Feynman Diagrams

- From Lagrangians to Feynman Diagrams
- Building Representations for Processes

Higher Order Correction

- Regularization and Renormalization
- Renormalization Schemes

From Textbook to Research

- The Decoupling Renormalization Scheme
- Two Higgs Doublets

Conclusion

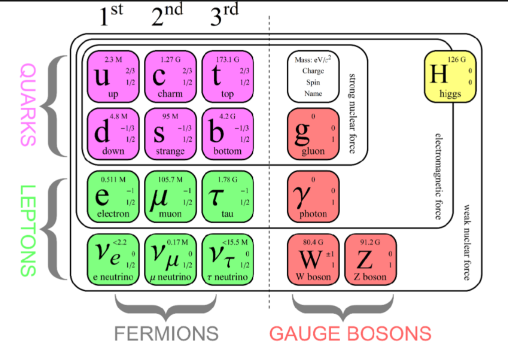
Why Quantum Field Theory?

The Standard Model

- Describes fundamental particles and interactions via QFT
- Combines quantum mechanics with special relativity
- Framework for all particle physics

Why precise calculations?

- LHC measures observables with high precision
- Need equally precise theory predictions
- QFT: systematic perturbative calculations



How to read Feynman Diagrams

Feynman Diagrams

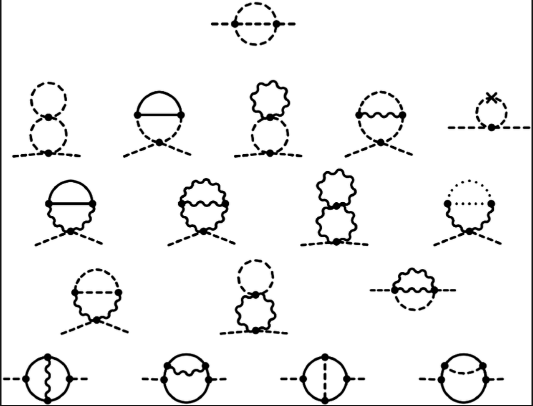
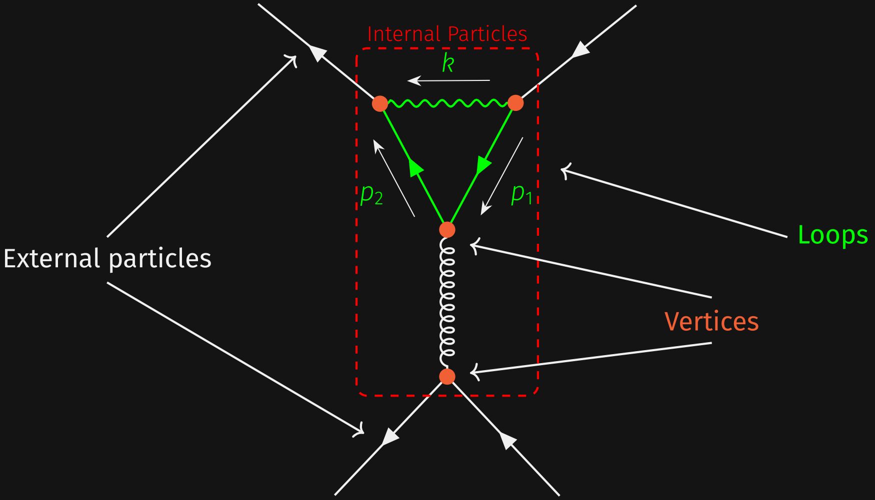


Figure take from [1]

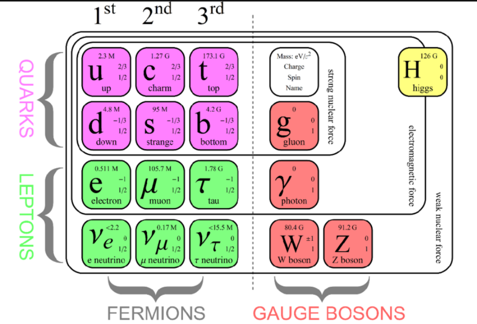
- Standard Tool in High Energy Physics
- Used to calculate cross-sections and decay widths
- But also the abundance of Dark Matter in the early universe
- Visual representation of formulas describing interactions of elementary particles

Incredients



Return of the Math Side

$$\mathcal{L}_{\mathrm{SM}} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi)$$
$$+ \sum_f \bar{\psi}_f i\gamma^\mu D_\mu \psi_f - \sum_f Y_f \left(\bar{\psi}_{fL} \Phi \psi_{fR} + \bar{\psi}_{fR} \Phi^\dagger \psi_{fL}\right)$$



Spontaneous Symmetry Breaking

$$\Phi = \begin{pmatrix} \phi^\pm \\ \frac{1}{\sqrt{2}}(v + H + i\phi_Z) \end{pmatrix}$$

How to unite theory and experiment?

- Goal: compare predictions with measurements to test the theory and search for new physics
- Experiments like the LHC collide particles at high energies
- Detectors measure: production cross sections, decay rates, branching ratios, scattering amplitudes

$$\langle T\left(\mathcal{O}(x_a)\mathcal{O}(x_b)\ldots\right)\rangle \propto \int \mathcal{D}[\Phi(x)]e^{iS[\Phi(x),\alpha]}\mathcal{O}(x_a)\mathcal{O}(x_b)\ldots$$

This is a weirdly written Gaussian integral

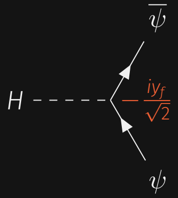
- where the free theory can be solved exactly,
- the addition of interactions requires a pertubative expansion in coupling constants

The calculation of such Greens functions is done through Feynman diagrams

Lets look at an example of a Higgs particle decaying to Leptons

$$\langle T\left(H\psi\overline{\psi}\right)\rangle \propto \int \mathcal{D}[H,\overline{\psi},\psi]e^{iS_{\text{free}}[H,\overline{\psi},\psi;\lambda,Y_f]}\left(-\frac{iy_f}{\sqrt{2}}\overline{\psi}H\psi - 3i\lambda\, vH^3 - \frac{i\lambda}{4}H^4\right)\left(H\psi\overline{\psi}\right)$$

The Greens functions are calculated using Wicks theorem: result is the sum of all pair-wise, connected contractions



Propagators

$\overline{HH}$ = - - - - - - - - -

$\overline{\psi(x)\overline{\psi}(y)}$ = y ——▶—— x

Vertices

Fermion-Higgs: $-\frac{iy_f}{\sqrt{2}}$

Triple Higgs: $-3i\lambda v$

Quartic Higgs: $-\frac{i\lambda}{4}$

Feynman Rules

1. Determine all non-vanishing and connected contractions and vertex factors
2. Each line gets an corresponding Propagator
3. Each vertex gets its appropriate factor
4. Momentum is conserved in each vertex
5. Undetermined momenta are integrated over
6. Don't forget statistical factors

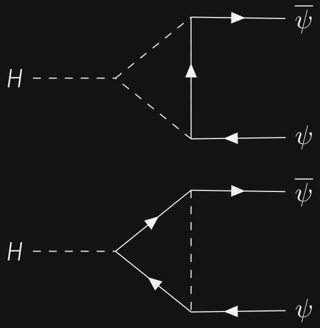
Higher Order Correction

What are Loops?

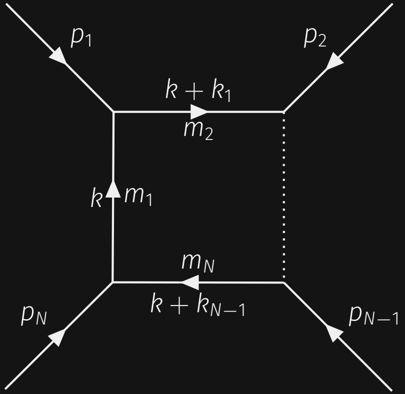
In the last example we constructed the **tree-level** contribution of the Higgs decay to leptons. Expanding the Path integral further results in more possibilites to internal contractions

$\Rightarrow$we get loops in diagrams

Loop diagrams are corrections to processes, and improve the result!



Why we deal with infinities?



$$\sim \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{A(k, k_i, m_i)}{(k^2 - m_1^2)((k+k_1)^2 - m_2^2) \cdots ((k+k_{N-1})^2 - m_N^2)}$$
$$\sim \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{A(k, k_i, m_i)}{k^{2N}}$$

There are two observations

- if $A = 1$, the tadpole and bubble diagrams diverge but triangle, box and diagrams with more external legs converge, and
- if $A = A(k, k_i, m_i)$, the last condition worsens!

How do we get reliable predictions?

This is a two step process

1. Regularization: extracting the divergence from the expressions
   - Cut-off regularization
   - Dimensional regularization
2. Renormalization: redefining theory parameters

Renormalization is fundamentally more important and is generally done in renormalized pertubation theory

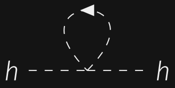
$$\left.\begin{aligned} \phi_0 &= \sqrt{Z_\phi}\phi \\ m_0^2 &= m^2 + \delta m^2 \\ \lambda_0 &= \lambda + \delta\lambda \end{aligned}\right\} \quad \mathcal{L} = \mathcal{L}_R + \mathcal{L}_{CT}$$

- we need to calculate counter terms, and
- we need to give a presecripton to fix the counter terms

The counter terms absorb the divergent terms we isolated during regularization!!!

How does a simplified calculation look like?

$$-i\Pi = \underbrace{\frac{i\lambda}{32\pi^2} m^2 \left(\frac{1}{\epsilon} - \log\left(\frac{m^2}{\mu^2} \right) + 1 \right)} \underbrace{-i((Z-1)p^2 + (Z-1)m^2 + Z\delta m^2)}$$



We can define different renormalization condiditions/schemes to fix the counter terms

On-Shell Scheme

$$\left.\frac{\partial \Pi(p^2)}{\partial p^2}\right|_{p^2=m^2} = 0$$

$$\Pi(p^2)|_{p^2=m^2} = 0$$

$\overline{\textbf{MS}} - Scheme$

Define the counter terms such that it absorbes only the divergent terms

From Textbook to Research

Why to care about renormalization schemes and more?

The OS- and $\overline{\text{MS}}$-scheme are so far standard in higher order loop calculations

The most common scheme ($\overline{\text{MS}}$) suffers from large contributions due to the presence of unknown BSM particles, which

- introduces large uncertainties,
- and makes the calculation problematic

The Higgs sector became more and more interesting

- measurement became increasingly more precise,
- many unsolved problems can be described by an extension of the scalar sector
  $\Rightarrow$flavour problems,..., baryogenesis

There is great potential in tightly constraining BSM models through the Higgs sector

- experimental precision must be matched by theory
- large variety of models must be worked through

Physical Intuition: We do not expect that quantum corrections including heavy BSM particles influence SM observables (Applequist-Carazzone Theorem)

Theoretically: We want to construct a renormalization scheme that differentiates between BSM and SM parameters and, especially for low-energy observables, do not mix them!

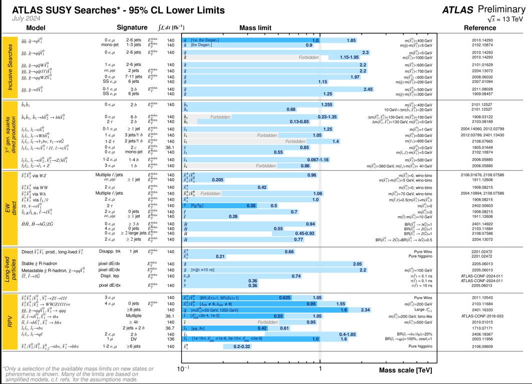


Figure take from [2]

How to setup the Decoupling Scheme?

We separate SM-like, P_{SM}, and BSM, P_{BSM}, parameters

SM-like parameters are fixed with the renormalization condition

$$P^{\text{dec}}_{\text{BSM}} = P^{\overline{\text{MS}}}_{\text{SM}}$$

How can we get it in a more usable way?

$$P_0 = P^{\text{dec}}_{\text{BSM}} + \delta P^{\text{dec}}_{\text{BSM}} = P^{\text{OS}}_{\text{BSM}} + \delta P^{\text{OS}}_{\text{BSM}}$$

$$P_0 = P^{\overline{\text{MS}}}_{\text{SM}} + \delta P^{\overline{\text{MS}}}_{\text{SM}} = P^{\text{OS}}_{\text{SM}} + \delta P^{\text{OS}}_{\text{SM}}$$

Expression in terms of renromalization constants

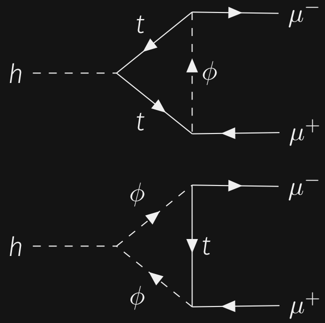
$$\delta P^{\text{dec}}_{\text{BSM}} = \delta P^{\overline{\text{MS}}}_{\text{SM}} + \delta P^{\text{OS}}_{\text{BSM}} - \delta P^{\text{OS}}_{\text{SM}}$$

A very short recap

We had one Leptoquark with an arbitrary mass m_{LQ}. Non decoupling effects are introduced through the first diagram. We also had the set of lepton renormaliztion constants

$$\delta Z_L, \delta Z_l, \delta Z_m$$

that removed the decoupling effect!



There are also non-decoupling contributions that cancel within the renormaliztion constants

Why the THDM? Problems with the Standard Model

The Standard Model is remarkably successful, but...

Open questions the THDM can address:

- Flavor structure and mass hierarchy: Why do fermions have such different masses?
  - SM provides no explanation for the pattern of Yukawa couplings
  - THDM allows different Higgs doublets to couple to different fermion sectors
  - Can provide texture for understanding flavor physics and CP violation
- Matter-antimatter asymmetry: Why is there more matter than antimatter?
  - SM CP violation is too weak to explain the observed asymmetry
  - THDM provides additional sources of CP violation needed for baryogenesis
- Dark matter: What is the 85% of matter we cannot see?
  - SM has no dark matter candidate
  - THDM can provide stable dark matter candidates (e.g., inert doublet model)

A phenomenological interesting model

We extend the SM with a second doublet Φ_2 with the same quantum numbers

$$
\begin{aligned}
V_{2\text{HDM}} = & \, m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2 \\
& - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}\right) \\
& + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) \\
& + \left(\frac{\lambda_5}{2}\left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right)
\end{aligned}
$$

If the mass matrix has a negative Eigenvalue both scalars develop a vacuum expectation value

$$
\langle \Phi_1 \rangle_0 = \begin{pmatrix} 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix} \qquad \langle \Phi_2 \rangle_0 = \begin{pmatrix} 0 \\ \frac{v_2}{\sqrt{2}} \end{pmatrix}
$$

The parametrization of the field is therefore:

$$
\Phi_i = \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \rho_i + i\varphi_i) \end{pmatrix}
$$

What is the spectrum of the THDM?

We plug in the parametrization and extract the quadratic terms of the potential:

$$V^2_{2\text{HDM}} = \frac{1}{2}\begin{pmatrix}\rho_1 & \rho_2\end{pmatrix} M^2_\rho \begin{pmatrix}\rho_1 \\ \rho_2\end{pmatrix} + \frac{1}{2}\begin{pmatrix}\varphi_1 & \varphi_2\end{pmatrix} M^2_\varphi \begin{pmatrix}\varphi_1 \\ \varphi_2\end{pmatrix} + \begin{pmatrix}\phi_1^+ & \phi_2^+\end{pmatrix} M^2_{\phi^\pm} \begin{pmatrix}\phi_1^- \\ \phi_2^-\end{pmatrix}$$

Two CP-even fields with masses M_h and M_H

$$\begin{pmatrix}\rho_1 \\ \rho_2\end{pmatrix} = R_\alpha \begin{pmatrix}H^0 \\ h^0\end{pmatrix}$$

A Nabu-Goldstone field and a CP-odd field with mass M_{A0}

$$\begin{pmatrix}\varphi_1 \\ \varphi_2\end{pmatrix} = R_\beta \begin{pmatrix}G^0 \\ A^0\end{pmatrix}$$

A Nabu-Goldstone field and a charged field with mass $M_{H\pm}$

$$\begin{pmatrix}\phi_1^\pm \\ \phi_2^\pm\end{pmatrix} = R_\beta \begin{pmatrix}G^\pm \\ H^\pm\end{pmatrix}$$

What about interactions with fermions

$$\mathcal{L}_{\text{Yukawa}} = -\overline{Q}\big(Y_{U,1}(i\sigma^2\Phi_1) + Y_{U,2}(i\sigma^2\Phi_2)\big)u - \overline{Q}(Y_{D,1}\Phi_1 + Y_{D,2}\Phi_2)d - \overline{L}(Y_{L,1}\Phi_1 + Y_{L,2}\Phi_2)l + \text{h.c.}$$

Classification of the THDM in terms of is lepton interactions:

	u_i	d_i	l_i
Type I	Φ_2	Φ_2	Φ_2
Type II	Φ_2	Φ_1	Φ_1
lepton-specific	Φ_2	Φ_2	Φ_1
flipped	Φ_2	Φ_1	Φ_2

Is there a decoupling Limit?

What to expect?

- one Higgs like state $M_h \sim \mathcal{O}(v)$
- heavy other scalars $M_H, M_{A^0}, M_{H^\pm} \gg \mathcal{O}(v)$

To see that we first realize there is only one dimensionful parameter we can change! We define

$$m_{\mathrm{D}}, m_{\mathrm{L}}, m_A \quad \text{and} \quad m_{\mathrm{S}}^2 \sim m_{12}^2$$

The masses of the CP-even scalars become

$$M_{H,h}^2 = \frac{1}{2}\left[m_{\mathrm{S}}^2 \pm \sqrt{m_{\mathrm{S}}^4 - 4m_{\mathrm{A}}^2 m_{\mathrm{L}}^2 - 4m_{\mathrm{D}}^4}\right] \qquad \cos^2(\beta - \alpha) = \frac{m_{\mathrm{L}}^2 - M_h^2}{M_H^2 - M_h^2}$$

The trajectory towards decoupling is very much convoluted

How do we finally parametrize the THDM?

Originally we describe the THDM with the 8 scalar, 9 fermion (and 2 gauge) parameters

$$
\begin{aligned}
m_{11}^2, m_{22}^2, m_{12}^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, v_1, v_2 \\
Y_U, Y_D, Y_L \\
g_1, g_2
\end{aligned}
$$

In principle we can choose any convenient set of parameters.
My personal favourite choice:

$$
\begin{aligned}
t_1, t_2, m_{12}^2, M_h, M_H, \lambda_3, \lambda_4, \lambda_5, v, \tan(\beta) \\
m_U, m_D, m_L \\
e, m_W
\end{aligned}
$$

Why not use the mixing angle?

For general calculations having the mixing angle α instead of λ_3 is benefitial because

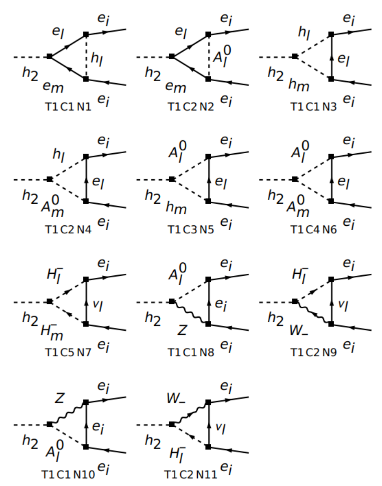
- expressions get simple,
- renormaliztion become easier to order, and
- numerical calulations become more stable.

BUT, our consideration of the decoupling limit suggests to choose a set with minimal parameters that vary in the decoupling limit.

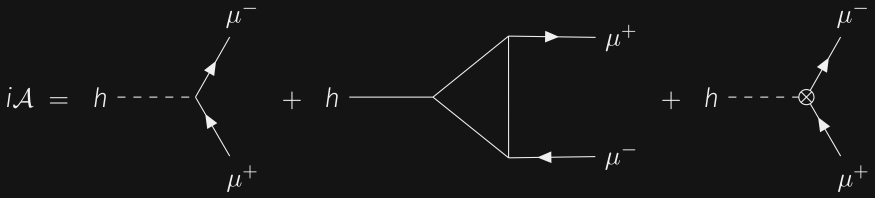
What do we require for a one-loop calculation?

For observables we calculate the diagrams on the right

The first two diagrams behave in a non-decoupling way and diverge logarithmically with increasing scalar particle mass



What happens with the divergencies?

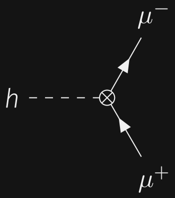


Luckily we setup the renormaliztion scheme that the divergencies are taken care of!
Exactly the same as in the Leptoquark model!

Amazing, the renormaliztion scheme works in the same way as explored for an easier model. But it is dangerous to think that this is the end!!!

Lets have a better look at the Yukawa Lagrangian!

$$\mathcal{L}_{\text{Yukawa}} \supset -\frac{m_\mu}{v\cos(\beta)}\left(\cos(\alpha)H^0 - \sin(\alpha)h^0\right)\bar{l}l$$



$$\supset \text{Lepton Stuff} + \frac{m_\mu \sin(\beta)}{v\cos(\beta)}\left(\frac{1}{2}\delta Z_{Hh} - \delta\alpha\right)\cot(\alpha)$$

The mixing field renormaliztion constant captures the initial change to the other scalar and subsequent decay

$$\delta Z_{Hh} = \frac{2\Sigma_{Hh}(M_h^2)}{M_H^2 - M_h^2}$$

The mixing angle renormaliztion constant has an similar effect. But this is just a dummy parameter and must be expressed through the input parameters!

Where do we get the mixing angle from?

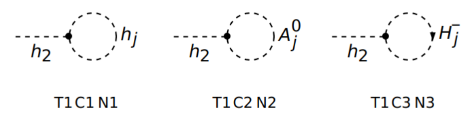
After rotating to the physical basis the off-diagonal part of the mass matrix satisfies

$$0 = \frac{m_{12}^2 \sin(2(\alpha-\beta))}{\sin(2\beta)} + \lambda_1 v^2 \sin(2\alpha)\cos(\beta)^2 - \lambda_2 v^2 \sin(2\alpha)\sin(\beta)^2 - \frac{1}{2}\lambda_{345} v^2 \cos(2\alpha)\sin(2\beta) + \frac{t_h \sin(2\alpha)\cos(\alpha-\beta)}{v\sin(2\beta)} + \frac{t_H \sin(2\alpha)\sin(\alpha-\beta)}{v\sin(2\beta)}$$

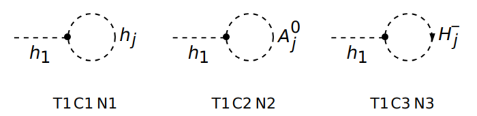
$$\Longrightarrow \delta\alpha \supset \frac{\delta t_h \sin(2\alpha)\cos(\alpha-\beta)}{v\sin(2\beta)} + \frac{\delta t_H \sin(2\alpha)\sin(\alpha-\beta)}{v\sin(2\beta)}$$

What might happen?

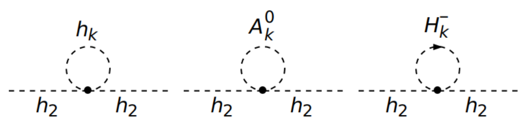
$\delta t_h \rightarrow$



$\delta t_H \rightarrow$



$\delta Z_{Hh} \rightarrow$



I hope/expect that the tadpole renormaliztion constants combine to remove the non-decoupling terms from the mixing field renormaliztion constant such that the total amplitude decouples!

There are many tools to automatize calculations, that is evaluating Feynman diagrams and calculating observables

- SARAH, FeynArts, FormCalc, LoopTools
- HDECAY, 2HDECAY, FeynHiggs,...

Problem: These tools are very model dependent

Develop FlexibleSUSY and the extention FlexibleDecay to extend the available models for automatized high-precision calculation of model properties!

FlexibleSUSY and FlexibleDecay

We have seen a nice example of the tedious math to show one simple result
No one wants to do these calculation by hand!

We want to develop FlexibleSUSY and FlexibleDecay to make the automatic high-precision calculation of SM parameters and observables easily available to anyone.

FlexibleSUSY is a spectrum-generator generator:

- → it generates codes for a large models
- → state-of-the-art Higgs mass prediction

FlexibleDecay adds the ability to calculate Higgs decays:

- → higher order SM effects are taken into account
- → the BSM effects are renormalized in the decoupling renormalization scheme

Conclusion

Summary and Outlook

- For precise prediction of observables we need regularization and renormalization
  1. isolate the divergence in loop diagrams $\Rightarrow$Regularization
  2. systematically define new parameters to absorb divergencies $\Rightarrow$Renormalization
- The Higgs Boson is important to constrain BSM models
- The decoupling renormalization scheme does not spoil higher order corrections with large BSM contributions
- FlexibleSUSY provides a framework to automatize the analyzis of many BSM models

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Backup Slides

How to deal with integrals?

We first need to get a grip on the divergent integrals, this process is called **Regularization**.

$$A(a,\Delta) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{(k^2 - \Delta + i\epsilon)^a} = i\,(-1)^a \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{(k^2 + \Delta)^a}$$

Cut-off Regularization:

We impose an upper limit to the Momentum Integral

$$A(a,\Delta) = \frac{i\,(-1)^a}{16\pi^2} \int_0^{\Lambda^2} \mathrm{d}k^2 \, \frac{k^2}{(k^2 + \Delta)^a}$$

- cut-off breaks momentum translation invariance
- there could be problems with gauge invariance of the final result

Dimensional Regularization:

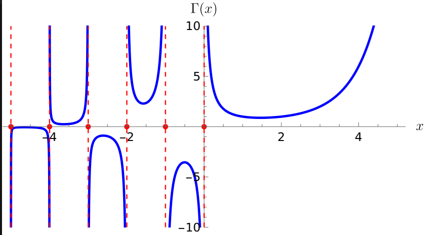
We assume that the space-time dimension is d. We solve the integral in for $d < 2a$ and then analytically continue the result to $d = 4$

$$\int \frac{\mathrm{d}^4 k}{(2\pi)^2} \longrightarrow \tilde{\mu}^{4-d} \int \frac{\mathrm{d}^d k}{(2\pi)^d}$$

- a non-integer space-time dimension seems artificial, and
- its non-trivial to extend the γ-matrices (γ_5-Problem)

How to take the limit?

$$A(a,\Delta) = \frac{i}{16\pi^2}\frac{\Gamma(a-\frac{d}{2})}{\Gamma(a)}\left(\frac{\Delta}{4\pi\tilde{\mu}^2}\right)^{\frac{d}{2}-2}(-\Delta)^{2-a}$$



The Gamma function is a analytic function with simple poles at negative integers. In the case $a-\frac{d}{2}\leq 0$ we expand $\Gamma(a-\frac{d}{2})$

$$A(1,\Delta) = \frac{i}{16\pi^2}\Delta\left(\frac{1}{\epsilon} - \log\left(\frac{\Delta}{\mu^2}\right) + 1\right)$$

$$A(2,\Delta) = \frac{i}{16\pi^2}\left(\frac{1}{\epsilon} - \log\left(\frac{\Delta}{\mu^2}\right)\right)$$

We encapsuled the divergence in the term $\frac{1}{\epsilon}$

How to remove the infinity?

Experimentally: Fundamental processes are local and relativistic invariant up to some energy E_{max} and relations between parameters and observables cannot depend on physics at much higher scales!

Theoretically this means that relations between physical quantites as well as observables in a theory described by $\mathcal{L}(\phi_0; m_0, \lambda_0)$, CAN NOT depend on the regularization

The following procedure should result in a unambiguous and testable predictions of the theory:

1. replace auxiliary parameters m_0, λ_0 by renormalized quantities m, λ
2. observables f expressed through physical quantities m, λ are independent of regularization
   $f = f(m, \lambda) = f(m_0(m, \lambda), \lambda_0(m, \lambda)) \Rightarrow$Regularization independent

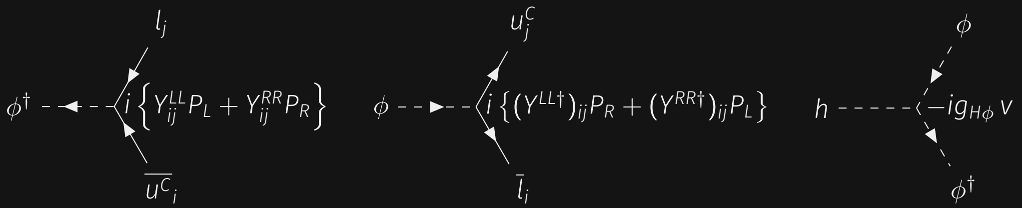
A Model to test the Decoupling scheme

We investigate the S1-Leptoquark model to see the Decoupling scheme in action! We need only one additional field, the Leptoquark ϕ which transforms as $(3, 1, -\frac{1}{3})$

We focus on the decay of a Higgs Boson into Leptons.

$$\mathcal{L}_{Y\phi} = Y^{LL}_{ij}\,(\overline{Q^C}_i^T\, i\sigma^2 L_j)\phi^\dagger + Y^{RR}_{ij}\,\overline{q^C_u}_i l_j \phi^\dagger + \text{h.c.}$$

$$\mathcal{L}_{H\phi} = -g_{H\phi}(\Phi^\dagger\Phi)\,\mathrm{Tr}\{\phi^\dagger\phi\}$$



How to calculate one-loop diagrams effectively?

For one-loop preocesses the general structure of loop integrals can be expressed in terms of Passarino-Veltman functions

$$B_{0;\mu;\mu\nu} = \frac{\tilde{\mu}^{4-D}}{i\pi^{\frac{D}{2}}} \int d^D k \frac{1; k_\mu; k_\mu k_\nu}{(k^2 - m_1^2)((k+k_1)^2 - m_2^2)}$$

$$C_{0;\mu;\mu\nu} = \frac{\tilde{\mu}^{4-D}}{i\pi^{\frac{D}{2}}} \int d^D k \frac{1; k_\mu; k_\mu k_\nu}{(k^2 - m_1^2)((k+k_1)^2 - m_2^2)((k+k_2)^2 - m_3^2)}$$

The tensor structure can be used to reduce difficulty of Integrals to scalar integrals

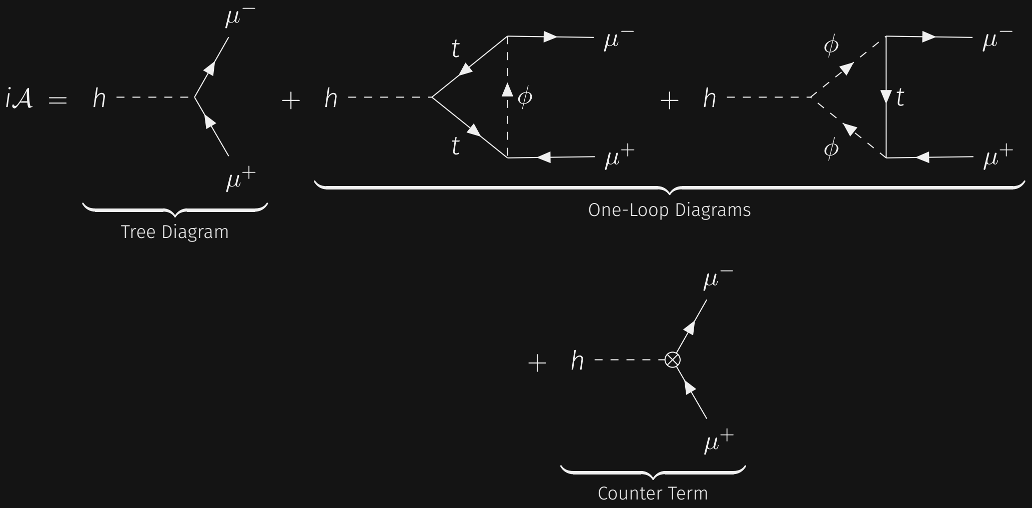
$$B^\mu = k_1^\mu B_1 \qquad C^\mu = k_1^\mu C_1 + k_2^\mu C_2$$

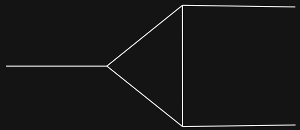
$$B^{\mu\nu} = \eta^{\mu\nu} B_{00} + k_1^\mu k_1^\nu B_{11} \qquad C^{\mu\nu} = \eta^{\mu\nu} C_{00} + (k_1^\mu k_2^\nu + k_2^\mu k_1^\nu) C_{12} + k_1^\mu k_1^\nu C_{11} + k_2^\mu k_2^\nu C_{22}$$

$$B_0(p, m_1, m_2) = \frac{1}{\epsilon} - \log\left(\frac{m_1^2}{\mu^2}\right) + 1 - \int_0^1 dx \log\left(1 + \frac{\bar{x}}{x}\alpha_1 - \bar{x}\beta_1\right), \quad \alpha_1 = \frac{m_2^2}{m_1^2}, \beta = \frac{p^2}{m_1^2}$$

What do we have to calculate?

We need to calculate the Amplitude $\mathcal{A}$ for the process $H \to l_i^+ \, l_j^-$





$\sim F_L\, P_L + F_R\, P_R$

In order to see the effects of the Decoupling scheme we need to analyze the form factors

$$F_L^1 = \frac{3m_t}{16\pi^2 v}\left\{\left[B_0\left(m_\mu, m_\phi, m_t\right) + 2m_t^2 C_0(a) + m_\mu^2 C_1(a) + m_H^2 C_0(a)\right]\left(Y^{RR\dagger}\right)_{23} Y_{32}^{LL} + \cdots\right\}$$

$$F_L^2 = \frac{3g_{H\phi}v}{16\pi^2}\left\{m_\mu[C_0(b) + C_1(b) + C_2(b)]\left(Y^{LL\dagger}\right)_{23} Y_{32}^{LL} - m_\mu C_2(b)\left(Y^{RR\dagger}\right)_{23} Y_{32}^{RR} + m_t C_0(b)\left(Y^{LL\dagger}\right)_{23} Y_{32}^{RR}\right\}$$

1. due to dimensional regularization
   - $\Rightarrow$ solved by choosing an appropriate renormalization scheme
2. due to large leptoquark mass
   - $\Rightarrow$ solved by the decoupling scheme

How to calculate renormalization constants?

Generally one can write self-energies as

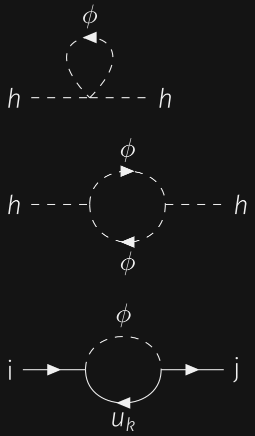
$$\Pi(p^2) = \Pi^{\text{BSM}}(p^2) + \Pi^{\text{SM}}(p^2) \longrightarrow \delta P^{\text{dec}}_{\text{BSM}} = \delta P^{\overline{MS}}_{\text{SM}} + \delta P^{\text{OS}}_{\text{BSM}} - \delta P^{\text{OS}}_{\text{SM}}$$

Beauty of this model: the SM constribution in the OS difference cancel, leaving only the BSM contributions to consider

$$\delta Z^{\text{dec}}_L = \delta Z^{\overline{MS}}_L + \frac{3}{16\pi^2}\left\{\left(Y^{LL\dagger}\right)_{23} Y^{LL}_{32}\, B_1(m_\mu, m_t, m_\phi) + \ldots\right\}$$

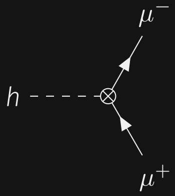
$$\delta Z^{\text{dec}}_l = \delta Z^{\overline{MS}}_l + \frac{3}{16\pi^2}\left\{\left(Y^{RR\dagger}\right)_{23} Y^{RR}_{32}\, B_1(m_\mu, m_t, m_\phi) + \ldots\right\}$$

$$\delta Z^{\text{dec}}_m = \delta Z^{\overline{MS}}_m + \frac{3m_t}{16\pi^2}\left(Y^{RR\dagger}\right)_{23} Y^{LL}_{32}\, B_0(m_\mu, m_t, m_\phi) - \frac{3}{32\pi^2}\left(\left(Y^{LL\dagger}\right)_{23} Y^{LL}_{32} + \left(Y^{RR\dagger}\right)_{23} Y^{RR}_{32}\right) B_1(m_\mu, m_t, m_\phi)$$



Putting everything together

First we need the expression for the counter term



$$\supset -\frac{im_\mu}{v}\left\{\frac{1}{2}\left(\delta Z_L^{\mathrm{dec}} + \delta Z_l^{\mathrm{dec}\dagger}\right)P_L + \delta Z_m^{\mathrm{dec}} P_L + \ldots\right\}$$

$$= -\frac{m_\mu}{2v}\left(\delta Z_L^{\overline{MS}} + \delta Z_l^{\overline{MS}}\right) - \frac{m_\mu}{2}\delta Z_m^{\overline{MS}} - \frac{3m_t}{16\pi^2 v}\left(Y^{RR\dagger}\right)_{23} Y_{32}^{LL}\, B_0(m_\mu, m_t, m_\phi) + \ldots$$

- by construction the loop divergencies are removed
- and the logarithms with the BSM mass are removed

What happens at different BSM masses?