Photon-Jet production in pA Collision in the Color Glass Condensate

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Under the supervision of **Tolga Altinoluk**,
the co-supervision of **Guillaume Beuf**,

with the help of the CGC-team **Alina Czajka**, **Swaleha Nisar Mulani**,
Etienne Blanco, **Kacper Gosławski** and **Michael Fucilla**.

Outline :

1 – Introduction :

High energy scattering events / Factorization in QCD ;

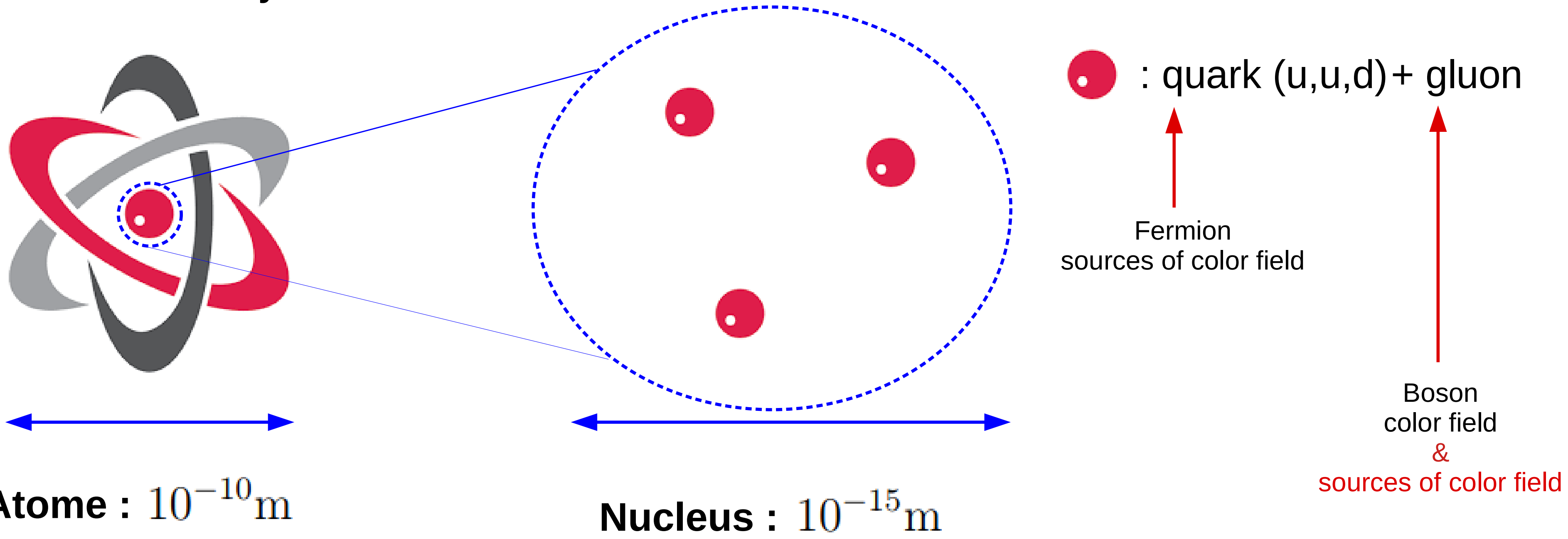
2 – Subeikonal approach to Saturation :

Saturation physics / Eikonal & Next-to-eikonal approximation ;

3 – The Photon-Jet Production

Neik effects of the gluons probe / LO + Neik + twist-3 differential cross section

Quantum ChromoDynamics :



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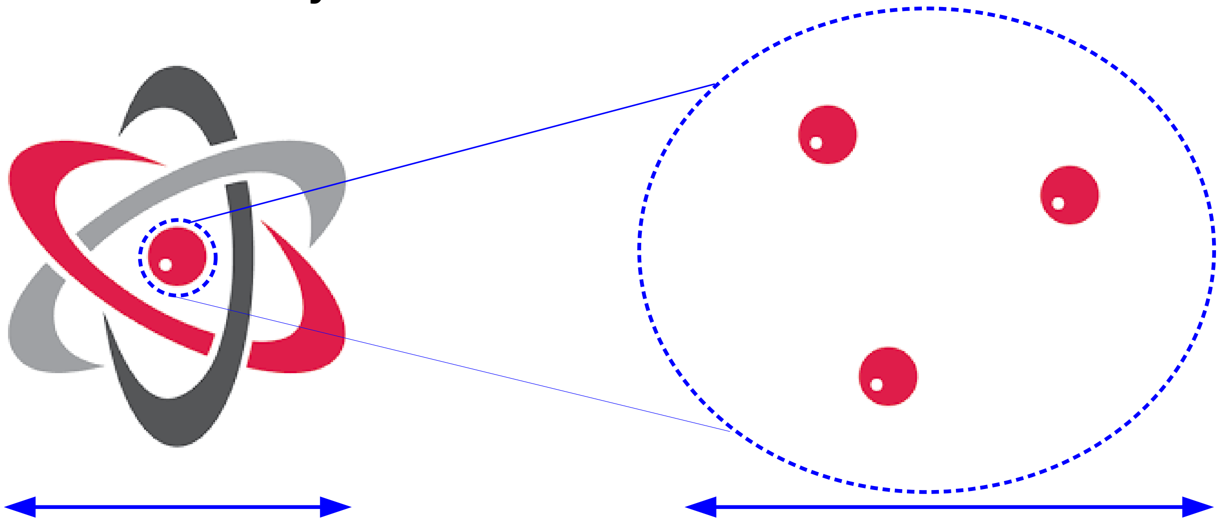


: quark (u,u,d) + gluon

Fermion
sources of color field

Boson
color field
&
sources of color field

Atome : $10^{-10}\mathrm{m}$

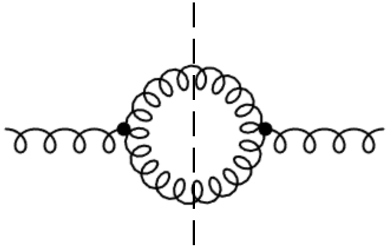
Nucleus : $10^{-15}\mathrm{m}$

The formal SU(3) symetry seems to describe relation among nucleus fundamental components

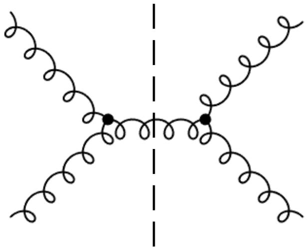
(at least at high energy - Asymptotic Freedom)

Nobel Price 2004

[D. Gross, F. Wilczek, (1973), Phys. Rev. Lett. 30, 1343]
[H.D D.Politzer, (1973), Phys. Rev. Lett. 30, 1346]

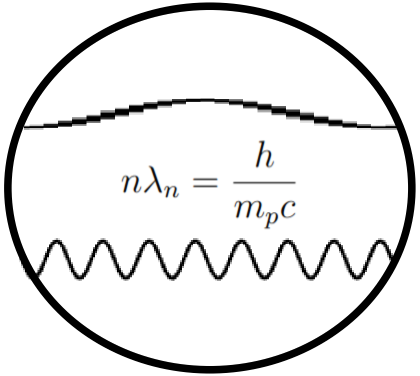


g → gg



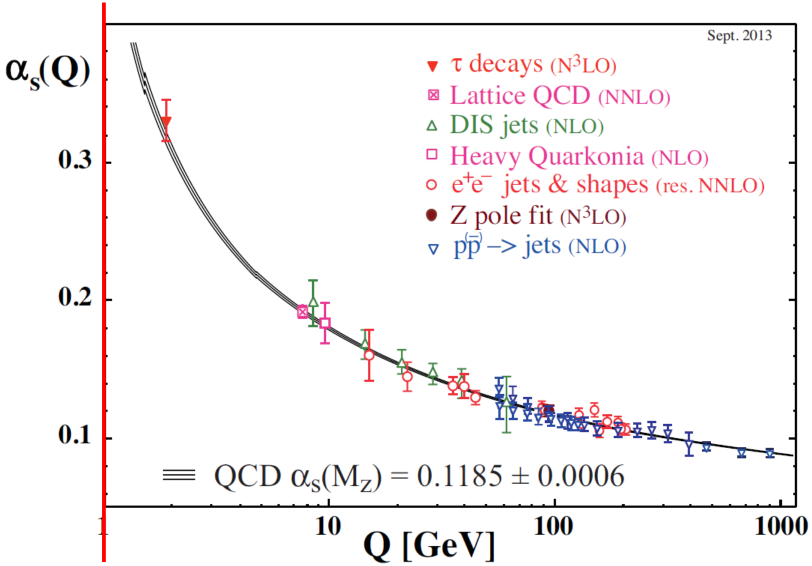
gg → g

Why high energy scattering in QCD ?



$$E_n = h\nu_n \ , \ \nu_n/c = \frac{m_p c}{h} \cdot n$$

$$n \in [|1; +\infty|[$$



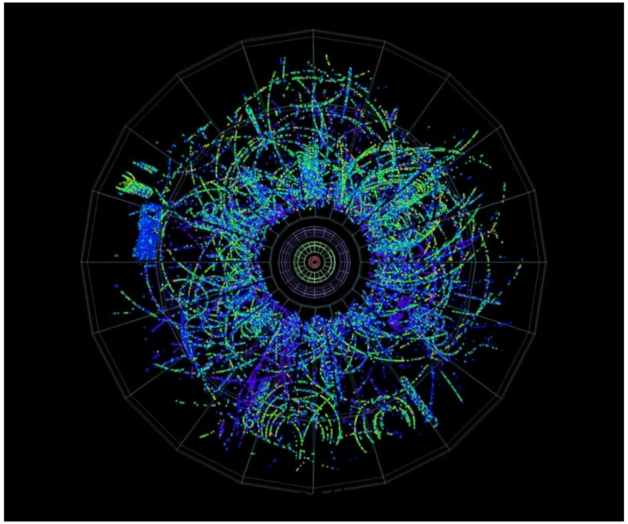
$$m_p c^2 \sim 1\mathrm{GeV}$$

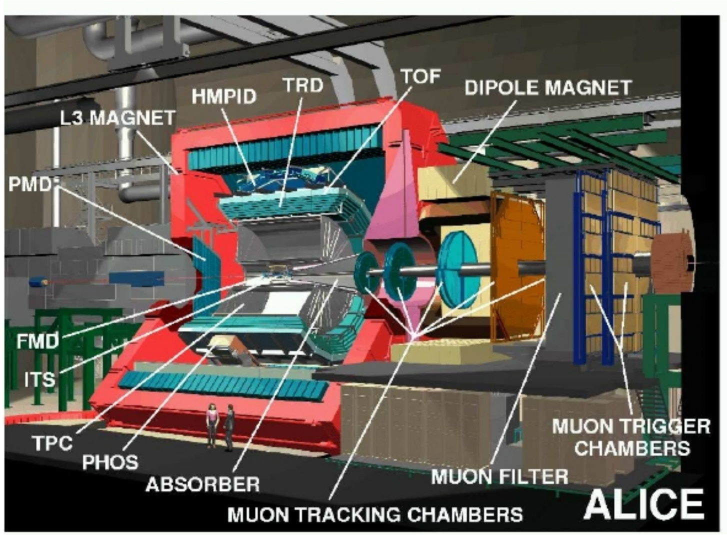
- No theoretical clue for pQCD below 1GeV ;
- Parton probes need high energy to be resolved ;
- Bound states are sources of IR safe oscillations.

Theoretical expectation for the strong coupling at one-loop :

$$\alpha_s^{lo}(Q^2) = \frac{1}{\beta_0 \log \frac{Q^2}{\Lambda^2}} , \quad Q^2 = -q^2$$

ALICE detector at the Large Hadron Collider





~ 3.5 TeV/proton in p-pb collisions

circonference : 26 659 m

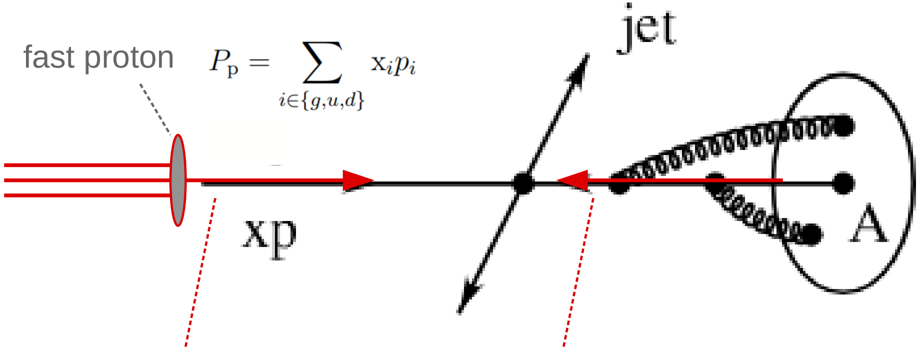
Can go up to 14 TeV in the C.O.M (p-p collisions)

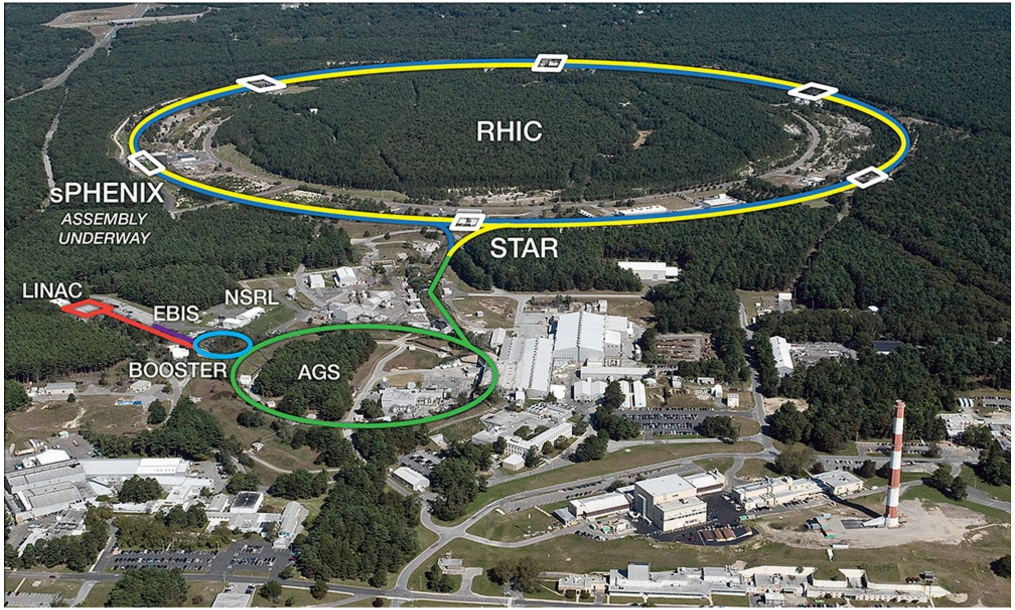
the Relativistic Heavy Ion Collider (RHIC)

circonference : 3834 m

delivers ~ 10 GeV/nucleon

fast proton $P_p = \sum_{i \in \{g,u,d\}} x_i p_i$





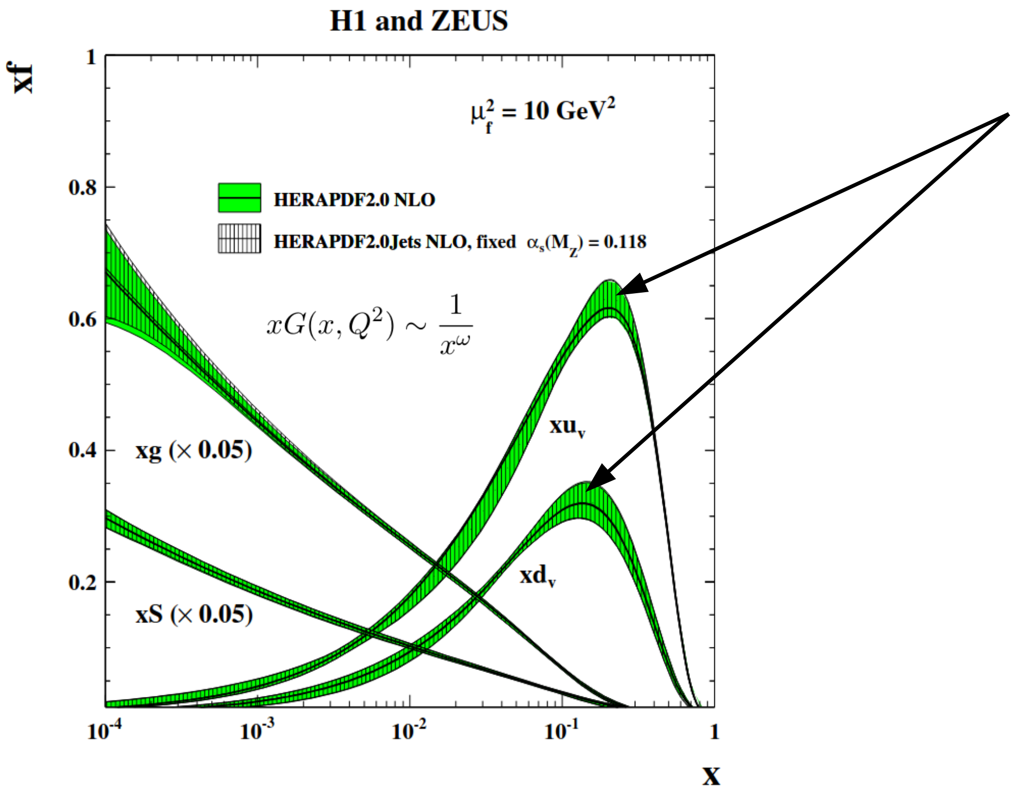
U.S. Department of Energy's Brookhaven National Laboratory

fast praton, taking a fraction x of the proton momenta.

Heavy and complex nuclei, represented by a color charge density **ρ(x)**.

Collinear PDF

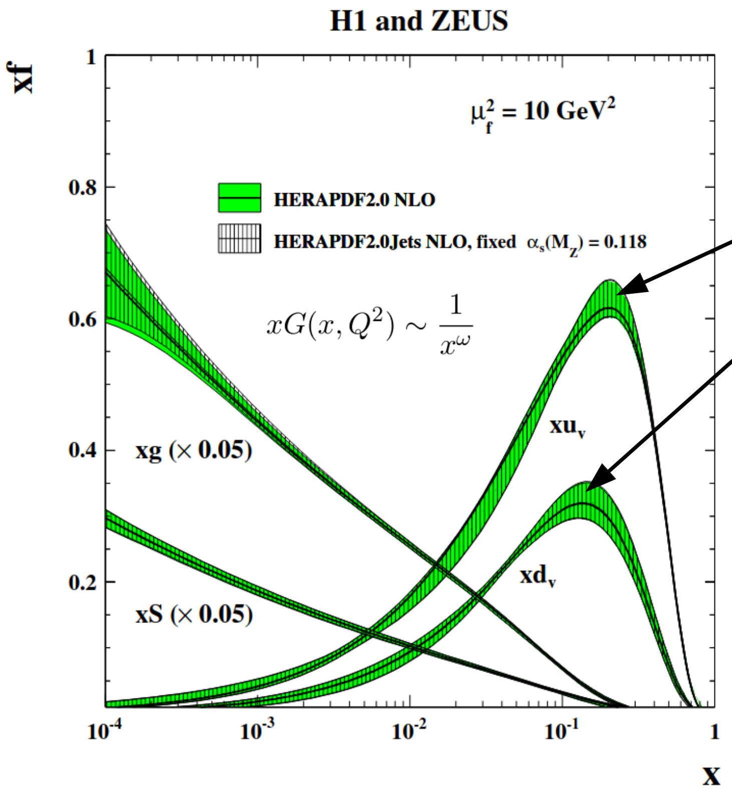
Defined in Bjorken limit : $\lim_{\substack{Q\to\infty \\ W\to\infty}} Q^2/W^2 = x$ (cst)



- Convenient to study valence quark ;
- Leave strong divergencies in xS and xg.

H1 and ZEUS Collaborations (2015)
DESY-15-039

Collinear PDF



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GLR equation (first hint of saturation from pdf factorization scheme)

$$\frac{\partial^2[\mathrm{x}g(\mathrm{x},Q^2)]}{\partial\log(1/\mathrm{x})\partial\log(Q^2)} = \frac{\alpha_s}{\pi}N_c[\mathrm{x}g(\mathrm{x},Q^2)] - \frac{\alpha_s^2}{Q^2\pi R_A^2}\tilde{\gamma}[\mathrm{x}g(\mathrm{x},Q^2)]^2$$

[L. V. Gribov, E. M. Levin and M. G. Ryskin: Phys.Rep. 100 (1983) 1]

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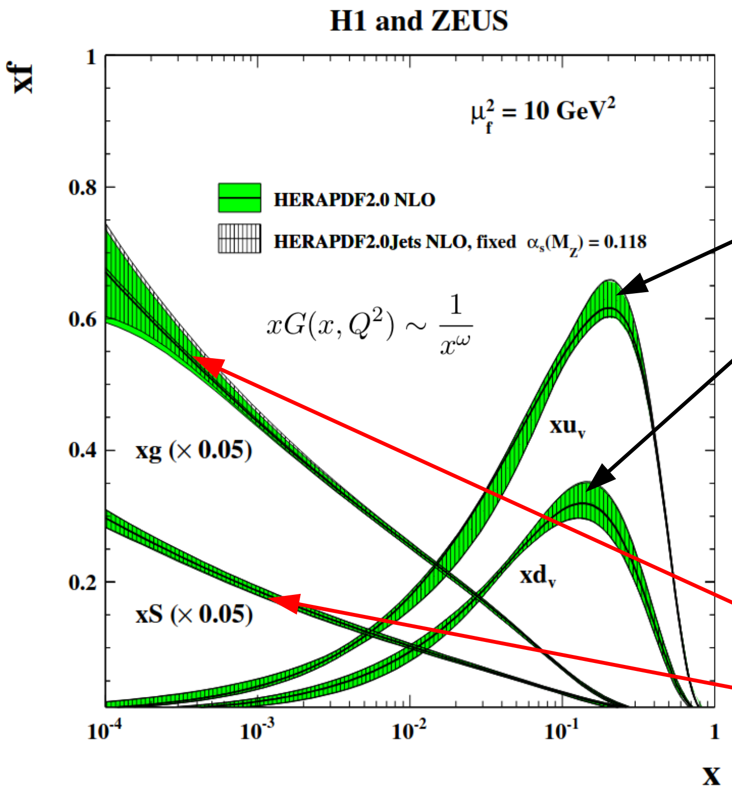

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Small-x formalism is defined from the Regge-Gribov limit :

$$\lim_{\substack{\mathrm{x}\to 0 \\ W\to\infty}} \mathrm{x}W^2 = Q^2 \quad \text{(cst)}$$

- To study sea-quark and gluons ;
- Staturation physics.

TMD in back-to-back kinematics

$$\tilde{W}^{\mu\nu}_{\text{low}} = \frac{6(2\pi)^2}{w^2 x_A^2 x_B^2} \text{Tr}\{\not{P}_A \gamma^\mu \not{P}_B \gamma^\nu\}$$
$$\times |H(w, w; g(C_2 w), C_2 w)|^2 \tilde{U}(b; g(C_1/b), C_1/b) \exp\{-S(w, b)\}$$
$$\times \sum_k e_j^2 \, [d_{A/j} + \text{corrections}][d_{B/\bar{j}} + \text{corrections}]$$
$$\times [1 + O(\text{mass}/w, 1/wb, \text{mass} \times b)] , \qquad (9.1)$$

Old fashion (pioneer),
Jets physics for
electron-positron colliders.

[J. C Collins, D. E. Soper, Nuclear Physics B193 (1981) 381-443] (see p. 439)

$$\left.\frac{d\sigma_{\gamma^*_{T,L}\to q_1\bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\right|_{\text{Eik+NEik}} = \alpha_{\text{em}} e_f^2\, \alpha_s \left\{ \mathcal{C}^{f_1^g}_{T,L}(z_1,\mathbf{P},\mathbf{k})\, \text{x}\, f_1^g(\text{x},\mathbf{k}) + \mathcal{C}^{h_1^{\perp g}}_{T,L}(z_1,\mathbf{P},\mathbf{k})\, \text{x}\, h_1^{\perp g}(\text{x},\mathbf{k}) \right.$$
$$\left.\left. + \frac{\mathbf{k}\cdot\mathbf{P}}{W^2}\, \mathcal{C}^{f^{\perp g}}_{T,L}(z_1,\mathbf{P})\, \text{x}\, f^{\perp g}(\text{x},\mathbf{k}) + \frac{\mathbf{k}\cdot\mathbf{P}}{W^2}\, \mathcal{C}^{\bar{g}^{\perp g}}_{T,L}(z_1,\mathbf{P})\, \text{x}\, \bar{g}^{\perp g}(\text{x},\mathbf{k}) \right\}\right|_{\text{x}=\frac{[\mathbf{P}^2+Q^2]}{z_1 z_2 W^2}}$$

Modern fashion,
Saturation physics for
EIC and related observables.

[T. Altinoluk, G. Beuf, A. Czajka, C. Marquet, (2024) http://arxiv.org/abs/2410.00612v1]

TMD in back-to-back kinematics

Hard factor

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Soft function

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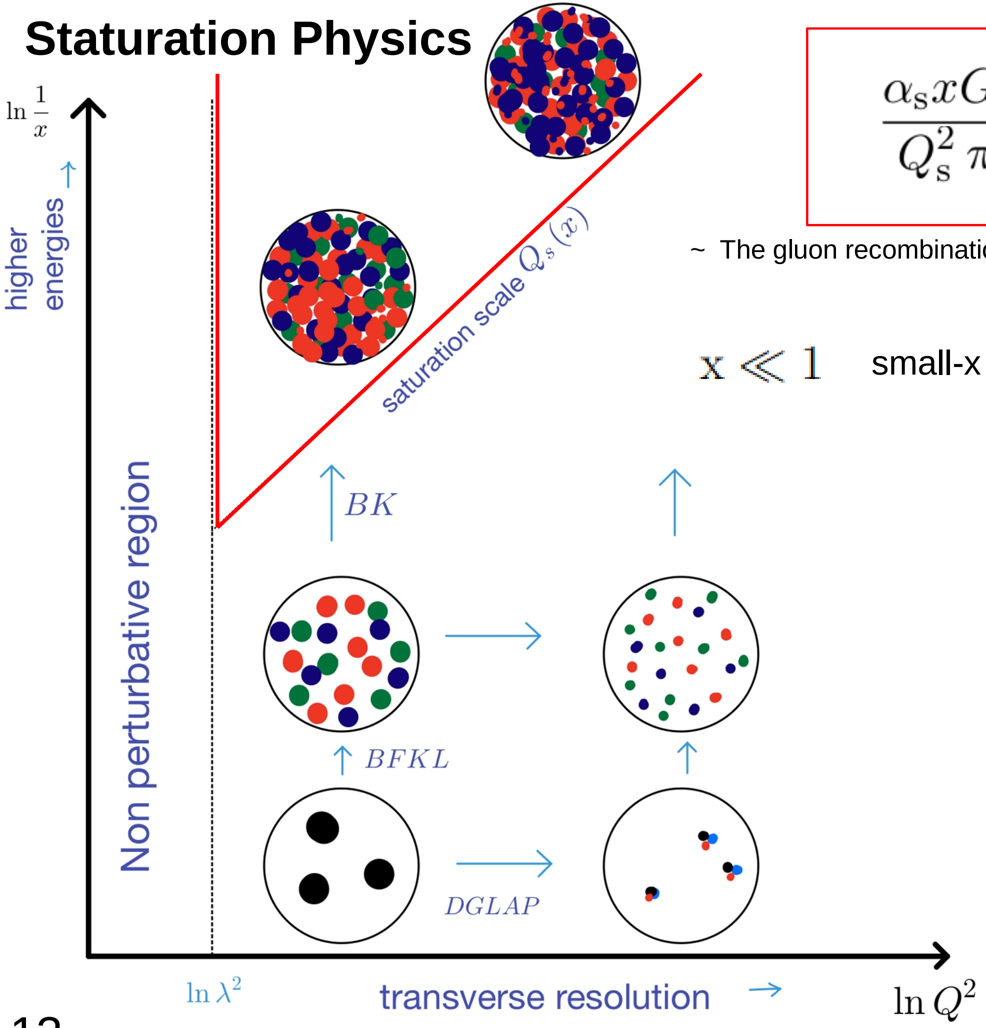
Staturation Physics

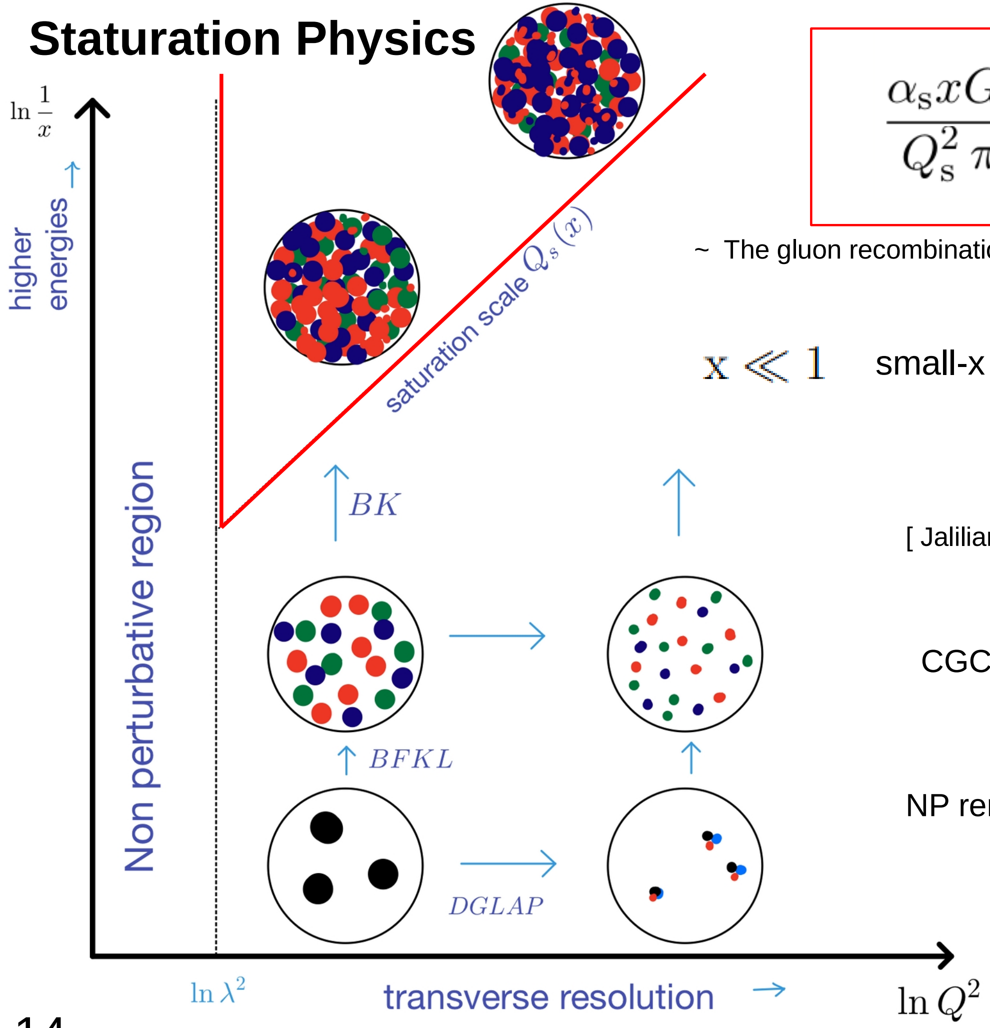


$$\frac{\alpha_s x G(x, Q_s^2)}{Q_s^2 \, \pi R_A^2} < 1$$

~ The gluon recombinations cross-section **x** Gluon's surface density

$x \ll 1$ small-x enhancement of the staturation at high energy $\alpha_s(Q_s^2(x)) \ll 1$

Staturation Physics



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Non-linear evolution equation :

[Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner, 1997-2002]

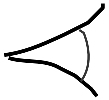
CGC observable : $\langle \mathcal{O} \rangle := \int [D\rho] W[\rho] \mathcal{O}[\rho]$

NP renormalisation : $\frac{\partial}{\partial \log x} W_x[\rho] = \mathcal{H}_{\text{JIMWLK}} \left[\rho, \frac{\delta}{\delta \rho} \right] W_x[\rho]$

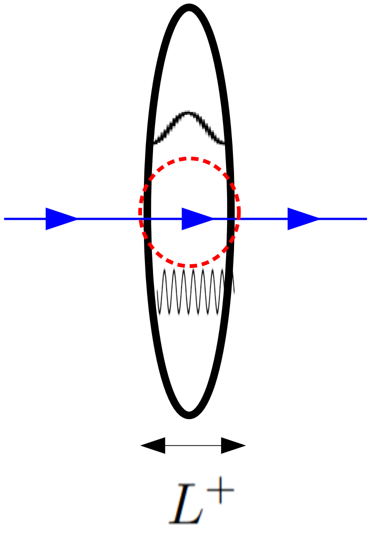
Boosted Color fields from the target :

$$A^\mu(x^\mu) \to (L^+A^+, A^-/L^+, A^\perp)(x^+/L^+, L^+x^-, x_\perp)$$

$$A^- = O(1/L^+) \gg A_j = O(1) \gg A^+ = O(L^+).$$



$$n\lambda_n \to \lambda_C\sqrt{1 - v_p^2/c^2}$$



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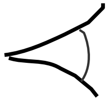

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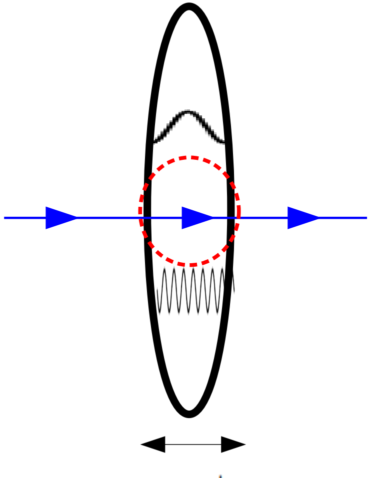


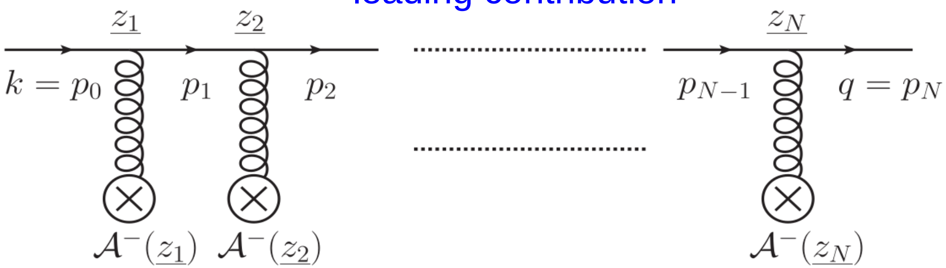
$$A^- = O(1/L^+) \gg A_j = O(1) \gg A^+ = O(L^+).$$

Eikonal Approximation $\quad \lim : L^+ = 0$

- High energy limit ;
- Infinitely flat target ;
- Staturated regime $\quad gA^- \sim 1$; (i.e high probability)
- A^- dominates the scattering inside the medium.

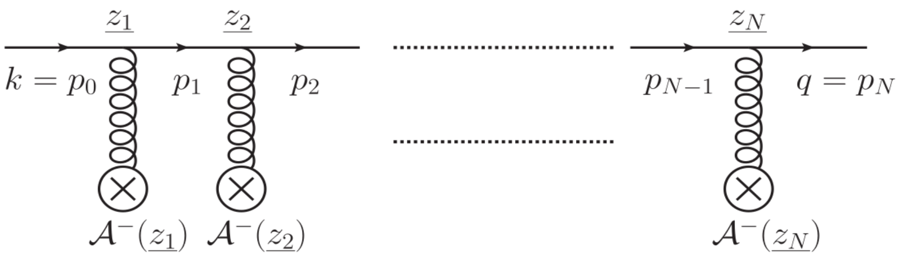
Condensated along x^+ and $x_\perp$
while Glassed (static) on x^-

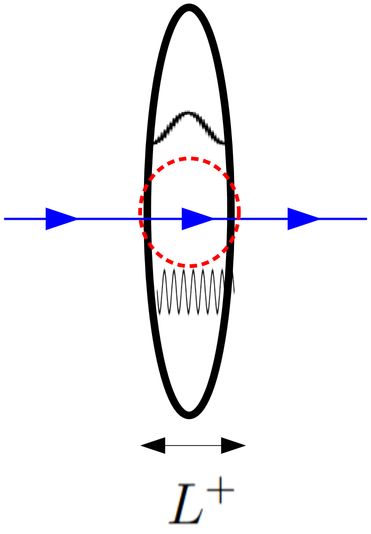
leading contribution



Random color average

Medium induced *"Wilson's lines"*





$$\mathcal{U}_F(x^+, y^+; \mathbf{z}) = \mathbf{1} + \sum_{N=1}^{+\infty} \int \left[\prod_{n=1}^{N} dz_n^+\right] \left(\prod_{n=0}^{N} \theta(z_{n+1}^+ - z_n^+)\right) \left\{\mathcal{P}_n \prod_{n=1}^{N} [-igt \cdot \mathcal{A}^-(z_n^+, \mathbf{z})]\right\}$$
$$= \mathbf{1} + \sum_{N=1}^{+\infty} \frac{1}{N!} \mathcal{P}_+ \left[-ig \int_{y^+}^{x^+} dz^+ t \cdot \mathcal{A}^-(z^+, \mathbf{z})\right]^N$$

- Fundamental object in QCD : rotation in the color space ;
- Model the multiple interaction within the target.

Subeikonal finit width effects

[T. Altinoluk, G. Beuf, A. Czajka and A. Tymowska 10.1103/PhysRevD.104.014019]
[T. Altinoluk, G. Beuf, and S. Mulani, Phys.Rev.D 111 (2025) 3, 034028]

- Target has a finite width ; $\lim : L^+ \neq 0$
- Include transverse components ;

$$\mathcal{U}^{(1)}_{F,+j}(\mathbf{v}) = ig \int_{-L^+/2}^{L^+/2} dv^+ (2v^+) U_F^{+\infty}(v^+,\mathbf{v}) F_j^{\;-}(v^+,\mathbf{v}) U_F^{-\infty}(v^+,\mathbf{v})$$

$$\mathcal{U}^{(2)}_{F}(\mathbf{v}) = (ig)^2 \int_{-L^+/2}^{L^+/2} dv^+ \int_{-L^+/2}^{L^+/2} du^+ \theta(v^+ - u^+)(v^+ - u^+) U_F^{+\infty}(u^+\mathbf{v}) F_j^{\;-}(u^+,\mathbf{v}) U_F(u^+,v^+,\mathbf{v}) F_j^{\;-}(v^+,\mathbf{v}) U_F^{-\infty}(v^+,\mathbf{v})$$

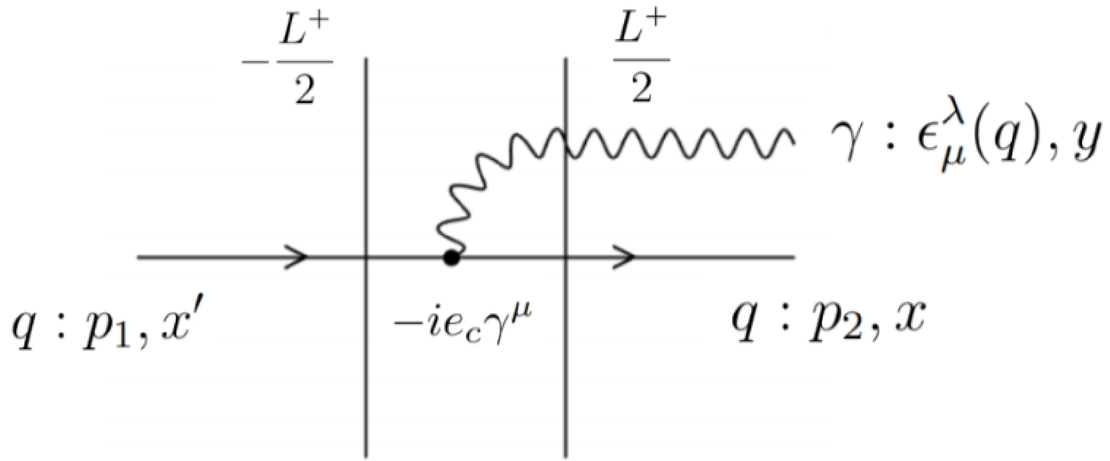
$$\mathcal{U}^{(3)}_{ij}(\mathbf{v}) = g \int_{L^+/2}^{L^+/2} dv^+ U_F^{+\infty}(v^+,\mathbf{v}) F_{ij}(v^+,\mathbf{v}) U_F^{-\infty}(v^+,\mathbf{v})$$

"3 decorations" induced by the transverse degrees of freedom of the target

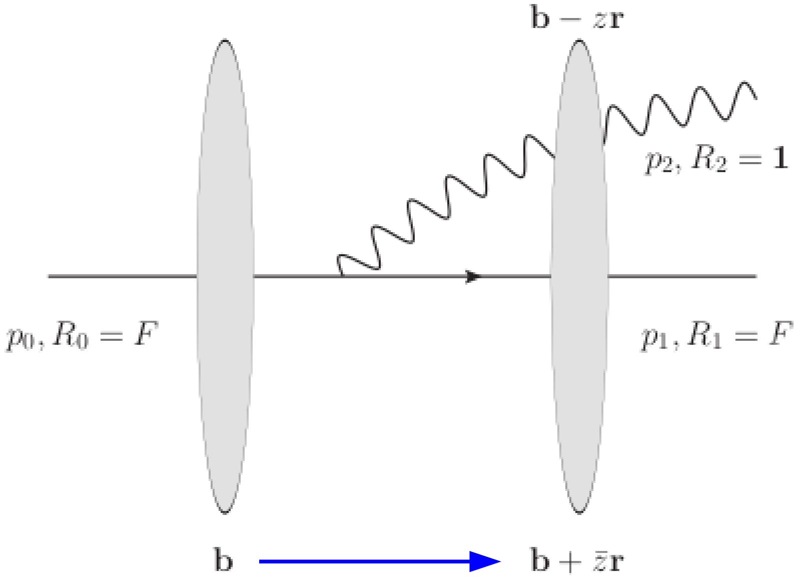
- dynamical effects are allowed. ;

$$U_F(\mathbf{x}) \to U_F(\mathbf{x}, x^-) = U_F(\mathbf{x}) + x^-(-ig) \int_{-L^+/2}^{L^+/2} dx^+ U_F^{+\infty}(\mathbf{x}, x^+) F^{+-}(\mathbf{x}, x^+) U_F^{-\infty}(\mathbf{x}, x^+) + O((x^-)^2)$$

Longitudinal Chromo-electric field


$-\frac{L^+}{2}$
$\frac{L^+}{2}$
$\gamma : \epsilon_\mu^\lambda(q), y$
$q : p_1, x'$
$-ie_c\gamma^\mu$
$q : p_2, x$

The Correlated basis



Transvers kick from the photo-emission

Relative momenta

$$\mathbf{P} := z\mathbf{p}_2 - \bar{z}\mathbf{q},$$

Momentum exchange with the target

$$\mathbf{k} := \mathbf{p}_2 + \mathbf{q} - \mathbf{p}_1;$$

Relative position

$$\mathbf{b} := \bar{z}\mathbf{x}_2 + z\mathbf{x}_1,$$

Dipole size

$$\mathbf{r} := \mathbf{x}_2 - \mathbf{x}_1.$$

Back-to-back limit = small dipole size expansion :

$$|\mathbf{r}| \ll |\mathbf{b}| \quad \Longleftrightarrow \quad \frac{2\pi}{\mathbf{b}} \leq \mathbf{k} \ll \frac{2\pi}{\mathbf{r}} \leq \mathbf{P}$$

From CGC to TMD :

Original expression in general kinematics

$$\frac{d_6\sigma_{pA\to\gamma+\text{jet}+A'}}{dz_2 d_2\mathbf{p}_2 dz_1 d_2\mathbf{p}_1} \propto \sum_{a,b} f_{a/p}(x_1) \sum_{i=1,2} \left\langle \text{TF}_{\mathbf{b}_{12},\mathbf{r}_{12}} \left[\mathcal{H}^{(i)}_{ag\to b\gamma}(\mathbf{r}_{12})\mathcal{O}^{(i)}_{ag\to b\gamma}(\mathbf{b}_{12},\mathbf{r}_{12})\right](\mathbf{P},\mathbf{k})\right\rangle_A$$

Deconvolution from the small dipole size expansion

$$\propto \sum_{a,b} f_{a/p}(x_1) \sum_{i=1,2} \left\langle \text{TF}_{\mathbf{b}_{12},\mathbf{r}_{12}} \left[\mathcal{H}^{(i)}_{ag\to b\gamma}(\mathbf{P},\mathbf{r}_{12})\left(\mathcal{O}^{(i)}_{ag\to b\gamma}(\mathbf{b}_{12},0) + O\left(\mathbf{r}_{12}^n \partial^n_{\mathbf{b}_{12}}\right)\right)\right]\right\rangle_A$$

Convert to higher twist corrections $|\mathbf{k}| \ll |\mathbf{P}| \ll W.$

$$\propto \sum_{a,b} f_{a/p}(x_1) \sum_{i=1,2} \left[\mathcal{H}^{(i)}_{ag\to b\gamma}(\mathbf{P},0) + O\left(\frac{\mathbf{k}^n}{\mathbf{P}^n}\right)\right] \left\langle \mathcal{O}^{(i)}_{ag\to b\gamma}(\mathbf{k})\right\rangle_A$$

Factorization of the hard factor and the NP color operator

pA Factorization for the photo-production

$$\frac{d_6\sigma_{pA\to\gamma+\text{jet}+A'}}{dz_2 d_2\mathbf{p}_2 dz_1 d_2\mathbf{p}_1} \propto \sum_{a,b} f_{a/p}(x_1) \sum_{i=1,2} \left[\mathcal{H}^{(i)}_{ag\to b\gamma}(\mathbf{P},\mathbf{k})\Phi_g^{(i)}(x_2,\mathbf{k}) + \mathcal{H}^{(i)}_{aq\to b\gamma}(\mathbf{P},\mathbf{k})(F_q^{(i)}(x_2,\mathbf{k})/W^2)\right]$$

Inspired form, [Kotko, Kutak, Marquet, Petreska, Sapeta, Van Hameren - arXiv:1503.03421]

Adapted to the photo-production and subeikonal apparition of quark TMD

$\Phi_g^{(i)}(x_2,\mathbf{k})$: linear combinations of unpolarized gluon TMDs ;

$F_q^{(i)}(x_2,\mathbf{k})$: linear combinations of quark TMDs ;

$\mathcal{H}^{(i)}_{ag\to b\gamma}(\mathbf{P},\mathbf{k})$: hard factor, kinetics effect of off-shell gauge invariant matrix elements.

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Projectile's side factorization (green box):

- Collinear factorization ;
- Parton Density Function ;
- Universal ;
- DGLAP evolution.

Target's side factorization (red box):

- Factorization in the back-to-back limit;
- Transverse Momentum Distribution ;
- Process dependent ;
- BK-JIMWLK evolution.

pA Factorization for the photo-production

Pure subeikonal contributions

$$\frac{d_6\sigma_{pA\to\gamma+\text{jet}+A'}}{dz_2 d_2\mathbf{p}_2 dz_1 d_2\mathbf{p}_1} \propto \sum_{a,b} f_{a/p}(x_1) \sum_{i=1,2} \left[\mathcal{H}^{(i)}_{ag\to b\gamma}(\mathbf{P},\mathbf{k})\Phi^{(i)}_g(x_2,\mathbf{k}) + \mathcal{H}^{(i)}_{aq\to b\gamma}(\mathbf{P},\mathbf{k})(F^{(i)}_q(x_2,\mathbf{k})/W^2)\right]$$

Inspired form, [Kotko, Kutak, Marquet, Petreska, Sapeta, Van Hameren - arXiv:1503.03421]

Adapted to the photo-production and subeikonal apparition of quark TMD

$\Phi^{(i)}_g(x_2,\mathbf{k})$: linear combinations of unpolarized gluon TMDs ;

$F^{(i)}_q(x_2,\mathbf{k})$: linear combinations of quark TMDs ;

$\mathcal{H}^{(i)}_{ag\to b\gamma}(\mathbf{P},\mathbf{k})$: hard factor, kinetics effect of off-shell gauge invariant matrix elements.

Projectile's side factorization :

- Collinear factorization ;
- Parton Density Function ;
- Universal ;
- DGLAP evolution.

Target's side factorization :

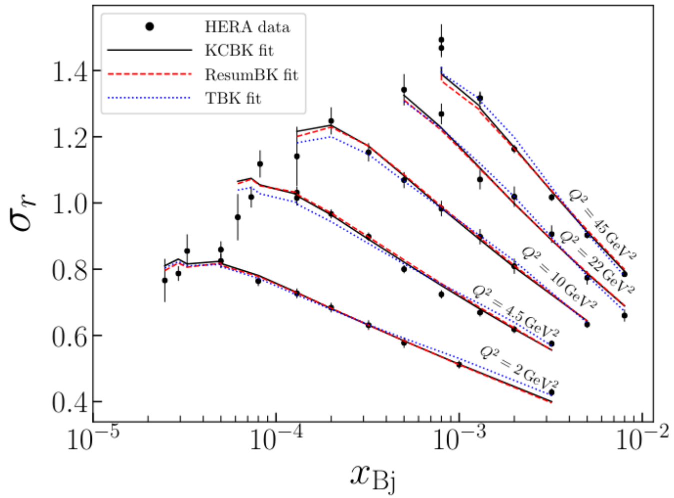
- Factorization in the back-to-back limit;
- Transverse Momentum Distribution ;
- Process dependent ;
- BK-JIMWLK evolution.

The gluon dipole

Elementary CGC operator

Eik+LO+NLO dipole cross section fit with Hera data

$$d(\mathbf{v},\mathbf{w}) = \frac{1}{N_c}\mathrm{Tr}\left\langle U_F^{\pm\infty}(\mathbf{v})U_F^{\pm\infty\dagger}(\mathbf{w})\right\rangle_A$$



small-x TMD-dipole relation :

$$\mathrm{x}G^{(2)}(\mathrm{x},\mathbf{k}) = \frac{N_c\mathbf{k}^2}{2\pi^2\alpha_s}S_\perp\int\frac{d_2\mathbf{r}}{(2\pi)^2N_c}e^{-i\mathbf{r}\cdot\mathbf{k}}\mathrm{Tr}\langle P'_{\mathrm{tar}}|U_F^{\pm\infty}(\mathbf{r})U_F^{\pm\infty\dagger}(\mathbf{0})|P_{\mathrm{tar}}\rangle$$

[G. Beuf, H. H¨anninen, T. Lappi, and H. M¨antysaari Phys.Rev.D 102 (2020) 074028]

From the dipole to the TMD

From Statistical to Quantum average

$$\langle \cdots \rangle_A \to \lim_{P'_{\rm tar}\to P_{\rm tar}} : \frac{\langle P'_{\rm tar}| \cdots |P'_{\rm tar}\rangle}{\langle P'_{\rm tar}|P'_{\rm tar}\rangle}$$

i.e Path integral averaging over color configuration to Gell-Mann and Low Interation pictur.

[Dominguez, Marquet, Xiao, Yuan - arXiv: 1101.0715]

From the dipole to the TMD

From Statistical to Quantum average

$$\langle \cdots \rangle_A \to \lim_{P'_{\rm tar} \to P_{\rm tar}} : \frac{\langle P'_{\rm tar} | \cdots | P'_{\rm tar} \rangle}{\langle P'_{\rm tar} | P'_{\rm tar} \rangle}$$

i.e Path integral averaging over color configuration to Gell-Mann and Low Interation pictur.

[Dominguez, Marquet, Xiao, Yuan - arXiv: 1101.0715]

$$i\mathbf{k}^j \int_{\mathbf{v}} U_F^{\pm\infty}(\mathbf{v}) e^{-i\mathbf{z}\cdot\mathbf{k}}. = -ig \int_{v^+,\mathbf{v}} U_F^{-\infty}(v^+, \mathbf{z}) F_j^{\ -}(v^+, \mathbf{v}) U_F^{+\infty}(v^+, \mathbf{v}).$$

$$\mathbf{k}^i \mathbf{k}^j \int_{\mathbf{v},\mathbf{w}} e^{-i(\mathbf{v}-\mathbf{w})\cdot\mathbf{k}} \mathrm{Tr} \left\langle U_F^{\pm\infty}(\mathbf{v}) U_F^{\pm\infty\dagger}(\mathbf{w}) \right\rangle_A$$

$$= 4\pi\, \alpha_s \int_{v^+,\mathbf{v},w^+,\mathbf{w}} e^{-i\mathbf{k}\cdot(\mathbf{v}-\mathbf{w})} \mathrm{Tr} \left\langle U_F^{+\infty}(v) F^{-i}(v) U_F^{-\infty}(v) U_F^{-\infty\dagger}(w) F^{-i}(w) U_F^{+\infty\dagger}(w) \right\rangle_A$$

$$\propto \alpha_s \frac{\mathbf{k}^i \mathbf{k}^j}{\mathbf{k}^2} [\mathrm{x}G(\mathrm{x}, \mathbf{k})]$$

From the dipole to the TMD

From Statistical to Quantum average

$$\langle \cdots \rangle_A \to \lim_{P'_{\rm tar} \to P_{\rm tar}} : \frac{\langle P'_{\rm tar} | \cdots | P'_{\rm tar} \rangle}{\langle P'_{\rm tar} | P'_{\rm tar} \rangle}$$

i.e Path integral averaging over color configuration to Gell-Mann and Low Interation pictur.

[Dominguez, Marquet, Xiao, Yuan - arXiv: 1101.0715]

$$i\mathbf{k}^j \int_{\mathbf{v}} U_F^{\pm\infty}(\mathbf{v}) e^{-i\mathbf{z}\cdot\mathbf{k}}. = -ig \int_{v^+,\mathbf{v}} U_F^{-\infty}(v^+,\mathbf{z}) F_j^{\;-}(v^+,\mathbf{v}) U_F^{+\infty}(v^+,\mathbf{v}).$$

$$\mathbf{k}^i\mathbf{k}^j \int_{\mathbf{v},\mathbf{w}} e^{-i(\mathbf{v}-\mathbf{w})\cdot\mathbf{k}} \mathrm{Tr}\left\langle U_F^{\pm\infty}(\mathbf{v}) U_F^{\pm\infty\dagger}(\mathbf{w})\right\rangle_A$$

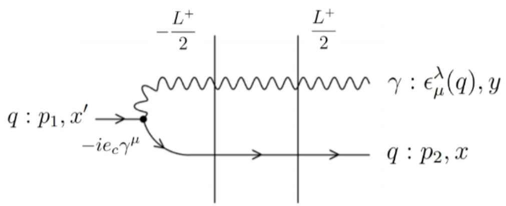
$$= 4\pi\,\alpha_s \int_{v^+,\mathbf{v},w^+,\mathbf{w}} e^{-i\mathbf{k}\cdot(\mathbf{v}-\mathbf{w})} \mathrm{Tr}\left\langle U_F^{+\infty}(v) F^{-i}(v) U_F^{-\infty}(v) U_F^{-\infty\dagger}(w) F^{-i}(w) U_F^{+\infty\dagger}(w)\right\rangle_A$$

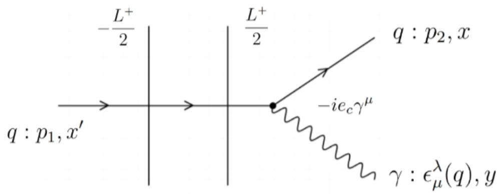
$$\propto \alpha_s \frac{\mathbf{k}^i\mathbf{k}^j}{\mathbf{k}^2}[\mathrm{x}G(\mathrm{x},\mathbf{k})]$$

dipole TMD

$$xG^{(2)}(x,\mathbf{k}) := 2\int \frac{dv^+ d_2\mathbf{v}}{(2\pi)^3 P^-_{\rm tar.}} e^{ixP^-_{\rm tar.}v^+ - i\mathbf{k}\cdot\mathbf{v}} \langle P'_{\rm tar.} | \mathrm{Tr}\{F^{-j}(v^+,\mathbf{v}) \mathrm{U}^{[-]} F^{-j}(0^+,\mathbf{0}) \mathrm{U}^{[+]\dagger}\} | P_{\rm tar.}\rangle$$

Dipole Factorization : Eikonal cross-section





Dipole Operator

$$\left.\frac{d_6\sigma_{\gamma p_2 \leftarrow pA}}{d_6\text{P.S}}\right|_{\text{Eik}} = 4\pi\alpha_e Q_f^2 2\pi\delta(k^+) H_{\text{Eik}}(\mathbf{p}_2, \mathbf{q}, \mathbf{k}) \int_{\mathbf{v},\mathbf{w}} e^{-i(\mathbf{v}-\mathbf{w})\cdot\mathbf{k}} \sum_{\text{col.}} \left\langle U_F^{\pm\infty}(\mathbf{v}) U_F^{\pm\infty\dagger}(\mathbf{w}) \right\rangle_A$$

we can define $\mathcal{H}_{\text{Eik}}^{i,j}(\mathbf{p}_2, \mathbf{q}, \mathbf{k}) = H_{\text{Eik}}(\mathbf{p}_2, \mathbf{q}, \mathbf{k}) \frac{\mathbf{k}^i \mathbf{k}^j}{\mathbf{k}^4}$

$$\left.\frac{d_6\sigma_{q\to q\gamma}}{d_6\text{P.S}}\right|_{\text{Eik}} = (2\pi^3) 16\pi^2 \alpha_e \alpha_s Q_f^2 2\pi\delta(k^+) \left(\mathcal{H}_{\text{Eik}}^{ij} \frac{\mathbf{k}^i \mathbf{k}^j}{\mathbf{k}^2}\right) \left. \text{x}G^{(2)}(\mathbf{x}, \mathbf{k})\right|_{\text{x}=0} .$$

[T. Altinoluk, R. Boussarie, P. Kotko
J. High Energ. Phys. 2019, 156 (2019)]

[F. Dominguez, C. Marquet, B. Xiao, F. Yuan
Phys.Rev.D 83 (2011) 105005]

Dipole Factorization in general kinematics

$$\frac{d_6\sigma_{q\to q\gamma}}{d_6\mathrm{P.S}}\Bigg|^{\text{dec. on q}}_{\text{NEik}} = 16\pi^2\alpha_e\alpha_s Q_f^2 2\pi\delta(k^+)\mathcal{H}^{ij}_{\text{dec. }1}(\mathbf{p}_2,\mathbf{q},\mathbf{k})\int_{v^+,w^+\mathbf{v},\mathbf{w}} e^{-i(\mathbf{v}-\mathbf{w})\cdot\mathbf{k}}$$

$$\times\sum_{\text{col.}}(2i(v^+-w^+))\left\langle U_F^{+\infty}(v^+,\mathbf{v})F_i^{\ -}(v^+,\mathbf{v})U_F^{-\infty}(v^+,\mathbf{v})U_F^{-\infty,\dagger}(w^+,\mathbf{w})\,{}^{\text{tr}}F_j^{\ -}(w^+,\mathbf{w})U_F^{+\infty,\dagger}(w^+,\mathbf{w})\right\rangle_A$$

$$\left[\int\frac{dx^+d_2\mathbf{v}}{2P^-_{\text{tar.}}}e^{ixP^-_{\text{tar.}}x^+-i\mathbf{k}\cdot\mathbf{x}}(2ix^+)\mathrm{Tr}\langle P'_{\text{tar.}}|F^{-i}(v^+,\mathbf{v})\mathrm{U}^{[-]}F^{-j}(0^+,\mathbf{0})\mathrm{U}^{[+]\dagger}|P_{\text{tar.}}\rangle\right]_{x=0}$$

$$=\frac{2(2\pi)^3}{P^-_{\text{tar.}}}\partial_x\left[\int\frac{dx^+d_2\mathbf{v}}{(2\pi)^3 2P^-_{\text{tar.}}}e^{ixP^-_{\text{tar.}}x^+-i\mathbf{k}\cdot\mathbf{x}}\mathrm{Tr}\langle P'_{\text{tar.}}|F^{-i}(x^+,\mathbf{v})\mathrm{U}^{[-]}F^{-j}(0^+,\mathbf{0})\mathrm{U}^{[+]\dagger}|P_{\text{tar.}}\rangle\right]_{x=0}$$

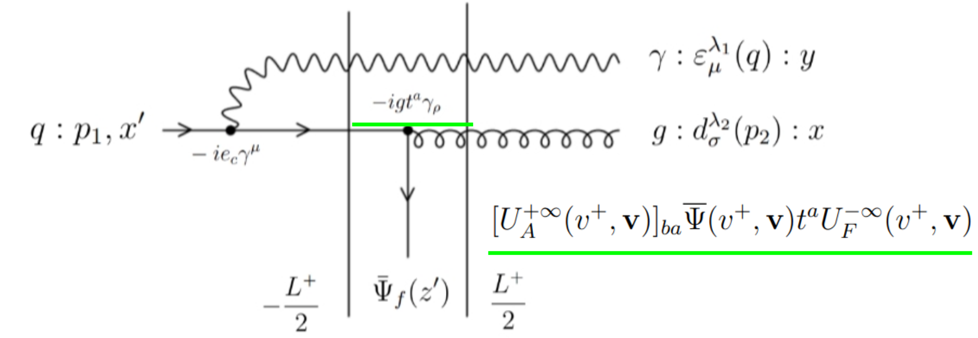
Next-to-Eikonal cross section
The born of non-zero x

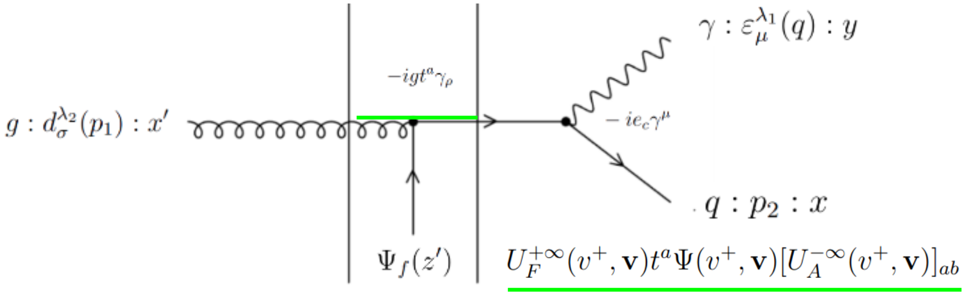
$$\frac{d_6\sigma_{q\to q\gamma}}{d_6\mathrm{P.S}}\Bigg|_{\text{NEik}} = (1/W^2)(2\pi^3)16\pi^2\alpha_e\alpha_s Q_f^2 2\pi\delta(k^+)\left(\mathcal{H}^{ij}_{\text{dec. }1}\frac{\mathbf{k}^i\mathbf{k}^j}{\mathbf{k}^2}\right)\frac{\partial}{\partial\mathrm{x}}\left[\mathrm{x}G^{(2)}(\mathrm{x},\mathbf{k})\right]_{\mathrm{x}=0}.$$

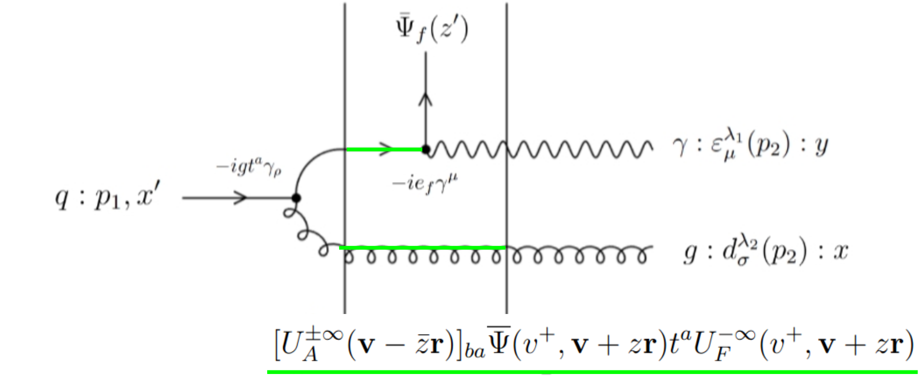
Next-to-Eik quark background *“Specjalność warszawska”*

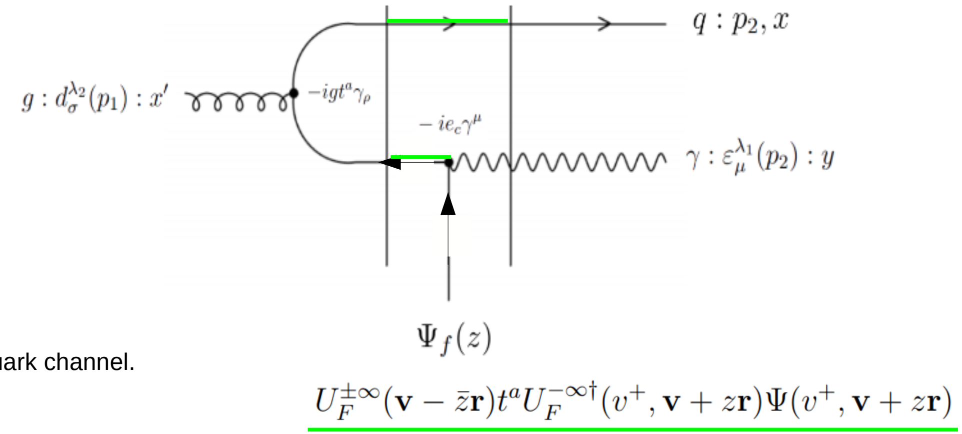
[T. Altinoluk, G. Beuf, N. Armesto, (2023), Phys. Rev. D 108, 074023]

Local | **Bilocal**









+ produced anti-quark channel.

- **Bilocal :** has to be expanded in the back-to-back limit to be factorized in TMD ;
- **Local :** can be factorized in TMD in general kinematics ;
- Both types of amplitudes for each channels has to be treated on the same footage.

Quark small-x TMDs appearing in photon-jet production twist-3 :

$q \to g\gamma$	$\frac{1}{2N_c}\left[N_c^2 f_q^{(2,+)}(x,\mathbf{k}) - f_q^{(1,-)}(x,\mathbf{k})\right]$
$g \to q/\bar{q}\gamma$	$\frac{1}{2N_c}\left[N_c^2 f_q^{(2,-)}(x,\mathbf{k}) - f_q^{(1,+)}(x,\mathbf{k})\right] + (q \leftrightarrow \bar{q}).$

where quark TMDs are defined :

$$f_q^{(1,\pm)}(x,\mathbf{k}) := \int \frac{dv^+ d_2\mathbf{v}}{(2\pi)^3} e^{-ixP^-_{\text{tar.}}v^+ - i\mathbf{k}\cdot\mathbf{v}} \langle P_{\text{tar.}}|\overline{\Psi}(v^+,\mathbf{v})\frac{\gamma^-}{2}\mathrm{U}^{[\pm]}\Psi(0^+,\mathbf{0})|P'_{\text{tar.}}\rangle$$

$$f_q^{(2,\pm)}(x,\mathbf{k}) := \frac{1}{N_c}\int \frac{dv^+ d_2\mathbf{v}}{(2\pi)^3} e^{-ixP^-_{\text{tar.}}v^+ - i\mathbf{k}\cdot\mathbf{v}} \langle P_{\text{tar.}}|\overline{\Psi}(v^+,\mathbf{v})\frac{\gamma^-}{2}\left[\mathrm{Tr}\{\mathrm{U}^{[\Box]}\}\mathrm{U}^{[\pm]}\right]\Psi(0^+,\mathbf{0})|P'_{\text{tar.}}\rangle$$

Photo-production from pA collision
Differential cross section at Next-to-Eikonal + twist-3 accuracy

$$\left(\frac{\mathbf{k}^i\mathbf{k}^j}{\mathbf{k}^2}\mathcal{H}^{ij}_{\text{Eik}}(\mathbf{P},\mathbf{k})\right) = 8p_1^+\bar{z}z^2[\bar{z}^2+1]\left[\frac{1}{\mathbf{P}^4}+\frac{2z(\mathbf{P}\cdot\mathbf{k})}{\mathbf{P}^6}\right] = \frac{2p_1^+z\bar{z}}{\mathbf{P}^2}\left(\frac{\mathbf{k}^i\mathbf{k}^j}{\mathbf{k}^2}\mathcal{H}^{ij}_{\text{dec. 1}}(\mathbf{P},\mathbf{k})\right)$$

(still some work to be proved)

$$\left.\frac{d_6\sigma_{q\to q\gamma}}{d_6\text{P.S}}\right|_{\text{Eik+NEik}} \propto \alpha_s\alpha_e\cdot\bar{z}z^2[\bar{z}^2+1]\left[\frac{1}{\mathbf{P}^4}+\frac{2z(\mathbf{P}\cdot\mathbf{k})}{\mathbf{P}^6}\right]\left[\text{x}G^{(2)}(\text{x},\mathbf{k})\right]_{\text{x}=\frac{\mathbf{P}^2}{z\bar{z}W^2}}$$

$$\left.\frac{d_6\sigma_{q\to g\gamma}}{d_6\text{P.S}}\right|_{\text{NEik}} + \left.\frac{d_6\sigma_{g\to q/\bar{q}\gamma}}{d_6\text{P.S}}\right|_{\text{NEik}} \propto \frac{\alpha_s\alpha_e}{W^2}\cdot\left[\frac{[2+z^2+\bar{z}^2]}{\mathbf{P}^2}+\frac{[\bar{z}^2-z^2]}{\mathbf{P}^4}(\mathbf{P}\cdot\mathbf{k})\right]\left[N_c f_q^{(2,+)}(\text{x},\mathbf{k})-\frac{1}{N_c}f_q^{(1,-)}(\text{x},\mathbf{k})\right]_{\text{x}=0}$$
$$+\frac{\alpha_s\alpha_e}{W^2}\cdot\left[\frac{\bar{z}[2\bar{z}^2+z^2+1]}{\mathbf{P}^2}-\frac{2z\bar{z}[\bar{z}^2+z^2](\mathbf{P}\cdot\mathbf{k})}{\mathbf{P}^4}\right]\left[\frac{1}{2N_c}\left[N_c^2 f_q^{(2,-)}(\text{x},\mathbf{k})-f_q^{(1,+)}(\text{x},\mathbf{k})\right]+(q\leftrightarrow\bar{q})\right]_{\text{x}=0}.$$

Results for SubEikonal studies in dijet production from pA collision

$$\frac{d\sigma^{\mathrm{b2b},\, m=0}_{g\to gq}}{d^2\mathbf{k}\, d^2\mathbf{P}\, dz} = \frac{\alpha_s^2}{2\pi}\frac{1}{W^2}\left[\overline{\mathcal{H}}^{+g}_{g\to gq}\; f^{+g}(\mathrm{x}=0, \mathbf{k}-\mathbf{q}) + \overline{\mathcal{H}}^{+\Box_g}_{g\to gq}\; f^{+\Box_g}(\mathrm{x}=0, \mathbf{k}-\mathbf{q})\right],$$

$$\frac{d\sigma^{\mathrm{b2b},\, m=0}_{q_f\to q_{f_1}\bar{q}_{f_2}}}{d^2\mathbf{k}\, d^2\mathbf{P}\, dz} = \frac{\alpha_s^2}{2\pi}\frac{1}{W^2}\left[\overline{\mathcal{H}}^{-}_{q_f\to q_{f_1}\bar{q}_{f_2}}\; f^{-}(\mathrm{x}=0, \mathbf{k}-\mathbf{q}) + \overline{\mathcal{H}}^{+\Box}_{q_f\to q_{f_1}\bar{q}_{f_2}}\; f^{+\Box}(\mathrm{x}=0, \mathbf{k}-\mathbf{q})\right],$$

$$\frac{d\sigma^{\mathrm{b2b},\, m=0}_{q\to gg}}{d^2\mathbf{k}\, d^2\mathbf{P}\, dz} = \frac{\alpha_s^2}{2\pi}\frac{1}{W^2}\left[\overline{\mathcal{H}}^{-}_{q\to gg}\; f^{-}(\mathrm{x}=0, \mathbf{k}-\mathbf{q}) + \overline{\mathcal{H}}^{-g}_{q\to gg}\; f^{-g}(\mathrm{x}=0, \mathbf{k}-\mathbf{q})\right],$$

$$\frac{d\sigma^{\mathrm{b2b},\, m=0}_{q_f\to q_{f_1}q_{f_2}}}{d^2\mathbf{k}\, d^2\mathbf{P}\, dz} = \frac{\alpha_s^2}{2\pi}\frac{1}{W^2}\left[\overline{\mathcal{H}}^{+\Box}_{q_f\to q_{f_1}q_{f_2}}\; f^{+\Box}(\mathrm{x}=0, \mathbf{k}-\mathbf{q}) + \overline{\mathcal{H}}^{+-+}_{q_f\to q_{f_1}q_{f_2}}\; f^{+-+}(\mathrm{x}=0, \mathbf{k}-\mathbf{q})\right]$$

Quark TMD back-ground contribution

[T. Altinoluk, G. Beuf, E. Blanco, S. Mulani, arXiv:2412.08485]

Conclusion and perspectives

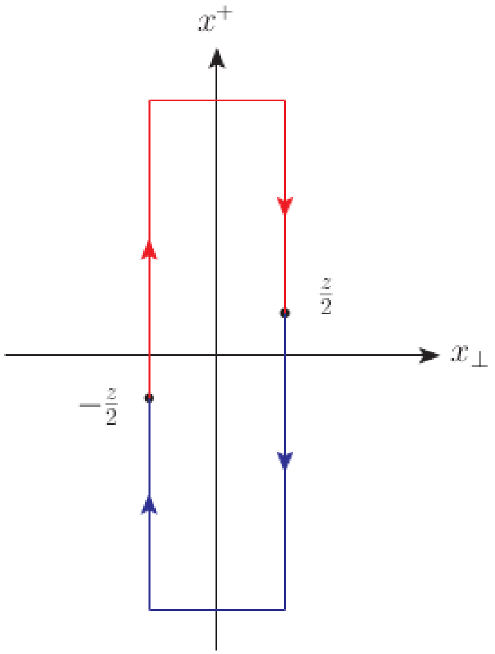
- The gluon distribution contributes at NEik with a value of x fixed by the theory $x = \frac{P^2}{z\bar{z}W^2}$
- Kinematics of the process factorise without twist expansion
- The back-to-back expansion is needed to factorise the quark TMD

- Semi-inclusive Cross section of a single Jet
- SDIS at NEik, EEC and other EIC observables
- NLO correction to a NEIk medium.

END.

Colored path

$$\mathrm{U}^{[-]} := U_F^{-\infty}(v^+, \mathbf{v}) U_F^{-\infty\dagger}(0^+, \mathbf{0})$$
$$\mathrm{U}^{[+]\dagger} := U_F^{+\infty}(0^+, \mathbf{0}) U_F^{+\infty\dagger}(v^+, \mathbf{v})$$



Litteratures definition of the Energy-Energy Correlator

Orginal work presenting the EEC [C.L Basham, L.S Brown, S.D Ellis, S.T Love, Phys. Rev. Lett. 41, 1585]

$$\frac{d^2\Sigma}{d\Omega\, d\Omega'} = \sum_{N=2}^{\infty} \int \prod_{a=1}^{N} E_a^{-1} d^3p_a \frac{d^N\sigma}{E_1^{-1} d^3p_1 \cdots E_N^{-1} d^3p_N} S_N \left[\sum_{b,c=1}^{N} \frac{E_b E_c}{W^2} \delta(\Omega_b - \Omega)\delta(\Omega_c - \Omega') \right].$$

The energy correlation function [3] is defined by

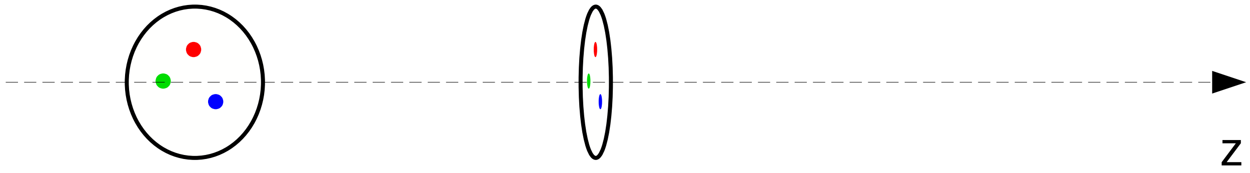
$$\frac{d\Sigma}{d\cos\theta} = \frac{1}{4} \sum_A \int_0^1 dx_A x_A \sum_B \int_0^1 dx_B x_B \frac{d\sigma}{dx_A dx_B d\cos\theta} . \qquad (5.1)$$

[J. C Collins D. E. Soper, Nuclear Physics B197 (1982) 446-476]

(in the series back-to-back Jets in QCD)

[J. C Collins D. E. Soper, Nuclear Physics B193 (1981) 381-443] (see p. 439)

Proton as a fixed color source for a dipole as probe

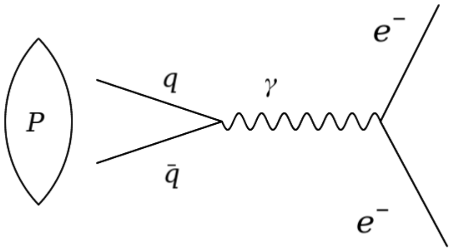


$$j^{\mu}(x) = \delta^{\mu,+}\delta(x^{-})\rho(x_{\perp})$$

$$\rho(\mathbf{x}_{\perp})$$

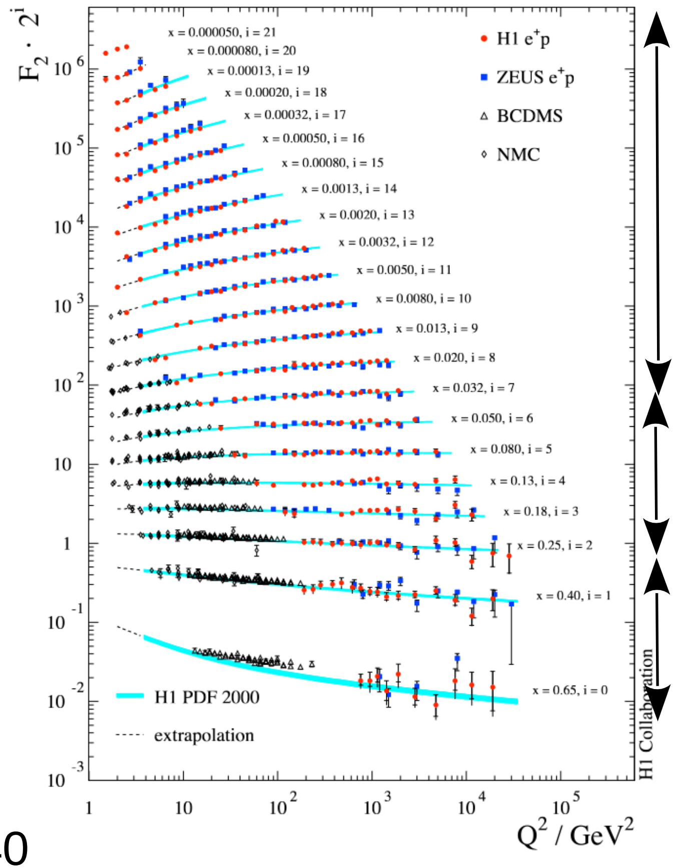
Distances are Lorentz contrated – Time Lorentz dilated

$$\begin{bmatrix} x'_{+} \\ x'_{-} \\ x'_{\perp} \end{bmatrix} = \begin{bmatrix} e^{\omega} & 0 & 0 \\ 0 & e^{-\omega} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{+} \\ x_{-} \\ x_{\perp} \end{bmatrix}$$



Internal proton's structure accessible to the measure

Old parton model and Bjorken scale



(3) low x positive scale deviation

(2) moderate x quasi scale invariance

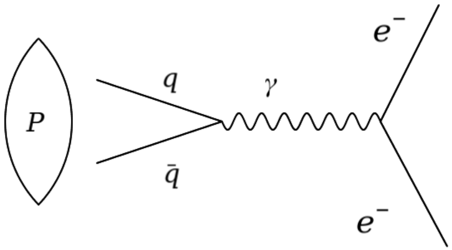
(1) large x low scale deviation

Proton as a fixed color source for a dipole as probe

$$\rho(\mathbf{x}_\perp)$$

Distances are Lorentz contrated – Time Lorentz dilated

$$\begin{bmatrix} x'_+ \\ x'_- \\ x'_\perp \end{bmatrix} = \begin{bmatrix} e^{\omega} & 0 & 0 \\ 0 & e^{-\omega} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_+ \\ x_- \\ x_\perp \end{bmatrix}$$



Internal proton's structure accessible to the measure

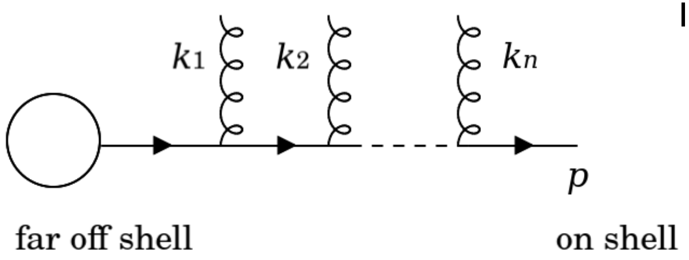
Unknow and different for each processes

We average on all the possible configurations

$$< O >= \int D[\rho] W_Y[\rho] O$$

e.g McLerran-Vanugopalan model → W is a gaussian distribution of ρ

Eikonal Scattering



In the soft approximation

$$\approx (ig)^n \frac{p\cdot\epsilon_1}{p\cdot k_1}\frac{p\cdot\epsilon_2}{p\cdot k_2}\cdots\frac{p\cdot\epsilon_n}{p\cdot k_n} T^{a,1}T^{a,2}..T^{a,n}$$

Phase factor i.e local

Can be resum in a Wilson line $U_i(\boldsymbol{x}_\perp) \equiv T_+ \exp\left[ig_i \int dx^+ \, A_a^-(x^+, 0, \boldsymbol{x}_\perp) t^a\right]$

→ **Reprensent an infinitesimal short interaction into the proton as external color field**

$$S_{\beta\alpha}^{(\infty)} = \sum_\delta \int \left[\prod_{i\in\delta} \frac{dk_i^+}{4\pi k_i^+} d^2\boldsymbol{x}_{i\perp}\right] \Psi_{\delta\beta}^\dagger(\{k_i^+, \boldsymbol{x}_{i\perp}\}) \left[\prod_{i\in\delta} U_i(\boldsymbol{x}_{i\perp})\right] \Psi_{\delta\alpha}(\{k_i^+, \boldsymbol{x}_{i\perp}\}),$$

S-matrx known to measure transition probability between asymptotic states