Quark-gluon plasma in magnetic fields

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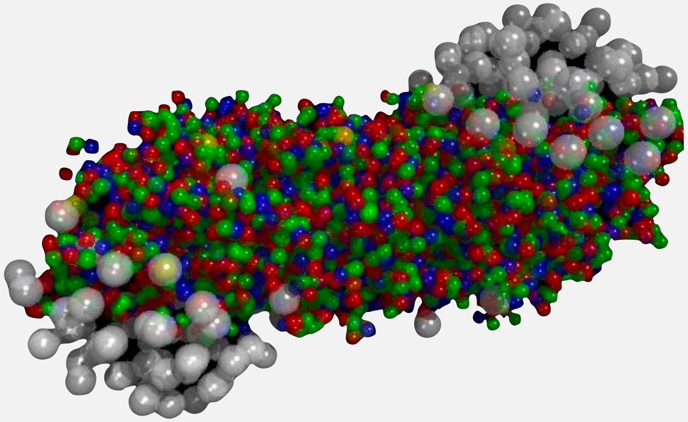
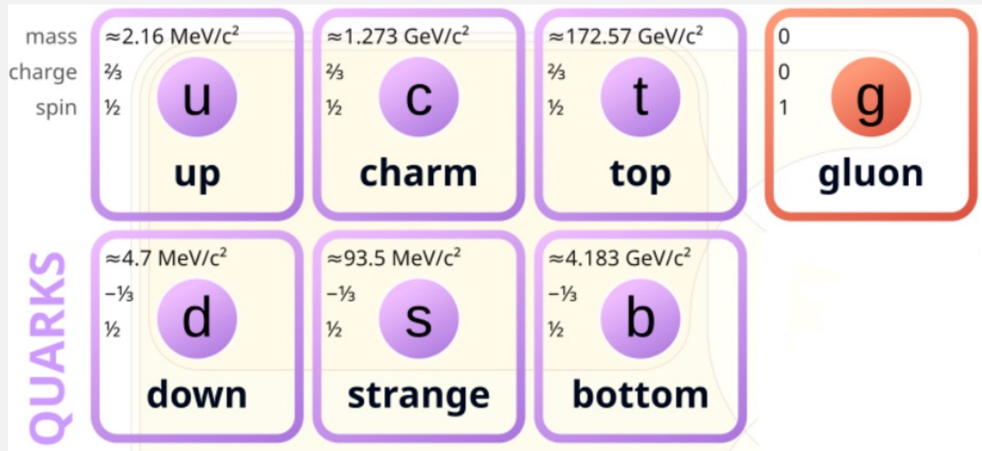
The quark-gluon plasma (QGP)



- Extremely hot (~300 MeV $\approx 3.5 \cdot 10^{12}$ K)

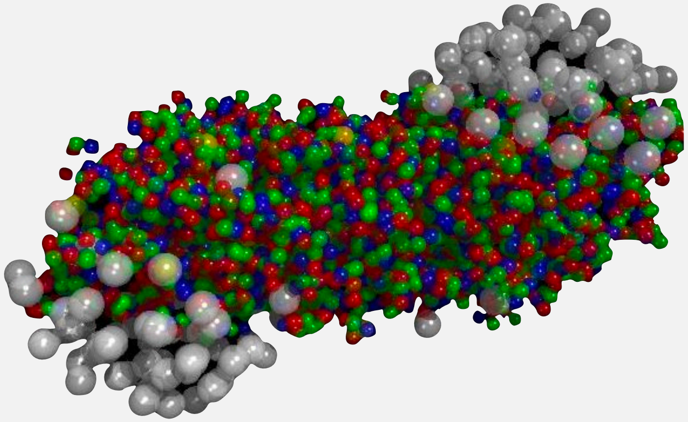
- Phase of matter consisting of deconfined quarks and gluons



- Asymptotic freedom - the strong force between quarks and gluons decreases with increasing relative momentum

- Colour confinement - colour-charged particles cannot be isolated

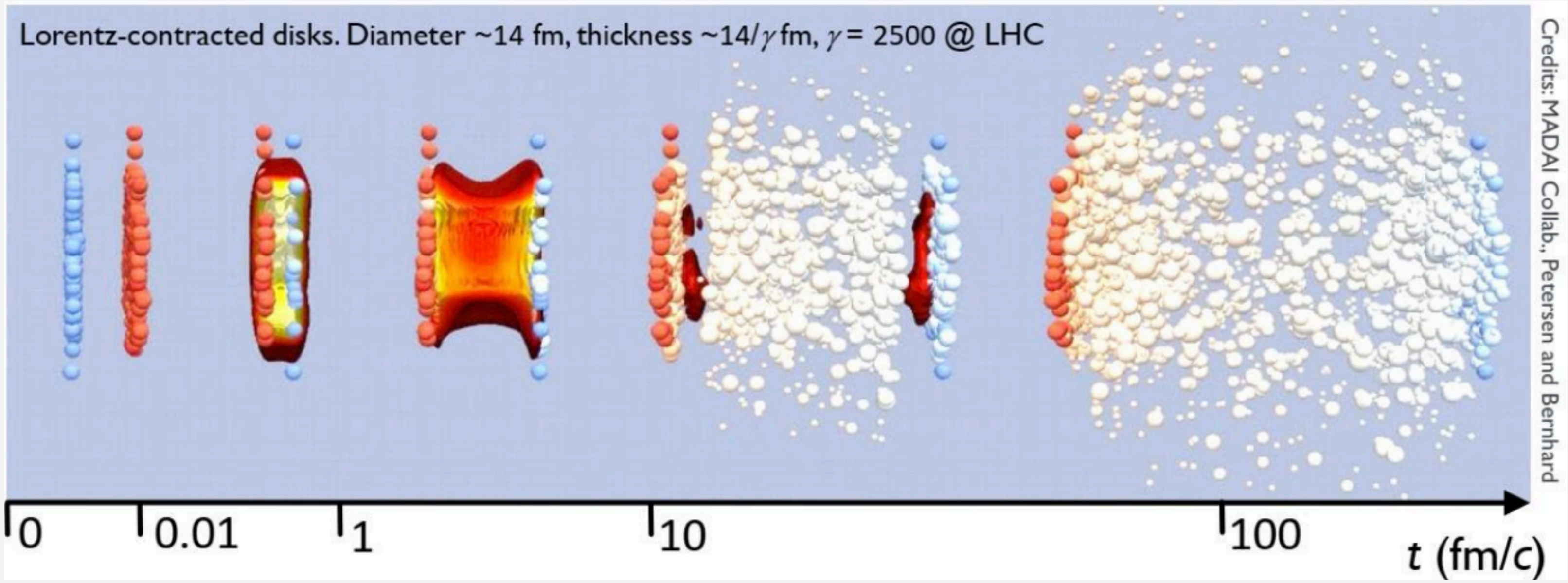
The quark-gluon plasma (QGP)



- Extremely hot (~300 MeV ≈ $3.5 \cdot 10^{12}$ K)

- The QGP existed in nature only soon after the Big Bang (10^{-12} to 10^{-6} s)

- We are able to study the QGP in the laboratory through heavy-ion collision experiments



Units: t [fm/c], $c = 1, \hbar = 1$

- Measuring time in fm$/c$ is convenient for describing the motion of high-speed particles in a laboratory setting.

- We understand it as time in which the light traveled a femtometer.

- In 1 second the light will travel $3 \cdot 10^8$ m
 ($1\text{ s} \cdot c = 3 \cdot 10^8$ m)

- The light will travel a 1 meter in $3.3 \cdot 10^{-9}$ s
 ($1\text{ m}/c = 1/(3 \cdot 10^8) \approx 3.3 \cdot 10^{-9}$ s)

- The light will travel a 1 femtometer in $3.3 \cdot 10^{-24}$ s
 ($1\text{ fm}/c = 10^{-15}\text{m}/c = 10^{-15}/(3 \cdot 10^8) \approx 3.3 \cdot 10^{-24}$ s)

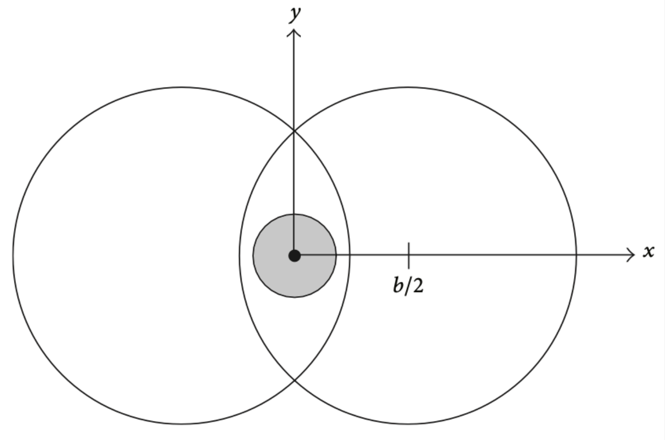
- A femtometer is a size characteristic for the problem of nuclei collision

- Nucleon size $\approx 10^{-15}\text{m} = 1$ fm

Magnetic field in the presence of strong interactions

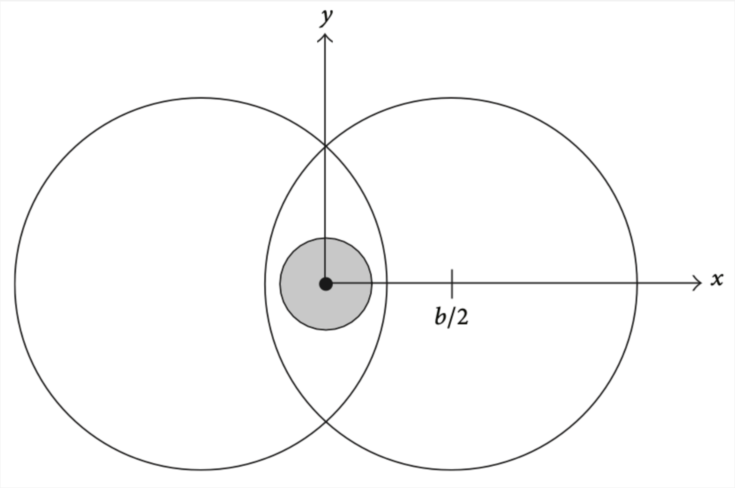
- Quarks interact with each other via the strong interactions (100 times stronger than the electromagnetic force)
- We cannot neglect the magnetic field when it is of a great magnitude

- We estimate the magnetic field in heavy-ion collisions using the Biot-Savart law (equation describing the magnetic field generated by a constant electric current).



$$B \sim \gamma Z e \frac{b}{R^3}$$

Magnetic field in heavy-ion collisions



Heavy-ion collision geometry as seen along the collision axis z

K. Tuchin, *Particle production in strong electromagnetic fields in relativistic heavy-ion collisions*, Adv. High Energy Phys. 2013, 490495

Biot-Savart law: $B \sim \gamma Ze \dfrac{b}{R^3}$

- R - the radius of two ions
- Ze - electric charge of the ions
- b - impact parameter
- $\gamma = \sqrt{s_{NN}}/2m_N$ - Lorentz factor.

$$eB \sim 10^{18}\ \mathrm{G} \ \approx m_\pi^2$$

- At RHIC heavy ions are collided at 200 GeV per nucleon, hence $\gamma = 100$. Using $Z = 79$ for gold and $b \sim R_A \approx 7$ fm.

Magnetic field in heavy-ion collisions

$$eB \sim 10^{18}\,\text{G} \approx m_\pi^2 \approx 1.96 \cdot 10^4\,\text{MeV}^2$$

- The influence of electromagnetic field is especially noticeable when it is of a size characteristic for strong interactions. For the magnetic field it's when it's proportional to the square of pion mass.

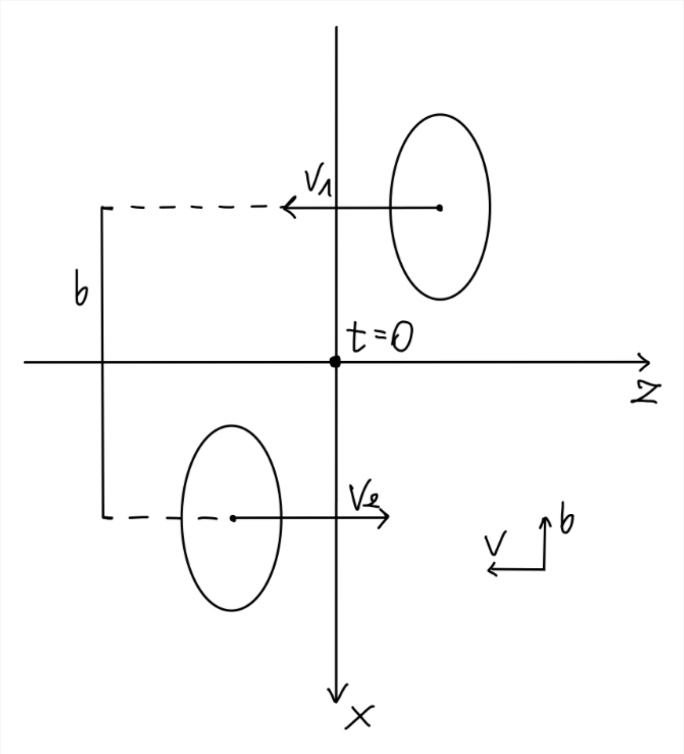
- the strongest magnetic field created on earth in a form of electromagnetic shock wave: $\sim 10^7$ G
- magnetic field of a neutron star: $10^{10} - 10^{13}$ G,
- magnetic field of a magnetar: up to 10^{15} G.

The magnetic field in a vacuum
- the point of reference -

- Two identical nuclei collide at $t = 0$ with an impact parameter b

$$\begin{cases} \mathbf{r}_{01} = (b/2, 0, 0), \\ \mathbf{v}_1 = (0, 0, v), \\ \mathbf{R}_1 = (x - b/2, y, z - vt), \end{cases} \qquad \begin{cases} \mathbf{r}_{02} = (-b/2, 0, 0), \\ \mathbf{v}_2 = (0, 0, -v), \\ \mathbf{R}_2 = (x + b/2, y, z + vt), \end{cases}$$

$$\mathbf{R} \equiv \mathbf{r} - \mathbf{r}_0 - \mathbf{v}t$$

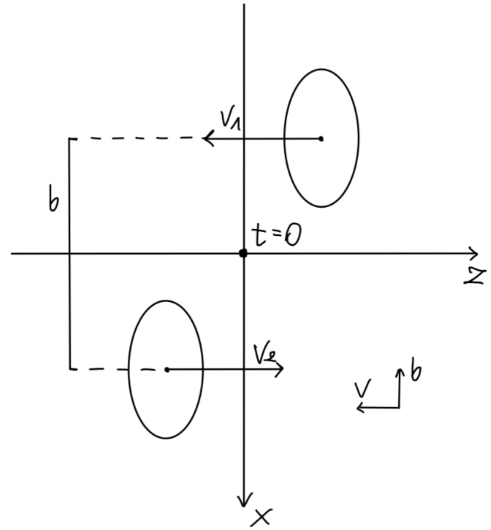


- The problem was already studied and the magnetic field can be calculated using the Li'enard-Wiechert potentials

$$\mathbf{B}(t, \mathbf{r}) = \frac{q(1 - \mathbf{v}_1^2)\mathbf{v}_1 \times \mathbf{R}_1}{\left(\mathbf{R}_1^2 - (\mathbf{R}_1 \times \mathbf{v}_1)^2\right)^{3/2}} + \frac{q(1 - \mathbf{v}_2^2)\mathbf{v}_2 \times \mathbf{R}_2}{\left(\mathbf{R}_2^2 - (\mathbf{R}_2 \times \mathbf{v}_2)^2\right)^{3/2}}$$

The magnetic field in a vacuum
- the point of reference -

$$\mathbf{B}(t,\mathbf{r}) = \frac{q(1-\mathbf{v}_1^2)\mathbf{v}_1 \times \mathbf{R}_1}{\left(\mathbf{R}_1^2 - (\mathbf{R}_1 \times \mathbf{v}_1)^2\right)^{3/2}} + \frac{q(1-\mathbf{v}_2^2)\mathbf{v}_2 \times \mathbf{R}_2}{\left(\mathbf{R}_2^2 - (\mathbf{R}_2 \times \mathbf{v}_2)^2\right)^{3/2}}$$



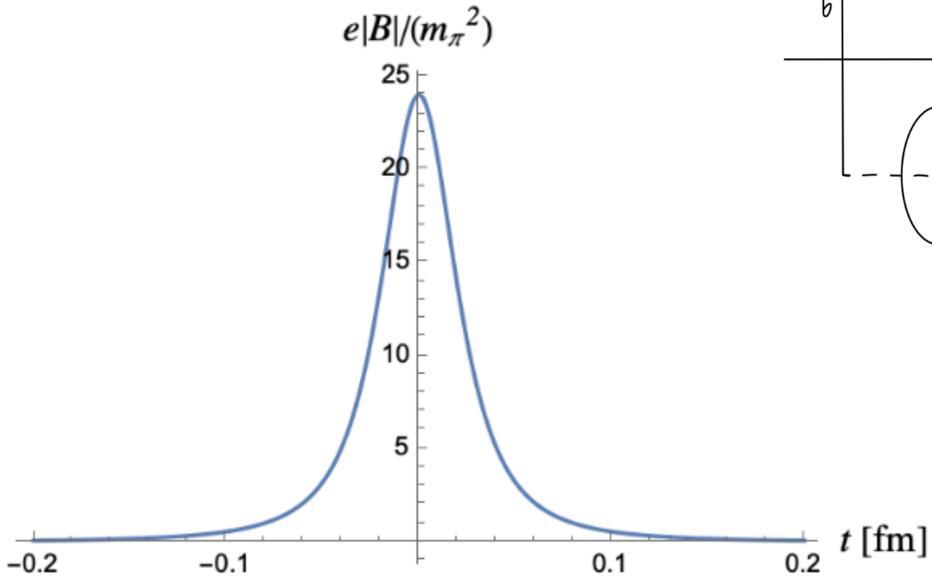


Fig. 2. Magnetic field as a function of time for Au-Au collision - reference

Electromagnetic field in heavy-ion collisions

- The Maxwell's equations

$$\begin{cases} \nabla \cdot \mathbf{D}(t,\mathbf{r}) = 4\pi\rho_{ext}(t,\mathbf{r}), & \text{- Gauss' law} \\ \nabla \cdot \mathbf{B}(t,\mathbf{r}) = 0, & \text{- Gauss' law for magnetism} \\ \nabla \times \mathbf{E}(t,\mathbf{r}) = -\dfrac{\partial \mathbf{B}(t,\mathbf{r})}{\partial t}, & \text{- Faraday's law} \\ \nabla \times \mathbf{B}(t,\mathbf{r}) = 4\pi\mathbf{j}_{ext}(t,\mathbf{r}) + \dfrac{\partial \mathbf{D}(t,\mathbf{r})}{\partial t}. & \text{- Modified Ampère's law} \end{cases}$$

  - **D** - the electric displacement field
  - $\dfrac{\partial \mathbf{D}(t,\mathbf{r})}{\partial t}$ - the displacement current - an additional source of the magnetic field. It is not an electric current of moving charges, but a time-varying electric field.

Dielectric permittivity tensor

- The electric displacement field $\mathbf{D}$

  $$\mathbf{D}(\omega, \mathbf{k}) = \hat{\varepsilon}(\omega, \mathbf{k})\mathbf{E}(\omega, \mathbf{k})$$

- $\hat{\varepsilon}(\omega, \mathbf{k})$ - the dielectric permittivity tensor - provides information about the medium in which the heavy-nuclei collide

- The medium properties are described with $(\omega, \mathbf{k})$. It requires us to transform the Maxwell equations from $(t, \mathbf{r})$ to $(\omega, \mathbf{k})$ dependance. We do that using the Fourier transformations.

Two-sided Fourier transformation

$$f(\omega, \mathbf{k}) \equiv \int_{-\infty}^{\infty} dt \int d^3r e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})} f(t, \mathbf{r})$$

$$f(t, \mathbf{r}) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-(\omega t - \mathbf{k}\cdot\mathbf{r})} f(\omega, \mathbf{k})$$

- Assumes that the medium in which the collision happens doesn't undergo any changes from time $t = -\infty$ to $t = +\infty$.

- In reality the production of quark-gluon plasma at $t = 0$ modifies the electromagnetic field due to the quark currents.

- The use of the two-sided Fourier transformation implies that those currents exist even before the plasma is present.

- The two-sided Fourier transformation is not sufficient for the initial condition problem.

One-sided Fourier transformation

Magnetic field in reality

- Before the collision ($t < 0$) there is no plasma. Only the approaching nuclei generate the magnetic field.
- The additional fields generated due to the plasma appear at $t = 0$
- At $t > 0$ we see the effects of the quark-gluon plasma on the electromagnetic field.

$$f(\omega, \mathbf{k}) \equiv \int_0^\infty dt \int d^3r e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})} f(t, \mathbf{r})$$

$$f(t, \mathbf{r}) = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-(\omega t - \mathbf{k}\cdot\mathbf{r})} f(\omega, \mathbf{k})$$

The real parameter $\sigma > 0$ is chosen is such a way that the integral over ω is taken along a straight line in the complex ω-plane, parallel to the real axis, above all singularities of $f(\omega, \mathbf{k})$.

The magnetic field in heavy-ion collisions
The two-sided Fourier transformation

$$f(\omega, \mathbf{k}) \equiv \int_{-\infty}^{\infty} dt \int d^3 r\, e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})} f(t, \mathbf{r})$$

$$f(t, \mathbf{r}) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^3 k}{(2\pi)^3} e^{-(\omega t - \mathbf{k}\cdot\mathbf{r})} f(\omega, \mathbf{k})$$

- The Maxwell's equations

$$\begin{cases} \mathbf{k}\cdot\mathbf{D}(\omega,\mathbf{k}) = 4\pi\rho_{ext}(\omega,\mathbf{k}), \\ \mathbf{k}\cdot\mathbf{B}(\omega,\mathbf{k}) = 0, \\ \mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}) = \omega\mathbf{B}(\omega,\mathbf{k}), \\ \mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) = -\,\omega\mathbf{D}(\omega,\mathbf{k}) - 4\pi i\mathbf{j}_{ext}(\omega,\mathbf{k}). \end{cases}$$

- Gauss' law
- Gauss' law for magnetism
- Faraday's law
- Modified Ampère's law

- The displacement field **D**

$$\mathbf{D}(\omega,\mathbf{k}) = \hat{\varepsilon}(\omega,\mathbf{k})\mathbf{E}(\omega,\mathbf{k})$$

$$\mathbf{B}(\omega,\mathbf{k}) = -\,4\pi i \frac{\mathbf{k}\times\mathbf{j}_{ext}(\omega,\mathbf{k})}{\omega^2\varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2}$$

- The current density $\mathbf{j}_{ext}$

$$\mathbf{j}_{ext}(t,\mathbf{r}) = q\mathbf{v}\delta(\mathbf{r} - \mathbf{r}_0 - \mathbf{v}t)$$

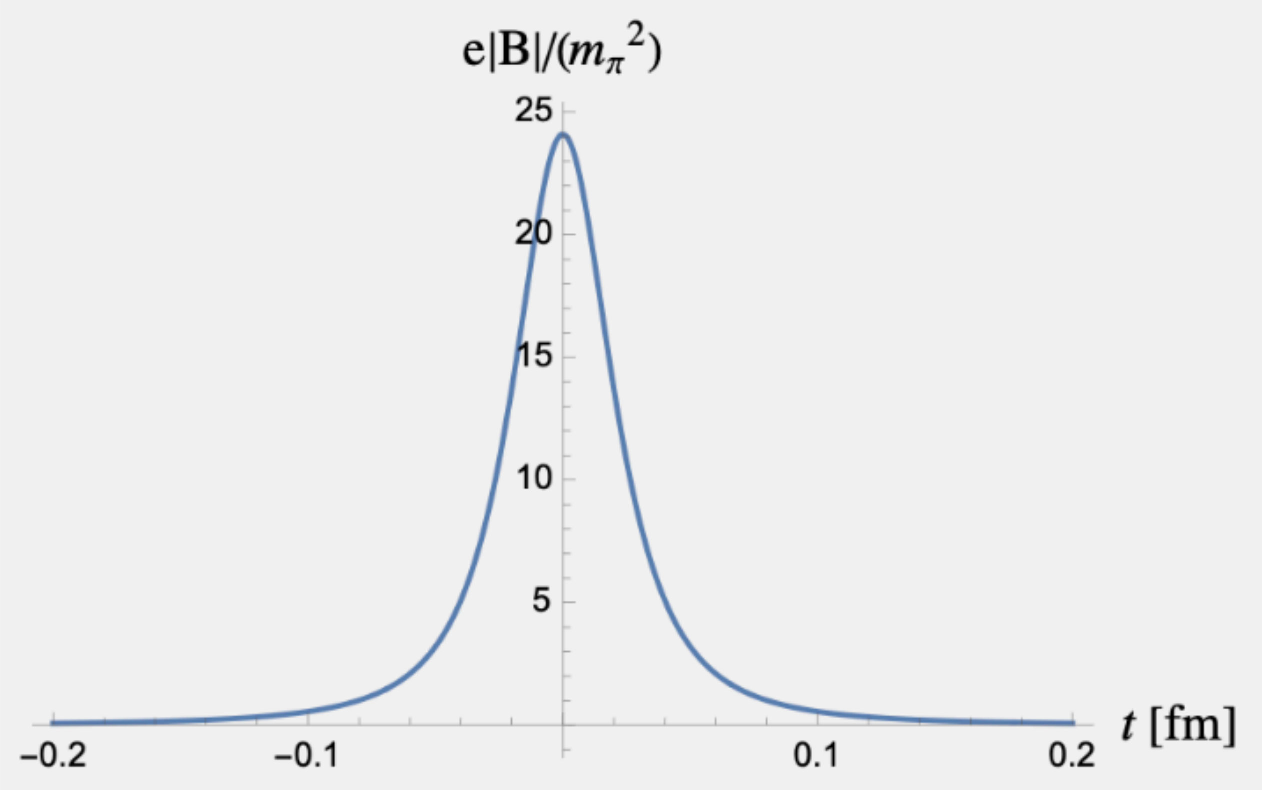
$$\mathbf{j}_{ext}(\omega,\mathbf{k}) = 2\pi q\mathbf{v}\,\delta(\omega - \mathbf{k}\cdot\mathbf{v})\,e^{-i\mathbf{k}\cdot\mathbf{r}_0}$$

Magnetic field in the vacuum

$$\varepsilon_T = 1$$

$$\mathbf{B}(\omega, \mathbf{k}) = -4\pi i \frac{\mathbf{k} \times \mathbf{j}_{ext}(\omega, \mathbf{k})}{\omega^2 - \mathbf{k}^2}$$

$$\mathbf{B}(t, \mathbf{k}) = -4\pi i Ze \frac{\mathbf{k} \times \mathbf{v}}{(\mathbf{k} \cdot \mathbf{v})^2 - \mathbf{k}^2} e^{-i(\mathbf{k}\cdot\mathbf{v}t + \mathbf{k}\cdot\mathbf{r}_0)}$$



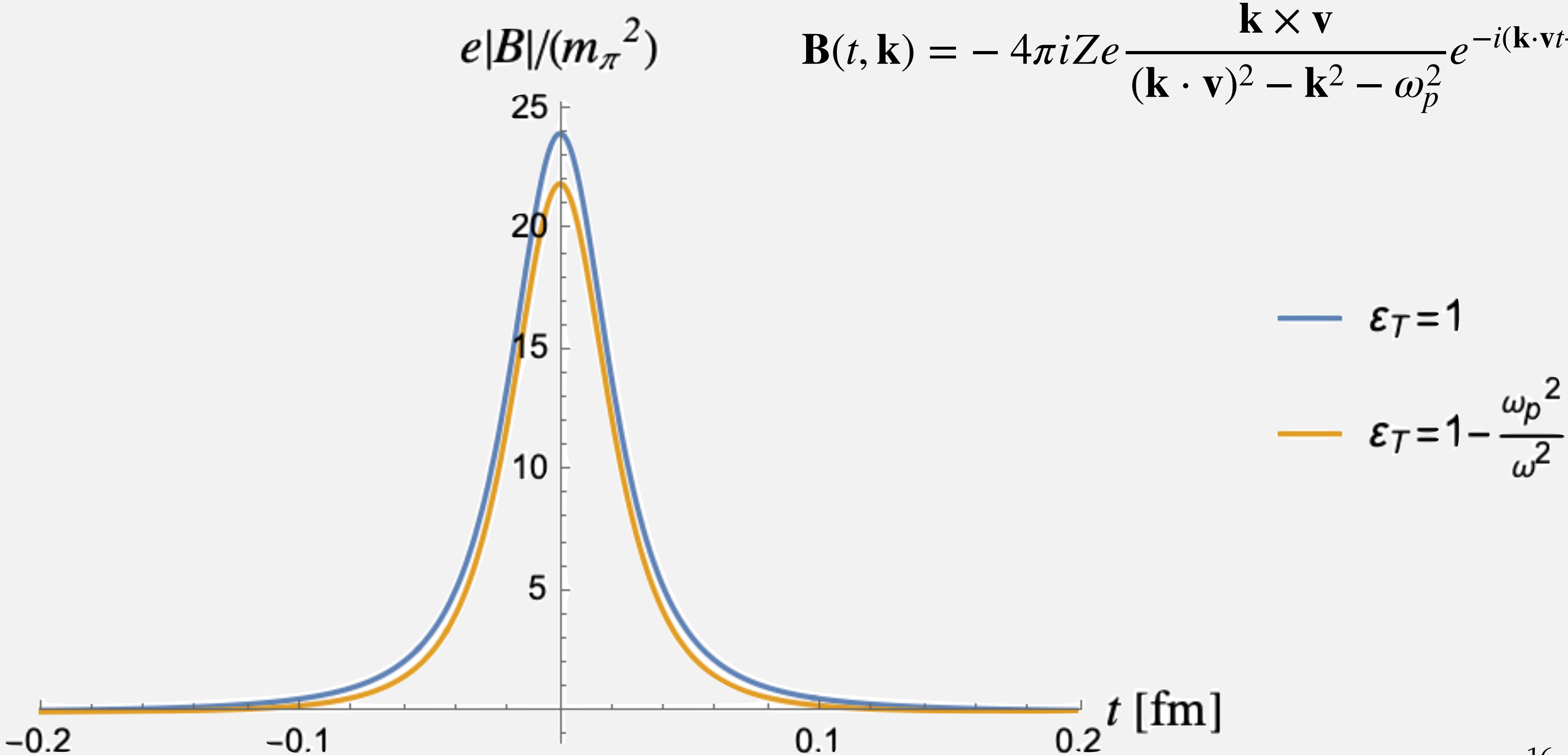
Magnetic field in the vacuum for the Au-Au collision

Magnetic field in the medium - collisionless plasma -

$$\mathbf{B}(\omega,\mathbf{k}) = -4\pi i \frac{\mathbf{k}\times\mathbf{j}_{ext}(\omega,\mathbf{k})}{\omega^2-\omega_p^2-\mathbf{k}^2}$$

$$\varepsilon_T = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\mathbf{B}(t,\mathbf{k}) = -4\pi i Z e \frac{\mathbf{k}\times\mathbf{v}}{(\mathbf{k}\cdot\mathbf{v})^2-\mathbf{k}^2-\omega_p^2} e^{-i(\mathbf{k}\cdot\mathbf{v}t+\mathbf{k}\cdot\mathbf{r}_0)}$$



Magnetic field in the medium for the Au-Au collision

Magnetic field in the medium - collisional plasma -

$$\varepsilon_T = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu)}$$

$$\mathbf{B}(\omega, \mathbf{k}) = -\,4\pi i \frac{\mathbf{k} \times \mathbf{j}_{ext}(\omega, \mathbf{k})}{\omega^2 - \frac{\omega\,\omega_p^2}{\omega + i\nu} - \mathbf{k}^2}$$

$$\mathbf{B}(t, \mathbf{k}) = -\,4\pi i Z e\, e^{-i(\mathbf{k}\cdot\mathbf{v}t + \mathbf{k}\cdot\mathbf{r}_0)} \Bigg[\frac{(\mathbf{k}\cdot\mathbf{v})^2\big((\mathbf{k}\cdot\mathbf{v})^2 - \mathbf{k}^2 - \omega_p^2\big) + \nu^2\big((\mathbf{k}\cdot\mathbf{v})^2 - \mathbf{k}^2\big)}{(\mathbf{k}\cdot\mathbf{v})^2\big((\mathbf{k}\cdot\mathbf{v})^2 - \mathbf{k}^2 - \omega_p^2\big)^2 + \nu^2\big((\mathbf{k}\cdot\mathbf{v})^2 - \mathbf{k}^2\big)^2} - i \frac{\nu(\mathbf{k}\cdot\mathbf{v})\omega_p^2}{(\mathbf{k}\cdot\mathbf{v})^2\big((\mathbf{k}\cdot\mathbf{v})^2 - \mathbf{k}^2 - \omega_p^2\big)^2 + \nu^2\big((\mathbf{k}\cdot\mathbf{v})^2 - \mathbf{k}^2\big)^2} \Bigg] \mathbf{k} \times \mathbf{v}$$

The magnetic field in heavy-ion collisions
The one-sided Fourier transformation

Magnetic field in reality

- Before the collision ($t < 0$) there is no plasma. Only the approaching nuclei generate the magnetic field.
- The additional fields generated due to the plasma appear at $t = 0$
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$$f(t, \mathbf{r}) = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-(\omega t - \mathbf{k}\cdot\mathbf{r})} f(\omega, \mathbf{k})$$

- The Maxwell's equations

$$\begin{cases} i\mathbf{k}\cdot\mathbf{D}(\omega,\mathbf{k}) = 4\pi\rho_{ext}(\omega,\mathbf{k}), \\ i\mathbf{k}\cdot\mathbf{B}(\omega,\mathbf{k}) = 0, \\ i\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}) = i\omega\mathbf{B}(\omega,\mathbf{k}) + \mathbf{B}_0(\mathbf{k}), \\ i\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) = 4\pi\mathbf{j}_{ext}(\omega,\mathbf{k}) - i\omega\mathbf{D}(\omega,\mathbf{k}) - \mathbf{D}_0(\mathbf{k}). \end{cases}$$

The magnetic field and the initial fields

$$\mathbf{B}(\omega,\mathbf{k}) = -\frac{i}{\omega^2\varepsilon_T(\omega,\mathbf{k})-\mathbf{k}^2}\left[\mathbf{k}\times\left(4\pi\mathbf{j}_{ext}(\omega,\mathbf{k})-\mathbf{E}_0(\mathbf{k})\right)-\frac{\mathbf{k}^2}{\omega}\mathbf{B}_0(\mathbf{k})\right]+\frac{i}{\omega}\mathbf{B}_0(\mathbf{k})$$

$$\mathbf{j}_{ext}(\omega,\mathbf{k}) = i\frac{Ze\mathbf{v}e^{-i\mathbf{k}\cdot\mathbf{r}_0}}{\omega-\mathbf{k}\cdot\mathbf{v}+i0^+}$$

- The initial fields generated at $t = 0$

$$\mathbf{E}_0(\mathbf{k}) = -4\pi i\int\frac{d\omega}{2\pi}\left[\frac{\mathbf{k}\left(\mathbf{k}\cdot\mathbf{j}_{ext}(\omega,\mathbf{k})\right)}{\omega(\omega^2-\mathbf{k}^2)}+\frac{\omega\mathbf{j}_{ext}(\omega,\mathbf{k})}{\omega^2-\mathbf{k}^2}\right]$$

$$\mathbf{B}_0(\mathbf{k}) = -4\pi i\int\frac{d\omega}{2\pi}\frac{\mathbf{k}\times\mathbf{j}_{ext}(\omega,\mathbf{k})}{\omega^2-\mathbf{k}^2}$$

$$\mathbf{j}_{ext}(\omega,\mathbf{k}) = 2\pi Ze\mathbf{v}\delta(\omega-\mathbf{k}\cdot\mathbf{v})e^{-i\mathbf{k}\cdot\mathbf{r}_0}$$

- We use the one-sided Fourier transformation for the magnetic field, but the two-sided transformation for the initial fields! (The same for their current densities)

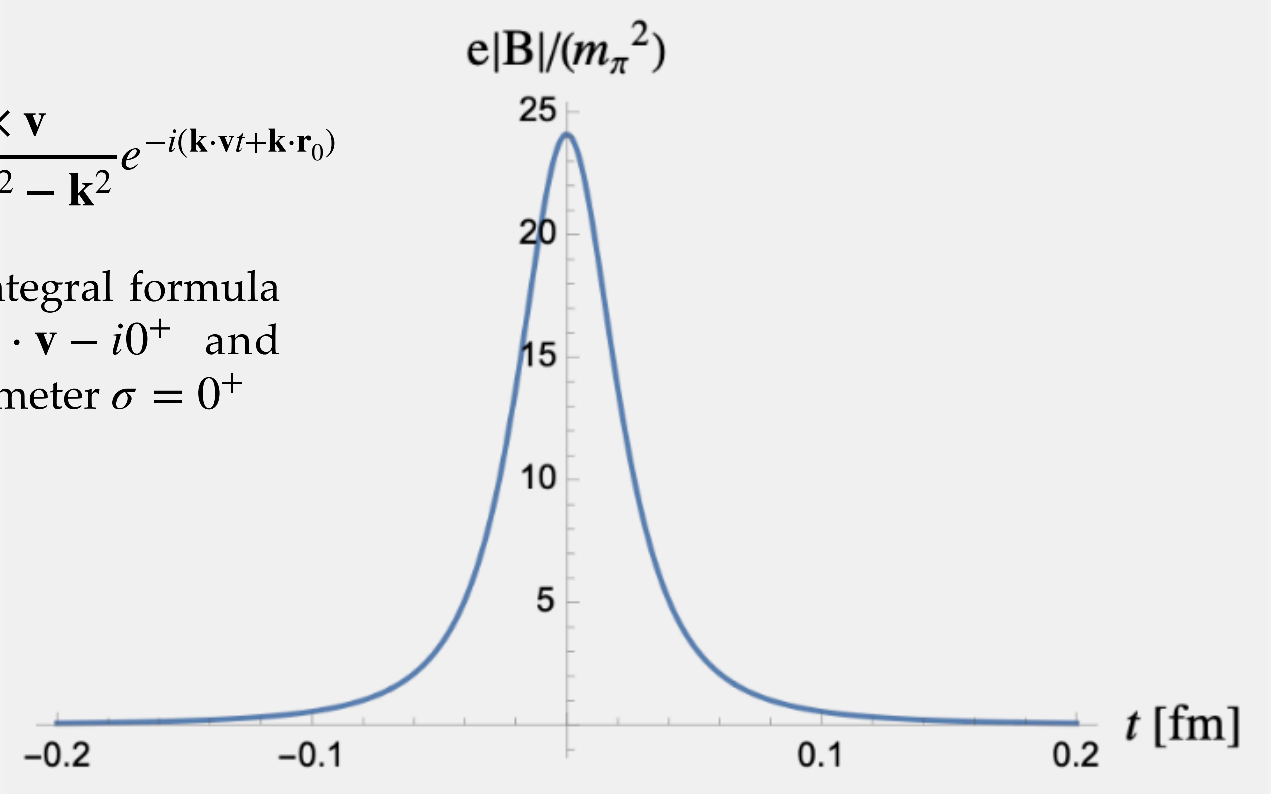
Magnetic field in the vacuum

$$\varepsilon_T = 1$$

$$\mathbf{B}(\omega,\mathbf{k}) = 4\pi Ze\, e^{-i\mathbf{k}\cdot\mathbf{r}_0}\frac{\mathbf{k}\times\mathbf{v}}{(\omega^2-\mathbf{k}^2)(\omega-\mathbf{k}\cdot\mathbf{v}+i0^+)} + i\frac{\mathbf{k}\times\mathbf{E}_0(\mathbf{k})}{\omega^2-\mathbf{k}^2} + i\frac{\mathbf{k}^2\,\mathbf{B}_0(\mathbf{k})}{\omega(\omega^2-\mathbf{k}^2)} + \frac{i}{\omega}\mathbf{B}_0(\mathbf{k})$$

$$\mathbf{B}(t,\mathbf{k}) = -4\pi i Ze\frac{\mathbf{k}\times\mathbf{v}}{(\mathbf{k}\cdot\mathbf{v})^2-\mathbf{k}^2}e^{-i(\mathbf{k}\cdot\mathbf{v}t+\mathbf{k}\cdot\mathbf{r}_0)}$$

- We used the Cauchy's integral formula with poles at $\omega = \mathbf{k}\cdot\mathbf{v} - i0^+$ and $\omega = \pm|\mathbf{k}|$, and the parameter $\sigma = 0^+$



Magnetic field in the vacuum for the Au-Au collision

Magnetic field in the medium

$$\varepsilon_T = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\mathbf{B}(\omega,\mathbf{k}) = 4\pi Ze\, e^{-i\mathbf{k}\cdot\mathbf{r}_0} \frac{\mathbf{k}\times\mathbf{v}}{(\omega^2-\mathbf{k}^2-\omega_p^2)(\omega-\mathbf{k}\cdot\mathbf{v}+i0^+)} + i\frac{\mathbf{k}\times\mathbf{E}_0(\mathbf{k})}{\omega^2-\mathbf{k}^2-\omega_p^2} + i\frac{\mathbf{k}^2\,\mathbf{B}_0(\mathbf{k})}{\omega(\omega^2-\mathbf{k}^2-\omega_p^2)} + \frac{i}{\omega}\mathbf{B}_0(\mathbf{k})$$

$$\mathbf{B}(t,\mathbf{k}) = -\,i4\pi Ze\, e^{-i\mathbf{k}\cdot\mathbf{r}_0}\Bigg[\frac{e^{-i\sqrt{\mathbf{k}^2+\omega_p^2}\,t}}{2\sqrt{\mathbf{k}^2+\omega_p^2}\,(\sqrt{\mathbf{k}^2+\omega_p^2}-\mathbf{k}\cdot\mathbf{v})} + \frac{e^{i\sqrt{\mathbf{k}^2+\omega_p^2}\,t}}{2\sqrt{\mathbf{k}^2+\omega_p^2}\,(\sqrt{\mathbf{k}^2+\omega_p^2}+\mathbf{k}\cdot\mathbf{v})}$$
$$+\frac{e^{-i\mathbf{k}\cdot\mathbf{v}t}}{(\mathbf{k}\cdot\mathbf{v})^2-\mathbf{k}^2-\omega_p^2} + \frac{e^{-i\sqrt{\mathbf{k}^2+\omega_p^2}\,t}-e^{i\sqrt{\mathbf{k}^2+\omega_p^2}\,t}}{2\sqrt{\mathbf{k}^2+\omega_p^2}}\frac{\mathbf{k}\cdot\mathbf{v}}{(\mathbf{k}\cdot\mathbf{v})^2-\mathbf{k}^2}$$
$$+\left(-\frac{1}{\mathbf{k}^2+\omega_p^2} + \frac{e^{-i\sqrt{\mathbf{k}^2+\omega_p^2}\,t}+e^{i\sqrt{\mathbf{k}^2+\omega_p^2}\,t}}{2(\mathbf{k}^2+\omega_p^2)}\right)\frac{\mathbf{k}^2}{(\mathbf{k}\cdot\mathbf{v})^2-\mathbf{k}^2} + \frac{1}{(\mathbf{k}\cdot\mathbf{v})^2-\mathbf{k}^2}\Bigg]\mathbf{k}\times\mathbf{v}$$

Future plans

- Finding a method to integrate highly oscillatory functions
- Calculating the magnetic field for the model more complex than that using point-like charges