

Many faces of decoherence

Wojciech Krzemień

From imaging algorithms to quantum methods

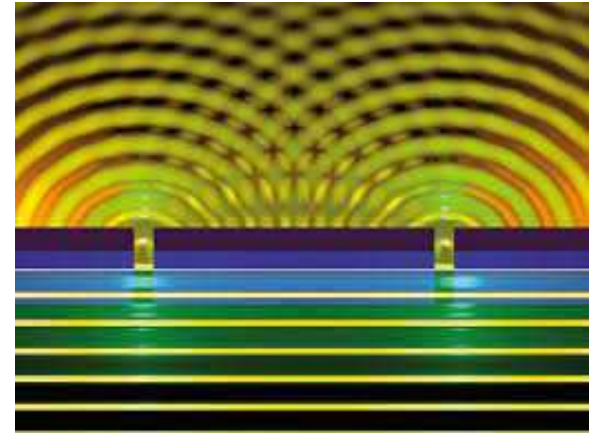
12.04.2025

Quantum Mechanics in one slide :-)

State: $|\Psi\rangle$

Superposition: $|\Phi\rangle = a|\Psi_1\rangle + b|\Psi_2\rangle \neq \Psi_1 \text{ or } \Psi_2$

Time evolution: $-i\hbar \frac{\partial}{\partial t} |\Psi\rangle = \hat{H} |\Psi\rangle$



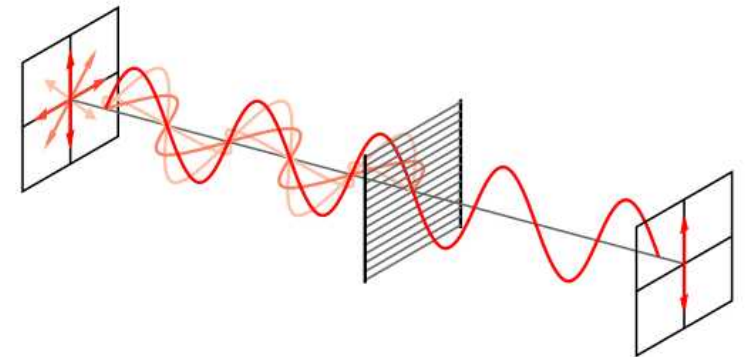
<https://physicsworld.com/a/double-slits-with-single-atoms/>

Measurement: $\hat{O} |\Psi\rangle = m |\Psi\rangle$

Probabilistic interpretation: $\hat{O} |\Psi\rangle = k |\Phi\rangle$

$$|\langle \Phi | \Psi \rangle|^2$$

$$\langle \Psi | \hat{O} | \Psi \rangle$$



<https://en.wikipedia.org/wiki/Polarizer>

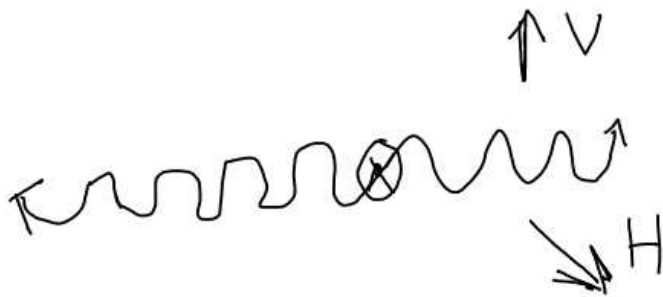
Pure state vs mixed state

Pure state: $|\Psi\rangle = \alpha |\Psi_1\rangle + \beta |\Psi_2\rangle$

Density operator: $\rho = |\Psi\rangle\langle\Psi| = \begin{pmatrix} \alpha \cdot \alpha^* & \alpha \beta^* \\ \beta \alpha^* & \beta \cdot \beta^* \end{pmatrix}$

Mixed state: $\rho = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|$

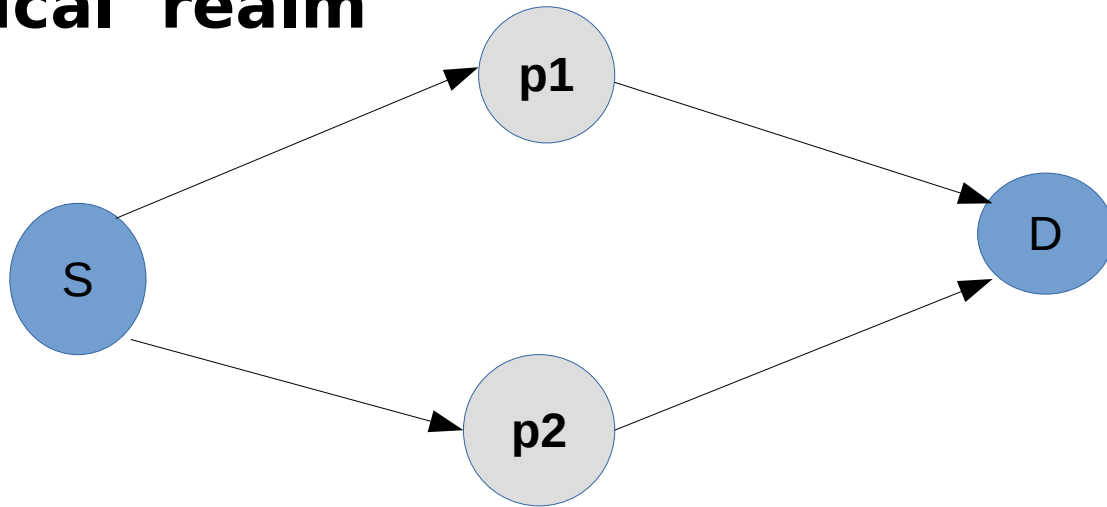
Example - a source of light in a partially mixed state:



$$\rho = \frac{1}{2} |HH\rangle\langle HH| + \frac{1}{2} |VV\rangle\langle VV|$$

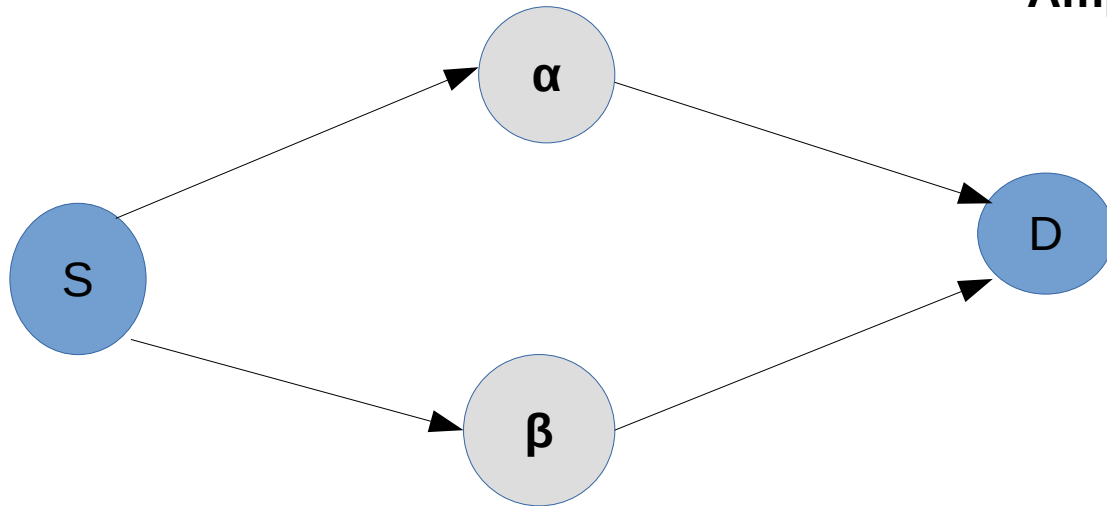
QM as a probability theory

Classical realm



$$P = p1 + p2$$

QM realm



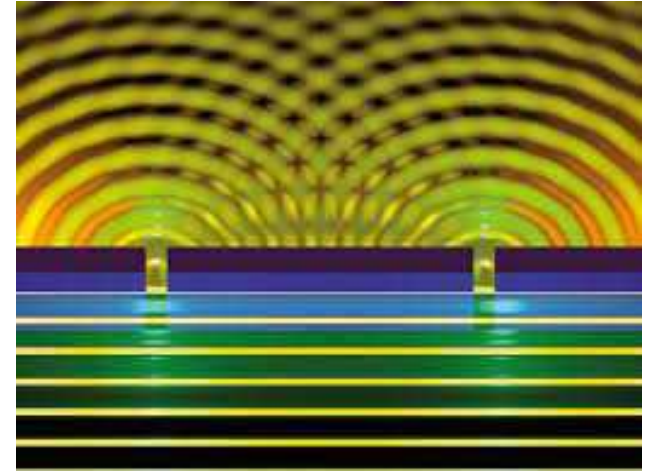
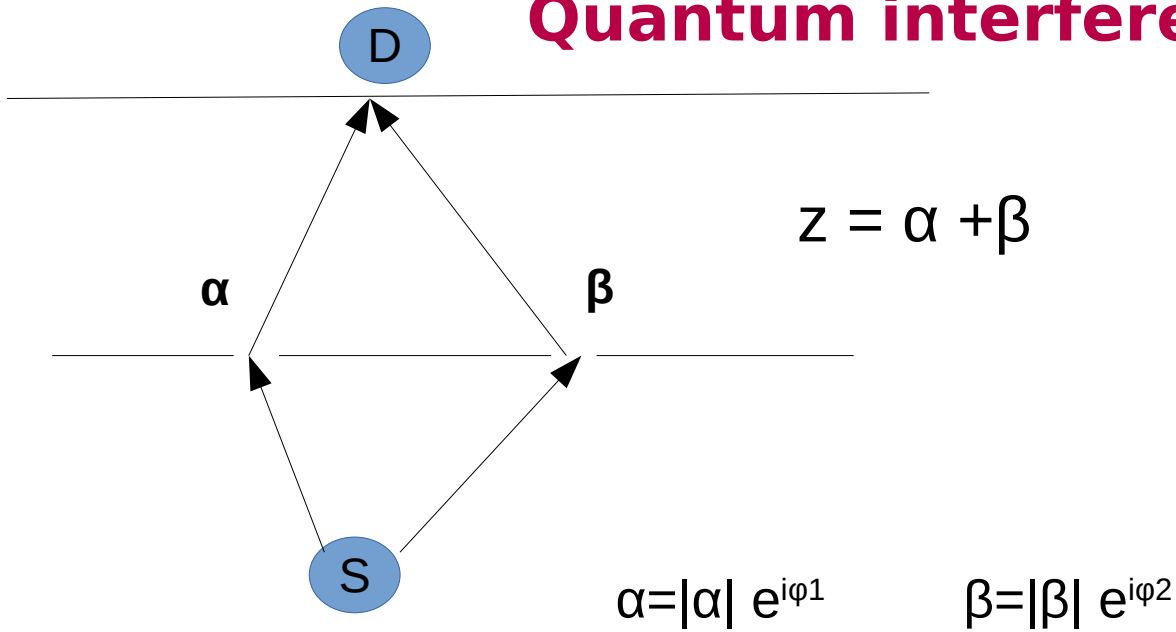
Amplitudes (Complex numbers)

A diagram showing the addition of amplitudes. A gray circle labeled 'α' has two arrows pointing downwards and to the right. One arrow points to a blue circle labeled 'z', and the other points to a blue circle labeled 'β'. Below this, the equation $z = \alpha + \beta$ is written.

$$z = \alpha + \beta$$

Born's rule: $P = |z|^2$

Quantum interference



$$P = |z|^2 = |\alpha + \beta|^2$$

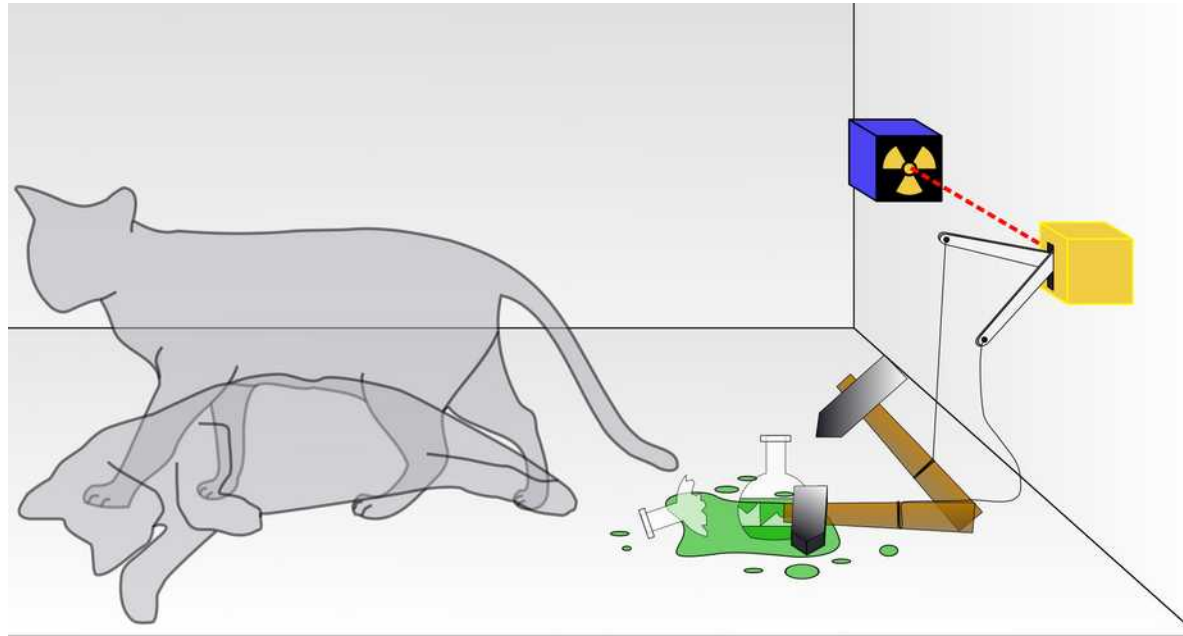
$$P = |\alpha|^2 + |\beta|^2 + 2|\alpha||\beta| \cos(\varphi_1 - \varphi_2)$$

$$P = p_1 + p_2 + 2 * \text{sqrt}(p_1 * p_2) \cos(\varphi_1 - \varphi_2)$$

← interference term

↑ ↑
classical probabilities

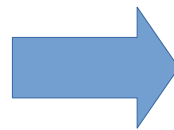
Why don't we see QM phenoma in the „macroscopic” world ?



https://en.wikipedia.org/wiki/File:Schrodingers_cat.svg

QM realm

$$|\Psi\rangle = \alpha |\text{dead cat}\rangle + \beta |\text{alive cat}\rangle$$



Classical realm

$|\text{dead cat}\rangle$

or

$|\text{alive cat}\rangle$

Measurement problem

Why don't we see QM phenomena in the „macroscopic” world II?

Environment

$$|\Psi\rangle = \alpha |\text{dead cat}\rangle + \beta |\text{alive cat}\rangle$$

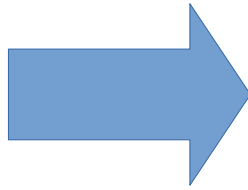
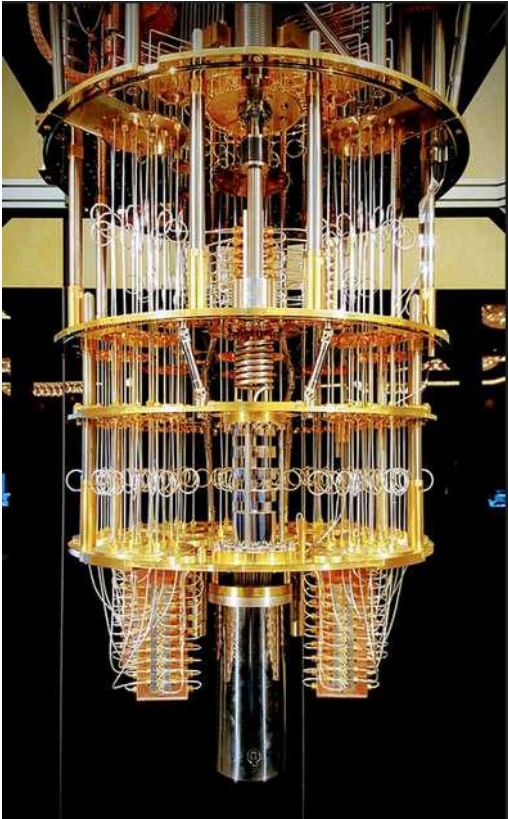
Quantum decoherence = environment-induced decoherence

The environment **filters** the possible states

→ quantum **interference** effects are (partially) **suppressed** → we are approaching the „**classical**” predictions

- Purely quantum phenomena (vs dissipation or stochastic fluctuations)
- Independent of any QM interpretations

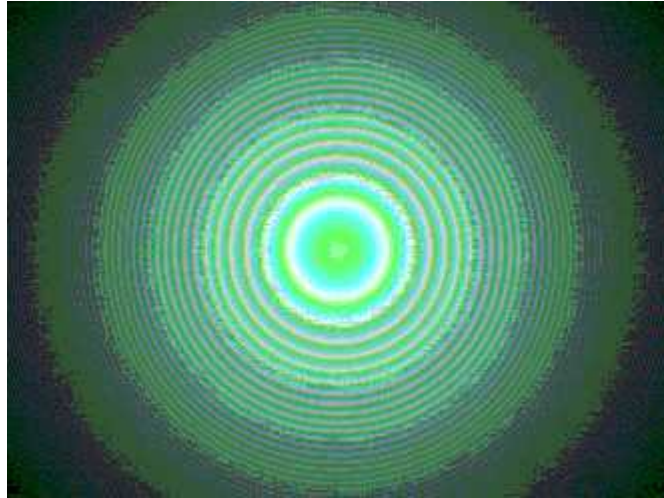
Decoherence and quantum computers



IBM Q quantum computer

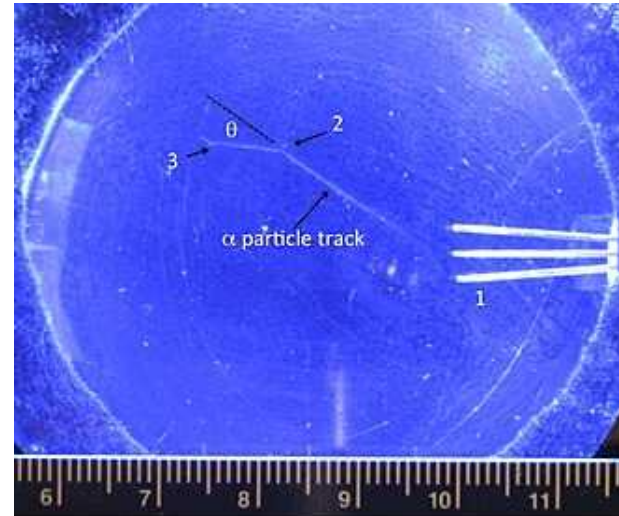
<https://flic.kr/p/23qF8wx>

First „decoherence model”



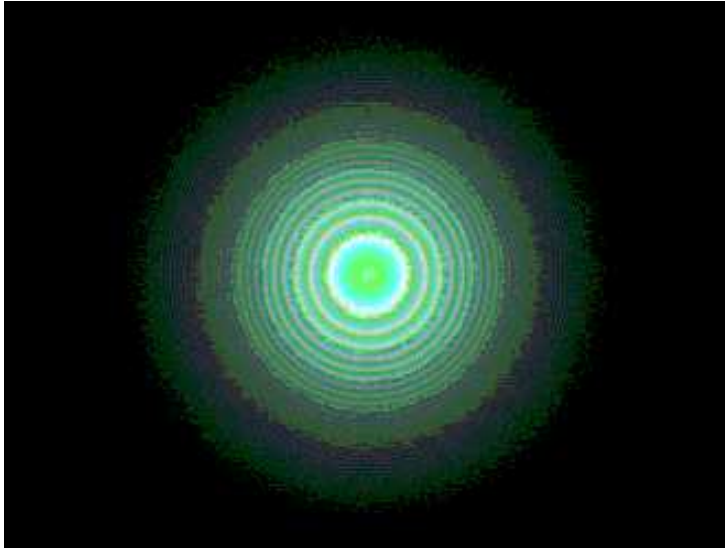
α -particle spherical wavefunction

[https://commons.wikimedia.org/wiki/
File:Indeterminacy_principle.gif](https://commons.wikimedia.org/wiki/File:Indeterminacy_principle.gif)



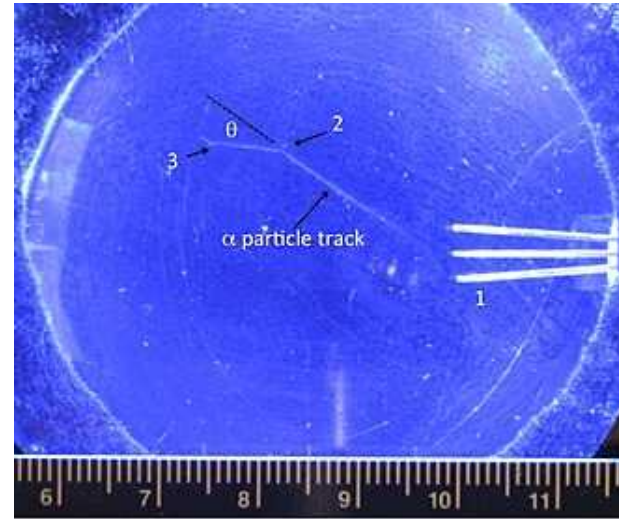
cloud chamber

First decoherence „model”

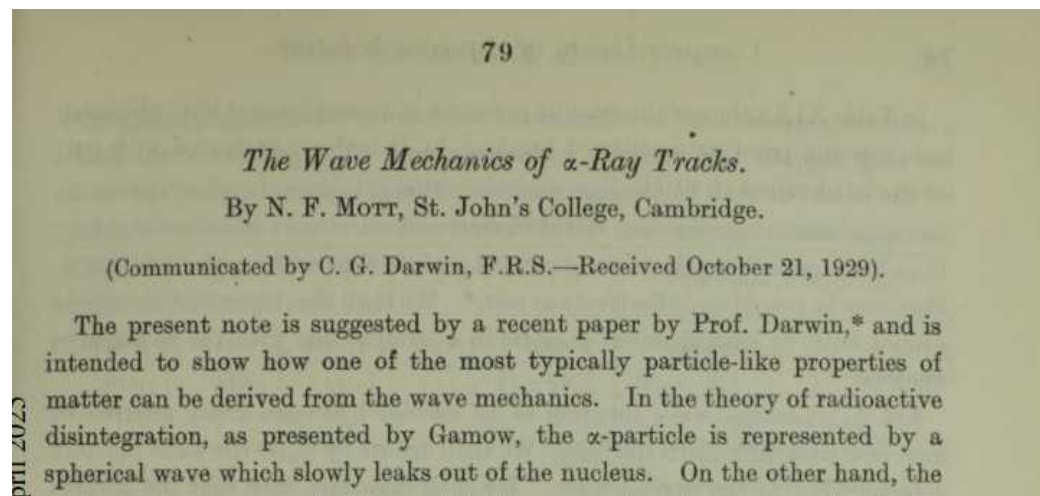


α -particle spherical wavefunction

[https://commons.wikimedia.org/wiki/
File:Indeterminacy_principle.gif](https://commons.wikimedia.org/wiki/File:Indeterminacy_principle.gif)



cloud chamber



<https://royalsocietypublishing.org/doi/pdf/10.1098/rspa.1929.0205>

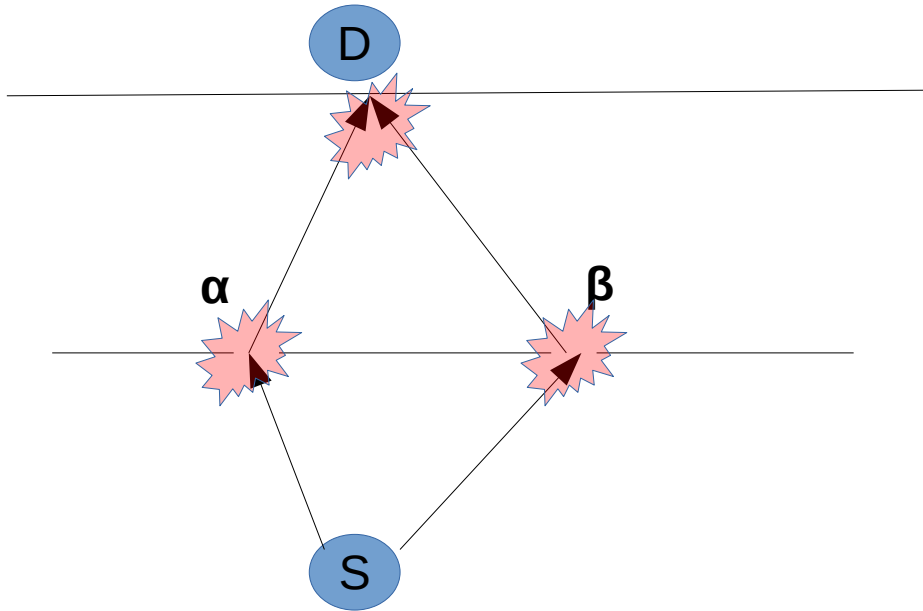


H. Dieter Zeh



Wojciech Żurek

Simple intuition



$$|\Psi\rangle = \alpha |\Psi_1\rangle + \beta |\Psi_2\rangle$$

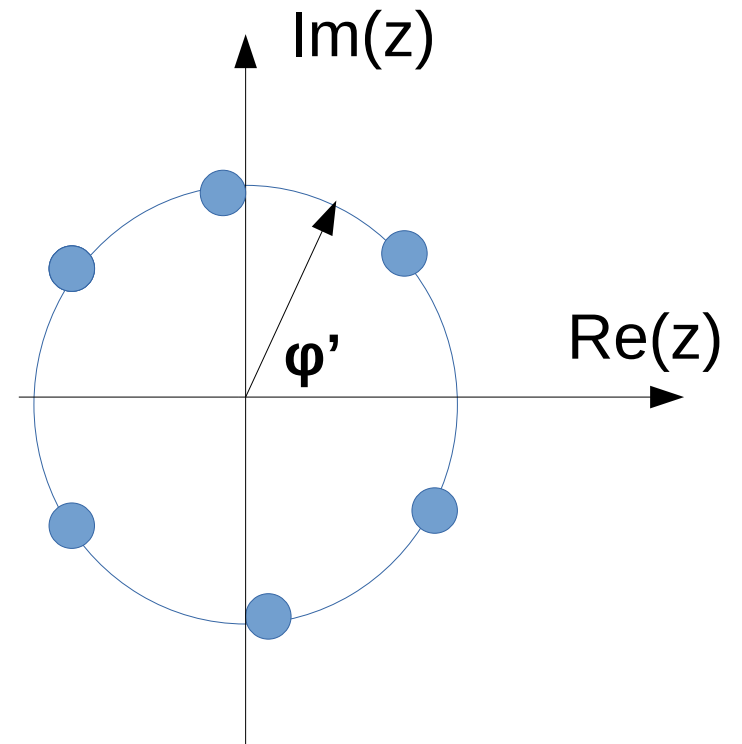
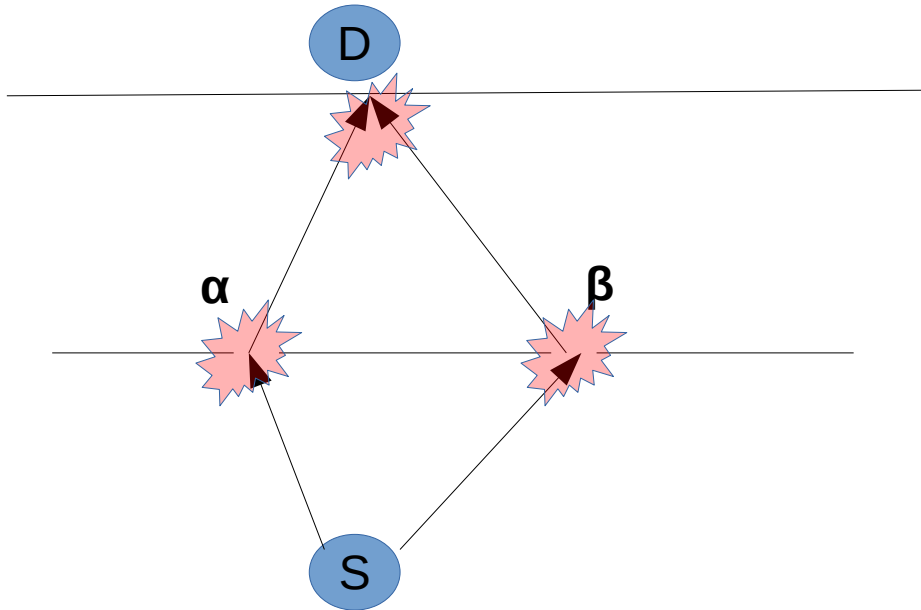
$$|\alpha|^2 + |\beta|^2 = 1 \quad \varphi_1 - \varphi_2 = \varphi$$

$$|\alpha|^2 = \frac{1}{2} \quad |\beta|^2 = \frac{1}{2}$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}} |\Psi_1\rangle + \frac{1}{\sqrt{2}} e^{i\varphi} |\Psi_2\rangle$$

$$S = |\Psi\rangle\langle\Psi| = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} e^{-i\varphi} \\ \frac{1}{2} e^{i\varphi} & \frac{1}{2} \end{pmatrix}$$

Simple intuition

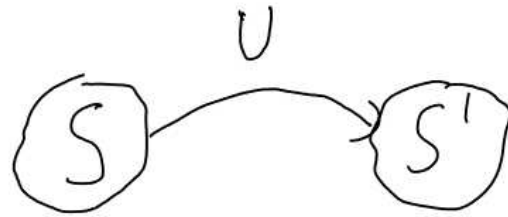


Multiple interactions with the environment each changes slightly the phase

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} e^{-i\varphi} \\ \frac{1}{2} e^{i\varphi} & \frac{1}{2} \end{pmatrix} \xrightarrow{\varphi \rightarrow 0} \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \quad \text{Fully mixed state}$$

Quantum interference vanishes!

Evolution of the quantum systems



$$UU^\dagger = U^\dagger U = \mathbb{1}$$

Schroedinger equation:

Pure State:

$$|\Psi\rangle$$

$$-i\hbar \frac{d}{dt} |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle$$

$$|\Psi(t)\rangle = U |\Psi(0)\rangle$$

Mixed state:

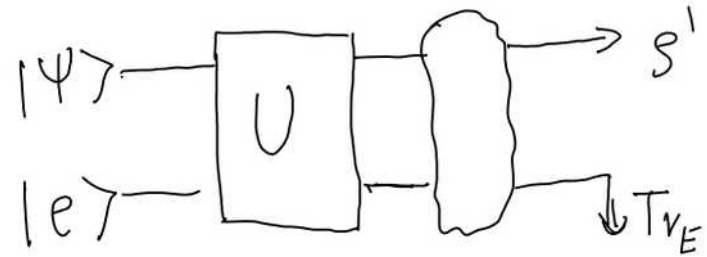
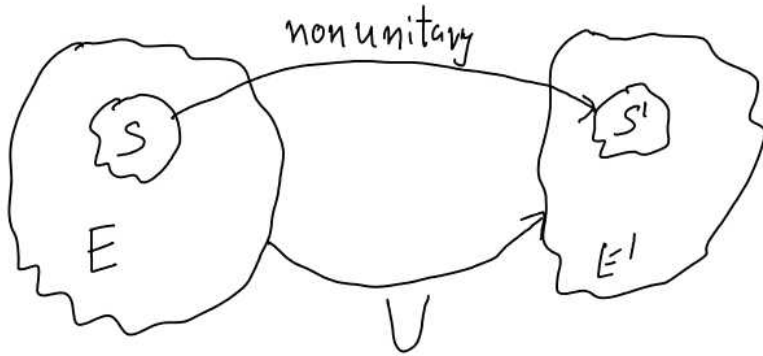
$$\rho$$

Von Neumann equation:

$$i\hbar \frac{d\rho}{dt} = [\hat{H}, \rho]$$

$$\rho(t) = U(t) \rho(0) U^\dagger(t)$$

Evolution of the quantum systems



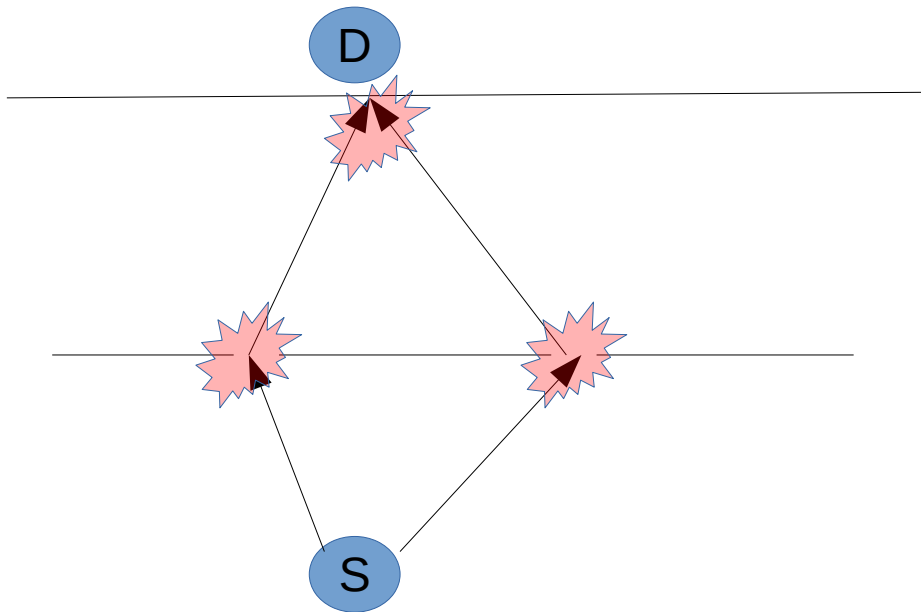
Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) equation:

$$\frac{ds}{dt} = -\frac{i}{\hbar} [H, s] + \sum_i \gamma_i (L_i s L_i^\dagger - \frac{1}{2} \{L_i^\dagger L_i, s\})$$

Krauz operators:

$$s \rightarrow s' = \sum_k E_k s E_k^\dagger$$

$$\sum_k E_k^\dagger E_k = \mathbb{1}$$



$$|\Psi\rangle = \alpha |\Psi_1\rangle |e\rangle + \beta |\Psi_2\rangle |e\rangle$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}} |\Psi_1\rangle |e\rangle + \frac{1}{\sqrt{2}} e^{i\varphi} |\Psi_2\rangle |e\rangle$$

decoherence

$$\frac{1}{\sqrt{2}} (|\Psi_1\rangle |e_1\rangle + e^{i\varphi} |\Psi_2\rangle |e_2\rangle)$$

Error model

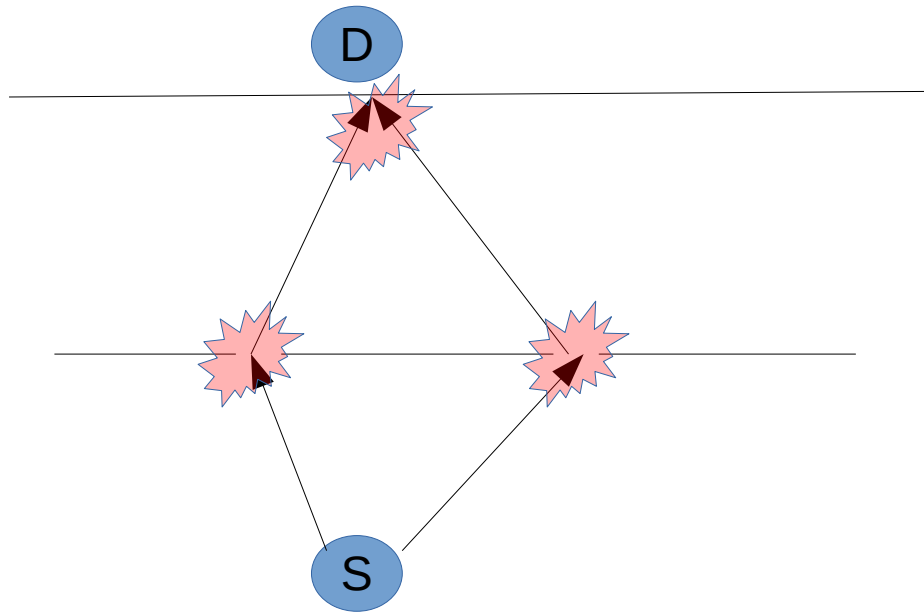
$$\begin{aligned} |\Psi_1\rangle |e\rangle &\rightarrow |\Psi_1\rangle |e_1\rangle \\ |\Psi_2\rangle |e\rangle &\rightarrow |\Psi_2\rangle |e_2\rangle \end{aligned}$$

$$\langle e_1 | e_2 \rangle = V e^{i\alpha}$$

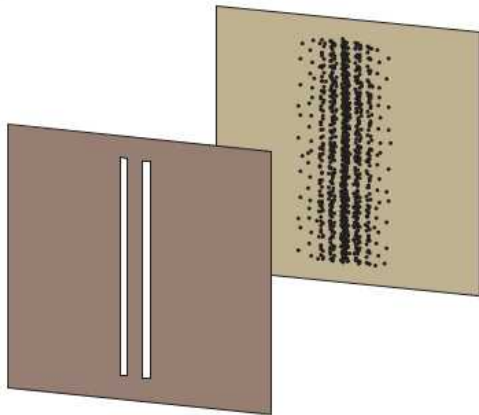
visibility

$$\varphi_1 - \varphi_2 = \varphi \quad |\alpha|^2 = \frac{1}{2} \quad |\beta|^2 = \frac{1}{2}$$

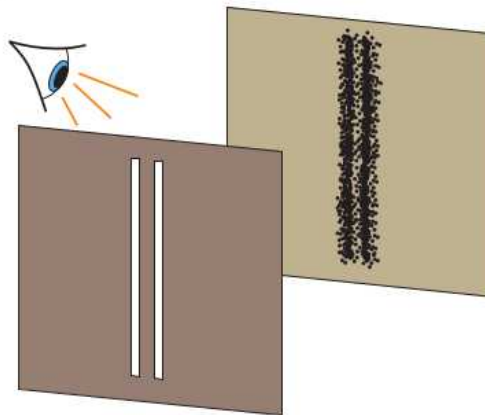
**Entanglement with the environment !!!
Environment learns something about
the system**



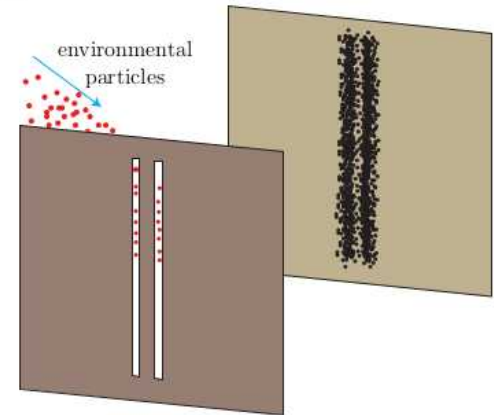
(a)



(b)



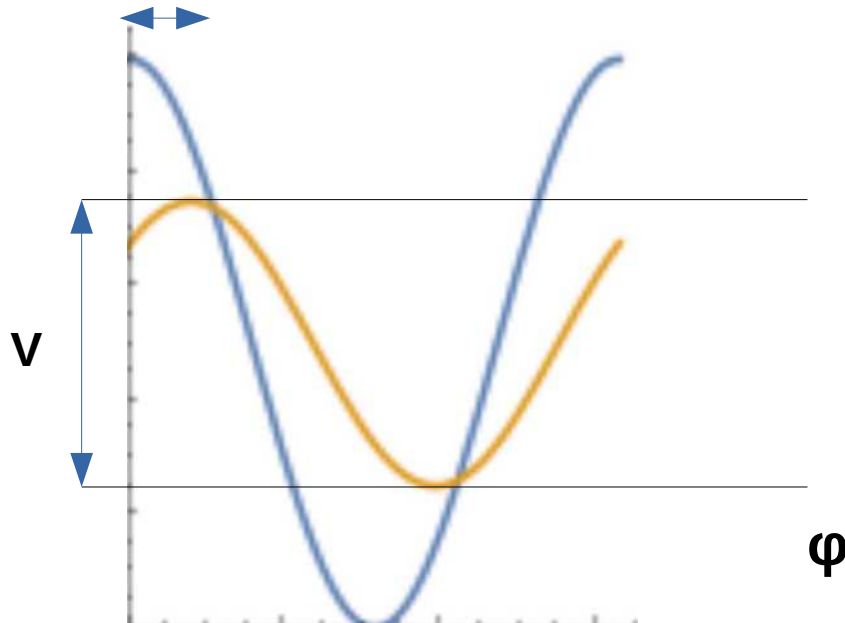
(c)



Taken from „Quantum Decoherence” Maximilian Schlosshauer

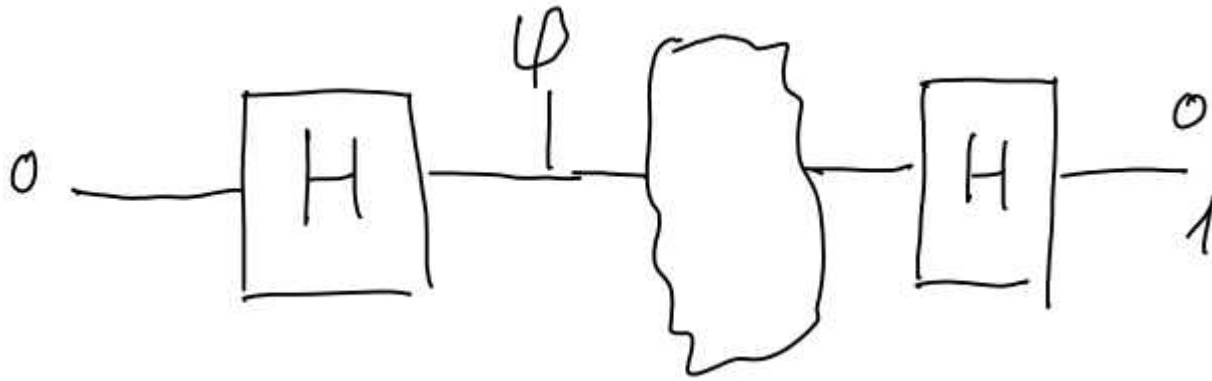
$$P_1 = \frac{1}{2} [1 + \langle e_1 | e_2 \rangle e^{i\varphi} + \text{c.c.}] = \frac{1}{2} [1 + V \cos(\varphi + \alpha)]$$

$$\langle e_1 | e_2 \rangle = V e^{i\alpha}$$



Visibility dumps the quantum interference term
 More environments „learns” more interference is suppressed!!

Same analysis for single qubit state

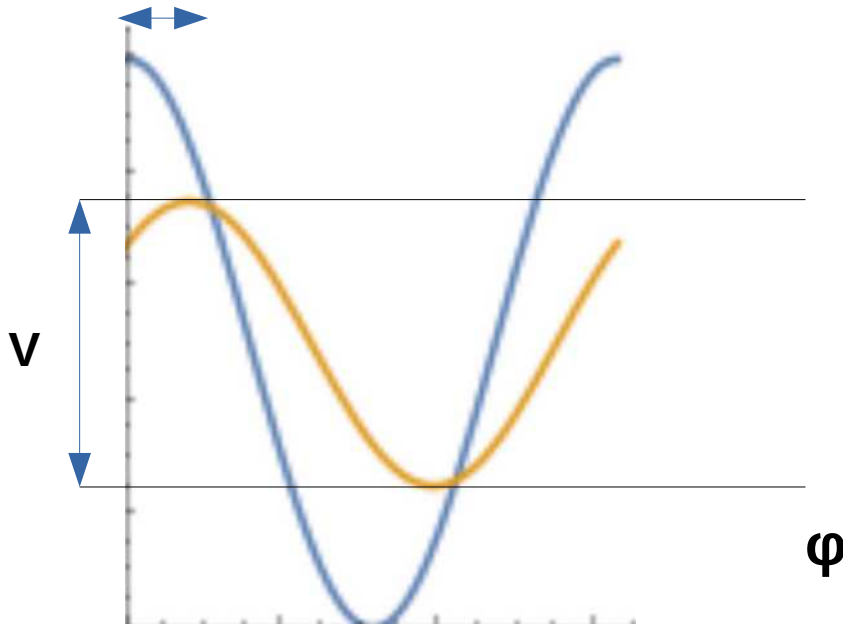


$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$P_0 = \frac{1}{2} [1 + V \cos(\varphi + \alpha)]$$

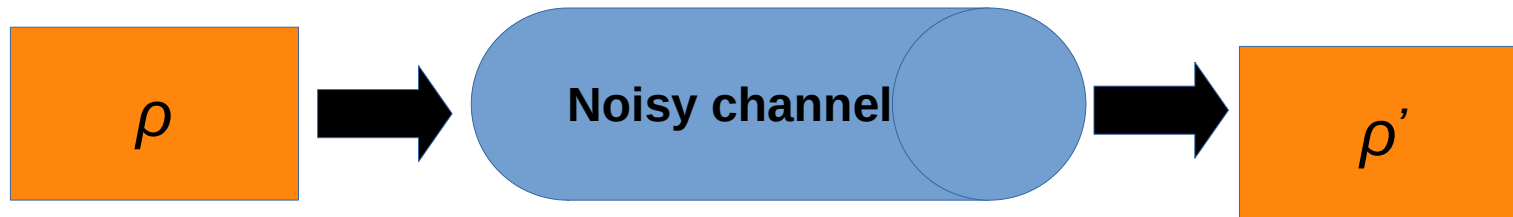
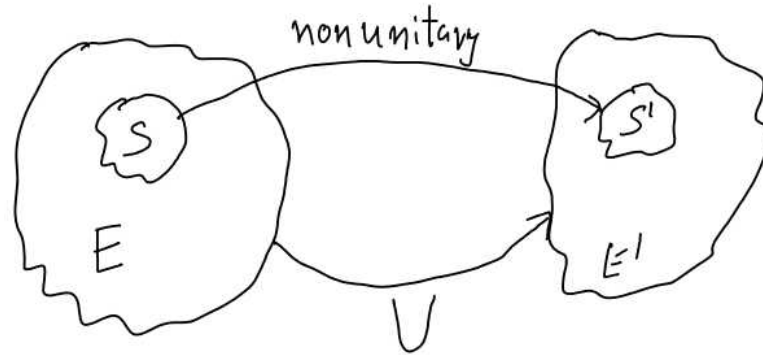
$$P_1 = \frac{1}{2} [1 + \langle e_1 | e_2 \rangle e^{i\varphi} + \text{c.c.}] = \frac{1}{2} [1 + V \cos(\varphi + \alpha)]$$

$$\langle e_1 | e_2 \rangle = V e^{i\alpha}$$



Visibility dumps the quantum interference term
 More environments „learns” more interference is suppressed!!

Krauz representation



$$\rho \rightarrow \rho' = \sum_k E_k \rho E_k^\dagger$$

$$\sum_k E_k^\dagger E_k = \mathbb{1}$$

- Non-unitary operators
- Preserve trace and positivity
- Non-unique decomposition in the Krauz representation
- In quantum computing language \rightarrow Quantum Errors (e.g. for qubit - flip the sign, or phase flip errors)

References

- „Quantum Decoherence” Maximilian Schlosshauer
(<https://arxiv.org/abs/1911.06282>)
- „Decoherence and the Transition from Quantum to Classical” W.H. Zurek
- „A short introduction to the Lindblad Master Equation”, D.Manzano
- Artur Ekert’s online course

separability vs entanglement

Separable state \leftrightarrow we can write it as tensor product of two states

$$|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$$

$$|\psi_1\rangle = |HH\rangle$$

$$|\psi_1\rangle = |H_A\rangle \otimes |H_B\rangle$$

Non-separable state \leftrightarrow entangled state

$$|\Phi_{\pm}\rangle = 1/\sqrt{2}(|HH\rangle \pm |VV\rangle)$$

