Production of Self Interacting Scalars in the early Universe

Esau Cervantes
Supervisor: **Andrzej Hryczuk**
Oct 16, 2025

Graduate seminar

NATIONAL CENTRE FOR NUCLEAR RESEARCH
ŚWIERK

Content

Freezing-in Cannibal Dark Sectors

DOI:10.1007/JHEP11(2024)050

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standard cosmology

Freezing-in Cannibals
with Low-reheating Temperature

DOI:10.1007/JHEP09(2025)083

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non-standard cosmology

Early Universe *dynamics* of Cannibal DM

Current collaboration with

Felix Kahlhoefer, **Jonas Matuszak** and **Rosellon Santiago** from KIT (Germany)

Cosmological (inverse) phase transitions

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- **All** required numerical implementations.
- **All** parametric scans.
- Results (relic abundance, bounds, etc).
- Analysis and paper writing.

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Same as above

Current collaboration with

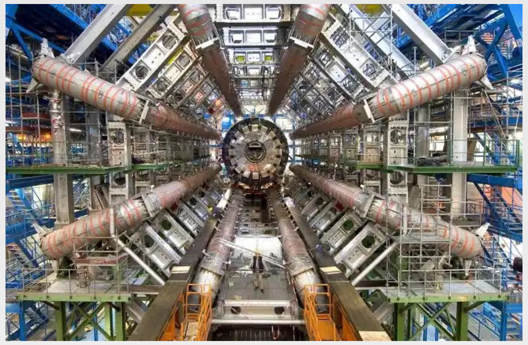
Felix Kahlhoefer, **Jonas Matuszak** and **Rosellon Santiago** from KIT (Germany)

- Deriving the correct equations to solve.
- **All** required numerical implementations.
- Analysis.
- Results (bubble profile, etc).

The study of Dark Matter

Experiments

(In)Direct Detection



http://www.ep.ph.bham.ac.uk/
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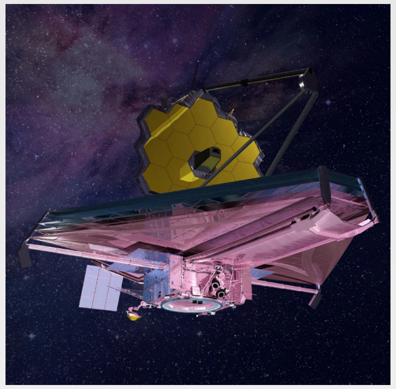

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The study of Dark Matter

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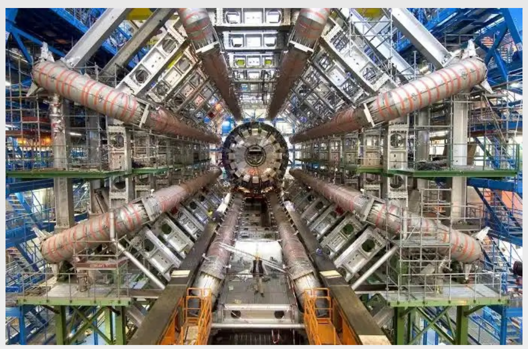

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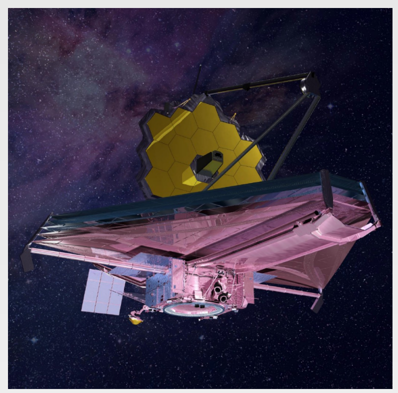

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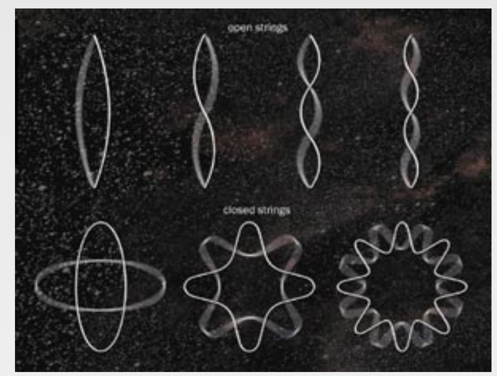
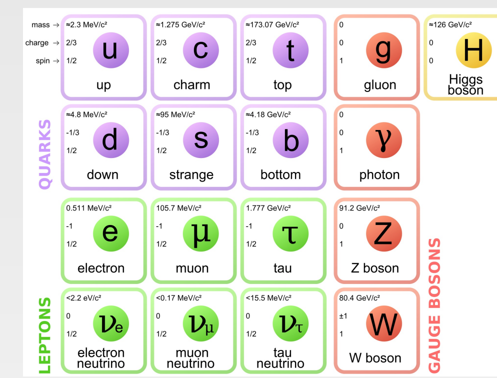
Model building

String theory (SUSY)



https://cerncourier.com/a/testing-times-for-strings/

Bottom-up



+DS

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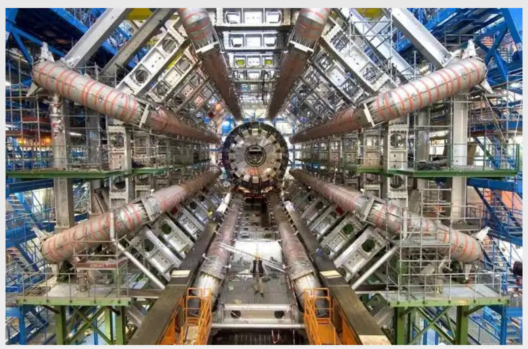

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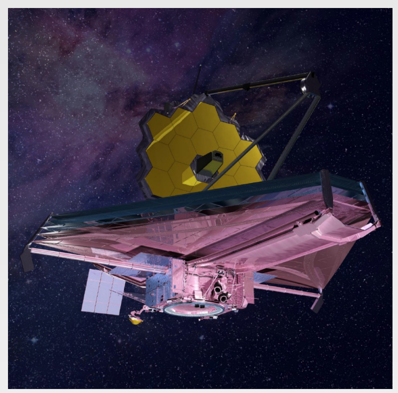

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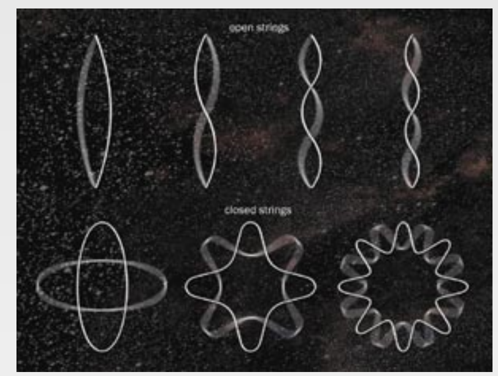

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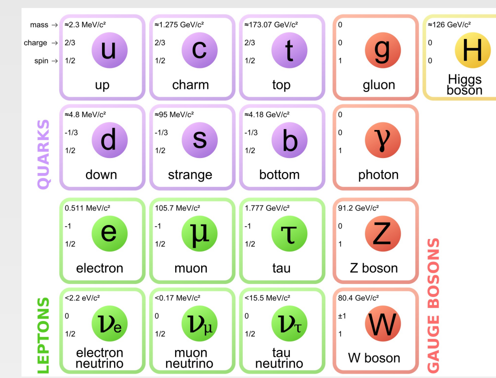

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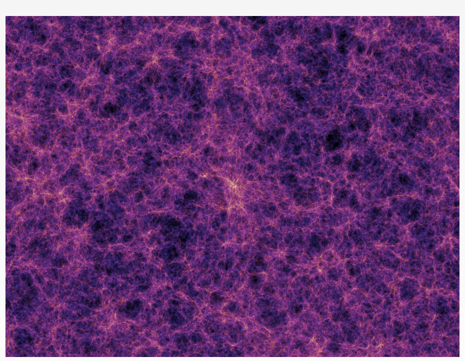
Astrophysics

Large structures

Distribution of DM



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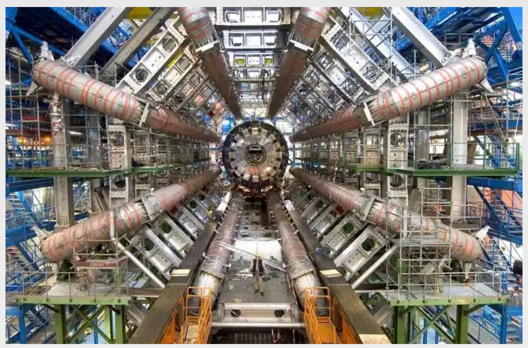

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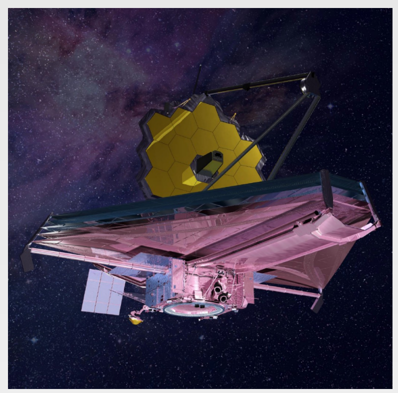

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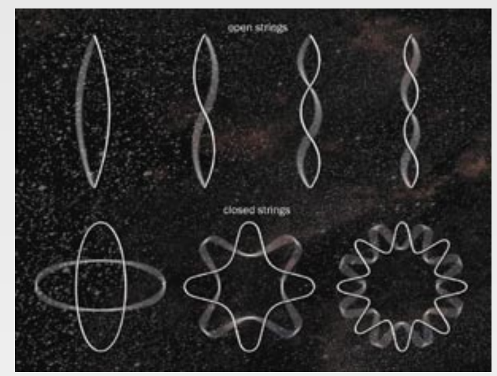

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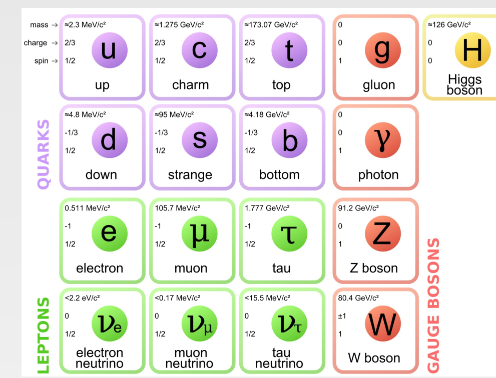

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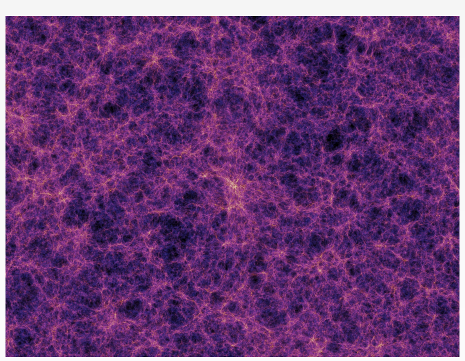

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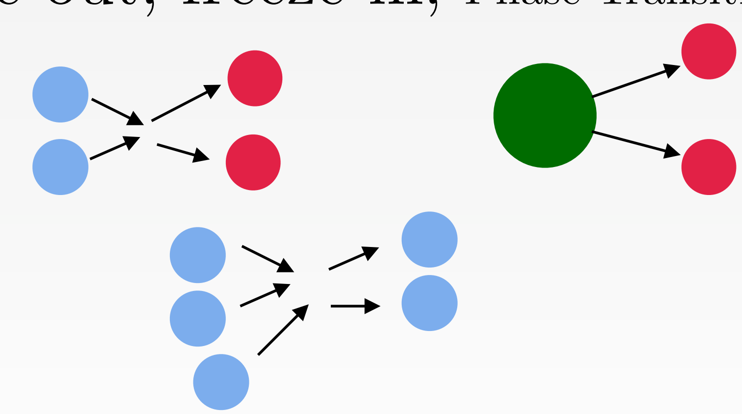
Early Universe dynamics

freeze-out, freeze-in, Phase Transitions



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non-standard cosmology

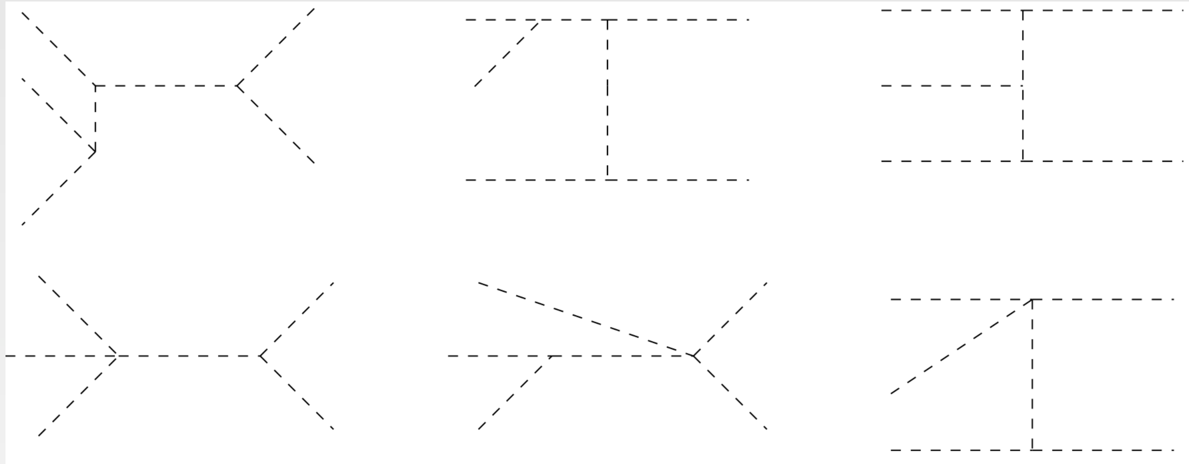
Early Universe *dynamics* of Cannibal DM

Cannibal Dark Matter

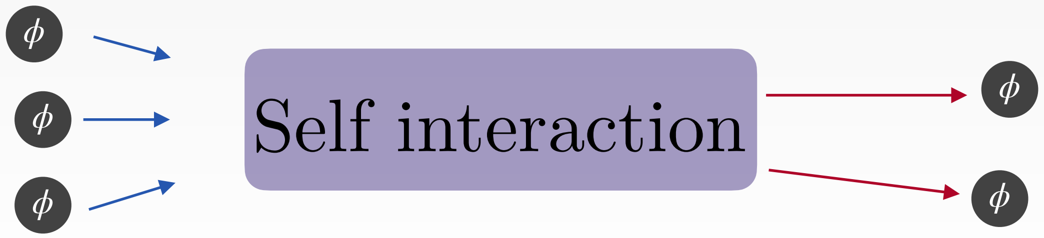
SELF-INTERACTING DARK MATTER

ERIC D. CARLSON
Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138
MARIE E. MACHACEK
Department of Physics, Northeastern University, Boston, MA 02115
AND
LAWRENCE J. HALL
Department of Physics, University of California; and Theoretical Physics Group, Physics Division, Lawrence Berkeley Laboratory, 1 Cyclotron Road, Berkeley, CA 94720
Received 1992 March 17; accepted 1992 April 20

Simple realisation with a scalar field: $\frac{g}{3!}\phi^3 + \frac{\lambda}{4!}\phi^4$



If DM is non-relativstic, $\Gamma_{3\to2} > \Gamma_{2\to3}$. The DM fluid **exchanges** particle number for kinetic energy!



Evolution of Cannibal DM

Absence of portals leads to $T_{DM} \neq T_{SM}$. Temperature evolution becomes relevant:

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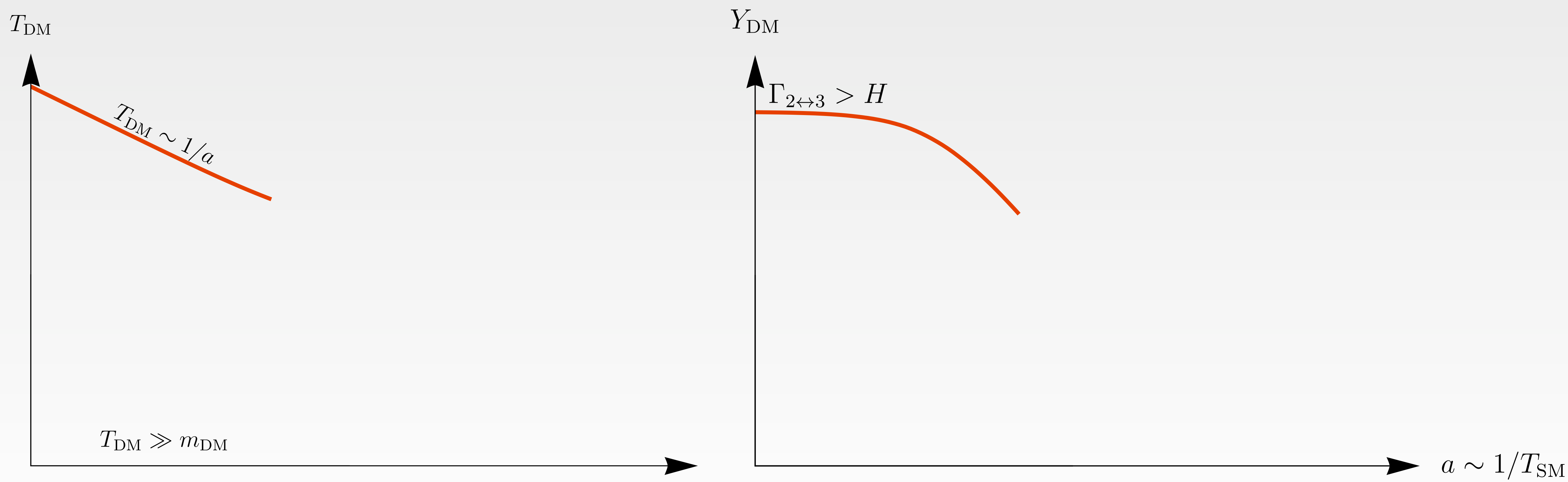
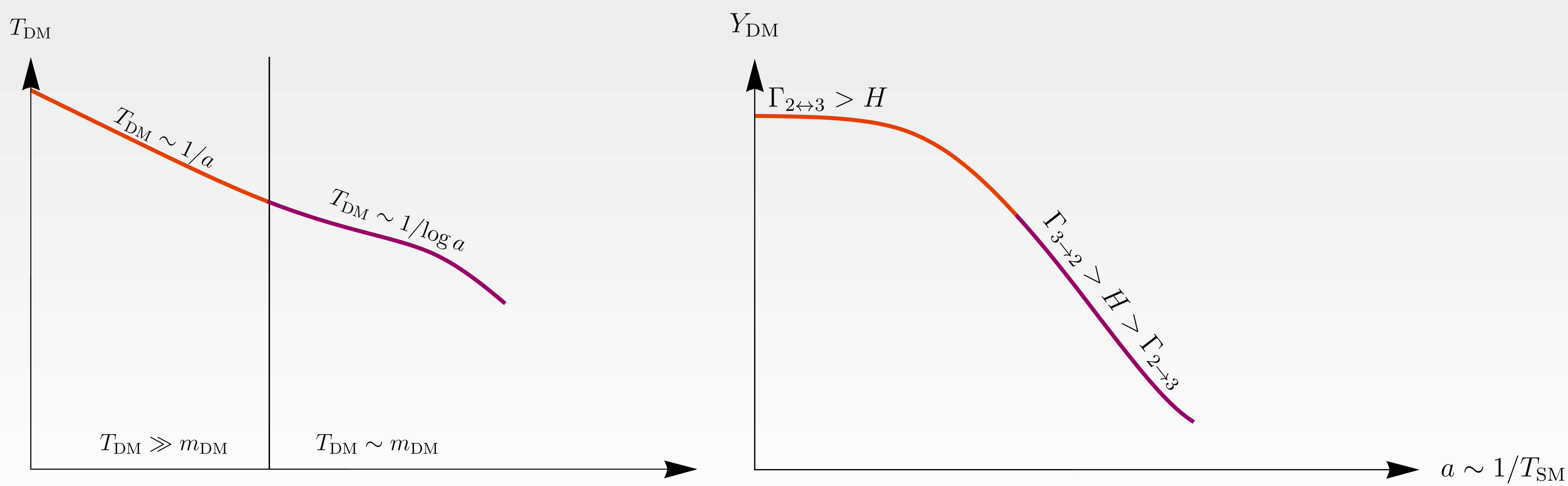


Evolution of Cannibal DM

Absence of portals leads to $T_{DM} \neq T_{SM}$. Temperature evolution becomes relevant:

- DM is initially *relativistic*;
- as the DM fluid cools down, the dark sector *exchanges* number of particles for kinetic energy;



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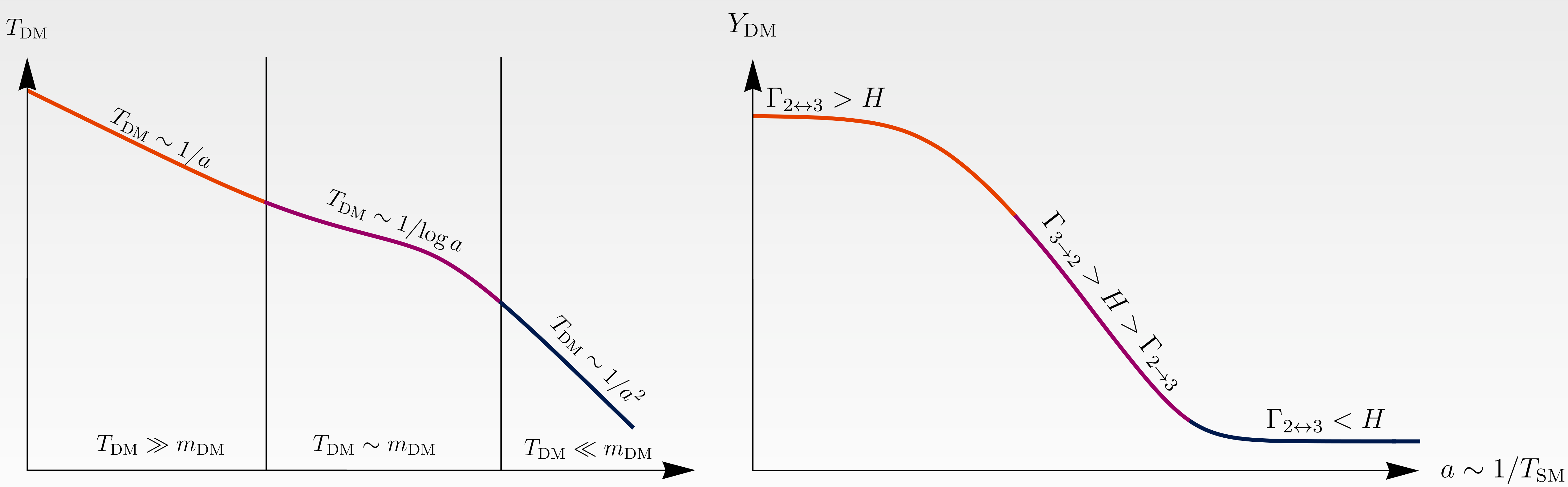

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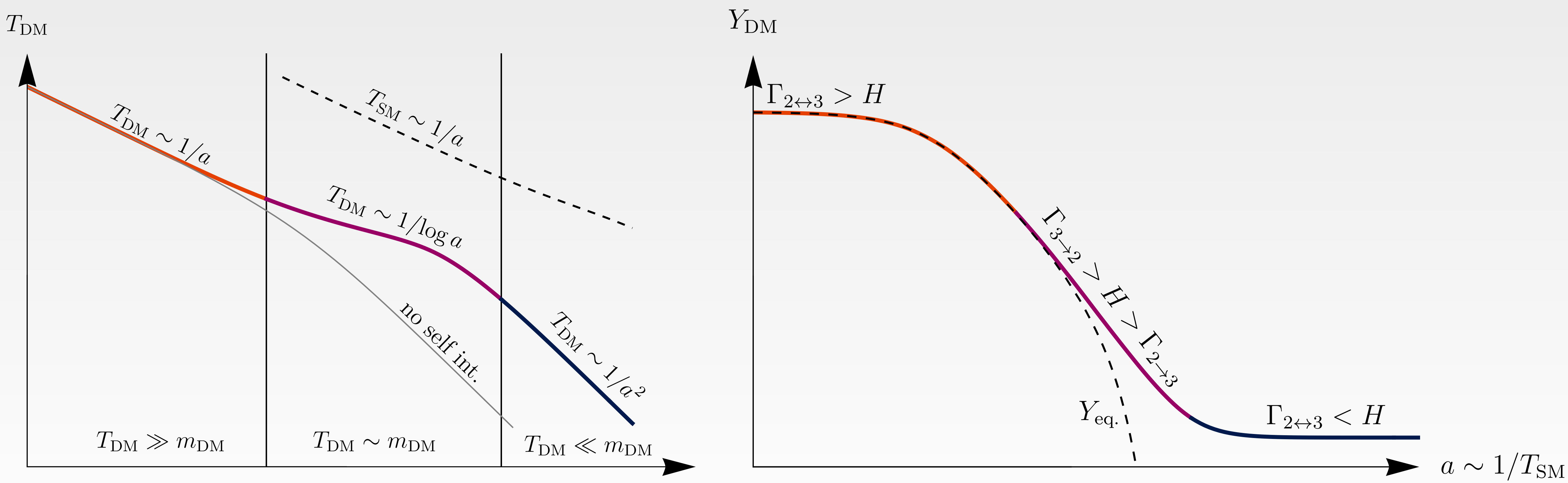
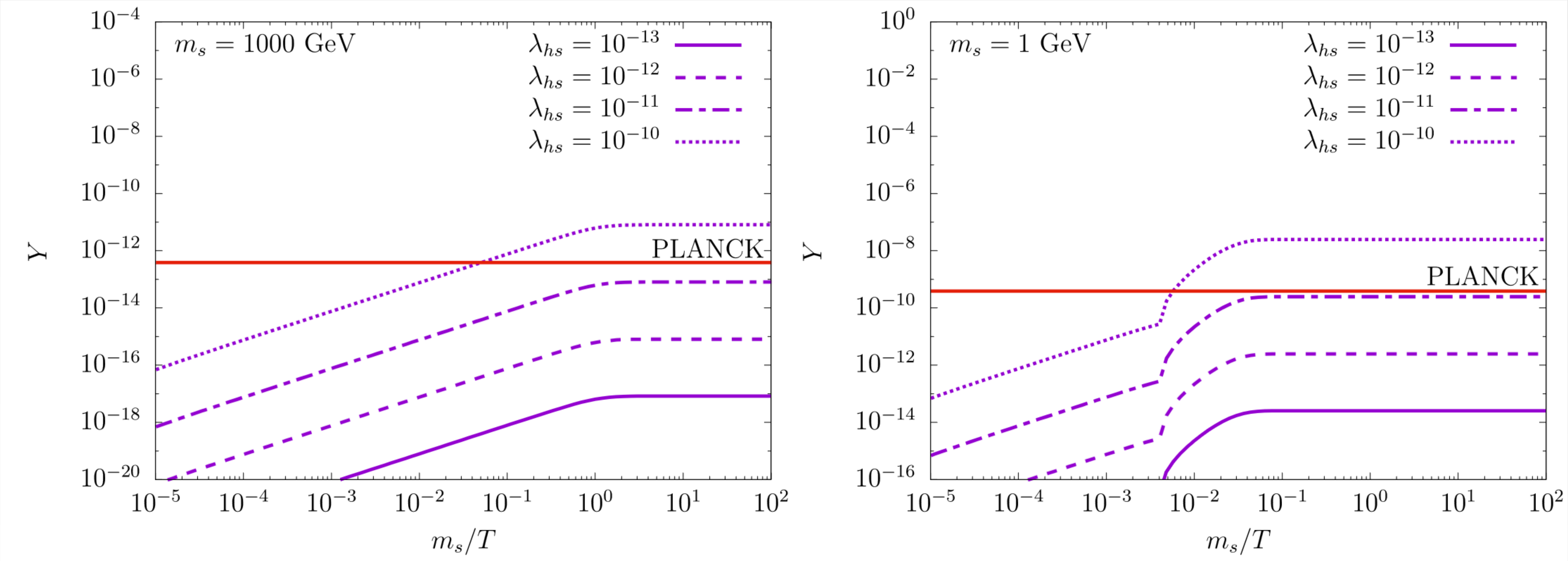


See also Hufnagel, Tygat 22 and Arcadi, Lebedev 19

The freeze-in mechanism

The freeze-in mechanism assumes $n_i = 0$, and **feeble** portals:



Lebedev, Toma 19

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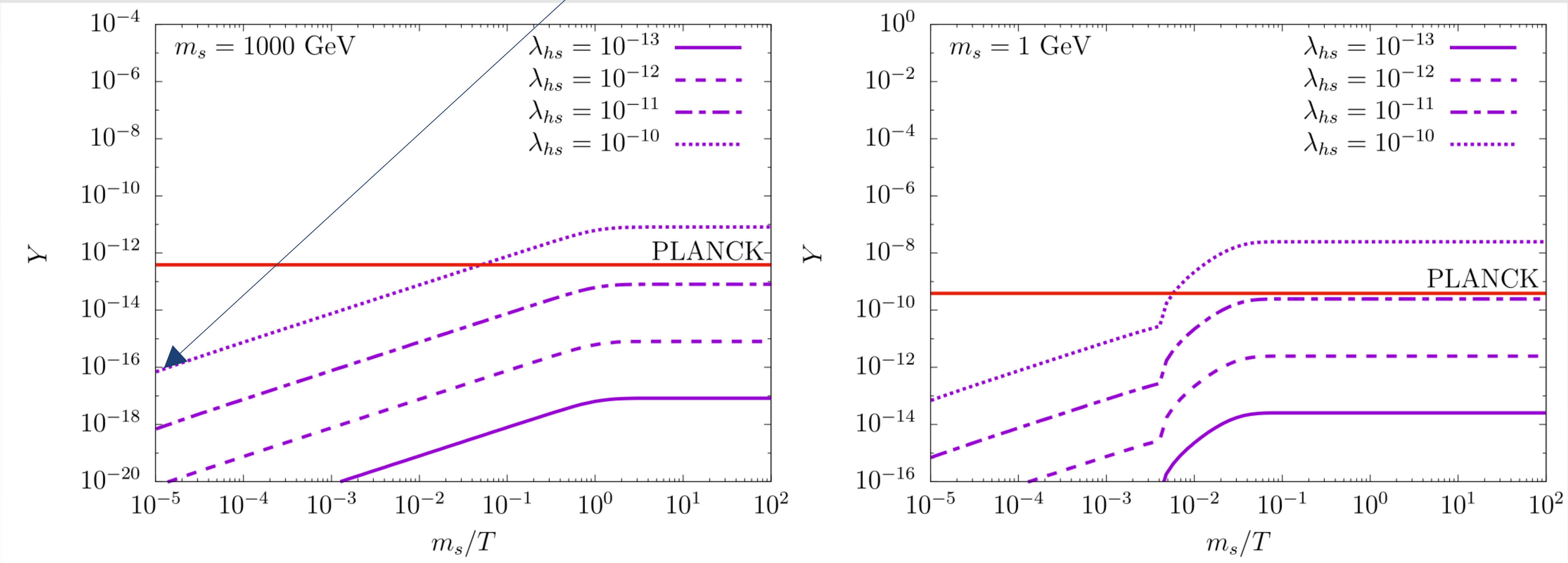

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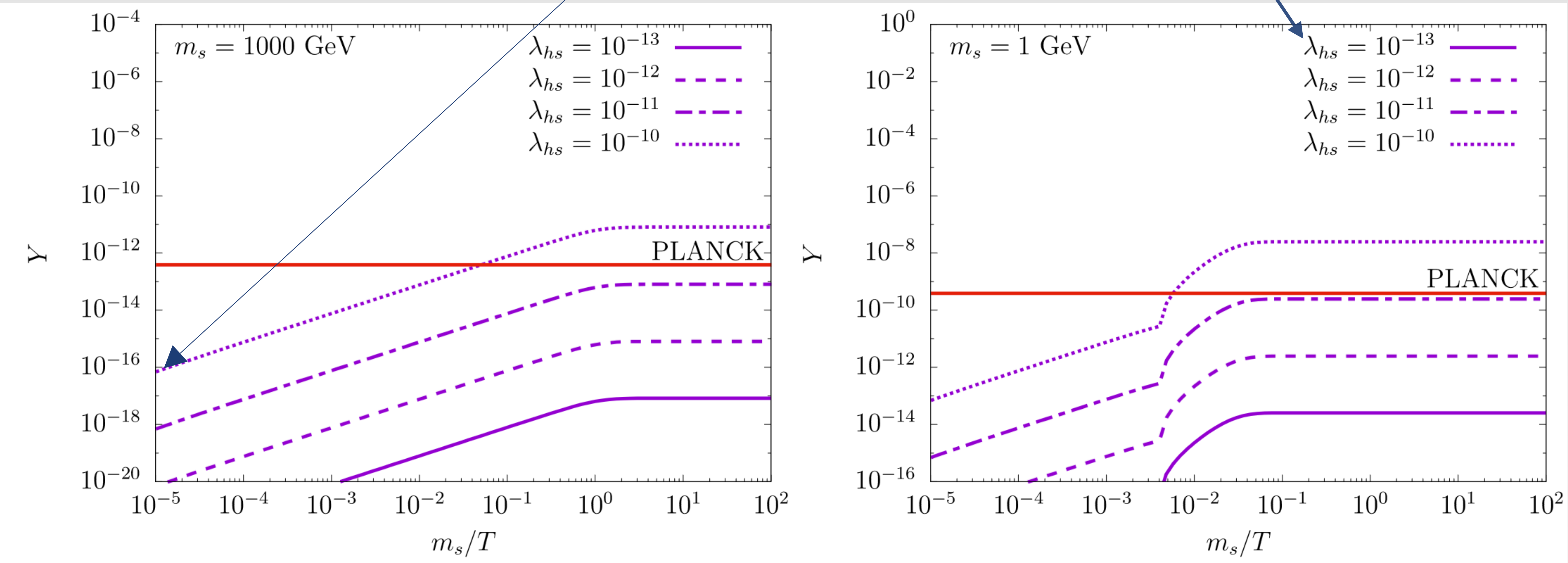
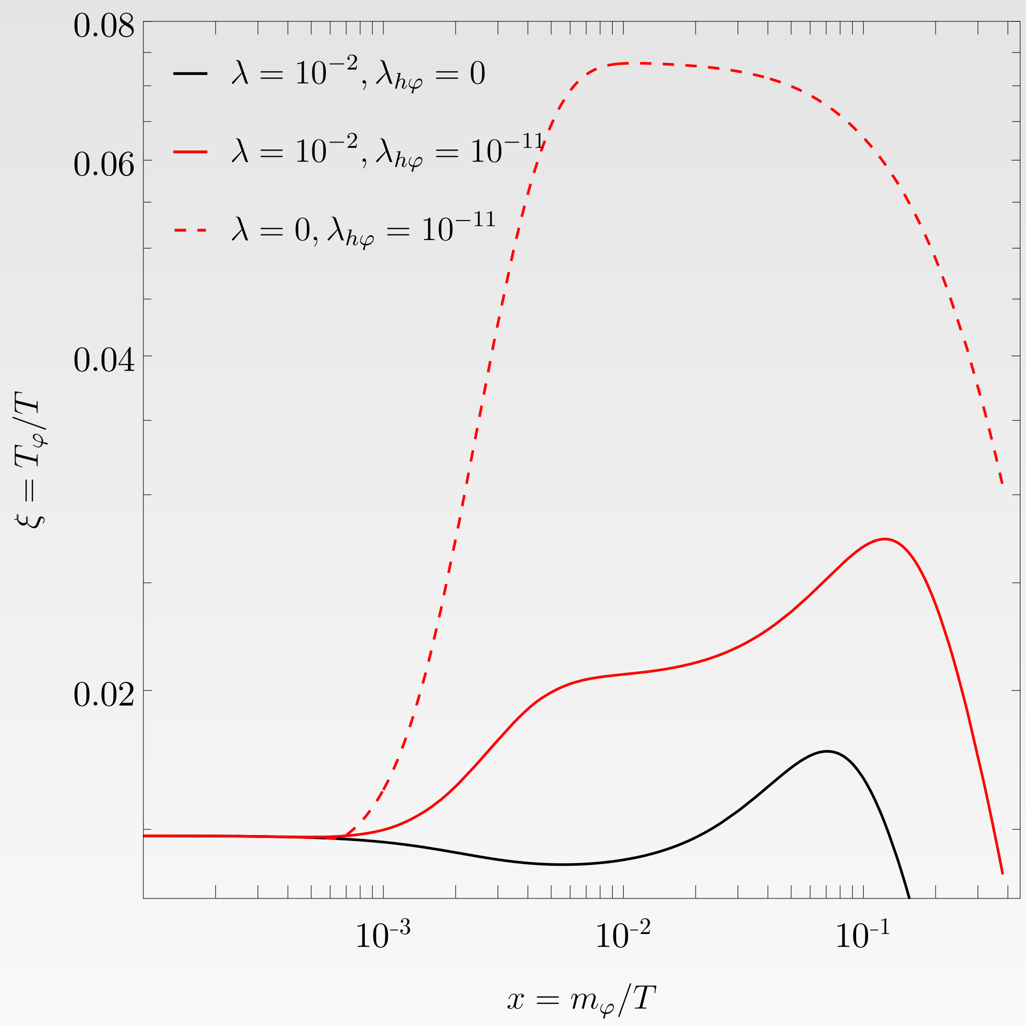
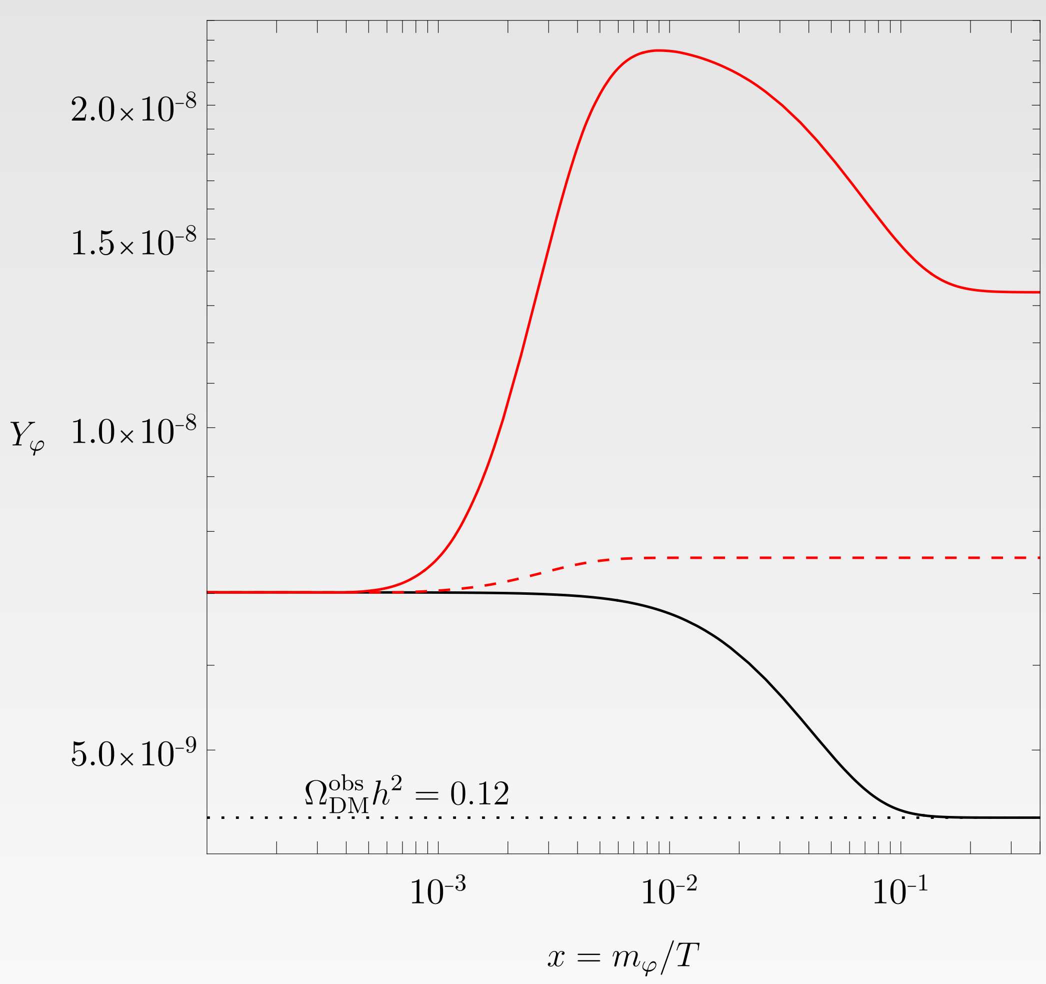


Lebedev, Toma 19

Cannibals produced via freeze-in

Consider $\mathscr{L} \supset -\lambda_{h\varphi}\varphi^2 H^\dagger H$, $\lambda_{h\varphi} \ll 1$, $\lambda_\varphi \geq 10^{-4}$ and initially cold DM; $T_{DM}/T_{SM} = 10^{-2}$:





See EC, A. Hryczuk 24

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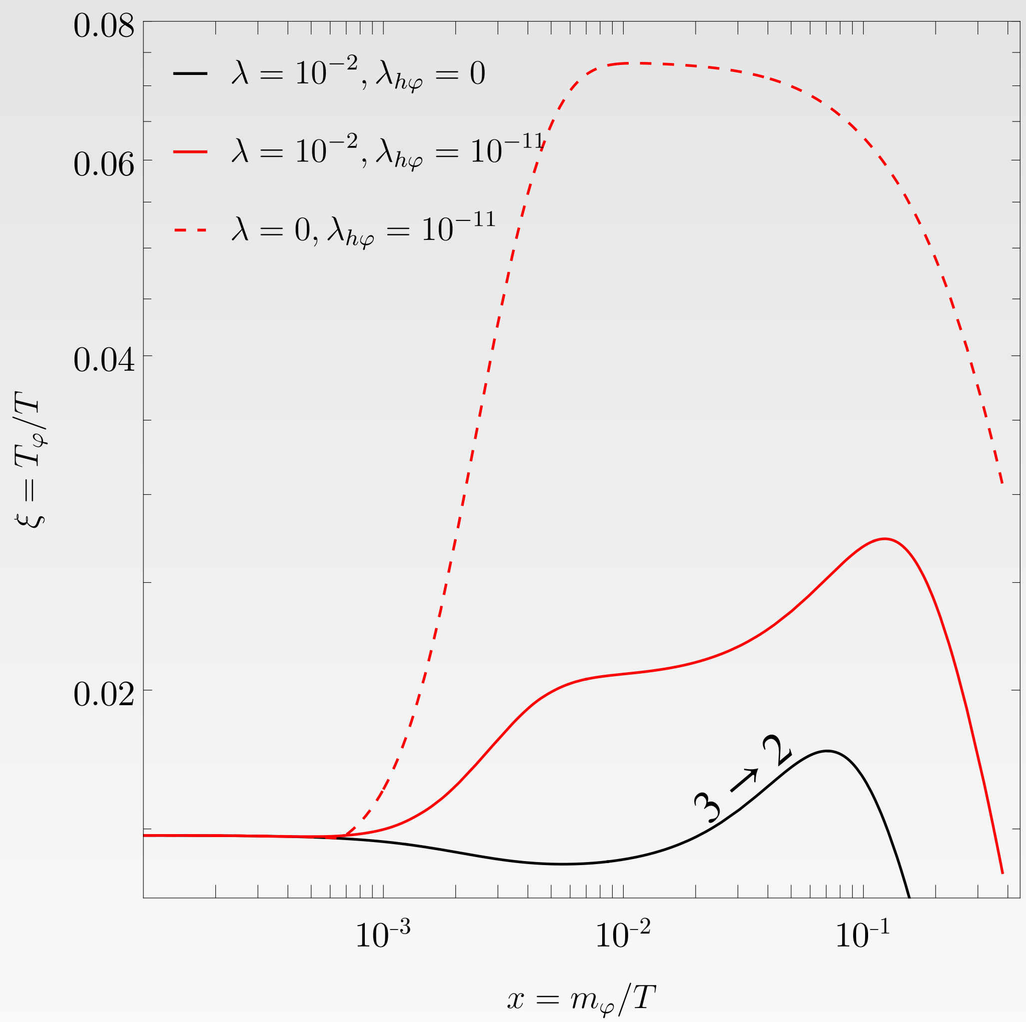

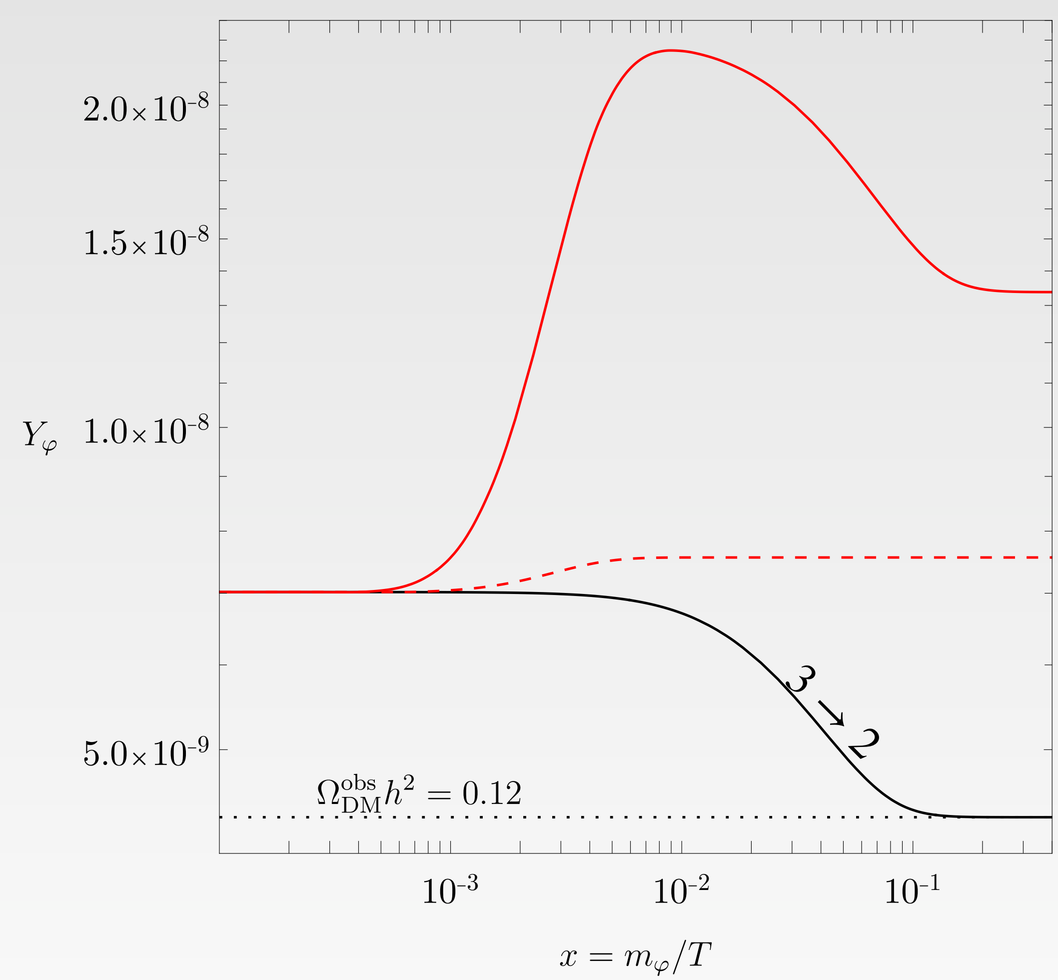

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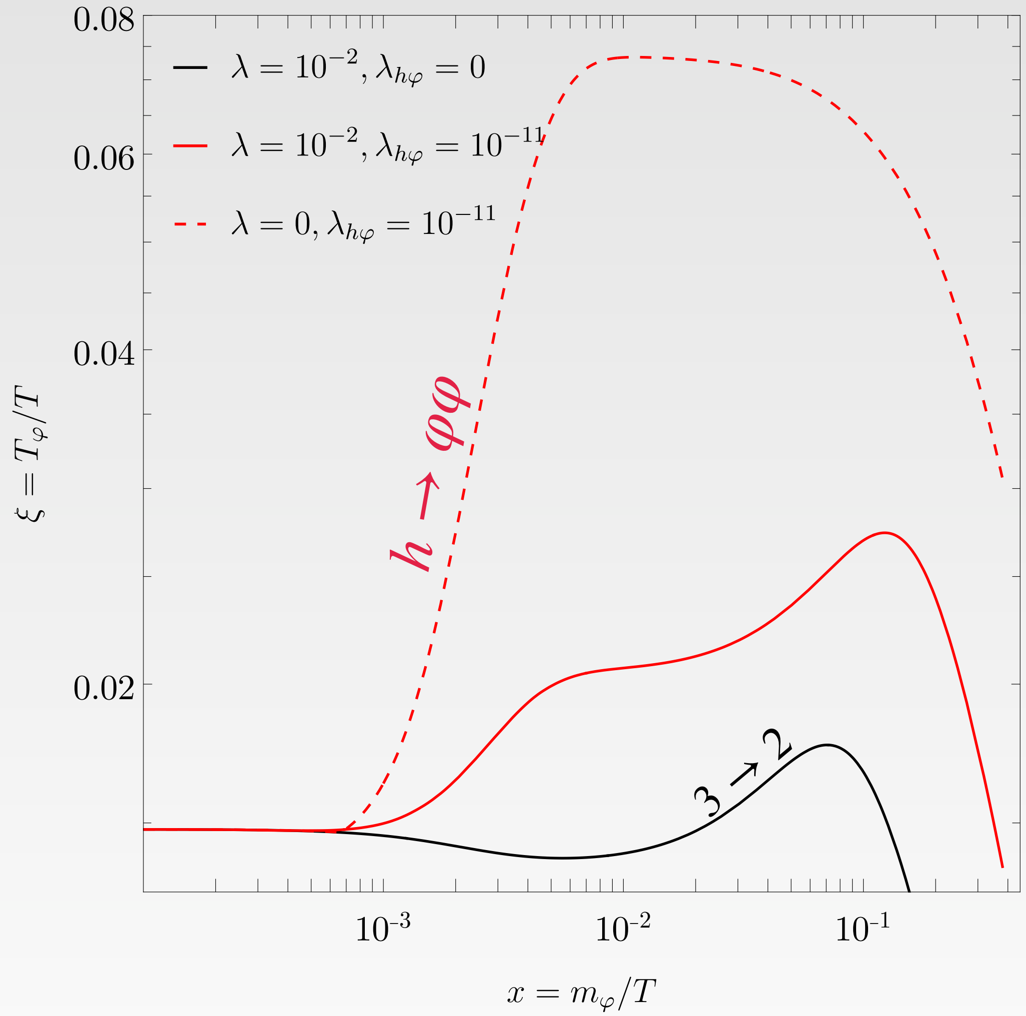

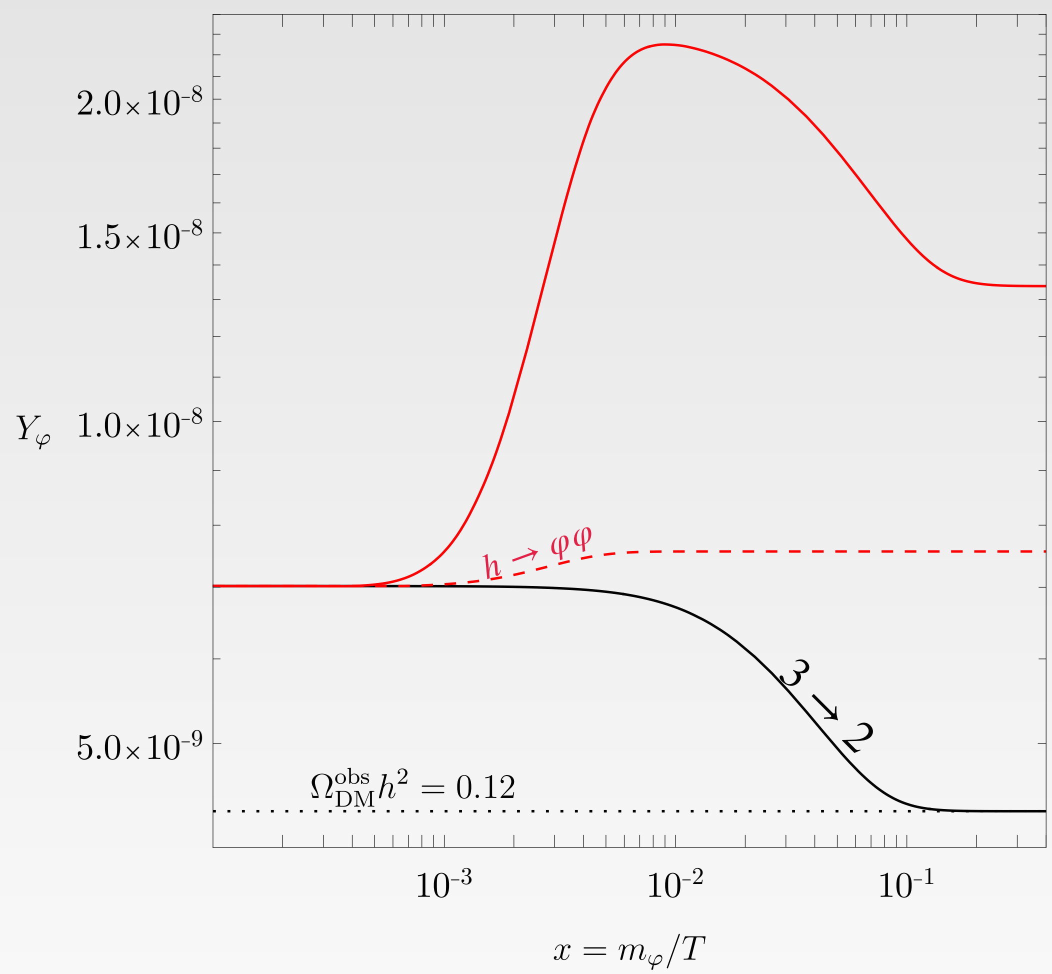

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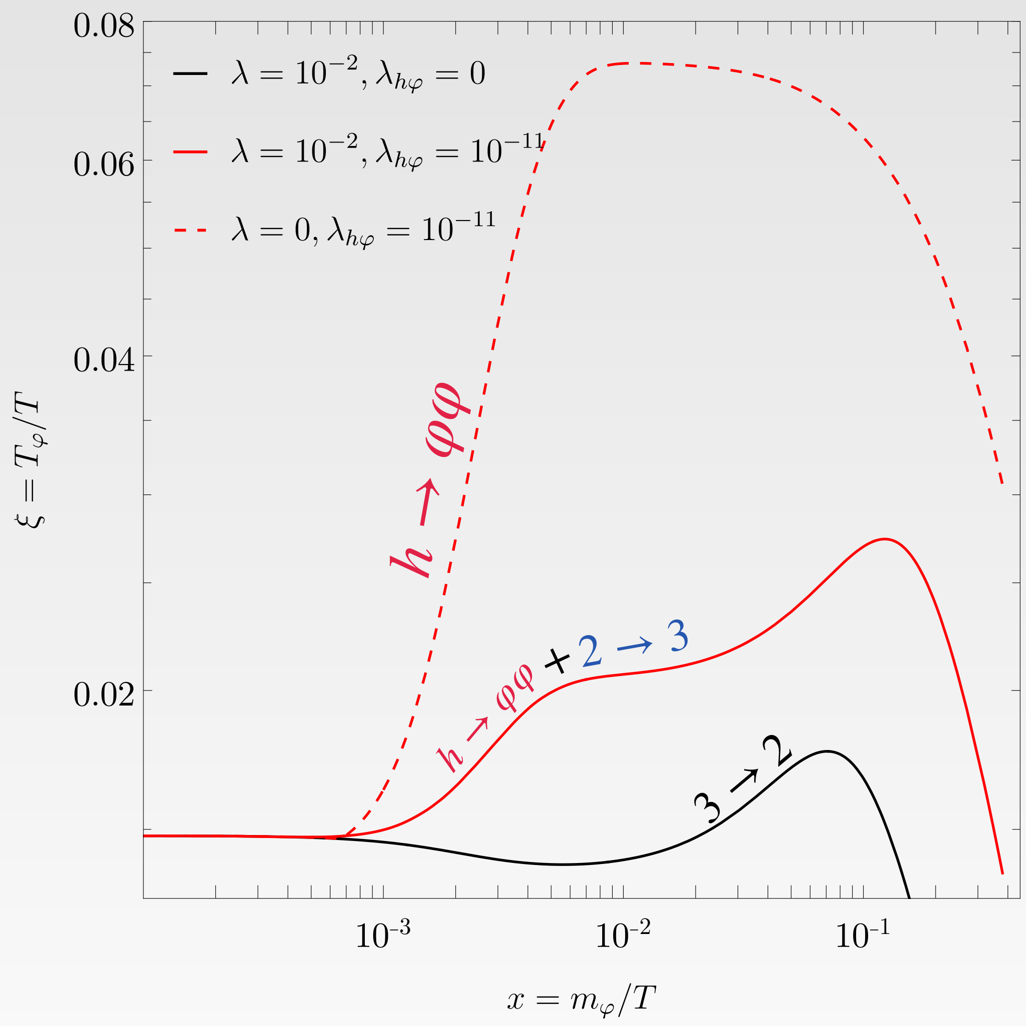
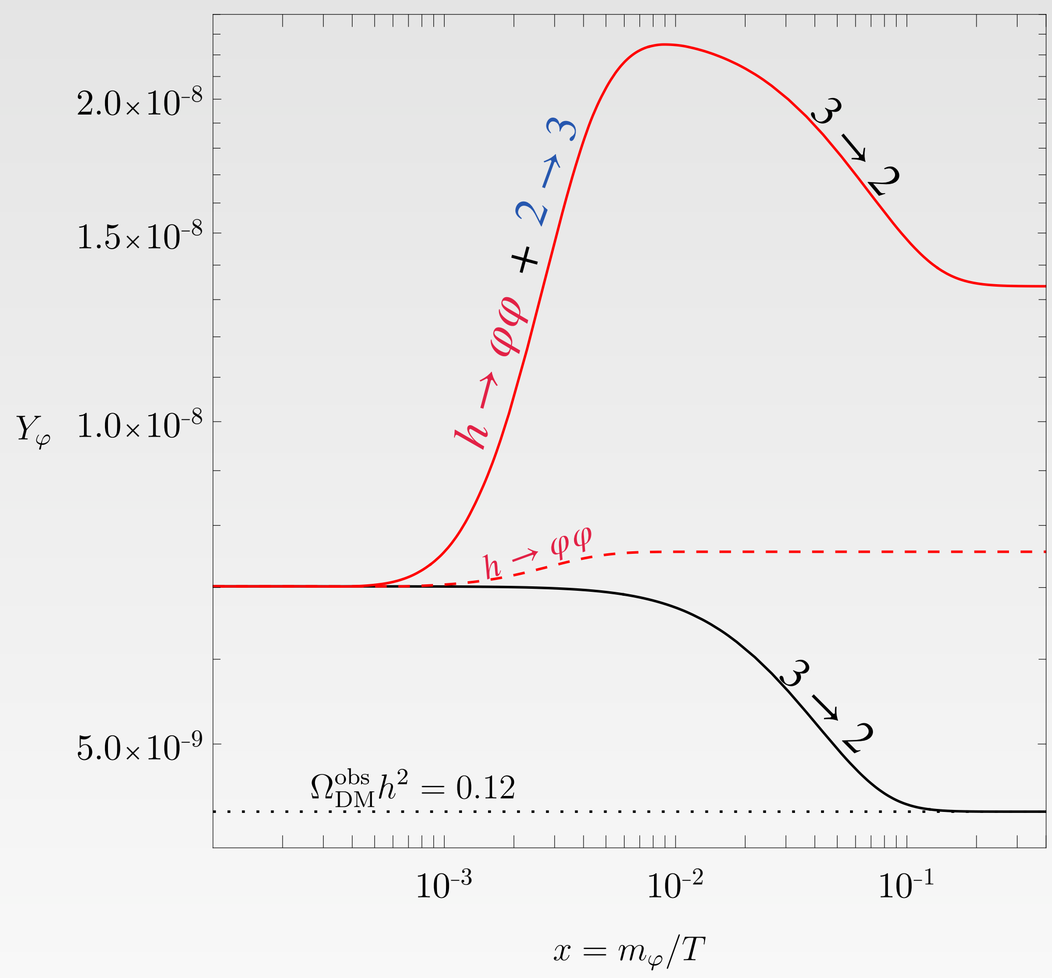




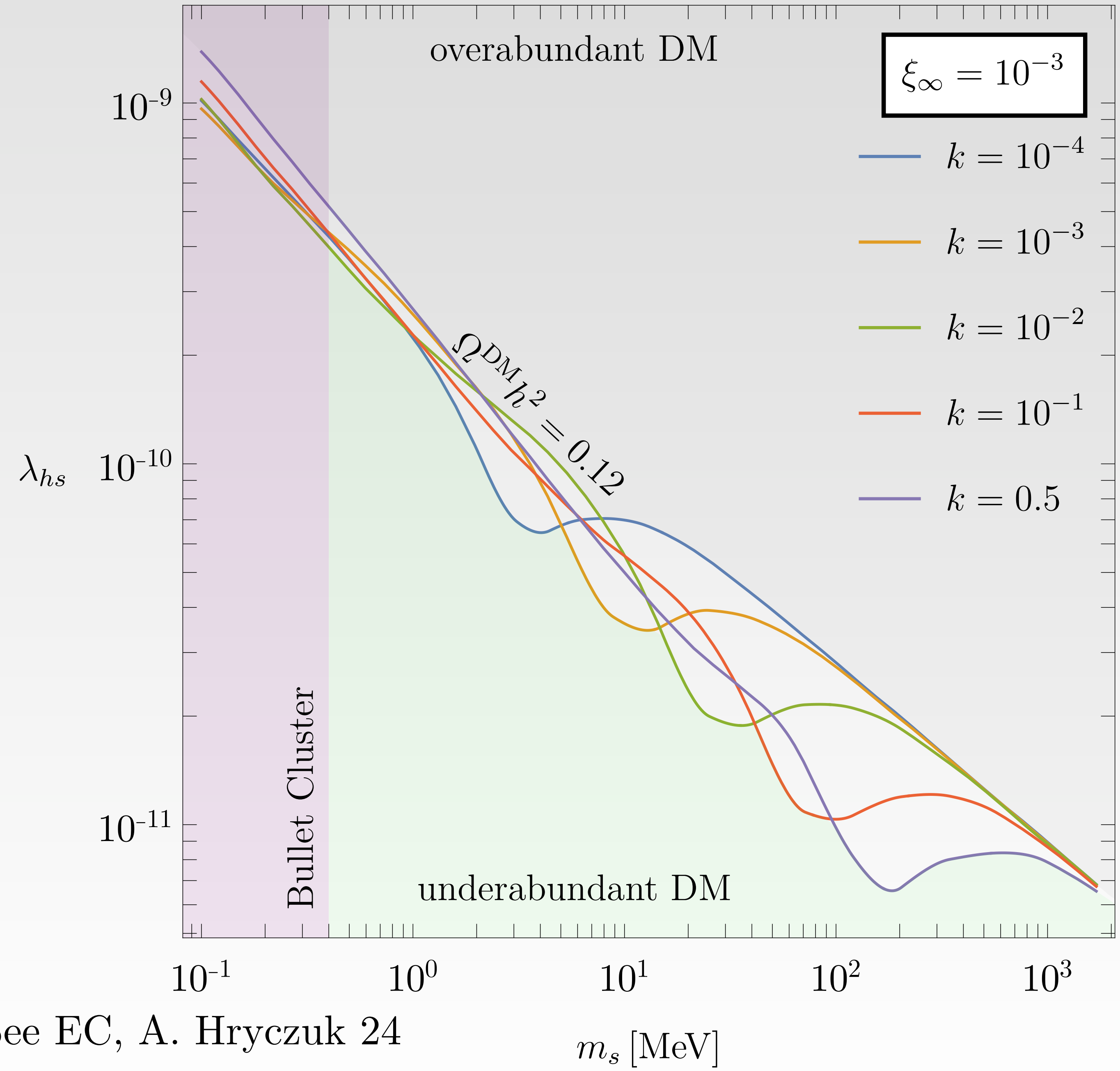
See EC, A. Hryczuk 24

and also Bernal, Chu 15 (SIMP $\mathbb{Z}_2$ DM)

Cannibals produced via freeze-in

Toy model: $$\mathscr{L} \supset -\frac{1}{3!}g(S^3 + (S^*)^3) - \frac{\lambda}{4}|S|^4 - \lambda_{hs}|S|^2|H|^2$$

DM self interactions (cannibal) Portal



$$k := g^2/(3\lambda m^2)$$

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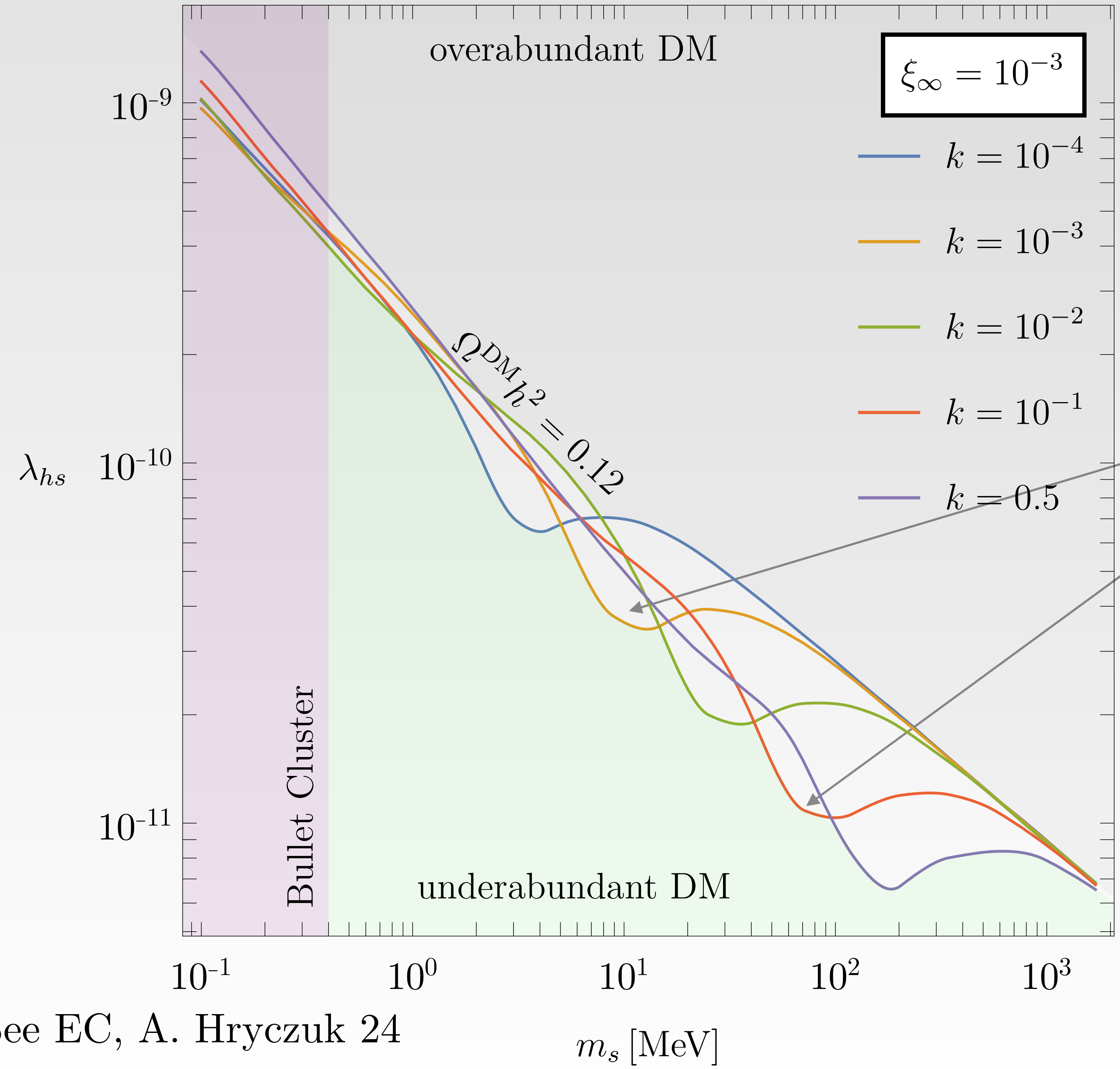

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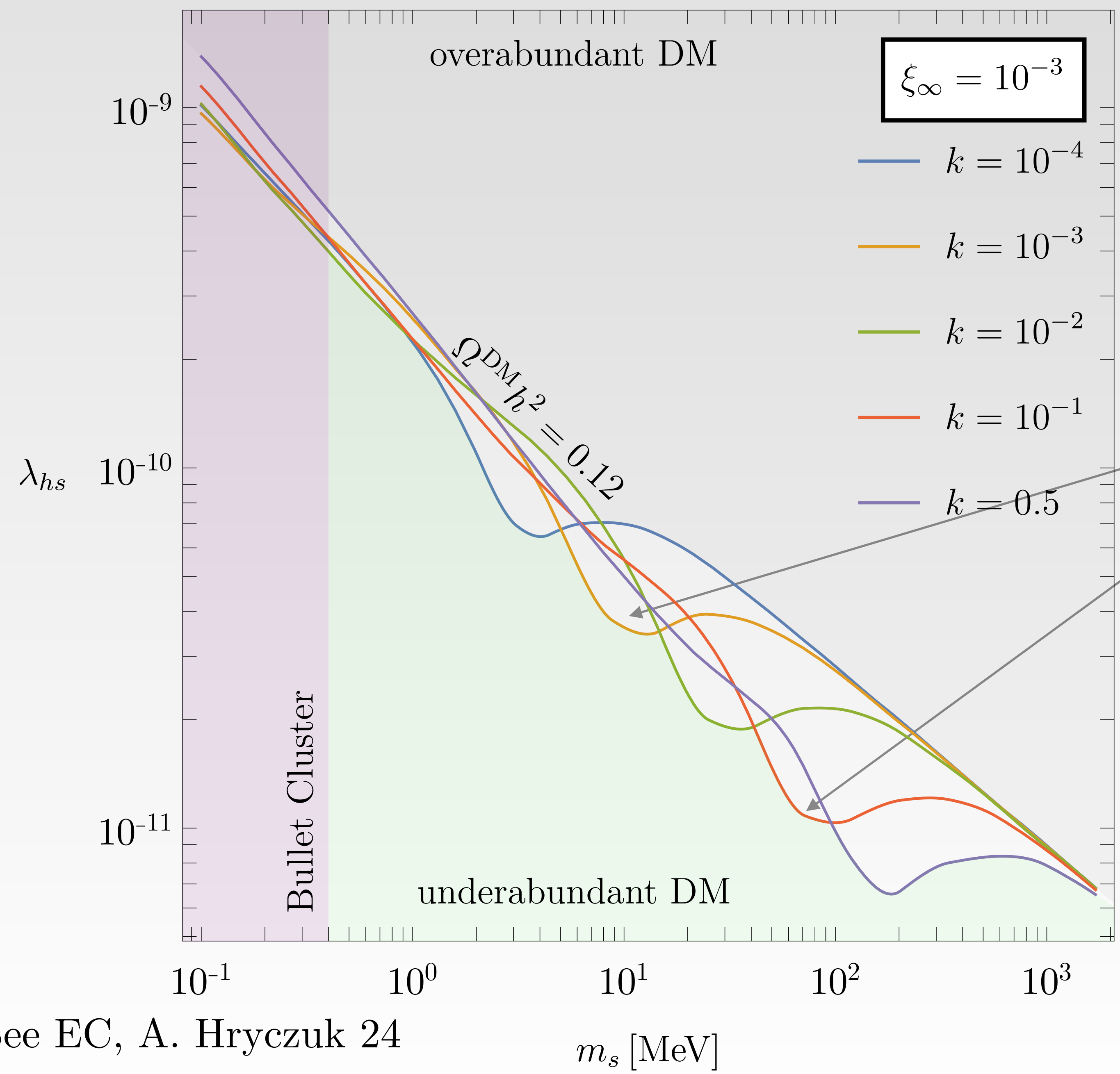

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Detectability prospects if *the reheating temperature* is **low**

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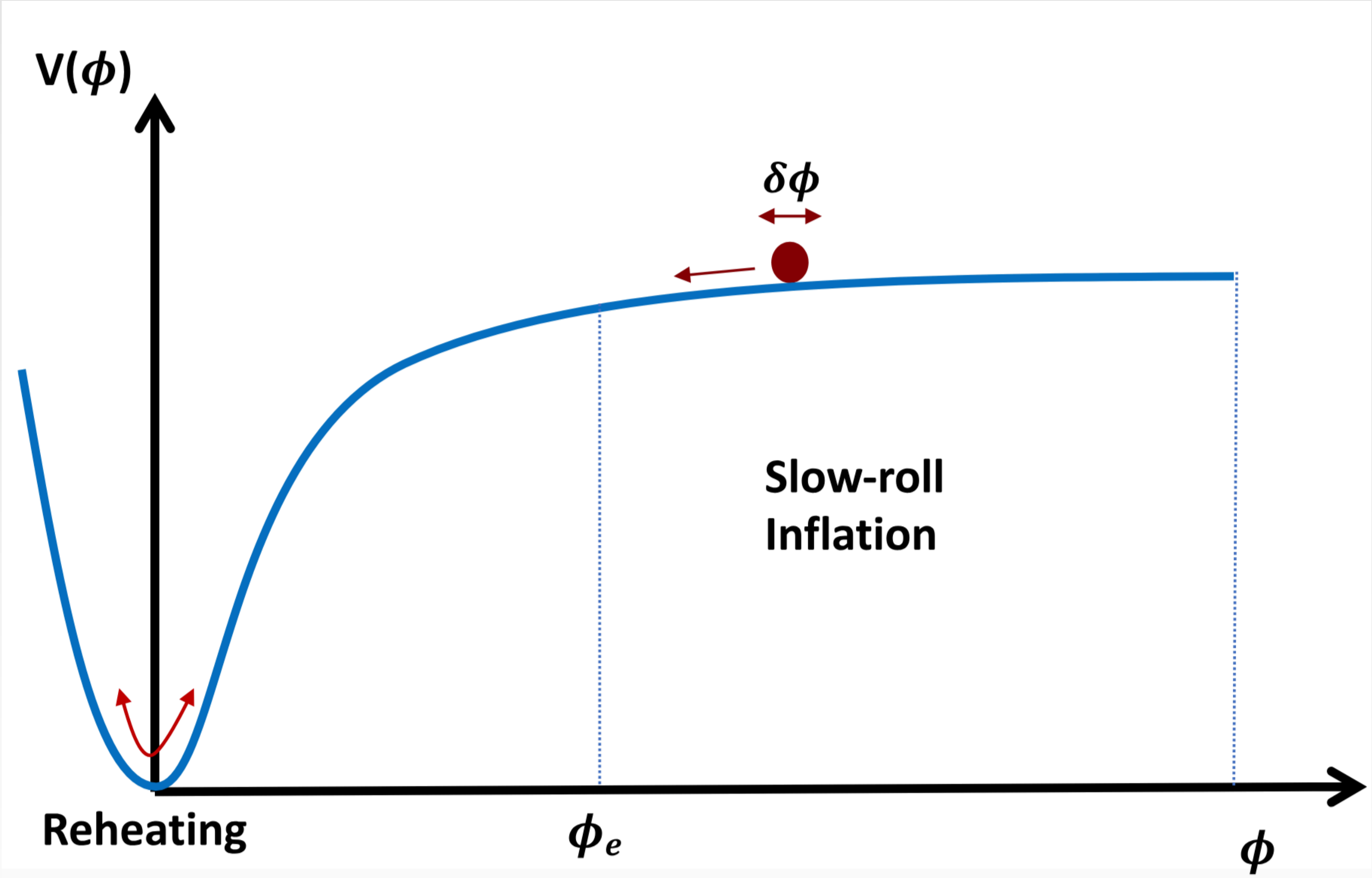
non-standard cosmology

Inflaton decay and reheating

Transition between ***non-standard*** and ***standard*** cosmology is due to a scalar (**inflaton**) field that rolls ($a \propto e^{Ht}$) in a potential and subsequently oscillates in the minimum **decaying** into SM states.



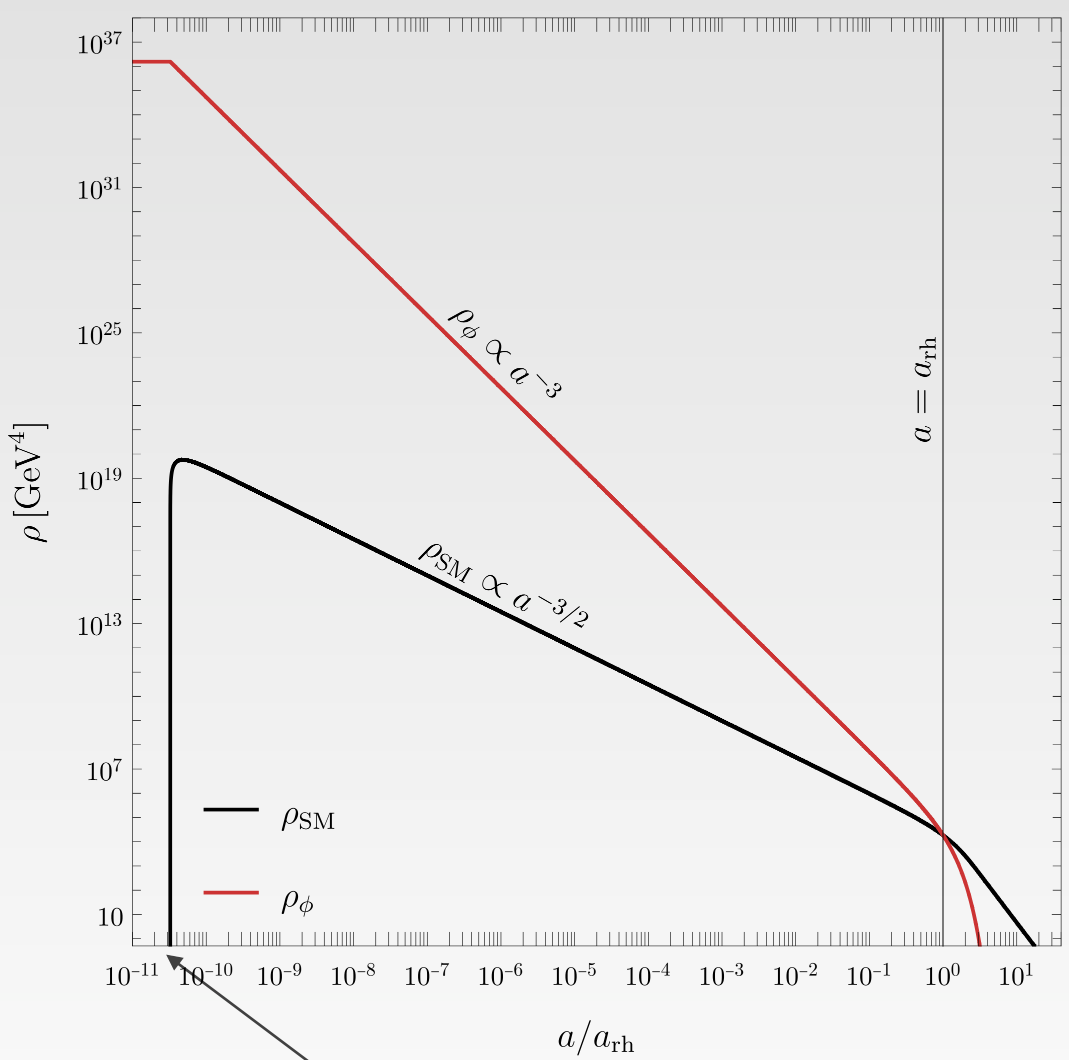
$$\frac{d\rho_\phi}{dt} + 3H\rho_\phi = -\Gamma\rho_\phi\,,$$

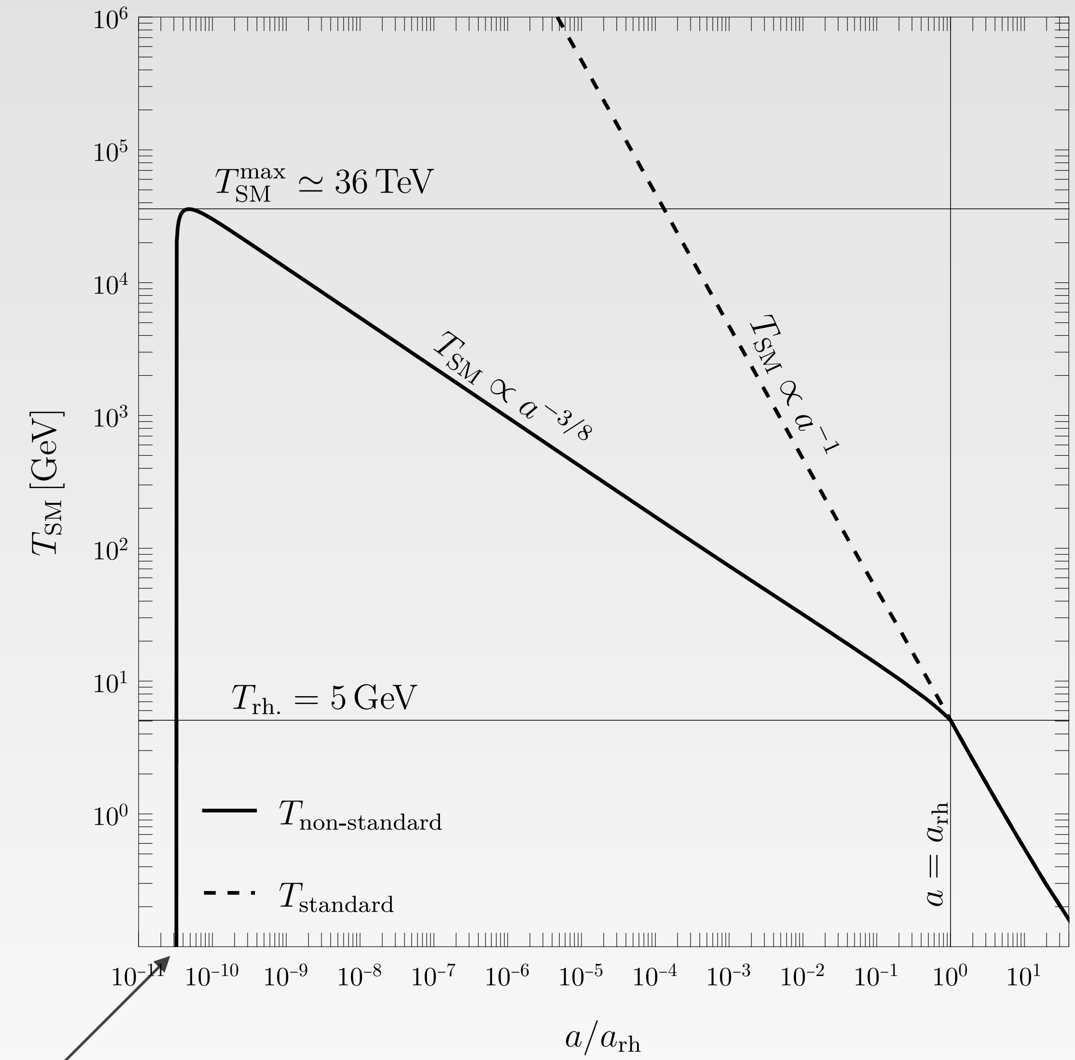
$$\frac{d\rho_R}{dt} + 4H\rho_R = +\Gamma\rho_\phi\,,$$

Cicoli, String Cosmology

Inflaton decay and reheating

During reheating $T \propto a^{-3/8}$ (matter domination), and $H \propto T^4$, i.e., rapid expansion of the universe

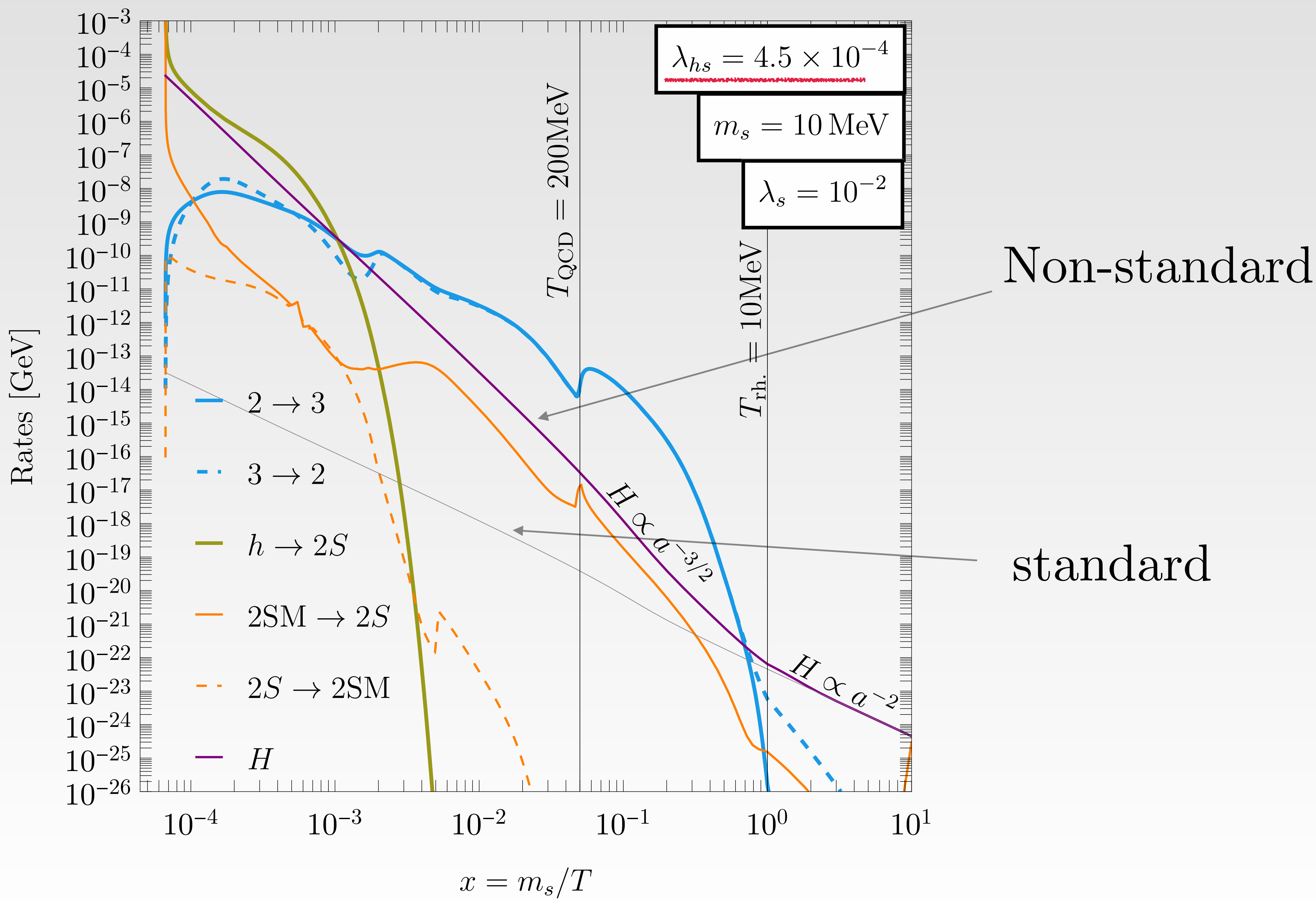




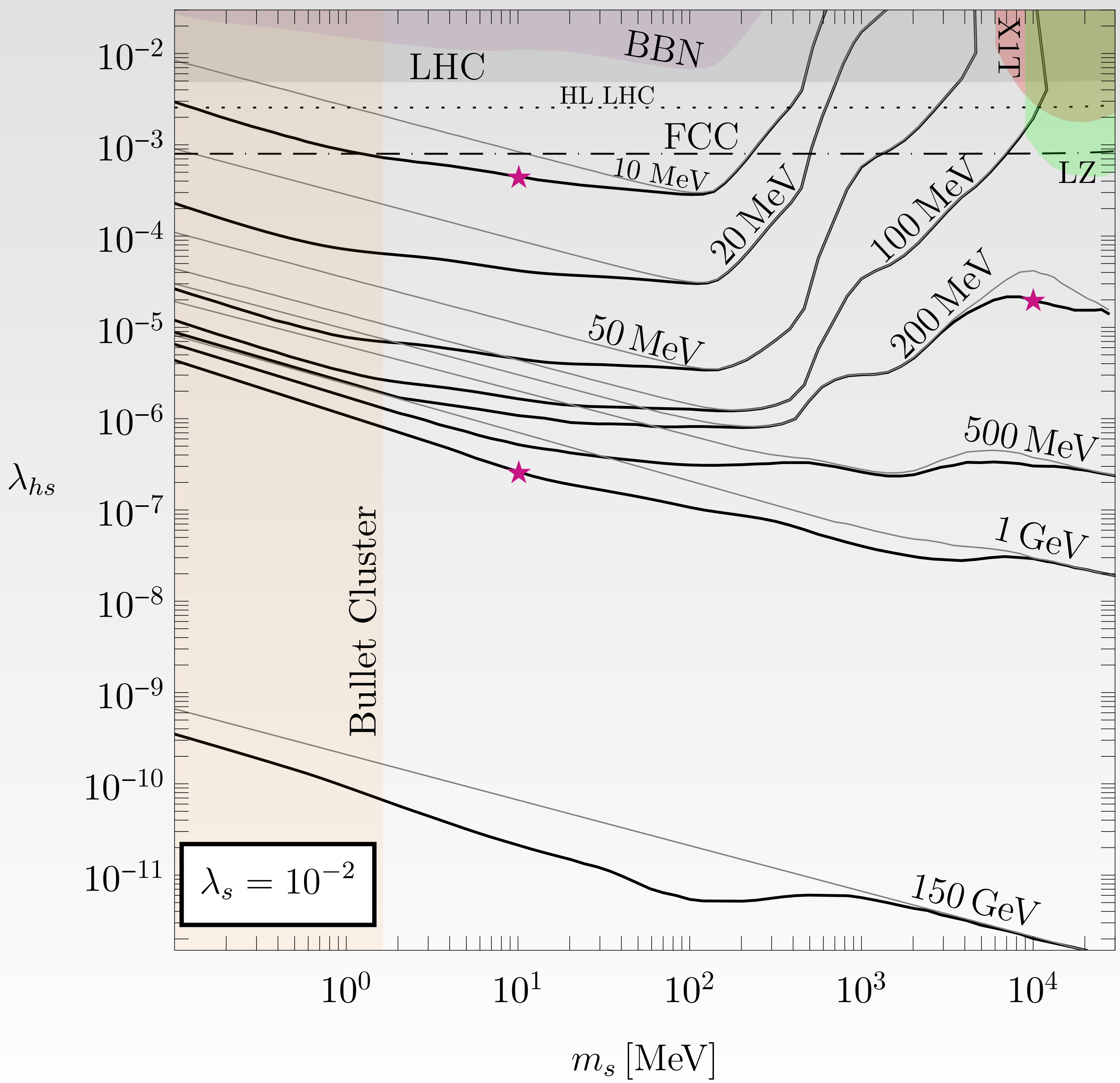
Freeze-in initial condition ($n_i^{DM} = 0$)

Production during reheating

Production rate from SM has to catch up with $H \propto T^4$, and ρ_{DM} **dilutes** during reheating.



Impact on collider phenomenology



- *Low* T_{rh} leads to **detectability**;
- The case of instantaneous reheating is studied in Lebedev, Morais, Oliveira, Pasechnik 24.

Current collaboration with

Felix Kahlhoefer, **Jonas Matuszak** and **Rosellon Santiago** from KIT (Germany)

Early Universe *dynamics* of a self interacting scalar

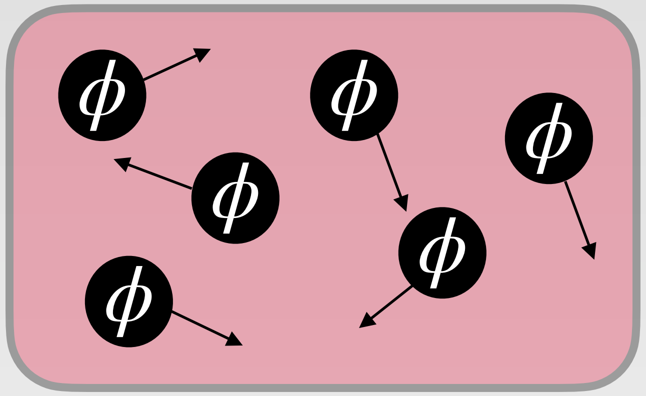
Cosmological (inverse) phase transitions

Thermal corrections

In the early universe there is a thermal (ensemble) background of states ϕ. This ensemble leads to thermal corrections:

$$V_{eff}(\phi_b) = V_{cl}(\phi_b) + V_{\mathrm{CW}}(\phi_b) + V_{\mathrm{ct}}(\phi_b) + V_T(\phi_b)$$

$$V_T(\phi_b) = \frac{T^4}{2\pi^2} g_\phi \int dy\, y^2 \ln\left(1 - e^{-\sqrt{y^2 + (m(\phi_b)/T)^4}}\right)$$



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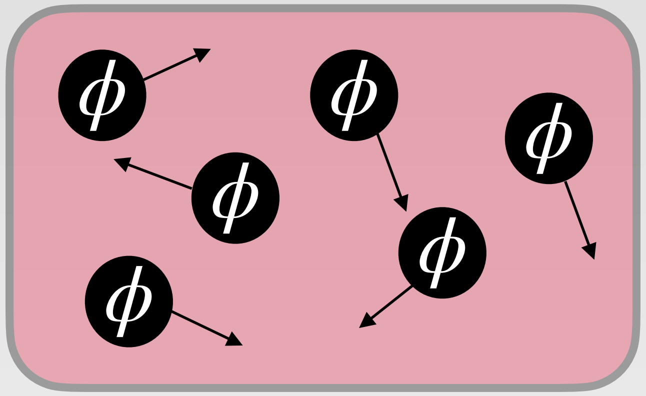

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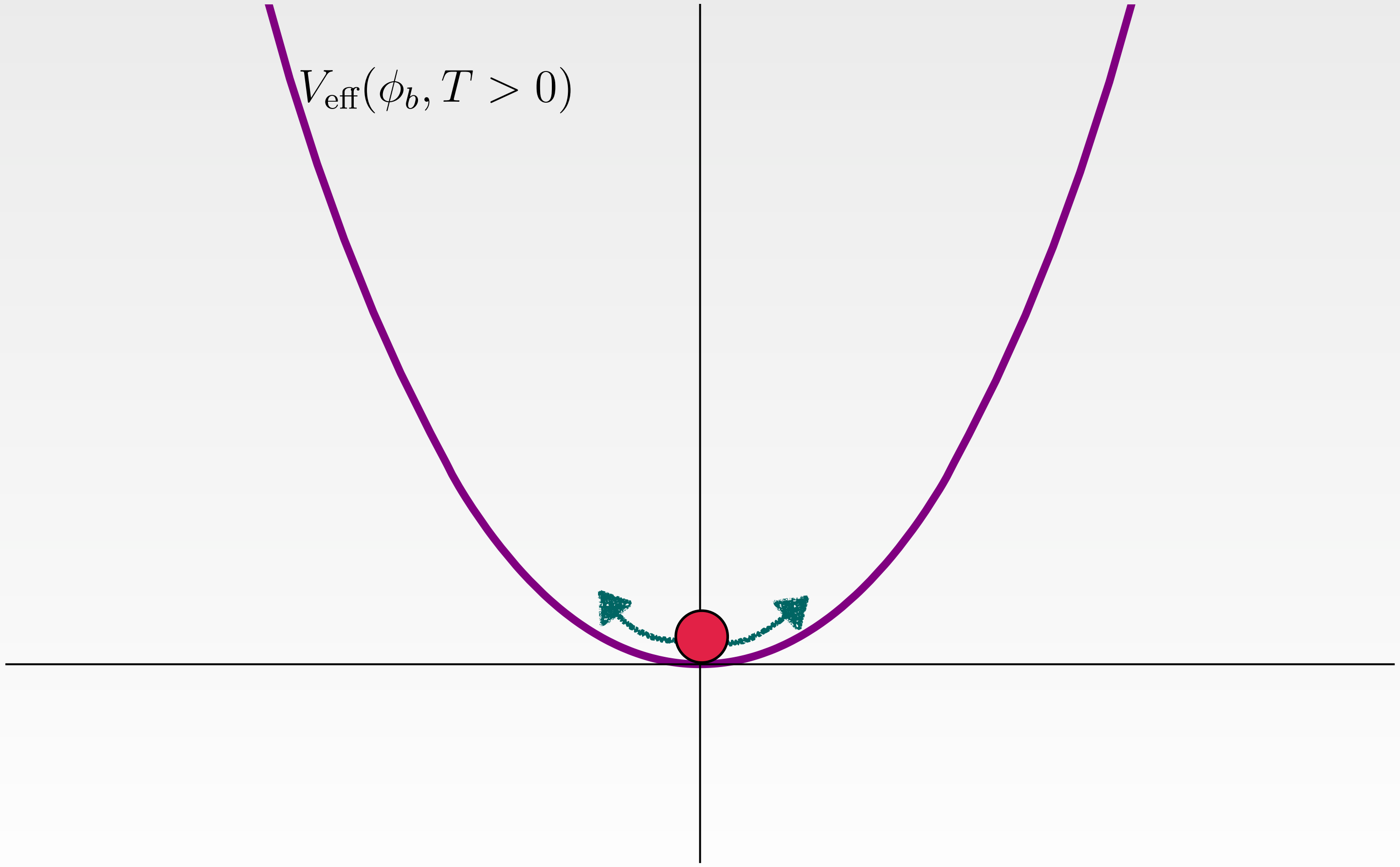

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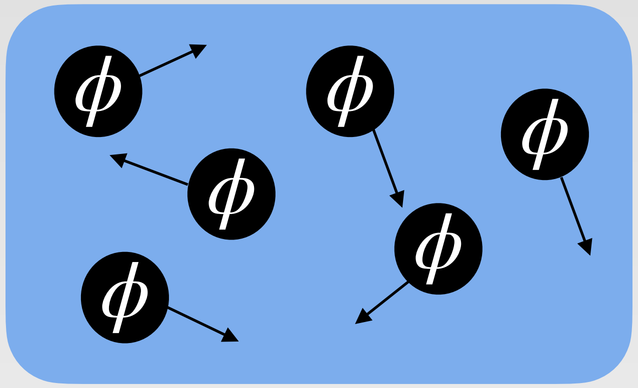

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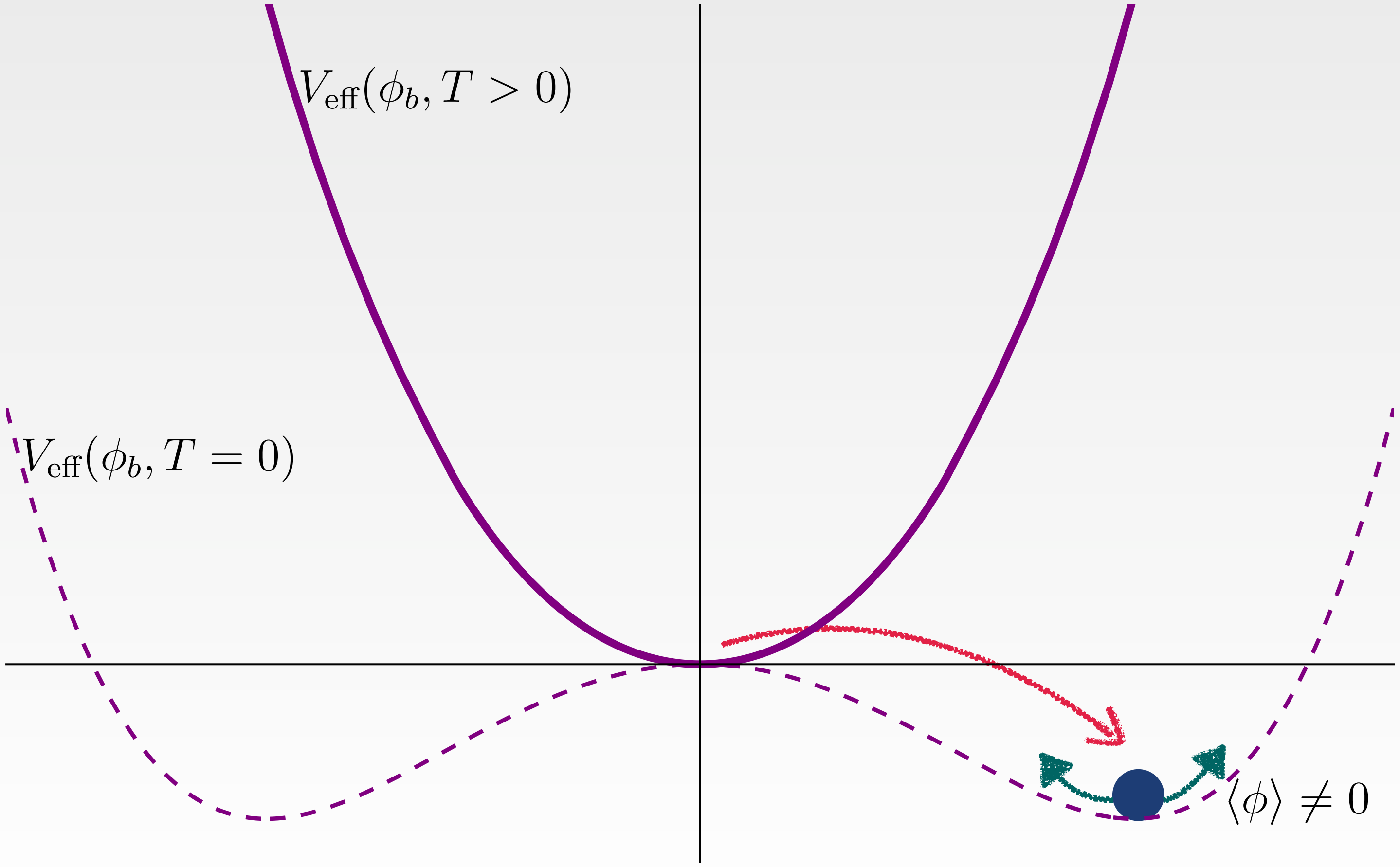

When the temperature ***drops***, the system will ***roll*** to the true minimum in a ***smooth*** transition.

U(1) theory

If the transition is more **abrupt**, it can leave imprints in the early universe. Simple realization: complex scalar field $\mathbf{\Phi}$ charged under a U(1) gauge symmetry

$$\mathscr{L} = |(\partial_\mu - igA'_\mu)\Phi|^2 - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \mu^2\Phi^*\Phi - \lambda(\Phi^*\Phi)^2 + \frac{\epsilon}{2\cos\theta_w}B_{\mu\nu}F'^{\mu\nu}\,.$$

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Dark gauge boson (dark photon)

Kinetic mixing

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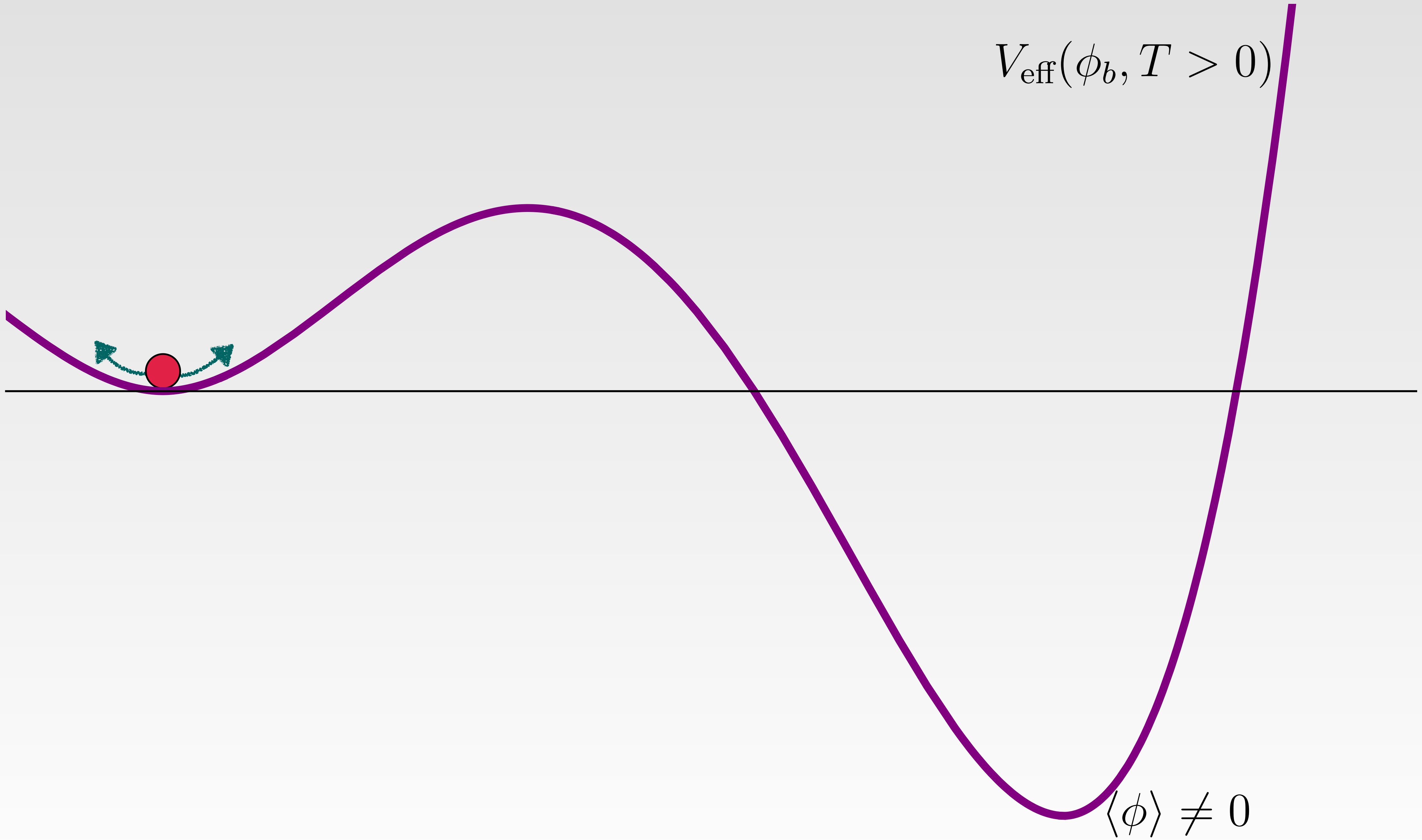
And we expand the scalar as $\mathbf{\Phi} \to (\phi + \phi_b + \mathrm{i}\varphi)$

Goldstone boson

$$\phi_b = \langle\phi\rangle \neq 0$$

First order phase transition

The appealing of this theory results in the interactions with the gauge boson, they can induce first order phase transitions:



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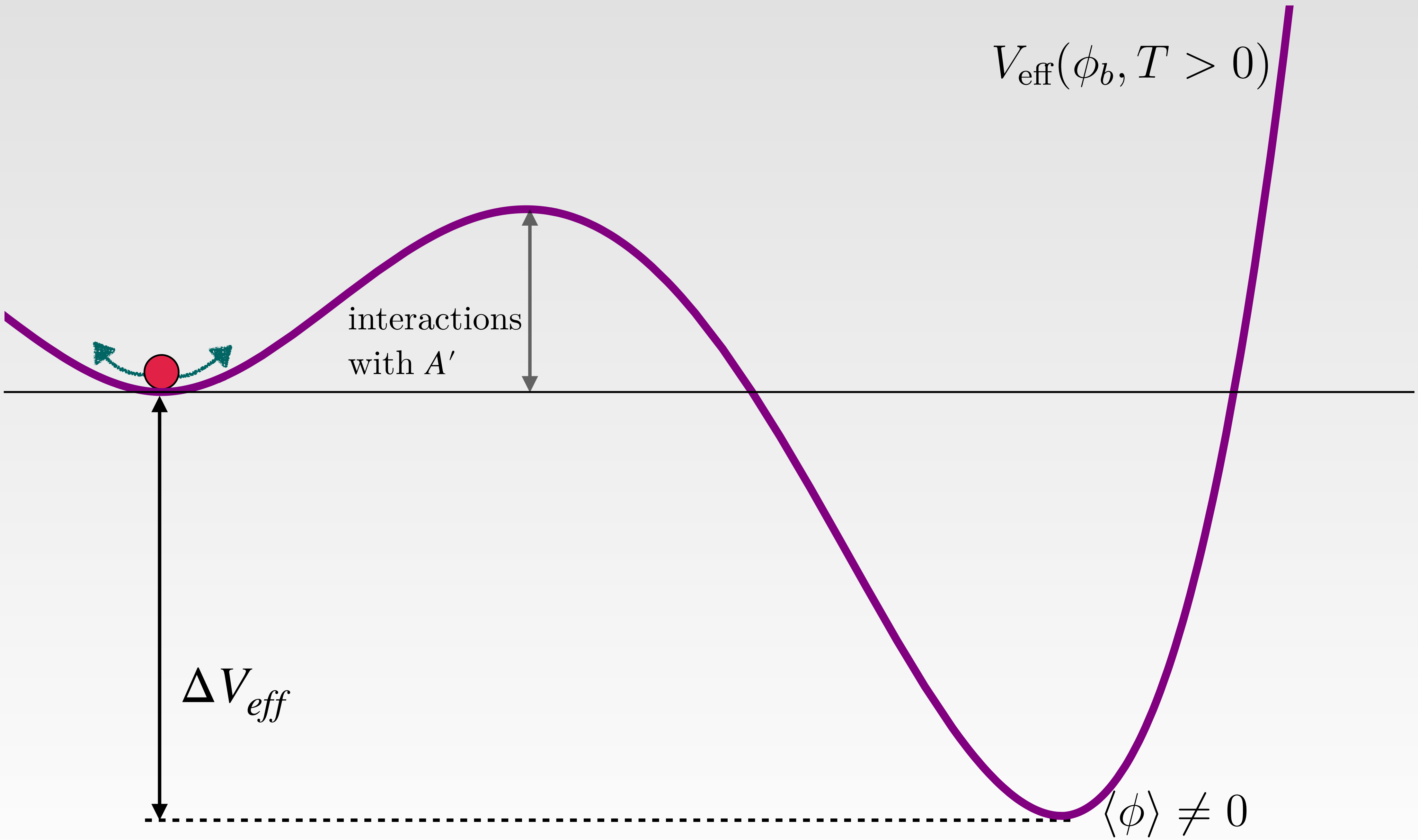

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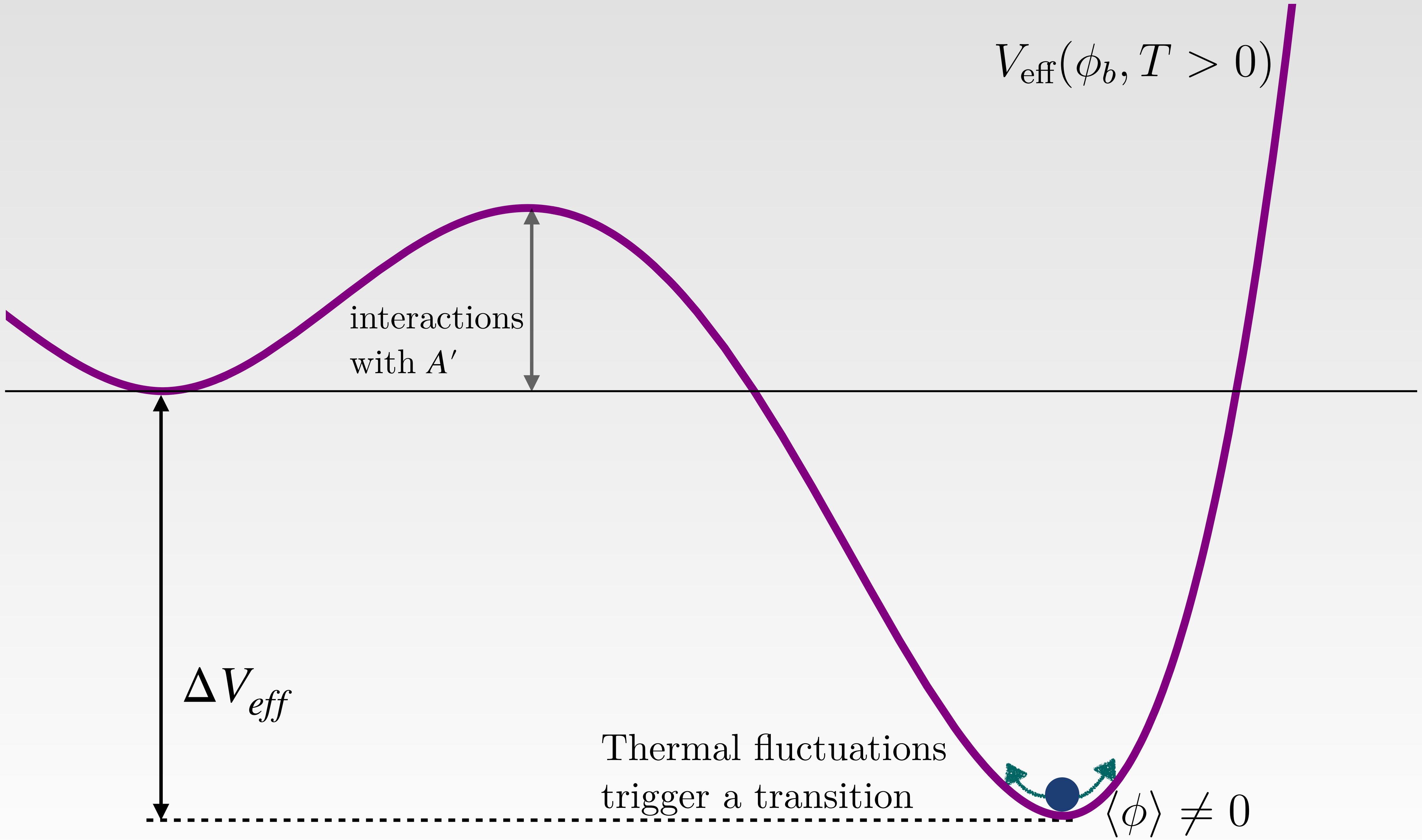

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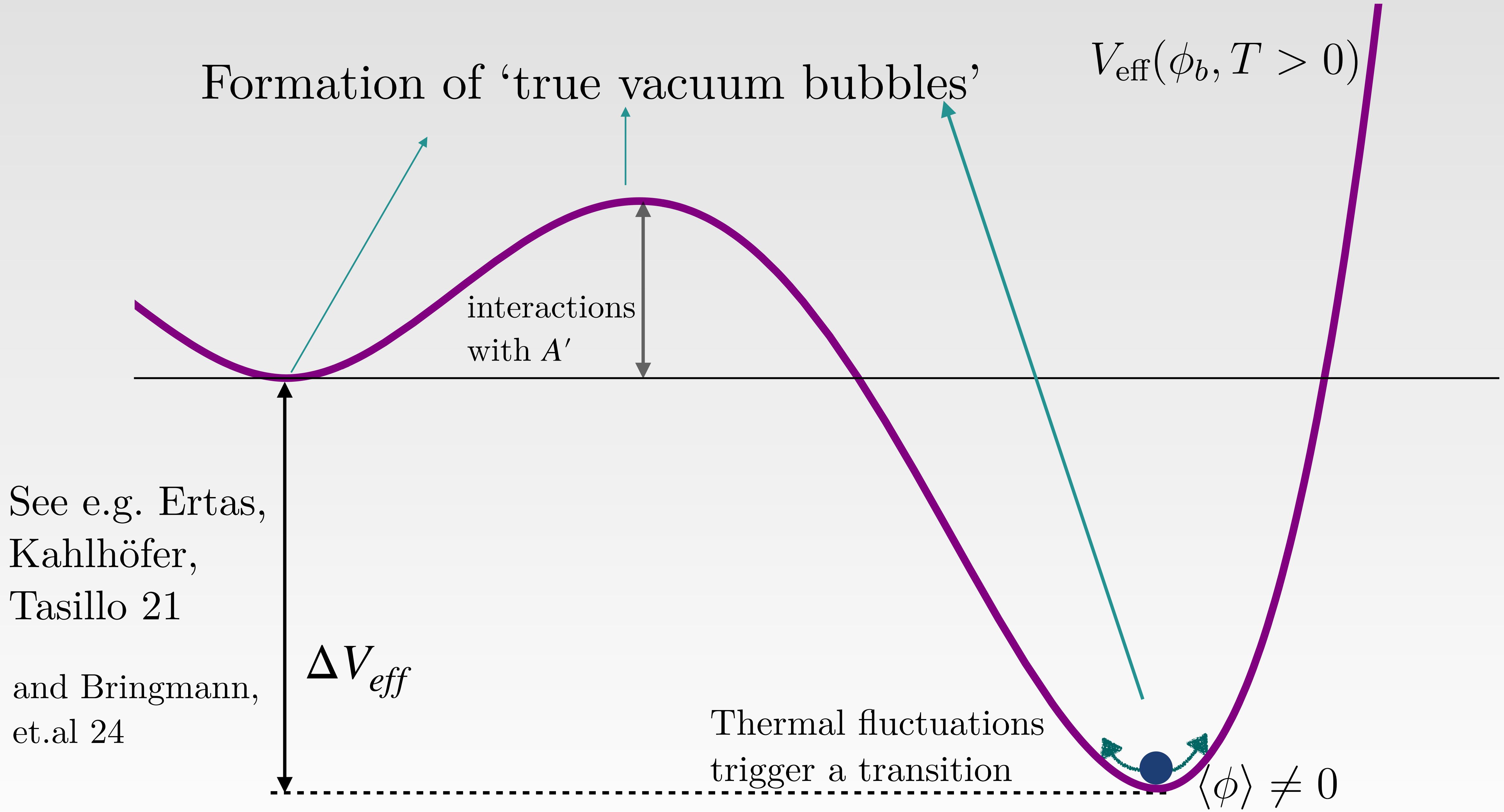
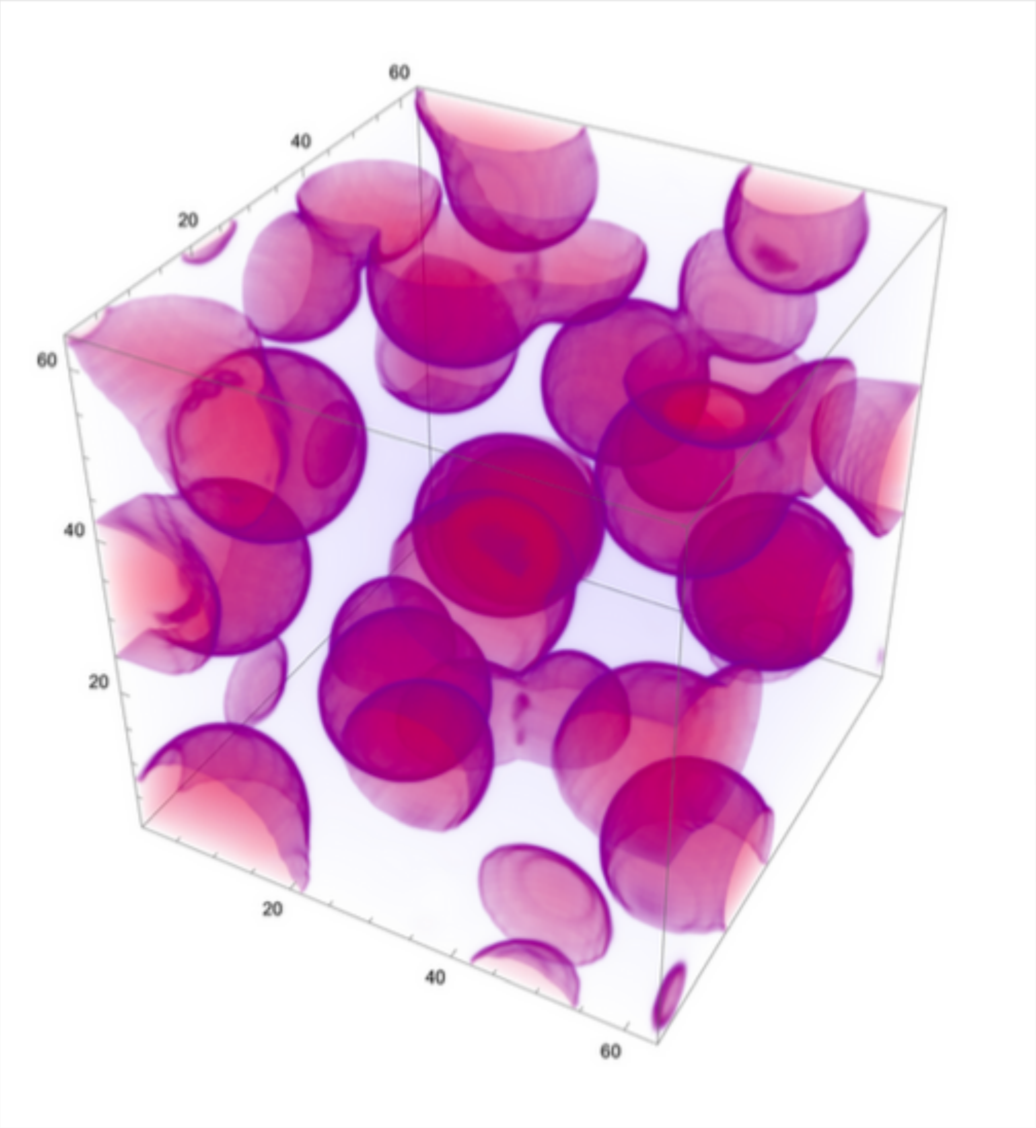
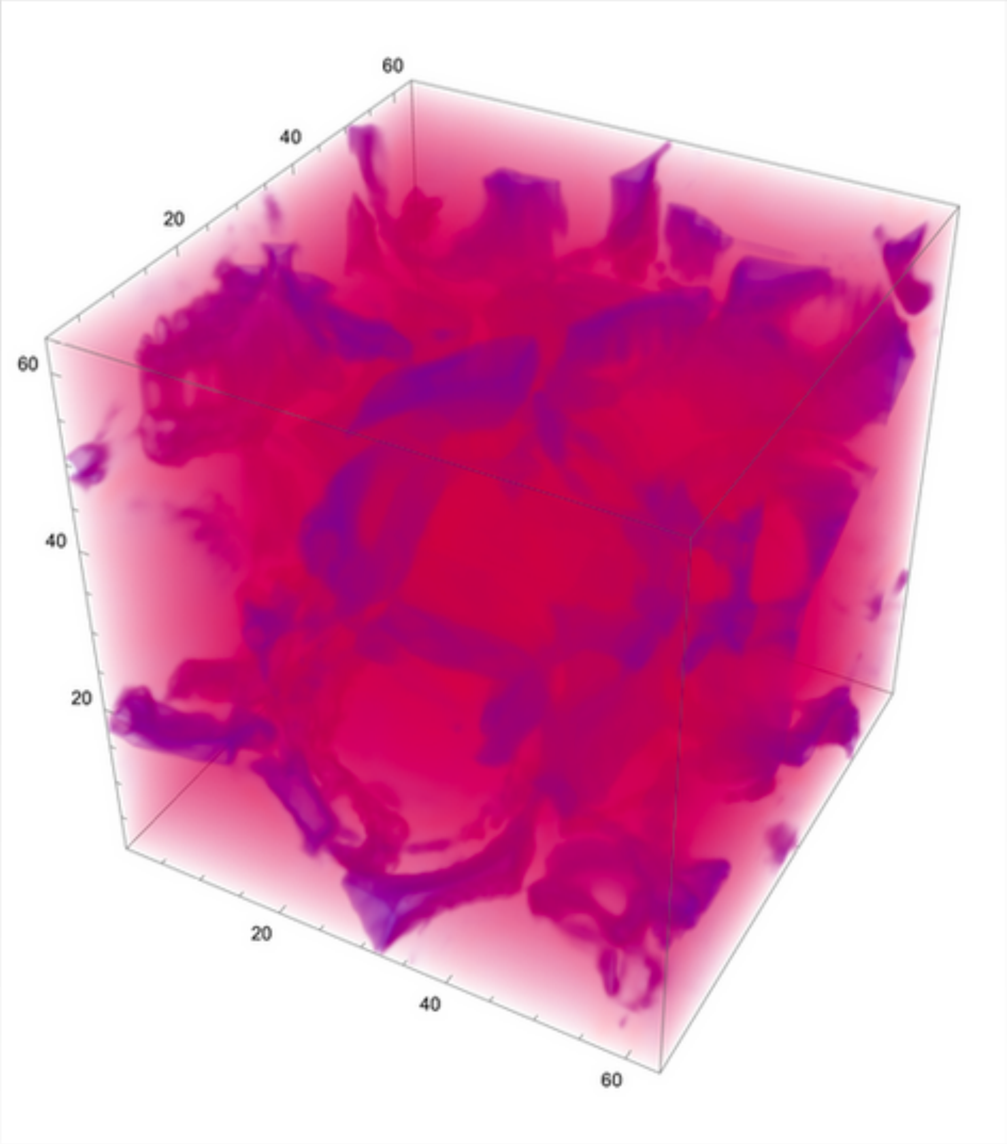


Vacuum bubbles

Vacuum bubbles nucleate and percolate (they ***form***, grow, ***collide*** and ***merge***)





Nucleation and percolation on the lattice. Image courtesy of Henda Mansour

Vacuum bubbles and GWs

Space-time **feels** the *percolation*: energy and momentum determine the **curvature** of space-time. If energy moves or oscillates **unevenly**, it **disturbs** the space-time fabric.

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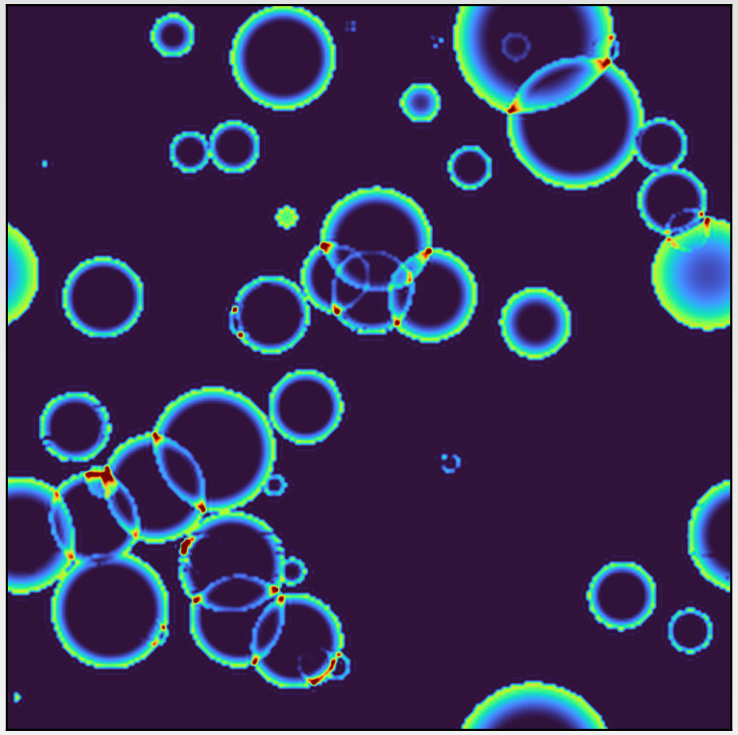

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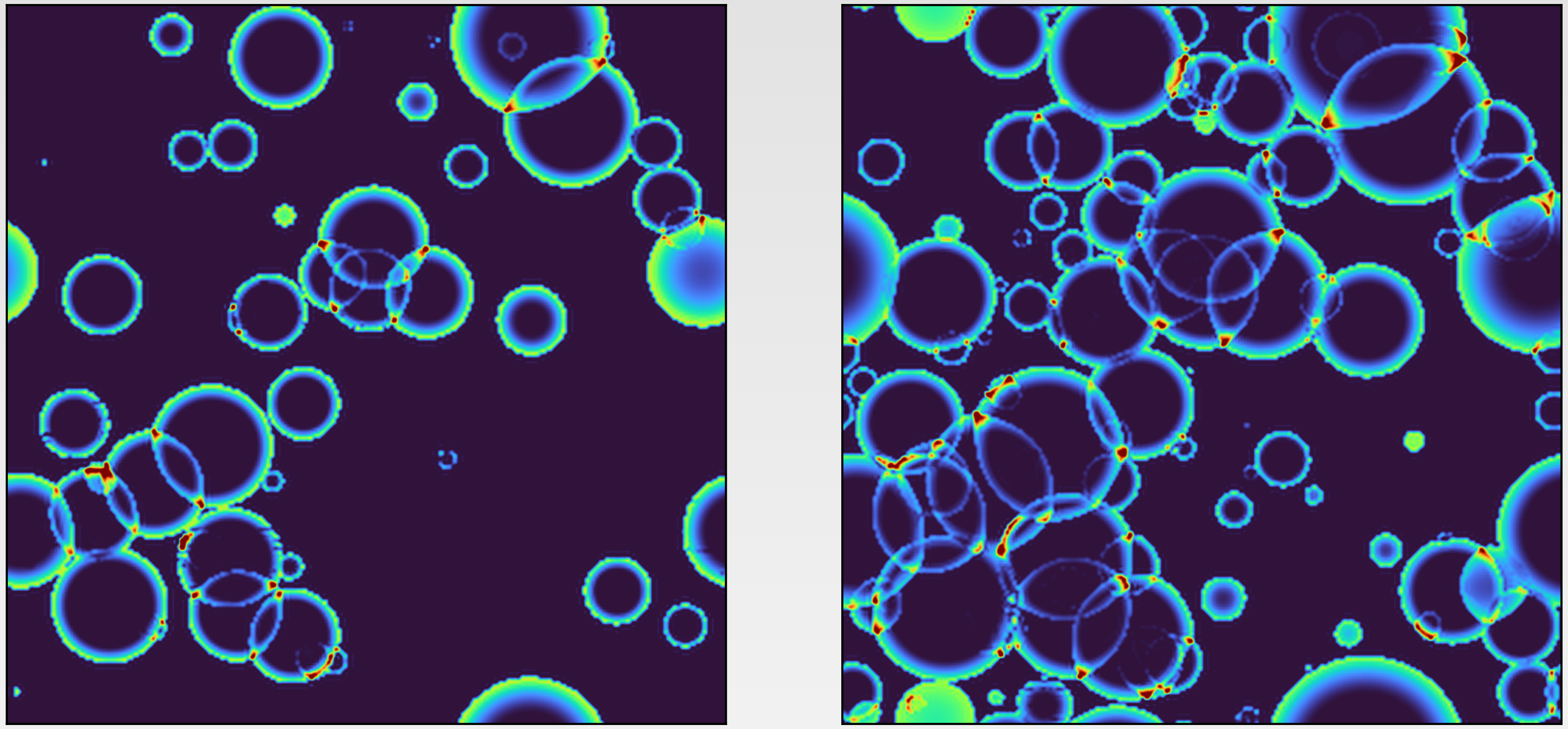

Nucleation Expansion

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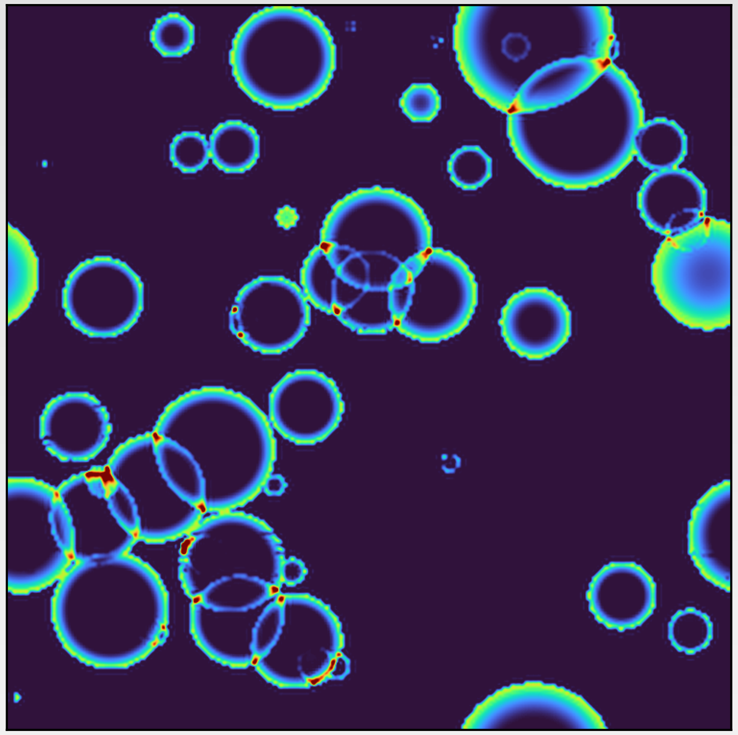
Balasz, et.al. Gravitational waves from cosmological first-order phase transitions with precise hydrodynamics



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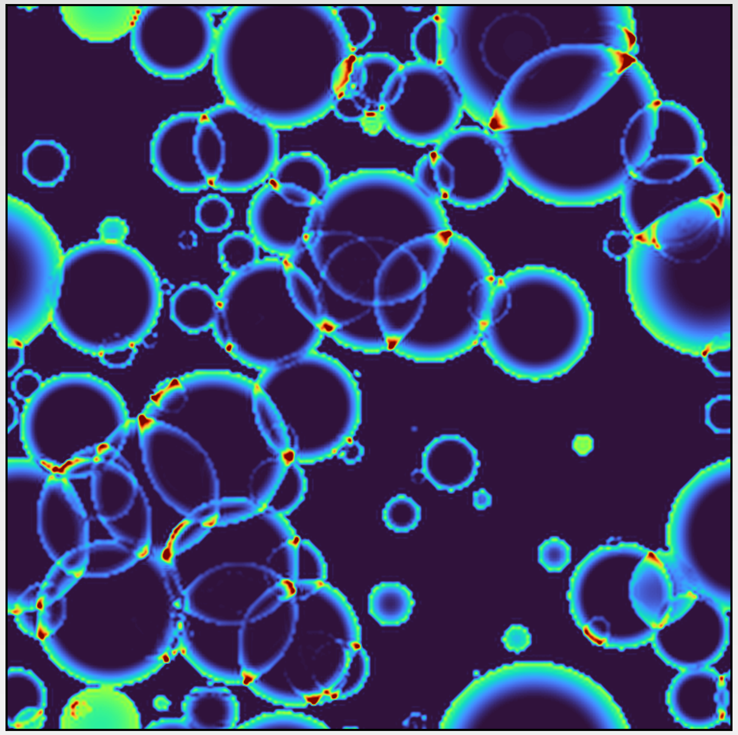
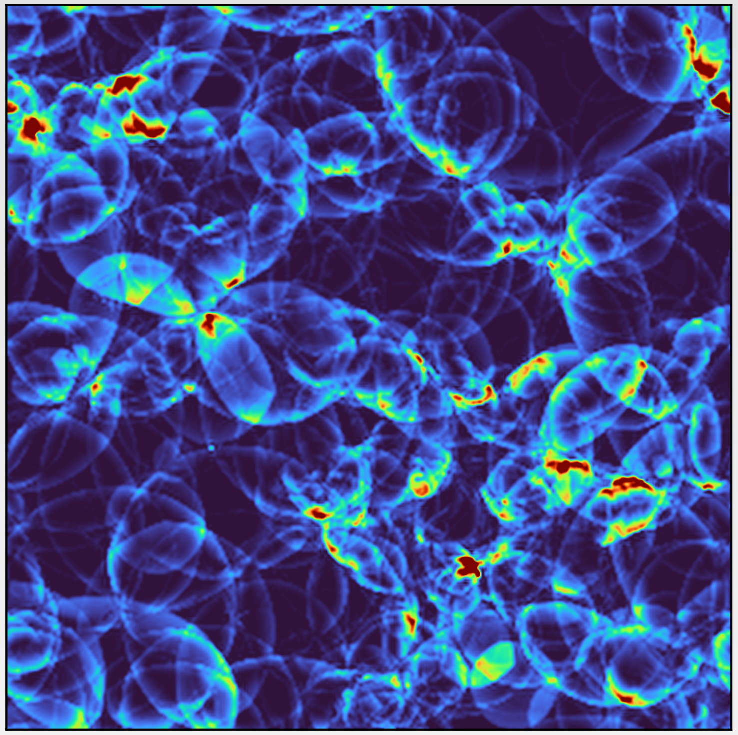


Expansion



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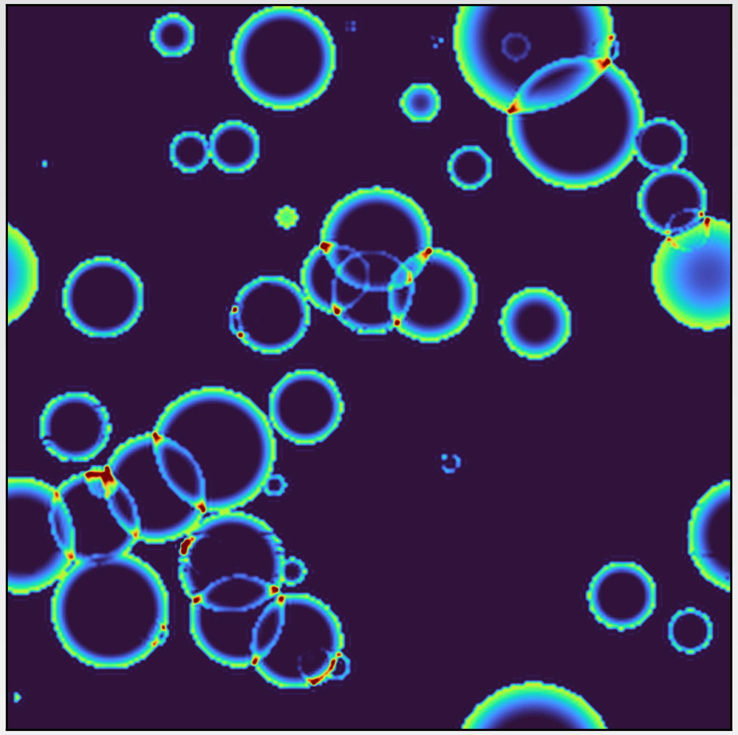

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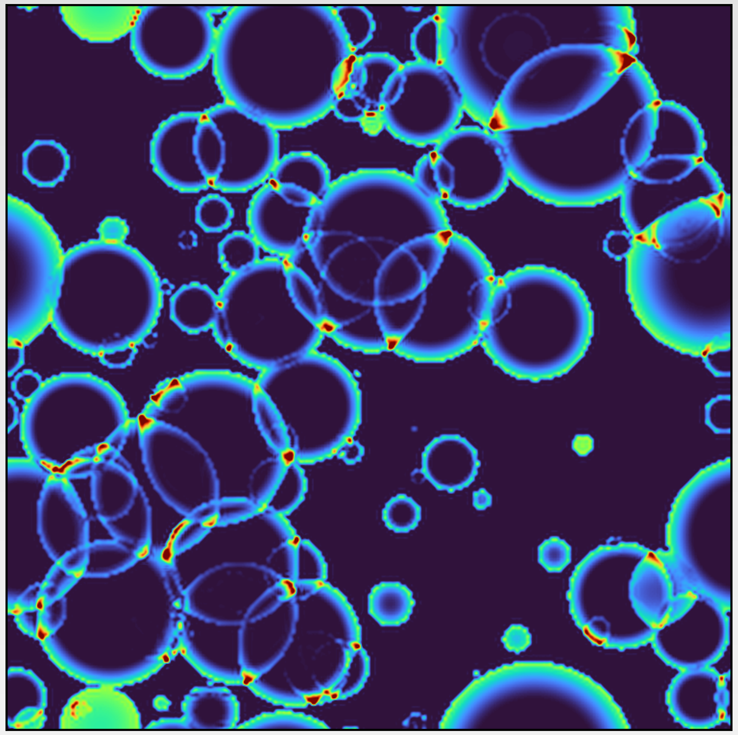

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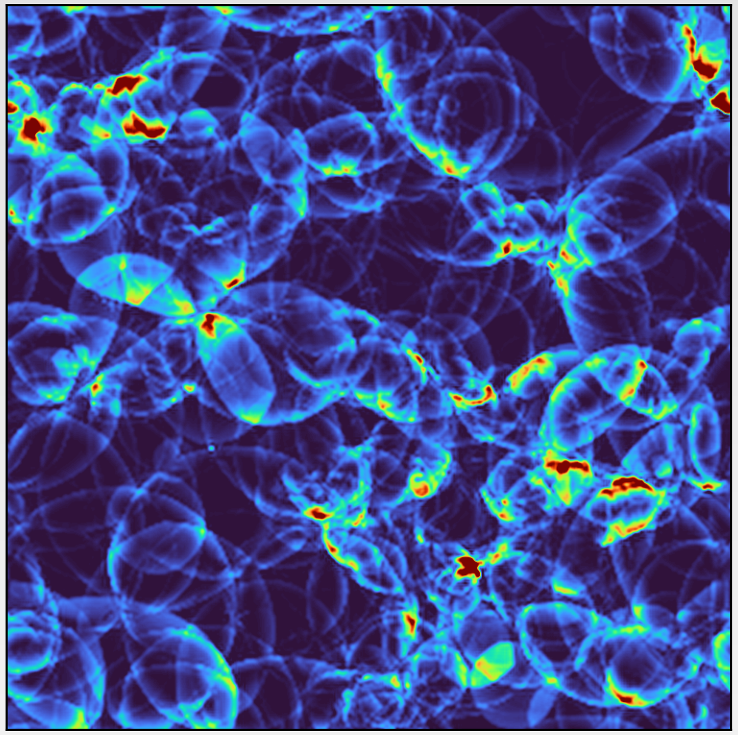

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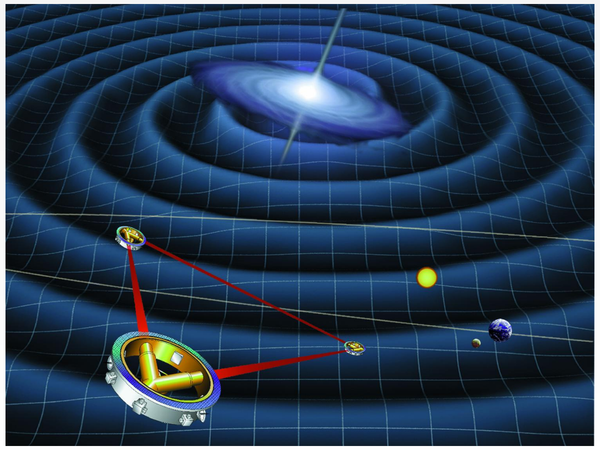


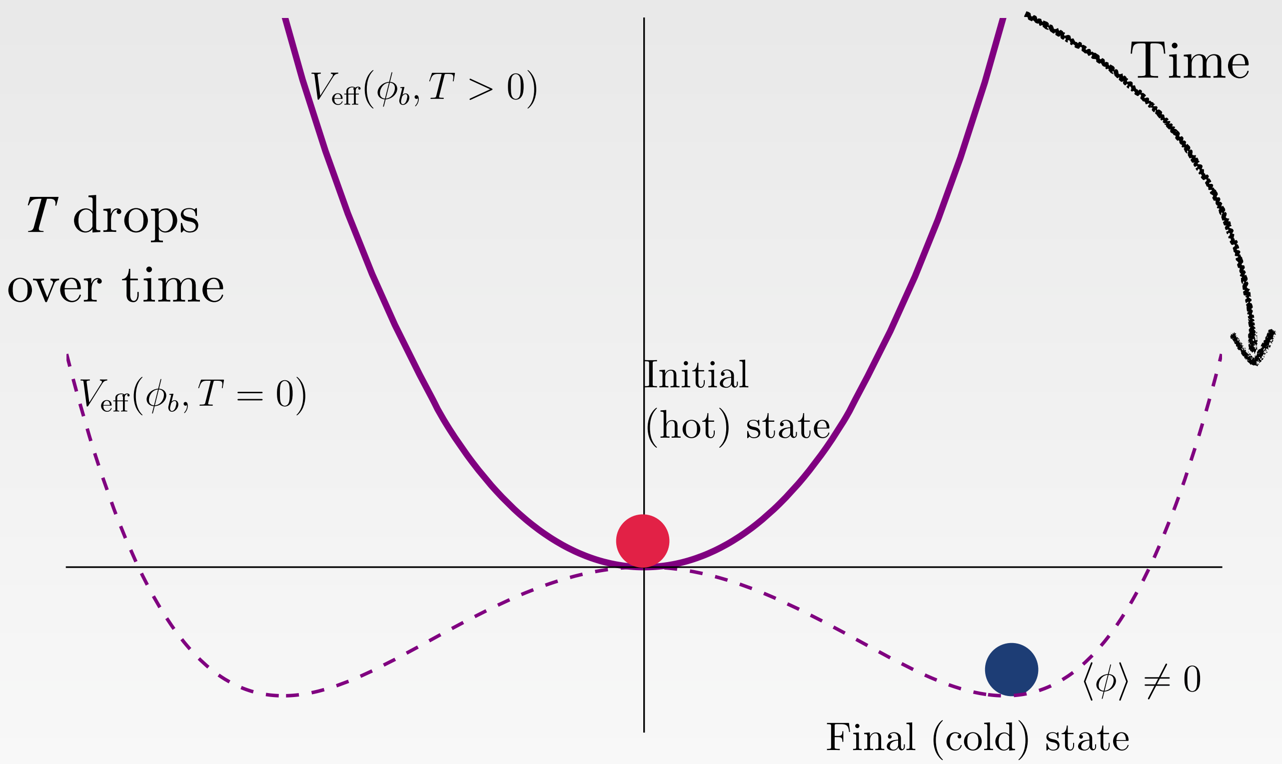
Wikipedia

Possible observable GW background with the **Laser Interferometer Space Antenna** (LISA)

Inverse First order phase transition

There are different scenarios leading to FOPTs. The system can go from a *broken* to a *symmetric* phase:

Standard assumption: **symmetric** to **broken** phase



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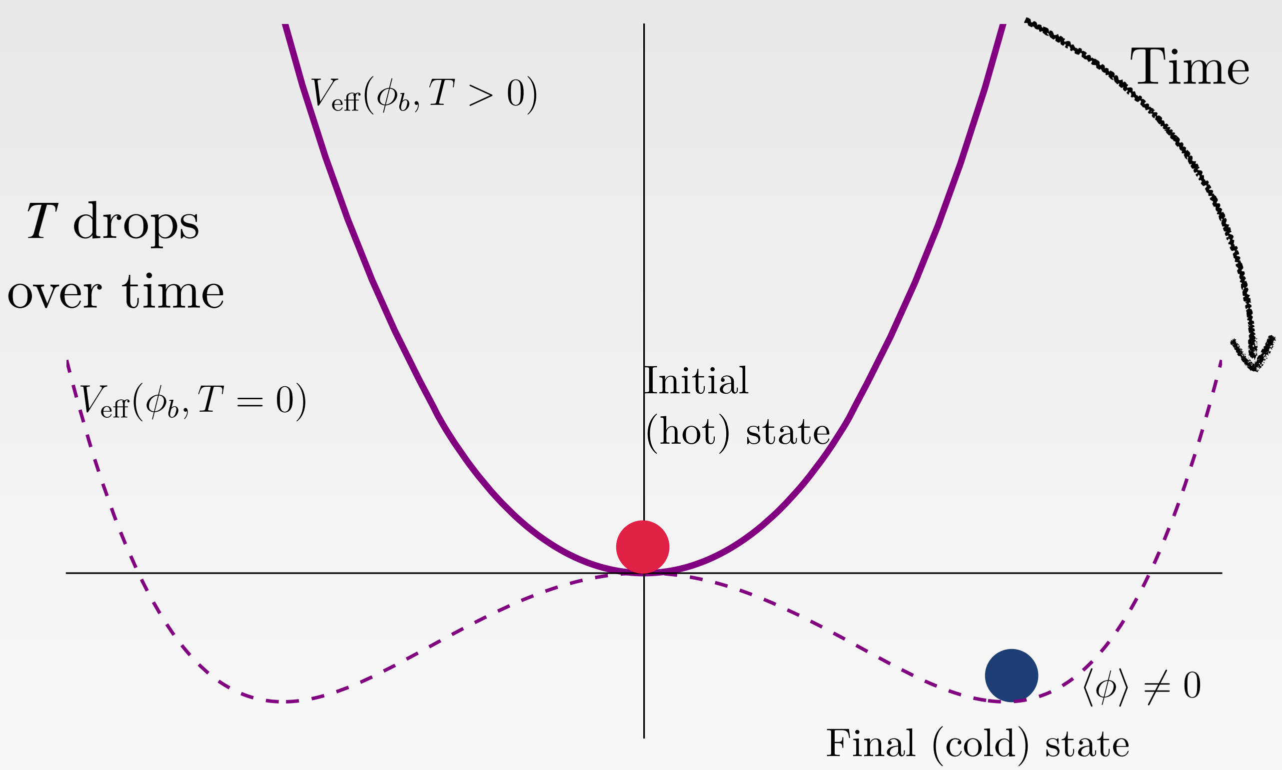
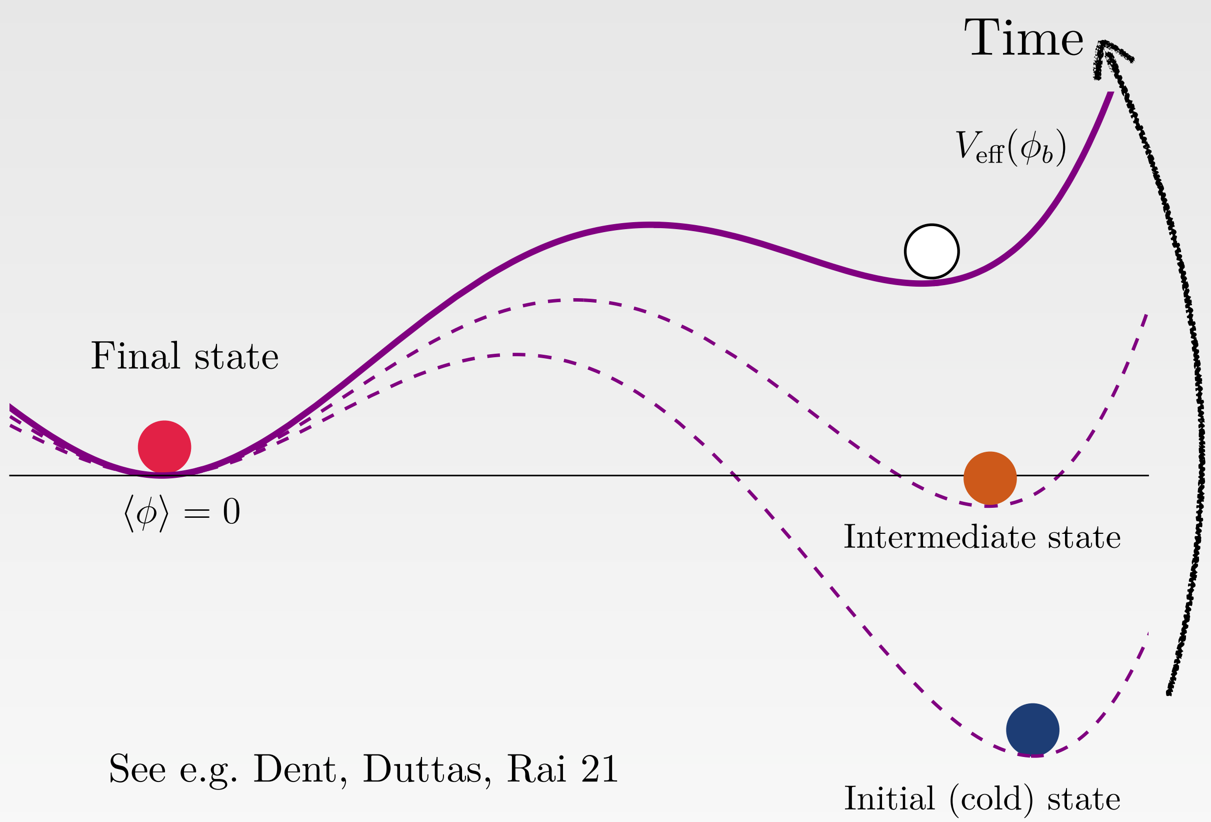


Inverse transition: **broken** to **symmetric** phase



See e.g. Dent, Duttas, Rai 21

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Realizable with injection of entropy from the SM plasma as in the freeze-in mechanism:

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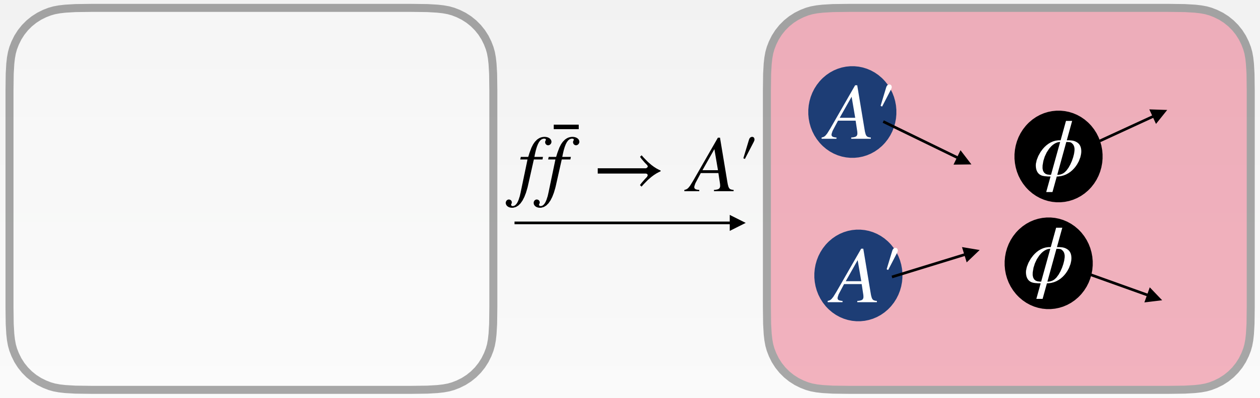

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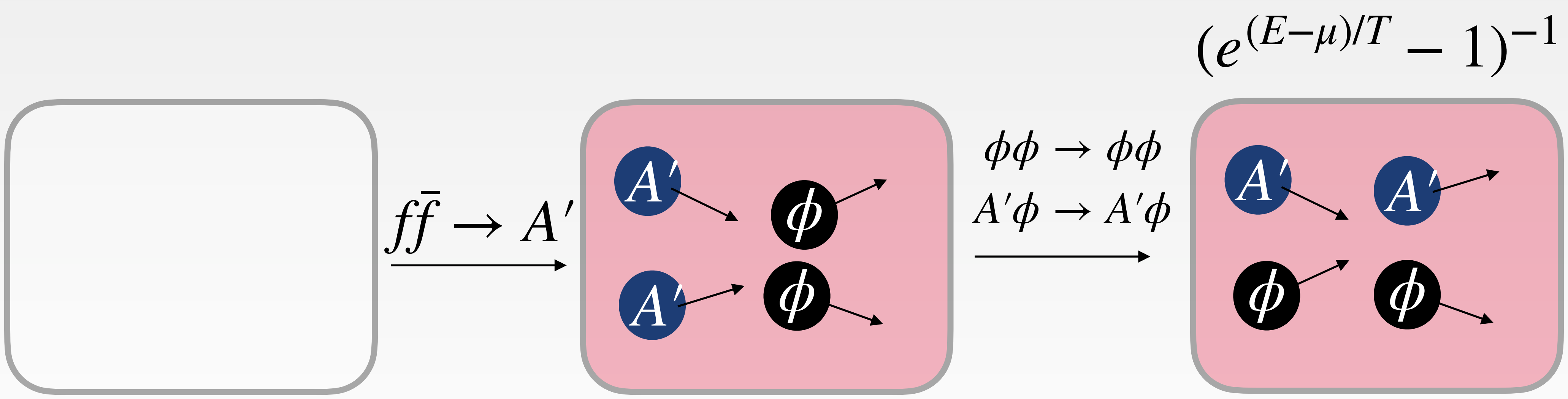

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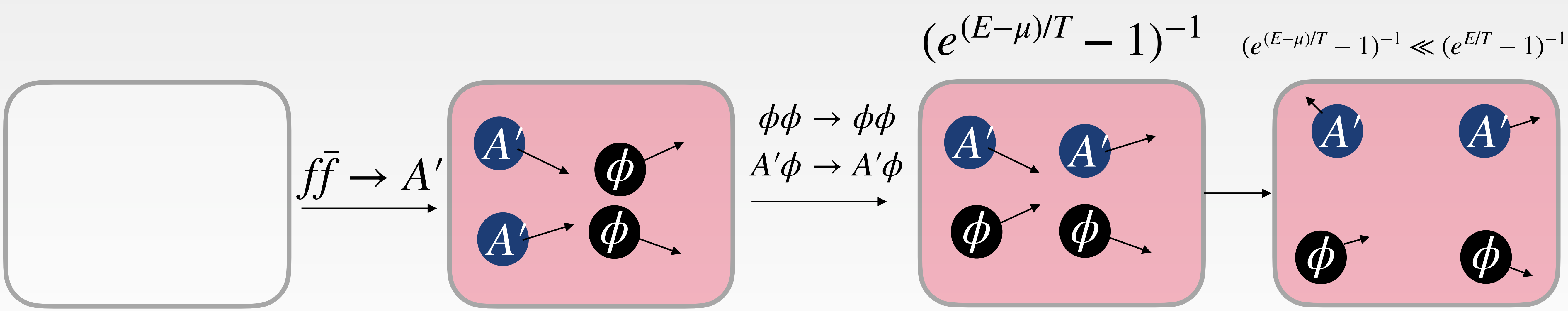

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In fact

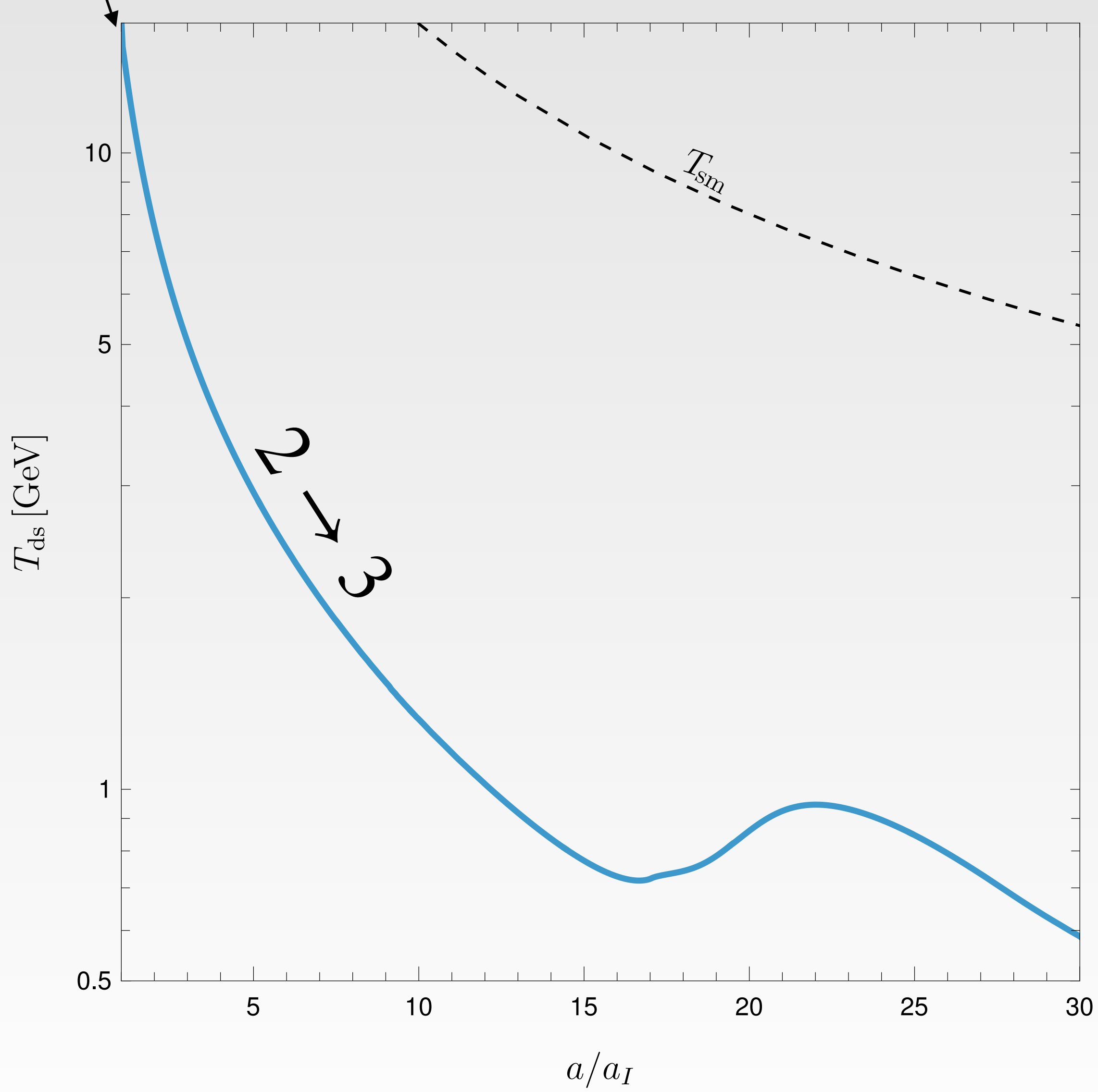
Free energy density

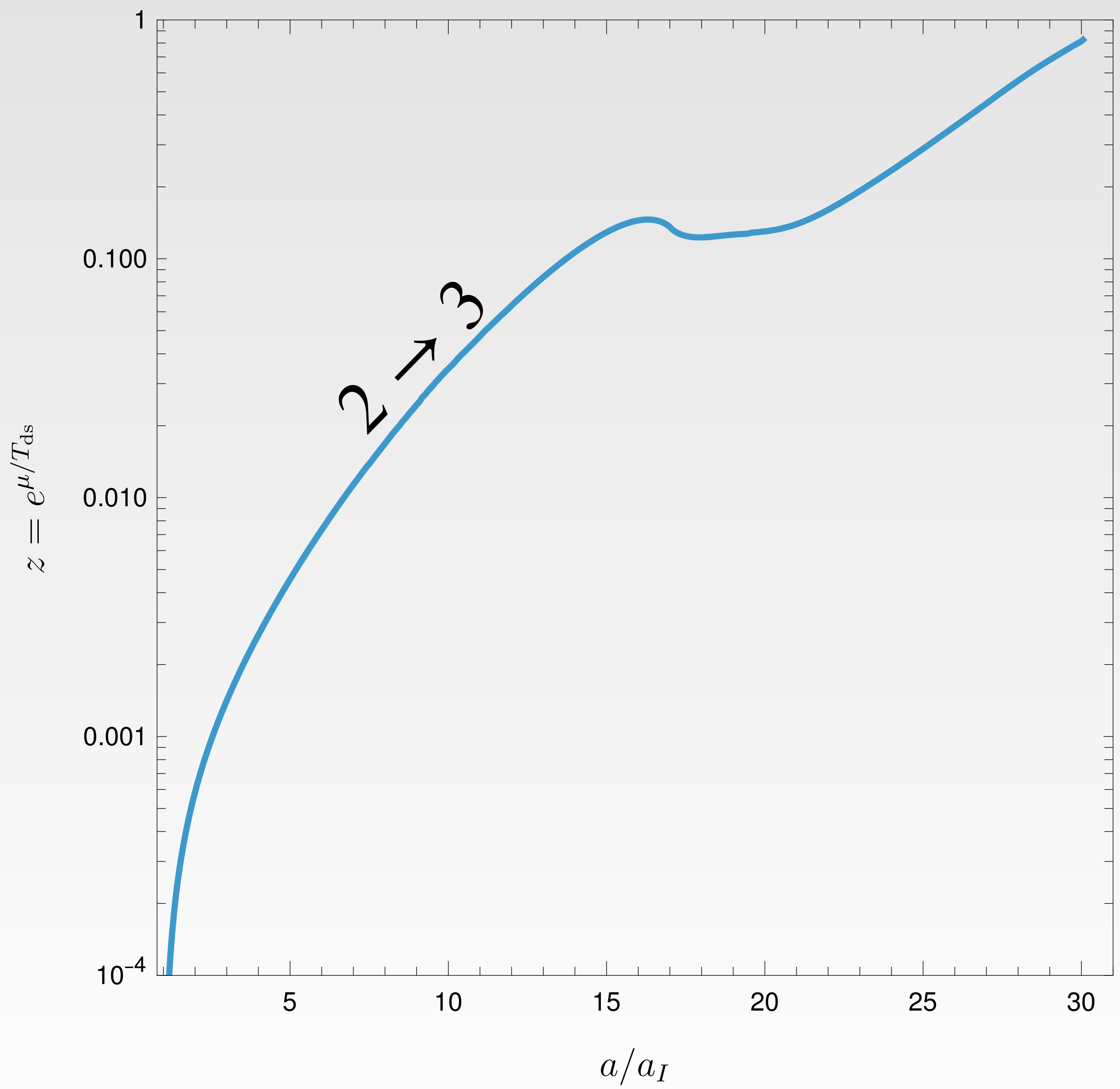
$$\frac{T^2}{2\pi^2} \int dy\, y^2 \ln\left(1 - \underbrace{e^{\mu/T}}_{z} e^{-\sqrt{y^2 + (m/T)^4}}\right) = -\int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} (e^{(E-\mu)/T} - 1)^{-1} = -p$$

Dynamics

Evolution of T_{ds} and z with $m_\phi = \text{few MeV}$ and $m_{A'} = 1\,\text{GeV}$ at $T = 0$.

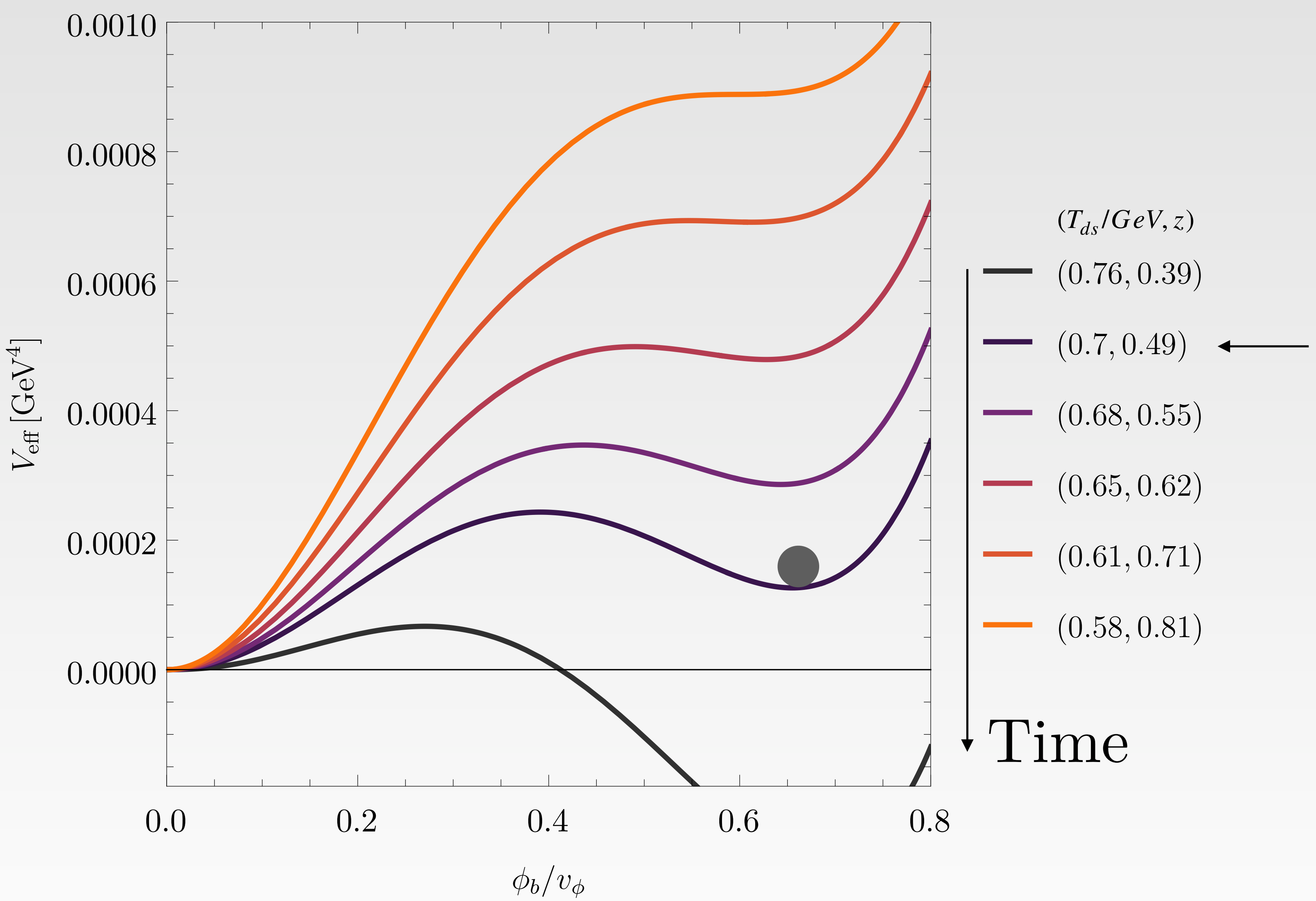
$T_{ds} > m_\phi, m_{A'}$ due to initial injection $\bar{f}f \to A'$





Results

The temperature ***drops***, while $z \to 1$ due to $2 \to 3$ reactions.



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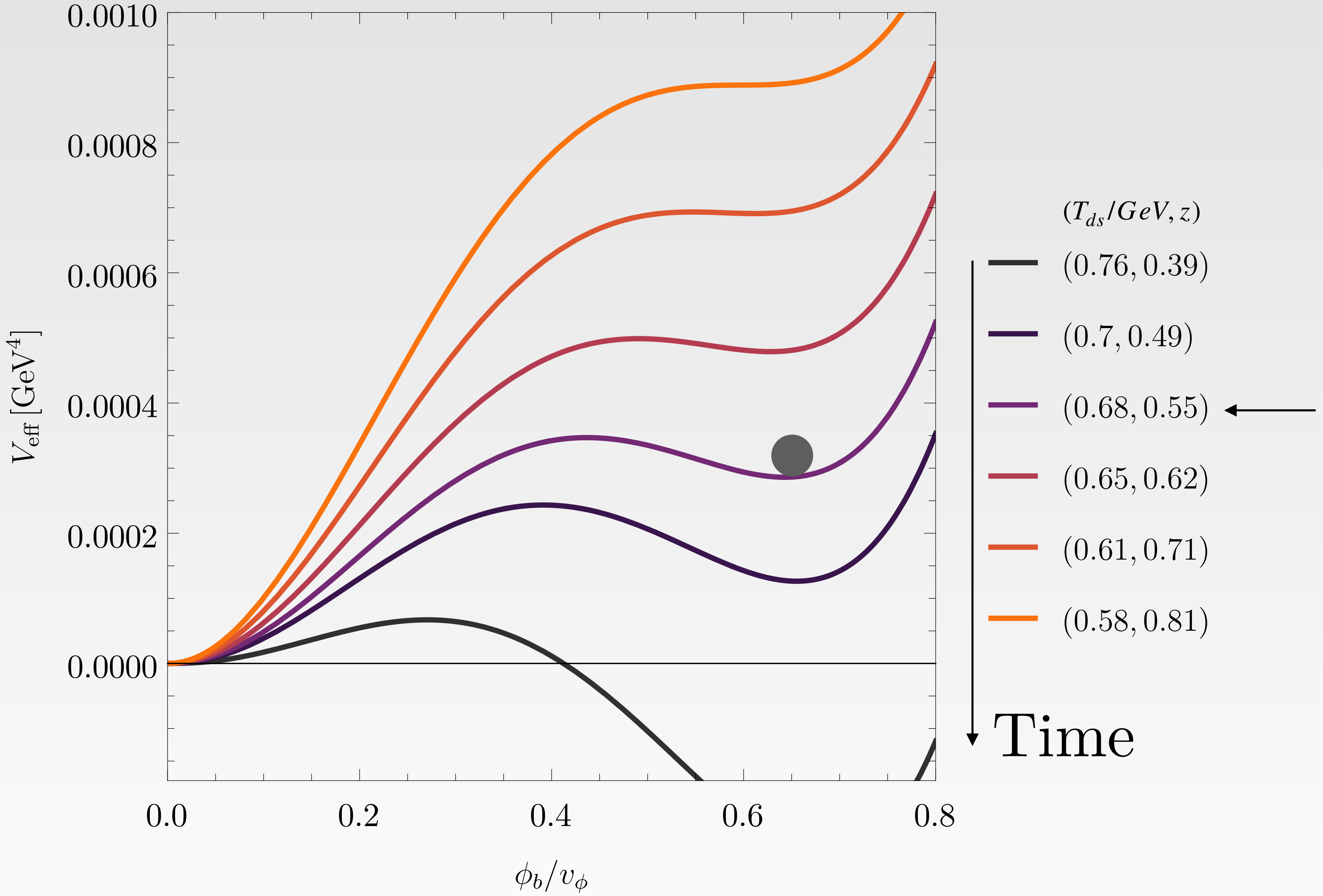

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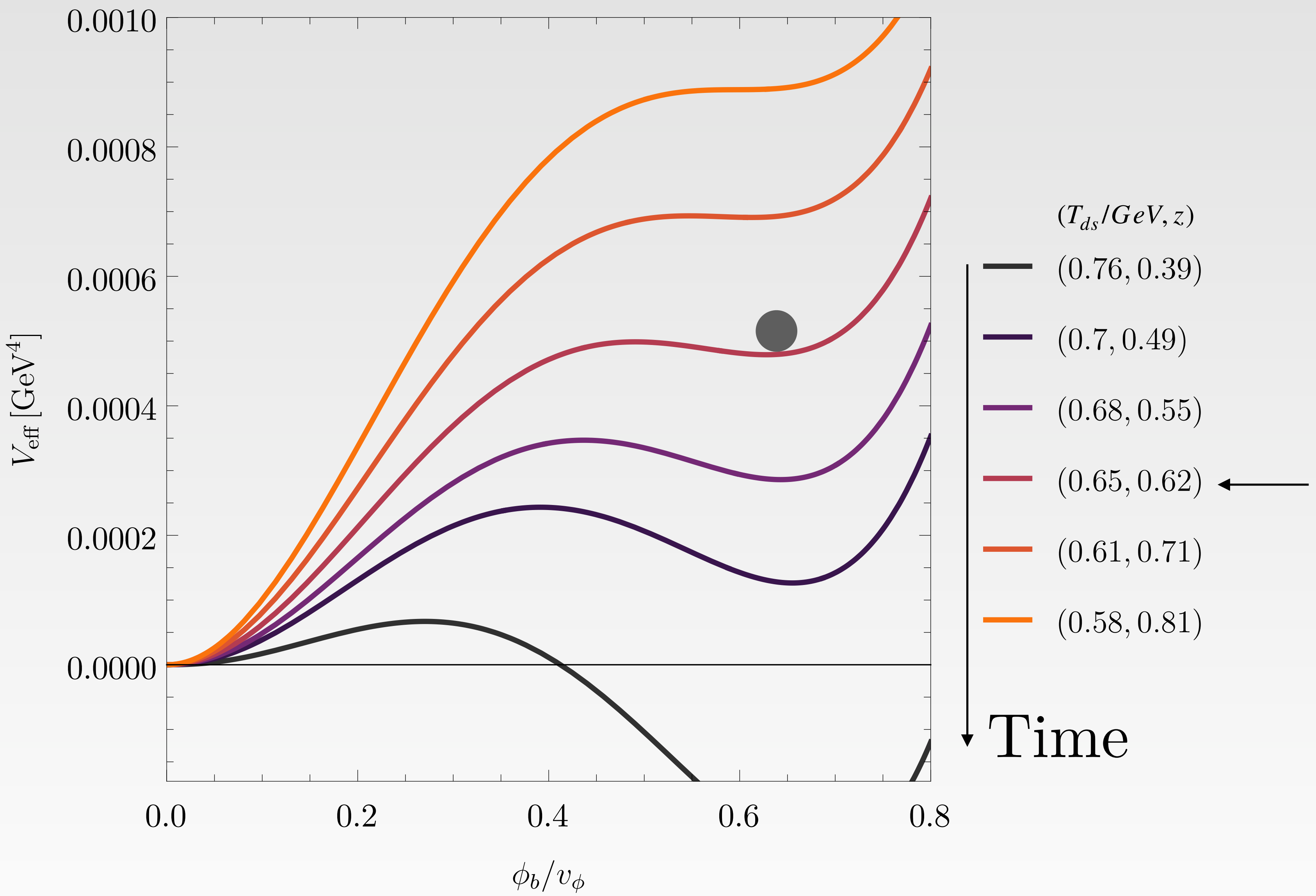

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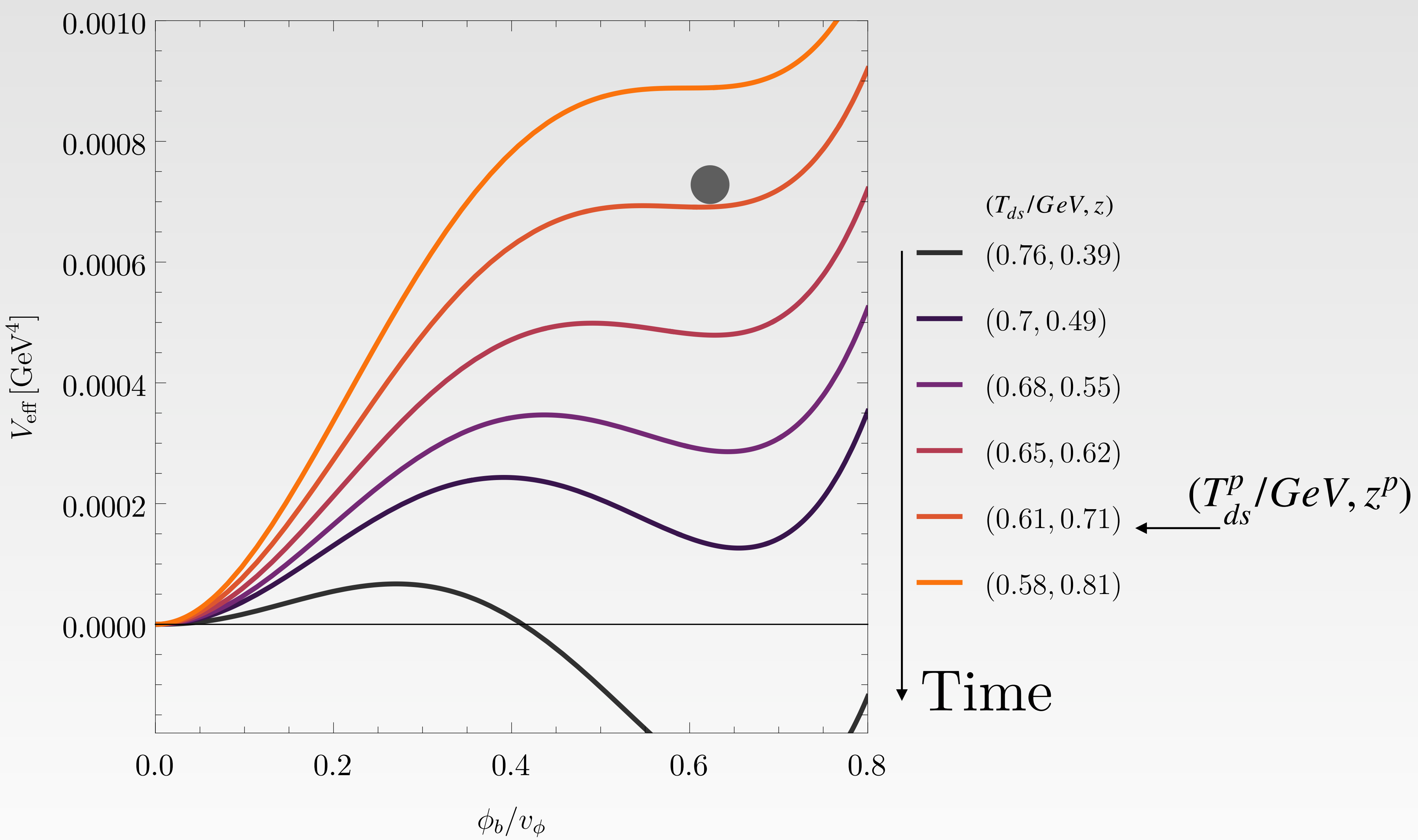

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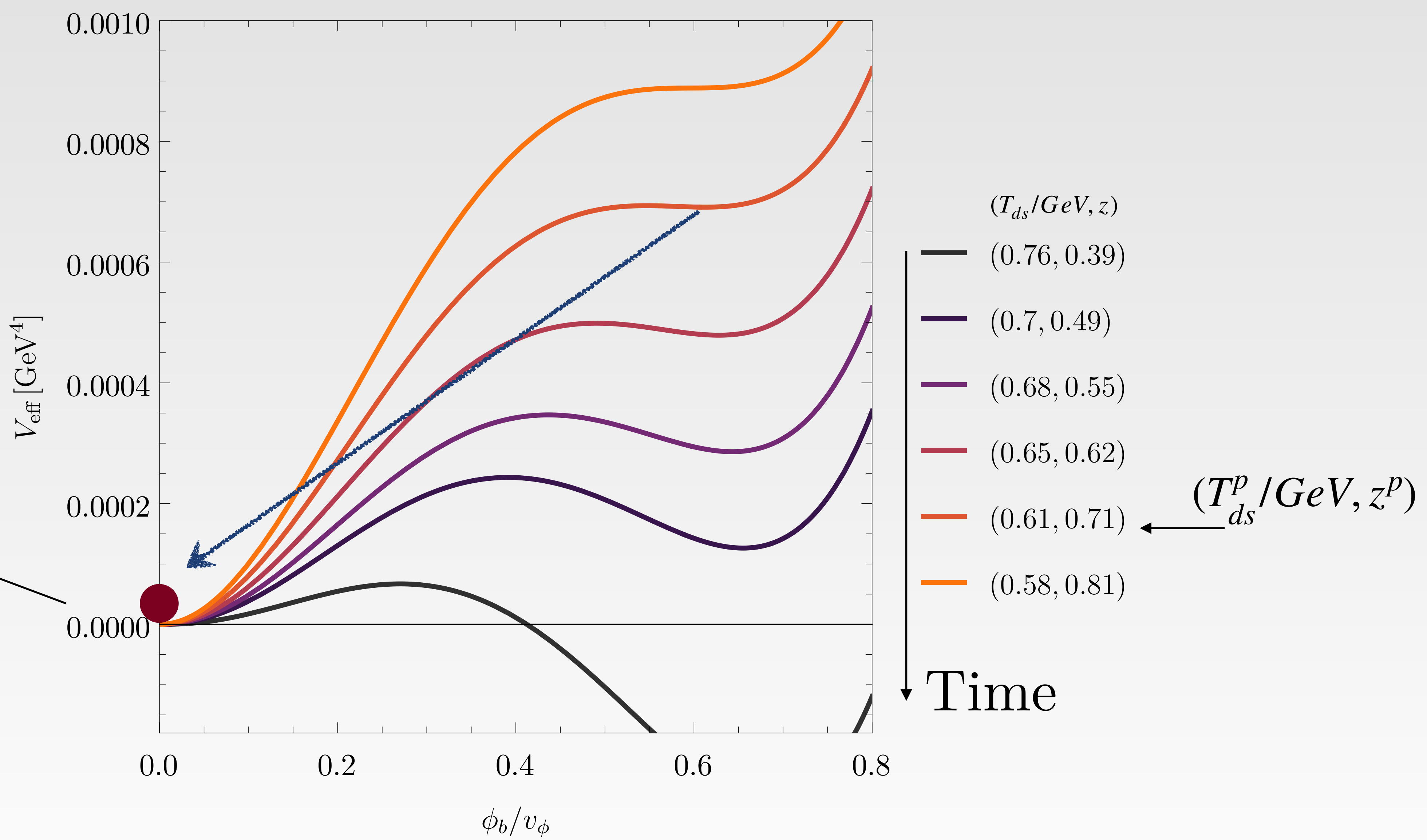
30% of universe is in this new phase.

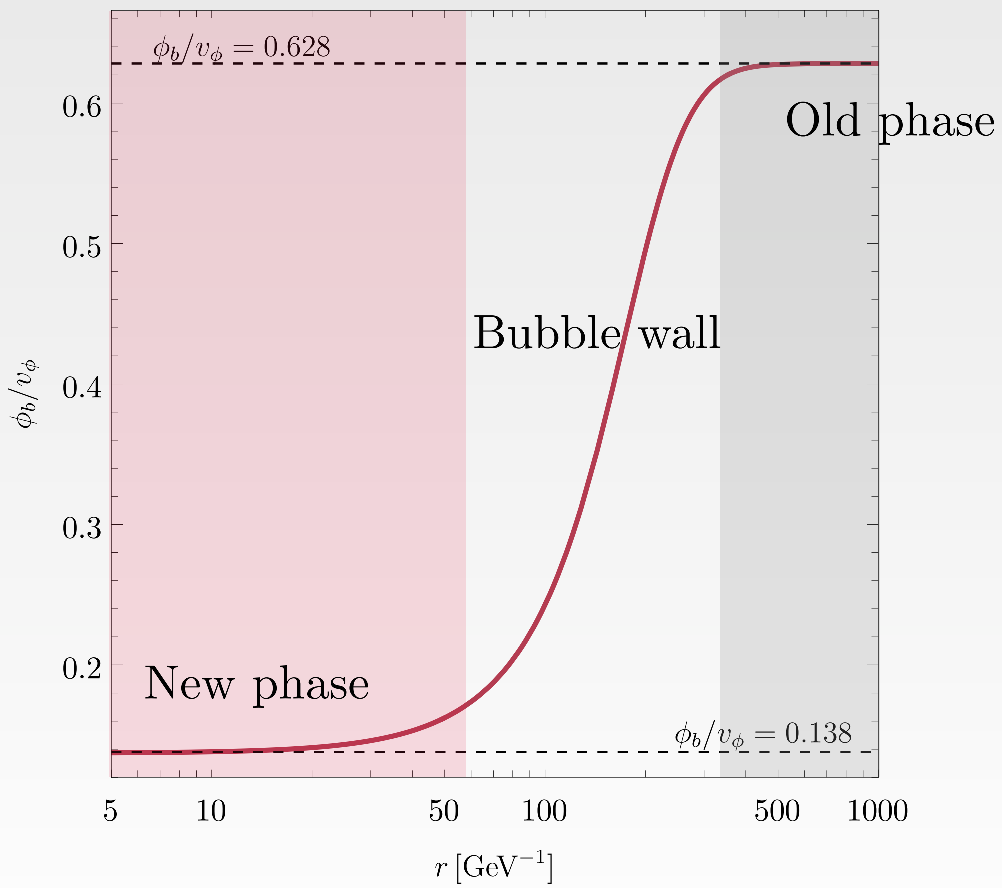


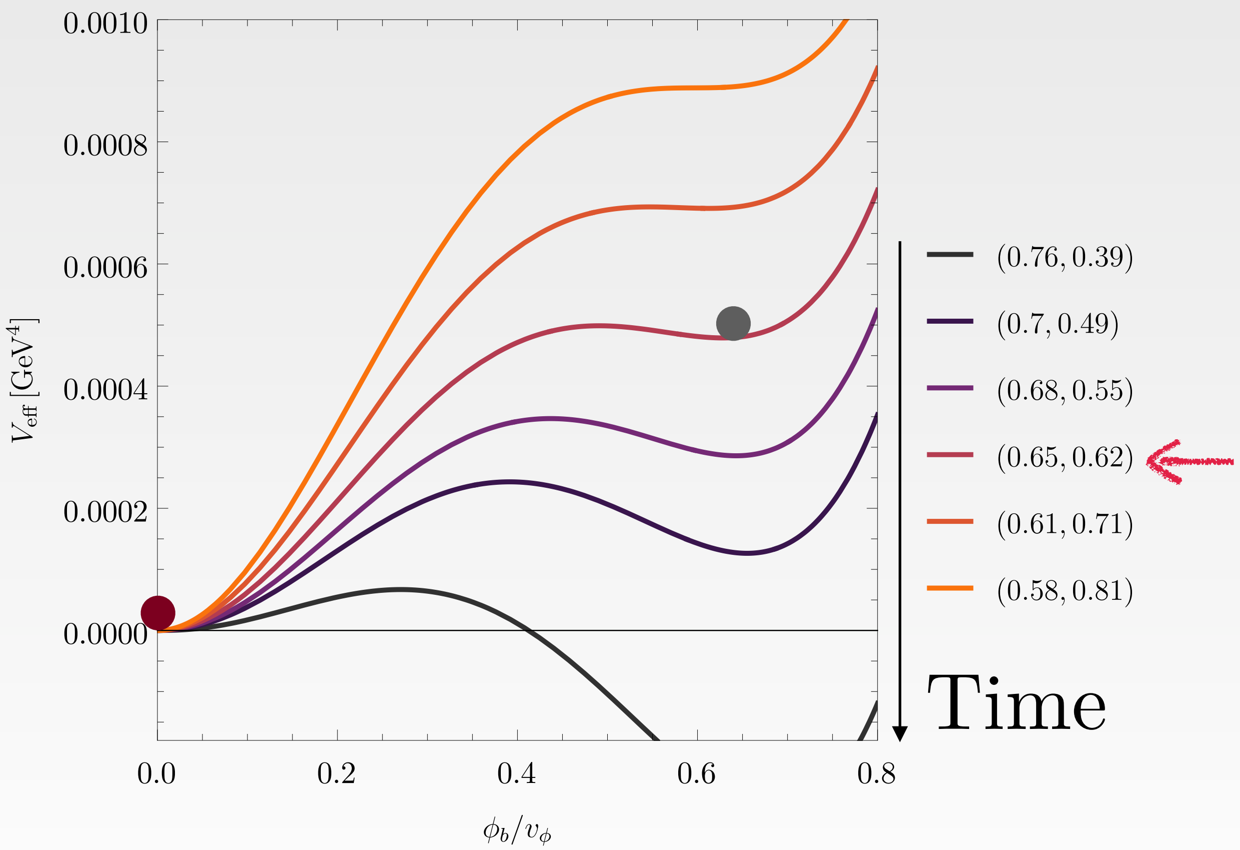
Bubble profile

The *shape* of the vacuum bubbles is given by the bounce equation (Klein Gordon equation)

$$(\partial_{rr} + \frac{2}{r}\partial_r)\phi_b(r) = \partial_{\phi_b} V_{eff}\,, \quad \partial_r \phi_b(r=0) = 0$$

Bubble profile





Conclusions

- **SIDM** produced via the **freeze-in** mechanism has a unique evolution in the Early Universe;

- Temperature can have a **non-trivial** impact in such scenarios and **need to be studied** carefully;

- The impact of **self-interactions** in **an inverse PT** requires a careful treatment of the **Boltzmann equation** coupled with the **effective potential**;

- Possible **gravitational waves** signatures might arise from such scenario (under current investigation).

Backup slides

Effective potential

Consider a simple QFT model with a real scalar field ϕ:

$$\mathscr{L}[\phi] = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4 .$$

Using the Feynman path integral definition:

$$Z[J] = e^{-iE[J]} = \int \mathscr{D}\phi \, \exp\left[i \int d^4x (\mathscr{L}[\phi] + J\phi) \right] .$$

We define a background field and effective potential

$$\phi_b \equiv \langle \phi \rangle = -\frac{\delta}{\delta J} E[J] = \frac{\int \mathscr{D}\phi \, \exp\left[i \int d^4x (\mathscr{L}[\phi] + J\phi) \right] \phi}{\int \mathscr{D}\phi \, \exp\left[i \int d^4x (\mathscr{L}[\phi] + J\phi) \right]}$$

i.e., averaged/ macroscopic behaviour of ϕ

$$V_{eff}(\phi_b) \equiv \frac{E[J] + \int d^4y \, J(y)\phi_b(y)}{Volume}$$

a.k.a. free energy density

Classical potential

Consider a classical theory:

$$V_{cl}(\phi_b) = \frac{1}{2}\mu^2\phi_b^2 + \frac{\lambda}{4!}\phi_b^4, \qquad \phi_b = \langle\phi\rangle$$

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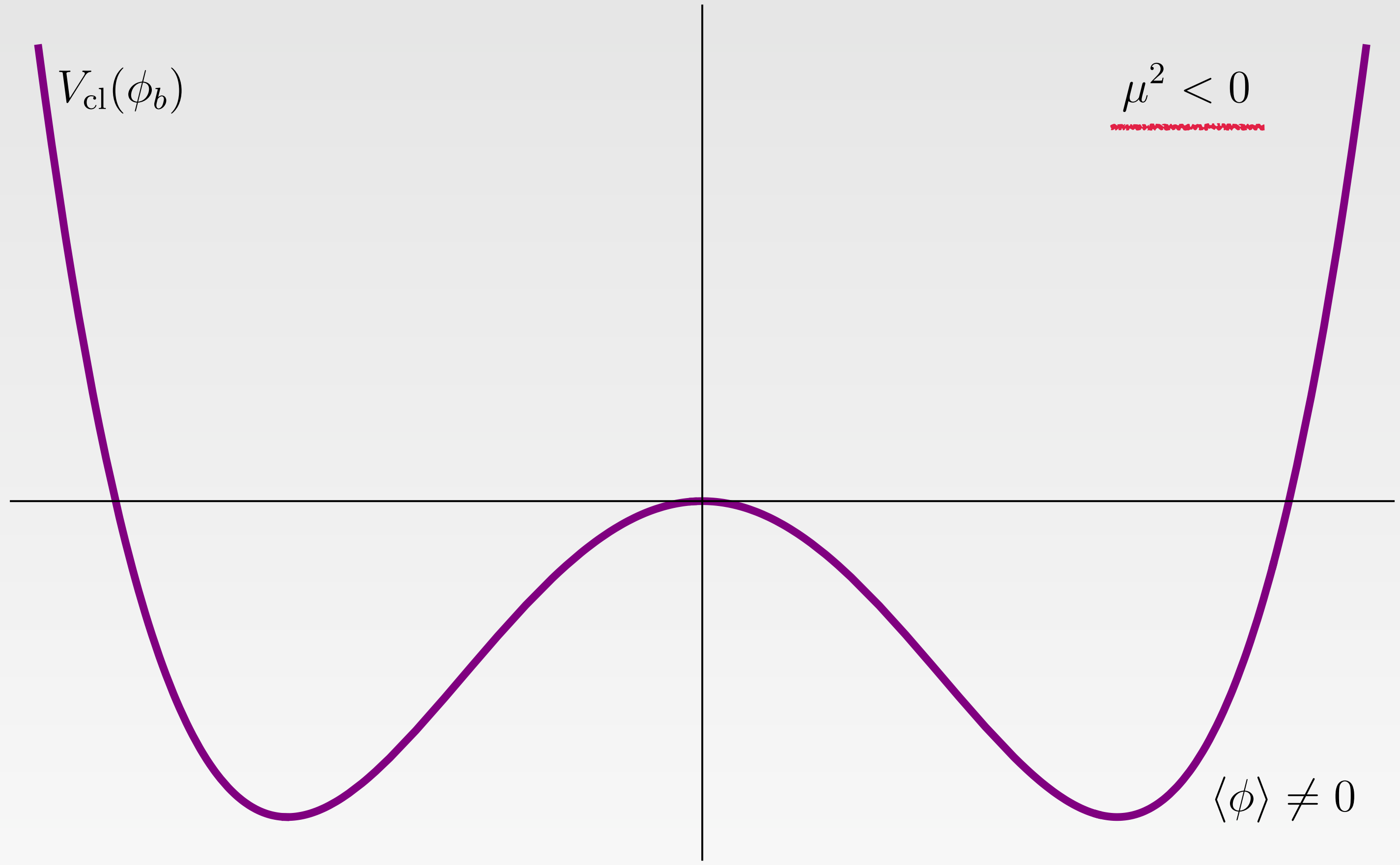

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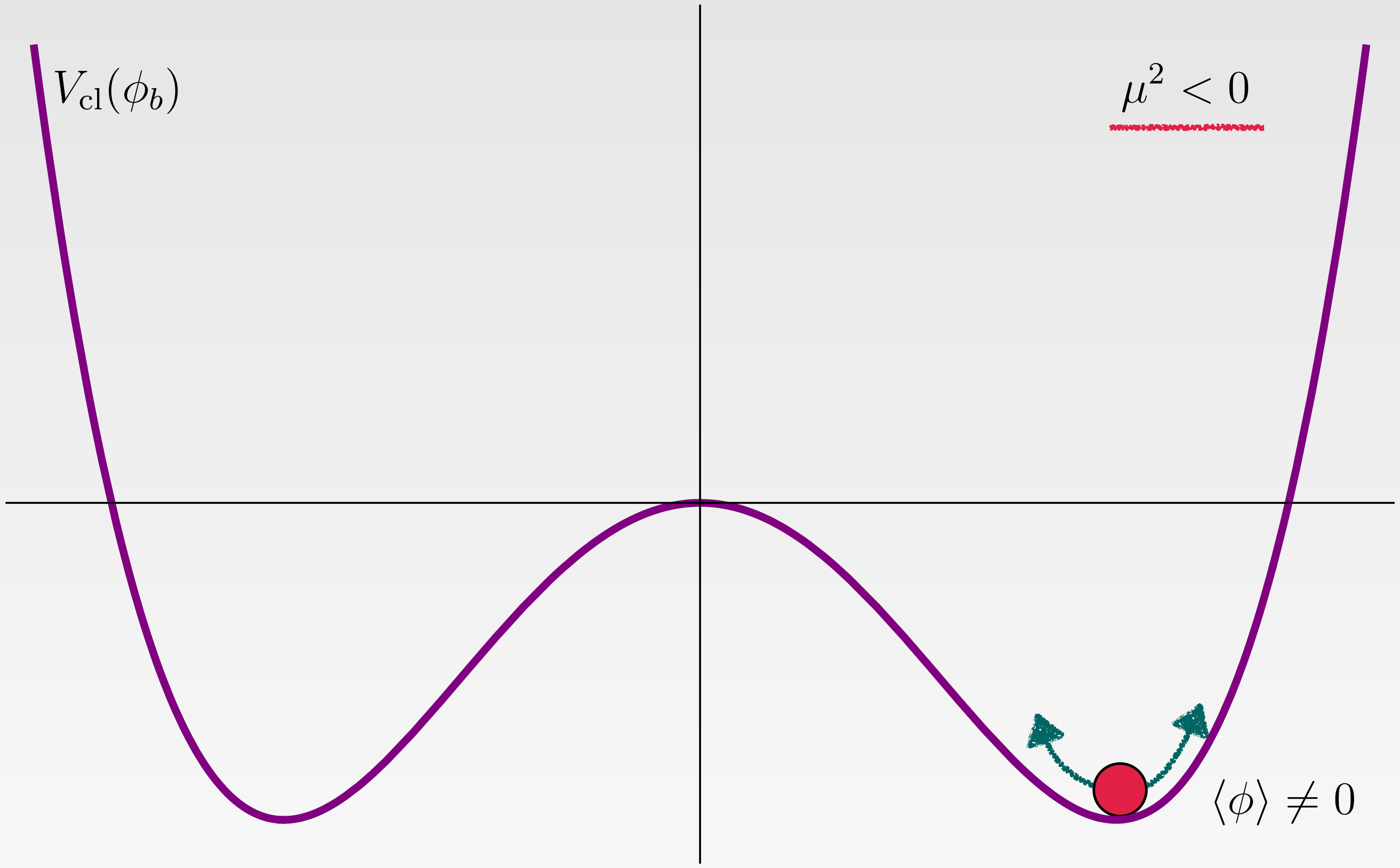

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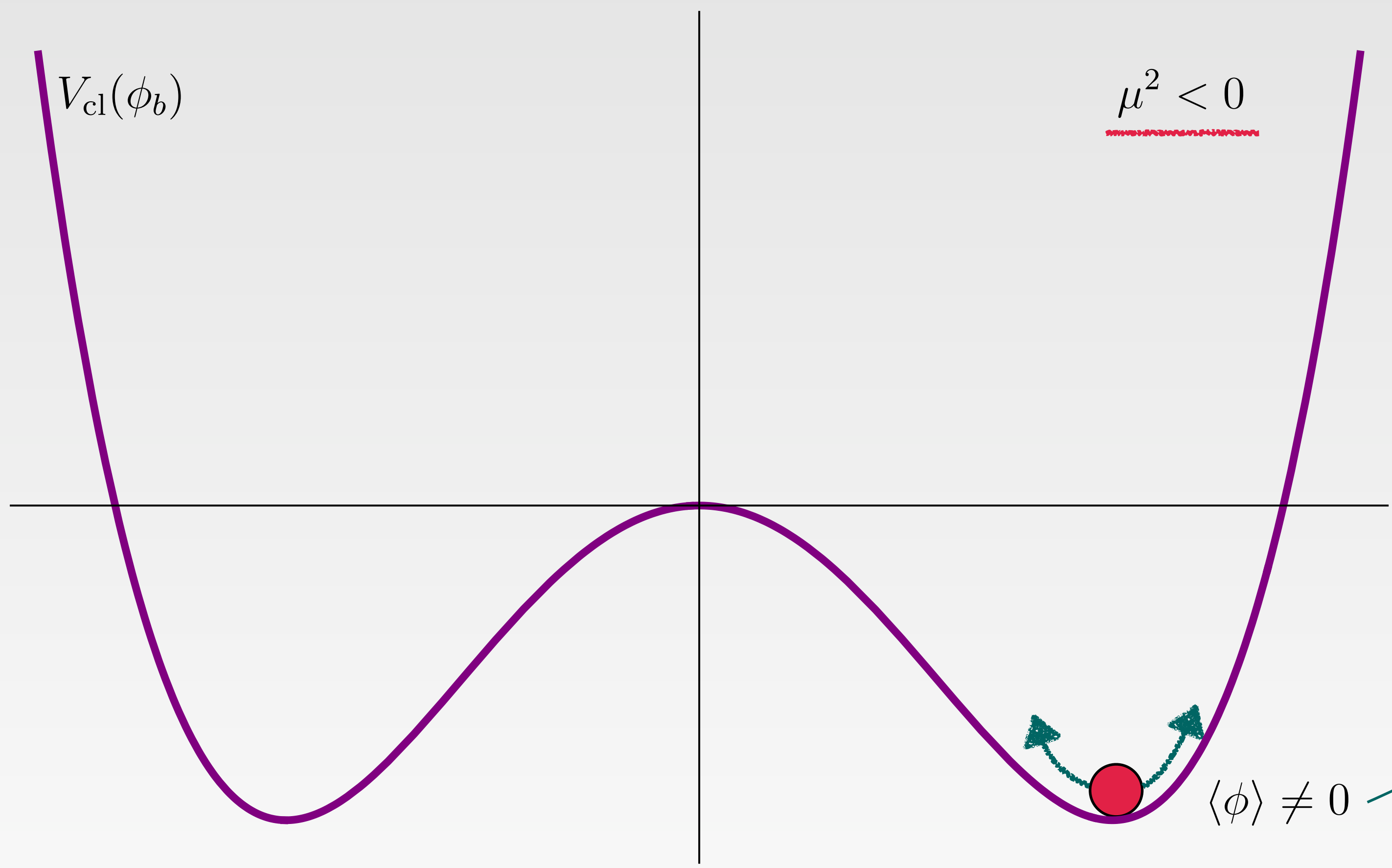

The theory has to be expanded around ϕ_b:

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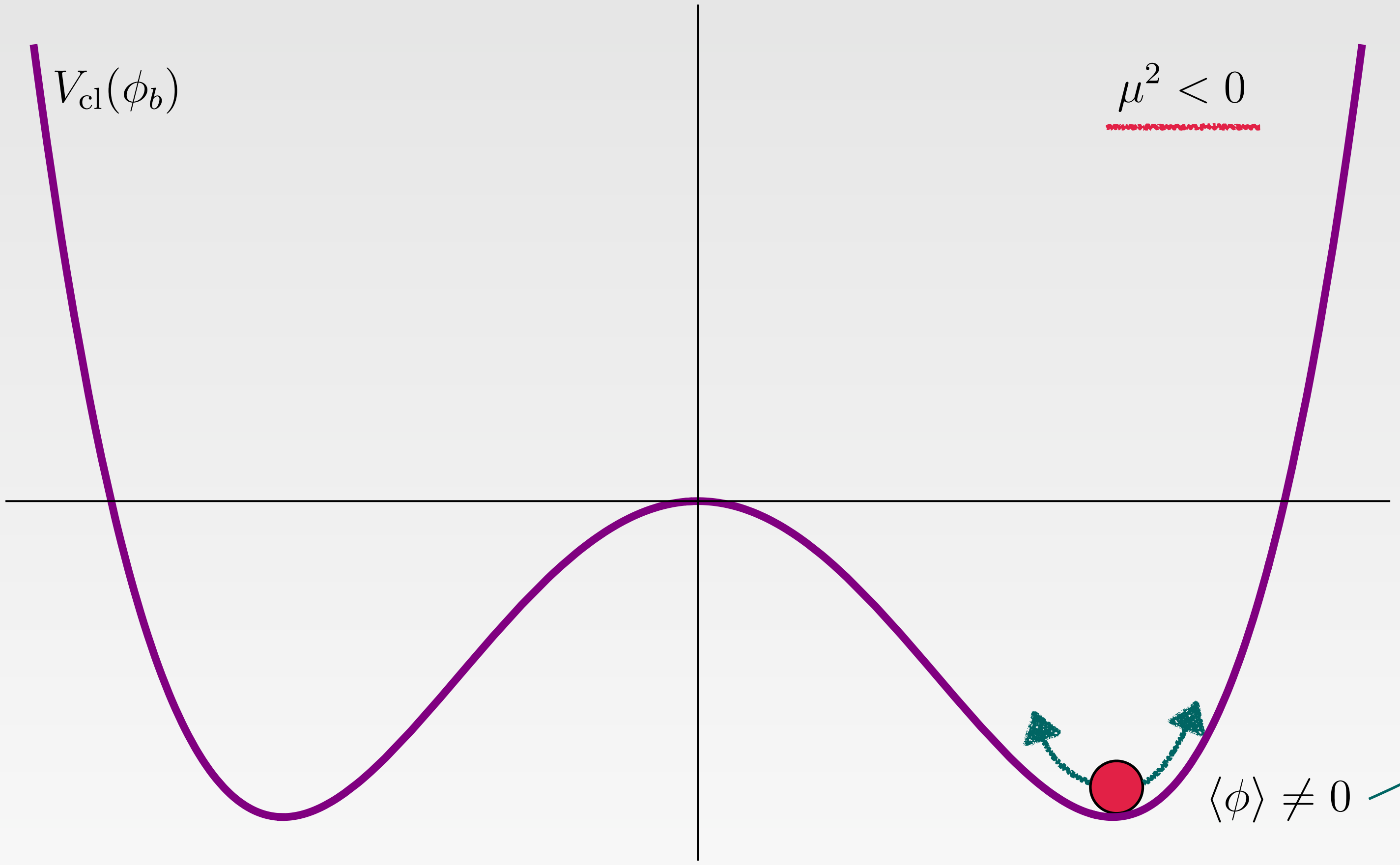

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$$\phi \to \phi + \phi_b$$

$$\mathscr{L}[\phi] \supset -\frac{1}{2}\mu^2(\phi+\phi_b)^2 - \frac{\lambda}{4!}(\phi+\phi_b)^4 = -\frac{1}{2}m^2(\phi_b)\phi^2 - \frac{1}{3!}g(\phi_b)\phi^3 - \frac{\lambda}{4!}\phi^4$$

Quantum corrections

The classical potential is not the full picture:

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$$V_{\mathrm{cw}}(\phi_b) = g_\phi \frac{m^4(\phi_b)}{64\pi^2}\left(\log\frac{m^2(\phi_b)}{\Lambda^2} - 3/2\right), \qquad m(\phi_b)^2 = 3\lambda\phi_b^2$$

$$V_{\mathrm{ct}}(\phi_b) = \delta\mu^2\phi_b^2 + \delta\lambda\phi_b^4$$

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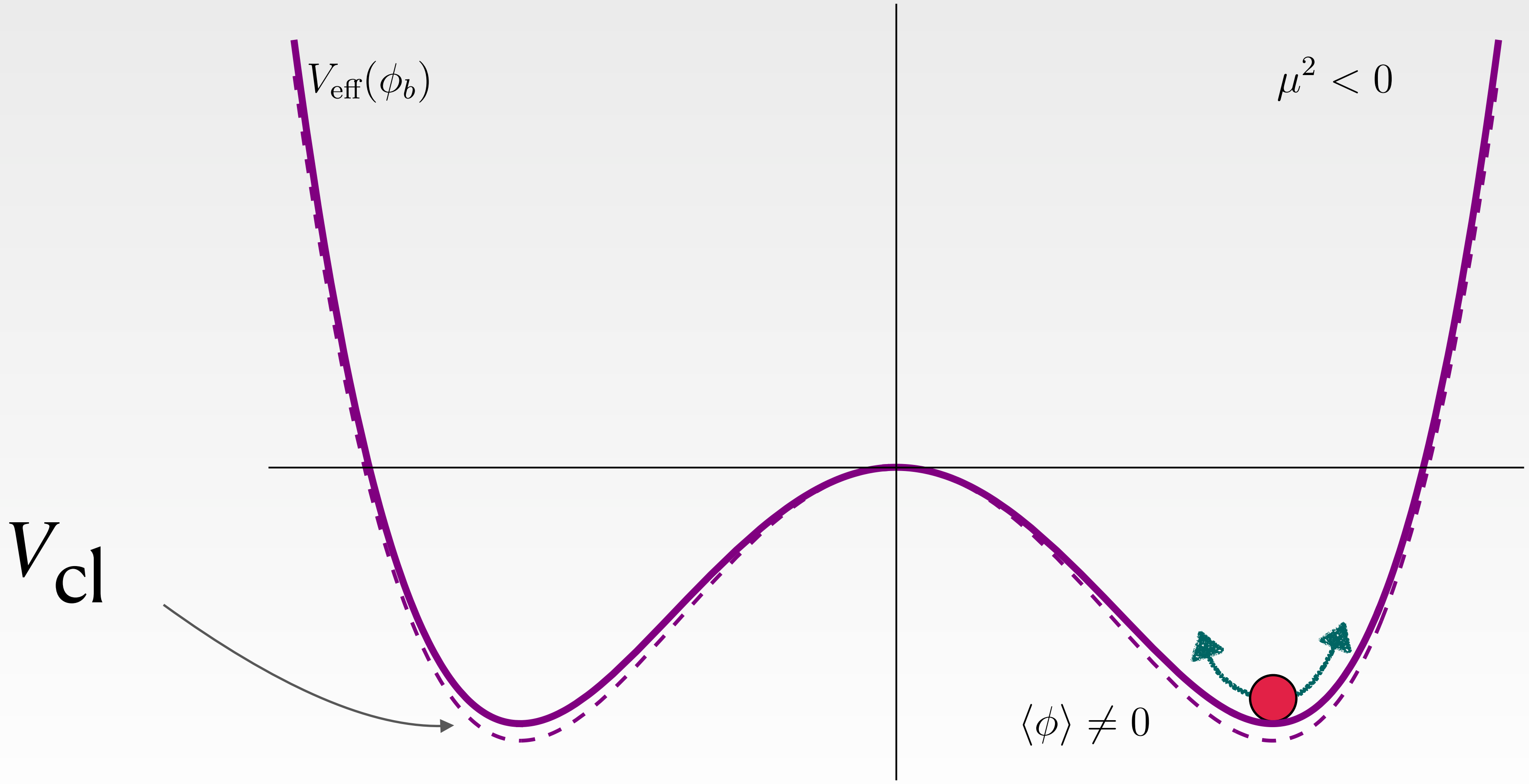
$$V_{eff}(\phi_b) = V_{cl}(\phi_b) + V_{\mathrm{CW}}(\phi_b) + V_{\mathrm{ct}}(\phi_b)$$

$$V_{\mathrm{CW}}(\phi_b) = g_\phi \frac{m^4(\phi_b)}{64\pi^2}\left(\log\frac{m^2(\phi_b)}{\Lambda^2} - 3/2\right), \qquad m(\phi_b)^2 = 3\lambda\phi_b^2$$

$$V_{\mathrm{ct}}(\phi_b) = \delta\mu^2\phi_b^2 + \delta\lambda\phi_b^4$$



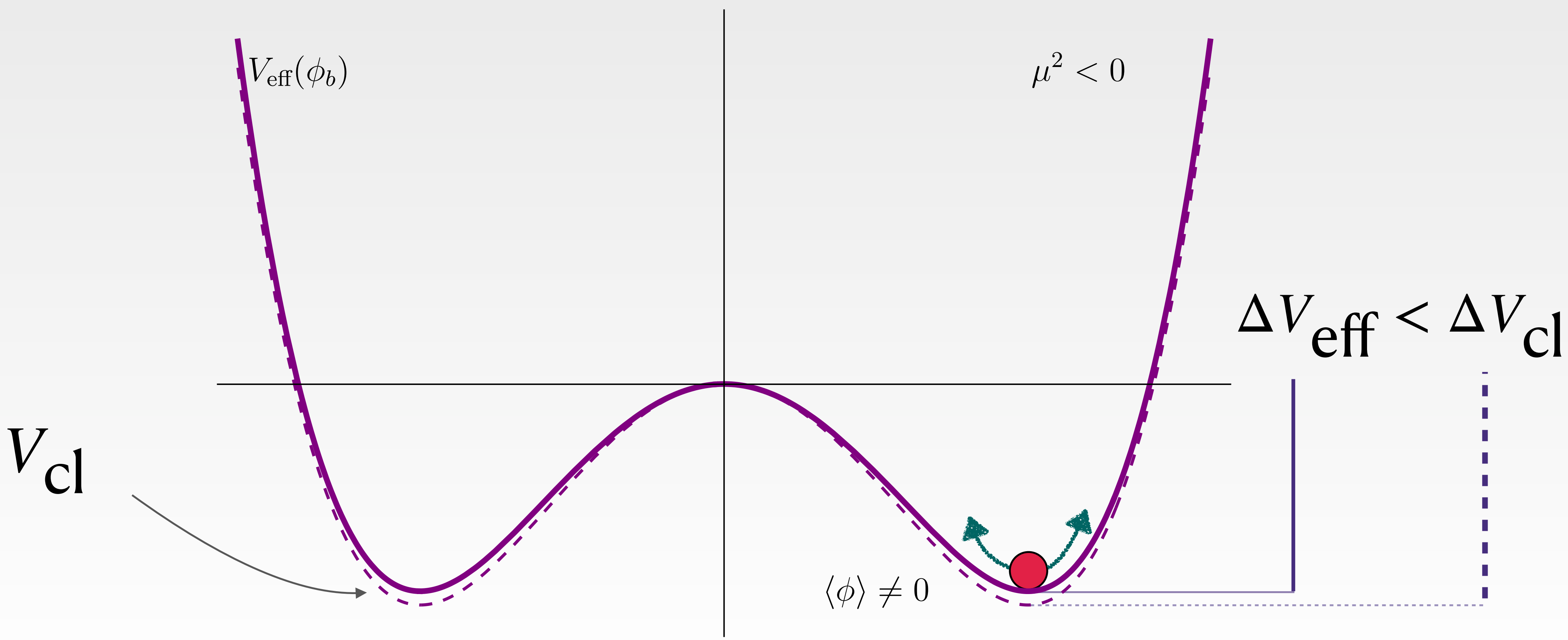
Quantum corrections

The classical potential is not the full picture:

$$V_{eff}(\phi_b) = V_{cl}(\phi_b) + V_{\mathrm{cw}}(\phi_b) + V_{\mathrm{ct}}(\phi_b)$$

$$V_{\mathrm{cw}}(\phi_b) = g_\phi \frac{m^4(\phi_b)}{64\pi^2}\left(\log\frac{m^2(\phi_b)}{\Lambda^2} - 3/2\right), \qquad m(\phi_b)^2 = 3\lambda\phi_b^2$$

$$V_{\mathrm{ct}}(\phi_b) = \delta\mu^2\phi_b^2 + \delta\lambda\phi_b^4$$



U(1) theory

The transition might be more violent, leaving imprints in the early universe. Simple realization: complex scalar field $\mathbf{\Phi}$ charged under a U(1) gauge symmetry

$$\mathscr{L} = |(\partial_\mu - igA'_\mu)\phi|^2 - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \mu^2\Phi^*\Phi - \lambda(\Phi^*\Phi)^2 + \frac{\epsilon}{2\cos\theta_w}B_{\mu\nu}F'^{\mu\nu}$$

$$\begin{aligned}
&= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_{A'}^2A'^2_\mu \\
&\quad -gA'_\mu(\varphi\partial^\mu\phi - \phi\partial^\mu\varphi - v_\phi\partial^\mu\varphi) + \frac{g^2}{2}\phi^2A'^2_\mu + g^2v_\phi\phi A'^2_\mu \\
&\quad -\lambda v_\phi\phi^3 - \lambda v_\phi\varphi^2\phi - \frac{\lambda}{4}\phi^2\varphi^2 - \frac{\lambda}{4}\phi^4 - \frac{\lambda}{4}\varphi^4 \\
&\quad +\frac{\epsilon}{2\cos\theta_w}B_{\mu\nu}F'^{\mu\nu}
\end{aligned}$$

Effective potential of a U(1) theory

The effective potential of ϕ will receive corrections from its interactions with φ and A':

$$V_{cl}(\phi_b) = \frac{1}{2}\mu^2\phi_b^2 + \frac{\lambda}{4!}\phi_b^4$$

$$V_{\mathrm{CW}}(\phi_b) = \sum_{a=\phi,\varphi,A'} g_a \frac{m_a^4(\phi_b)}{64\pi^2}\left(\log\frac{m_a^2(\phi_b)}{\Lambda^2} - 3/2\right)$$

$$V_T(\phi_b) = \frac{T^4}{2\pi^2}\sum_{a=\phi,\varphi,A'} g_a \int dy\, y^2 \ln\left(1 - e^{-\sqrt{y^2 + (m_a(\phi_b)/T)^4}}\right)$$

$$V_{\mathrm{daisy}}(\phi_b) = -\frac{T}{12\pi}\sum_{a=\phi,\varphi,A'_L} g_a \left[(m_a^2(\phi_b) + \Pi_a(T))^{3/2} - (m_a^2(\phi_b))^{3/2}\right]$$

$$m_\phi^2(\phi_b) = 3\lambda\phi_b^2 - \mu^2, \qquad m_\varphi^2(\phi_b) = \lambda\phi_b^2 - \mu^2, \qquad m_{A'}^2(\phi_b) = g^2\phi_b^2\,.$$

What about a chemical potential?

Not surprising ($V_{eff} = -\,pressure$), but useful:

$$\int \frac{d^3p}{(2\pi)^3}\frac{p^2}{3E}\left(e^{\frac{E-\mu}{T}}-1\right)^{-1} = \frac{m(\phi_b)^2T^2}{2\pi^2}\sum_{k=1}^{\infty}\frac{z^k}{k^2}K_2(k\,m(\phi_b)/T)\,,$$

which converges for $z \leq 1$. The ***evolution*** for T and z are given by the ***Boltzmann equation***

$$\left(\partial_t - H\,p\,\partial_p\right)f(p,t) = C[f] \longrightarrow \begin{array}{l}\text{Collision}\\ \text{operator}\end{array} \longrightarrow \begin{array}{l}\text{Freeze-in and}\\ \text{Self-int.}\end{array} \longrightarrow \begin{array}{l}\bar{f}f \to A' \text{ and}\\ 3\phi \leftrightarrow 2\phi\end{array}$$

$$\begin{matrix}\text{0th}\\ \text{moment}\\[3em] \text{2nd}\\ \text{moment}\end{matrix}\left\{\begin{aligned} \frac{dN}{da} &= \frac{a^2}{H}\,C_0\,,\\ \frac{dT_{ds}}{da} &= \frac{1}{1+\kappa_1}\left(-\frac{2\,T_{ds}}{a} + \frac{1}{3\,a}\left\langle\frac{p^4}{E^3}\right\rangle + \frac{a^2}{3\,H\,N}\,C_2 - \frac{N'}{N}T_{ds} + \kappa_2\right)\end{aligned}\right.$$

Coupled Boltzmann equations

To obtain 'temperature' Boltzmann equation:

first we define $T' := \dfrac{g_{dm}}{3n} \displaystyle\int \dfrac{d^3p}{(2\pi)^3} \dfrac{p^2}{E} f(p);$

we integrate $g(2\pi)^{-3} \displaystyle\int d^3p \, \dfrac{p^2}{E} (\partial_t - H\vec{p} \cdot \vec{\nabla}_p) f = g(2\pi)^{-3} \displaystyle\int d^3p \, \dfrac{p^2}{E} C[f] =: C_2;$

to obtain $\dfrac{dT'}{da} = -\dfrac{2\,T'}{a} + \dfrac{1}{3\,a} \left\langle \dfrac{p^4}{E^3} \right\rangle + \dfrac{a^2}{3\,H\,N} C_2 - \dfrac{a^2\,T'}{H\,N} C_0;$

along with the usual nBE: $\dfrac{dN}{da} = \dfrac{a^2}{H} g \displaystyle\int \dfrac{d^3p}{(2\pi)^3} C[f] =: \dfrac{a^2}{H} C_0 \,, \; N = na^3;$

we close the system assuming $f(E) = \dfrac{n}{n_{\rm eq}} \exp\left[-\dfrac{E}{T'}\right]$ (local thermal eq).

Coupled Boltzmann equations

When the mass changes over time (due to the effective potential)

the integral $\int d^3p\, \frac{p^2}{E}(\partial_t - H\vec{p}\cdot\vec{\nabla}_p)f$ contains the term;

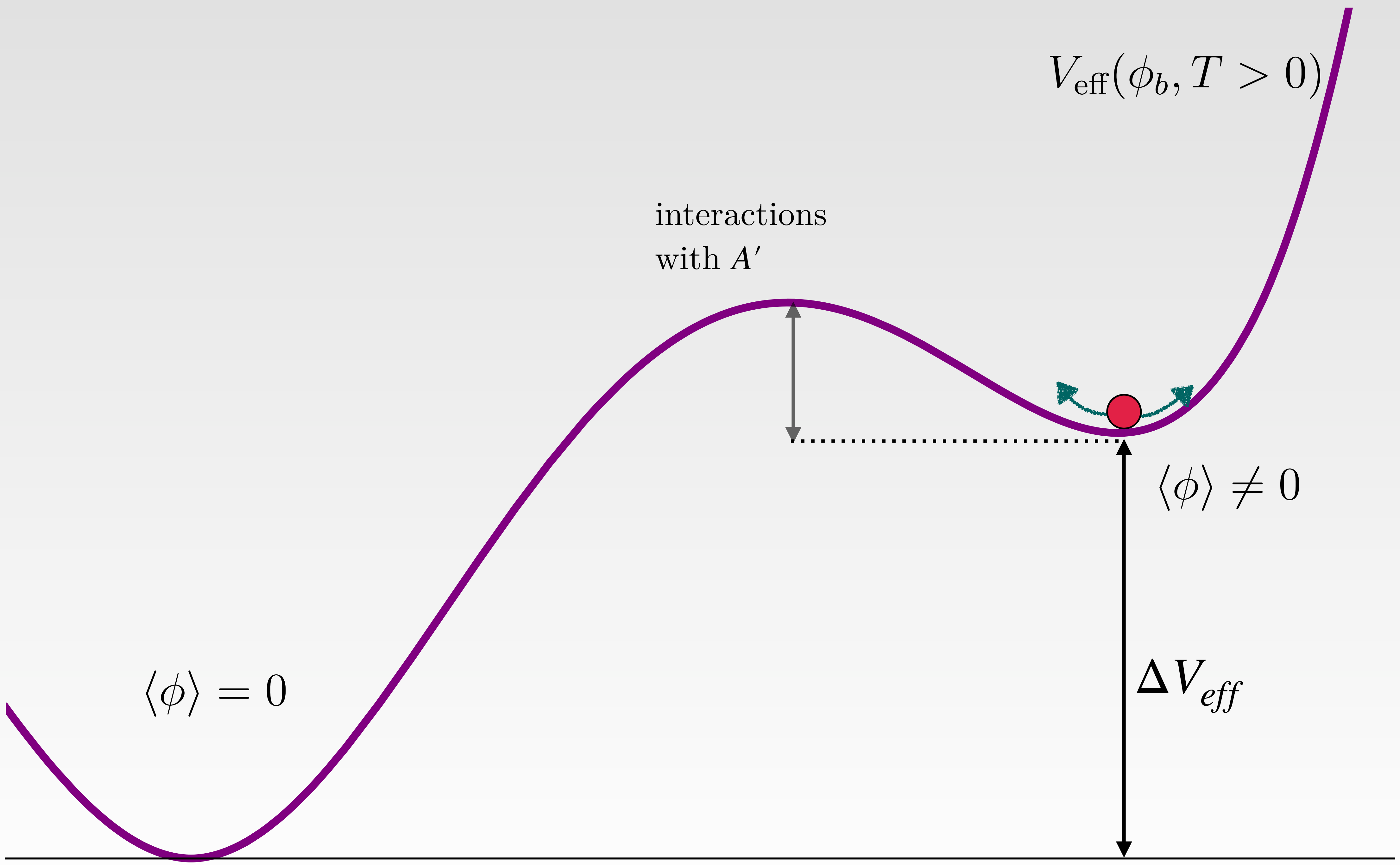
$\frac{d(m^2)}{dt}\int \frac{d^3p}{(2\pi)^3}\frac{p^2}{E^3}\, f$ because $E = (p^2 + m(t)^2)$.

therefore $\frac{dT_{ds}}{da} = \frac{1}{1+\kappa_1}\left(-\frac{2\,T_{ds}}{a} + \frac{1}{3\,a}\left\langle\frac{p^4}{E^3}\right\rangle + \frac{a^2}{3\,H\,N}C_2 - \frac{N'}{N}T_{ds} + \kappa_2\right)$;

with $\kappa_1 = \frac{1}{6}\langle p^2/E^3\rangle\left(V_{\phi\phi T} - \frac{V_{\phi\phi\phi}V_{\phi T}}{m^2}\right)$ and $\kappa_2 = -\frac{1}{6}\langle p^2/E^3\rangle\left(V_{\phi\phi z} - \frac{V_{\phi\phi\phi}V_{\phi z}}{m^2}\right)\frac{N'}{N_{eq}}$.

Inverse phase transition

The system is *initially* in a **broken** phase and is ‘pushed’ towards a **symmetric** phase:



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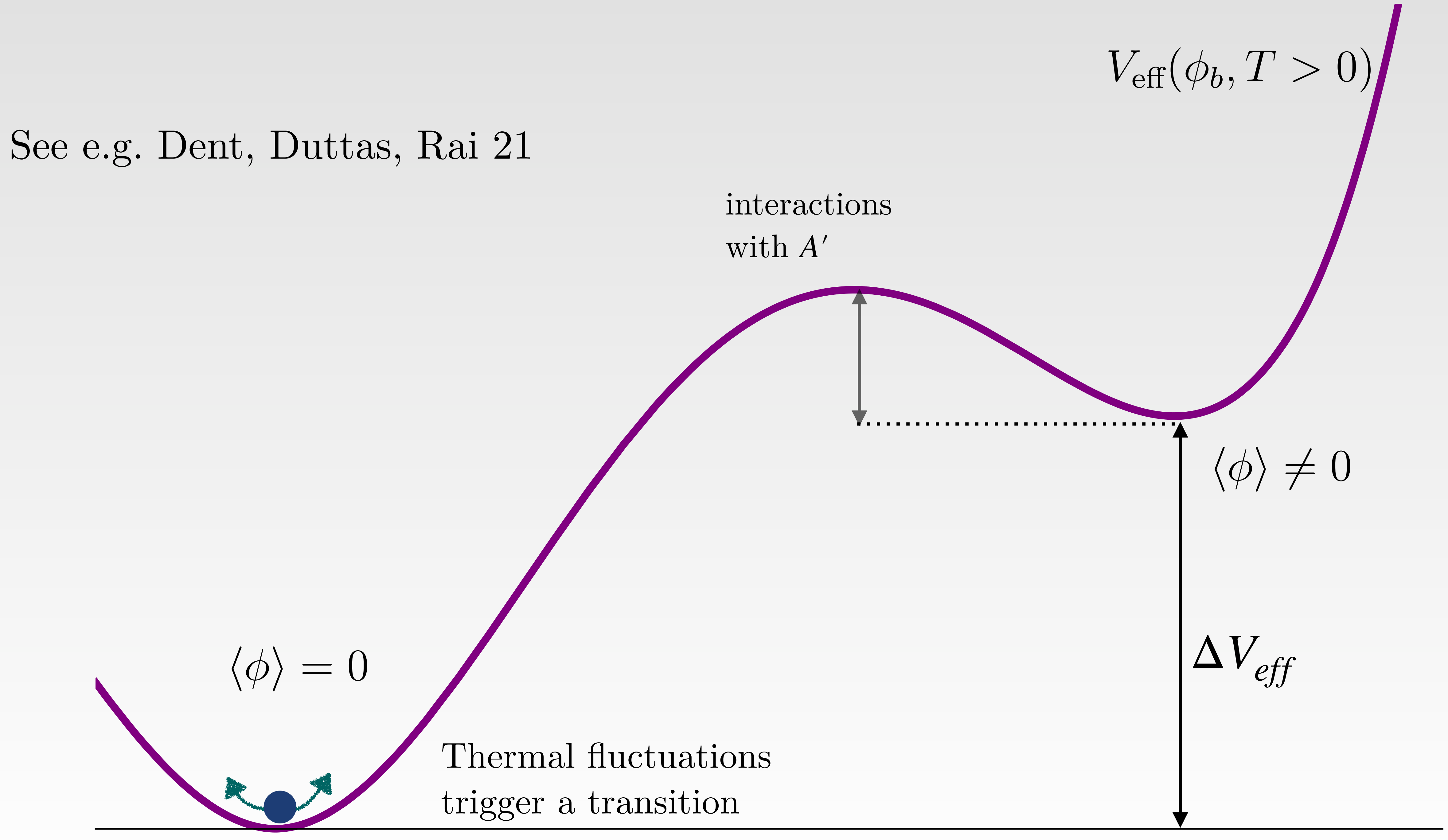


GW spectrum

The spectrum is calculated via

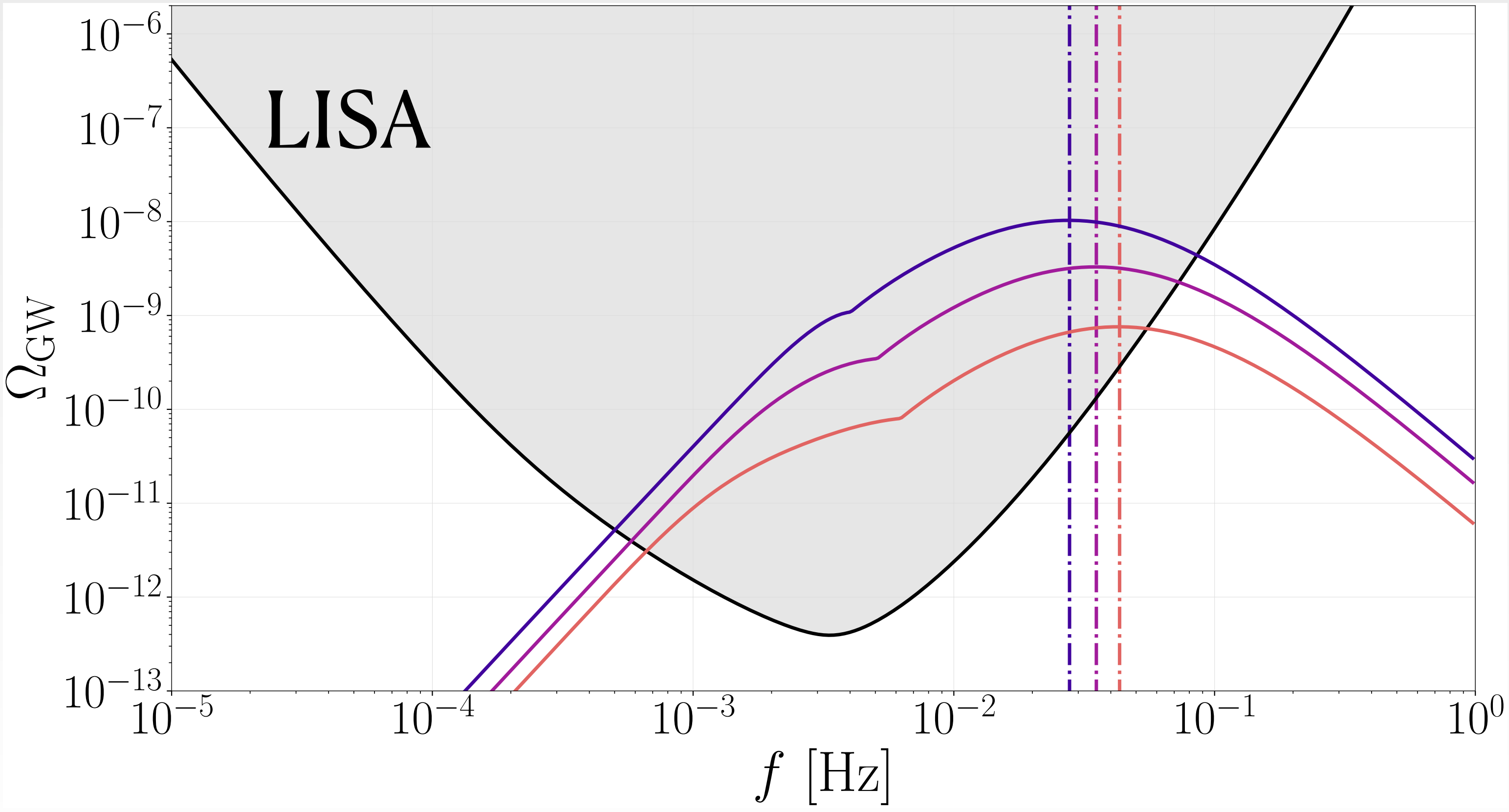
$$\Omega_{GW}h^2(f) = Rh^2\,\tilde{\Omega}\left(\frac{\kappa_{sw}\alpha}{\alpha+1}\right)\left(\beta/H\right)^{-1}\Upsilon\,S\,,$$

$$S(f,f_p) = \left(\frac{f}{f_p}\right)^3\left(\frac{7}{4+3(f/f_p)^2}\right)^{7/2}$$

Latent heat (ΔV_{eff})

$$\alpha = \frac{\Delta\theta}{4(\rho_{DS}+\rho_{sm})}$$

$$\kappa_{sw} = \frac{\Delta\Theta}{4\rho_{DS}}$$



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