Studying vector-like fermions as candidates for New Physics from different perspectives

In collaboration with

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Graduate Seminar NCBJ

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20/03/2025

Legend

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Simple Slide:
understandable by everyone,
assuming you were paying attention
to the previous “smiley” slides.

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The Standard Model

Gauge Bosons

$SU(3) \times SU(2)_L \times U(1)_Y$

Interactions

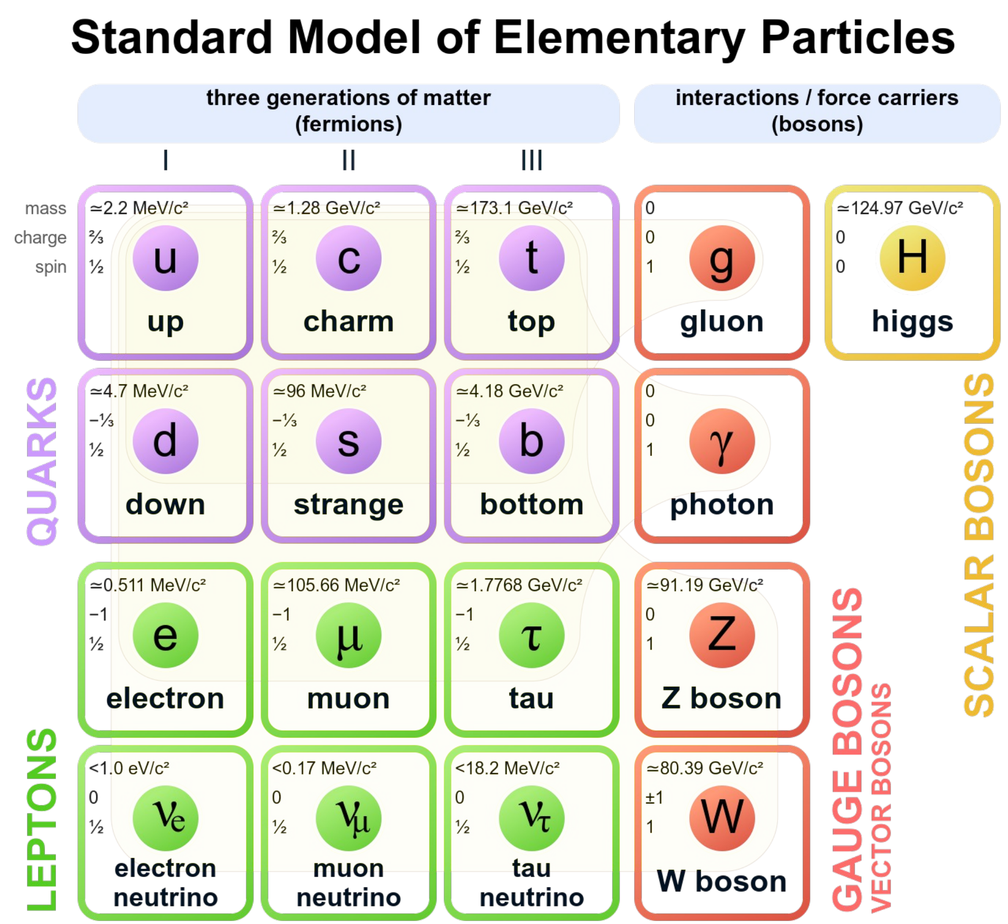
Strong

Electromagnetic

Flavor conserving neutral

Flavor conserving charged

Three generations, why?



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Theoretical problems of The Standard Model

- Gravity
- Mass generation
- Dark matter
- UV completion
- Hierarchy
- Naturalness
- Nature of Neutrinos

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What is a vector-like fermion?

Mass generation

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What is a vector-like fermion?

Mass generation

Dirac spinor in chiral representation

$$\psi_D = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

Lagrangian of a Dirac spinor

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$$

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U(1) Transformation

Vectorial

$$\psi_D \rightarrow \mathrm{e}^{i\,\alpha}\psi_D$$

Axial

$$\psi_D \rightarrow \mathrm{e}^{i\,\gamma_5\,\beta}\psi_D$$

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Vectorial | Axial

U(1) Transformation

$$\psi_D \rightarrow \mathrm{e}^{i\,\alpha}\psi_D \qquad \psi_D \rightarrow \mathrm{e}^{i\,\gamma_5\,\beta}\psi_D$$

Noether conserved current (massless particle)

$$J^\mu_V = \bar{\psi}\gamma^\mu\psi \qquad J^\mu_A = \bar{\psi}\gamma^\mu\gamma_5\psi$$

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Vectorial	Axial
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$\psi_D \rightarrow e^{i\,\alpha}\psi_D$	$\psi_D \rightarrow e^{i\,\gamma_5\,\beta}\psi_D$
Noether conserved current (massless particle)	
$J_V^\mu = \bar{\psi}\gamma^\mu\psi$	$J_A^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi$
Massive particle	
$\partial_\mu J_V^\mu = 0$	$\partial_\mu J_A^\mu = 2i\,m\,(\bar{\psi}\gamma_5\psi)$

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$$J_V^\mu = \bar{\psi}\gamma^\mu\psi \qquad J_A^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi$$

Massive particle

$$\partial_\mu J_V^\mu = 0 \qquad \partial_\mu J_A^\mu = 2i\,m\,(\bar{\psi}\gamma_5\psi)$$

$$\downarrow$$

$$m = 0$$

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The Higgs Mechanism

Mass generation

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The Higgs Mechanism

Mass generation

$$\mathcal{L}_Y = y\,\bar{f}\,\phi\,f$$

The Higgs Mechanism

Mass generation

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

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The Higgs Mechanism

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$$\mathcal{L}_Y = \frac{yv}{\sqrt{2}}\, \bar{f} f + \frac{y}{\sqrt{2}} \bar{f}\, H\, f$$

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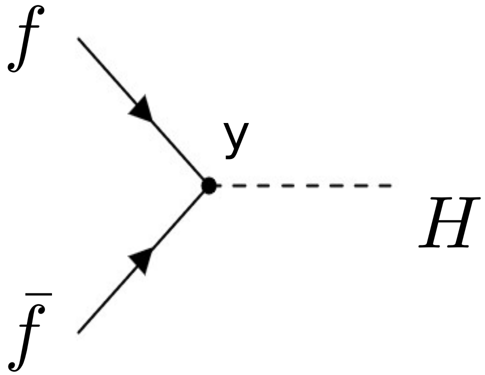

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The Higgs Mechanism

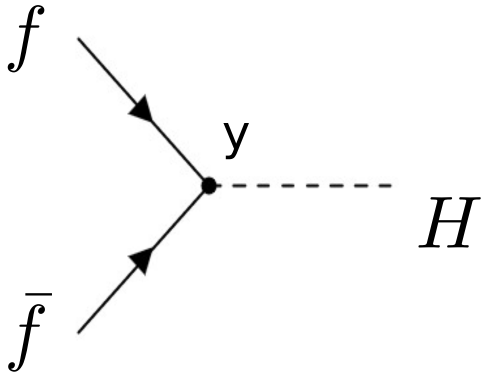
Mass generation

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

$$\mathcal{L}_Y = y \, \bar{f} \, \phi \, f$$

$$m_f = \frac{y \, v}{\sqrt{2}}$$

$$\mathcal{L}_Y = \frac{yv}{\sqrt{2}} \bar{f} f + \frac{y}{\sqrt{2}} \bar{f} \, H \, f$$



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The seesaw mechanism

Mass generation

The seesaw mechanism

Mass generation

$$A = \begin{pmatrix} 0 & M \\ M & B \end{pmatrix}$$

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The seesaw mechanism

Mass generation

$$A = \begin{pmatrix} 0 & M \\ M & B \end{pmatrix}$$

$$\lambda_- = \frac{B - \sqrt{B^2 + 4M^2}}{2} \approx \frac{M^2}{B} + O\left(\frac{2M^2}{B}\right)$$

$$\lambda_+ = \frac{B + \sqrt{B^2 + 4M^2}}{2} \approx B + \frac{M^2}{B} + O\left(\frac{2M^2}{B}\right)$$

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How have I studied vector-like fermions during my PhD?

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How have I studied vector-like fermions during my PhD?

Phenomenologically

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How have I studied vector-like fermions during my PhD?

Phenomenologically

Theoretically

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How have I studied vector-like fermions during my PhD?

Phenomenologically

Mix of the two

Theoretically

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How have I studied vector-like fermions during my PhD?

Phenomenologically

- Build a model (most likely extension of the SM)
- Impose physical conditions (*i.e.* the model must reproduce the SM at low energy)
- Scan the remaining parameter space in the IR

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- Build a model (most likely extension of the SM)
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I will put these in the top-right corner of the slide so you know where we are!!

The model

- Build a model
- Impose physical conditions
- Scan the parameter space

Field	Q_{iL}	u_{iR}	d_{iR}	L_{iL}	e_{iR}	Q_{kL}	u_{kR}	d_{kR}	L_{kL}	e_{kR}	ν_{kR}	$\widetilde{Q}_{kR}$	$\widetilde{u}_{kL}$	$\widetilde{d}_{kL}$	$\widetilde{L}_{kR}$	$\widetilde{e}_{kL}$	$\widetilde{\nu}_{kR}$	ϕ	H_u	H_d
$SU(3)_C$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$
$U(1)_Y$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	0	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
$U(1)'$	0	0	0	0	0	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	-1

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- Build a model
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$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$
$U(1)_Y$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	0	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
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$SU(2)_L$	**2**	**1**	**1**	**2**	**1**	**2**	**1**	**1**	**2**	**1**	**1**	**2**	**1**	**1**	**2**	**1**	**1**	**1**	**2**	**2**
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Mass Matrix for the fermions

$$\mathrm{M}_D = \left(\begin{array}{c|ccccc} & d_{1R} & d_{2R} & d_{3R} & d_{4R} & \widetilde{Q}_{4R} \\ \hline Q_{1L} & 0 & 0 & 0 & y^d_{14}\langle H^0_d\rangle & 0 \\ Q_{2L} & 0 & 0 & 0 & y^d_{24}\langle H^0_d\rangle & 0 \\ Q_{3L} & 0 & 0 & 0 & y^d_{34}\langle H^0_d\rangle & x^d_{34}\langle\phi\rangle \\ Q_{4L} & 0 & 0 & y^d_{43}\langle H^0_d\rangle & 0 & M^Q_4 \\ \widetilde{d}_{4L} & 0 & x^d_{42}\langle\phi\rangle & x^d_{43}\langle\phi\rangle & M^d_4 & 0 \end{array}\right)$$

This element is 0 in the up and charged lepton mass matrices

CKM matrix analytically computed for the first time in our work

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Global analysis of a vector-like extension with extra scalars

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The model was studied for the first time in *S.F.King, JHEP 09, 069 (2018)*

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and then later again with different type of experimental motivations

- *A.Cárcamo Hernández, S.F.King, H.Lee, S.J.Rowley, Phys. Rev. D 101, 115016 (2020)*
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- *A.C.Hernández, S.F.King, H.Lee, Phys. Rev. D 105, 015021 (2022)*
- *H.Lee, A.Cárcamo Hernández, 2207.01710 (2022)*

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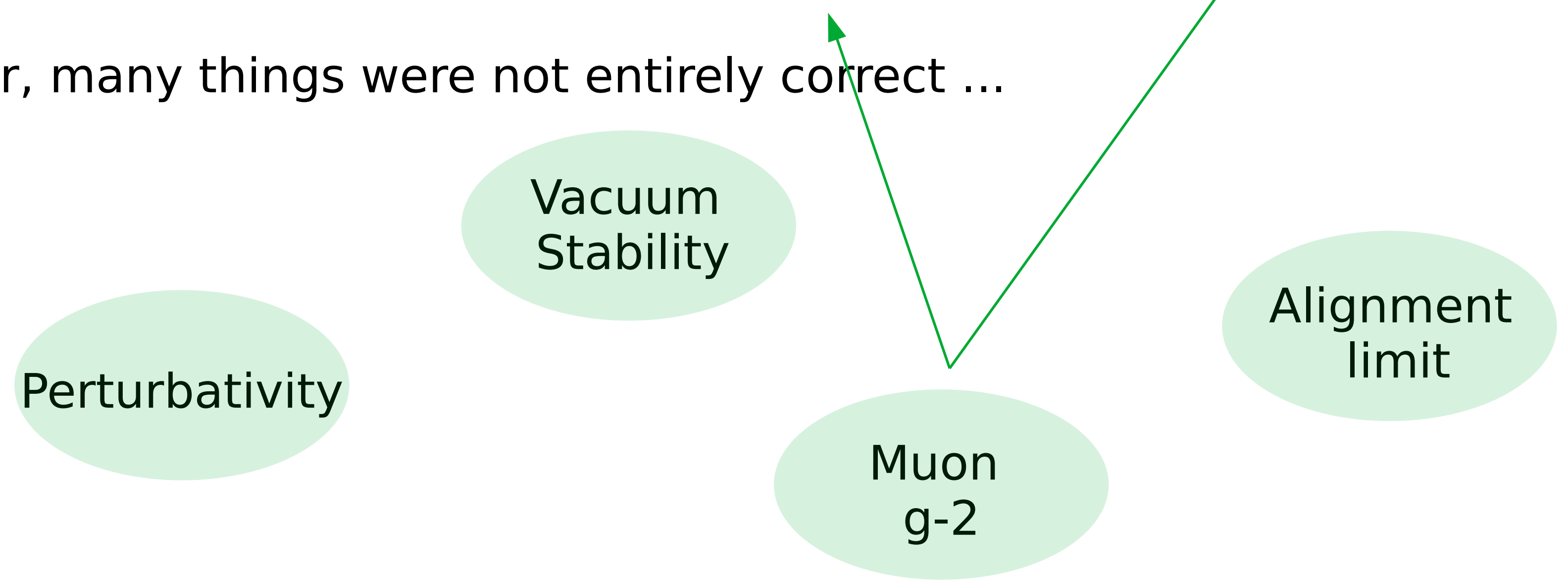
However, many things were not entirely correct ...



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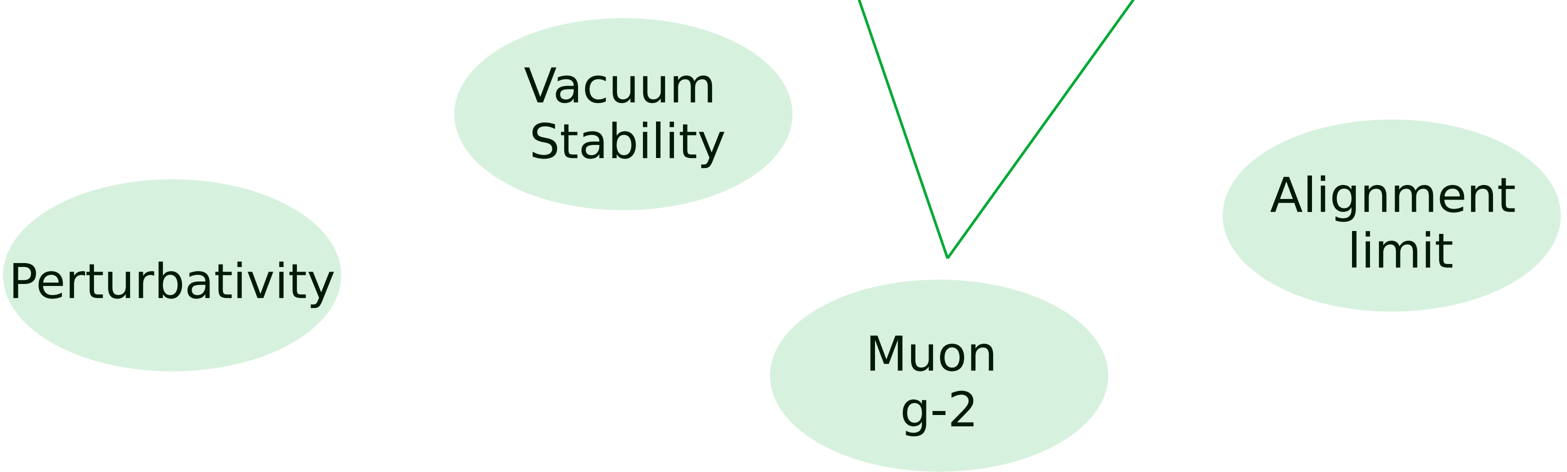

In this work, we **re-assess** the previous analysis and add some **new results**.

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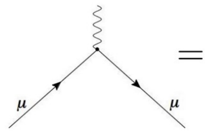
Phenomenological problems of The Standard Model

Muonic (g-2)

The magnetic moment of charged leptons (e, μ, τ): $\vec{\mu} = g\frac{e}{2m}\vec{S}$

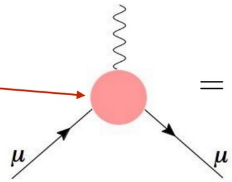
Dirac (leading order): $g = 2$



$= (-ie)\,\bar{u}(p')\gamma^{\mu}u(p)$

Quantum effects (loops):

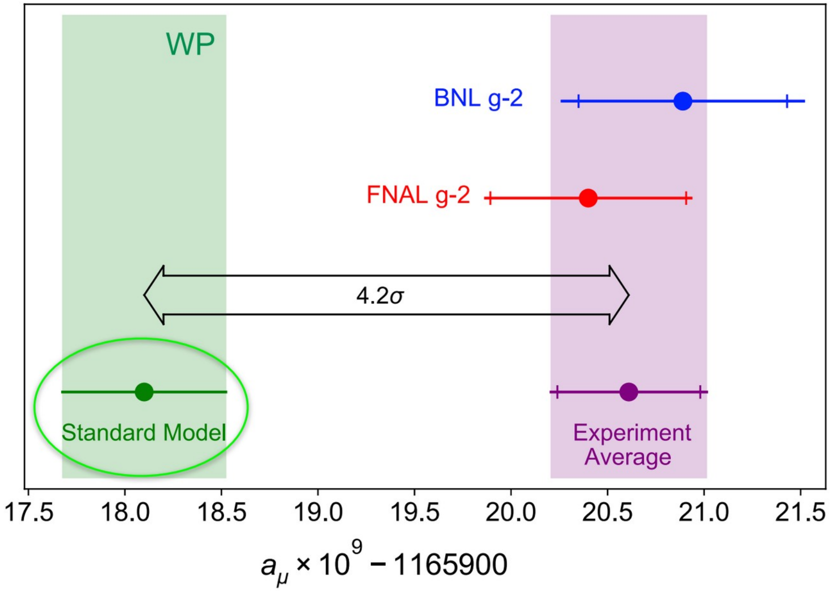
All SM particles contribute



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2)\right]u(p)$$

Note: $F_1(0) = 1$ and $g = 2 + 2\,F_2(0)$

Anomalous magnetic moment: $a \equiv \frac{g-2}{2} = F_2(0)$



$$\Delta a_{\mu} = a_{\mu}^{\mathrm{Exp}} - a_{\mu}^{\mathrm{SM}} = (26.1 \pm 8.0) \times 10^{-10}$$
$$\Delta a_{e} = a_{e}^{\mathrm{Exp}} - a_{e}^{\mathrm{SM}} = (-0.88 \pm 0.36) \times 10^{-12}$$

The status of muon g-2 theory: Aida El-Khadra
Higgs, Flavor and Beyond – DESY Theory Workshop

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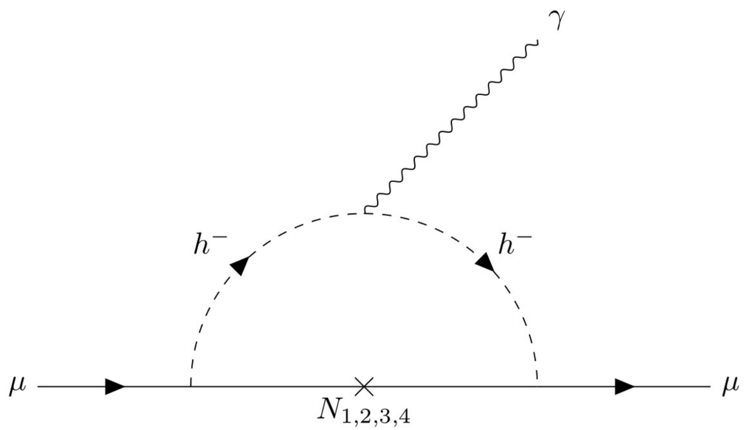
New Physics contributions to g-2

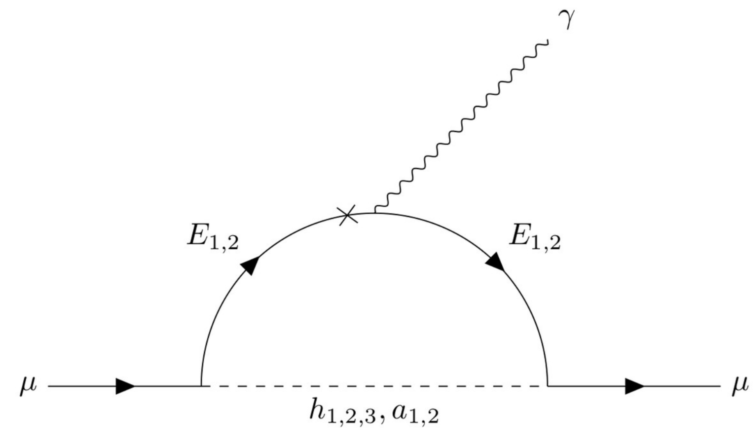
- Build a model
- Impose physical conditions
- Scan the parameter space

Contributions given by this diagram have **not** been considered in previous works



Loop mediated by
neutrinos and charged scalar

Contributions given by these diagrams have been computed already in previous works



Loop mediated by
charged lepton and neutral scalar

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New Physics contributions to g-2

- Build a model
- Impose physical conditions
- Scan the parameter space

Contributions to $\Delta a_\mu \times 10^9$							
Charged scalars				CP-even scalars			
Loop	BP1	BP2	BP3	Loop	BP1	BP2	BP3
$h^\pm, N_{1,2}$	-1.076	-0.792	-0.942	h_1, E_1	-0.003	-0.001	-0.009
$h^\pm, N_{3,4}$	3.300	2.898	3.153	h_1, E_2	0.003	0.001	0.009
$h^\pm, N_{\text{tot}}$	2.225	2.106	2.211	h_2, E_1	-0.409	-0.520	-0.969
CP-odd scalars				h_2, E_2	0.437	0.548	0.994
a_1, E_1	0.425	0.528	0.938	h_3, E_1	0.018	0.115	0.076
a_1, E_2	-0.544	-0.611	-1.529	h_3, E_2	-0.017	-0.127	-0.076
a_2, E_1	-0.033	-0.135	-0.071	h, E_{tot}	0.032	0.027	0.025
a_2, E_2	0.110	0.196	0.621	Total			
a, E_{tot}	-0.015	-0.023	-0.041	Δa_μ	2.215	2.101	2.196

The deviation from the experimental measurement of g-2 can be explained within this model.

The main contribution to g-2 in mediated by **charged scalars** and **neutrinos**!

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New Physics Spectra

- Build a model
- Impose physical conditions
- Scan the parameter space

VL Quarks

$U_1 \to \sim 1500$ GeV $\quad \approx \sqrt{(M_4^Q)^2 + \frac{1}{2}(v_\phi x_{34}^Q)^2 - \frac{(M_4^Q y_{43}^u v_u)^2}{(x_{34}^Q v_\phi)^2 + 2(M_4^Q)^2}}$

$D_1 \to \sim 1500$ GeV $\quad \approx \sqrt{(M_4^Q)^2 + \frac{1}{2}(v_\phi x_{34}^Q)^2}$

$U_2 \to \sim 1700\text{-}1900$ GeV

$D_2 \to \sim 2900\text{-}3600$ GeV

Almost degenerative because depend on the mass of the VL doublet

VL Leptons

$N_{1,2} \to \sim 200$ GeV

$N_{3,4} \to \sim 500\text{-}600$ GeV

$E_1 \to \sim 500\text{-}600$ GeV

($N_{3,4}$, E_1) $\approx \sqrt{(M_4^L)^2 + \frac{1}{2}(v_\phi x_{34}^L)^2}$

$E_2 \to \sim 550\text{-}650$ GeV

Strongly constrained by g-2

CP-Even Scalars

$h_1 \to 125$ GeV

$h_2 \to \sim 400$ GeV

$h_3 \to \sim 600\text{-}800$ GeV

CP-Odd Scalars

$a_1 \to \sim 400$ GeV

$a_2 \to \sim 450\text{-}600$ GeV

Charged Scalars

$h_\pm \to \sim 400$ GeV

Can be tested in Run 3

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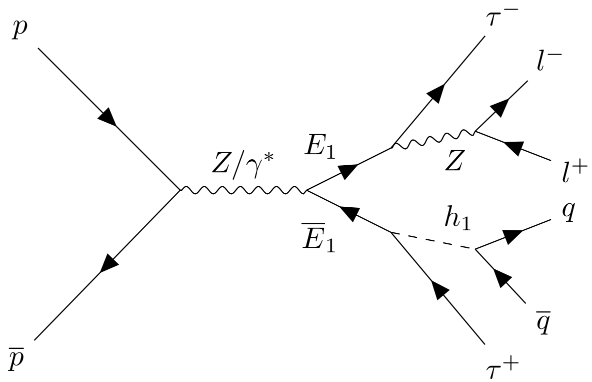
LHC Bounds on vector-like fermions

- Build a model
- Impose physical conditions
- Scan the parameter space

Leptons

Our leptons decay predominantly to muons, but there are **no** dedicated experimental analysis.

The best we can do is study:



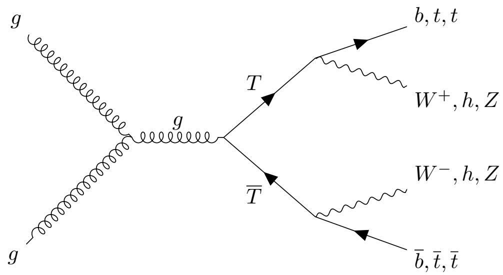
ATLAS: JHEP 07, 118 (2023)
CMS: Phys. Rev. D 100, 052003 (2019)

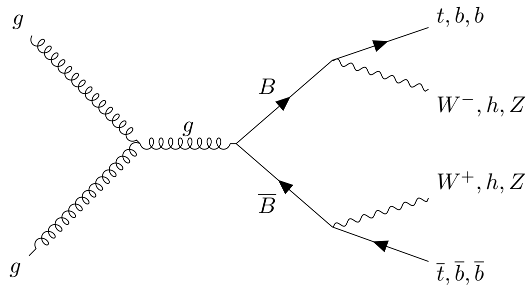
BR(LL→ττ) < 10% and x-section is 3-4 orders of magnitude **smaller** than current bounds.

Can **not** be tested in Run 3

Quarks

Two possible channels can be studied:





ATLAS: Eur. Phys. J. C 83, 719 (2023)
CMS: JHEP 07, 020 (2023)

The x-section is one order of magnitude smaller than the current bounds.

Can be tested in Run 3

How have I studied vector-like fermions during my PhD?

Phenomenologically

Mix of the two

Theoretically

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How have I studied vector-like fermions during my PhD?

I will put these in the top-right corner of the slide so you know where we are!!

Theoretically

- Build a toy-model
- Perform non-perturbative calculations
- Look for UV Fixed Points in the couplings’ running

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The Litim-Sannino (LiSa) Model

- Build a toy-model
- Non-perturbative calculations
- Look for UV Fixed Points

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The Litim-Sannino (LiSa) Model

- Build a toy-model
- Non-perturbative calculations
- Look for UV Fixed Points

Gauge $F^a_{\mu\nu}\,(a = 1, ..., N_C^2 - 1)$

Fermions $Q_i\,(i = 1, ..., N_F)$

Scalars $H \in N_F \times N_F$

Veneziano parameter

$$\epsilon \equiv \frac{N_F}{N_C} - \frac{11}{2}$$

$$0 < \epsilon \ll 1$$

The couplings in the theory are

$$(g,\ y,\ u,\ v)$$

The Lagrangian of the model:

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} + \mathrm{Tr}\left(\partial_\mu H^\dagger \partial^\mu H\right) + \mathrm{Tr}\left(\bar{Q}\,i\not{D}\,Q\right)$$
$$-y\,\mathrm{Tr}\left(\bar{Q}_L H Q_R + \bar{Q}_R H^\dagger Q_L\right) - u\,\mathrm{Tr}\left(H^\dagger H\right)^2 - v\left(\mathrm{Tr}\,H^\dagger H\right)^2$$

Litim, Sannino (2014)

Running of Coupling Constants

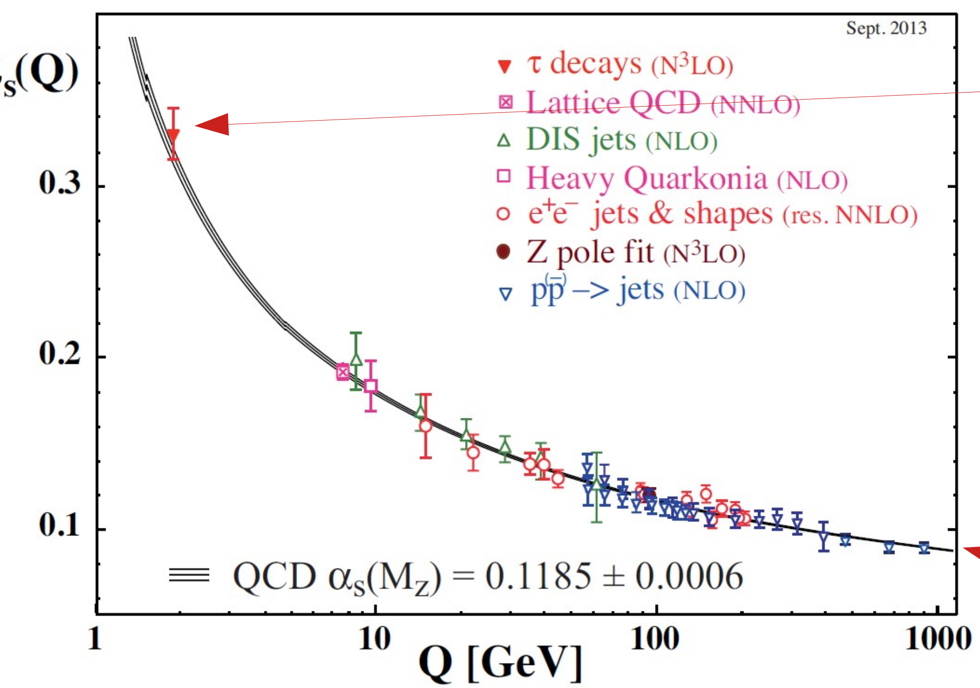
In quantum field theory coupling constants are not constant, they depend on the energy scale of the process under consideration

Beta Function

$$\beta(g) \equiv \frac{dg}{d \log \mu}$$

$$\frac{g_c^2}{4\pi} \equiv \alpha_s(Q)$$

Fine structure constant in QCD



At low energies, the interaction between quarks and gluons is incredibly strong

At high energies, quarks and gluons do not interact

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Asymptotic Behaviors

$$\beta(g) \equiv \frac{dg}{d\log\mu} = A\, g^2$$

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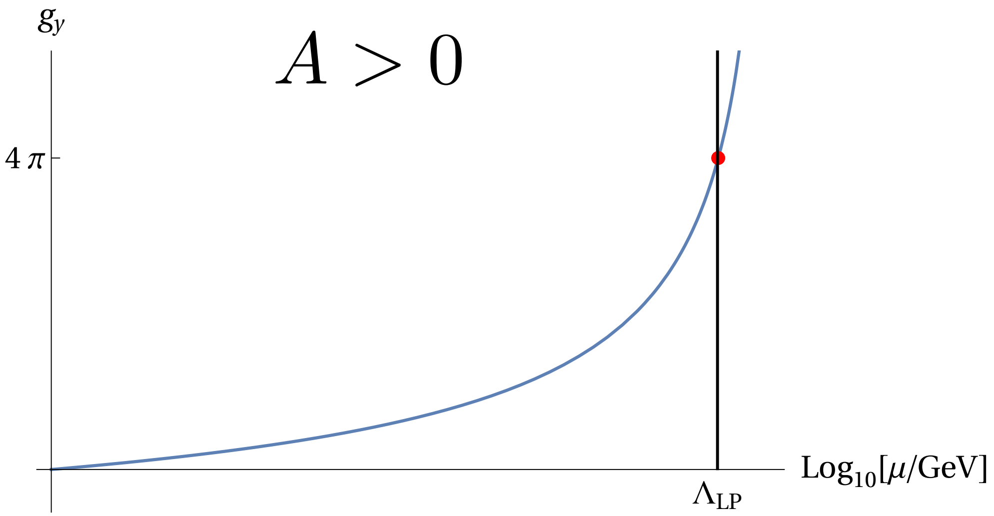

Landau pole

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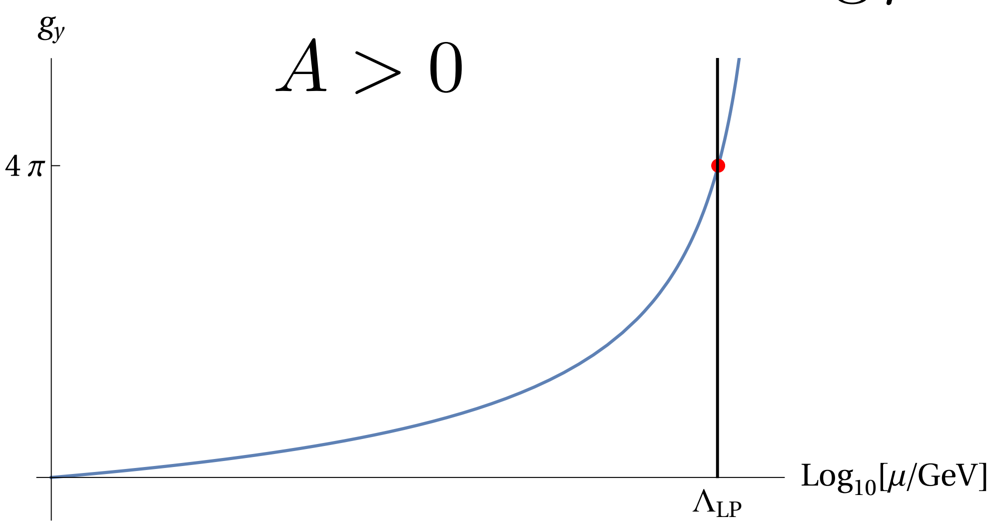
Asymptotic Behaviors

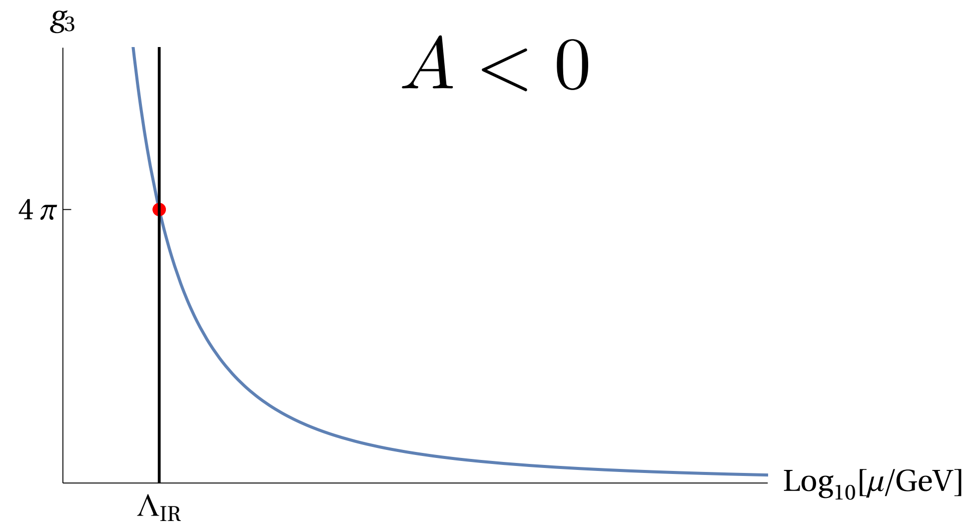
$$\beta(g) \equiv \frac{dg}{d\log\mu} = A\, g^2$$

$A > 0$



Landau pole

$A < 0$



Asymptotic freedom

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Asymptotic Safety

$$\beta(g) \equiv \frac{dg}{d\log\mu} = A\,g^2 + B\,g^3$$

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Asymptotic Safety

$$\beta(g) \equiv \frac{dg}{d\log\mu} = A\,g^2 + B\,g^3$$

There is a specific value $g^* = -B/A$

$$\beta(g^*) = 0$$

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Asymptotic Safety

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Fixed Point!

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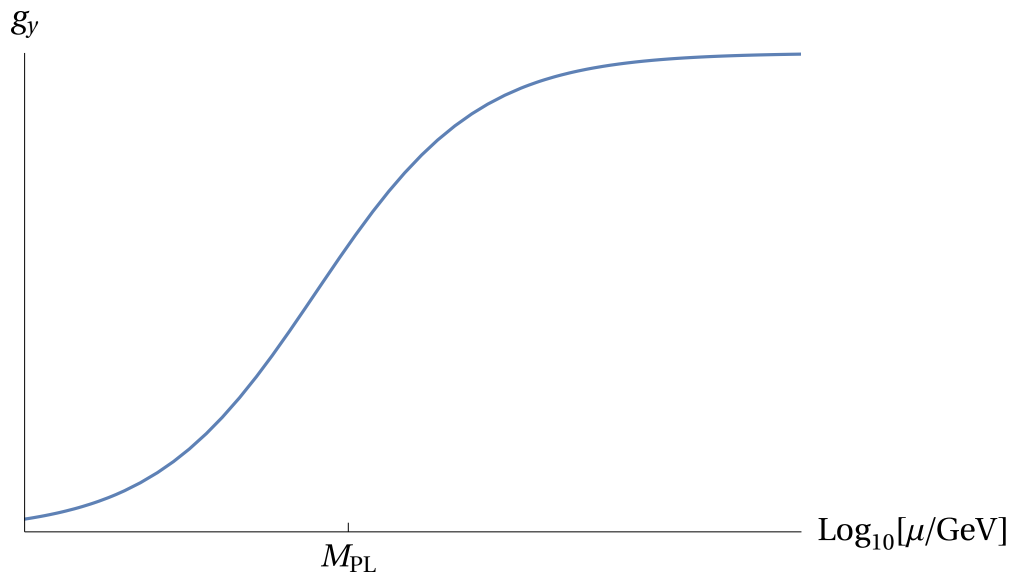

Asymptotic safety

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The Litim-Sannino (LiSa) Model

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- Look for UV Fixed Points

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Fermions $Q_i\,(i = 1, \ldots, N_F)$

Scalars $H \in N_F \times N_F$

Veneziano parameter $\epsilon \equiv \frac{N_F}{N_C} - \frac{11}{2}$

$$0 < \epsilon \ll 1$$

AT THE FP THE WHOLE MODEL DEPENDS ONLY ON THE VENEZIANO PARAMETER

The couplings in the theory are

$$(g,\ y,\ u,\ v)$$

The Lagrangian of the model:

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} + \mathrm{Tr}\left(\partial_\mu H^\dagger \partial^\mu H\right) + \mathrm{Tr}\left(\bar{Q}\,i\not{D}\,Q\right)$$

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Litim, Sannino (2014)

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Results from Perturbation Theory

Under perturbative expansion, the theory has an ultraviolet Fixed Point:

$$g^* = +0.456\epsilon + 0.781\epsilon^2 + 6.610\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$y^* = +0.211\epsilon + 0.508\epsilon^2 + 3.322\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$u^* = +0.200\epsilon + 0.440\epsilon^2 + 2.693\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$v^* = -0.137\epsilon - 0.632\epsilon^2 - 4.313\epsilon^3 + \mathcal{O}(\epsilon^4)$$

Litim, Sannino (2014)

Litim, Riyaz, Stamou, Steudtner (2023)

Bond, Litim, Vazquez, Steudtner (2017)

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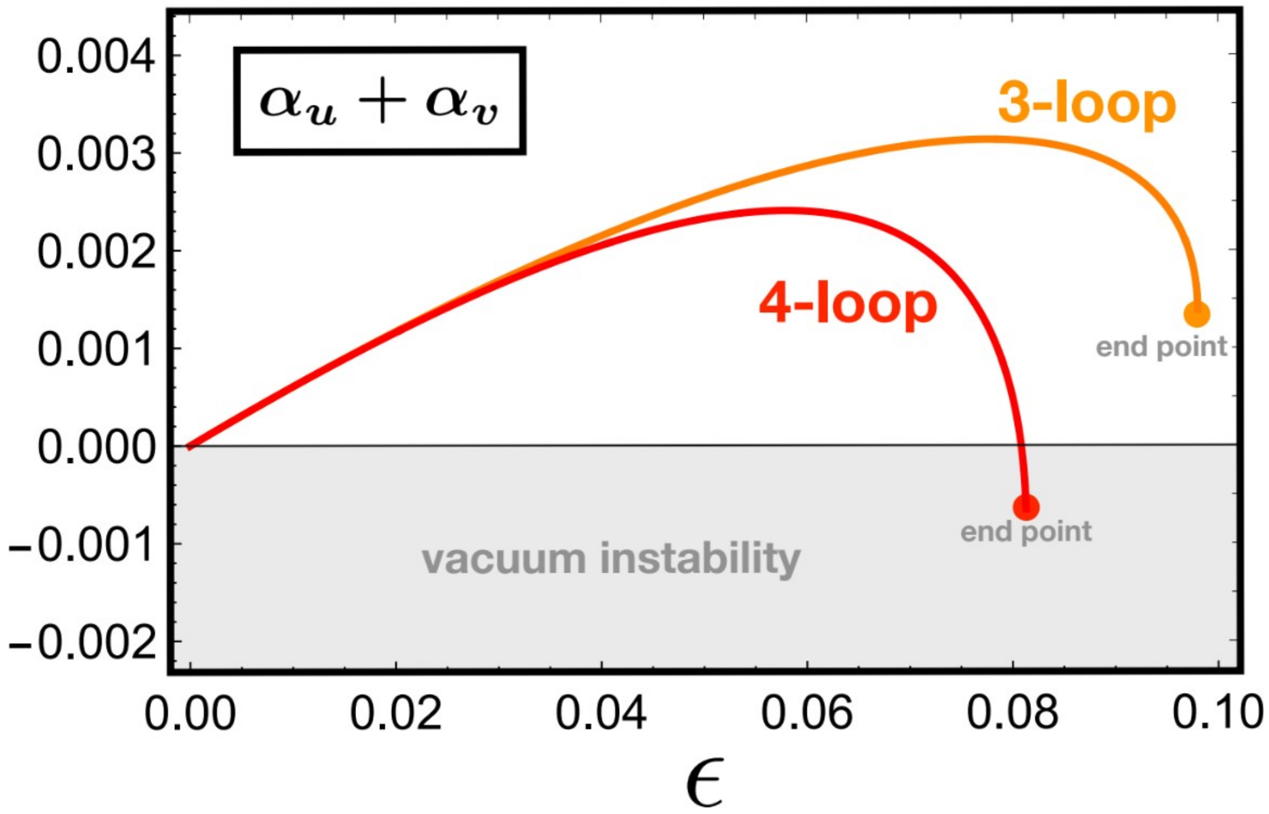
Conformal Window

Is the conformal window closing because of Vacuum Stability or a Fixed Point merger?



Plot kindly shared by Nahzaan Riyaz.

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Beyond marginal operators

- Build a toy-model
- Non-perturbative calculations
- Look for UV Fixed Points

$$v\left(\mathrm{Tr}\,H^\dagger H\right)^2 \xrightarrow{\partial_t U(\mathrm{Tr}H^\dagger H)} \underbrace{\sum_n \gamma_n (\mathrm{Tr}H^\dagger H)^{n-2}}_{U(\mathrm{Tr}H^\dagger H)}(\mathrm{Tr}H^\dagger H)^2$$

$$u\,\mathrm{Tr}\left(H^\dagger H\right)^2 \xrightarrow{\partial_t C(\mathrm{Tr}H^\dagger H)} \underbrace{\sum_m \alpha_m (\mathrm{Tr}H^\dagger H)^{m-2}}_{C(\mathrm{Tr}H^\dagger H)}\,\mathrm{Tr}(H^\dagger H)^2$$

$$y\,\mathrm{Tr}(\bar{Q}HQ) \xrightarrow{\partial_t Y(\mathrm{Tr}H^\dagger H)} \underbrace{\sum_l Y_l\,(\mathrm{Tr}H^\dagger H)^l}_{Y(\mathrm{Tr}H^\dagger H)}\,\mathrm{Tr}(\bar{Q}HQ)$$

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Fixed Point

- Build a toy-model
- Non-perturbative calculations
- Look for UV Fixed Points

Coupling	FP	Coupling	FP	Coupling	FP
γ_1	$+0.199781\epsilon$	α_1	$+0.0625304\epsilon$	y_0	$+0.458831\sqrt{\epsilon}$
γ_2	$-0.404135\epsilon^3$	α_2	$-0.0844283\epsilon^3$	y_1	$+0.318417\sqrt{\epsilon^5}$
γ_3	$+0.558651\epsilon^4$	α_3	$+0.0721923\epsilon^4$	y_2	$-0.468528\sqrt{\epsilon^7}$
γ_4	$-0.812282\epsilon^5$	α_4	$-0.0699564\epsilon^5$	y_3	$+0.626392\sqrt{\epsilon^9}$
γ_5	$+1.16104\epsilon^6$	α_5	$+0.0706016\epsilon^6$	y_4	$-0.798058\sqrt{\epsilon^{11}}$
	$\vdots$		$\vdots$		$\vdots$

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Litim, Sannino (2014)

Litim, Riyaz, Stamou, Steudtner (2023)

FRG
(this work)

Bond, Litim, Vazquez, Steudtner (2017)

How have I studied vector-like fermions during my PhD?

Phenomenologically

- Build a model (most likely extension of the SM)
- Impose physical conditions (*i.e.* the model must reproduce the SM at low energy)
- Scan the remaining parameter space in the IR

Mix of the two

Theoretically

- Build a toy-model
- Perform non-perturbative calculations
- Look for UV Fixed Points in the couplings' running

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Asymptotic Safety in Quantum Gravity

- Build a model
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Renormalization Group Equations in the Sub-Planckian regime

$$\beta_g = \beta_g^{\mathrm{SM+NP}}$$

$$\beta_y = \beta_y^{\mathrm{SM+NP}}$$

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Renormalization Group Equations in the Trans-Planckian regime

$$\beta_g = \beta_g^{\mathrm{SM+NP}} - g\, f_g$$

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$$\beta_y = \beta_y^{\mathrm{SM+NP}} - y\, f_y$$

In our approach they are determined by matching the low-energy data.

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Asymptotic Safety in Quantum Gravity

- Build a model
- Assume a UV Fixed Point
- Predict the couplings’ value

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

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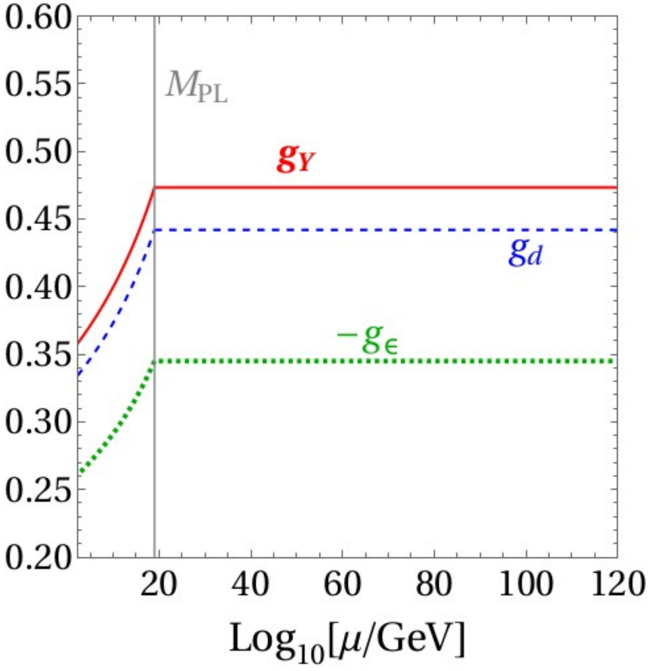
Gauge couplings



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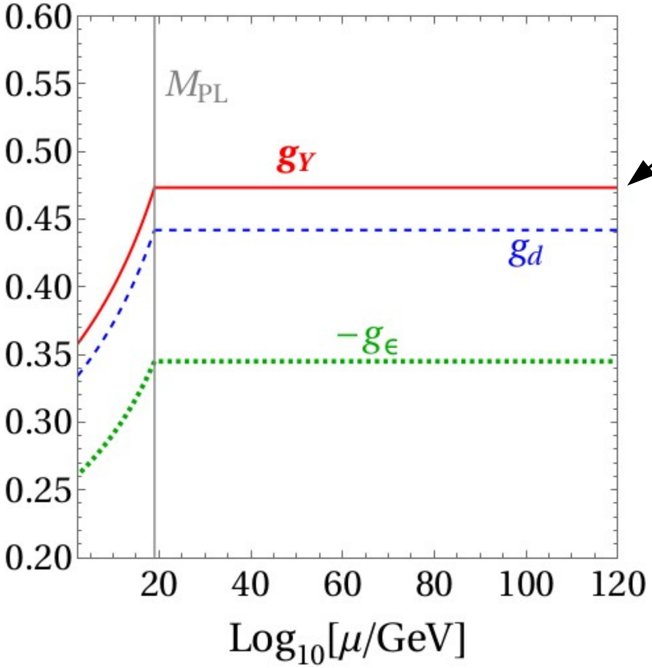

Get f_g

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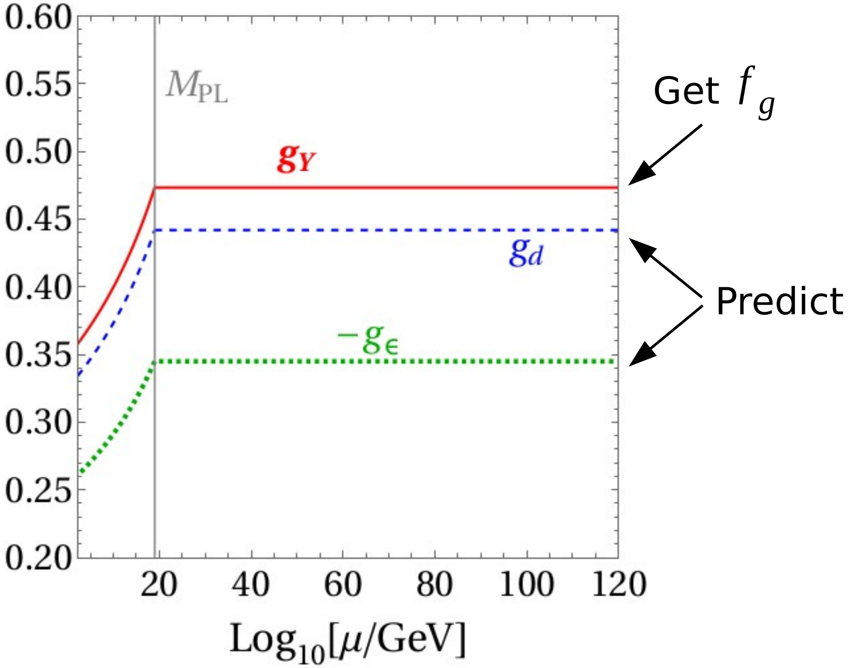

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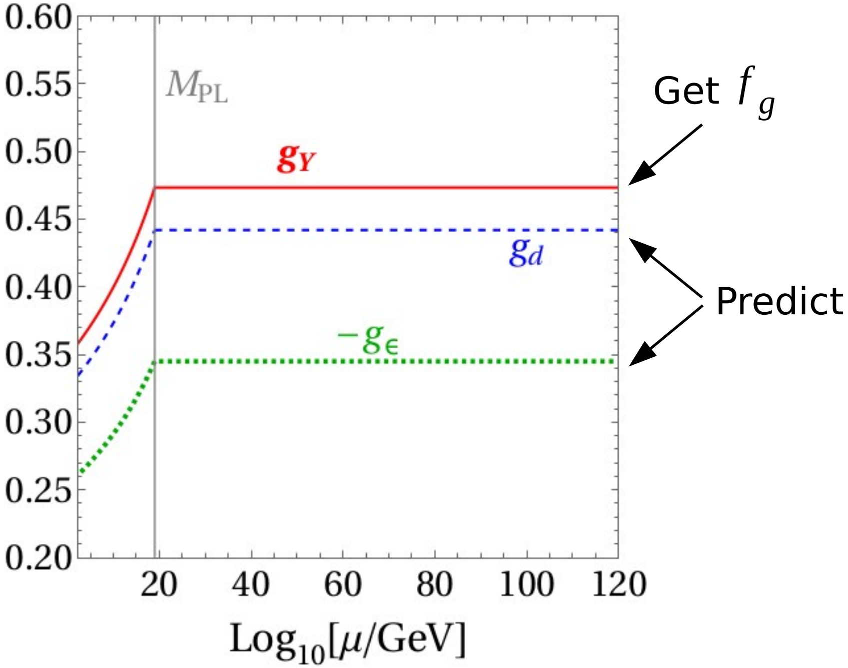
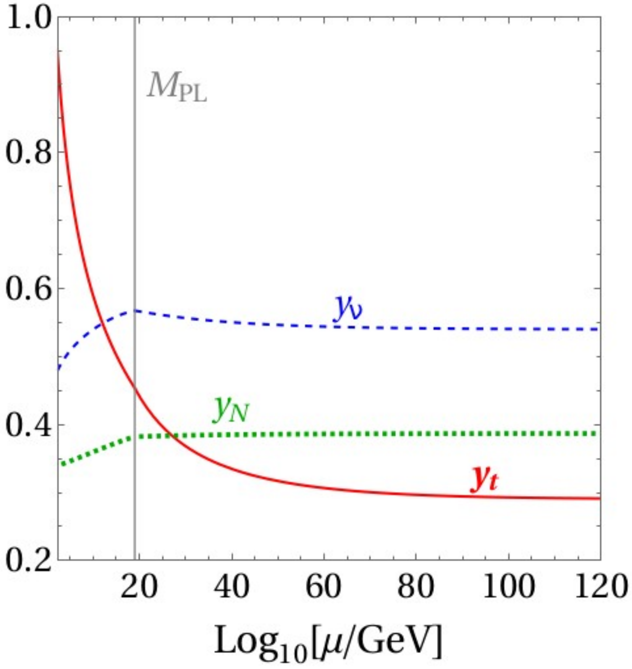
Gauge couplings



Yukawa couplings



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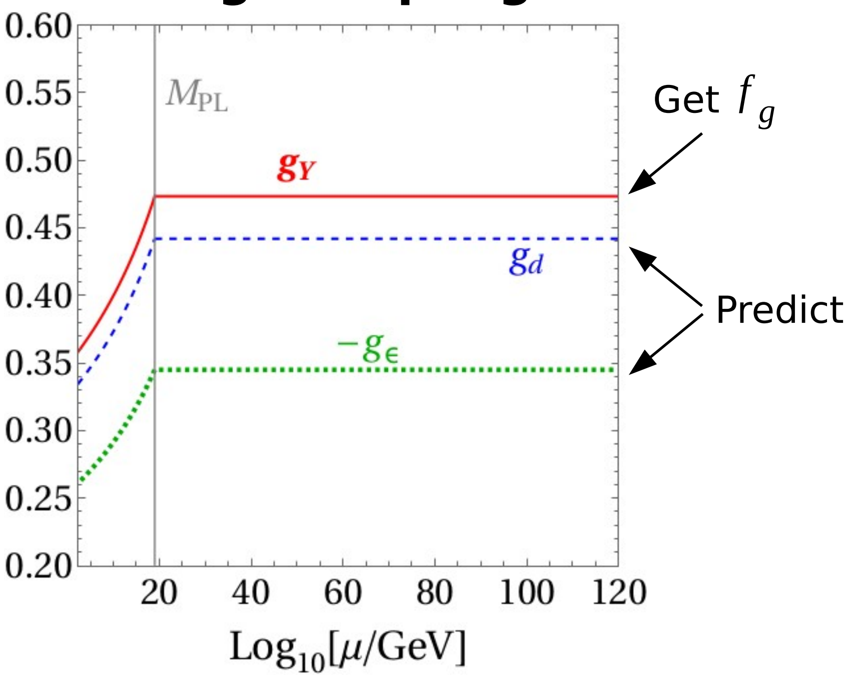
Gauge couplings



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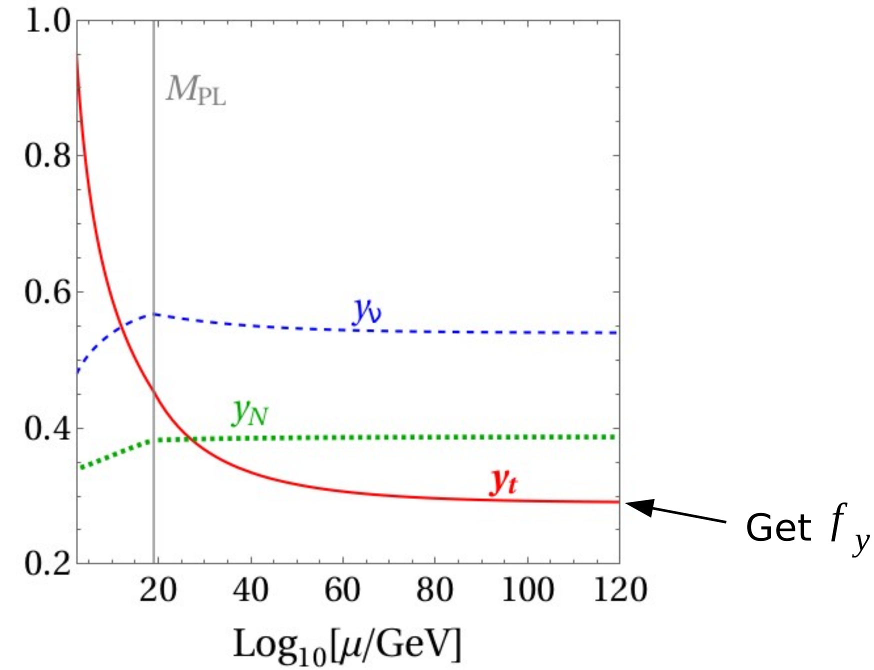

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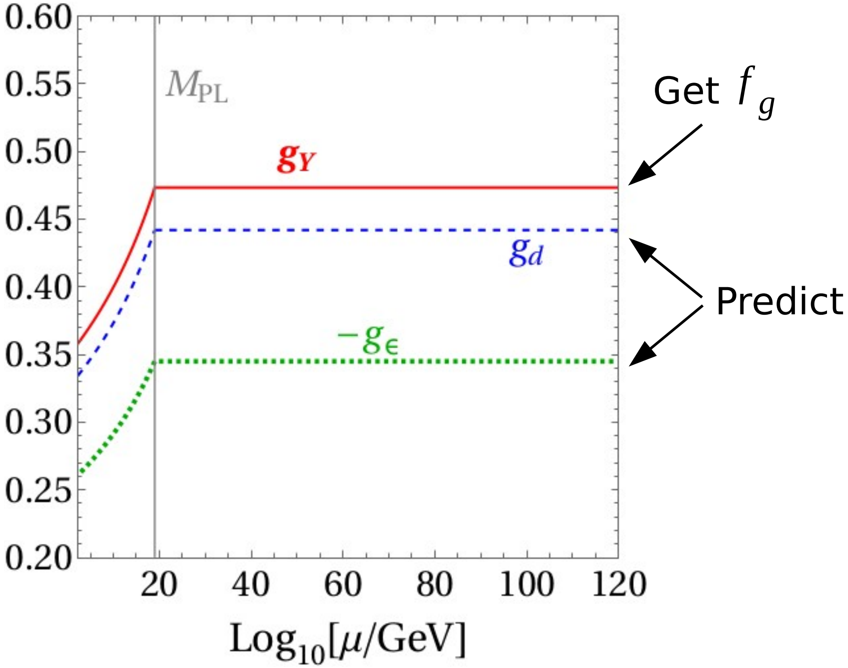

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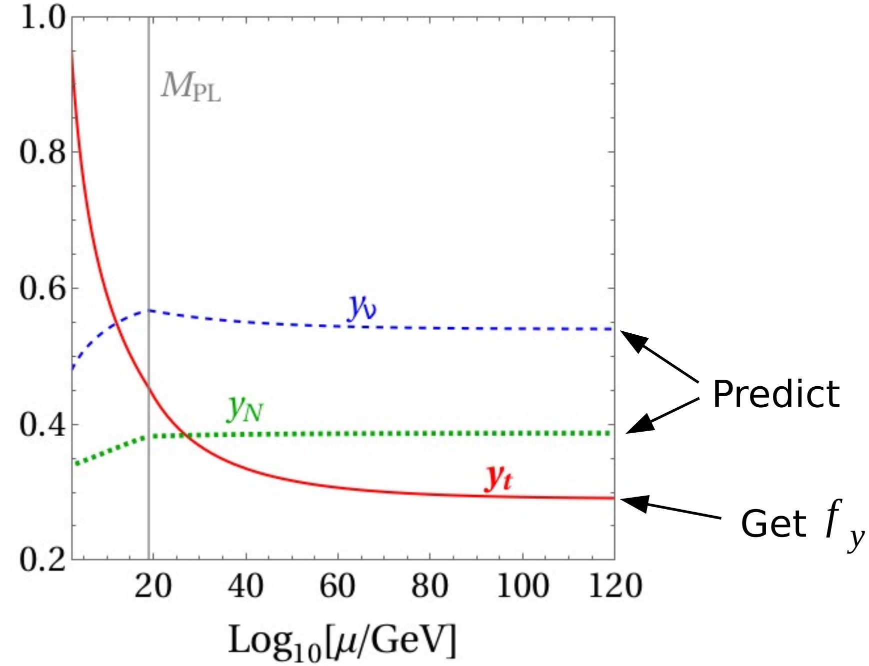

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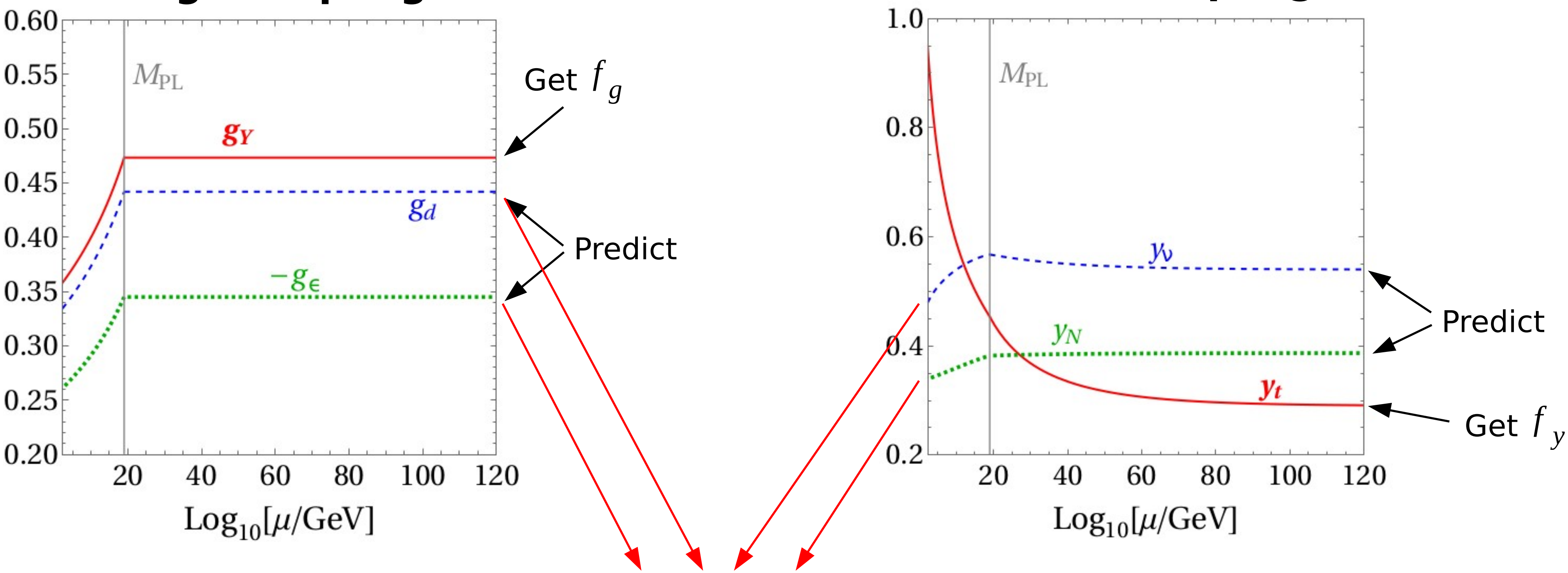
Asymptotic Safety in Quantum Gravity

$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

$$\mathcal{L} \supset -Y_\nu N\left(\tilde{\epsilon}H^*\right)^\dagger L - \frac{1}{2}Y_N S N N + \text{H.c.}$$

Gauge couplings | **Yukawa couplings**



Phenomenology!

cf. *e.g.* Chikkaballi, Kotlarski, Kowalska, **DR**, Sessolo JHEP (2023).

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An example: ASQG to explain flavour anomalies

$$\text{SM}: \quad g_3\,,\ g_2\,,\ g_Y\,,\ y_t\,,\ y_b\,,\ V_{33}\,,$$
$$\text{NP}: \quad g_D,\ g_\epsilon,\ \lambda_{Q,2},\ \lambda_{Q,3},\ \lambda_{L,2}\,,$$

Particle content

$$S : (\mathbf{1}, \mathbf{1}, 0, Q_S)$$

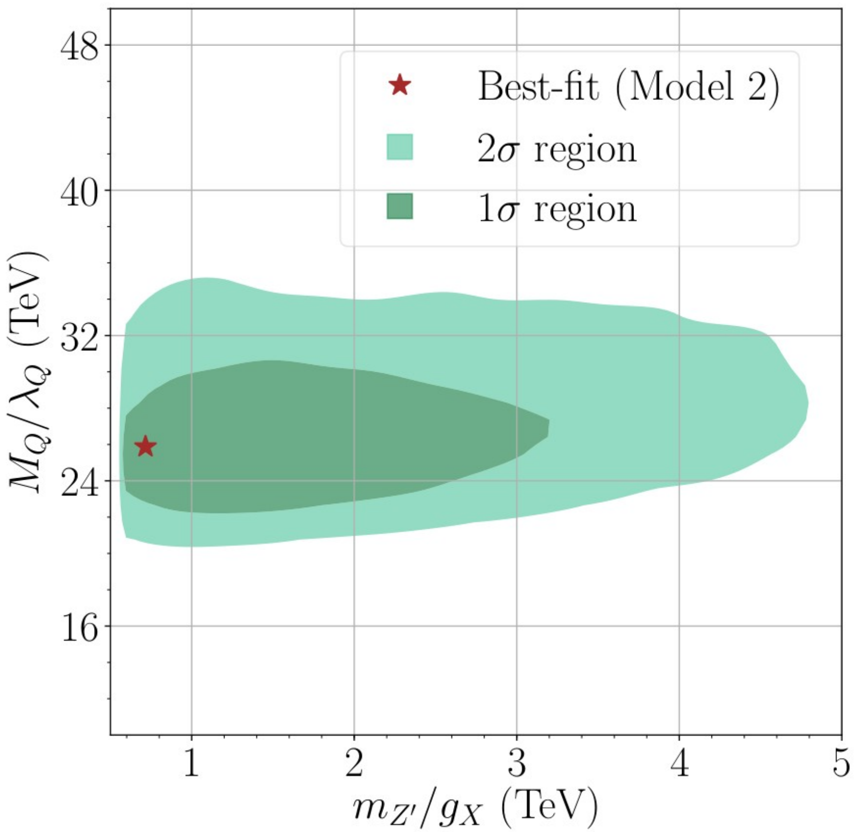
$$Q : (\mathbf{3}, \mathbf{2}, 1/6, Q_S) \qquad Q' : (\bar{\mathbf{3}}, \bar{\mathbf{2}}, -1/6, -Q_S)$$

$$L : (\mathbf{1}, \mathbf{2}, -1/2, Q_L) \qquad L' : (\mathbf{1}, \bar{\mathbf{2}}, 1/2, -Q_L)\,.$$

Interaction

$$\mathcal{L} \supset (-\lambda_{Q,i}\, S\, Q'\, q_i + \text{H.c.}) - M_Q Q' Q\,.$$

$$\mathcal{L} \supset \lambda_{L,i} S^{(*)} L' l_i + m_L L' L + \text{H.c.}\,,$$



Kowalska, Kumar, Sessolo. arXiv: 1903.10932

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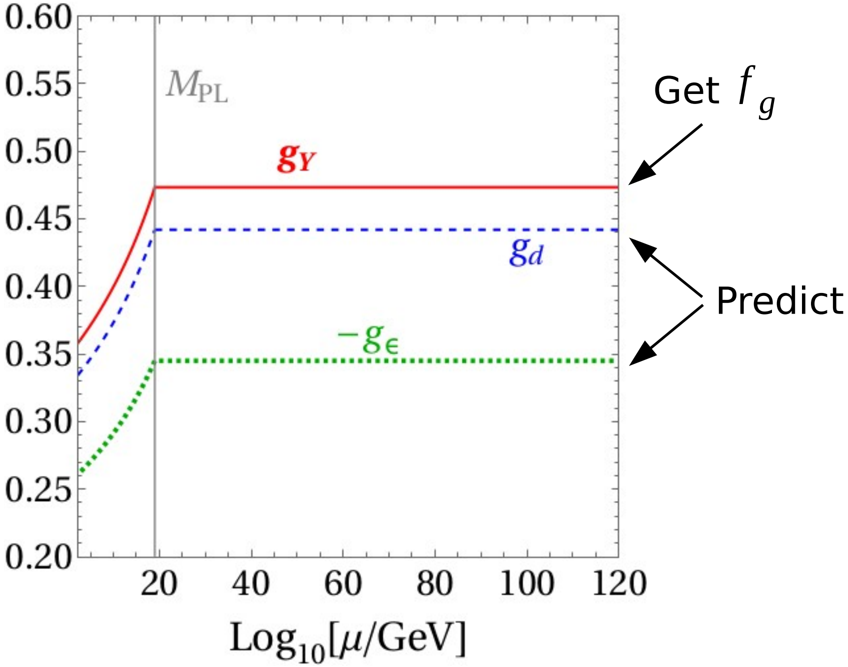
How robust are particle physics predictions in asymptotic safety?

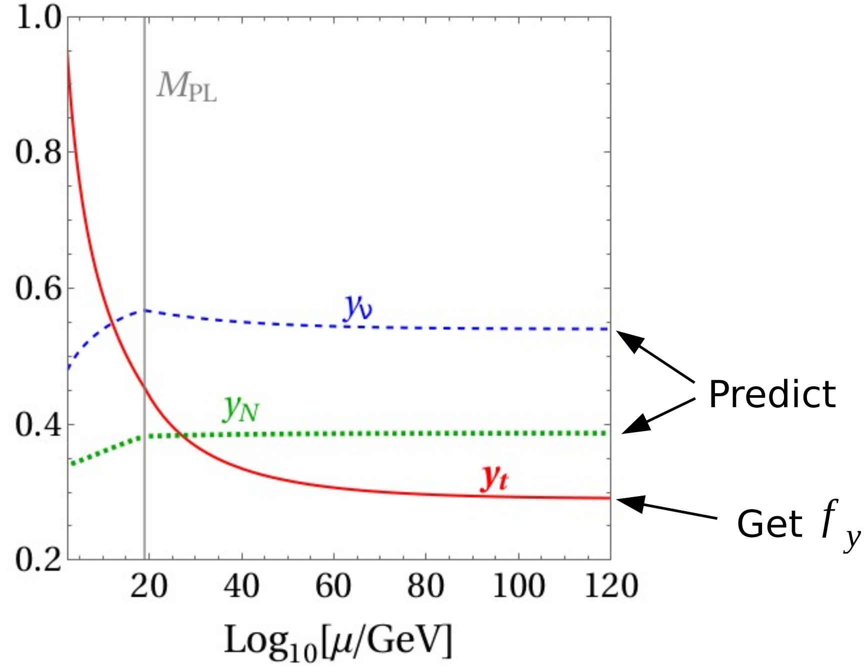
$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

$$\mathcal{L} \supset -Y_\nu N\,(\tilde{\epsilon}H^*)^\dagger L - \frac{1}{2}Y_N SNN + \text{H.c.}$$

Gauge couplings



Yukawa couplings



Sources of uncertainties →

1 - Computations of the beta functions are performed at 1-loop level.
2 - Planck scale is set arbitrarily at 10^{19} GeV.
3 - Gravity decouples instantaneously at the Planck scale.

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Higher loops computations: Gauge Sector

Renormalization Group Equations:

$$\frac{dg_Y}{dt} = \frac{1}{16\pi^2}\tilde{b}_Y g_Y^3 - f_g\, g_Y$$

Known from experiments

$$\frac{dg_d}{dt} = \frac{1}{16\pi^2}\left[\tilde{b}_Y g_d g_\epsilon^2 + \tilde{b}_d g_d^3 + \tilde{b}_\epsilon g_d^2 g_\epsilon\right] - f_g\, g_d$$

$$\frac{dg_\epsilon}{dt} = \frac{1}{16\pi^2}\left[\tilde{b}_Y\left(g_\epsilon^3 + 2g_Y^2 g_\epsilon\right) + \tilde{b}_d g_d^2 g_\epsilon + \tilde{b}_\epsilon\left(g_Y^2 g_d + g_d g_\epsilon^2\right)\right] - f_g\, g_\epsilon\,.$$

$$\tilde{b}_y = \left(b_Y + \Pi^{(Y)}_{n\geq 2}\right)$$

$$\tilde{b}_d = \left(b_d + \Pi^{(Y)}_{n\geq 2}\right)$$

$$\tilde{b}_\epsilon = \left(b_\epsilon + \Pi^{(Y)}_{n\geq 2}\right)$$

At the fixed point, the ratio of gauge couplings does not depend on f_g :

$$r^*_{g,d}(n \text{ loops}) \equiv \frac{g_d^*}{g_Y^*}(n \text{ loops}) \approx \frac{2\tilde{b}_Y}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}$$

$$r^*_{g,\epsilon}(n \text{ loops}) \equiv \frac{g_\epsilon^*}{g_Y^*}(n \text{ loops}) \approx -\frac{\tilde{b}_\epsilon}{\sqrt{4\tilde{b}_Y\tilde{b}_d - \tilde{b}_\epsilon^2}}$$

Uncertainties:

$\delta g_Y^*/g_Y^*$	$\delta g_d^*/g_d^*$	$\delta g_\epsilon^*/g_\epsilon^*$	$\delta g_d/g_d(M_t)$	$\delta g_\epsilon/g_\epsilon(M_t)$
0.3%	-0.1%	-0.1%	-0.4%	-0.5%

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In progress projects not mentioned for time reasoning

Vector-Like Fermion contribution to Gravitational Waves signals

In collaboration with K. Kowalska; E.M. Sessolo

FRG beta function of QCD at 2-loop using Proper-time regulatisation

In collaboration with G. Giacometti; D. Zappala

Gauge independent FRG calculations in Quantum Gravity with matter

In collaboration with G. Giacometti; K. Kowalska; E.M. Sessolo; D. Zappala

Conclusions

- Vector-like fermions are a type of fermions that can “naturally” be much heavier than the SM fermions, which makes them the perfect candidates for physics beyond the Standard Model.

- During My PhD I have studied vector-like fermions within three type of approaches: a purely phenomenological approach, a more theoretical one, and a mixture of the two.

- In the phenomenological approach we performed the scan of the parameter space of an extension of the SM with scalars and vector-like fermions.

- In the theoretical approach we used the assumption of Asymptotic Safety and the non-perturbative technology of the Functional Renormalization Group to study a model of quarks and gluons at the Fixed Point.

- In the mixed approach we used Asymptotic Safety induced by Quantum Gravity to put constraints on the parameter space of an extension of the standard model.

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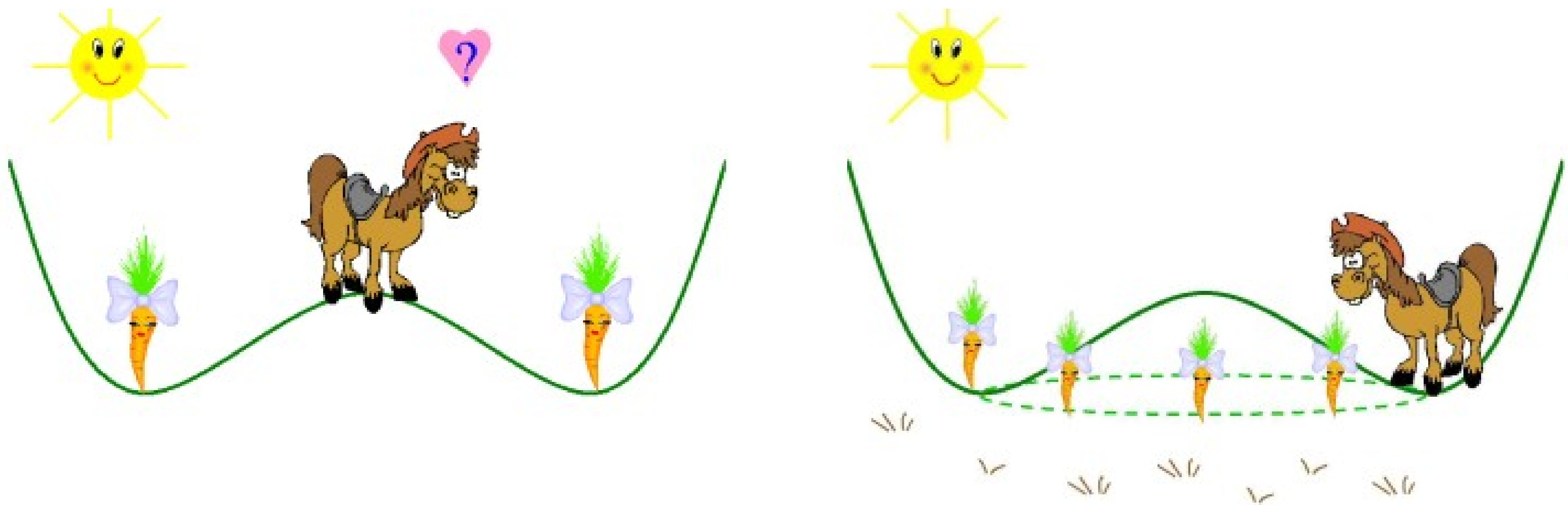
Thank for your attention

Backup

The Higgs Mechanism

Mass generation



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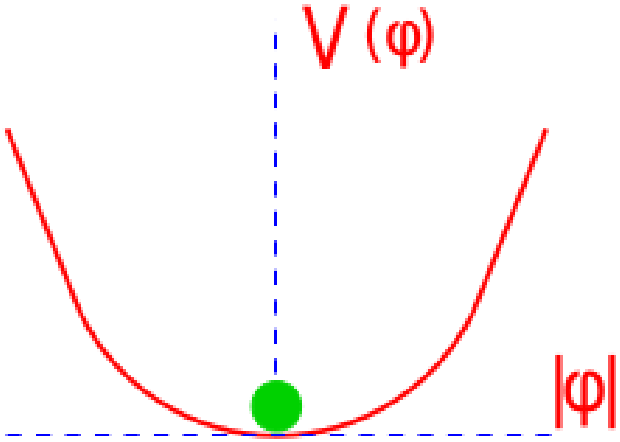
The Higgs Mechanism

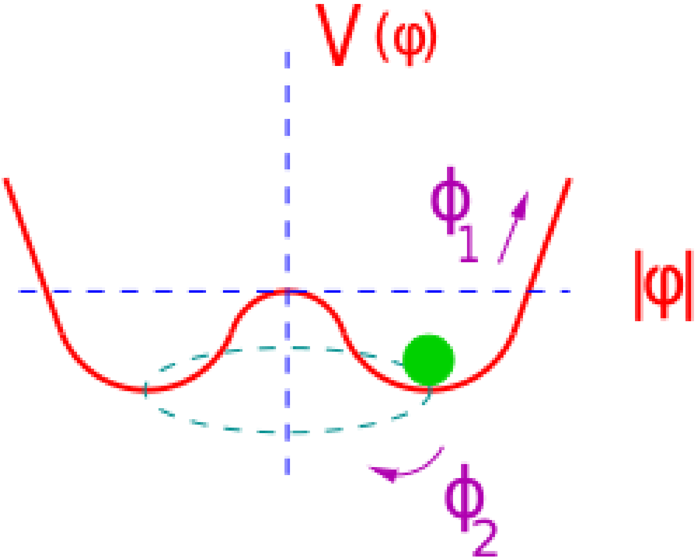
Mass generation

$$\mathcal{L} = T - V = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \left(\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\right)$$

$\mu^2 > 0$



$\mu^2 < 0$



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The Higgs Mechanism

Mass generation

Fermion	**Mass (GeV)**	$y_f = \sqrt{2}\, m_f / v$
Electron (e)	0.000511	$\approx 2.9 \times 10^{-6}$
Muon (μ)	0.10566	$\approx 6.1 \times 10^{-4}$
Tau (τ)	1.77686	$\approx 1.0 \times 10^{-2}$
Up (u)	~ 0.0022	$\approx 1.3 \times 10^{-5}$
Charm (c)	~ 1.27	$\approx 7.3 \times 10^{-3}$
Top (t)	~ 173	≈ 1.0
Down (d)	~ 0.0047	$\approx 2.7 \times 10^{-5}$
Strange (s)	~ 0.096	$\approx 5.5 \times 10^{-4}$
Bottom (b)	~ 4.18	$\approx 2.4 \times 10^{-2}$

The seesaw mechanism

Mass of 3rd generation fermions

$$m_t \approx \frac{1}{\sqrt{2}} \frac{y^u_{43} x^Q_{34} v_\phi v_u}{\sqrt{(x^Q_{34} v_\phi)^2 + 2(M^Q_4)^2}}$$

Mass of 2nd generation fermions

$$m_c \approx \frac{y^u_{24} x^u_{42} v_\phi v_u}{2\, M^u_4}$$

Mass Matrix for the fermions

$$\mathrm{M}_D = \left(\begin{array}{c|ccccc} & d_{1R} & d_{2R} & d_{3R} & d_{4R} & \widetilde{Q}_{4R} \\ \hline Q_{1L} & 0 & 0 & 0 & y^d_{14}\langle H^0_d\rangle & 0 \\ Q_{2L} & 0 & 0 & 0 & y^d_{24}\langle H^0_d\rangle & 0 \\ Q_{3L} & 0 & 0 & 0 & y^d_{34}\langle H^0_d\rangle & x^d_{34}\langle\phi\rangle \\ Q_{4L} & 0 & 0 & y^d_{43}\langle H^0_d\rangle & 0 & M^Q_4 \\ \widetilde{d}_{4L} & 0 & x^d_{42}\langle\phi\rangle & x^d_{43}\langle\phi\rangle & M^d_4 & 0 \end{array}\right)$$

This element is 0 in the up and charged lepton mass matrices

CKM matrix analytically computed for the first time in our work

S.F.King, JHEP 09, 069 (2018)

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Perturbativity & g-2

- Build a model
- Impose physical conditions
- Scan the parameter space

$$g, y(\mathrm{NP}) < \sqrt{4\pi}$$
$$\lambda(\mathrm{NP}) < 4\pi$$

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Perturbativity & g-2

- Build a model
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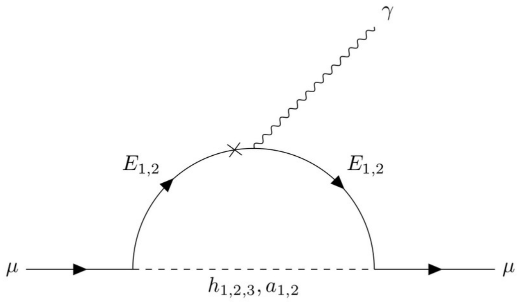
We define the cutoff energy for the model by requiring that any New Physics wrt the model is at such an energy scale that the corrections to g-2 are negligible.

$$\Delta a_\mu^\Lambda \sim \frac{1}{16\pi^2}\frac{m_\mu\, v}{\Lambda^2} y_L(\Lambda)\, y_R(\Lambda)$$



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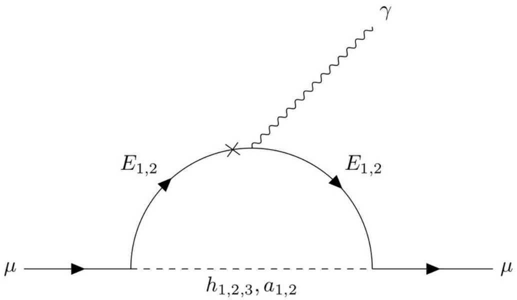

By requiring that such correction is smaller than 3σ and in the most pessimistic scenario → $\Lambda \gtrsim 50 \text{ TeV}$

$$y_L(\Lambda) = y_R(\Lambda) = \sqrt{4\pi}$$

~~$g, y(\text{NP}) < \sqrt{4\pi}$~~ $\quad g, y(\text{NP}) \lesssim 1$

~~$\lambda(\text{NP}) < 4\pi$~~ $\quad \lambda(\text{NP}) \lesssim 2$

All benchmark point from previous works are this way excluded.

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Conformal Window

$$g^* = +0.456\epsilon + 0.781\epsilon^2 + 6.610\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$y^* = +0.211\epsilon + 0.508\epsilon^2 + 3.322\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$u^* = +0.200\epsilon + 0.440\epsilon^2 + 2.693\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$v^* = -0.137\epsilon - 0.632\epsilon^2 - 4.313\epsilon^3 + \mathcal{O}(\epsilon^4)$$

Let us be a little bit more quantitative and ask the questions:

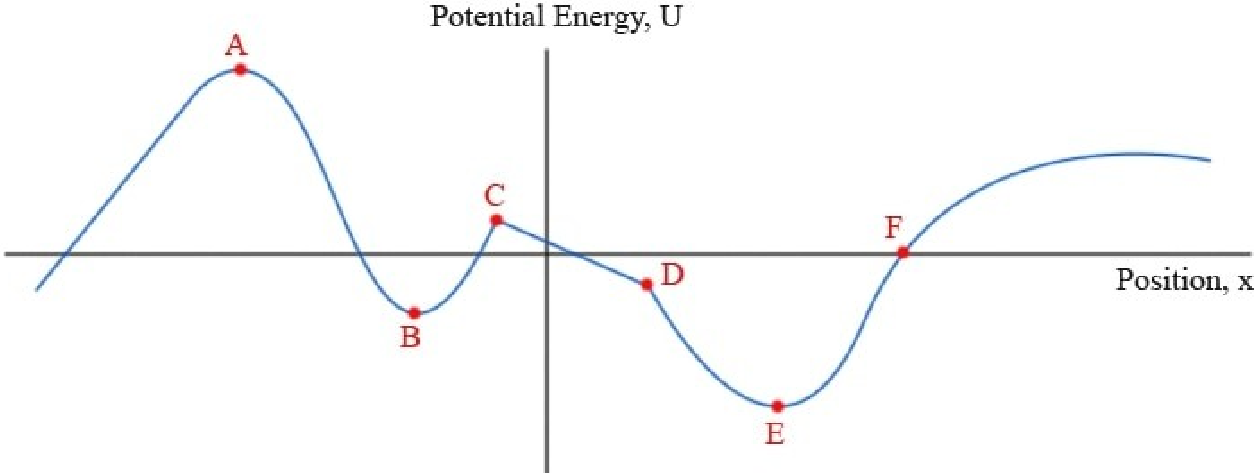
- For what values of the Veneziano parameter do we actually have a fixed point?
- What can cause a fixed point to disappear?

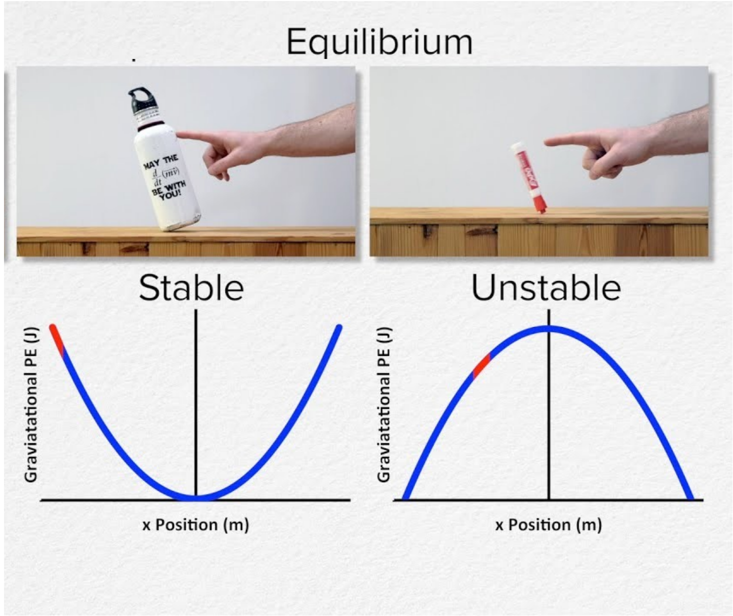
The values of the Veneziano parameter for which the fixed point exist is called

CONFORMAL WINDOW

Vacuum Stability

The conformal window can “close” because of vacuum stability.





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Fixed Point Merger

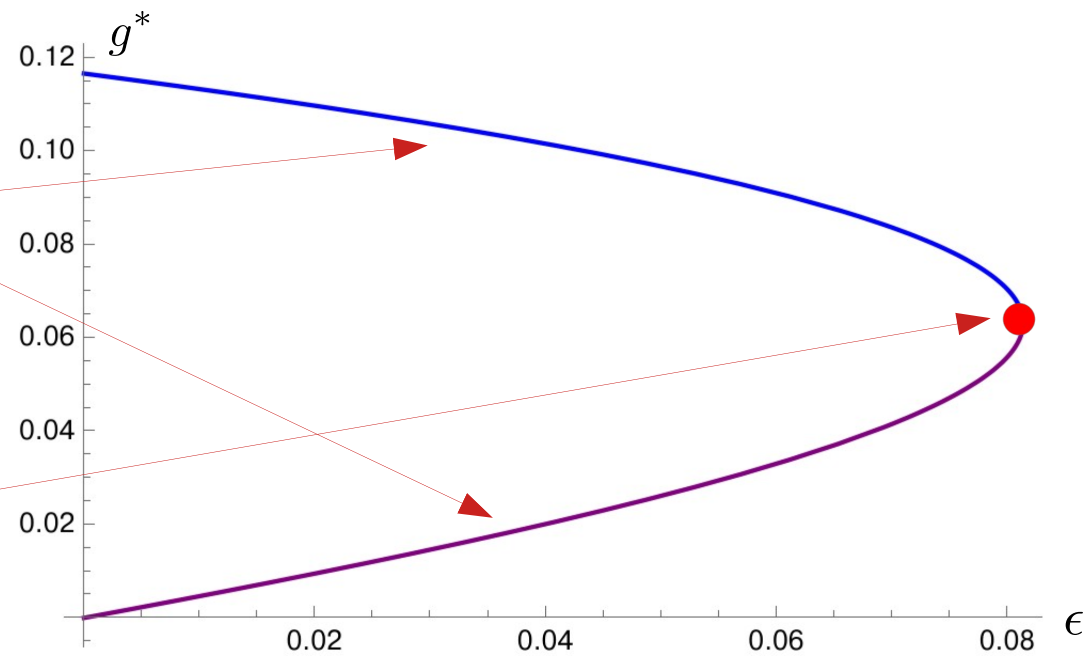
$$A + B\,g + g^2 = 0$$

$$g^*_\pm = \frac{-B \pm \sqrt{B^2 - 4A}}{2}$$

- If the expression inside the squared root is negative, we have a pair of complex conjugate poles.
- On the other hand, if the expression inside squared root is positive, we have two real solutions, with a split given by the squared root term.

For small values of the Veneziano parameter the FPs are real

At a certain value, the solutions become complex and the FPs disappear



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Functional Renormalisation

- Build a toy-model
- Non-perturbative calculations
- Look for UV Fixed Points

$$\partial_t U(\mathrm{Tr} H^\dagger H) \qquad \partial_t C(\mathrm{Tr} H^\dagger H) \qquad \partial_t Y(\mathrm{Tr} H^\dagger H)$$

Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \left[\partial_t R_k \cdot \left(\Gamma_k^{(2)} + R_k \right)^{-1} \right]$$

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Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left[\partial_t R_k \cdot \left(\Gamma_k^{(2)} + R_k\right)^{-1}\right]$$

Regulator

$$R_k = Z_k\,(k^2 - q^2)\,\Theta(k^2 - q^2)$$

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Running of Beyond Marginal Operators

- Build a toy-model
- Non-perturbative calculations
- Look for UV Fixed Points

$$\partial_t u = -4u + (2+\eta_H)\rho u' + \frac{1}{2}\left(\frac{1}{1+u'+4\rho\, c} + \frac{1}{1+u'}\right) - \frac{2N_C}{N_F}\frac{1}{1+\rho\, y^2}$$

$$\partial_t c = 2\eta_H c + (2+\eta_H)\rho c' - \frac{2N_C}{N_F}\frac{y^4}{(1+\rho y^2)^3} + \frac{1}{2}\left(-\frac{128\rho^3 c^5}{(1+u')^3(1+4\,\rho\, c+u')^3} + \frac{64\rho^2 c^3(c-\rho c')}{(1+u')^2(1+4\,\rho\, c+u')^3} - \frac{8\rho c c'}{(1+4\,\rho\, c+u')^3} - \frac{48\rho^2 c^2 c'}{(1+u')(1+4\,\rho\, c+u')^3} + \frac{16c^2}{(1+4\,\rho\, c+u')^3} - \frac{2c'}{(1+4\,\rho\, c+u')^2}\right)$$

Tuğba Büyükbeşe, PhD Thesis

$$\partial_t y = -3\alpha_g y(0) + \frac{1}{2}(2\eta_\psi + \eta_H)y + (2+\eta_\phi)\rho y' - \frac{1}{2}\left(\frac{y'}{(1+4\,\rho\, c+u')^2} + \frac{y'}{(1+u')^2}\right) + \frac{y^3}{2(1+\rho y^2)(1+4\rho\, c+u')}\left(\frac{1}{1+4\rho\, c+u'} + \frac{1}{1+\rho y^2}\right) - \frac{y^3}{2(1+u')(1+\rho y^2)}\left(\frac{1}{1+\rho y^2} + \frac{1}{1+u'}\right)$$

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Fixed Point

- Build a toy-model
- Non-perturbative calculations
- Look for UV Fixed Points

$$\partial_t u = 0 \qquad \partial_t c = 0 \qquad \partial_t y = 0$$

$$u(\rho) = \sum_{n=0}^{\infty} \alpha_n \rho^{n+1} \qquad c(\rho) = \sum_{n=1}^{\infty} \gamma_n \rho^{n-1} \qquad y(\rho) = \sum_{n=0}^{\infty} y_n \rho^{n}$$

$$\partial_t \alpha_n = 0 \qquad \partial_t \gamma_n = 0 \qquad \partial_t y_n = 0$$

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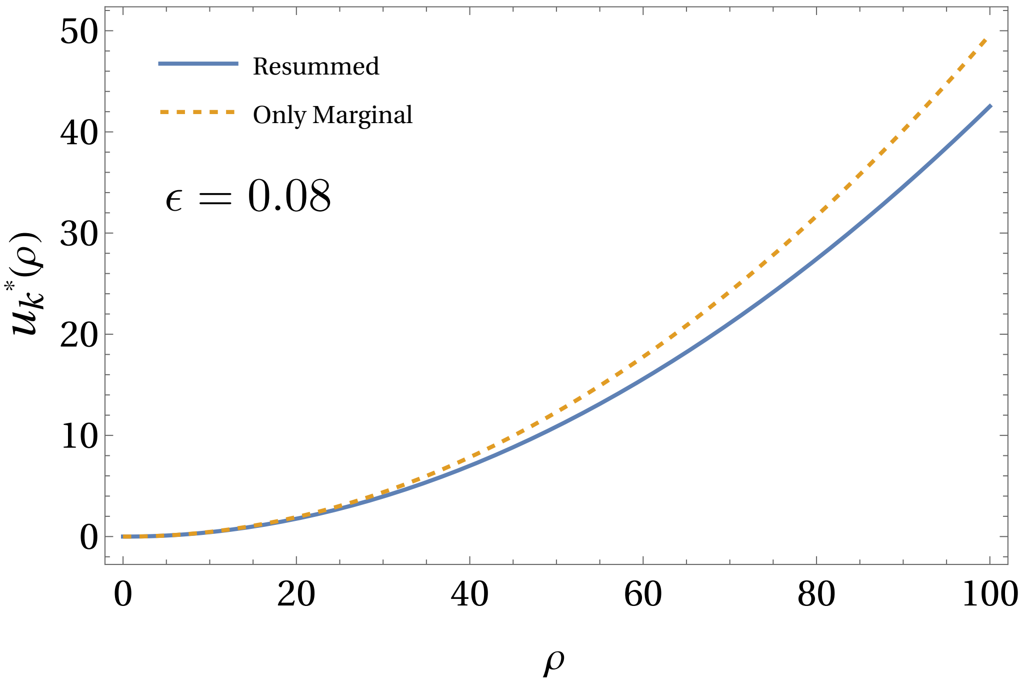
Resummation

At leading order in ε a re-summation of the couplings can be performed:

$$u^*(\rho) = \alpha_1^* \rho^2 + \frac{A^2\,\rho^2}{4}\log(1 + A\,\rho) + \frac{B^2\,\rho^2}{4}\log(1 + B\,\rho) - \frac{N_c}{N_F} D^2\,\rho^2 \log(1 + D\,\rho)$$



$$A \equiv 2\alpha_1^* \qquad B \equiv 2\alpha_1^* + 4\gamma_1^*$$

$$D \equiv \frac{N_F}{N_C}\alpha_y^*$$

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Higher loops computations: Yukawa Sector

Known from experiments

Renormalization Group Equations:

$$\frac{dy_1}{dt} = \frac{y_1}{16\pi^2}\left(a_1^{(1)} y_1^2 + a_2^{(1)} y_2^2 - a'^{(1)} g_1^2 + \sum_{n\geq 2} \widetilde{\Pi}_n^{(1)}\right) - f_y y_1,$$

$$\frac{dy_2}{dt} = \frac{y_2}{16\pi^2}\left(a_1^{(2)} y_1^2 + a_2^{(2)} y_2^2 - a'^{(2)} g_1^2 + \sum_{n\geq 2} \widetilde{\Pi}_n^{(2)}\right) - f_y y_2.$$

At the fixed point, solve the first equation for the gravity parameter f_y and insert it in the second equation to get:

1 loop Higher loops

$$y_2^*(2\text{ loops}) \approx \left[\frac{\left(a_1^{(2)} - a_1^{(1)}\right) y_1^{*2}(1\text{ loop}) + \left(a'^{(1)} - a'^{(2)}\right) g_1^{*2}}{a_2^{(1)} - a_2^{(2)}} + \frac{\left(a_1^{(2)} - a_1^{(1)}\right) \delta y_1^{*2} + \left(\widetilde{\Pi}_2^{(2)*} - \widetilde{\Pi}_2^{(1)*}\right)}{a_2^{(1)} - a_2^{(2)}}\right]^{1/2}$$

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Higher loops computations: Yukawa Sector - Large Yukawa

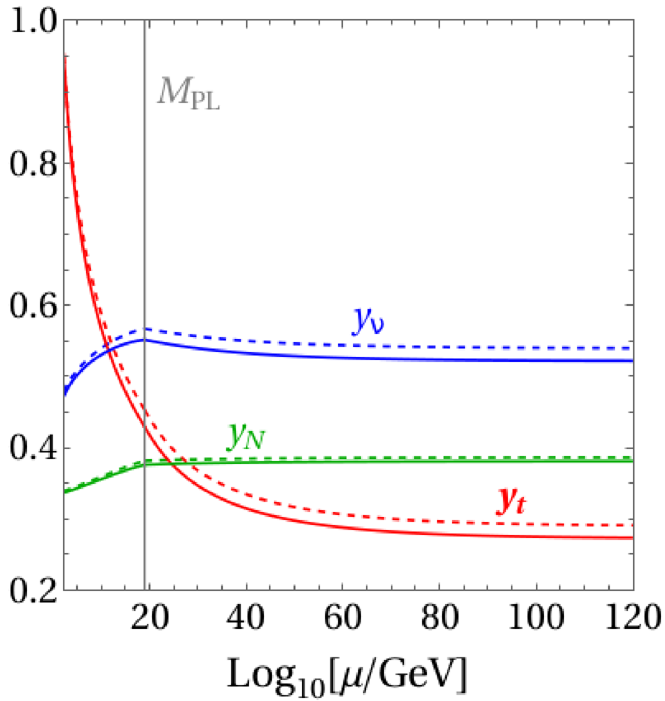
$$y_2^*(2\text{ loops}) \approx \left[\underbrace{\frac{\left(a_1^{(2)} - a_1^{(1)}\right) y_1^{*2}(1\text{ loop}) + \left(a'^{(1)} - a'^{(2)}\right) g_1^{*2}}{a_2^{(1)} - a_2^{(2)}}}_{O(1)} + \underbrace{\frac{\left(a_1^{(2)} - a_1^{(1)}\right) \delta y_1^{*2} + \left(\widetilde{\Pi}_2^{(2)*} - \widetilde{\Pi}_2^{(1)*}\right)}{a_2^{(1)} - a_2^{(2)}}}_{\text{Small}} \right]^{1/2}$$

Higher loops are negligible → predictions are very stable.

$\delta y_t^*/y_t^*$	$\delta y_\nu^*/y_\nu^*$	$\delta y_N^*/y_N^*$
-6.0%	-3.3%	-1.4%

Focusing in the infrared → uncertainties are reduced.

$\delta y_\nu/y_\nu(M_t)$	$\delta y_N/y_N(M_t)$
-1.4%	-0.8%



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Higher loops computations: Yukawa Sector - Small Yukawa

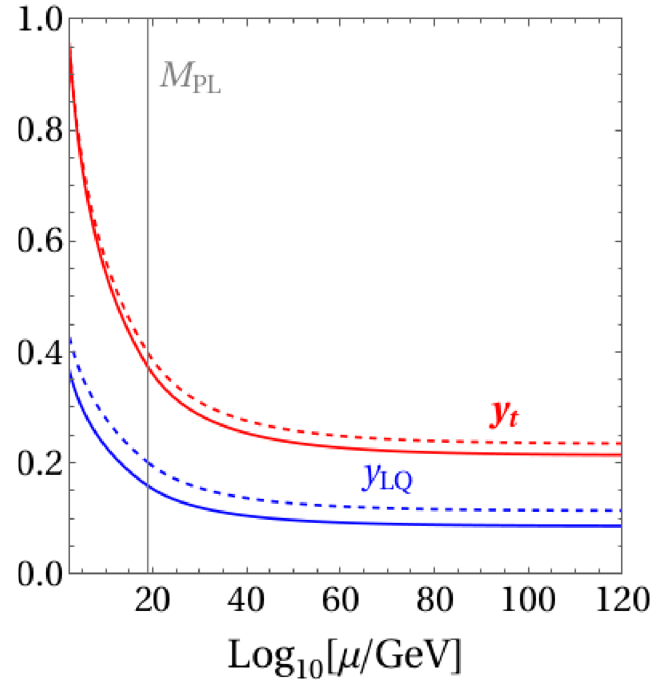
$$y_2^*(2\text{ loops}) \approx \left[\overbrace{\frac{\left(a_1^{(2)} - a_1^{(1)}\right) y_1^{*2}(1\text{ loop}) + \left(a'^{(1)} - a'^{(2)}\right) g_1^{*2}}{a_2^{(1)} - a_2^{(2)}}}^{\text{Cancel out}} + \overbrace{\frac{\left(a_1^{(2)} - a_1^{(1)}\right) \delta y_1^{*2} + \left(\widetilde{\Pi}_2^{(2)*} - \widetilde{\Pi}_2^{(1)*}\right)}{a_2^{(1)} - a_2^{(2)}}}^{\text{Main contribution}} \right]^{1/2}$$

Higher loops are important → predictions are unstable.

$\delta y_t^* / y_t^*$	$\delta y_{\rm LQ}^* / y_{\rm LQ}^*$
-8.8%	-24.5%

Focusing in the infrared → uncertainties are reduced.

$\delta y_{\rm LQ} / y_{\rm LQ}(M_t)$
-14.3%



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The Scalar Potential

- Build a model
- Impose physical conditions
- Scan the parameter space

$$V = \mu_u^2(H_u^\dagger H_u) + \mu_d^2(H_d^\dagger H_d) + \mu_\phi^2(\phi^*\phi) - \frac{1}{2}\mu_{sb}^2\left(\phi^2 + \phi^{*2}\right)$$
$$+ \frac{1}{2}\lambda_1(H_u^\dagger H_u)^2 + \frac{1}{2}\lambda_2(H_d^\dagger H_d)^2 + \lambda_3(H_u^\dagger H_u)(H_d^\dagger H_d) + \lambda_4(H_u^\dagger H_d)(H_d^\dagger H_u)$$
$$- \frac{1}{2}\lambda_5(\epsilon_{ij}H_u^i H_d^j \phi^2 + H.c.) + \frac{1}{2}\lambda_6(\phi^*\phi)^2 + \lambda_7(\phi^*\phi)(H_u^\dagger H_u) + \lambda_8(\phi^*\phi)(H_d^\dagger H_d)$$

Spectrum:
3 massive CP-Even
2 massive CP-Odd + 1 Goldstone
1 massive charged + 1 Goldstone

Alignment limit:

$$\lambda_2 = \lambda_3 + \tan^2\beta(\lambda_1 - \lambda_3)$$

$$\lambda_8 = -\tan\beta\left(\lambda_7 \tan\beta + \lambda_5\right)$$

$$\lambda_3 \approx \lambda_1 + \mathcal{O}(1/\tan^2\beta)$$

$$\lambda_7 \sim \mathcal{O}(1/\tan^2\beta) \qquad \lambda_5 \sim \mathcal{O}(1/\tan\beta)$$

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- Impose physical conditions
- Scan the parameter space

$$V = \mu_u^2 (H_u^\dagger H_u) + \mu_d^2 (H_d^\dagger H_d) + \mu_\phi^2 (\phi^* \phi) - \frac{1}{2}\mu_{sb}^2 \left(\phi^2 + \phi^{*2}\right)$$
$$+ \frac{1}{2}\lambda_1 (H_u^\dagger H_u)^2 + \frac{1}{2}\lambda_2 (H_d^\dagger H_d)^2 + \lambda_3 (H_u^\dagger H_u)(H_d^\dagger H_d) + \lambda_4 (H_u^\dagger H_d)(H_d^\dagger H_u)$$
$$- \frac{1}{2}\lambda_5 (\epsilon_{ij} H_u^i H_d^j \phi^2 + H.c.) + \frac{1}{2}\lambda_6 (\phi^* \phi)^2 + \lambda_7 (\phi^* \phi)(H_u^\dagger H_u) + \lambda_8 (\phi^* \phi)(H_d^\dagger H_d)$$

“Boundedness from below”

$$\lambda_3 + \sqrt{\lambda_2 \lambda_1} > 0$$
$$\lambda_3 + \lambda_4 + \sqrt{\lambda_2 \lambda_1} > 0$$

$$\lambda_8 + \sqrt{\lambda_2 \lambda_6} > 0$$
$$\lambda_7 + \sqrt{\lambda_1 \lambda_6} > 0$$

$$-\frac{1}{4}\frac{(\mathrm{Re}\lambda_5)^2 + (\mathrm{Im}\lambda_5)^2}{\lambda_a} + \lambda_4 > 0$$
$$4\lambda_b^2 - (\mathrm{Re}\lambda_5)^2 + \mathrm{Re}\lambda_5 \mathrm{Im}\lambda_5 > 0$$
$$4\lambda_b^2 - (\mathrm{Im}\lambda_5)^2 + \mathrm{Re}\lambda_5 \mathrm{Im}\lambda_5 > 0$$

These were already implemented in previous works ← 2HDM Type II

These conditions were not considered in previous work


My job here is done
But you didn’t do anything