Photon-Jet production in pA Collision in the Color Glass Condensate

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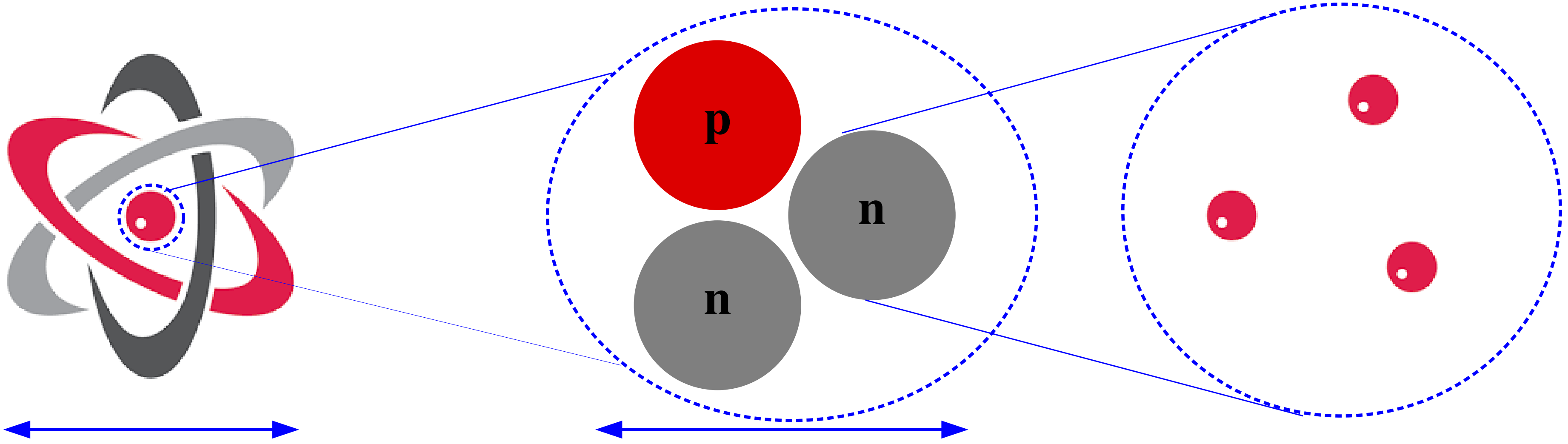
In Collaboration with
Tolga Altinoluk & **Guillaume Beuf**,

Article to appear soon

Quantum ChromoDynamics :



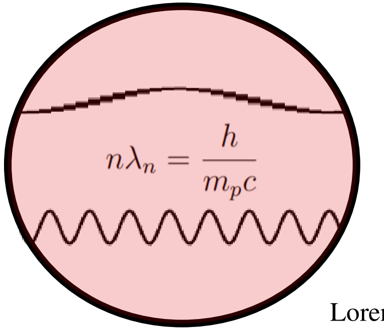
Atome : $10^{-10}\mathrm{m}$

Nucleus : $10^{-15}\mathrm{m}$

Fermion sources of color field — quark (u,d,d) + Boson sources of color field — gluon

1898 – M. Skłodowska-Curie : radiation is an intrinsic property of atoms ;
1932 – W. Heisenberg : the concept of isospin (strong interaction);
1934 – E. Fermi : phenomenology of weak interaction ;
1935 – H. Yukawa : heavy boson for short interaction ;
1953 – C. N. Yang and R. Mills : Theory of non-Abelian gauge symetry for the nuclear interactions ;
1960 – P. Higgs : masses of fermions and W/Z bosons ;
1969 – R. Feynman : parton model to describe the internal degrees of freedom of hadrons;
1973 – D. Gross, F. Wilczek and H.D D.Politzer : Asymptotic freedom in QCD ;
1977-78 – E. Kuarev, V. Fadin & Y. Balitsky L. Lipatov, low-x gluon distribution equation ;
1998-99 – Y. Balitsky & Y. Kovchegov, the saturation equation.

Why high energy scattering in QCD ?

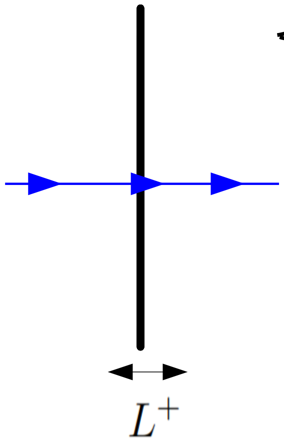


$$n \in [|1; +\infty|[$$

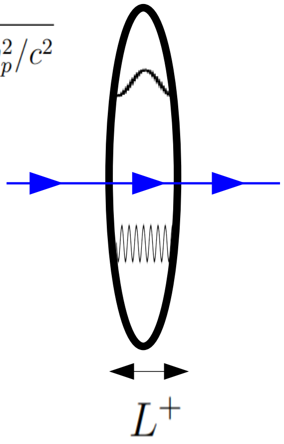
$$E_n = h\nu_n \; , \; \nu_n / c = \frac{m_p c}{h} \cdot n$$

Lorentz contraction at high energy

$$n\lambda_n \to \lambda_C \sqrt{1 - v_p^2/c^2}$$



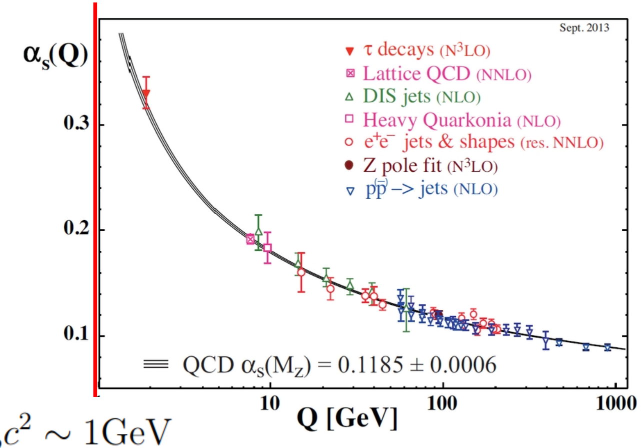
Eikonal Approximation



Beyond the flat width (subeikonal effects)

Strong coupling at one-loop :

$$\alpha_s^{lo}(Q^2) = \frac{1}{\beta_0 \log \frac{Q^2}{\Lambda^2}}, \quad Q^2 = -q^2$$



$m_p c^2 \sim 1\text{GeV}$

- No theoretical clue for pQCD below 1GeV ;
- Parton probes need high energy to be resolved ;
- Bound states are sources of IR safe oscillations.

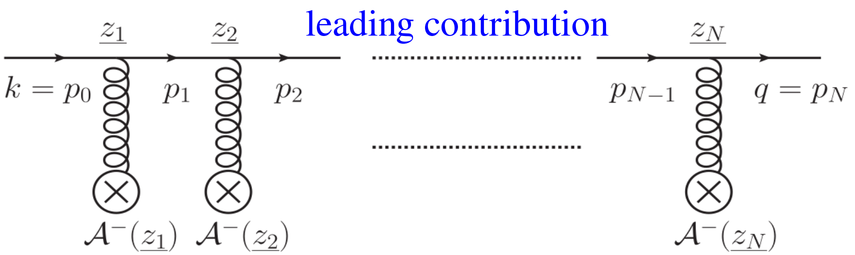
Boosted Color fields from the target :

$$A^\mu(x^\mu) \to \Lambda^\mu_{\ \nu} A^\nu((\Lambda^{-1})^\mu_{\ \nu} x^\nu) = \left(\gamma^{-1}A^+,\, A^\perp,\, \gamma A^-\right)\left(\gamma\, x^+,\, x_\perp,\, \gamma^{-1}x^-\right)$$

$$A^- = O(1/L^+) \gg A_j = O(1) \gg A^+ = O(L^+).$$

Condensated along x^+ and $x_\perp$
while Glassed (static) on x^-

Medium induced *"Wilson's lines"*



$$\mathcal{U}_F(x^+, y^+, \mathbf{z}) = \mathcal{P}_+ \exp\left(-ig\int_{y^+}^{x^+} dz^+ A^-(z^+, \mathbf{z})\right)$$

- e.g, Eikonal Quark propagator :

[T. Altinoluk, G. Beuf, A. Czajka and A. Tymowska 10.1103/PhysRevD.104.014019 (2021)]

$$S_F(x,y)|_{\text{Eik}} = \int \frac{d_4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \frac{i(\not k + m)}{[k^2 - m^2 + i\epsilon]}$$

$$+ \int \frac{d_3\underline{q}}{(2\pi)^3} \int \frac{d_3\underline{k}}{(2\pi)^3} 2\pi\delta(q^+ - k^+) e^{-ix\cdot q + iy\cdot k} \frac{(\not q + m)\gamma^+(\not k + m)}{(2k^+)^2}$$

$$\times \int d_2\mathbf{z} e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \left\{\theta(k^+)\theta(x^+ - y^+)[\mathcal{U}_F(y^+, x^+, \mathbf{z}) - 1] - \theta(-k^+)\theta(y^+ - x^+)[\mathcal{U}_F^\dagger(y^+, x^+, \mathbf{z}) - 1]\right\}$$

For the Eikonal Gluon propagator see : [T. Altinoluk, G. Beuf, and S. Mulani, Phys.Rev.D 111 (2025) 3, 034028]

Subeikonal finit width effects $\lim : L^+ \neq 0$

- Allow transverse d.o.f of the gluon background ;
- Observable are sensible to target's polarization ;
- Relax the static limit of the internal structur of the target;
- Offer the possibility of quark background insertions.

- e.g, Quark propagator at Next-to-Eikonal accuracy from *before to after* $x^+ \leq -\frac{L^+}{2}\,,\ \frac{L^+}{2} \leq y^+$ the target :

Unpolarized contribution :

$$
{}^{\neg\pi}S_F(x,y)\big|_{\rm NEik} = \int \frac{d_3\underline{q}}{(2\pi)^3}\int \frac{d_3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+ - k^+)\, e^{-ix\cdot q + iy\cdot k}\,\frac{(\not{q}+m)\gamma^+(\not{k}+m)}{(2k^+)^2}
$$

$$
\times \int d_2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}\left\{\mathcal{U}_F\left(\frac{L^+}{2}, -\frac{L^+}{2}, \mathbf{z}\right) - \frac{\mathbf{q}^j + \mathbf{k}^j}{4k^+}\mathcal{U}^{(1)}_{F,j}(\mathbf{z}) - \frac{i}{4k^+}\mathcal{U}^{(2)}_{F}(\mathbf{z})\right\}
$$

3 types of Wilson's lines *"decorations"*

Polarized contribution :

$$
{}^{\pi}S_F(x,y)\big|_{\rm NEik} = \int \frac{d_3\underline{q}}{(2\pi)^3}\int \frac{d_3\underline{k}}{(2\pi)^3}\, 2\pi\delta(q^+ - k^+)\, e^{-ix\cdot q + iy\cdot k}
$$

$$
\times \frac{(\not{q}+m)\gamma^+[\gamma^i, \gamma^j](\not{k}+m)}{4(2k^+)^3} \int d_2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})}\mathcal{U}^{(3)}_{F,ij}(\mathbf{z})
$$

- 3 types of Wilson's lines "*decorations*" :

induced by the transverse d.o.f of the target

$$\mathcal{U}^{(1)}_{F,j}(\mathbf{v}) = ig\int_{-L^+/2}^{L^+/2} dv^+(2v^+)\mathcal{U}_F(\frac{L^+}{2},v^+,\mathbf{v})F_j^{\;-}(v^+,\mathbf{v})\mathcal{U}_F(v^+,-\frac{L^+}{2},\mathbf{v})$$

$$\mathcal{U}^{(2)}_{F}(\mathbf{v}) = (ig)^2\int_{-L^+/2}^{L^+/2} dv^+\int_{-L^+/2}^{L^+/2} du^+\theta(v^+-u^+)(v^+-u^+)\,\mathcal{U}_F(\frac{L^+}{2},u^+,\mathbf{v})F_j^{\;-}(u^+,\mathbf{v})\mathcal{U}_F(u^+,v^+,\mathbf{v})F_j^{\;-}(v^+,\mathbf{v})\mathcal{U}_F(v^+,-\frac{L^+}{2},\mathbf{v})$$

$$\mathcal{U}^{(3)}_{F,ij}(\mathbf{v}) = g\int_{L^+/2}^{L^+/2} dv^+\mathcal{U}_F(\frac{L^+}{2},v^+,\mathbf{v})F_{ij}(v^+,\mathbf{v})\mathcal{U}_F(v^+,-\frac{L^+}{2},\mathbf{v}).$$

Beyond the static approximation of the target ;

$$\mathcal{U}_F(\mathbf{v})\rightarrow\mathcal{U}_F(\mathbf{v},v^-) = \mathcal{U}_F(\mathbf{v}) + v^-(-ig)\int_{-L^+/2}^{L^+/2} dz^+\mathcal{U}_F(\frac{L^+}{2},v^+,\mathbf{v})F^{+-}(v^+,\mathbf{v})\mathcal{U}_F(v^+,-\frac{L^+}{2},\mathbf{v}) + O((x^-)^2)$$

- Boosted quark background : $J^\mu(x^\mu) = \overline{\Psi}(x^\mu)\gamma^\mu\Psi(x^\mu)\rightarrow\Lambda^\mu_{\;\nu}J^\nu((\Lambda^\mu_{\;\nu})^{-1}x^\nu)$

It identified enhanced components of the wave function $J^+\propto L^+ \ll J^i\propto(L^+)^0 \ll J^- = \overline{\Psi}^{(-)}\gamma^-\Psi^{(-)}\propto(L^+)^{-1}$

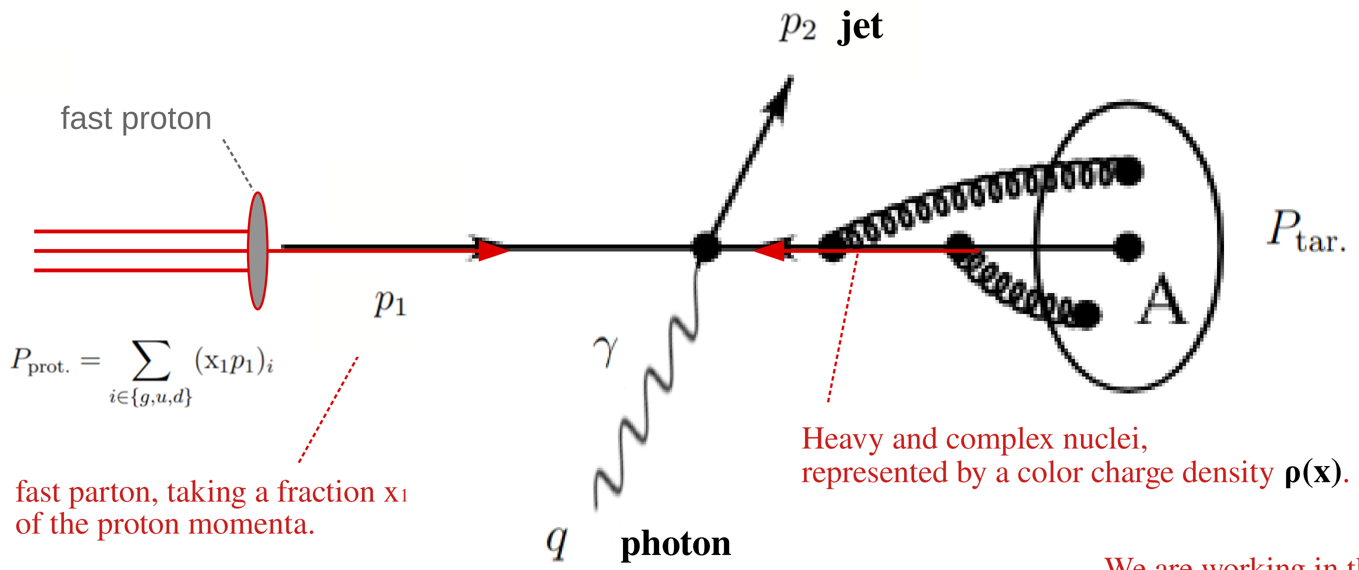
$\Psi^{(-)}\propto(L^+)^{-\frac{1}{2}}$

Dynamic spinor
in the target
seen by the projectile

$\Psi^{(+)}\propto(L^+)^{\frac{1}{2}}$

Constraint spinor
in the target
seen by the projectile

Proton-Heavy Nuclei Collision to photon-Jet production



During our discussion, we will need to define :

- Relative momenta : $\mathbf{P} := z\mathbf{p}_2 - \bar{z}\mathbf{q}$
- Momentum exchange with the target : $\mathbf{k} := \mathbf{p}_2 + \mathbf{q} - \mathbf{p}_1$

With longitudinal momentum fractions : $z = \frac{p_2^+}{p_1^+}, \quad \bar{z} = \frac{q^+}{p_1^+}$

We are working in the Regge-Gribov limit :

$$\lim_{W\to\infty,\ x_{\text{tar.}}\to 0} W^2 x_{\text{tar.}} \to Q^2 \quad \text{(fixed)}$$

With :

- The transverse resolution : $Q^2 = (p_2 + q)^2 = \frac{\mathbf{P}^2}{z\bar{z}}$
- Centre of mass energy : $W^2 = (p_1 + P_A) \approx 2p_1 \cdot P_A$
- With the target probe at : $x_{\text{tar.}} = \frac{\mathbf{P}^2}{z\bar{z}W^2}$

Back-to-back limit for CGC : $|\mathbf{k}| \ll |\mathbf{P}| \ll W.$

Ordering of the covariant derivatives

$$\mathcal{D}_+ = O\left(P^-_{\text{tar.}}\right) \qquad \mathcal{D}_- = O\left(\frac{|\mathbf{k}|^2}{P^-_{\text{tar.}}}\right) \qquad \mathcal{D}_{\mathbf{b}^j} = O\left(|\mathbf{b}|^{-1}\right) = O\left(|\mathbf{k}|\right)$$

Ordering of the field strength

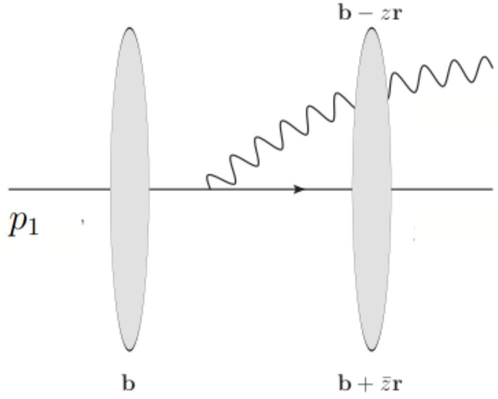
$$F_i^{\ -} = O\left(|\mathbf{k}|P^-_{\text{tar.}}\right) \ ; \ F_i^{\ +} = O\left(\frac{|\mathbf{k}|^3}{P^-_{\text{tar.}}}\right) ;$$

$$F_{ij} = O\left(\mathbf{k}^2\right) \ ; \ F_{+-} = O\left(\mathbf{k}^2\right)$$

Ordering of the decoration

$$\mathcal{U}^{(1)}_{F,j} = O\left(|\mathbf{k}|/P^-_{\text{target}}\right) \qquad \mathcal{U}^{(2)}_{F} = O\left(\mathbf{k}^2/P^-_{\text{target}}\right)$$

$$\mathcal{U}^{(3)}_{F,ij} = O\left(\mathbf{k}^2/P^-_{\text{target}}\right)$$



Relative position Dipole size

$$\mathbf{b} := \bar{z}\mathbf{x}_2 + z\mathbf{x}_1, \quad \mathbf{r} := \mathbf{x}_2 - \mathbf{x}_1.$$

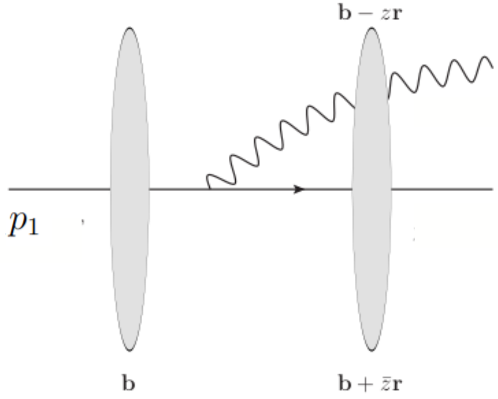
Back-to-back limit = small dipole size expansion :

$$\frac{2\pi}{\mathbf{b}} \leq \mathbf{k} \ll \frac{2\pi}{\mathbf{r}} \leq \mathbf{P} \Longleftrightarrow |\mathbf{r}| \ll |\mathbf{b}|$$

Back-to-back limit for CGC : $|\mathbf{k}| \ll |\mathbf{P}| \ll W.$

Original expression in general kinematics

$$\frac{d_6\sigma_{pA\to\gamma+\text{jet}+A'}}{dz_2 d_2\mathbf{p}_2 dz_1 d_2\mathbf{p}_1} \propto \sum_{a,b} f_{a/p}(x_1) \sum_{i=1,2} \left\langle \text{TF}_{\mathbf{b}_{12},\mathbf{r}_{12}} \left[\mathcal{H}^{(i)}_{ag\to b\gamma}(\mathbf{r}_{12})\mathcal{O}^{(i)}_{ag\to b\gamma}(\mathbf{b}_{12},\mathbf{r}_{12})\right](\mathbf{P},\mathbf{k})\right\rangle_A$$



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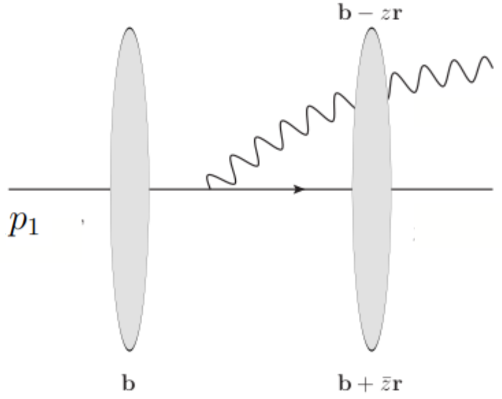
Deconvolution from the small dipole size expansion

$$\propto \sum_{a,b} f_{a/p}(x_1) \sum_{i=1,2} \left\langle \mathrm{TF}_{\mathbf{b}_{12},\mathbf{r}_{12}} \left[\mathcal{H}^{(i)}_{ag\to b\gamma}(\mathbf{P},\mathbf{r}_{12}) \left(\mathcal{O}^{(i)}_{ag\to b\gamma}(\mathbf{b}_{12},0) + O\left(\mathbf{r}^n_{12}\partial^n_{\mathbf{b}_{12}}\right)\right)\right]\right\rangle_A$$



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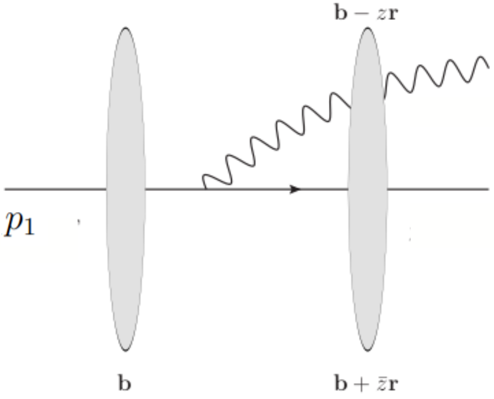
Convert to higher twist corrections

$$\propto \sum_{a,b} f_{a/p}(x_1) \sum_{i=1,2} \left[\mathcal{H}^{(i)}_{ag\to b\gamma}(\mathbf{P},0) + O\left(\frac{\mathbf{k}^n}{\mathbf{P}^n}\right)\right] \left\langle \mathcal{O}^{(i)}_{ag\to b\gamma}(\mathbf{k})\right\rangle_A$$

The hard factor is factorized form the Non-Perturbative color structure



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From the CGC observables to the Transverse Momentum Distributions

Starting from the CGC expression of the dipole,

$$\int_{\mathbf{v},\mathbf{w}} e^{-i(\mathbf{v}-\mathbf{w})\cdot\mathbf{k}} \mathrm{Tr}\left\langle \mathcal{U}_F(\frac{L^+}{2}, v^+, \mathbf{v}) \mathcal{U}_F^\dagger(v^+, -\frac{L^+}{2}, \mathbf{v}) \right\rangle_A$$

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$$= \frac{\mathbf{k}^i\mathbf{k}^j}{\mathbf{k}^4}\left(\mathbf{k}^i\mathbf{k}^j\int_{\mathbf{v},\mathbf{w}} e^{-i(\mathbf{v}-\mathbf{w})\cdot\mathbf{k}}\mathrm{Tr}\left\langle \mathcal{U}_F(\frac{L^+}{2},v^+,\mathbf{v})\mathcal{U}_F^\dagger(v^+,-\frac{L^+}{2},\mathbf{v})\right\rangle_A\right)$$

, using : $i\mathbf{k}^j\int_{\mathbf{v}}\mathcal{U}_F(\mathbf{v})e^{-i\mathbf{v}\cdot\mathbf{k}}. = -ig\int_{v^+,\mathbf{v}}\mathcal{U}_F(\frac{L^+}{2},v^+,\mathbf{v})F_j^{\;-}(v^+,\mathbf{v})\mathcal{U}_F(v^+,-\frac{L^+}{2},\mathbf{v})$

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$$=4\pi\,\alpha_s\frac{\mathbf{k}^i\mathbf{k}^j}{\mathbf{k}^4}\int_{v^+,w^+,\mathbf{v},\mathbf{w}} e^{-i(\mathbf{v}-\mathbf{w})\cdot\mathbf{k}}\;\mathrm{Tr}\left\langle \mathcal{U}_F(\frac{L^+}{2},v^+,\mathbf{v})F_i^{\;-}(v^+,\mathbf{v})\mathcal{U}_F(v^+,-\frac{L^+}{2},\mathbf{v})\mathcal{U}_F^\dagger(w^+,-\frac{L^+}{2},\mathbf{w})F_j^{\;-}(w^+,\mathbf{w})\mathcal{U}_F^\dagger(\frac{L^+}{2},w^+,\mathbf{w})\right\rangle_A$$

Note that the dipole is only linearly polarized and using the relation :

From Statistical to Quantum average

$$\langle\cdots\rangle_A = \lim_{P'_{\mathrm{tar.}}\to P_{\mathrm{tar.}}} : \frac{\langle P'_{\mathrm{tar.}}|\cdots|P_{\mathrm{tar.}}\rangle}{\langle P'_{\mathrm{tar.}}|P_{\mathrm{tar.}}\rangle}$$

i.e Path integral averaging over color configuration to Gell-Mann and Low Interation pictur.

[Dominguez, Marquet, Xiao, Yuan - arXiv: 1101.0715]

From the CGC observables to the Transverse Momentum Distributions

Starting from the CGC expression of the dipole,

$$\int_{\mathbf{v},\mathbf{w}} e^{-i(\mathbf{v}-\mathbf{w})\cdot\mathbf{k}} \mathrm{Tr}\left\langle \mathcal{U}_F(\frac{L^+}{2},v^+,\mathbf{v})\mathcal{U}_F^\dagger(v^+,-\frac{L^+}{2},\mathbf{v})\right\rangle_A$$

$$= \frac{\mathbf{k}^i\mathbf{k}^j}{\mathbf{k}^4}\left(\mathbf{k}^i\mathbf{k}^j \int_{\mathbf{v},\mathbf{w}} e^{-i(\mathbf{v}-\mathbf{w})\cdot\mathbf{k}} \mathrm{Tr}\left\langle \mathcal{U}_F(\frac{L^+}{2},v^+,\mathbf{v})\mathcal{U}_F^\dagger(v^+,-\frac{L^+}{2},\mathbf{v})\right\rangle_A\right)$$, using : $$i\mathbf{k}^j\int_{\mathbf{v}}\mathcal{U}_F(\mathbf{v})e^{-i\mathbf{v}\cdot\mathbf{k}}. = -ig\int_{v^+,\mathbf{v}}\mathcal{U}_F(\frac{L^+}{2},v^+,\mathbf{v})F_j^{\;-}(v^+,\mathbf{v})\mathcal{U}_F(v^+,-\frac{L^+}{2},\mathbf{v})$$

$$=4\pi\,\alpha_s\frac{\mathbf{k}^i\mathbf{k}^j}{\mathbf{k}^4}\int_{v^+,w^+,\mathbf{v},\mathbf{w}} e^{-i(\mathbf{v}-\mathbf{w})\cdot\mathbf{k}}\;\mathrm{Tr}\left\langle \mathcal{U}_F(\frac{L^+}{2},v^+,\mathbf{v})F_i^{\;-}(v^+,\mathbf{v})\mathcal{U}_F(v^+,-\frac{L^+}{2},\mathbf{v})\mathcal{U}_F^\dagger(w^+,-\frac{L^+}{2},\mathbf{w})F_j^{\;-}(w^+,\mathbf{w})\mathcal{U}_F^\dagger(\frac{L^+}{2},w^+,\mathbf{w})\right\rangle_A$$

$$= \boxed{\frac{4\pi\,\alpha_s}{\mathbf{k}^2}(2\pi^3)\left[\mathrm{x}G^{(2)}(\mathrm{x},\mathbf{k})\right]}$$

dipole TMD

Note that the dipole is only linearly polarized and using the relation :

$$\boxed{\mathrm{x}G^{(2)}(\mathrm{x},\mathbf{k}) := 2\int\frac{dv^+d_2\mathbf{v}}{(2\pi)^3P^-_{\mathrm{tar.}}}e^{i\mathrm{x}P^-_{\mathrm{tar.}}v^+-i\mathbf{k}\cdot\mathbf{v}}\langle P_{\mathrm{tar.}}|\mathrm{Tr}\{F^{-j}(v^+,\mathbf{v})\mathrm{U}^{[-]}F^{-j}(0^+,\mathbf{0})\mathrm{U}^{[+]\dagger}\}|P_{\mathrm{tar.}}\rangle}$$

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i.e Path integral averaging over color configuration to Gell-Mann and Low Interation pictur.

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Differential cross-section pA for the photo-production in the back-to-back limit

$$\frac{d_6\sigma_{pA\to\gamma+\text{jet}+A'}}{dz_2 d_2\mathbf{p}_2 dz_1 d_2\mathbf{p}_1} \propto \sum_{a,b} f_{a/p}(x_1) \sum_{i=1,2} \left[\mathcal{H}^{(i)}_{ag\to b\gamma}(\mathbf{P},\mathbf{k})\Phi^{(i)}_g(x_2,\mathbf{k}) + \mathcal{H}^{(i)}_{aq\to b\gamma}(\mathbf{P},\mathbf{k})(F^{(i)}_q(x_2,\mathbf{k})/W^2)\right]$$

[Kotko, Kutak, Marquet, Petreska, Sapeta, Van Hameren - arXiv:1503.03421]

$\mathcal{H}^{(i)}_{ag\to b\gamma}(\mathbf{P},\mathbf{k})$: hard factor, kinetics effect of off-shell gauge invariant matrix elements.

$\Phi^{(i)}_g(x_2,\mathbf{k})$: linear combinations of unpolarized gluon TMDs ;

$F^{(i)}_q(x_2,\mathbf{k})$: linear combinations of quark TMDs ;

TMD obtained for the photo-production at NEik accuracy

(also present at Eik accuracy)

$$\mathrm{x}G^{(2)}(\mathrm{x},\mathbf{k}) := 2\int \frac{dv^+ d_2\mathbf{v}}{(2\pi)^3 P^-_{\text{tar.}}} e^{i\mathrm{x}P^-_{\text{tar.}}v^+ - i\mathbf{k}\cdot\mathbf{v}} \langle P_{\text{tar.}}|\mathrm{Tr}\{F^{-j}(v^+,\mathbf{v})\mathrm{U}^{[-]}F^{-j}(0^+,\mathbf{0})\mathrm{U}^{[+]\dagger}\}|P_{\text{tar.}}\rangle$$

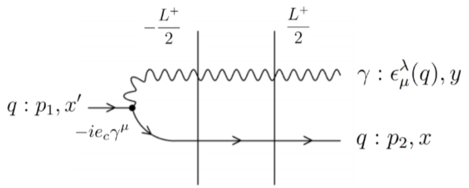
(pure NEik contributions)

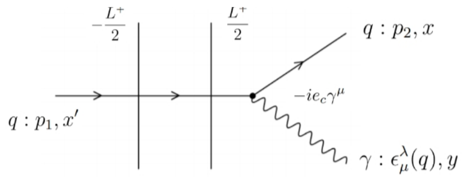
+ anti-quark TMD

$$f^{(1,\pm)}_q(\mathrm{x},\mathbf{k}) := \int \frac{dv^+ d_2\mathbf{v}}{(2\pi)^3} e^{-i\mathrm{x}P^-_{\text{tar.}}v^+ - i\mathbf{k}\cdot\mathbf{v}} \langle P_{\text{tar.}}|\overline{\Psi}(v^+,\mathbf{v})\frac{\gamma^-}{2}\mathrm{U}^{[\pm]}\Psi(0^+,\mathbf{0})|P_{\text{tar.}}\rangle$$

$$f^{(2,\pm)}_q(\mathrm{x},\mathbf{k}) := \frac{1}{N_c}\int \frac{dv^+ d_2\mathbf{v}}{(2\pi)^3} e^{-i\mathrm{x}P^-_{\text{tar.}}v^+ - i\mathbf{k}\cdot\mathbf{v}} \langle P_{\text{tar.}}|\overline{\Psi}(v^+,\mathbf{v})\frac{\gamma^-}{2}\left[\mathrm{Tr}\{\mathrm{U}^{[\square]}\}\mathrm{U}^{[\pm]}\right]\Psi(0^+,\mathbf{0})|P_{\text{tar.}}\rangle$$

Gluon background in the photo-production





Eikonal cross section :

$$\frac{d_6\sigma_{q\to q\gamma}}{d_6\text{P.S}}\Big|_{\text{Eik}} \propto \alpha_e\alpha_s\left(\mathcal{H}_{\text{Eik}}\frac{\mathbf{k}^i\mathbf{k}^j}{\mathbf{k}^4}\right)\int_{v^+,w^+,\mathbf{v},\mathbf{w}} e^{-i(\mathbf{v}-\mathbf{w})\cdot\mathbf{k}}$$
$$\times\text{Tr}\left\langle \mathcal{U}_F(\frac{L^+}{2},v^+,\mathbf{v})F_i^{\;-}(v^+,\mathbf{v})\mathcal{U}_F(v^+,-\frac{L^+}{2},\mathbf{v})\mathcal{U}_F^\dagger(w^+,-\frac{L^+}{2},\mathbf{w})F_j^{\;-}(w^+,\mathbf{w})\mathcal{U}_F^\dagger(\frac{L^+}{2},w^+,\mathbf{w})\right\rangle_A$$

[T. Altinoluk, R. Boussarie, P. Kotko J. High Energ. Phys. 2019, 156 (2019)]

[F. Dominguez, C. Marquet, B. Xiao, F. Yuan Phys.Rev.D 83 (2011) 105005]

Next-to-Eikonal cross section :

$$\frac{d_6\sigma_{q\to q\gamma}}{d_6\text{P.S}}\Big|^{\text{dec. on q}}_{\text{NEik}} \propto \alpha_e\alpha_s\left(\mathcal{H}^j_{\text{dec. 1}}\frac{\mathbf{k}^i}{\mathbf{k}^2}\right)\int_{v^+,w^+} 2i(v^+-w^+)\int_{\mathbf{v},\mathbf{w}} e^{-i(\mathbf{v}-\mathbf{w})\cdot\mathbf{k}}$$
$$\times\text{Tr}\left\langle \mathcal{U}_F(\frac{L^+}{2},v^+,\mathbf{v})F_i^{\;-}(v^+,\mathbf{v})\mathcal{U}_F(v^+,-\frac{L^+}{2},\mathbf{v})\mathcal{U}_F^\dagger(w^+,-\frac{L^+}{2},\mathbf{w})F_j^{\;-}(w^+,\mathbf{w})\mathcal{U}_F^\dagger(\frac{L^+}{2},w^+,\mathbf{w})\right\rangle_A$$

Gluon contribution to the cross section :

$$\frac{d_6\sigma_{q\to q\gamma}}{d_6\text{P.S}}\Big|^{\text{g}}_{\text{Eik+NEik}} \propto \alpha_e\alpha_s(2\pi^3)\left(\mathcal{H}_{\text{Eik}}\frac{1}{\mathbf{k}^2}-\frac{2\mathcal{H}^j_{\text{dec. 1}}\mathbf{k}^j}{P^-_{\text{tar.}}\mathbf{k}^2}\frac{\partial}{\partial \text{x}}\right)\text{x}G^{(2)}(\text{x},\mathbf{k})\big|_{\text{x}=0}\,.$$

Next-to-Eikonal + kinematic twist-3 accuracy

$$\left(\mathcal{H}^{j}_{\text{dec. }1}\frac{2\mathbf{k}^j}{P^-_{\text{tar.}}}\right) = \frac{\mathbf{P}^2}{z\bar{z}W^2}\mathcal{H}_{\text{Eik}}$$

: in the back-ot-back limit the 1st decoration contribution appear as the 1st order Taylor expansion in $\mathrm{x}_{\text{tar.}} = \frac{\mathbf{P}^2}{z\bar{z}W^2}$ of the Eikonal cross-section.

Behavior found in dijet production, see : [T. Altinoluk, G. Beuf, A. Czajka, C. Marquet, (2024) http://arxiv.org/abs/2410.00612v1]

$$\left.\frac{d_5\sigma_{q\to q\gamma}}{dz d_2\mathbf{P} d_2\mathbf{k}}\right|^{\text{g.}}_{\text{Eik+NEik}} = \alpha_e\alpha_s e_f^2\left(2z[(1-z)^2+1]\right)\left[\frac{1}{\mathbf{P}^2}+\frac{2z(\mathbf{P}\cdot\mathbf{k})}{\mathbf{P}^4}\right]\left(\mathrm{x}G^{(2)}(\mathrm{x},\mathbf{k})+\mathrm{x}_{\text{tar.}}\frac{\partial}{\partial \mathrm{x}}\left[\mathrm{x}G^{(2)}(\mathrm{x},\mathbf{k})\right]\right)$$

$$\left.\frac{d_5\sigma_{q\to q\gamma}}{dz d_2\mathbf{P} d_2\mathbf{k}}\right|^{\text{g.}}_{\text{Eik+NEik}} = \alpha_e\alpha_s Q_f^2\left(2z[(1-z)^2+1]\right)\left[\frac{1}{\mathbf{P}^2}+\frac{2z(\mathbf{P}\cdot\mathbf{k})}{\mathbf{P}^4}\right]\left[\mathrm{x}G^{(2)}(\mathrm{x},\mathbf{k})\right]_{\mathrm{x}_{\text{tar.}}}$$

What CGC contributions are omitted for the TMD matching …

Collinear PDF like contribution

Beyond the static limit : $$\frac{d_6\sigma_{q\to q\gamma}}{d_6\text{P.S}}\Big|_{\text{gen. Eik}} \propto \alpha_e\alpha_s H_{\text{Eik}}(\mathbf{P},\mathbf{k})$$
$$\times\Big\{2\pi\delta(k^+)\int_{\mathbf{v},\mathbf{w}} e^{-i(\mathbf{v}-\mathbf{w})\cdot\mathbf{k}}\sum_{\text{col.}}\Big\langle\mathcal{U}_F(\mathbf{v},b^-)\mathcal{U}_F^\dagger(\mathbf{w},b^-)\Big\rangle_A + i\pi\delta'(k^+)\int_{\mathbf{v},\mathbf{w}} e^{-i(\mathbf{v}-\mathbf{w})\cdot\mathbf{k}}\sum_{\text{col.}}\Big\langle\mathcal{U}_F(\mathbf{v},b^-)\overleftrightarrow{\partial_{b^-}}\mathcal{U}_F^\dagger(\mathbf{w},b^-)\Big\rangle_A\Big\}_{b^-=0}.$$

$$\frac{d_6\sigma_{q\to q\gamma}}{d_6\text{P.S}}\Big|_{\text{dyn. targ.}} \propto \alpha_e\alpha_s\,\pi\delta'(k^+)\left(H_{\text{Eik}}(\mathbf{P},\mathbf{k})\frac{\mathbf{k}^i}{\mathbf{k}^2}\right)2\text{Im}:\int_{\mathbf{v},\mathbf{w},v^+,z^+} e^{-i(\mathbf{v}-\mathbf{w})\cdot\mathbf{k}}$$
$$\times\sum_{\text{col.}}\Big\{i\Big\langle\mathcal{P}_+\Big[\mathcal{U}_F(\frac{L^+}{2},z^+,\mathbf{v})F^{+-}(z^+,\mathbf{v})\mathcal{U}_F(z^+,v^+)F_i^{\ -}(v^+,\mathbf{v})\mathcal{U}_F(v^+,-\frac{L^+}{2},\mathbf{v})\Big]\mathcal{U}_F^\dagger(\mathbf{w})\Big\rangle_A - \frac{n_f}{g}\cdot\Big\langle\mathcal{U}_F(\frac{L^+}{2}v^+,\mathbf{v})J^i(v^+,\mathbf{v})\mathcal{U}_F(v^+,-\frac{L^+}{2},\mathbf{v})\mathcal{U}_F^\dagger(\mathbf{w})\Big\rangle_A\Big\}.$$

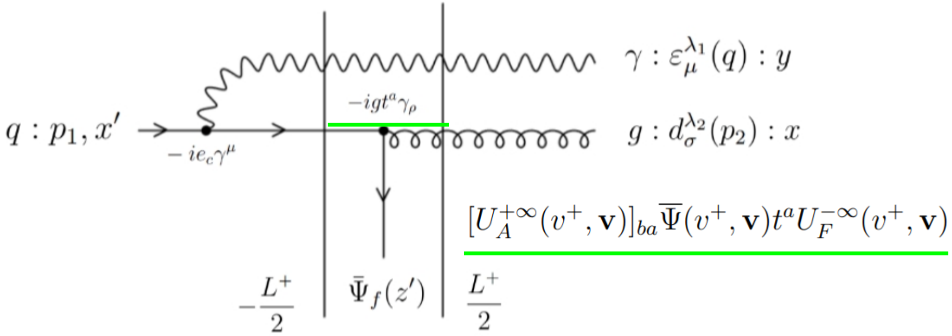
$$\Big(\ \partial^+F_{+j}+\partial^iF_{ij}+\partial^-F_{-j}=n_f\cdot\overline{\Psi}\gamma^j\Psi-gf^{abc}(A_b^iF_c^{ij}+A_b^-F_c^{+j})\ \Longrightarrow\ \partial^+F_{+j}=n_f\cdot\overline{\Psi}\gamma^j\Psi+O(\mathbf{k}^2).\Big)$$

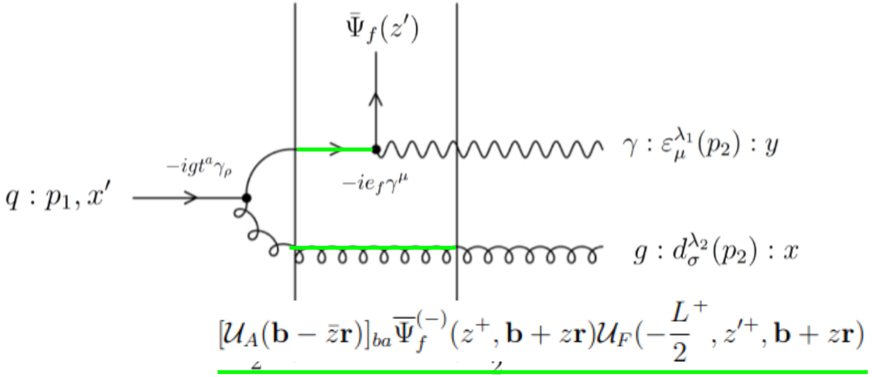
The second decoration : $$\frac{d_6\sigma_{q\to q\gamma}}{d_6\text{P.S}}\Big|^{\text{dec. 2 on q}}_{\text{NEik}} \propto \alpha_e\alpha_s\mathcal{H}_{\text{dec. 2}}(\mathbf{P},\mathbf{k})/W^2$$
$$\times(-2)\text{ReTr}\langle P_{\text{tar.}}|\widetilde{\mathcal{U}}_F^\dagger(\mathbf{k}):\int_{x^+\geq 0}(2x^+)F_j^{\ -}(0^+,\mathbf{0})\mathcal{U}_F(0^+,x^+,\mathbf{0})F_j^{\ -}(x^+,\mathbf{0})\mathcal{U}_F(x^+,-\frac{L^+}{2},\mathbf{0})|P_{\text{tar.}}\rangle.$$

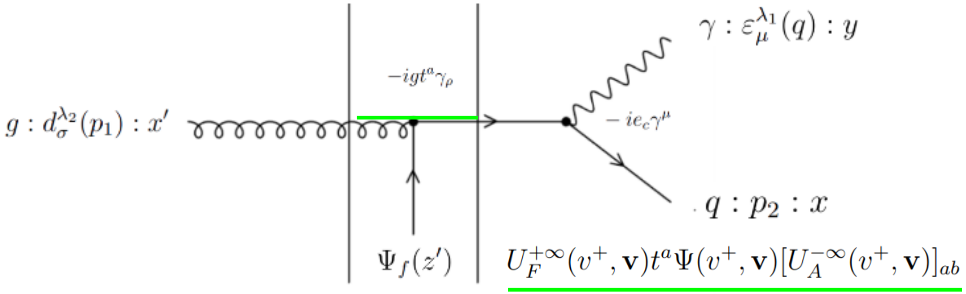
Next-to-Eik quark background *“Specjalność warszawska”*

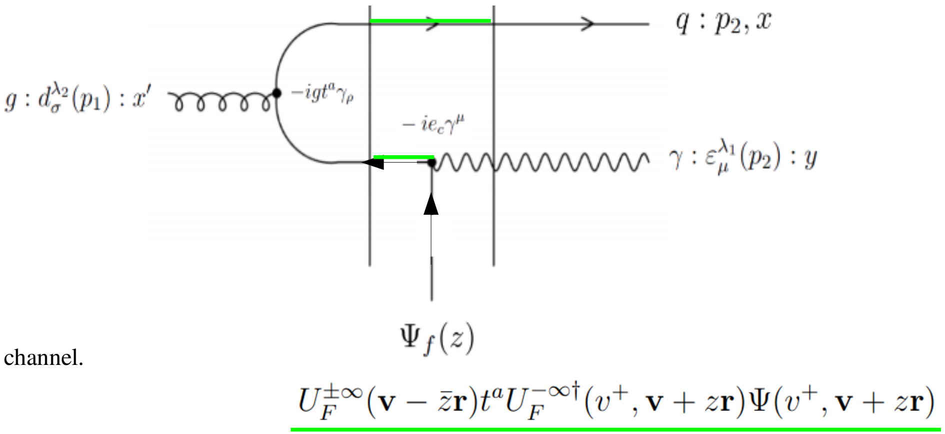
[T. Altinoluk, G. Beuf, N. Armesto, (2023), Phys. Rev. D 108, 074023]

Local | **Bilocal**









+ produced anti-quark channel.

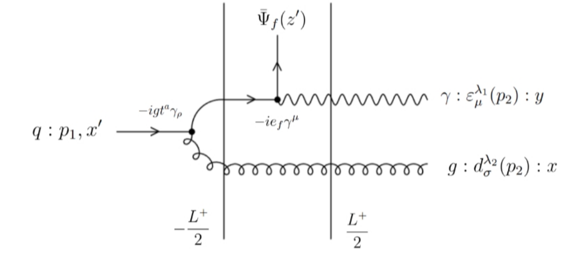
- **Bilocal :** has to be expanded in the back-to-back limit to be factorized in TMD ;
- **Local :** can be factorized in TMD in general kinematics ;
- Both types of amplitudes for each channels has to be treated on the same footage.

Typical small dipole expansion :

$$f(\mathbf{v}-z\mathbf{r})g(\mathbf{v}+\bar{z}\mathbf{r}) = f(\mathbf{v})g(\mathbf{v}) + f(\mathbf{v})(\bar{z}\mathbf{r}\cdot\partial_{\mathbf{v}} g) + (-z\mathbf{r}\cdot\partial_{\mathbf{v}} f)\, g(\mathbf{v}) + O(\mathbf{r}^2\cdot\partial_{\mathbf{v}}^2)$$

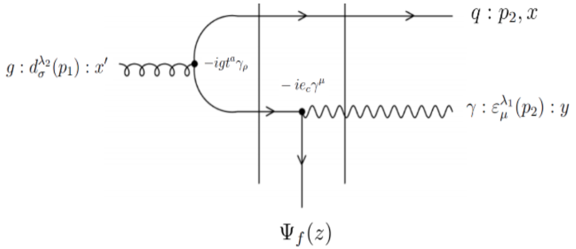
$$= (1 - z\mathbf{r}\cdot\partial_{\mathbf{v}}) f(\mathbf{v})g(\mathbf{v}) + O(\mathbf{r}^2\cdot\partial^2_{\mathbf{v}}) + O(f\partial_{\mathbf{v}} g)$$

← *"genuine twist correction"* to operators



$$\mathcal{O}_{g\gamma\leftarrow q}[f] = \int_{\mathbf{b},\mathbf{r}} e^{-i\mathbf{b}\cdot\mathbf{k}-i\mathbf{r}\cdot\mathbf{P}} f(\mathbf{r})[\mathcal{U}_A(\mathbf{b}-\bar{z}\mathbf{r})]_{ba}\overline{\Psi}_f^{(-)}(z^+,\mathbf{b}+z\mathbf{r})\mathcal{U}_F(-\frac{L^+}{2},z'^+,\mathbf{b}+z\mathbf{r})$$

$$= \int_{\mathbf{b},\mathbf{r}} e^{-i\mathbf{b}\cdot\mathbf{k}-i\mathbf{r}\cdot\mathbf{P}} f(\mathbf{r})\left\{((1+iz\mathbf{k}\cdot\mathbf{r})[\mathcal{U}_A(\mathbf{b})]_{ba} - \mathbf{r}\cdot\partial_{\mathbf{b}}\{[\mathcal{U}_A(\mathbf{b})]_{ba}\})\,\overline{\Psi}_f^{(-)}(z'^+,\mathbf{b})\mathcal{U}_F(\frac{L^+}{2},z'^+,\mathbf{b})\right\} + O(\mathbf{r}\cdot\partial_{\mathbf{b}})^2$$



$$\mathcal{O}_{q\gamma\leftarrow g}[f] = \int_{\mathbf{b},\mathbf{r}} e^{-i\mathbf{b}\cdot\mathbf{k}-i\mathbf{r}\cdot\mathbf{P}} f(\mathbf{r})\mathcal{U}_F(\mathbf{b}-z\mathbf{r})t^a\mathcal{U}_F^\dagger(-\frac{L^+}{2},z'^+,\mathbf{b}+z\mathbf{r})\Psi_f^{(-)}(z'^+,\mathbf{b}+z\mathbf{r})$$

$$= \int_{\mathbf{b},\mathbf{r}} e^{-i\mathbf{b}\cdot\mathbf{k}-i\mathbf{r}\cdot\mathbf{P}} f(\mathbf{r})\left\{((1+i\mathbf{k}\cdot\mathbf{r})\mathcal{U}_F(\mathbf{b}) - \mathbf{r}\cdot\partial_{\mathbf{b}}\{\mathcal{U}_F(\mathbf{b})\})\,t^a\mathcal{U}_F^\dagger(-\frac{L^+}{2},z'^+,\mathbf{b})\Psi_f^{(-)}(z'^+,\mathbf{b})\right\} + O(\mathbf{r}\cdot\partial_{\mathbf{b}})^2$$

In the Chiral limit : quark and anti-quark share the same hard factor.

Using the Fritz identity : $(t^a)_{i_1 j_1}(t^a)_{i_1 j_2} = \frac{1}{2N_c}\delta_{i_1 j_1}\delta_{i_2 j_2} - \delta_{i_1 j_1}\delta_{i_2 j_2}$

All the Color strutures about the quark background simplify greatly to TMDs :

$$q \to g\gamma : \quad F^{\gamma}_{qg}(\mathrm{x}, \mathbf{k}) = N_c f_q^{(2,+)}(\mathrm{x}, \mathbf{k}) - \frac{1}{N_c} f_q^{(1,-)}(\mathrm{x}, \mathbf{k})$$

$$g \to q/\bar{q}\gamma : \ F^{\gamma}_{gq}(\mathrm{x}, \mathbf{k}) = N_c f_q^{(2,-)}(x, \mathbf{k}) - \frac{1}{N_c} f_q^{(1,+)}(x, \mathbf{k}) + (q \leftrightarrow \bar{q}).$$

Theoreticaly “distinghuisable” from the colored paths

$$f_q^{(1,\pm)}(\mathrm{x}, \mathbf{k}) := \int \frac{dv^+ d_2\mathbf{v}}{(2\pi)^3} e^{-i\mathrm{x}P^-_{\mathrm{tar.}}v^+ - i\mathbf{k}\cdot\mathbf{v}} \langle P_{\mathrm{tar.}}|\overline{\Psi}(v^+, \mathbf{v})\frac{\gamma^-}{2}\mathrm{U}^{[\pm]}\Psi(0^+, \mathbf{0})|P_{\mathrm{tar.}}\rangle$$

$$f_q^{(2,\pm)}(\mathrm{x}, \mathbf{k}) := \frac{1}{N_c}\int \frac{dv^+ d_2\mathbf{v}}{(2\pi)^3} e^{-i\mathrm{x}P^-_{\mathrm{tar.}}v^+ - i\mathbf{k}\cdot\mathbf{v}} \langle P_{\mathrm{tar.}}|\overline{\Psi}(v^+, \mathbf{v})\frac{\gamma^-}{2}\left[\mathrm{Tr}\{\mathrm{U}^{[\square]}\}\mathrm{U}^{[\pm]}\right]\Psi(0^+, \mathbf{0})|P_{\mathrm{tar.}}\rangle$$

Next-to-Eikonal + kinematic twist-3 accuracy

$$\left.\frac{d_5\sigma_{q\to g\gamma}}{dz d_2\mathbf{P} d_2\mathbf{k}}\right|_{\mathrm{NEik}} = e_f^2 \frac{\alpha_e \alpha_s}{W^2}\left[\frac{1 - z\bar{z}}{z\bar{z}\mathbf{P}^2} + \frac{2(\mathbf{P}\cdot\mathbf{k})}{\mathbf{P}^4}\right] F^{\gamma}_{qg}(\mathrm{x}, \mathbf{k})$$

$$\left.\frac{d_5\sigma_{g\to q/\bar{q}\gamma}}{dz d_2\mathbf{k} d_2\mathbf{P}}\right|_{\mathrm{NEik}} = e_f^2 \frac{\alpha_e \alpha_s}{W^2}\left[\frac{[2\bar{z}^2 - \bar{z}z + 1]}{z\mathbf{P}^2} - \frac{2\bar{z}[\bar{z} - z](\mathbf{P}\cdot\mathbf{k})}{\mathbf{P}^4}\right]\left[F^{\gamma}_{gq}(\mathrm{x}, \mathbf{k}) + (q \leftrightarrow \bar{q})\right]$$

Conclusion :

- The gluon distribution contributes at NEik with a value of x fixed by the theory ; $x = \frac{P^2}{z\bar{z}W^2}$
- Kinematics of the process within a pure gluon background factorizes in general kinematics ;
- The back-to-back expansion is needed to factorise the quark background as TMD.

Subeikonal perspectives for photo-production :

- Semi-inclusive Cross section of a single Jet
  - soft and collinear divergencies in the photon phase space ;

- NLO corrections and evolutions.

END.