Hierarchical Cosmology Constraints Through Strong Gravitational Lensing

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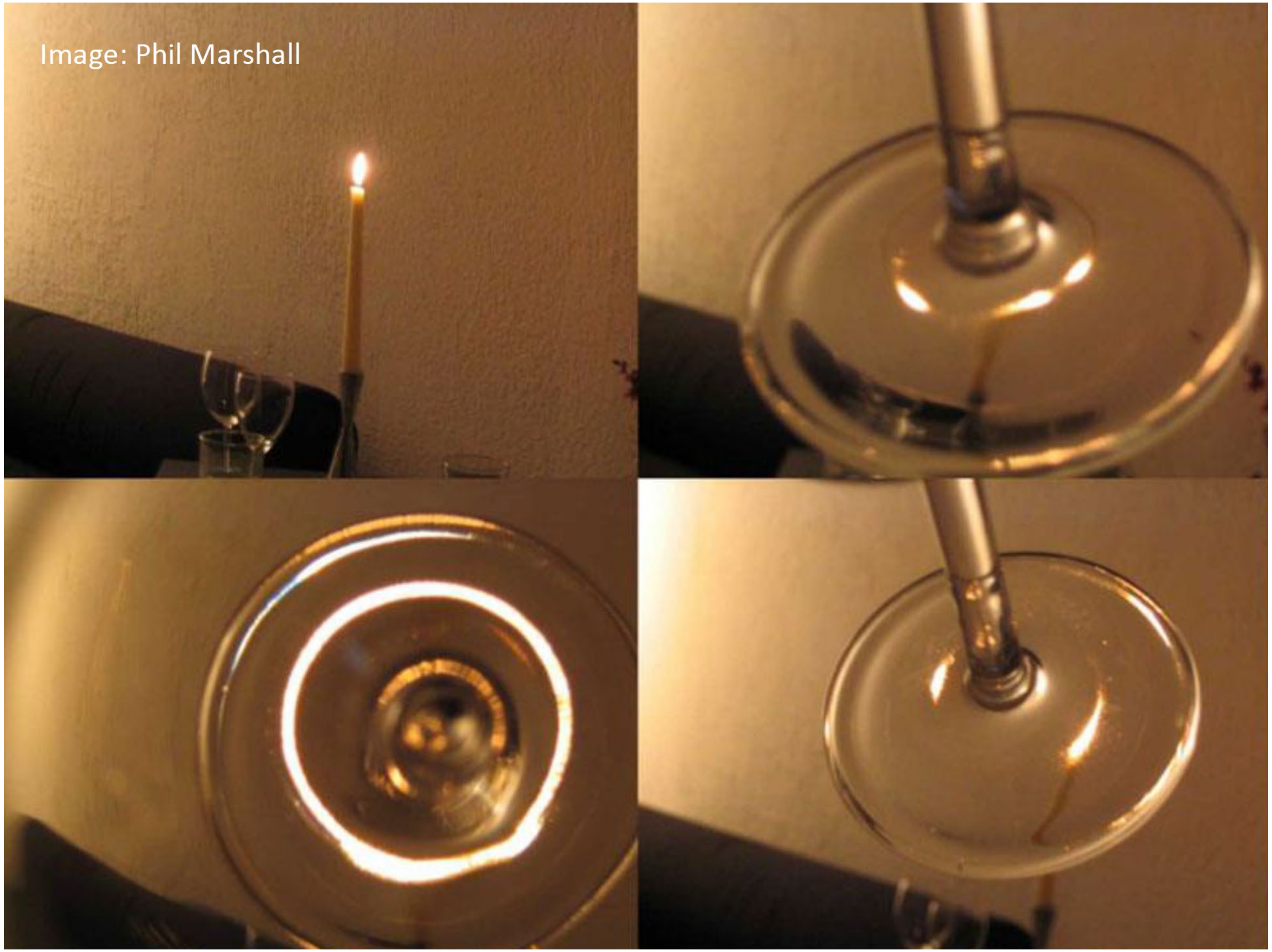
(National Centre for Nuclear Research)

Supervisor: Prof. Marek Biesiada

Contents

1. Backgrounds

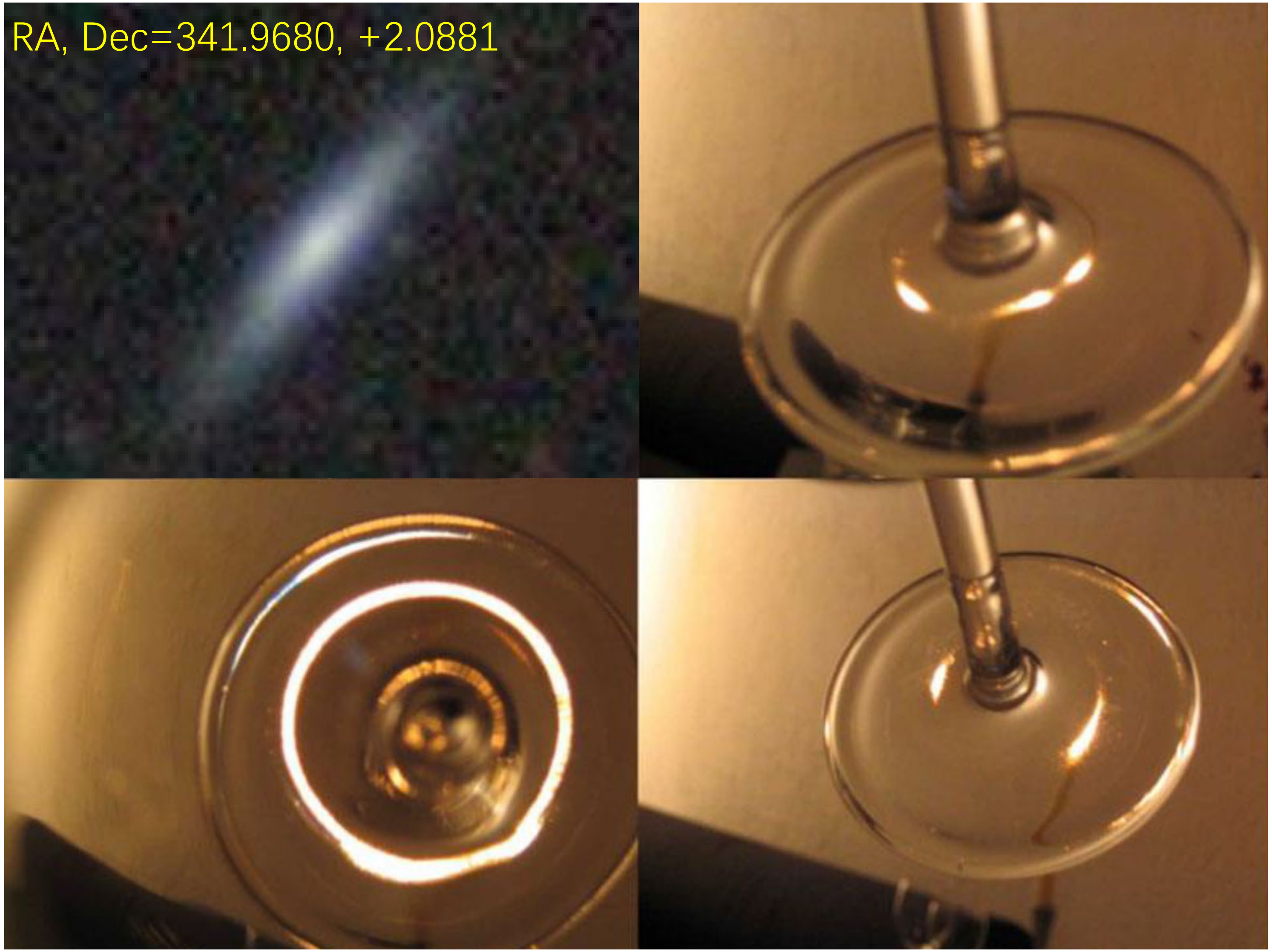
Lensing Effects



Different configurations

Different lensed images

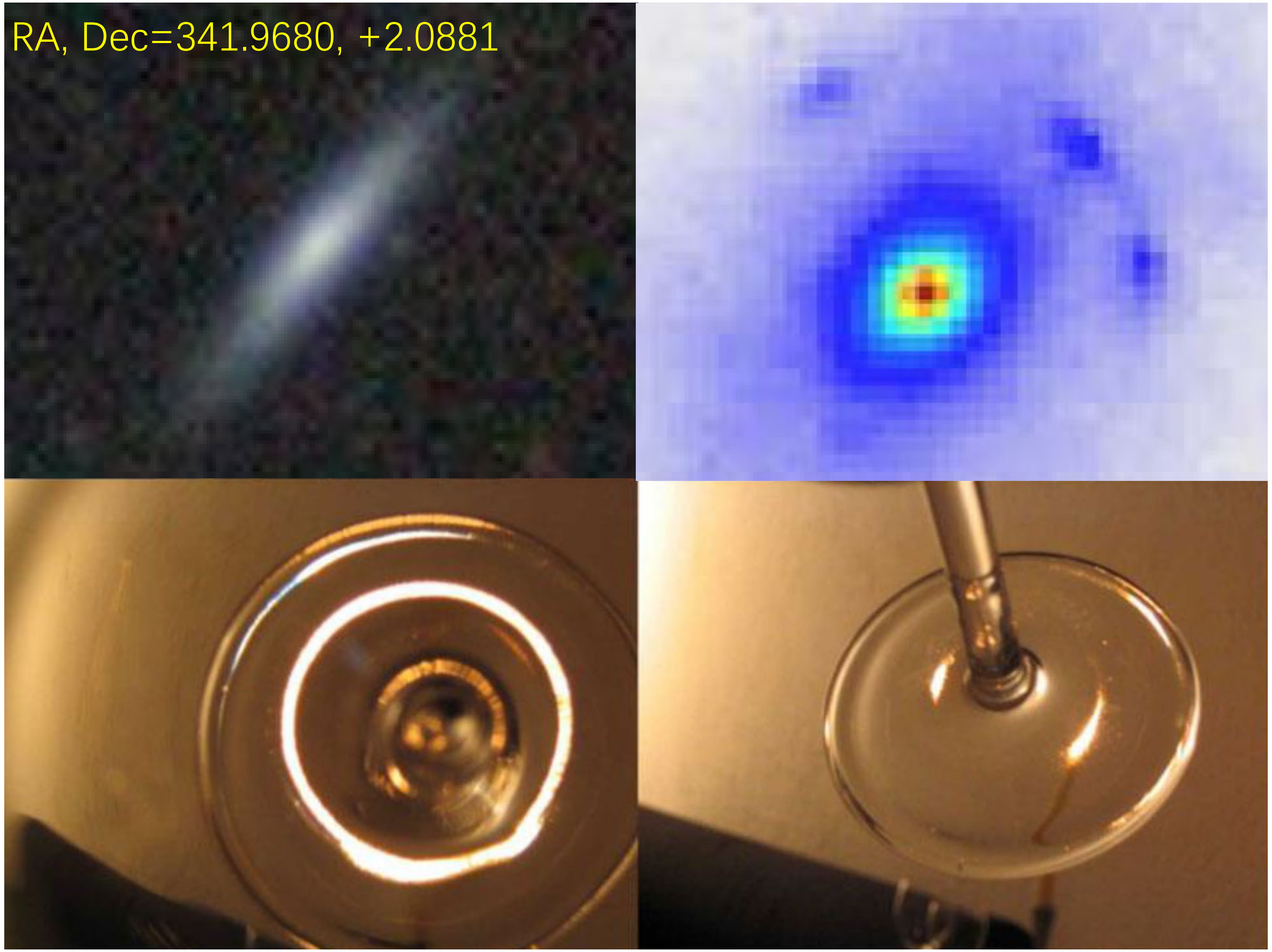
Lensing Effects in the Universe

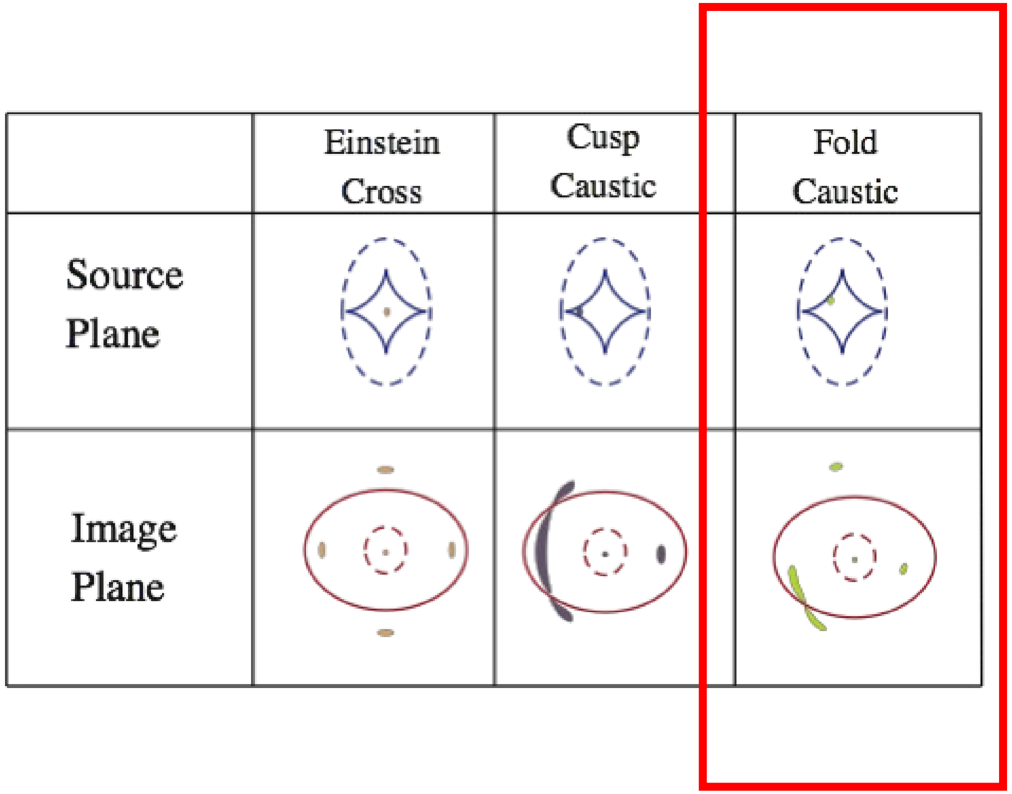


General Relativity

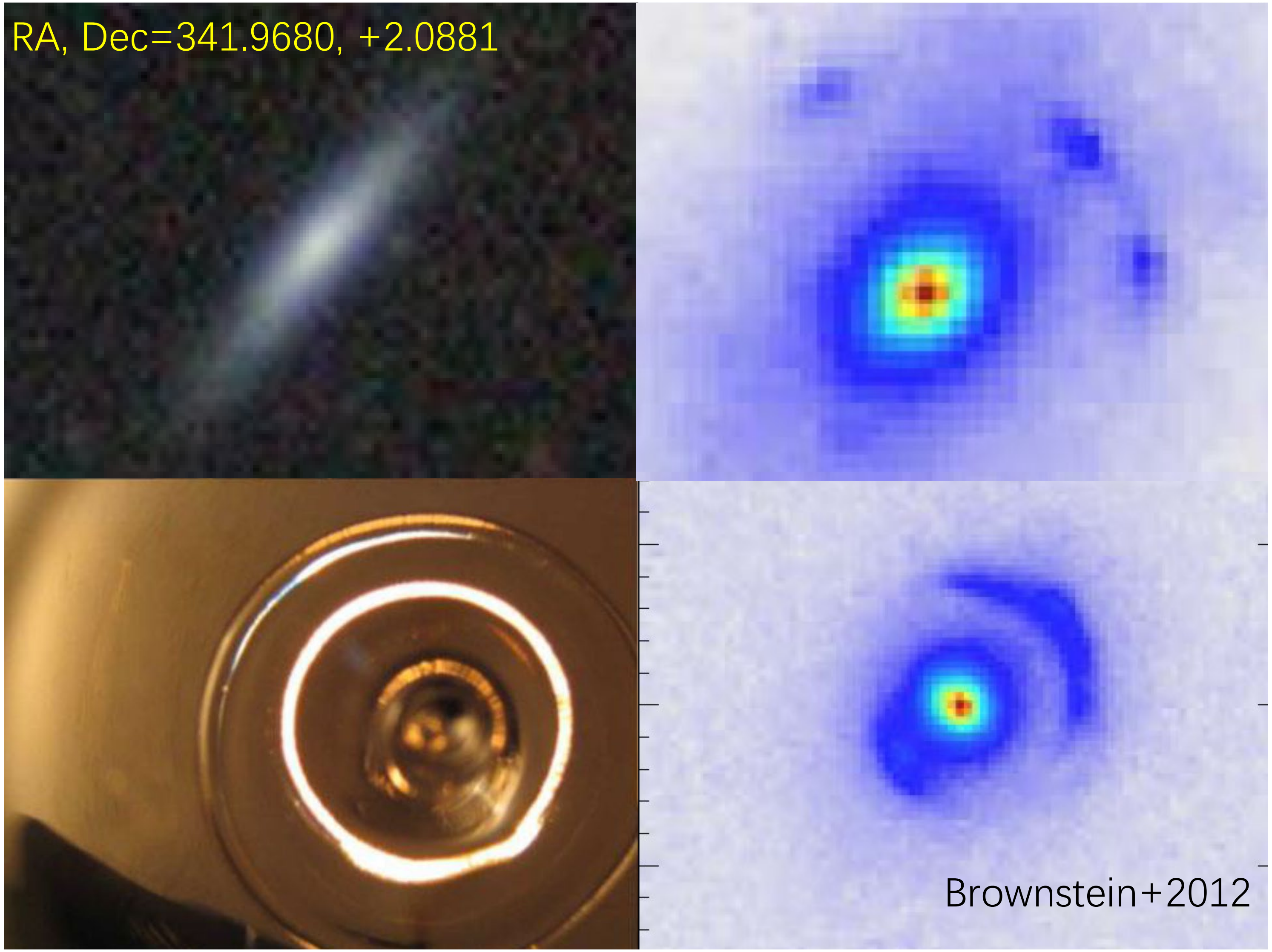
Massive galaxies $\Longrightarrow$ Lenses

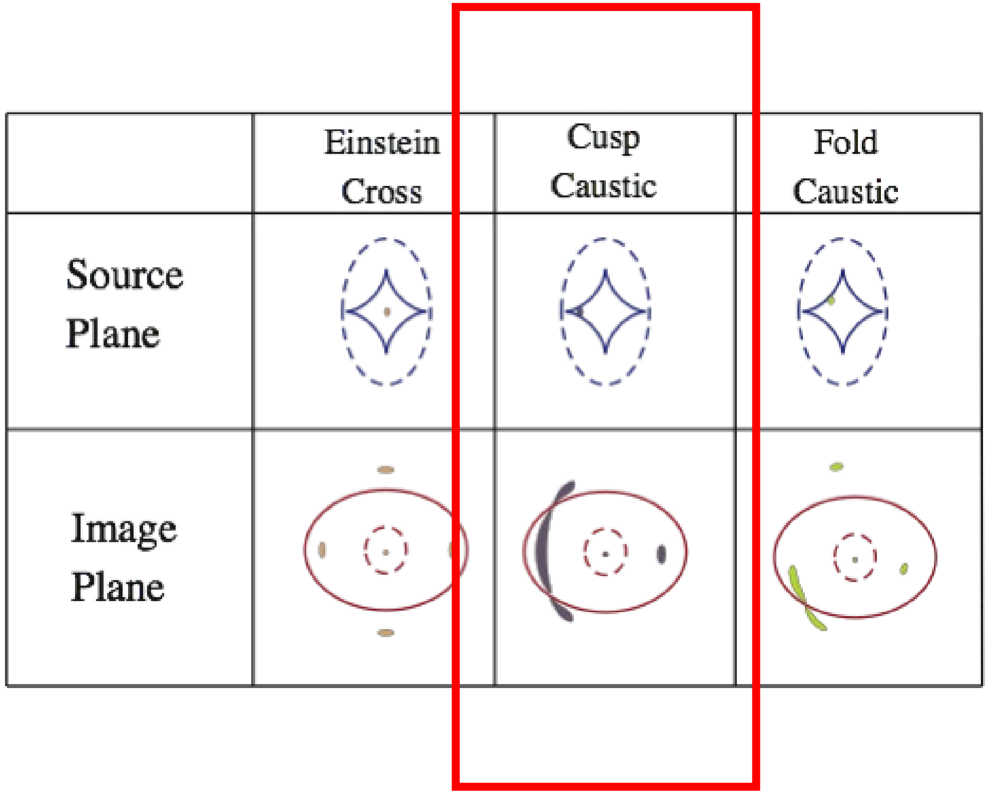
Lensing Effects in the Universe



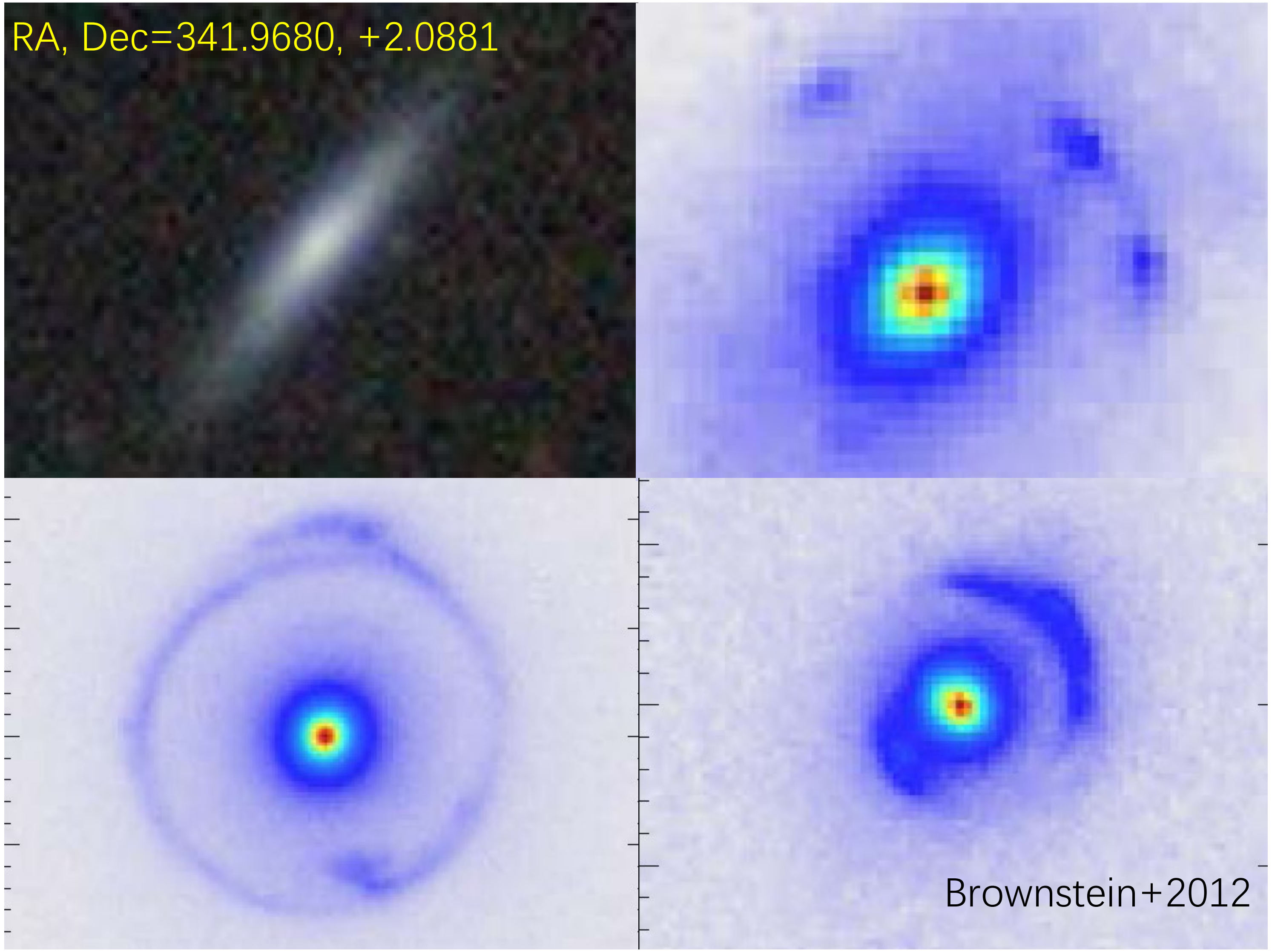


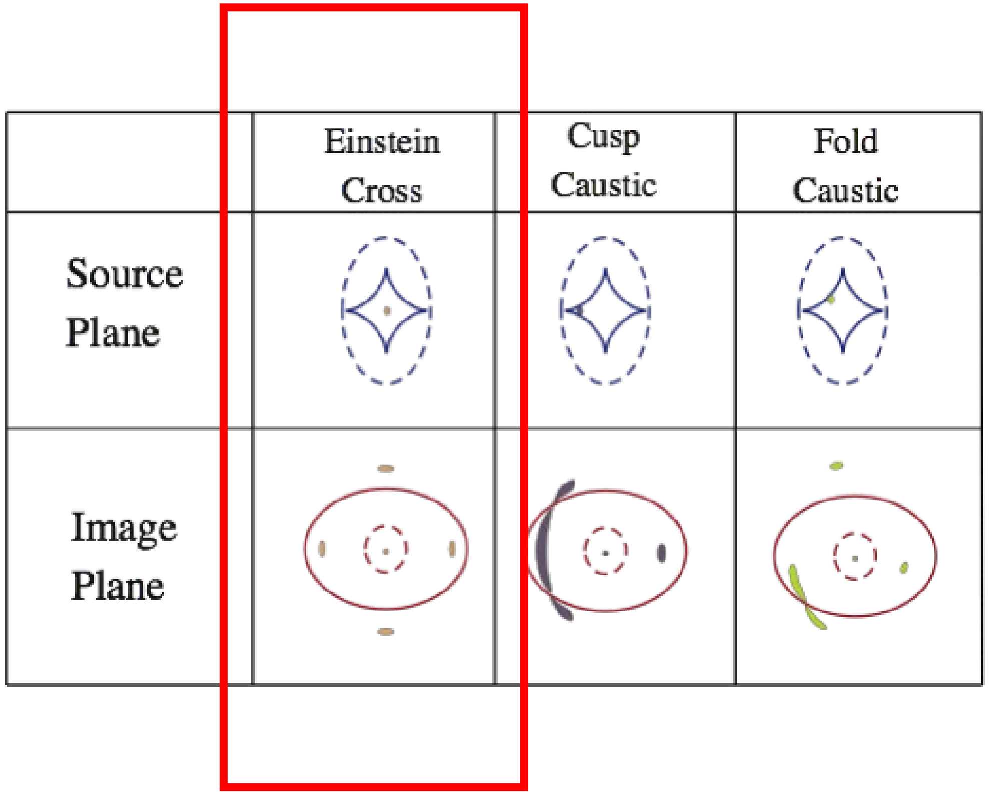
Lensing Effects in the Universe



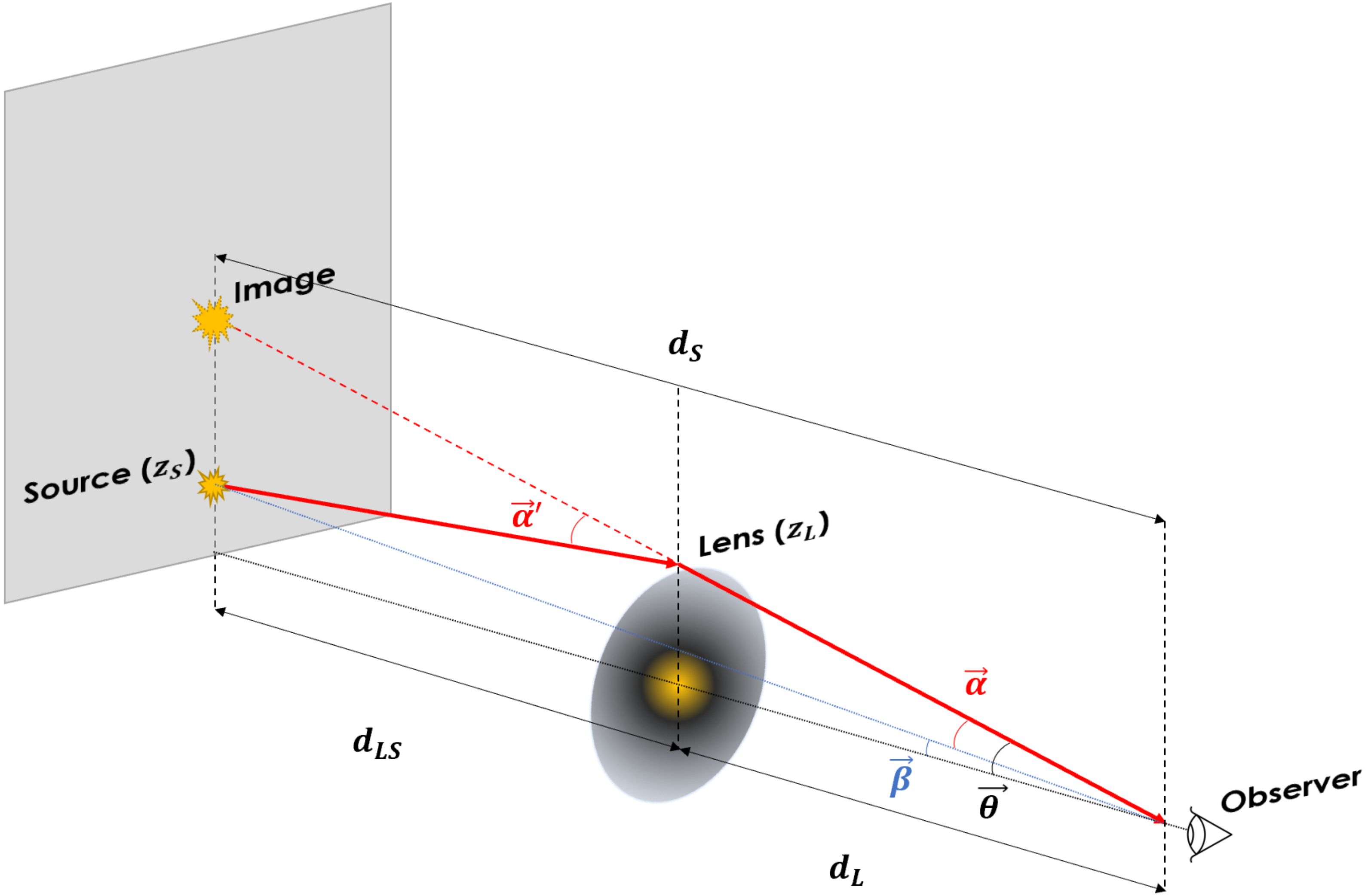


Lensing Effects in the Universe





Observables



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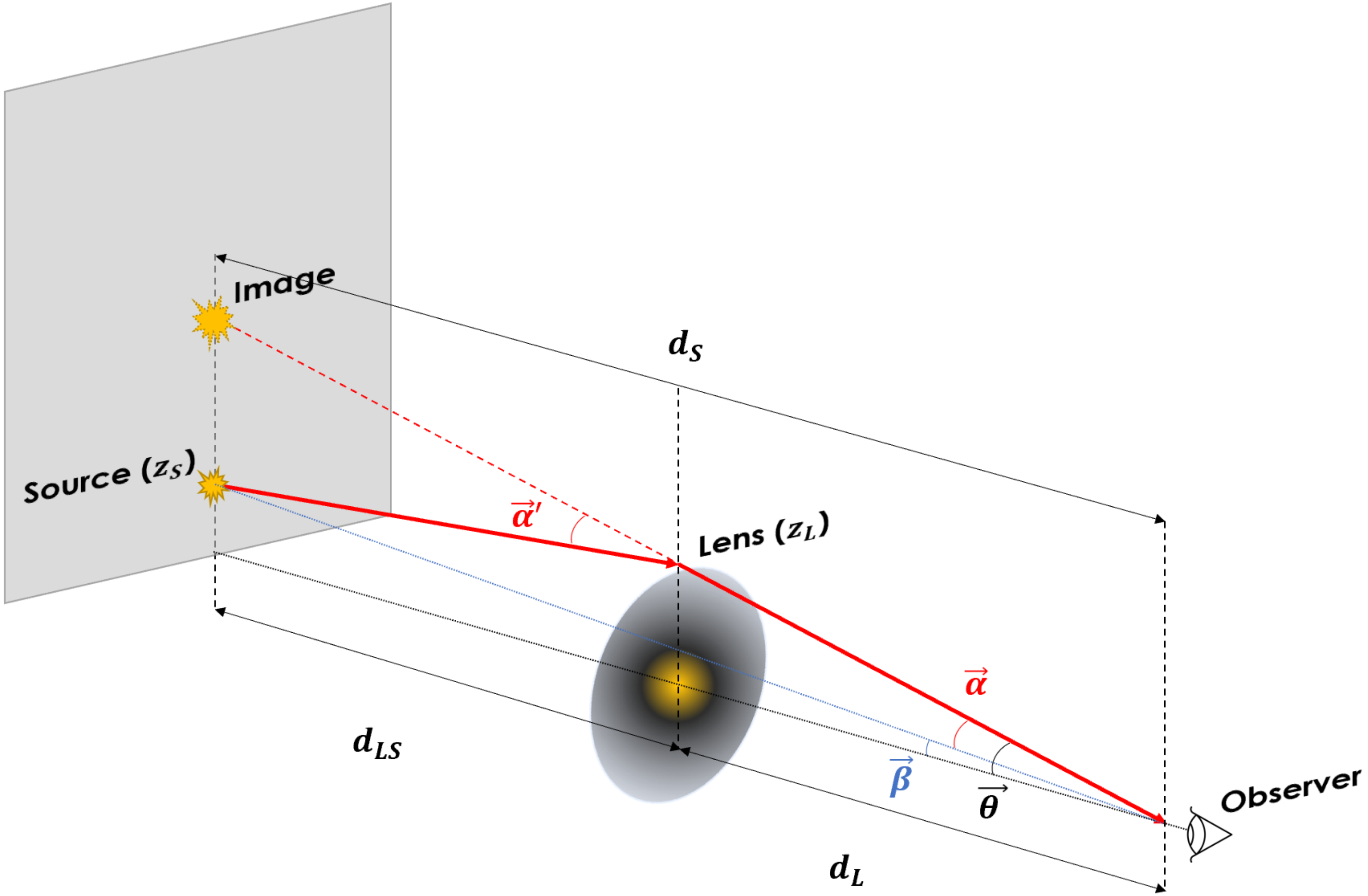
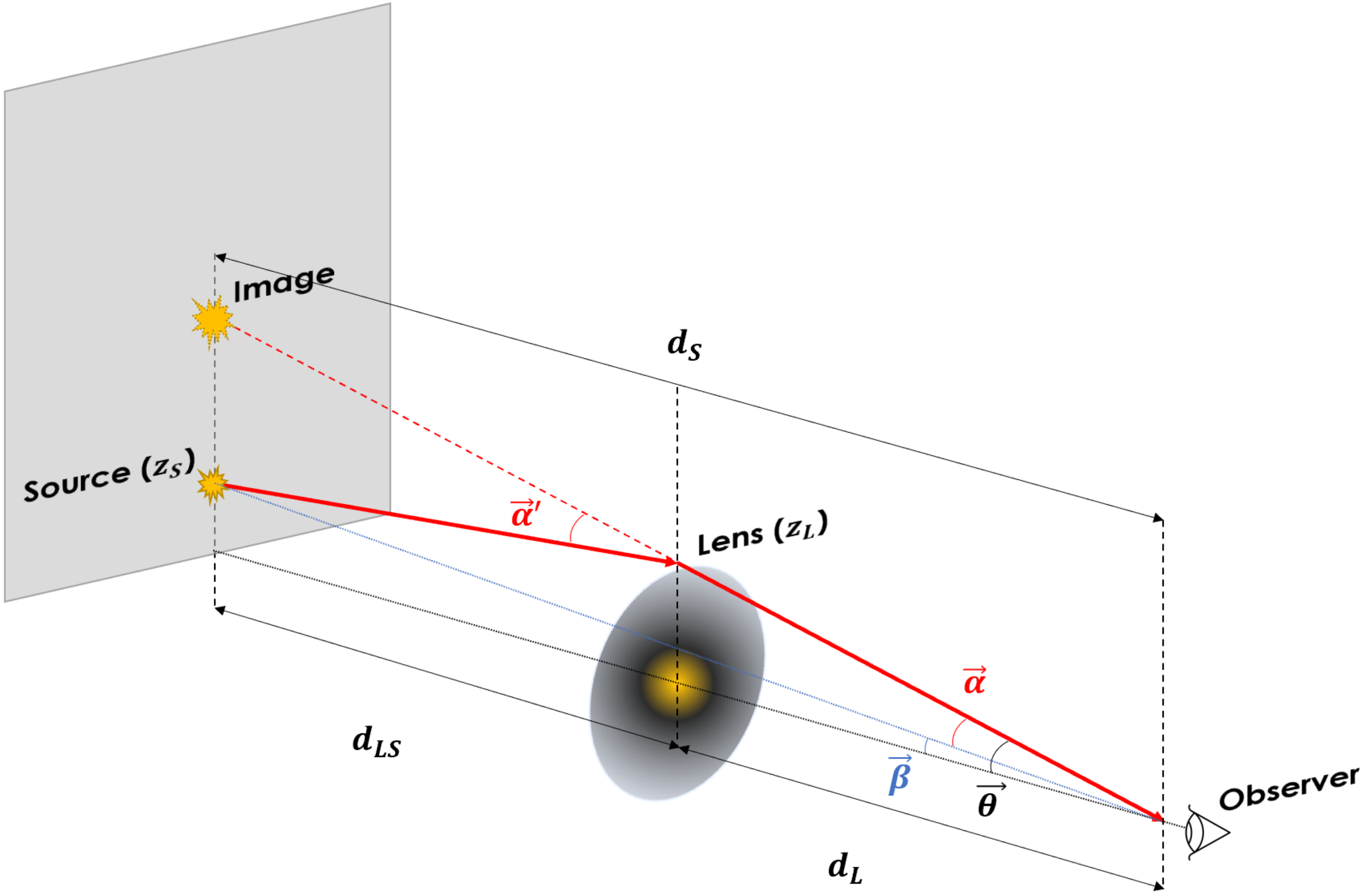


Lens:

- Lens redshift ($\boldsymbol{z_l}$)
- Velocity dispersion ($\boldsymbol{\sigma_{ap}}$)
- ……

Observables



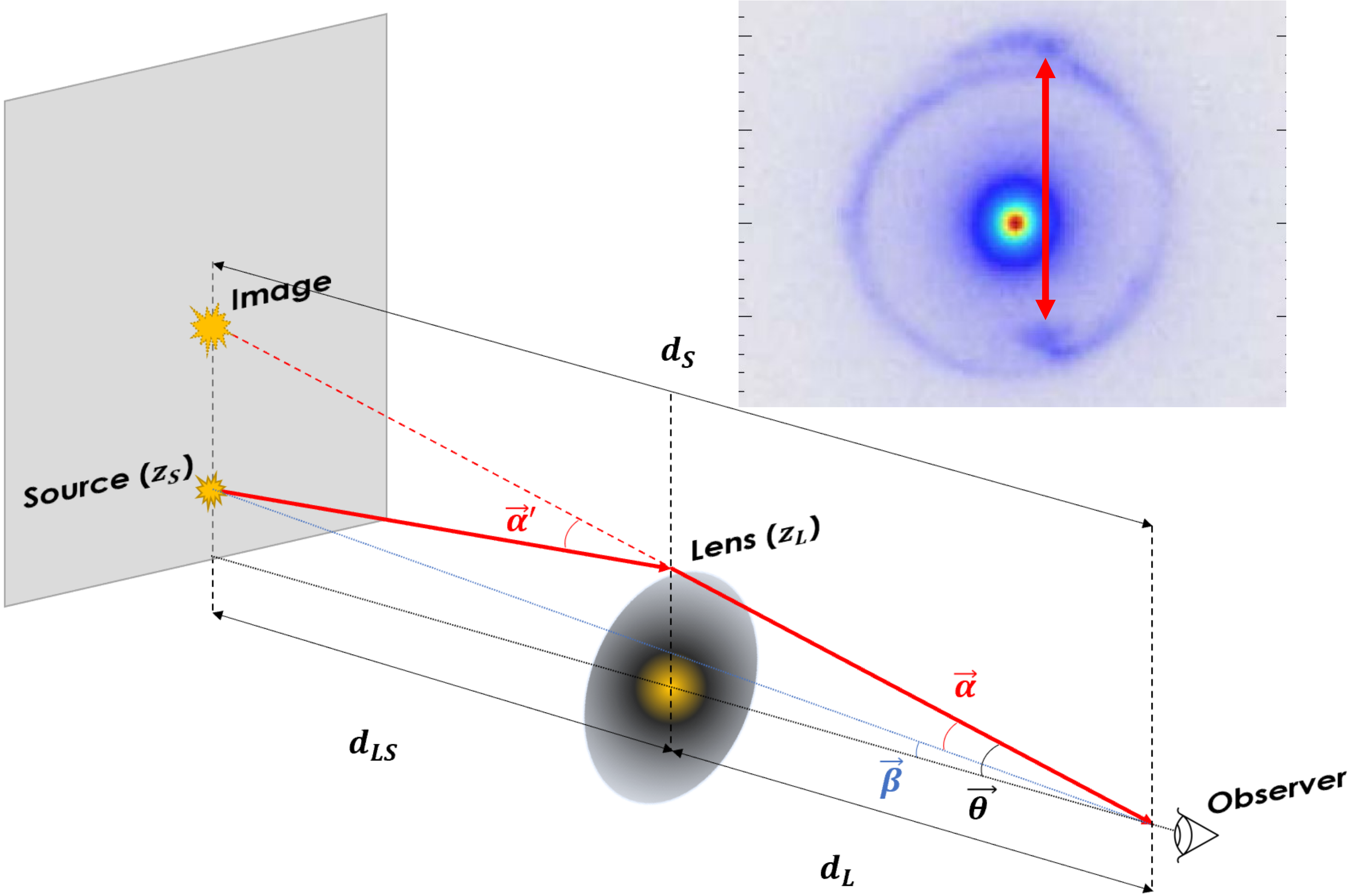
Lens:

- Lens redshift ($\boldsymbol{z_l}$)
- Velocity dispersion ($\boldsymbol{\sigma_{ap}}$)
- ……

Source Images on the lens plane:

- Source redshift ($\boldsymbol{z_s}$)
- Images' separations ($2\theta_E$)
- Flux & flux ratio of the image
- Time-delays
- ……

Observables



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Source Images on the lens plane:

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- **Images' separations ($\boldsymbol{2\theta_E}$)**
- Flux & flux ratio of the image
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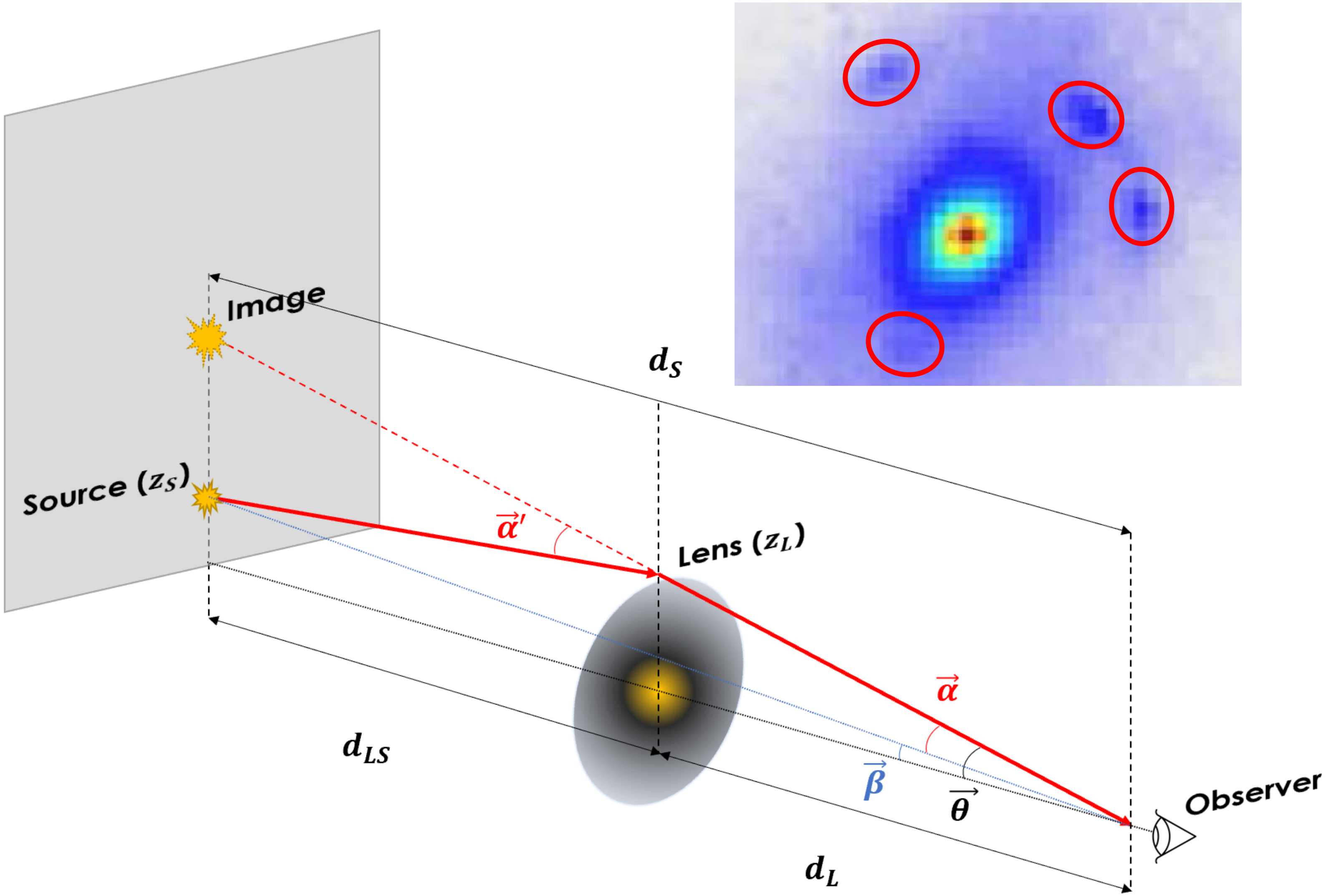
- ……

$$\theta_{\mathrm{E}}^{\mathrm{SIS}} = 4\pi(\frac{\sigma_{ap}}{c})^2\frac{D_{ls}}{D_s}$$

Observables



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Source Images on the lens plane:

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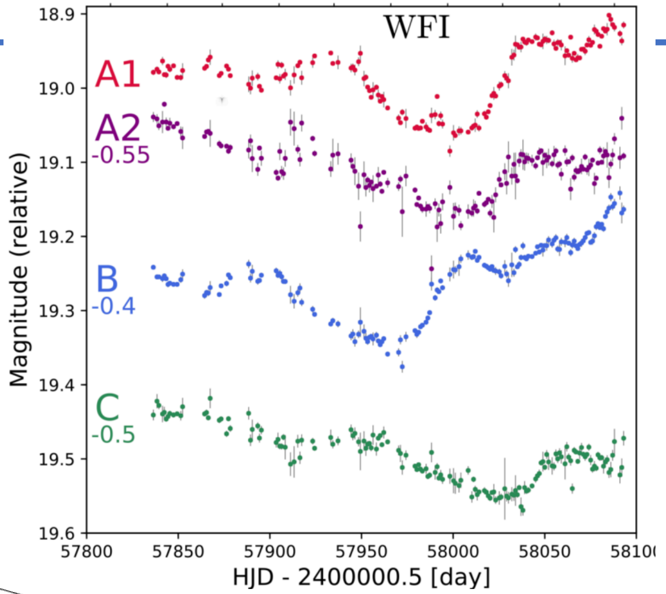
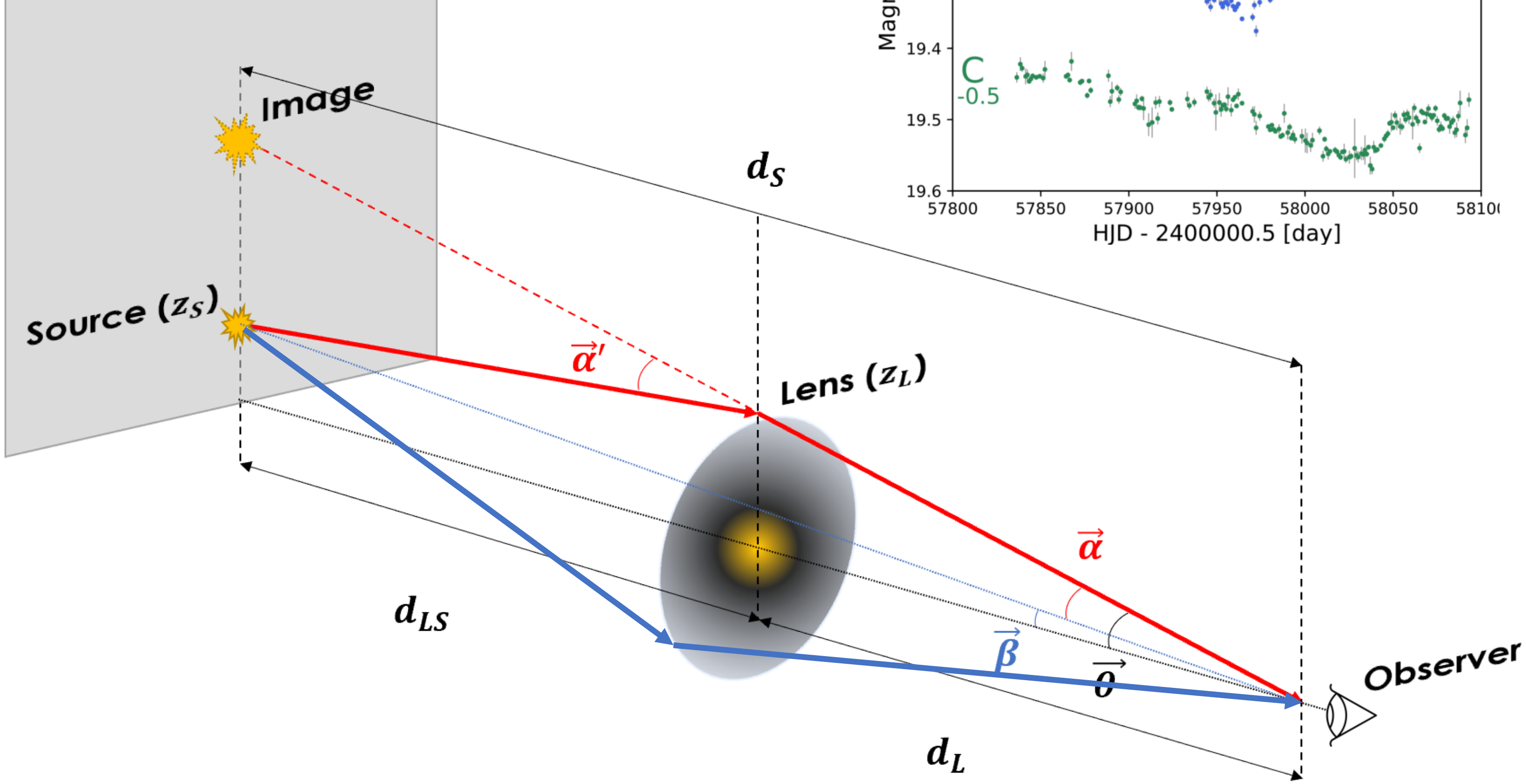
- **Flux & flux ratio of the image**
- Time-delays
- ……

$$\mu \sim \frac{1}{(1 - \frac{\Sigma(\vec{\theta})}{\Sigma_{\rm cr}})^2}$$

Observables

Birrer+2022





Lens:

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- ……

Source Images on the lens plane:

- Source redshift ($\boldsymbol{z_s}$)
- Images' separations ($2\theta_E$)
- Flux & flux ratio of the image
- **Time-delays**
- ……

$$\Delta t_{AB} = (1 + z_L) \frac{D_L D_S}{D_{LS}} \frac{\Delta \tau_{AB}}{c}$$

Strong Lensing Cosmology

Distance ratio

$$\mathcal{D}_{DR} = \frac{D_{LS}}{D_S}$$

Time-delay distance

$$\mathcal{D}_{\Delta t} = (1+z_L)\frac{D_L D_S}{D_{LS}}$$

Double source plane ratio

$$\mathcal{D}_{DSPDR} = \frac{D_{LS_1}/D_{S_1}}{D_{LS_2}/D_{S_2}}$$

Only observables + lens model

$$\Leftrightarrow \frac{c^2}{4\pi}\frac{\theta_E}{\sigma_{ap}^2}\left(\frac{\theta_E}{\theta_{ap}}\right)^{\gamma-2} f^{-1}(\gamma,\delta,\beta)$$

$$\Leftrightarrow \frac{1}{(1-\kappa_{\rm ext})\lambda_{\rm int}}\frac{c\Delta t_{AB}}{\Delta\tau_{AB}}$$

$$\Leftrightarrow \left(\frac{\theta_{E,1}}{\theta_{E,2}}\right)^{\gamma-1}$$

Cosmology beyond ΛCDM

$$D^A(z) = \frac{c}{(1+z)H_0}\int_0^z \frac{dz'}{E(z')}$$

Angular diameter distance

$\Lambda\mathrm{CDM}: \quad w = -1,$

$E^2_{\Lambda\mathrm{CDM}} = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda$

$w\mathrm{CDM}: \quad w = \mathrm{const},$

$E^2_{\Lambda\mathrm{CDM}} = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_{\mathrm{DE}}(1+z)^{3(1+w)}$

$w_0w_a\mathrm{CDM}: \quad w(z) = w_0 + w_a\frac{z}{1+z},$

$E^2_{\Lambda\mathrm{CDM}} = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_{\mathrm{DE}}(1+z)^{3(1+w_0+w_a)}\exp\left[-3w_a\frac{z}{1+z}\right]$

$w_\phi\mathrm{CDM}: \quad w_\phi(z) = -1 + (1+w_0)\left(\frac{1}{1+z}\right)^\alpha,$

$E^2_{\Lambda\mathrm{CDM}} = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\phi\exp\left[\frac{3(1+w_0)}{\alpha}\left(1-(1+z)^{-\alpha}\right)\right]$

Strong Lensing Cosmology

Distance ratio

$$\mathcal{D}_{DR} = \frac{D_{LS}}{D_S}$$

H0-independent

Time-delay distance

$$\mathcal{D}_{\Delta t} = (1 + z_L)\frac{D_L D_S}{D_{LS}}$$

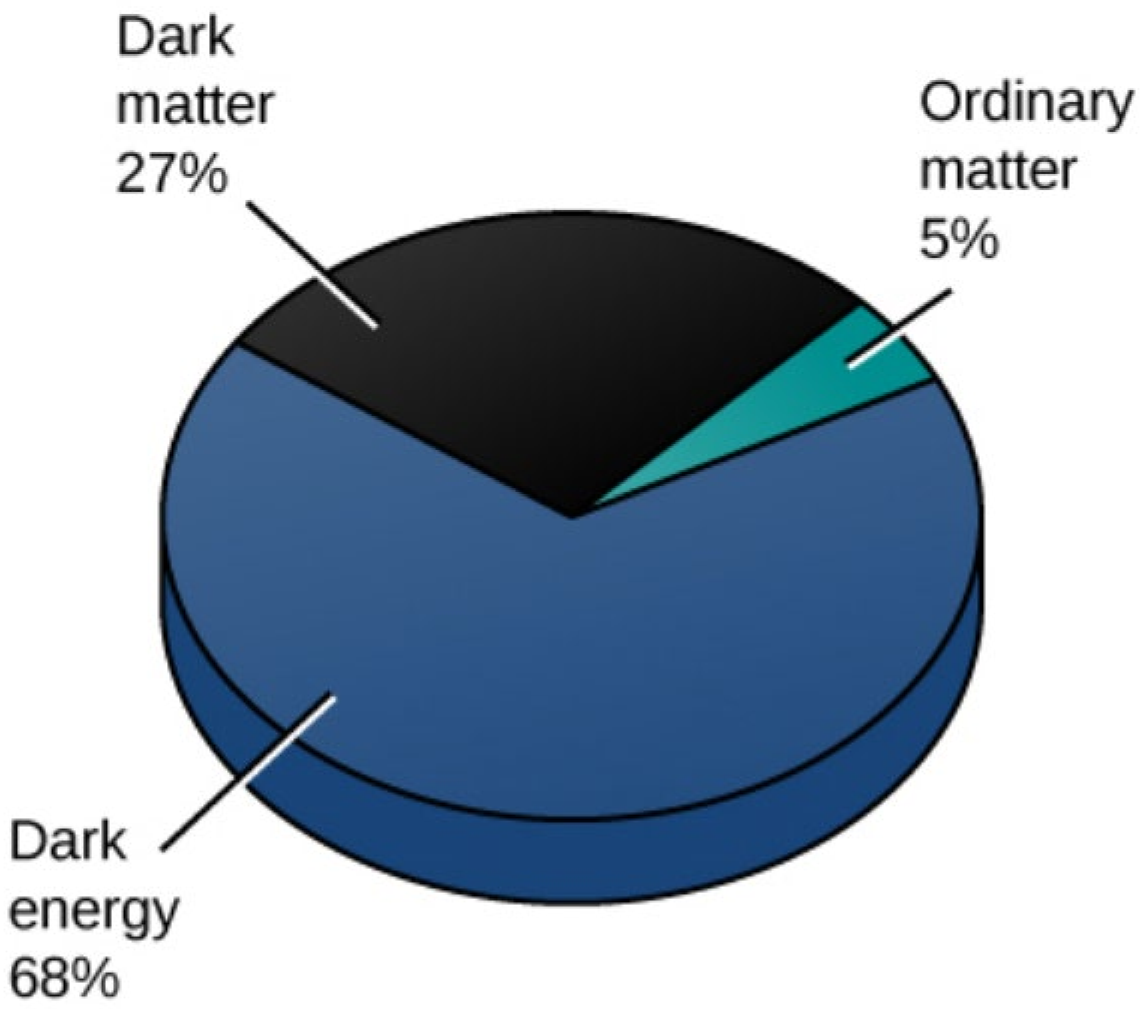
$\propto 1/\mathrm{H0}$

Double source plane ratio

$$\mathcal{D}_{DSPDR} = \frac{D_{LS_1}/D_{S_1}}{D_{LS_2}/D_{S_2}}$$

H0-independent

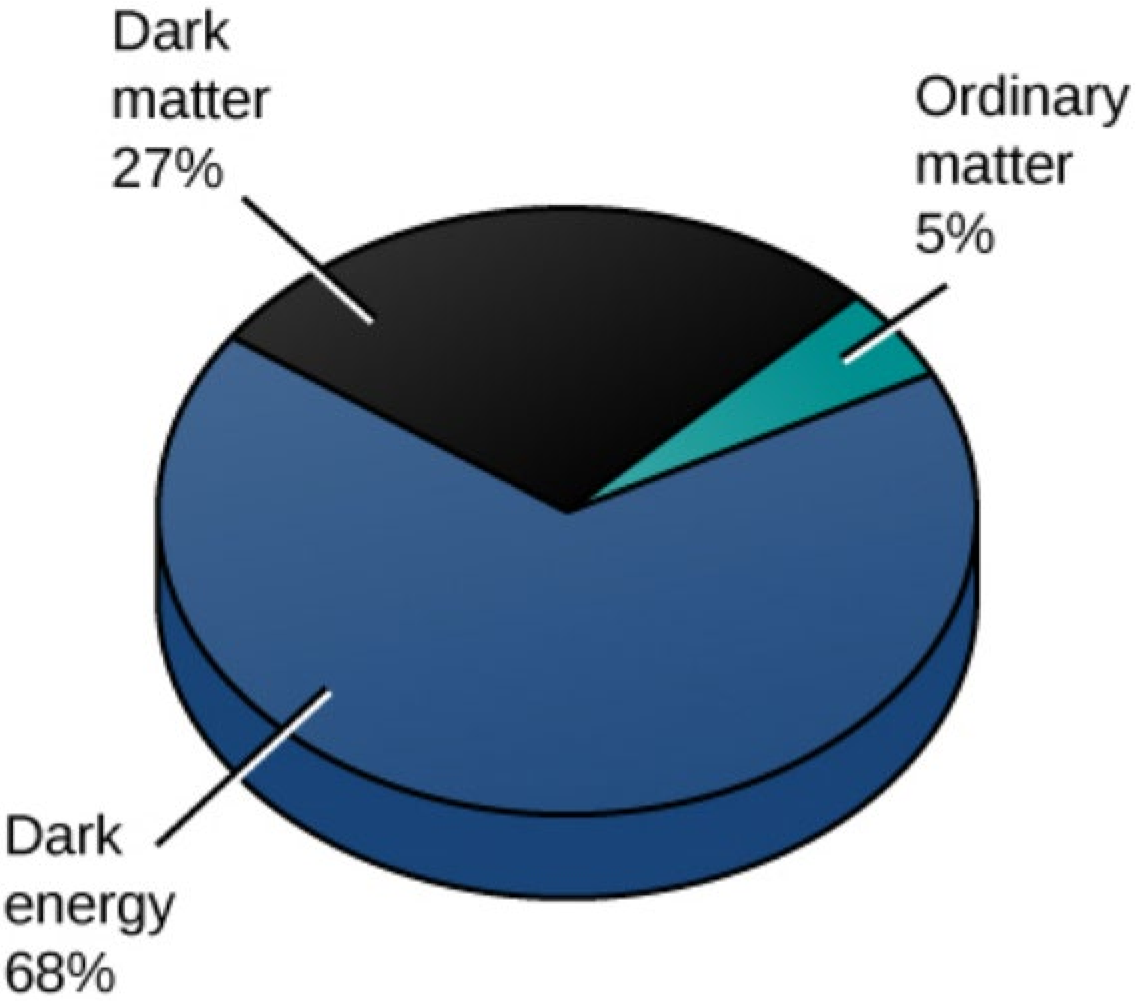
Why strong lensing is important?

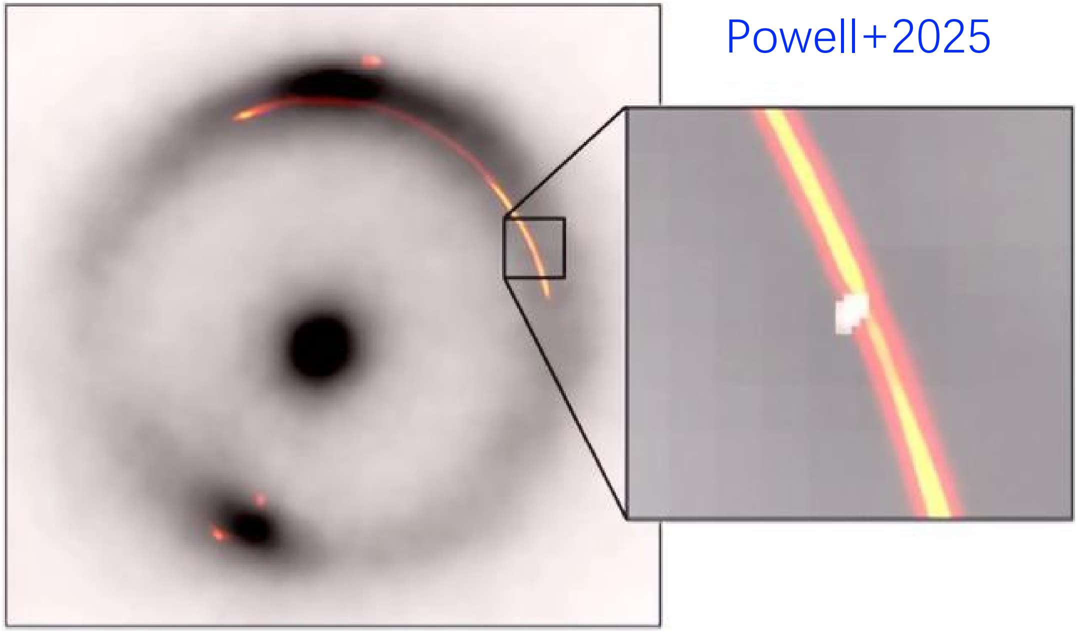


- Dark Matter
- Dark Energy

Why strong lensing is important?

$$\mu \sim \frac{1}{(1 - \frac{\Sigma(\vec{\theta})}{\Sigma_{cr}})^2}$$





Powell+2025

JVAS B1938+666 (Keck/EVN/GBT/VLBA)

- **Dark Matter**
- Dark Energy

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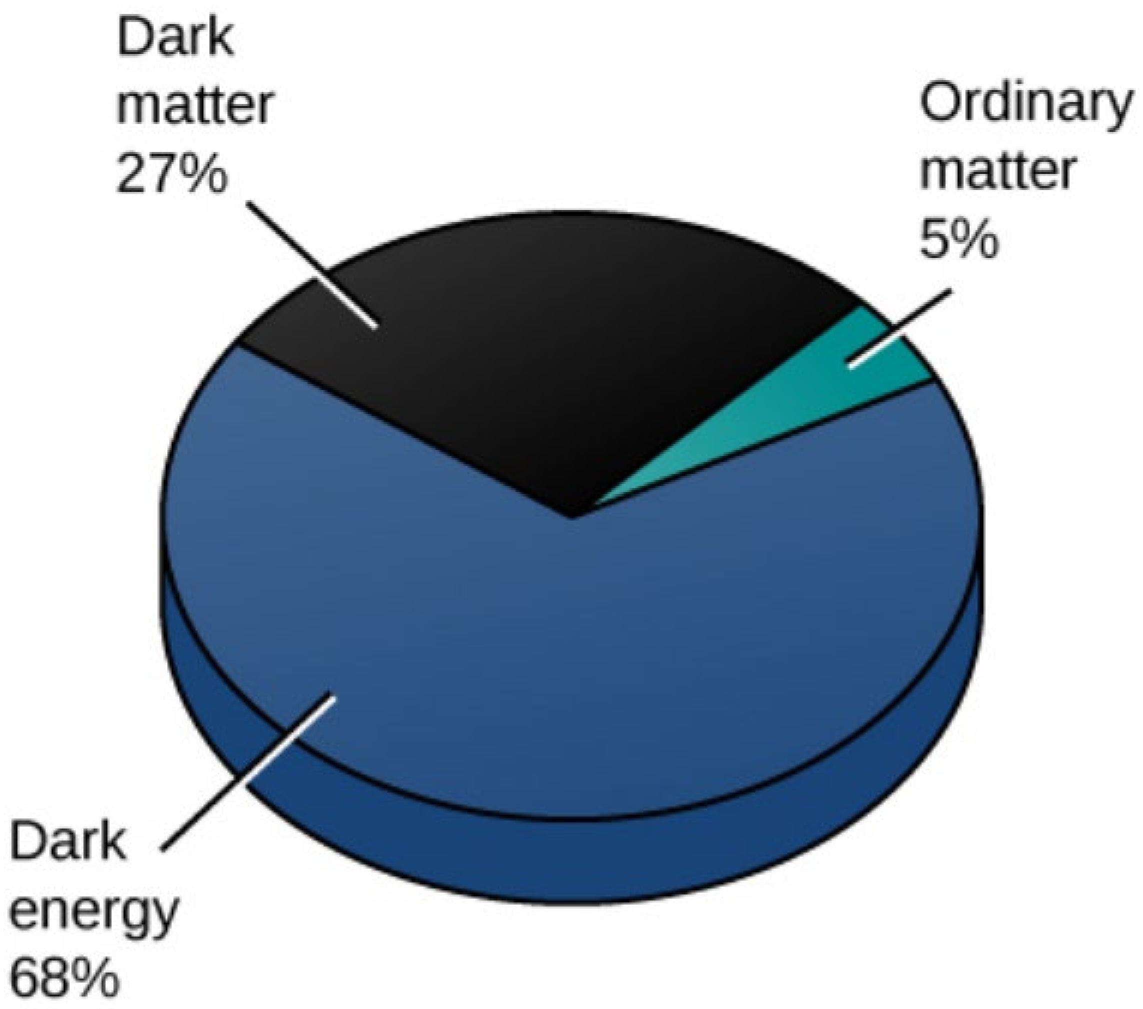
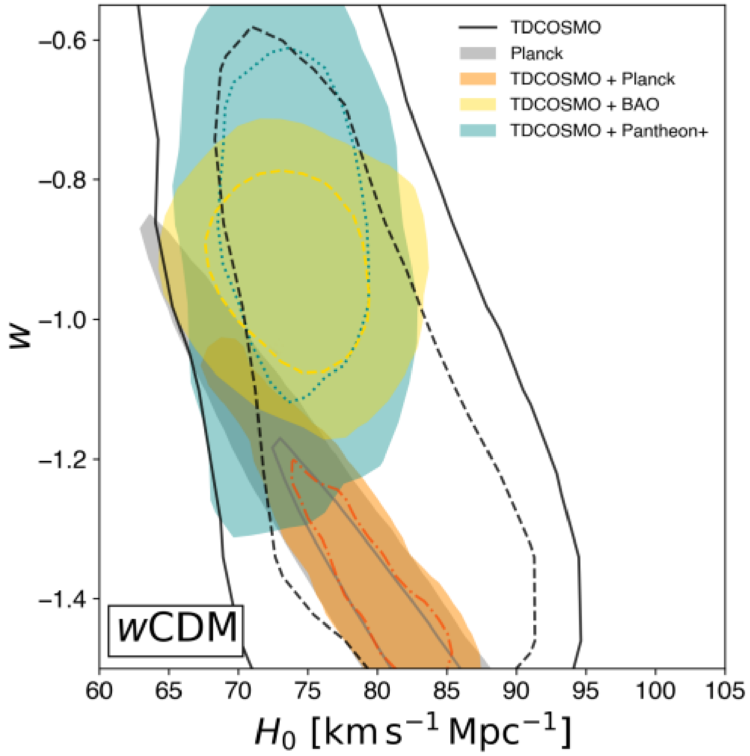
$$\Delta t_{AB} = (1 + z_L)\,\frac{D_L D_S}{D_{LS}}\,\frac{\Delta \tau_{AB}}{c}$$



TDCOSMO2025



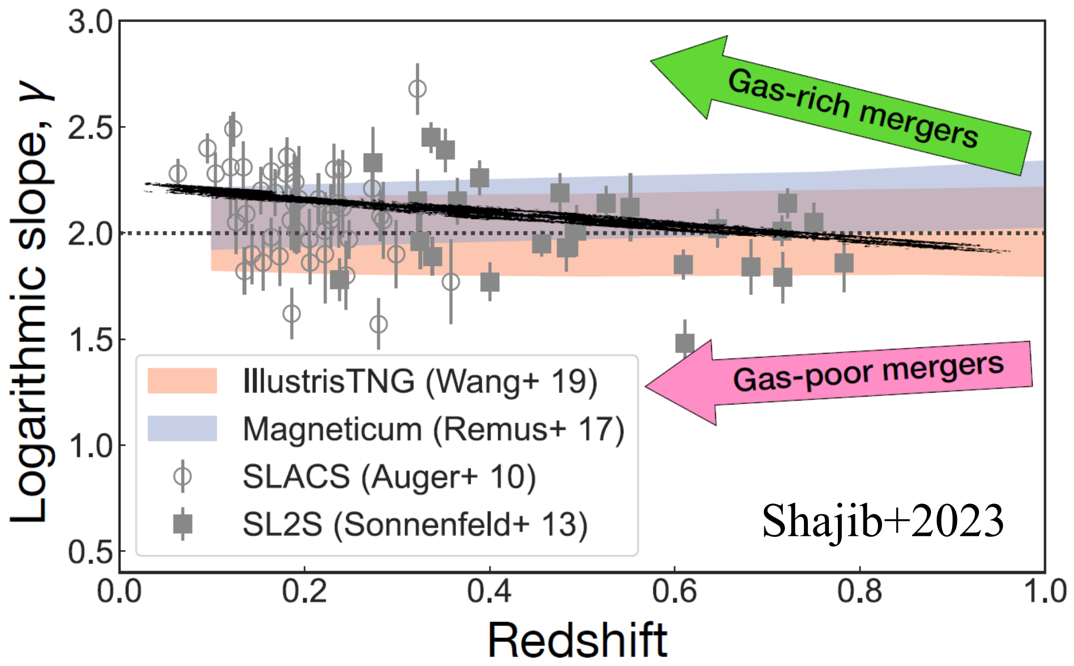
- Dark Matter
- **Dark Energy**

Problems to solve

How does the mass distribution of lens galaxies evolve with redshift (cosmic time)?

How can we fully exploit distance-ratio measurements from large galaxy-galaxy lens samples?

Can distance ratios provide robust constraints on extensions to ΛCDM?



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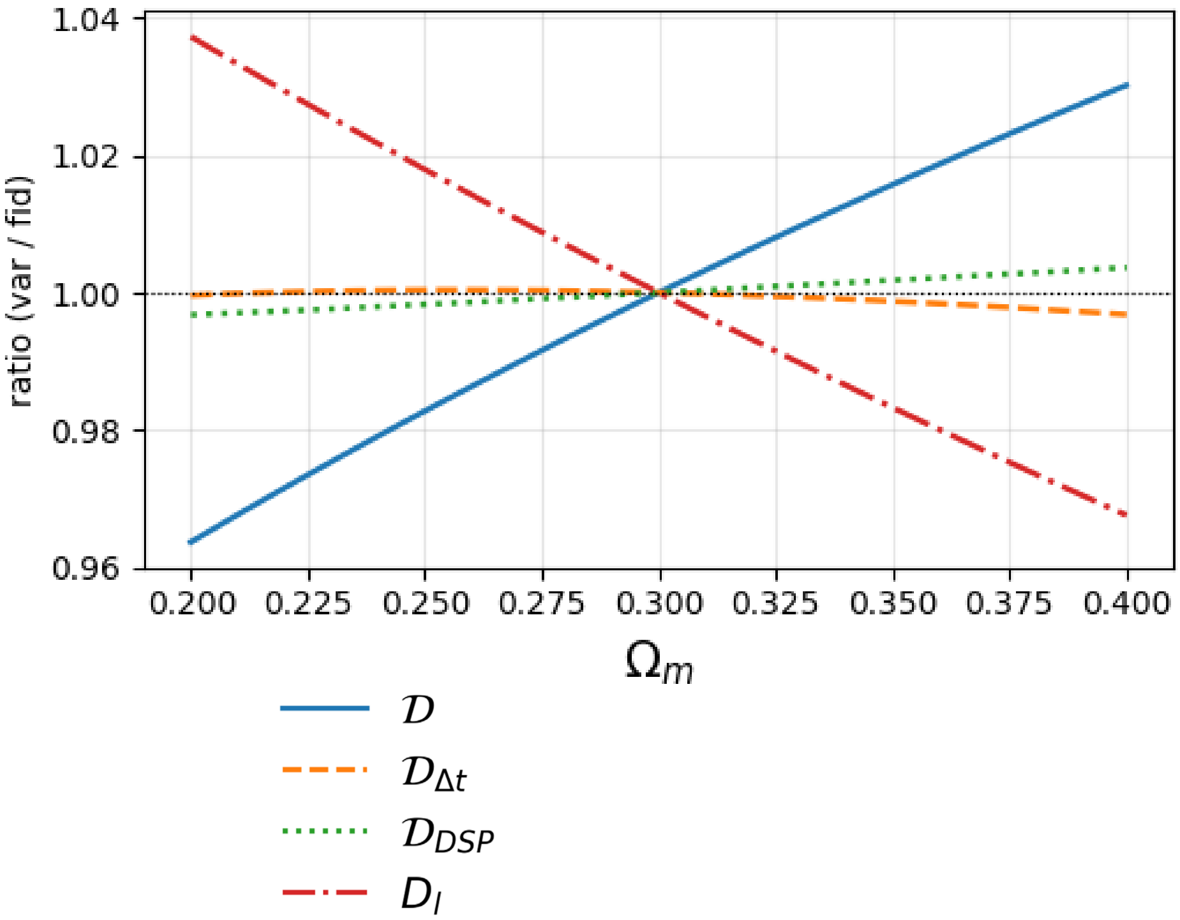

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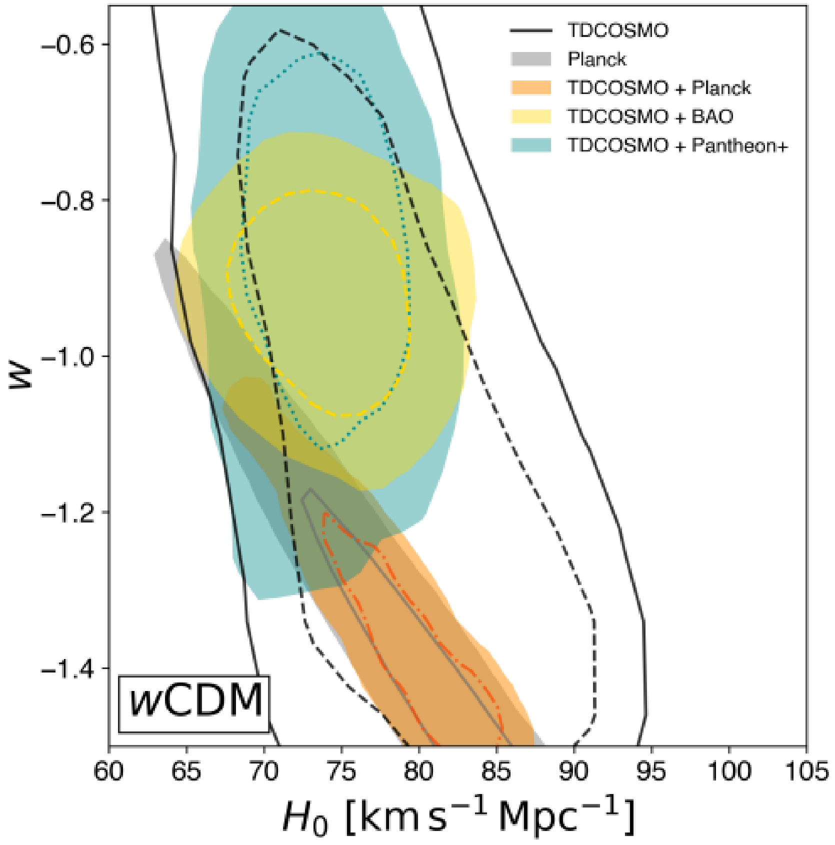

My Contribution

How does the mass distribution of lens galaxies evolve with redshift (cosmic time)?

I developed a **dark-energy–model-independent approach** to measure the redshift evolution of the total mass-density slope in strong-lensing galaxies.

How can we fully exploit distance-ratio measurements from large galaxy-galaxy lens samples?

Can distance ratios provide robust constraints on extensions to ΛCDM?

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**How can we fully exploit distance-ratio measurements
from large galaxy-galaxy lens samples?**

I built a **hierarchical Bayesian framework** that **simultaneously** constrains cosmological parameters and **self-calibrates** the redshift evolution of lens density slopes.

Can distance ratios provide robust constraints on
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My Contribution

How does the mass distribution of lens galaxies evolve with redshift (cosmic time)?

I developed a dark-energy–model-independent approach to measure the redshift evolution of the total mass-density slope in strong-lensing galaxies.

How can we fully exploit distance-ratio measurements from large galaxy-galaxy lens samples?

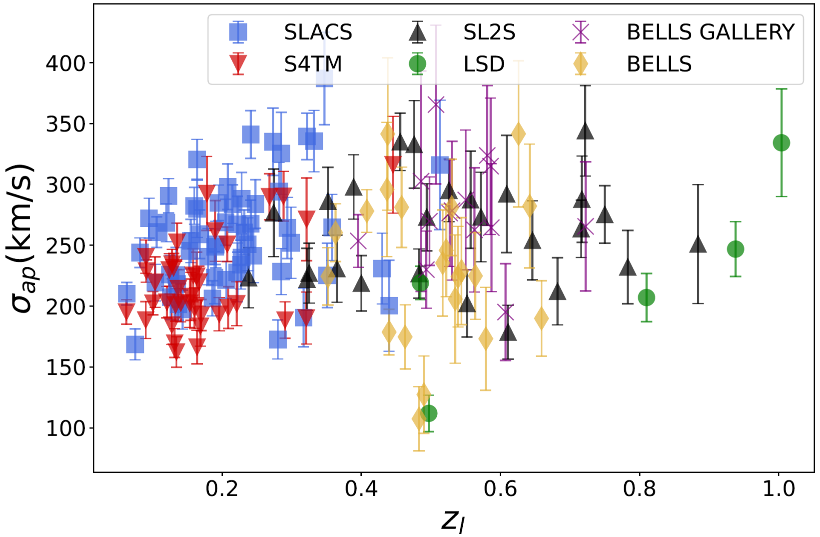
I built a hierarchical Bayesian framework that simultaneously constrains cosmological parameters and self-calibrates the redshift evolution of lens density slopes.

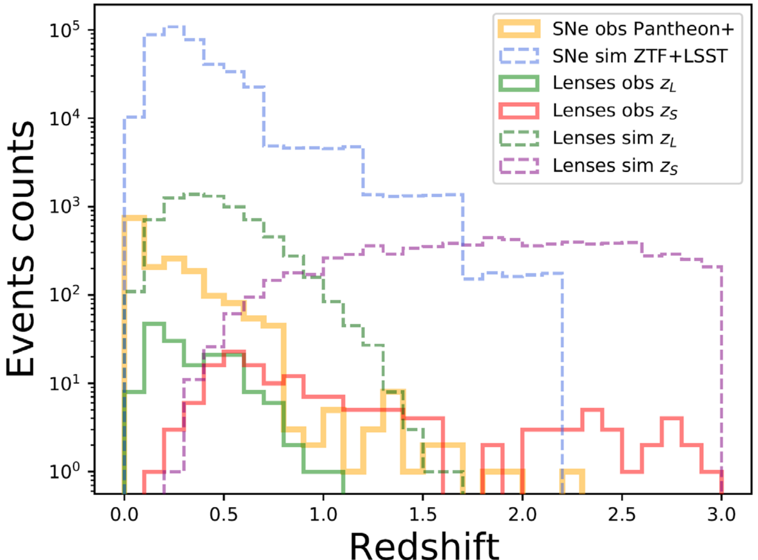
Can distance ratios provide robust constraints on extensions to ΛCDM?

I applied this framework to a suite of **beyond-ΛCDM models** to assess the **cosmological constraining power** of distance ratios.

2. Data and Methods

Data





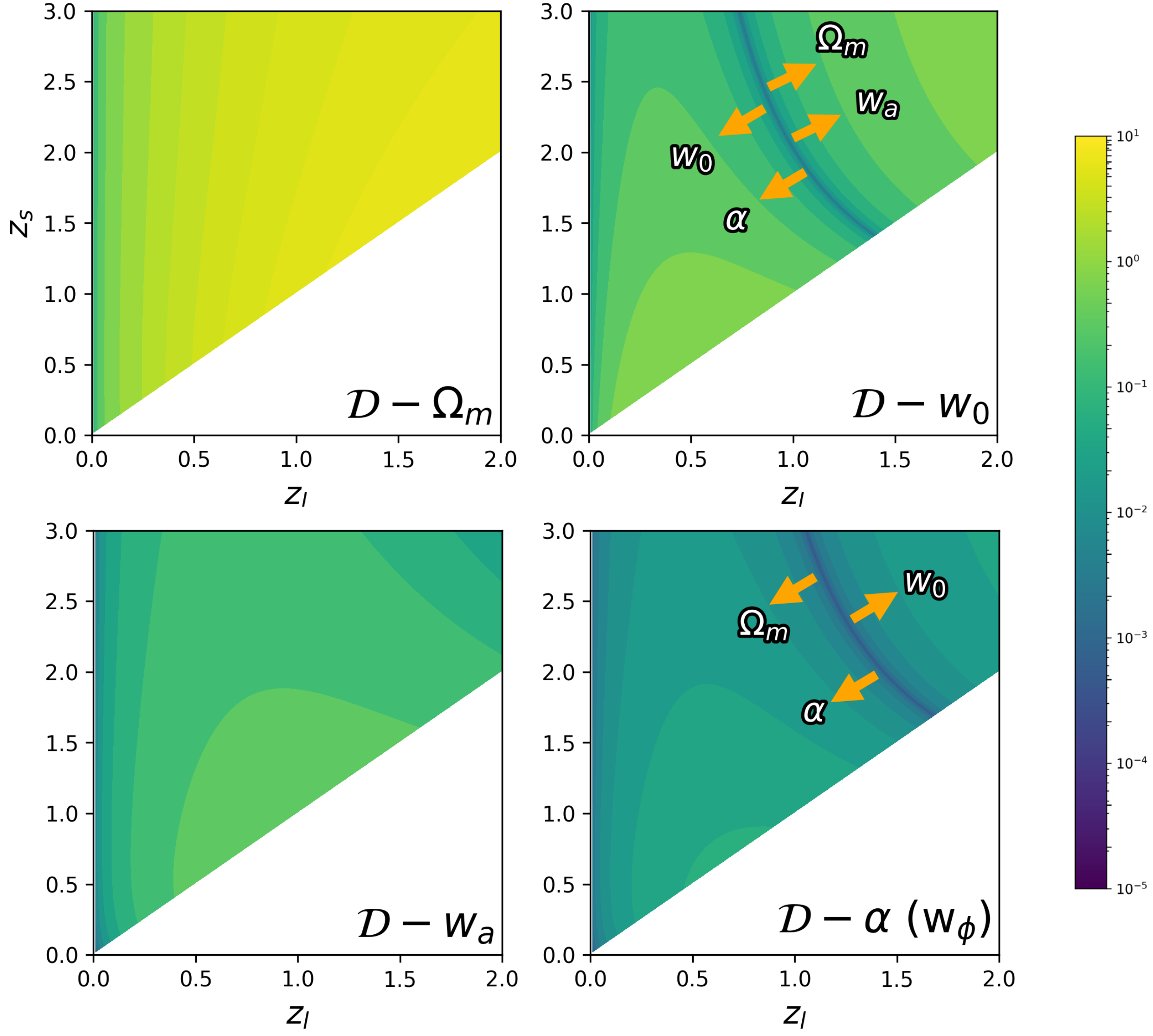
Observed data (Chen+2019)

LSD: 5 BELLS GALLERY: 14
SL2S: 26 S4TM: 38
SLACS: 57 BELLS:21

Simulation data

- Power-law model
- ~7000 lensing systems with velocity dispersion information
- No redshift evolution

Theoretical Sensitivity Analysis

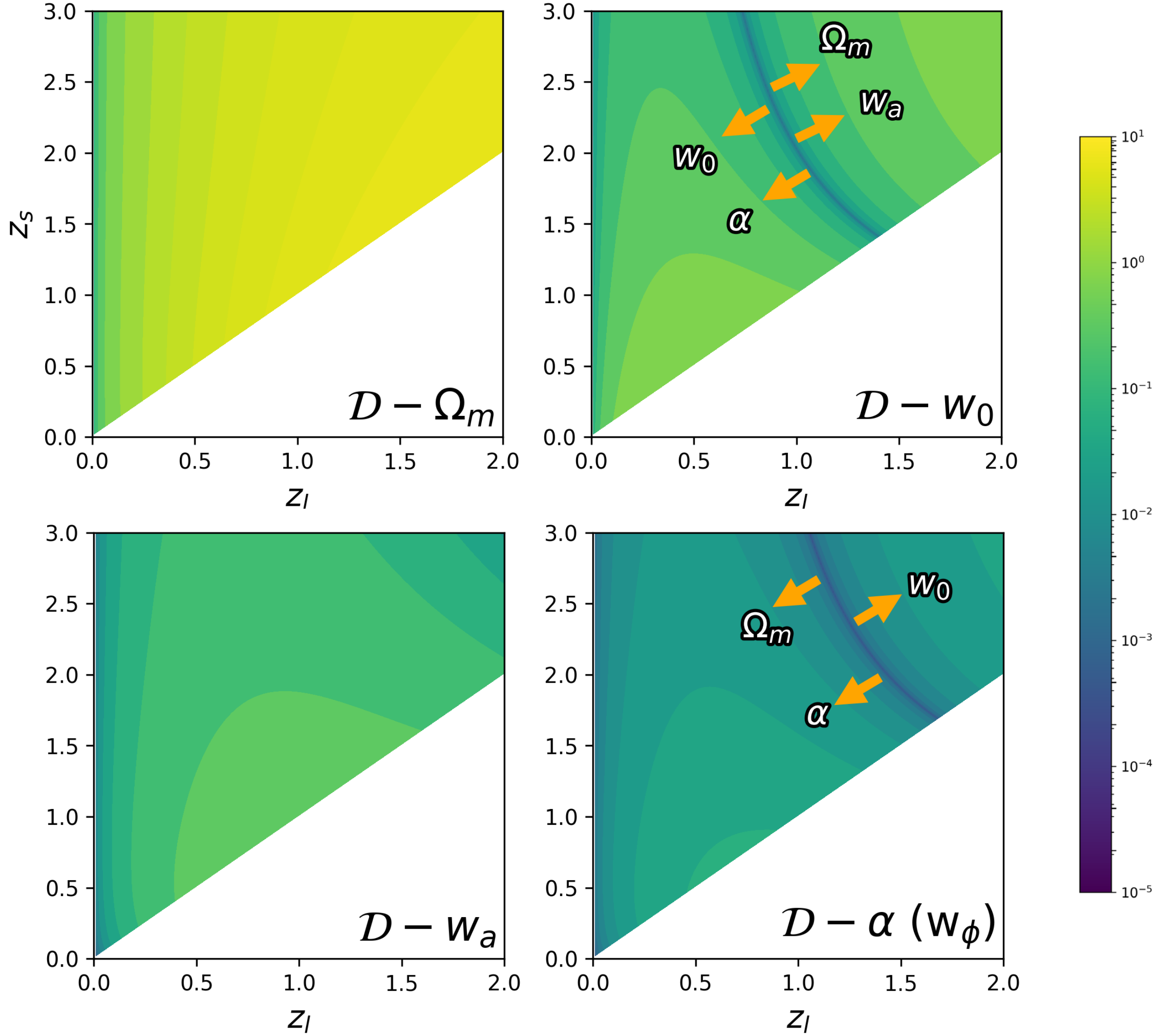


$$\mathcal{F}_s = \frac{1}{\sigma(p_i)} \sim \frac{\left|\frac{\partial \ln \mathcal{D}_{DR}}{\partial p_i}\right|}{\sigma_{\ln \mathcal{D}_{DR}}}$$

$$\sigma_{\ln \mathcal{D}_{DR}} = \frac{\sigma_{\mathcal{D}_{DR}}}{\mathcal{D}_{DR}}, \quad p_i = \Omega_m, w, w_a, \ldots$$

$$\mathcal{D}_{DR} = \frac{D_{LS}}{D_S}$$

Theoretical Sensitivity Analysis



$$\mathcal{F}_s = \frac{1}{\sigma(p_i)} \sim \frac{\left|\frac{\partial \ln \mathcal{D}_{DR}}{\partial p_i}\right|}{\sigma_{\ln \mathcal{D}_{DR}}}$$

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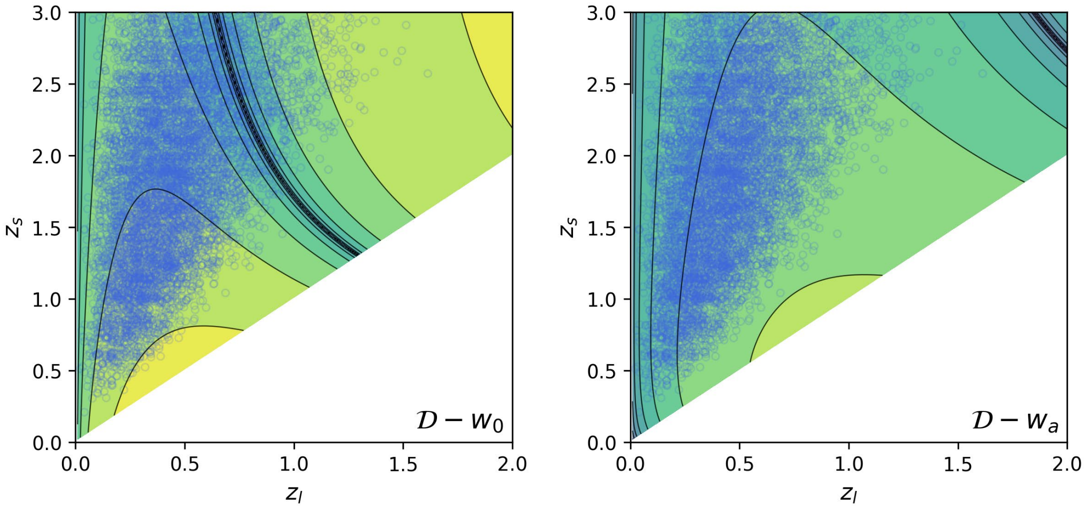
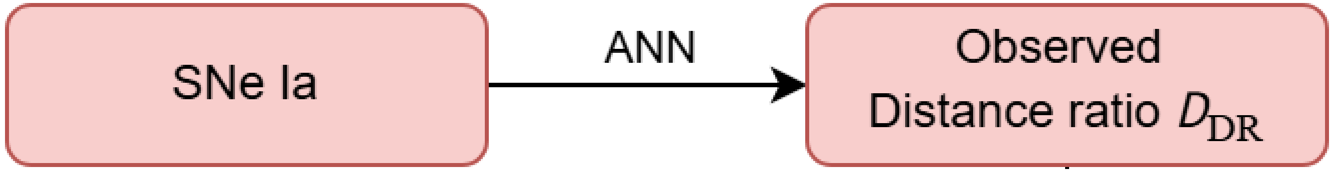


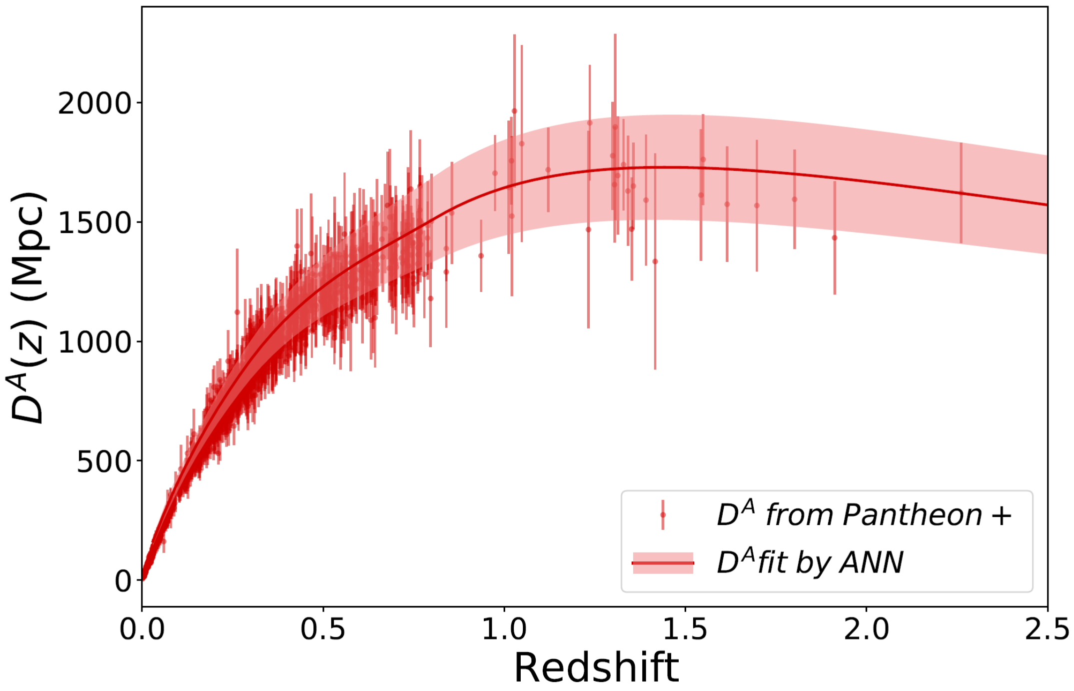
LSST simulation data

Model-independent approach for redshift evolution of the lens density slope

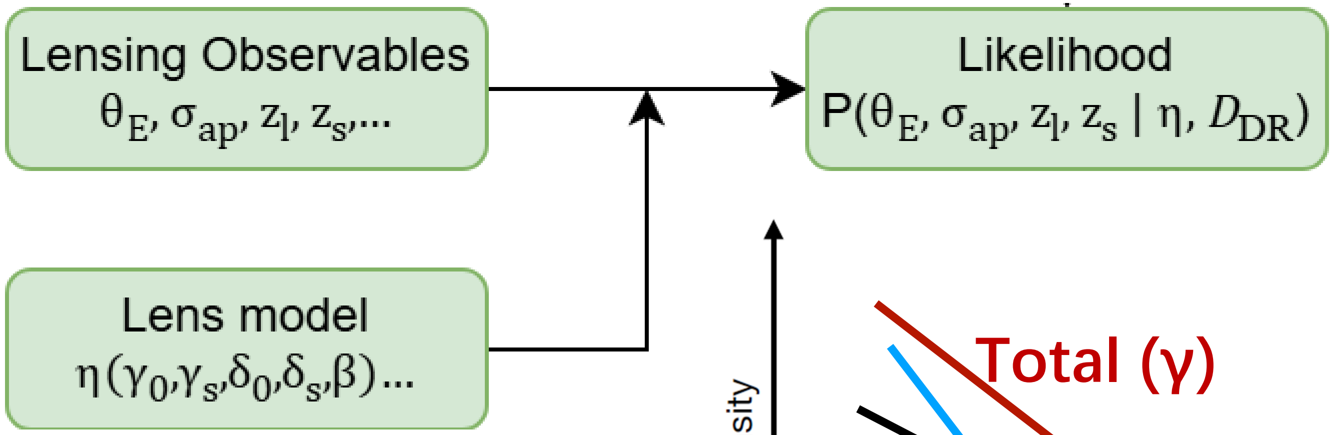


- Data driven
  (no cosmology model assumption)
- H0-indenpendent

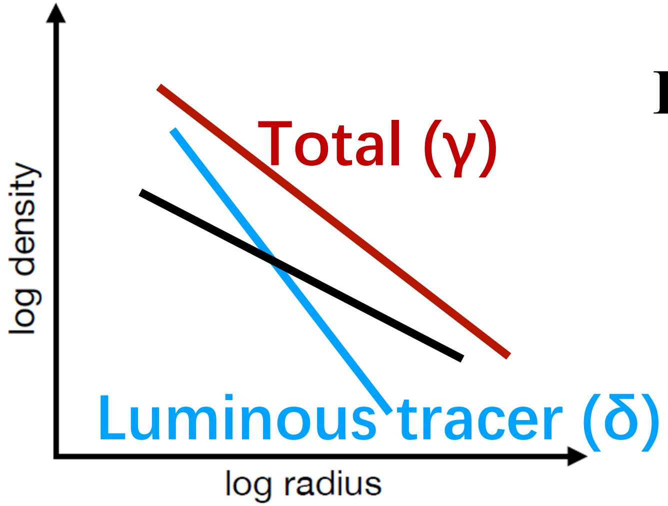
$$\mathcal{D}_{DR} = \frac{D_{LS}}{D_S}$$



Model-independent approach for redshift evolution of the lens density slope



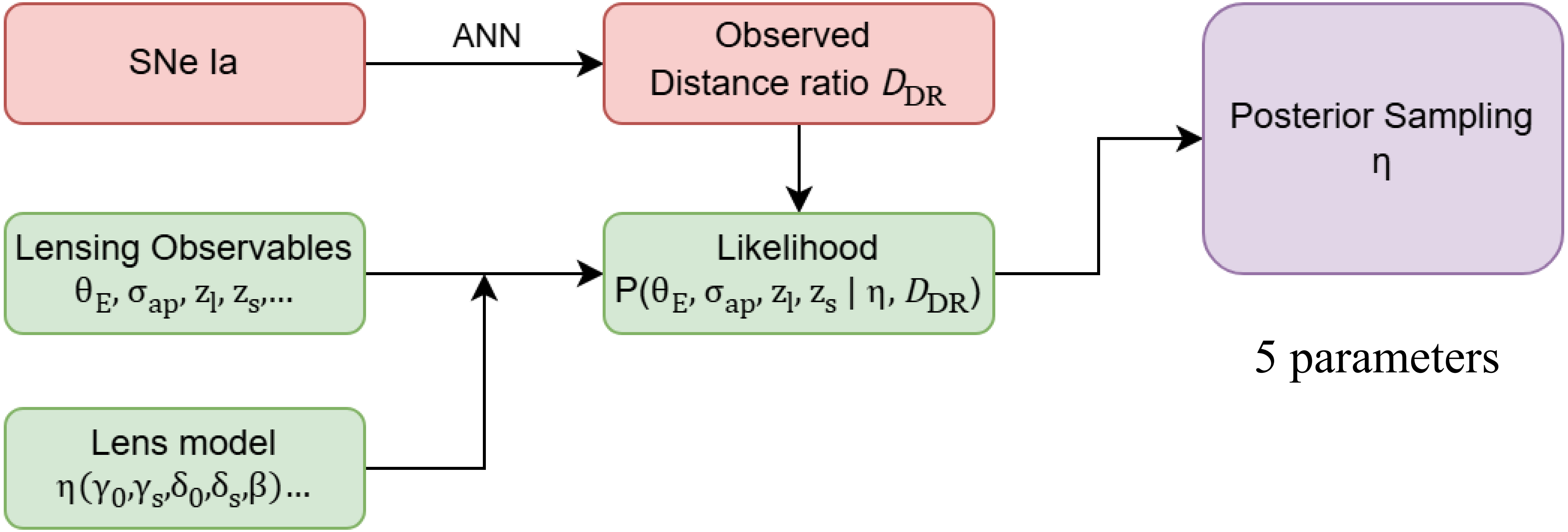
$$\gamma = \gamma_0 + \gamma_s \times z_l$$
$$\delta = \delta_0 + \delta_s \times z_l$$



Extended Power-law Model

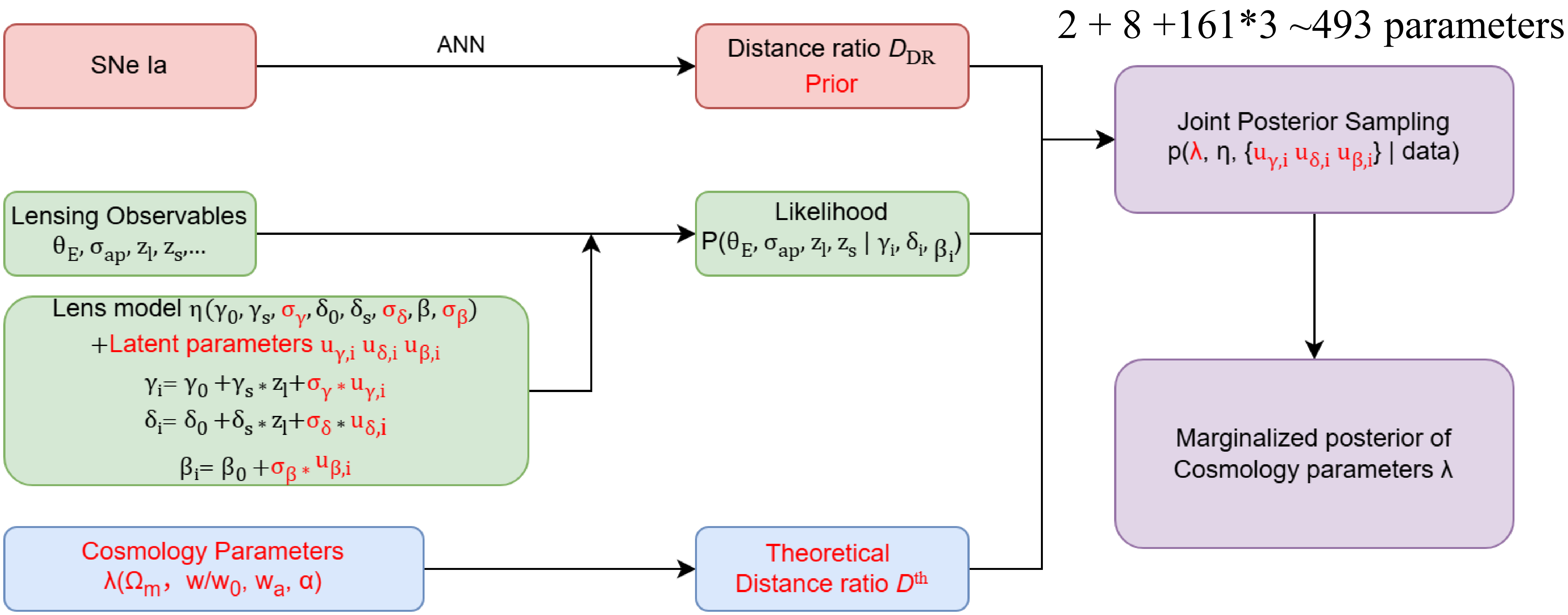
$$\rho_L(r) = \rho_L r^{-\delta}$$
$$\rho_{tot}(r) = \rho_{tot} r^{-\gamma}$$
$$\beta(r) = 1 - \frac{\langle \sigma_\theta^2 \rangle}{\langle \sigma_r^2 \rangle},$$

Model-independent approach for redshift evolution of the lens density slope



5 parameters

Hierarchical Bayesian Framework for Cosmology analysis



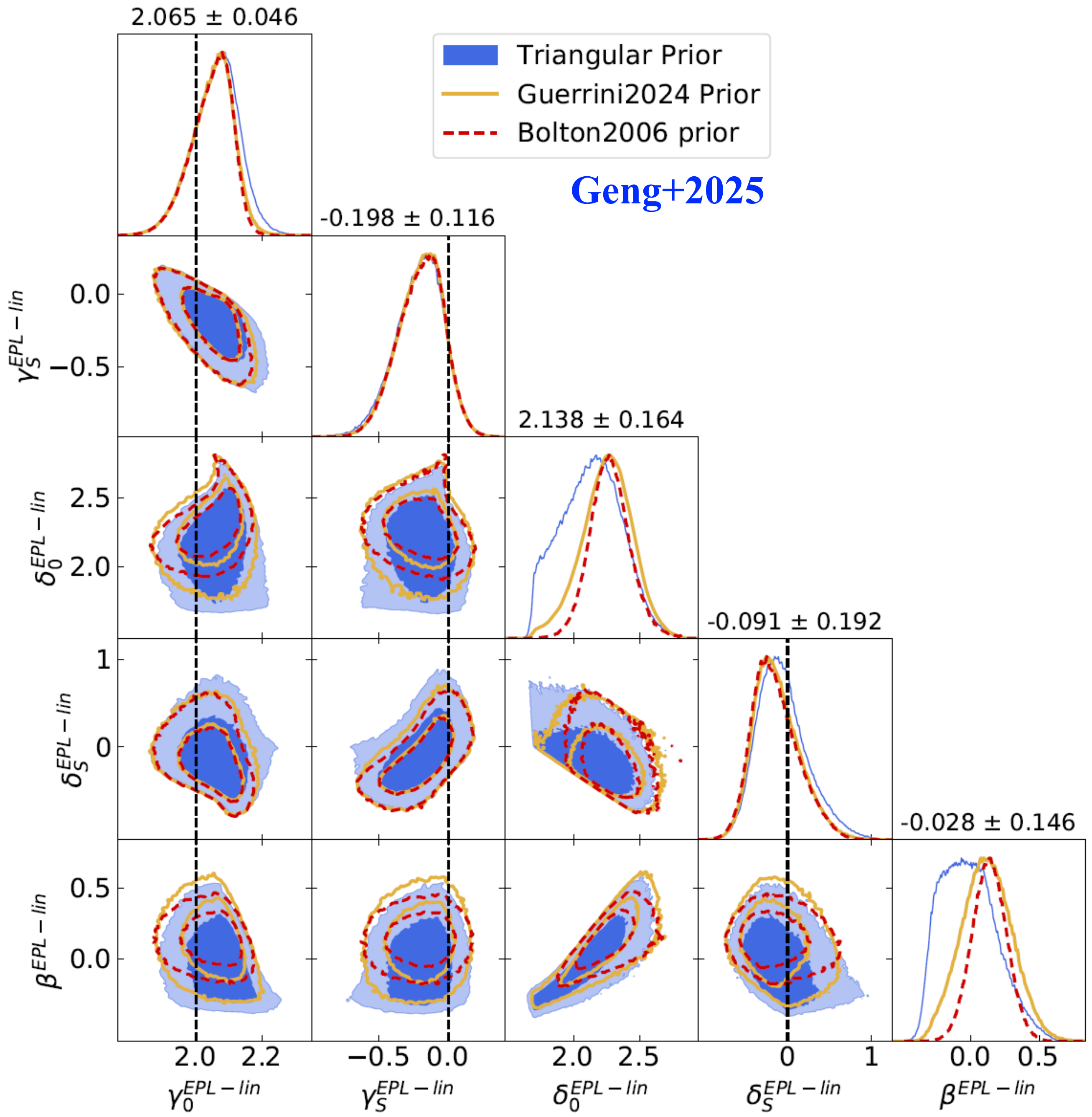
3. Results

Redshift Evolution

$$\gamma^{EPL} = \gamma_0^{lin} + \gamma_s^{lin} \times z_l$$
$$\delta^{EPL} = \delta_0^{lin} + \delta_s^{lin} \times z_l$$

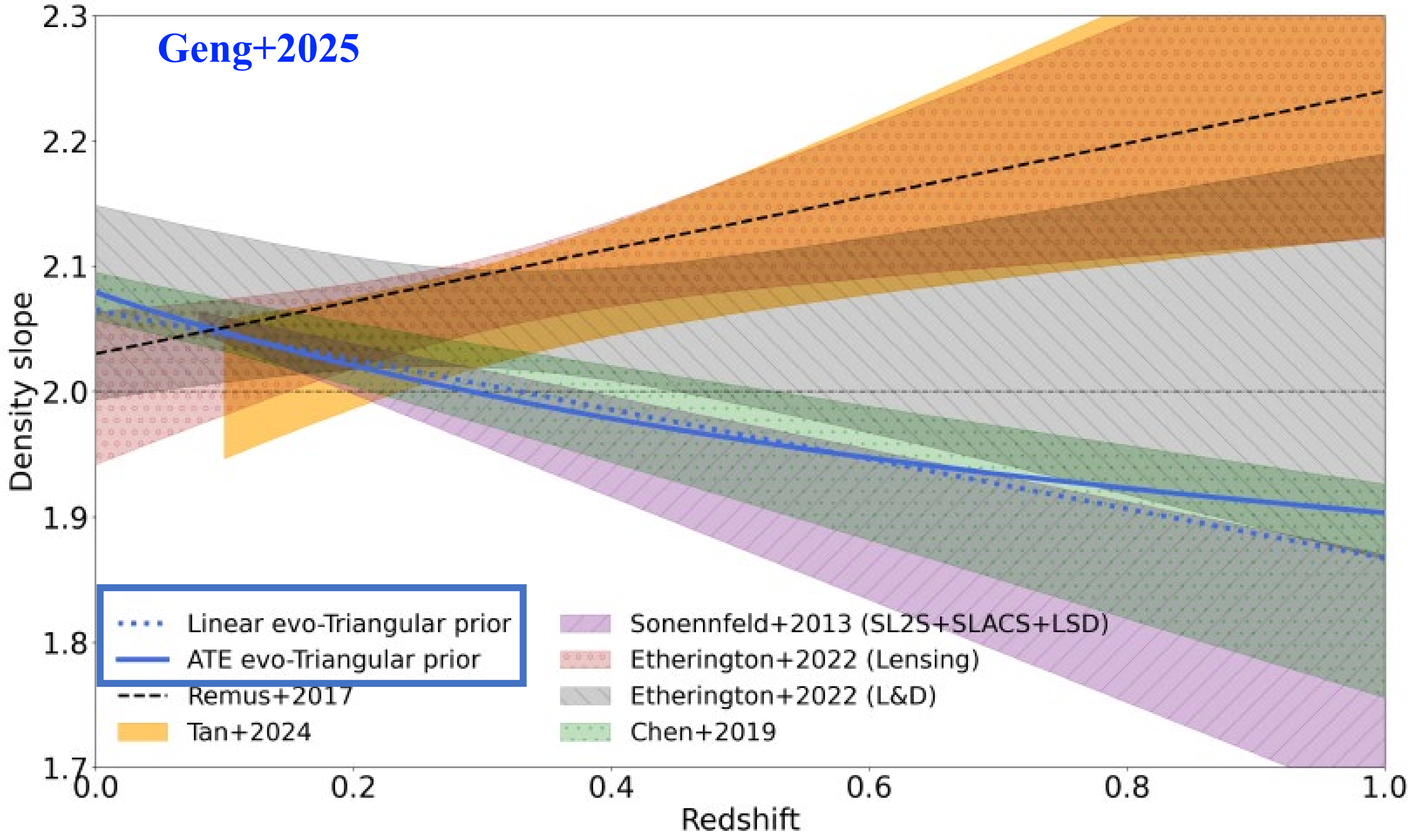
Linear evolution + Triangular prior
Tri(-0.5, 0.656, mode=0.102)

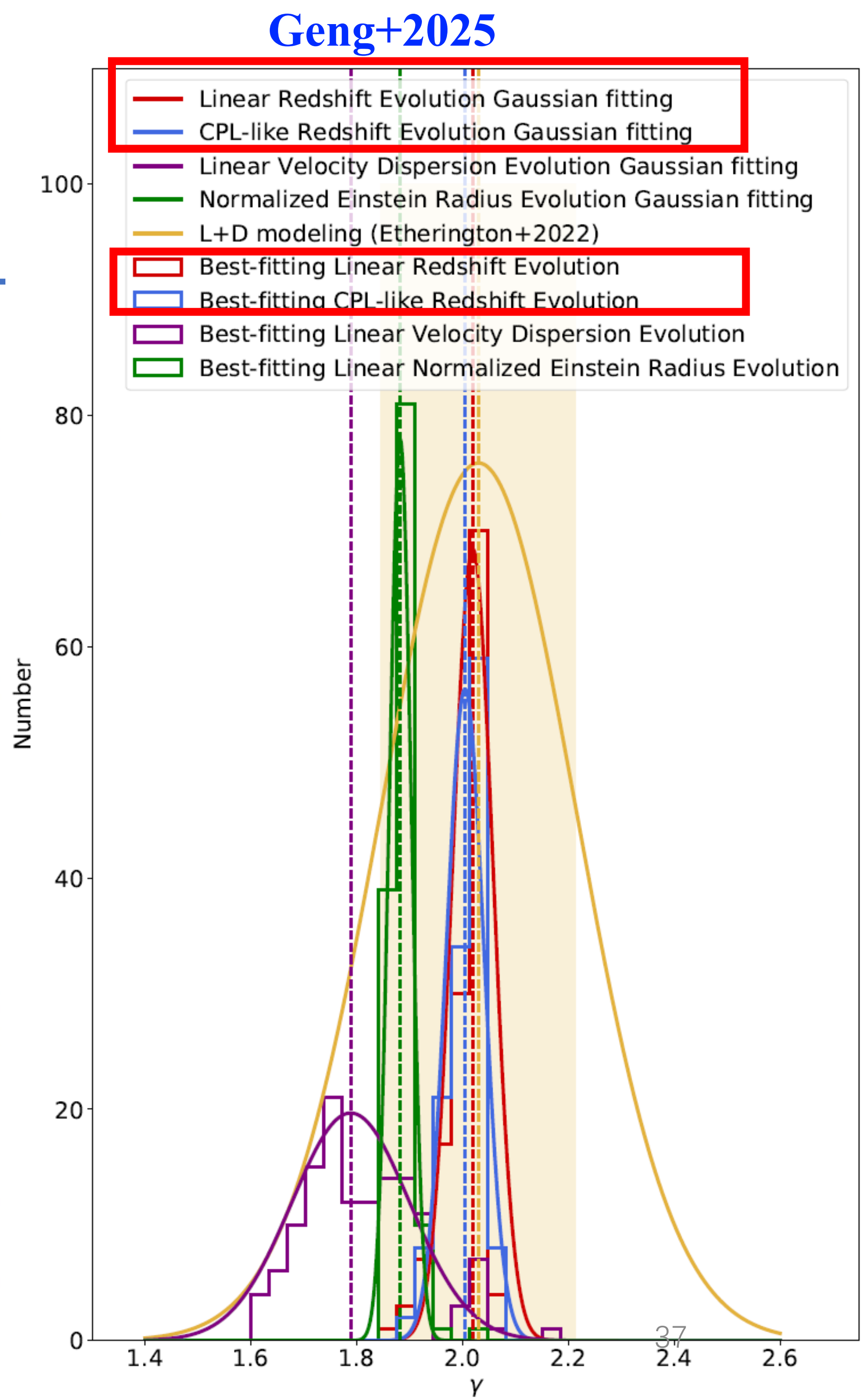
$$\gamma_0^{\rm glin} = 2.065 \pm 0.046$$
$$\gamma_s^{\rm glin} = -0.20 \pm 0.12$$
$$\delta_0^{\rm glin} = 2.14 \pm 0.16$$
$$\delta_s^{\rm glin} = -0.09 \pm 0.19$$



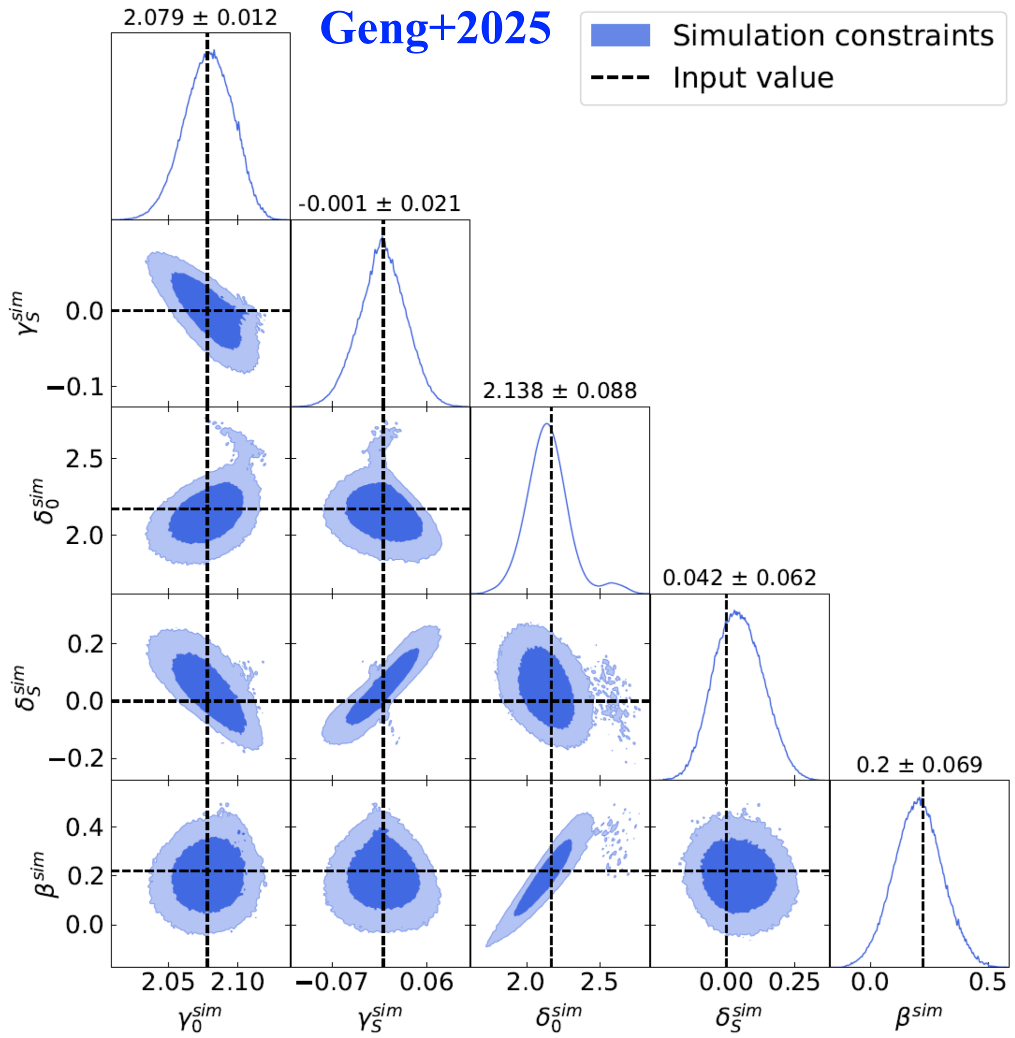
Results

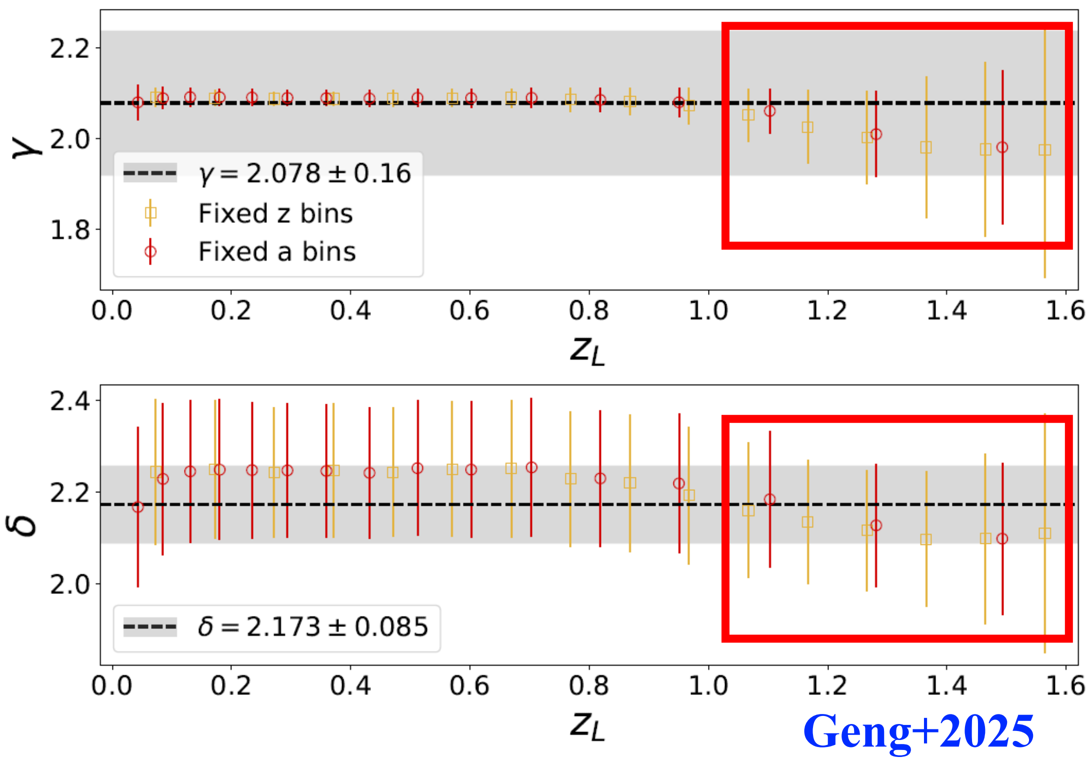
My results align well with Lensing+Dynamics works





What do we expect from LSST + Space Telescopes?

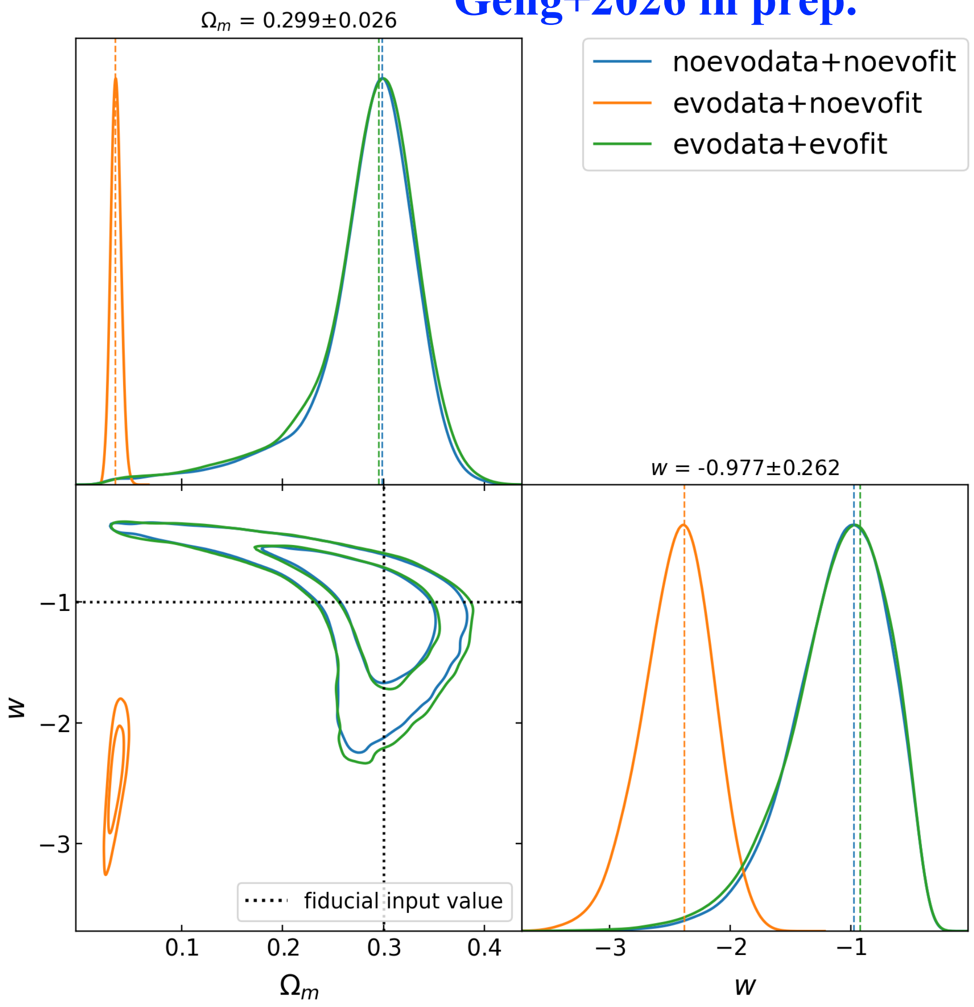




- High-redshift lenses (JWST, Euclid)
- Spatially resolved dynamic measurements (IFU)

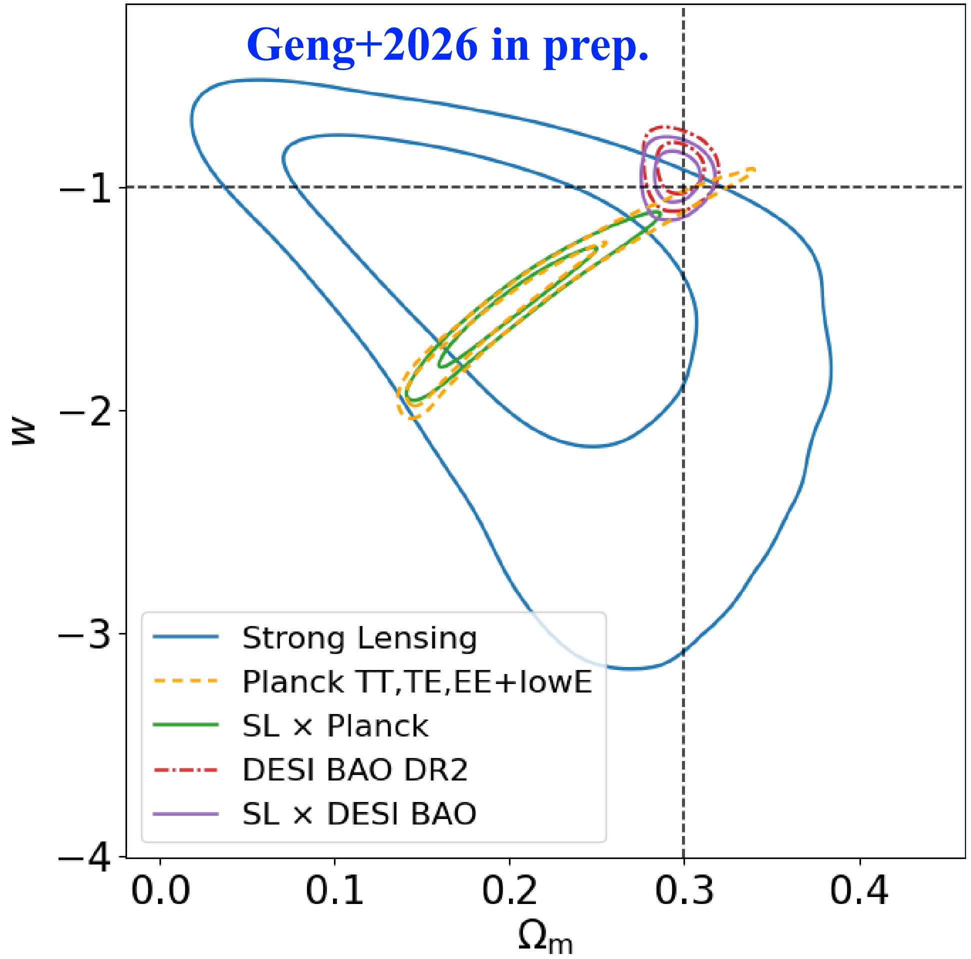
Results

Geng+2026 in prep.



Neglecting slope evolution can may cause cosmological parameter Ωm to deviate by more than 10σ.

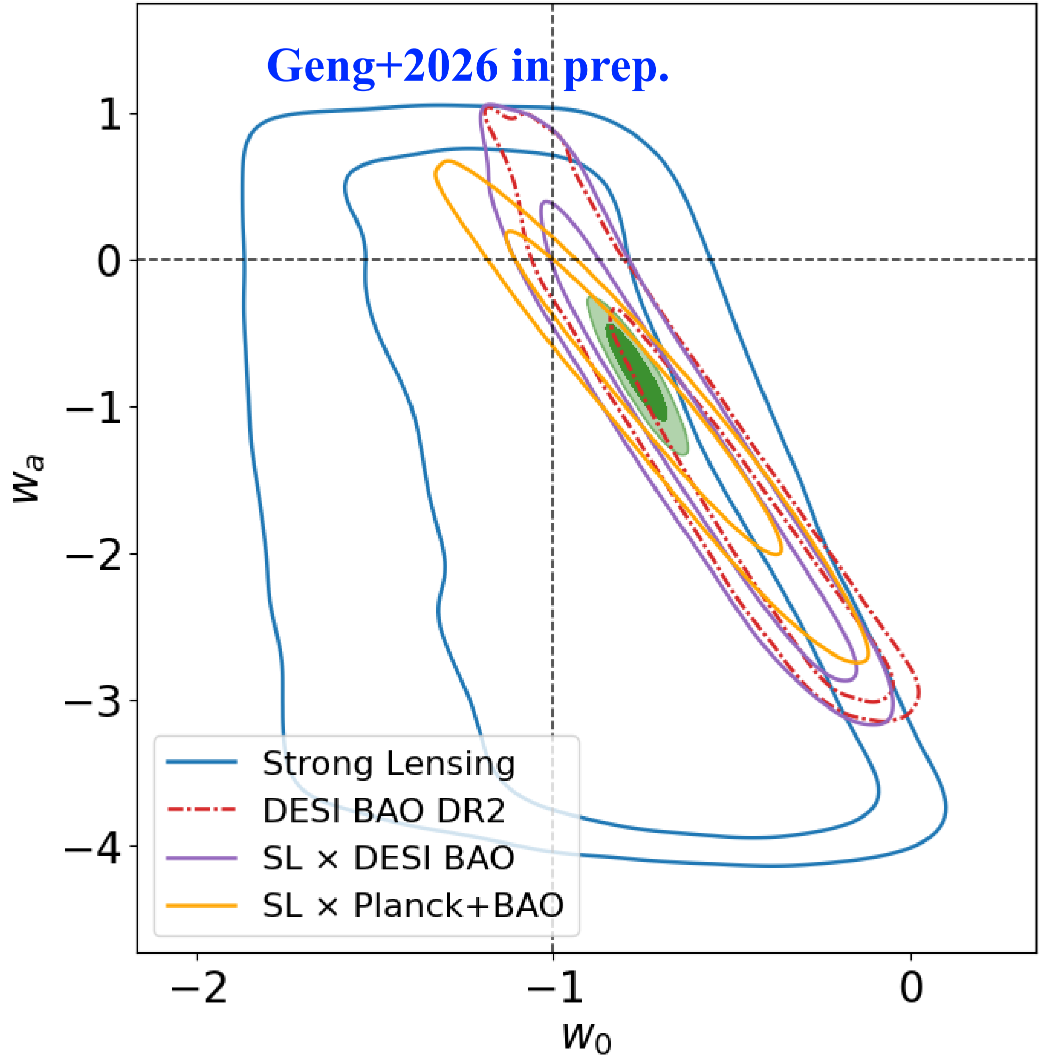
*w*CDM



For the 161 strong lensing sample:

- Accounting for the redshift evolution of the lens mass-density power law yields competitive constraints on Ω_m and w.
- Joint analysis with Planck further tightens the constraints effectively.

$w_0 w_a$CDM



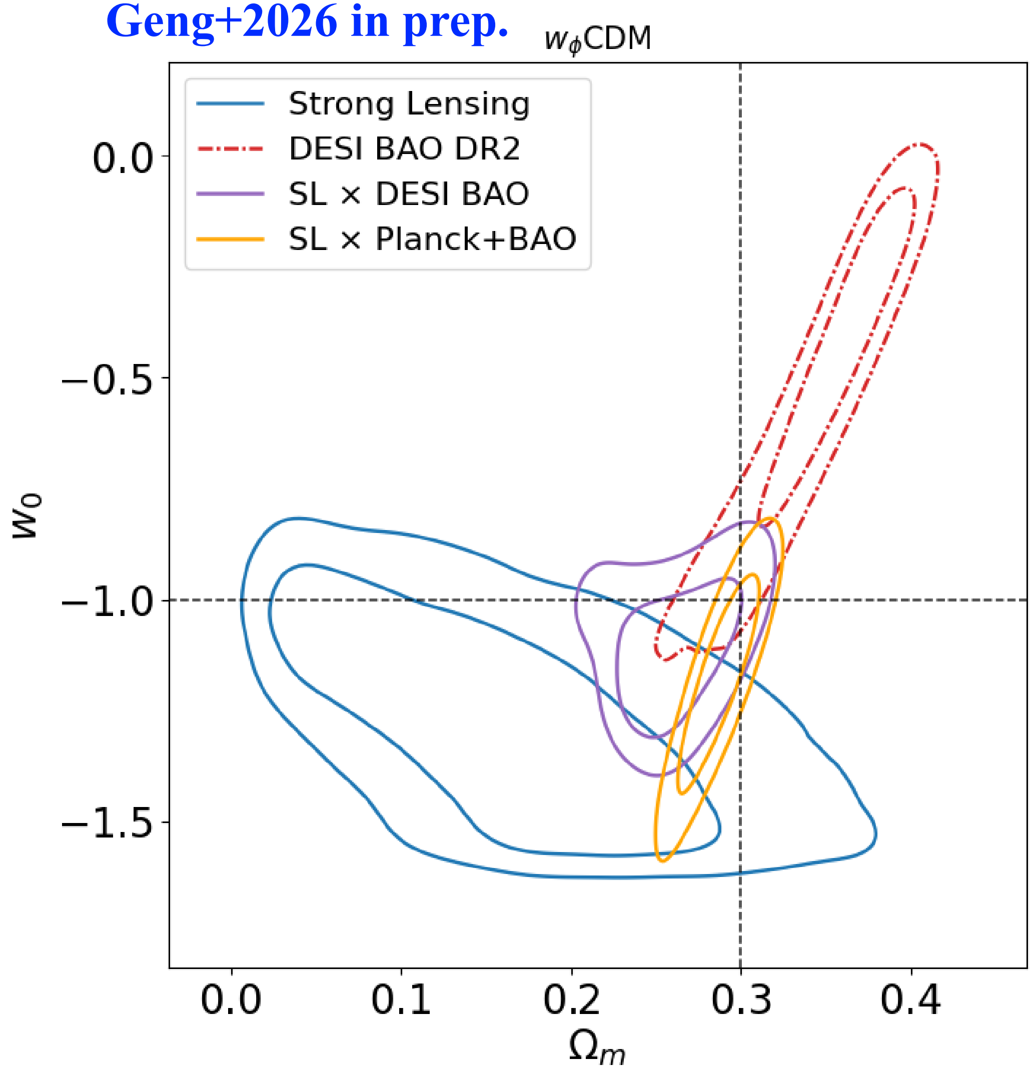
For the 161 strong lensing sample:

- Accounting for the redshift evolution of the lens mass-density power law yields competitive constraints on Ω_m and w_0, but weak leverage on w_a .

- Joint analysis with Planck+BAO closely reproduces DESI+CMB+DESY5 constraints.

w_ϕCDM

Geng+2026 in prep.



For the 161 strong lensing sample:

- Accounting for the theoretical bound on α yields a narrow constraint on w_0, but weak leverage on Ω_m.
- Joint analysis with Planck+BAO and DESI compensates for the deficiency of constraints on Ω_m.

4. Summary and Outlooks

Summary

- **Redshift evolution of the mass-density slope**
  I detect a **negative redshift evolution** in the total mass-density slope of lens galaxies using a **model-independent** approach. This trend is consistent with expectations from **gas-rich merger** scenarios. For an LSST-like sample of ~7,000 strong lenses, the expected precision on the evolution parameter is $\mathbf{\Delta(\partial\gamma/\partial z) \approx 0.021}$.

- **Hierarchical distance-ratio cosmography**
  I developed a **hierarchical Bayesian framework** that fully exploits **lensing distance ratios** for cosmological inference. With this approach, distance-ratio measurements can achieve Ωm constraints **comparable in precision to those from time-delay distances**, reaching the same order of magnitude.

- **Beyond-ΛCDM constraints from distance ratios**
  The hierarchical framework enables distance-ratio tests of cosmological models beyond standard ΛCDM. In particular, the inferred constraints on w0waCDM **closely reproduce** the results from **DESI + CMB + DES Y5** joint analyses, indicating a possible evolution of dark energy.

Publications

Published

[1]	**Investigating the redshift evolution of lensing galaxy density slopes via model-independent distance ratios** Authors: **Shuaibo Geng**, Margherita Grespan, Hareesh Thuruthipilly, Sreekanth Harikumar, Agnieszka Pollo, and Marek Biesiada A&A, 694, A196 (2025) https://doi.org/10.1051/0004-6361/202451894
[2]	**Revisiting the Hubble constant, spatial curvature, and cosmography with strongly lensed quasar and Hubble parameter observations** Authors: Tonghua Liu, Shuo Cao, Marek Biesiada, and **Shuaibo Geng** Astrophys.J. 939 (2022) 1, 37 https://iopscience.iop.org/article/10.3847/1538-4357/ac93f3

Submitted

[1]	*Hierarchical cosmological constraints through strong lensing distance ratio* Authors: **Shuaibo Geng** and Marek Biesiada
[2]	*The mass density slope of early-type galaxies: newest results from the redshift distribution of strong gravitational lenses* Authors: Xinyue Jiang, **Shuaibo Geng**, Shuo Cao, and Marek Biesiada

Outlooks

- **Advancing the hierarchical framework**
  I will refine the ANN-based distance-ratio reconstruction by incorporating the full SN Ia covariance to reduce reconstruction uncertainties. I will also quantify how lens sub-population hyperparameters (e.g., environmental components) propagate into cosmological constraints.

- **High-precision photometric redshifts for lenses and sources**
  I will develop a high-precision photo-z pipeline for strong-lensing systems by combining PAUS narrow-band imaging with public broad-band photometry, enabling more accurate redshift estimates for both lenses and sources and improving downstream lensing inference.

- **Probing the graviton mass with lensed gravitational waves**
  I will assess the prospects for constraining the graviton mass using inspiral signals from binary white dwarfs gravitationally lensed by the Milky Way central supermassive black hole, forecasting the achievable sensitivity under realistic detector and lensing configurations.