From Gravitational Symmetries to the Area Law

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Motivation

Gravity is described by General Relativity (1915)

$$S_{\mathrm{EH}} = \frac{M_{\mathrm{Pl}}^2}{2} \int \mathrm{d}^4 x \sqrt{-g} R \qquad\qquad M_{\mathrm{Pl}} = (8\pi G_N)^{-\frac{1}{2}}$$

- Works well for energy scale $E << M_{\mathrm{Pl}} c^2$

For an object of mass M, quantum effects (1925) become important at Compton wave length

$$\lambda_C = \frac{\hbar}{Mc}$$

- When the Schwarzschild radius $r_s = 2GMc^{-2}$ is comparable to $\lambda_C \longrightarrow$ Quantum Gravity

$$M \sim M_{\mathrm{Pl}}$$

Motivation

- 100 years and still no satisfying Quantum Gravity
- Most quantum theories are quantized classical theory

$$\text{Classical} \longrightarrow \text{Quantum}$$

- Nature works the other way!

$$\text{Quantum} \xrightarrow{\hbar \to 0} \text{Classical}$$

$\longrightarrow$ What can we say about Quantum Gravity?

- Symmetries organize the possible quantum states

Spin $SO(3)$, E&M $U(1)$, QCD $SU(3)$, Particles $ISO(1,3)$

Symmetries of gravity

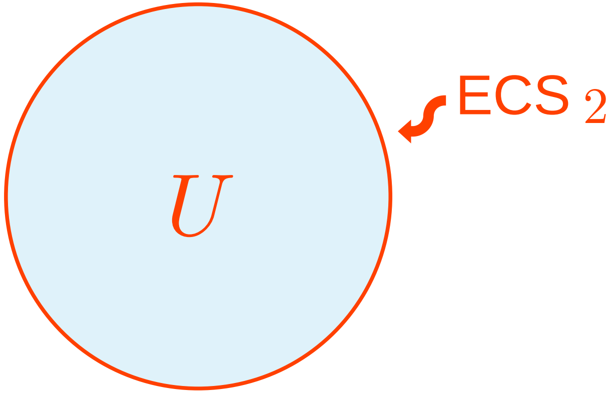
- What are the symmetries of gravity?

  $\longrightarrow$ Gravity is build on diffeomorphism invariance (Equivalence Principle)

- Noether: "*To every continuous symmetry, there exists an associated conserved quantity*" (Translation $\leftrightarrow$ Momentum, U(1) $\leftrightarrow$ Electric charge,...)

- For a subregion U in spherically symmetric spacetime, the conserved Noether charges on the boundary: [Kowalski-Glikman & **LV,** 2510.25851][Ciambelli & Leigh, 2104.07643, *Phys.Rev.D 104 (2021) 4*]

$$\mathrm{ECS}_2 = \mathrm{SL}(2,\mathbb{R}) \ltimes \mathbb{R}^2$$



Spherically Symmetric Gravity $\leftrightarrow$ ECS$_2$

Spherically symmetric corner program

[Ciambelli & Kowalski-Glikman & **LV**, 2507.16800, *accepted in JHEP*]

[Kowalski-Glikman & **LV**, 2510.25843]

[Neri & **LV**, 2507.16800, *accepted in Phys.Rev.D*]

Quantum Corners

- ECS representation theory
- Corner Coherent states
- Entanglement entropy of U

$$[\hat{H}^a, \hat{H}^b] = c^{ab}_c \hat{H}^c$$

$$\hat{\rho}_U$$

$$S_U = -\mathrm{Tr}\,(\rho_U \ln(\rho_U))$$

[**LV**, 2409.10624, *Phys.Rev.D 111 (2025) 8*]

[Ciambelli & Kowalski-Glikman & **LV**, 2406.07101, *Phys.Lett.B 866 (2025) 139544*]

Classical charges

- Covariant phase space
- ECS Noether charges
- Classical Casimir = Area

$$H[g_{\mu\nu}]$$

$$C_{\mathrm{ECS}}[g] \sim A^2_{\partial U}$$

[Assanioussi & Kowalski-Glikman & Mäkinen & **LV**, 2312.01918, Class.Quant.Grav. 41 (2024) 11]

[Ciambelli & **LV** et al., 2312.01918, PoS QG-MMSchools (2024) 010]

Coadjoint orbits

- Classical: moment maps
- Quantum: Berezin symbols
- Orbit method: classical to quantum

$$\langle \hat{H} \rangle = H$$

$$\langle \hat{C} \rangle = \frac{A^2_U}{l^4_p} + \mathcal{O}(\hbar)$$

[**LV**, 2510.25843]

Quantum Corners

- Look at the Quantum Corner Symmetry group (central extension of ECS)

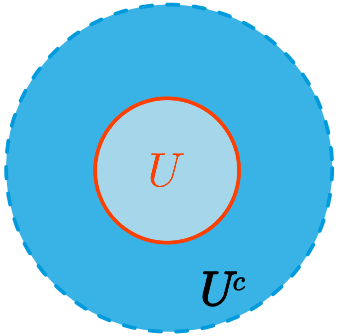
$$\mathrm{QCS} = \mathrm{SL}\,(2,\mathbb{R}) \ltimes H_3$$

- Sectors (rep) are labeled by $s \in \mathbb{R}_+$ and $c \in \mathbb{R}$

[**LV**, 2409.10624] [Neri & **LV**, 2507.16800]

- States are labeled by $E \in \mathbb{R}_+$ and $p \in \mathbb{R}$

$$\hat{\mathcal{C}}_{\mathrm{QCS}}|E,p\rangle = cs^2|E,p\rangle$$



$$|B\rangle = |E,p\rangle_U \otimes |E,p\rangle_{U^c}$$

[Ciambelli & Kowalski-Glikman & **LV**, 2507.16800]

[Ciambelli & Kowalski-Glikman & **LV**, 2406.07101]

- QCS coherent states (most classical)

$$|\zeta,\alpha\rangle = \int \mathrm{d}E\mathrm{d}p\,\psi_\zeta(E)\phi_\alpha(p)|E,p\rangle$$

- Take boundary B is in a coherent state $\rho^{\zeta,\alpha}$

$$\rho_U^{\zeta,\alpha} = \mathrm{Tr}_{U^c}\left(\rho^{\zeta,\alpha}\right)$$

- Entanglement entropy between U and U^c

$$S_U^{\zeta,\alpha} = -\mathrm{Tr}\left(\rho_U^{\zeta,\alpha}\ln(\rho_U^{\zeta,\alpha})\right)$$

$$S_U^{\zeta,\alpha} \sim 2s + \frac{1}{2}\ln(s), \quad s >> 1$$

Classical charges

- Einstein-Hilbert action

$$S = \int \mathrm{d}x^4 \sqrt{-g} R$$

- Spherically symmetric metric

$$\mathrm{d}s^2 = g_{ab}(x^c)\mathrm{d}x^a \mathrm{d}x^b + \rho^2(x^a)\mathrm{d}\Omega_S^2$$

- Covariant phase space $\longrightarrow$ Noether charges

  [Kowalski-Glikman & **LV**, 2510.25843]

$$t_a = \frac{1}{\sqrt{-g^{(0)}}} \frac{\epsilon^{cb}}{2G} \left[g^{(0)}_{ba} \rho_{(0)} \rho^{(1)}_c + \frac{\rho_{(0)}}{2} \left(\rho^{(1)}_b g^{(0)}_{ca} + \rho_{(0)} g^{(1)}_{cab} \right) \right]$$

$$N^b_a = \frac{\epsilon^{cb}}{\sqrt{-g^{(0)}}} \frac{\rho^2_{(0)}}{4G} g^{(0)}_{ca}$$

- ECS algebra

$$\{N^a_b, N^c_d\} = \delta^c_b N^a_d - \delta^a_d N^c_b$$
$$\{N^a_b, t_c\} = \frac{1}{2}\delta^a_b t_c - \delta^a_c t_b$$
$$\{t_a, t_b\} = 0.$$

- Deform the algebra

$$\{t_a, t_b\} = c\, \epsilon_{ab}$$

- Classical limit: $c \mapsto 0$

- Classical charges vs quantum charges?

Classical limit

- Coherent states to the rescue [**LV**, 2510.25843]

$$\langle\zeta,\alpha|\hat{J}^a_b|\zeta,\alpha\rangle = N^a_b,\ \langle\zeta,\alpha|\hat{P}_a|\zeta,\alpha\rangle = t_a$$

- Casimir operator [Kowalski-Glikman & **LV**, 2510.25843]

$$\langle\zeta,\alpha|\hat{C}_{\mathrm{QCS}}|\zeta,\alpha\rangle = \frac{c}{2}N^a_bN^b_a + \frac{1}{2}\epsilon^{ab}t_cN^c_bt_a + \mathcal{O}(\hbar)$$

$$s^2 = \frac{\rho^2_{(0)}}{4l_p^2}\left(\frac{\rho^2_{(0)}}{4l_p^2} + \frac{1}{2c\hbar}g^{(0)ab}t_at_b\right) + \mathcal{O}(\hbar)$$

- Classical limit $c \mapsto 0$

$$s = \frac{4\pi\rho^2_{(0)}}{4cl_p^2} + \mathcal{O}(c)$$

$$\langle\alpha|\hat{X}(t)|\alpha\rangle = x(t)$$
$$\langle\alpha|\hat{P}(t)|\alpha\rangle = p(t)$$

$$\langle\hat{C}_{\mathrm{SU}(2)}\rangle = \hbar^2j^2 + \hbar^2j$$
$$= r^2 + \hbar r$$

Schwarzschild example

- Schwarzschild solution in Eddington-Finkelstein coordinates (v,r,θ,ϕ)

$$g_{\mathrm{Sch}} = -\left(1-\frac{2GM}{r}\right)\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}r + r^2\mathrm{d}\Omega_S^2 \qquad g_{ab} = -\left(1-\frac{2GM}{r}\right)\mathrm{d}v^2 + 2\,\mathrm{d}v\,\mathrm{d}r, \quad \rho(v,r) = r$$

- We pick the corner as the horizon $r=r_s=2GM$ and $c=-\frac{\epsilon^2}{32\pi G}$

$$s^2 = \frac{\rho_{(0)}^2}{4l_p^2}\left(\frac{\rho_{(0)}^2}{4l_p^2} + \frac{1}{2c\hbar}g^{(0)ab}t_a t_b\right) = \frac{4\pi G^2M^4}{\hbar^2\epsilon^2} + \mathcal{O}(\epsilon^0)$$

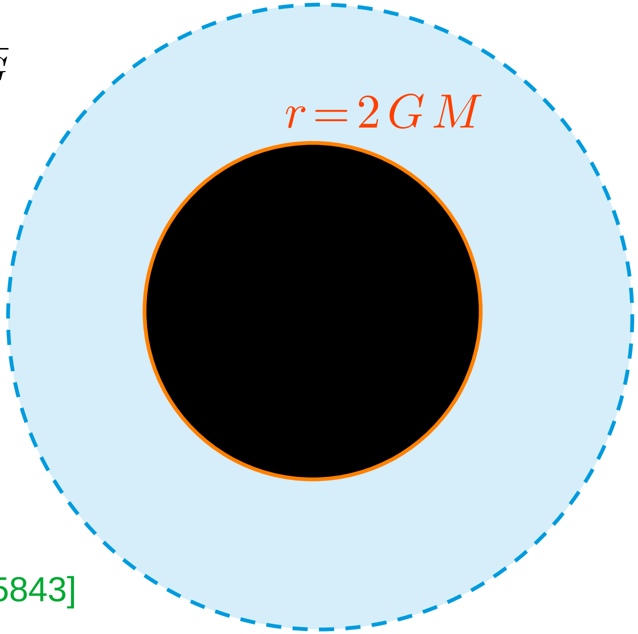
- If the boundary is in a coherent state $|\zeta,\alpha\rangle$

$$S_{\mathrm{Sch}} = \frac{A_{r_s}}{4l_p^2\epsilon} + \frac{1}{2}\ln\left(\frac{A_{r_s}}{4l_p^2\epsilon}\right) + \mathcal{O}(\epsilon)$$

$$A_{r_s} = 4\pi r_s^2$$

Also true for deSitter, and Minkowski!

[Kowalski-Glikman & **LV**, 2510.25843]



Conclusion and further work

- The entanglement entropy of a spherical subregion gives the Bekenstein-Hawking area law and logarithmic quantum corrections in the semi-classical limit
- This gives a symmetry based understanding of the famous area law, without needing to quantize a particular theory
- Strong evidence for the corner program for quantum gravity $\rightarrow$ Non-pertubative playground to understand entropy in quantum gravity

<u>**Further work**</u>

- Generalize to higher dimensions (non spherically symmetric)
- Compute quantum fluctuations of the area and connect with Verlinde-Zurek formula

$$(\Delta A)^2 \propto A$$

- Further the corner program: Emergence of geometry, dynamics, quantum cosmology, ...