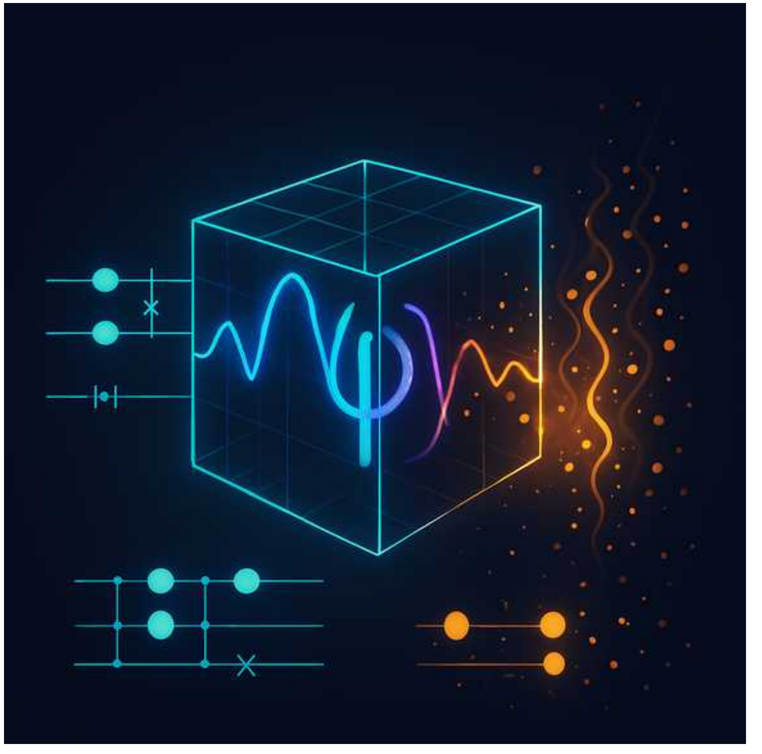


Many faces of decoherence

Wojciech Krzemień

From imaging algorithms to quantum methods

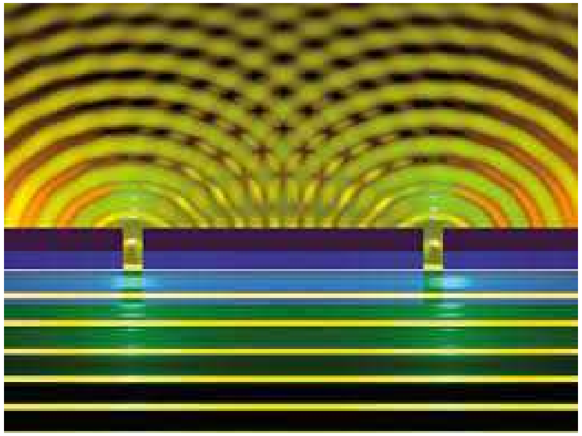
14.04.2025

Quantum Mechanics in one slide :-)

State: $$|\Psi\rangle$$

Superposition: $$|\Phi\rangle = a\,|\Psi_1\rangle + b\,|\Psi_2\rangle \neq \Psi_1 or \Psi_2$$

Time evolution: $$-i\hbar\frac{\partial}{\partial t}\,|\Psi\rangle = \hat{H}\,|\Psi\rangle$$



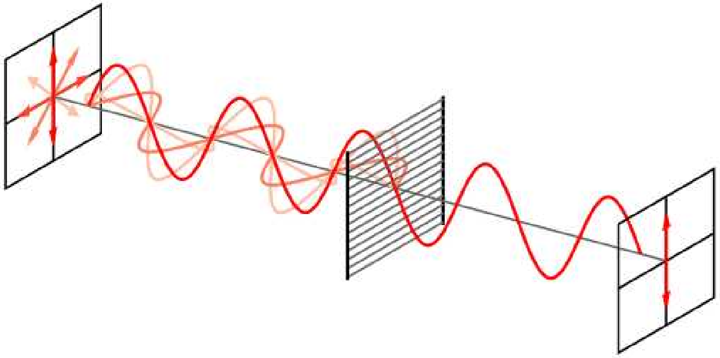
https://physicsworld.com/a/double-slits-with-single-atoms/

Measurement: $$\hat{O}\,|\Psi\rangle = m\,|\Psi\rangle$$

Probabilitic interpretation: $$\hat{O}\,|\Psi\rangle = k\,|\Phi\rangle$$

$$|\langle\Phi|\Psi\rangle|^2$$

$$\langle\Psi|\hat{O}\,|\Psi\rangle$$



https://en.wikipedia.org/wiki/Polarizer

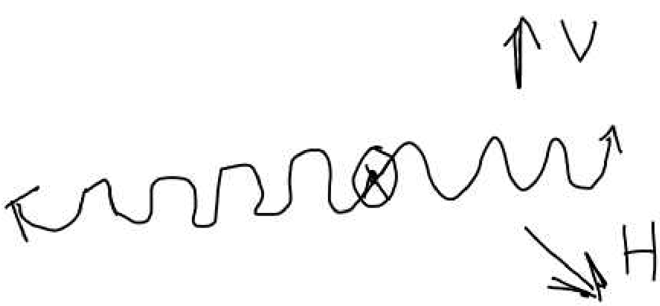
Pure state vs mixed state

Pure state: $|\Psi\rangle = \alpha|\Psi_1\rangle + \beta|\Psi_2\rangle$

Density operator: $\rho = |\Psi\rangle\langle\Psi| = \begin{pmatrix} \alpha\cdot\alpha^* & \alpha\beta^* \\ \beta\alpha^* & \beta\cdot\beta^* \end{pmatrix}$

Mixed state: $\rho = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|$

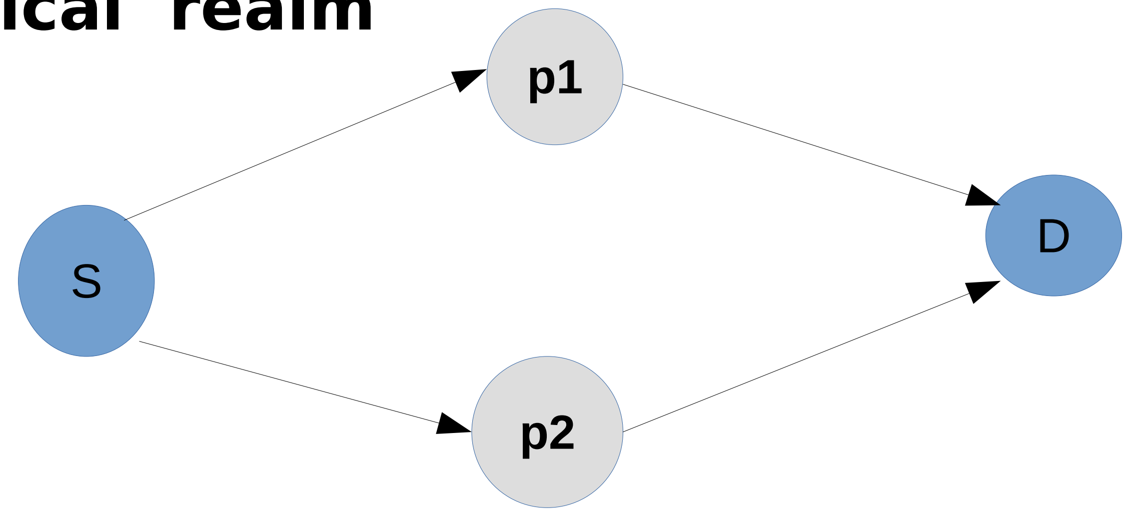
Example – a source of light in a partially mixed state:



$\rho = \frac{1}{2}|HH\rangle\langle HH| + \frac{1}{2}|VV\rangle\langle VV|$

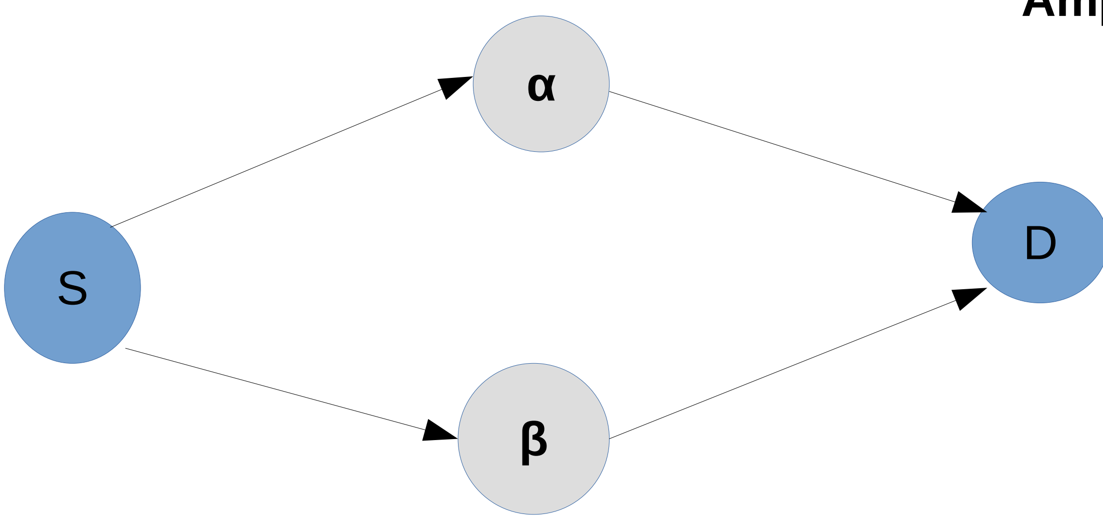
QM as a probability theory

Classical realm



$P = p1 + p2$

QM realm

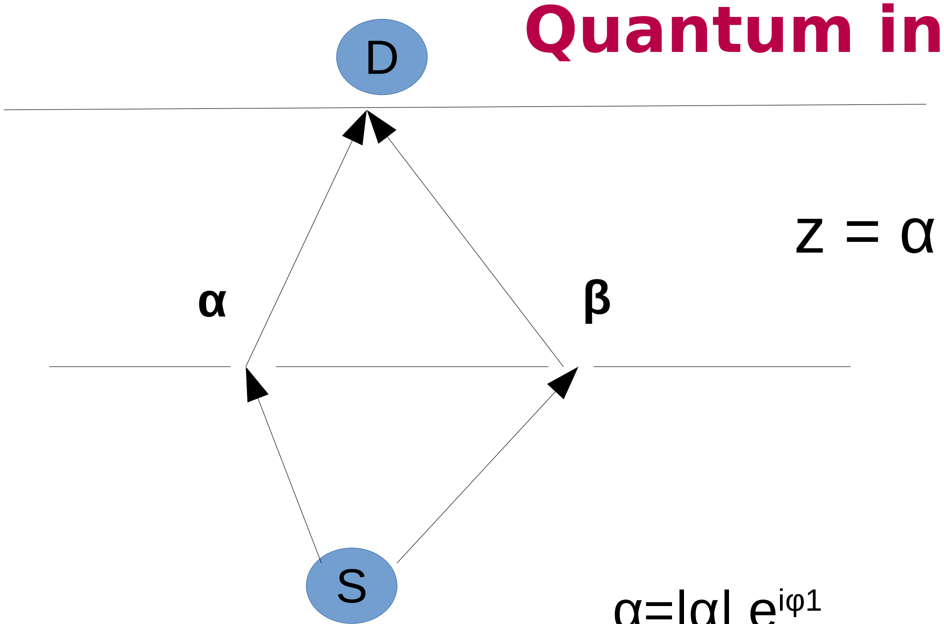


Amplitudes (Complex numbers)

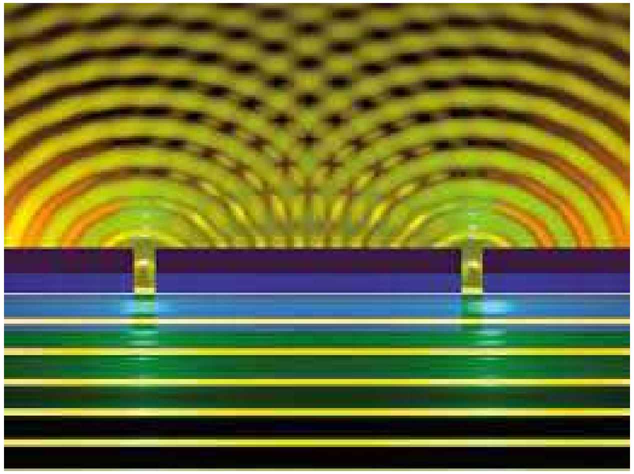
$z = \alpha + \beta$

Born's rule: $P = |z|^2$

Quantum interference



$z = \alpha + \beta$



$\alpha = |\alpha|\, e^{i\varphi 1}$ $\beta = |\beta|\, e^{i\varphi 2}$

$$P = |z|^2 = |\alpha + \beta|^2$$

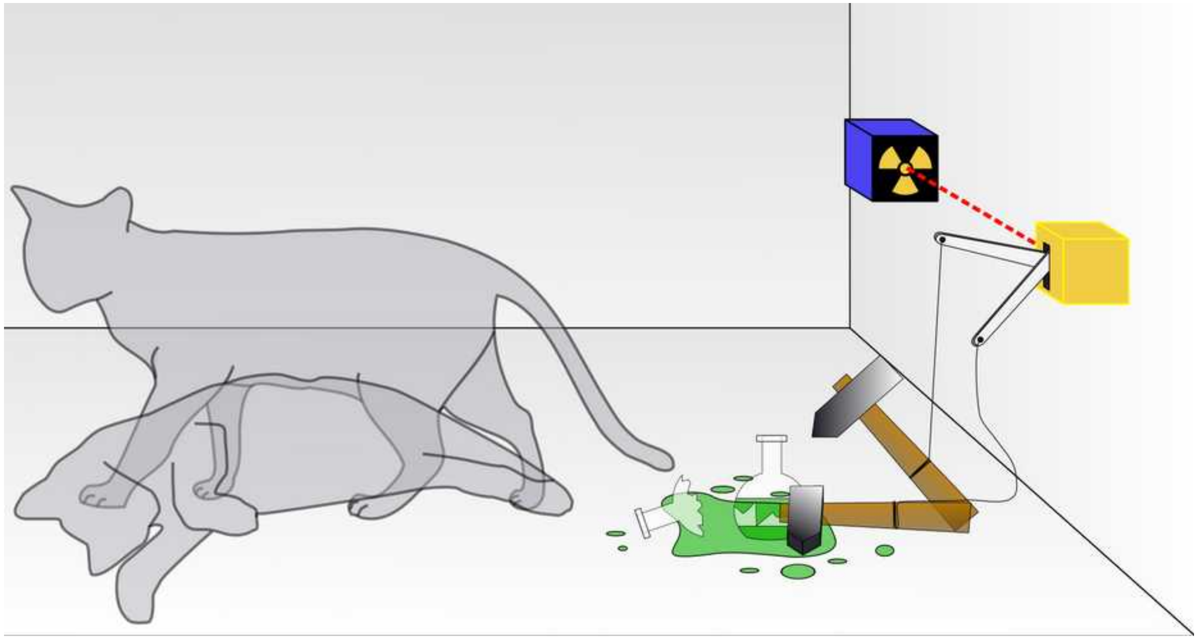
$$P = |\alpha|^2 + |\beta|^2 + 2|\alpha||\beta| \cos(\varphi 1 - \varphi 2)$$

$$P = p1 + p2 + 2 * \mathrm{sqrt}(p1 * p2) \cos(\varphi 1 - \varphi 2)$$

interference term

classical probabilities

Why don’t we see QM phenoma in the „macroscopic” world ?



https://en.wikipedia.org/wiki/File:Schrodingers_cat.svg

QM realm

$|\Psi\rangle = \alpha\,|\text{dead cat}\rangle + \beta\,|\text{alive cat}\rangle$

Classical realm

|dead cat>

or

|alive cat>

Measurement problem

Why don’t we see QM phenoma in the „macroscopic” world II?

Environment

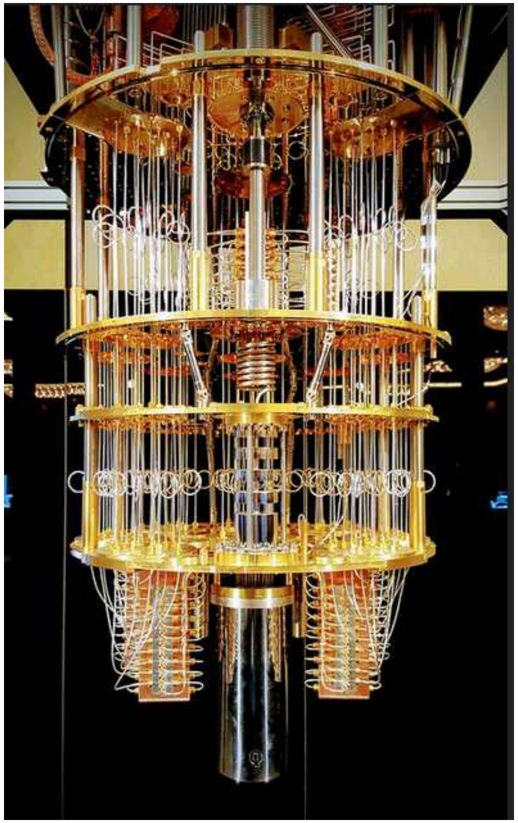
$$|\Psi\rangle = \alpha\,|\text{dead cat}\rangle + \beta\,|\text{alive cat}\rangle$$

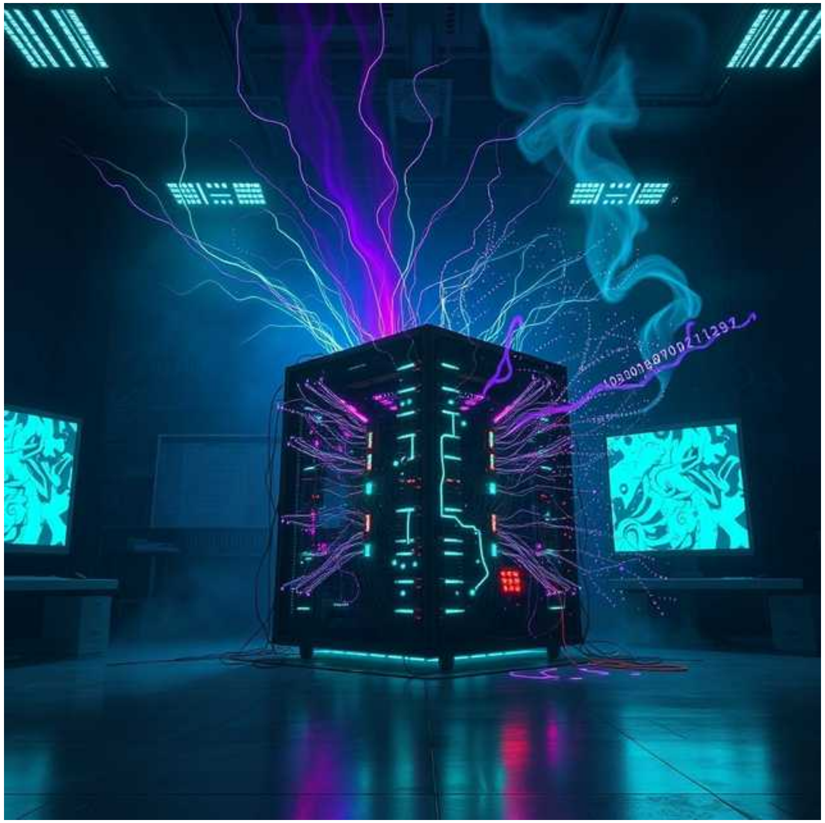
Quantum decoherence = environment-induced decoherence

The environment **filters** the possible states
→quantum **interference** effects are (partially) **suppressed** → we are approaching the „**classical**” predictions

- Purely quantum phenomena (vs dissipation or stochastic fluctuations)
- Independent of any QM interpretations

Decoherence and quantum computers

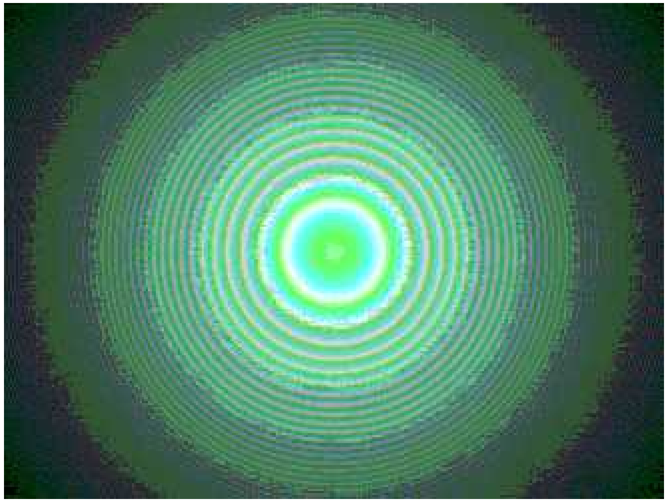




IBM Q quantum computer

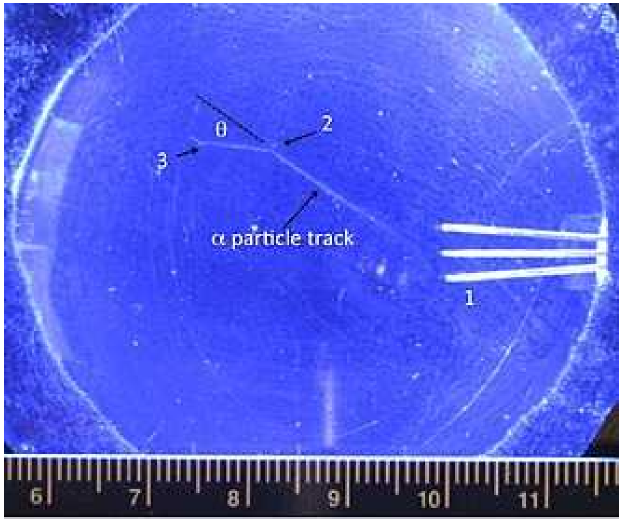
https://flic.kr/p/23qF8wx

First „decoherence model”



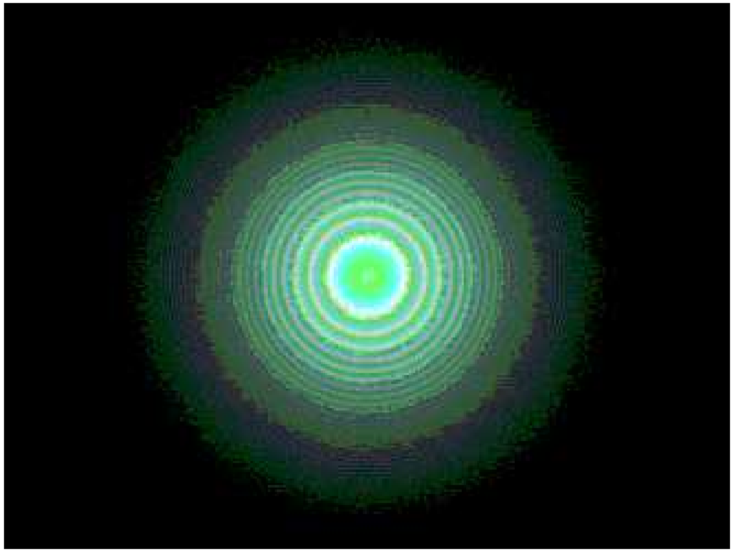
α-particle spherical wavefunction

https://commons.wikimedia.org/wiki/
File:Indeterminacy_principle.gif



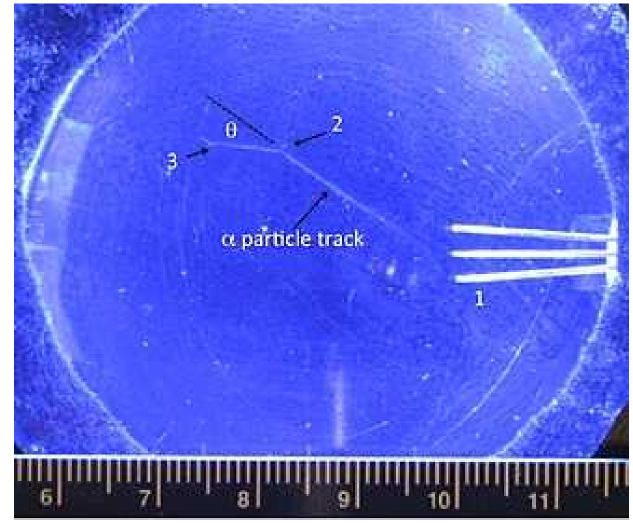
cloud chamber

First decoherence „model”

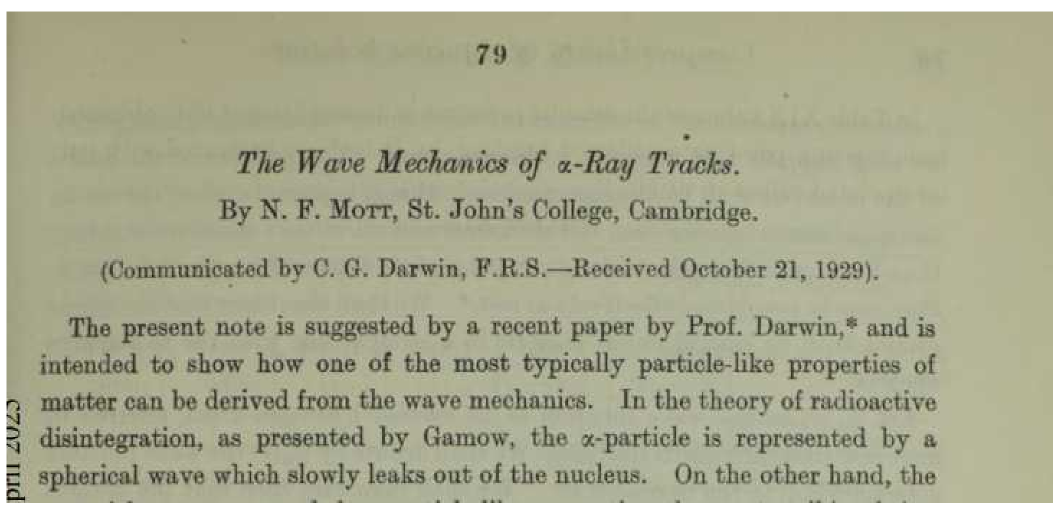


α-particle spherical wavefunction

https://commons.wikimedia.org/wiki/
File:Indeterminacy_principle.gif



cloud chamber



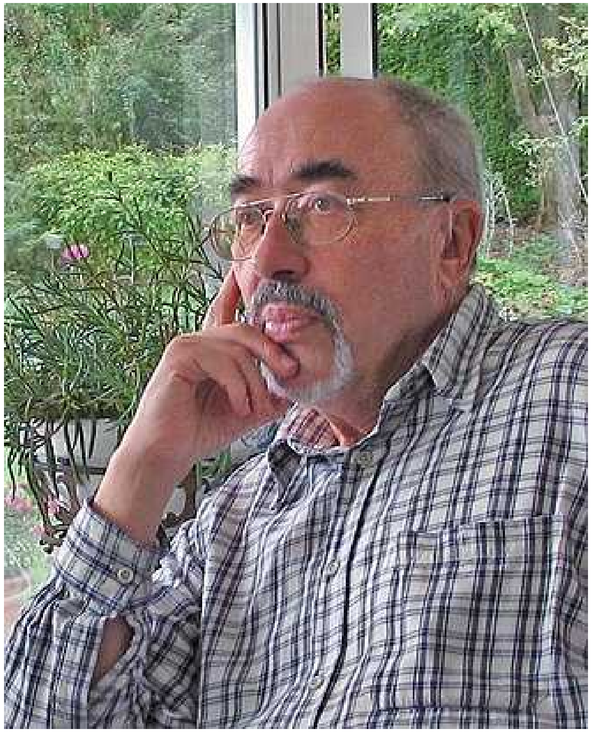
The Wave Mechanics of α-Ray Tracks.

By N. F. Mott, St. John's College, Cambridge.

(Communicated by C. G. Darwin, F.R.S.—Received October 21, 1929).

The present note is suggested by a recent paper by Prof. Darwin,* and is intended to show how one of the most typically particle-like properties of matter can be derived from the wave mechanics. In the theory of radioactive disintegration, as presented by Gamow, the α-particle is represented by a spherical wave which slowly leaks out of the nucleus. On the other hand, the

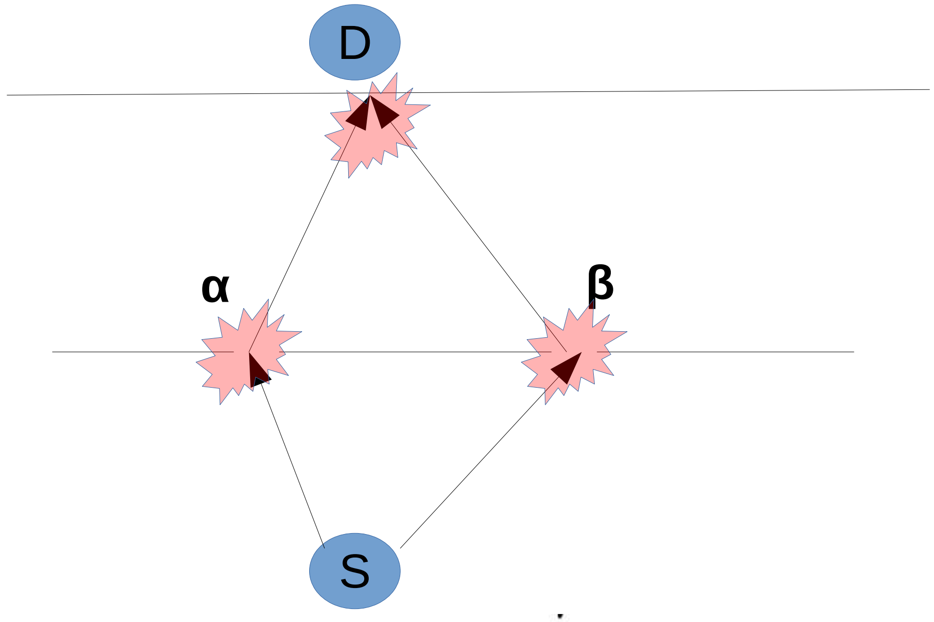
https://royalsocietypublishing.org/doi/pdf/10.1098/rspa.1929.0205



H. Dieter Zeh

Wojciech Żurek

Simple intuition



$$|\Psi\rangle = \alpha|\Psi_1\rangle + \beta|\Psi_2\rangle$$

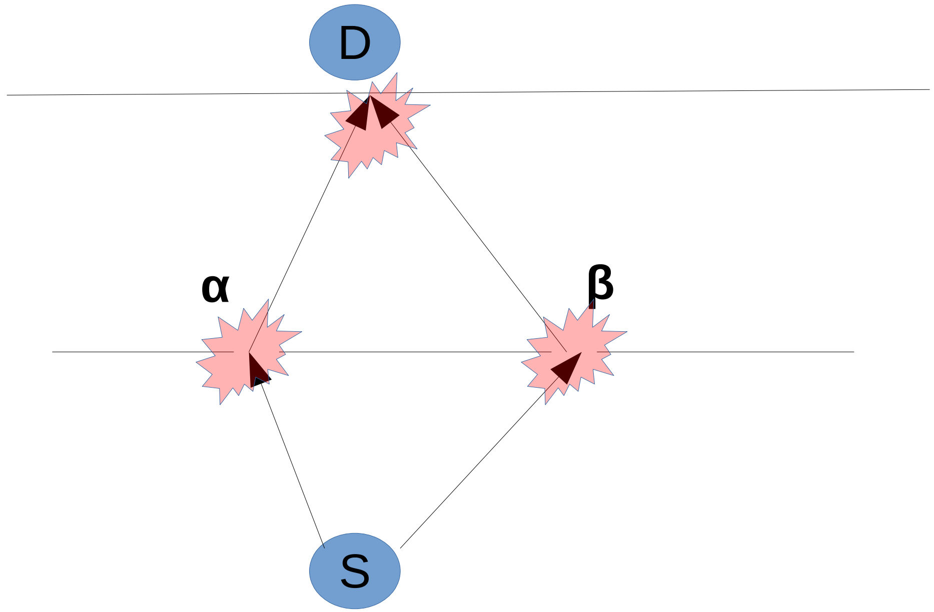
$$|\alpha|^2 + |\beta|^2 = 1 \qquad \varphi_1 - \varphi_2 = \varphi$$

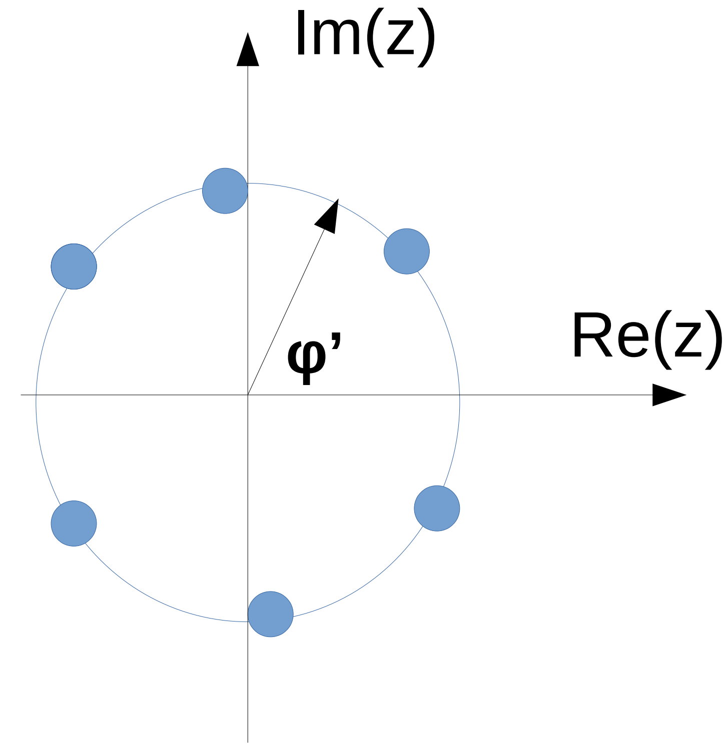
$$|\alpha|^2 = \frac{1}{2} \qquad |\beta|^2 = \frac{1}{2}$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|\Psi_1\rangle + \frac{1}{\sqrt{2}} e^{i\varphi}|\Psi_2\rangle$$

$$\rho = |\Psi\rangle\langle\Psi| = \begin{pmatrix} \frac{1}{2} & \frac{1}{2}e^{-i\varphi} \\ \frac{1}{2}e^{i\varphi} & \frac{1}{2} \end{pmatrix}$$

Simple intuition





Simple „model" - multiple interactions with the environment each changes the phase randomly

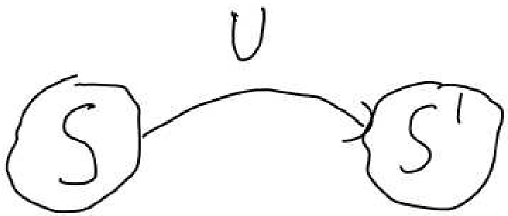
$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2}e^{-i\varphi} \\ \frac{1}{2}e^{i\varphi} & \frac{1}{2} \end{pmatrix} \xrightarrow{\text{Average}} \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

Fully mixed state!

Pure state

Quantum interference vanishes!

Evolution of the quantum systems



$$UU^{\dagger} = U^{\dagger}U = 1$$

Pure State: $|\Psi\rangle$

Schroedinger equation:

$$-i\hbar \frac{d}{dt}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle$$

$$|\Psi(t)\rangle = U|\Psi(0)\rangle$$

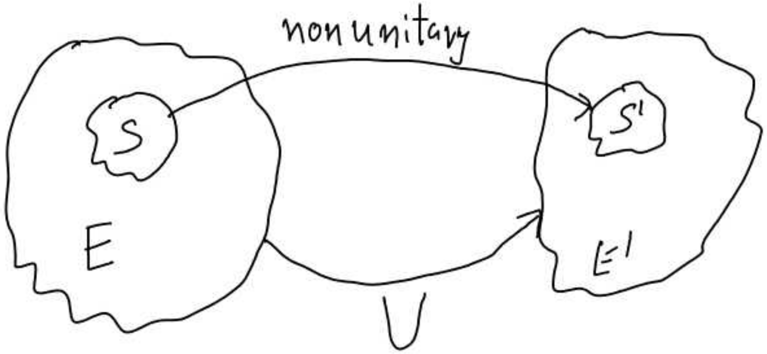
Mixed state: ρ

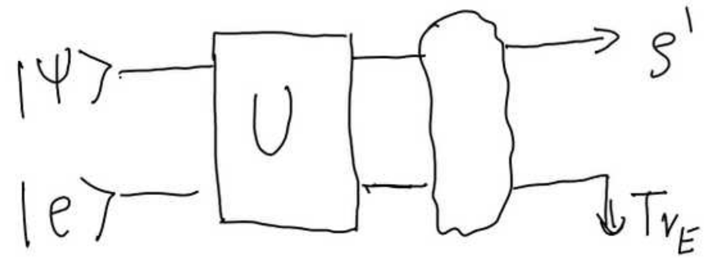
Von Neumann equation:

$$i\hbar \frac{d\rho}{dt} = [H, \rho]$$

$$\rho(t) = U(t)\,\rho(0)\,U^{\dagger}(t)$$

Evolution of the quantum systems



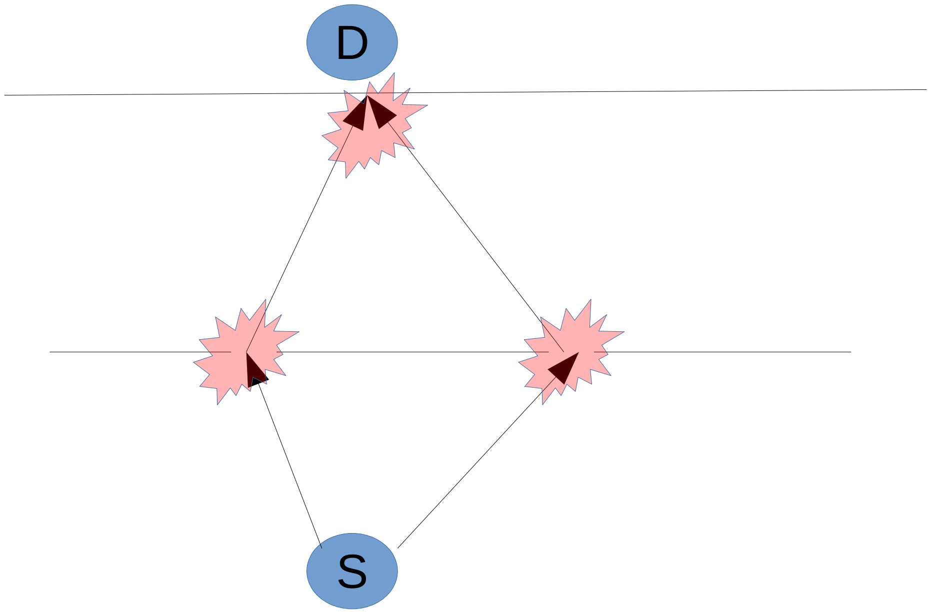


Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) equation:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \sum_i \gamma_i \left(L_i \rho L_i^\dagger - \frac{1}{2}\{L_i^\dagger L_i, \rho\} \right)$$

Kraus operators:

$$\rho \longrightarrow \rho' = \sum_k E_k \rho E_k^\dagger \qquad \sum E_k^\dagger E_k = \mathbb{1}$$



Error model

$$|\psi_1\rangle|e\rangle \to |\psi_1\rangle|e_1\rangle$$
$$|\psi_2\rangle|e\rangle \to |\psi_2\rangle|e_2\rangle$$

$$\langle e_1|e_2\rangle = V e^{i\alpha}$$

visibility

$$|\psi\rangle = \alpha|\psi_1\rangle|e\rangle + \beta|\psi_2\rangle|e\rangle$$

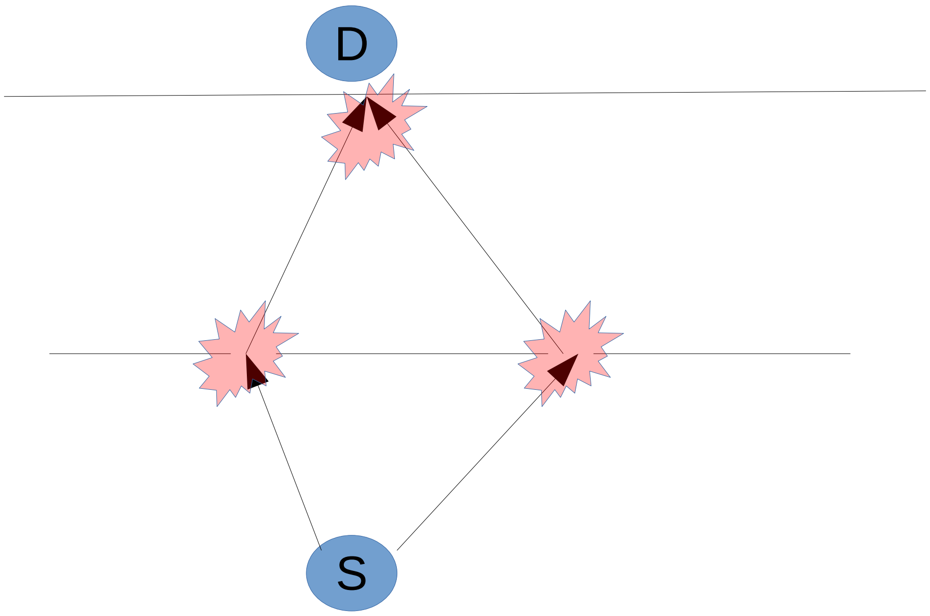
$$\varphi_1 - \varphi_2 = \varphi \qquad |\alpha|^2 = \frac{1}{2} \qquad |\beta|^2 = \frac{1}{2}$$

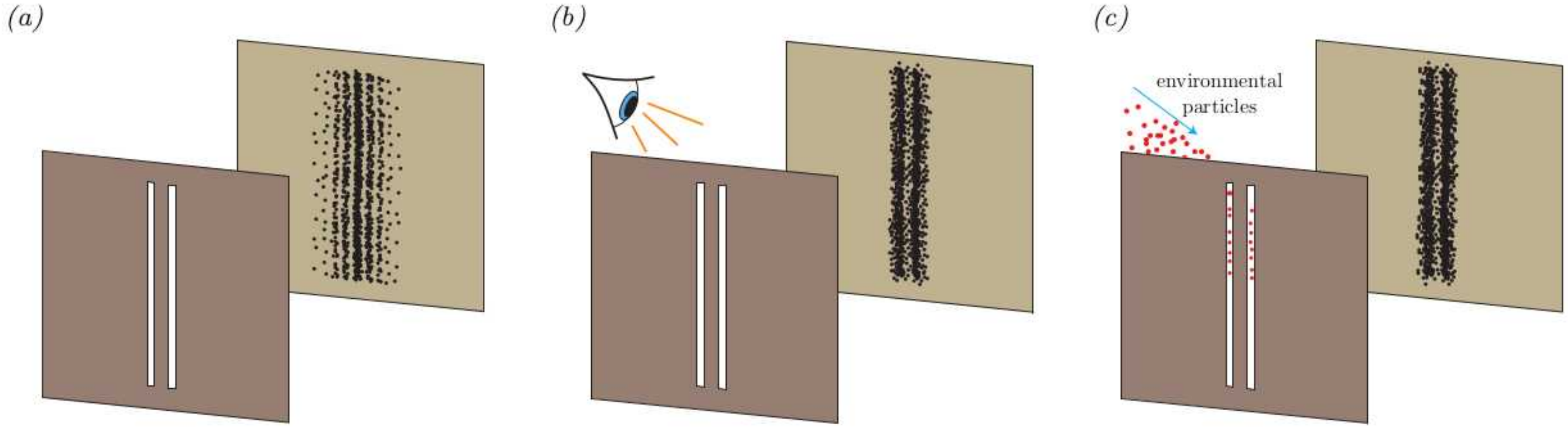
$$|\psi\rangle = \frac{1}{\sqrt{2}}|\psi_1\rangle|e\rangle + \frac{1}{\sqrt{2}}e^{i\varphi}|\psi_2\rangle|e\rangle$$

decoherence

$$\frac{1}{\sqrt{2}}\left(|\psi_1\rangle|e_1\rangle + e^{i\varphi}|\psi_2\rangle|e_2\rangle\right)$$

Entanglement with the environment !!!
Environment learns something about the system

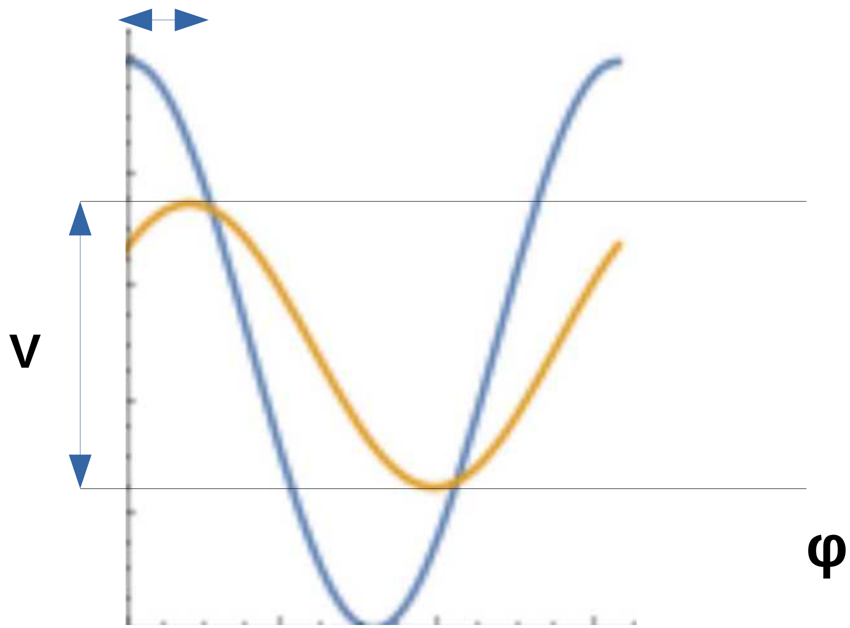




Taken from „Quantum Decoherence” Maximilian Schlosshauer

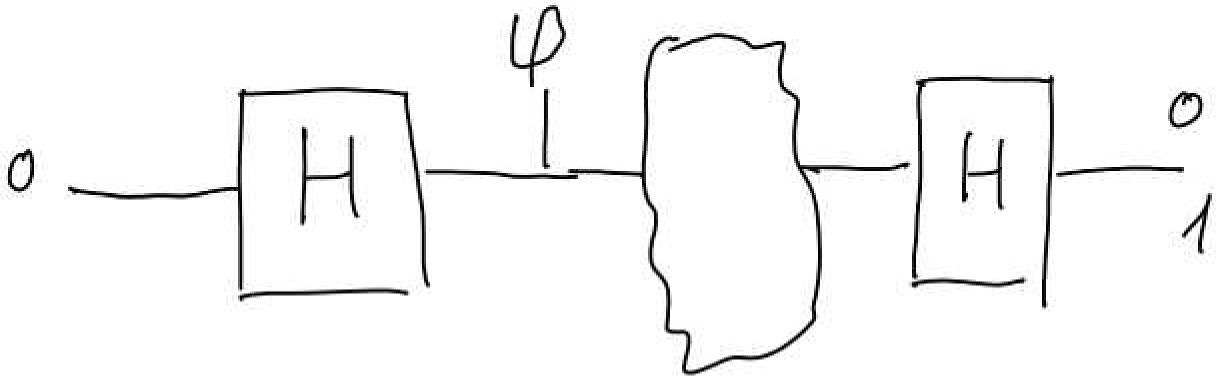
$$P = \frac{1}{2}\left[1 + \langle e_1|e_2\rangle e^{i\varphi} + c.c.\right] = \frac{1}{2}\left[1 + V\cos(\varphi + \alpha)\right]$$

$$\langle e_1|e_2\rangle = V e^{i\alpha}$$



Visibility damps the quantum intereference term
More environments „learns” more interference is suppressed!!

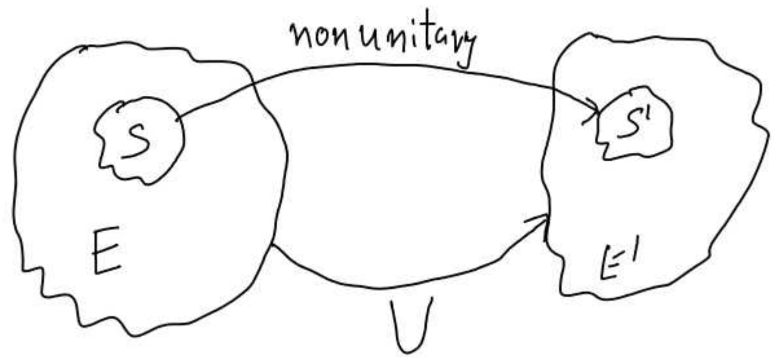
Same analysis for a single qubit state



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$P_0 = \frac{1}{2}\left[1 + V\cos(\varphi + \alpha)\right]$$

Kraus representation



$$\rho \longrightarrow \rho' = \sum_k E_k \rho E_k^\dagger$$

$$\sum E_k^\dagger E_k = \mathbb{1}$$

- Non-unitary operators
- Preserve trace and positivity
- Non-unique decomposition in the Kraus representation
- In quantum computing language → Quantum Errors
  (e.g. for qubit - flip the sign, or phase flip errors)

References

- „Quantum Decoherence” Maximilian Schlosshauer (https://arxiv.org/abs/1911.06282)
- „Decoherence and the Transition from Quantum to Classical” W.H. Żurek
- „A short introduction to the Lindblad Master Equation”, D.Manzano
- Artur Ekert’s online course