Graduate Physics Seminar

NCBJ

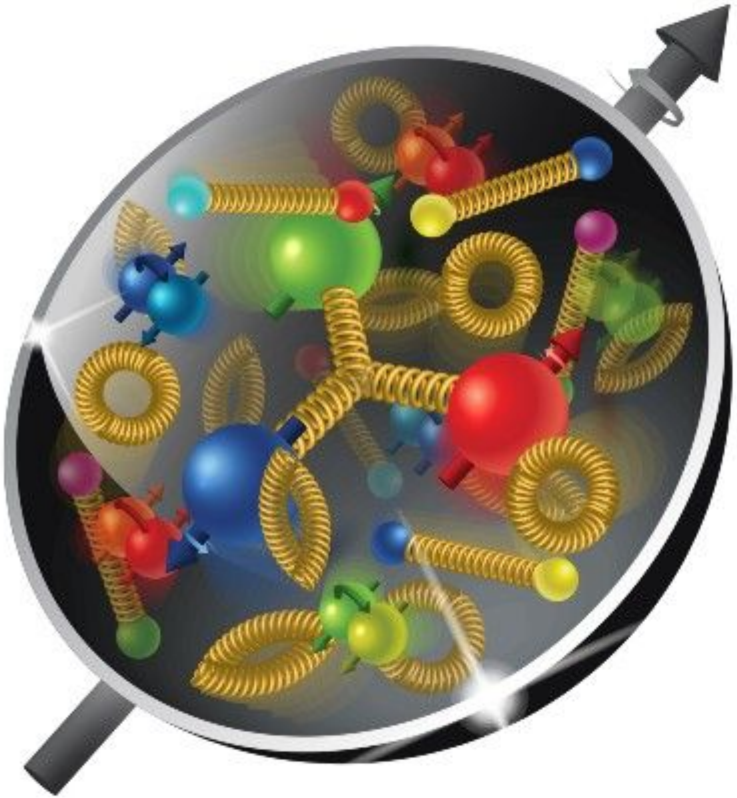
Precision calculations for high energy scattering: SIDIS and Inclusive DIS at next-to-eikonal order

-Swaleha Mulani

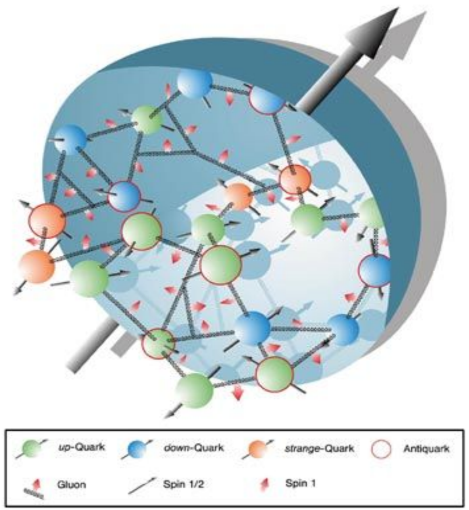
Supervisors: Dr. hab Tolga Altinoluk, Dr. Guillaume Beuf

Outline

- Introduction
  - QCD and QED
  - Deep Inelastic Scattering
  - Saturation Physics
  - Color Glass condensate
  - Eikonal Approximation
- Beyond Eikonal Order
- SIDIS
- Inclusive DIS
- Summary

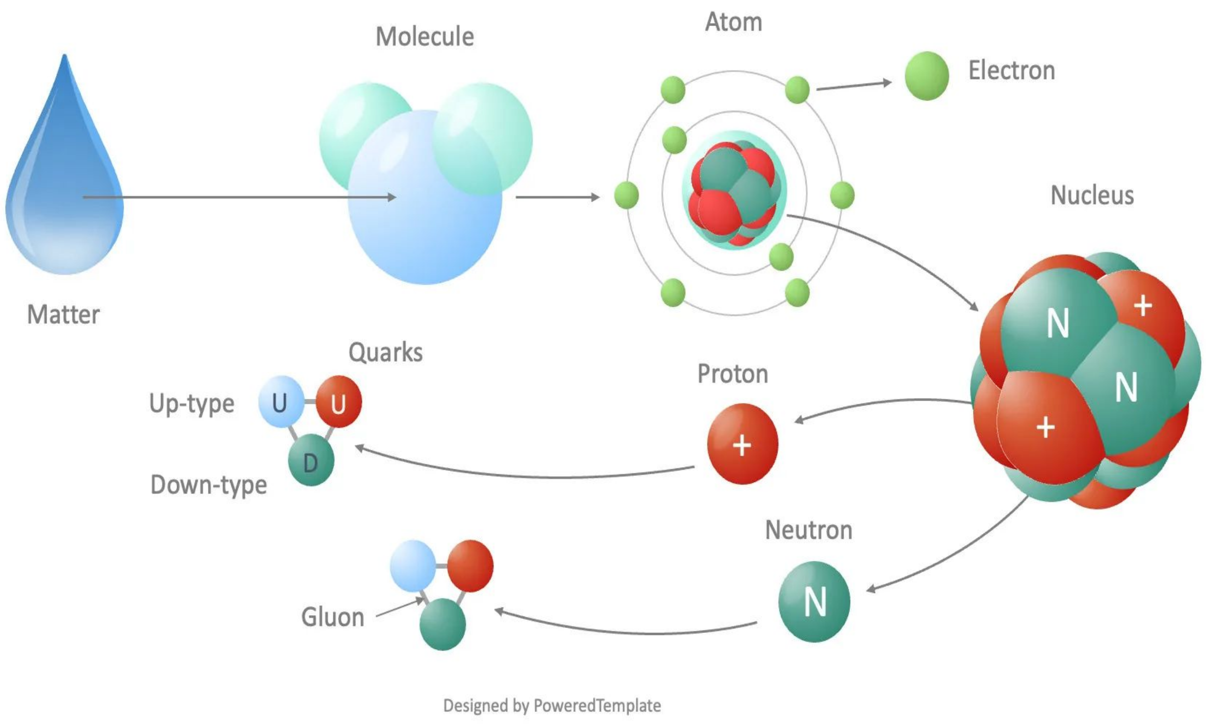


"How many quarks in a gallon again?"



Credit: CERN

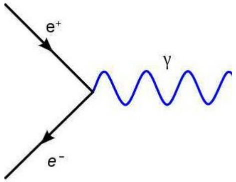
Matter from Molecule to Quark



QED and QCD

QED

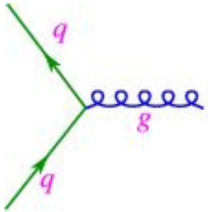
- Describes the interactions of charged particles with **electromagnetic fields**
- Particles exchange **photons** between them



- Photos are massless bosons, with spin 1, and are chargeless, do not self interact
- Coupling constant, strength of interaction, α~1/137

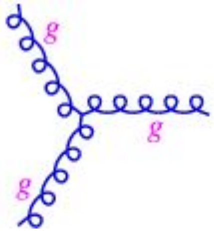
QCD

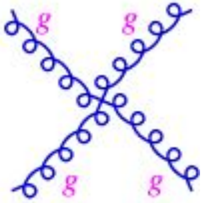
- Natural fundamental interaction that occurs between subatomic particles, **strong force**
- Particles exchange **gluons** between them



- Gluons are massless vector bosons, spin1, have color charge, self-interact
- Running coupling constant, α~α(Q)

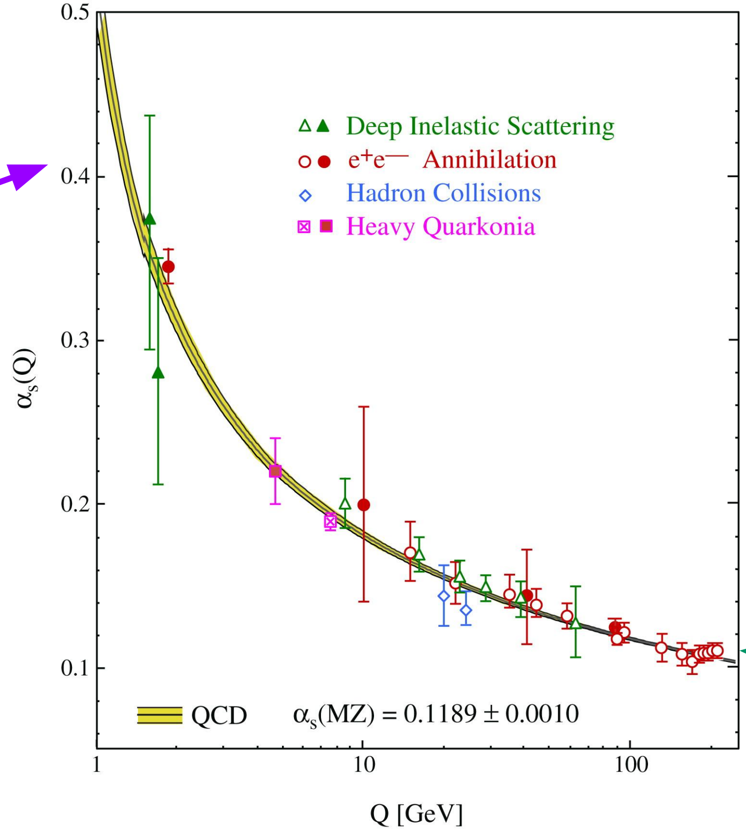
QCD Only





Coupling Constant (QCD)

Confinement



strong force confining quarks inside a proton (and keeping protons inside a nucleus)

Asymptotic Freedom

Credit: arXiv:hep-ex/0606035

Cross section

To study inside of hadrons

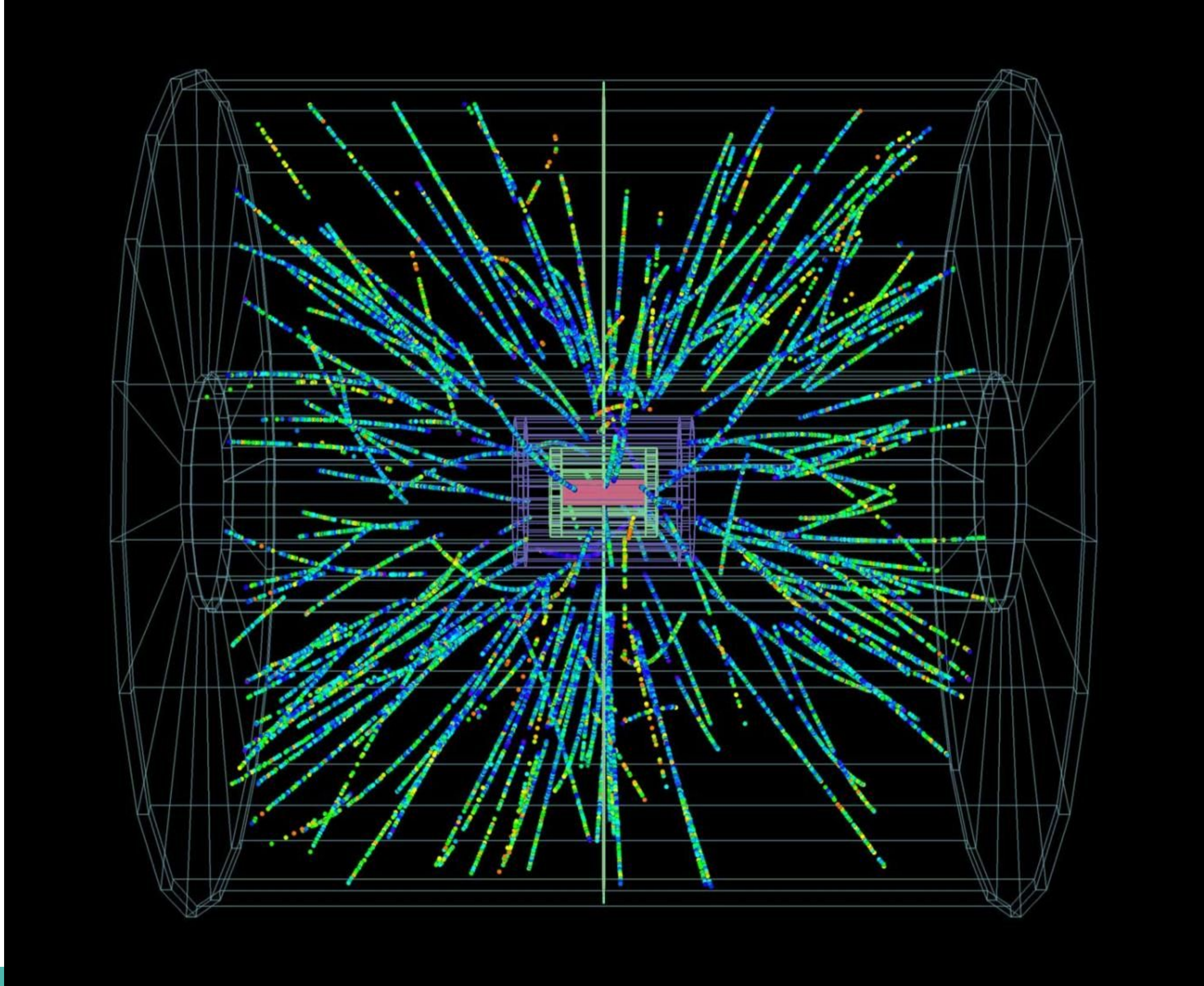
↓

We have to probe it

↓

Perform collision and measure the produced particles

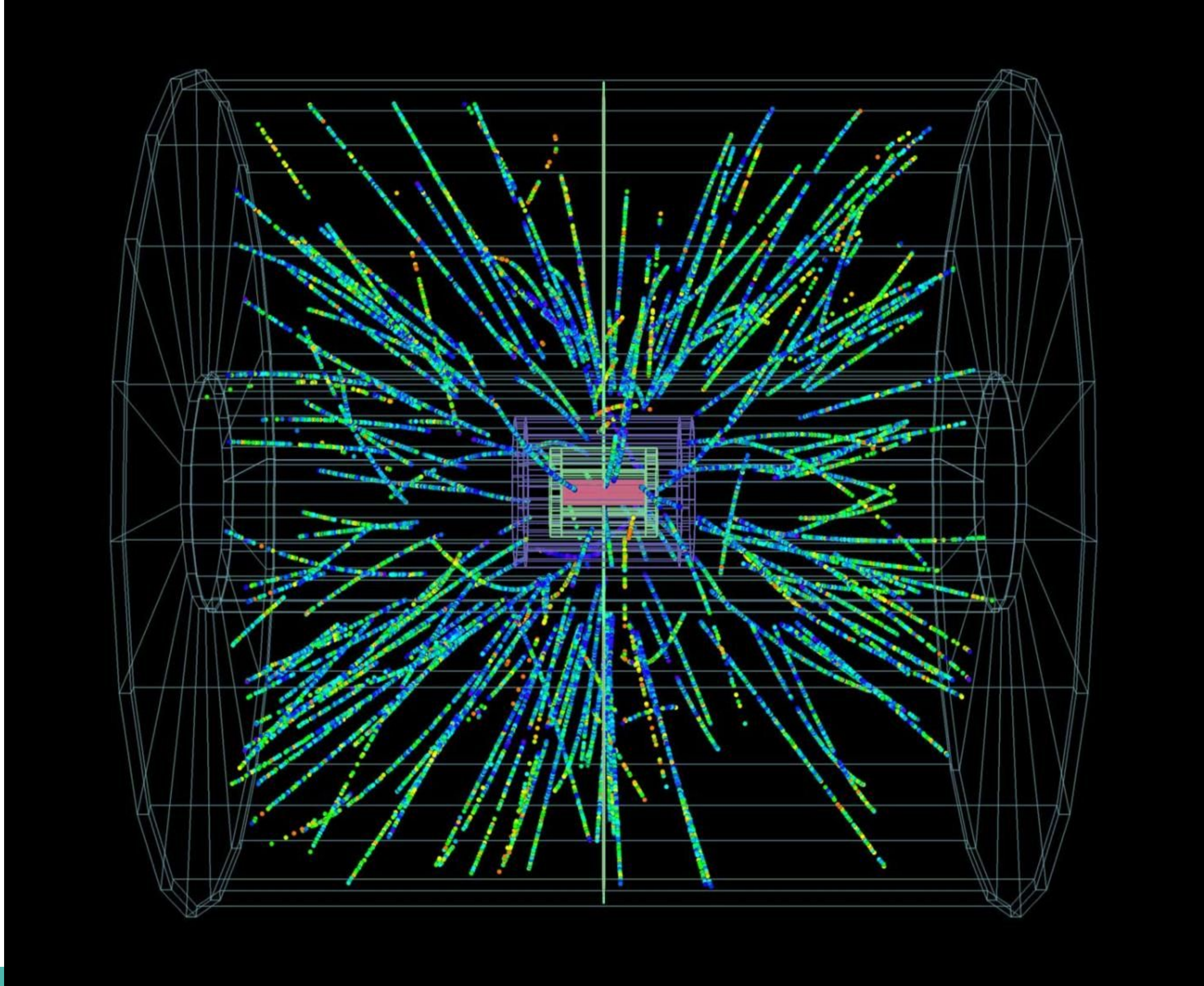
The first proton-lead collisions of 2013 send showers of particles through the ALICE detector (Image: CERN)



Cross section

probability of particles interacting or scattering off each other as they collide

Helps understanding properties, dynamics and interior of hadrons



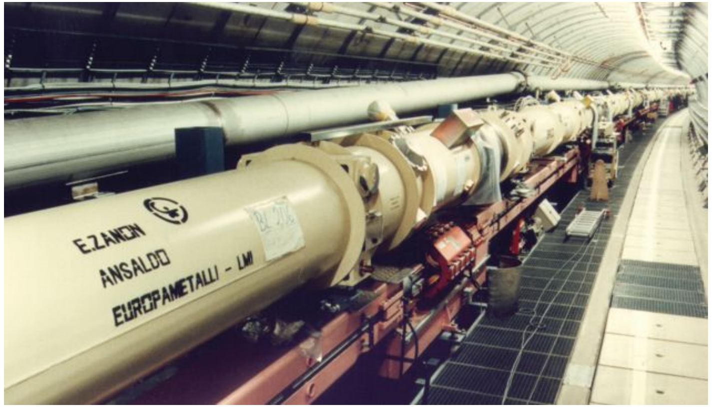
The first proton-lead collisions of 2013 send showers of particles through the ALICE detector (Image: CERN)

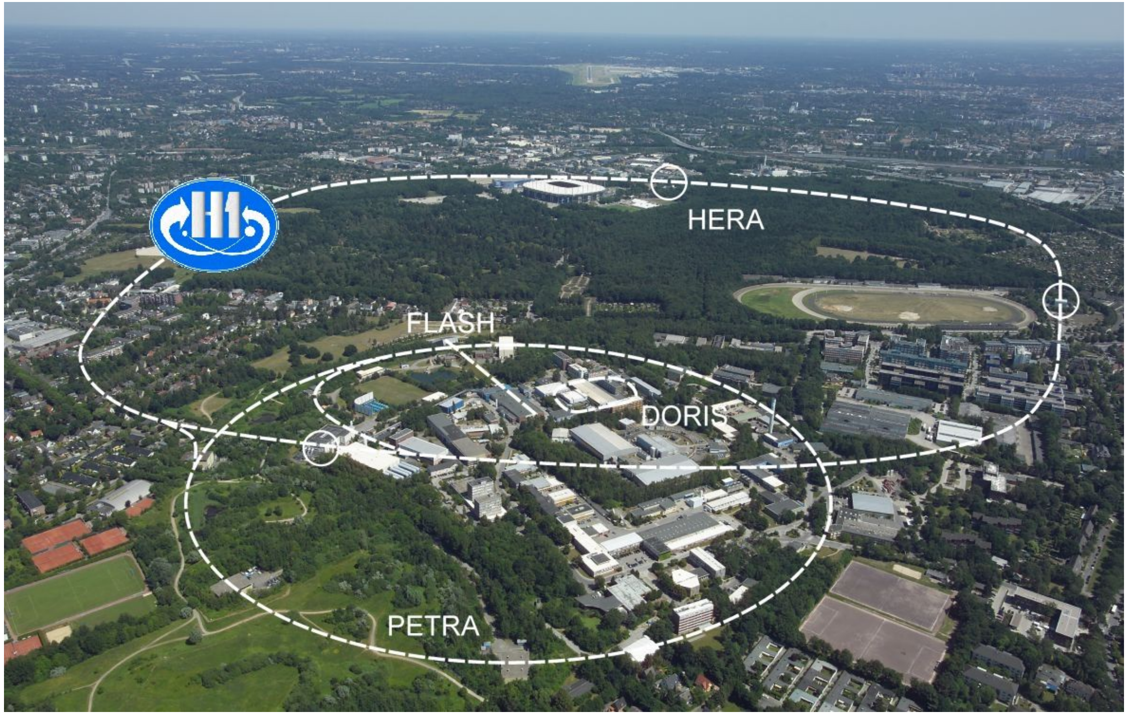
Collision Experiment: e-p collision at HERA (1992-2007)

Centre of mass energy= 320 GeV

E_e = 27.5 GeV,

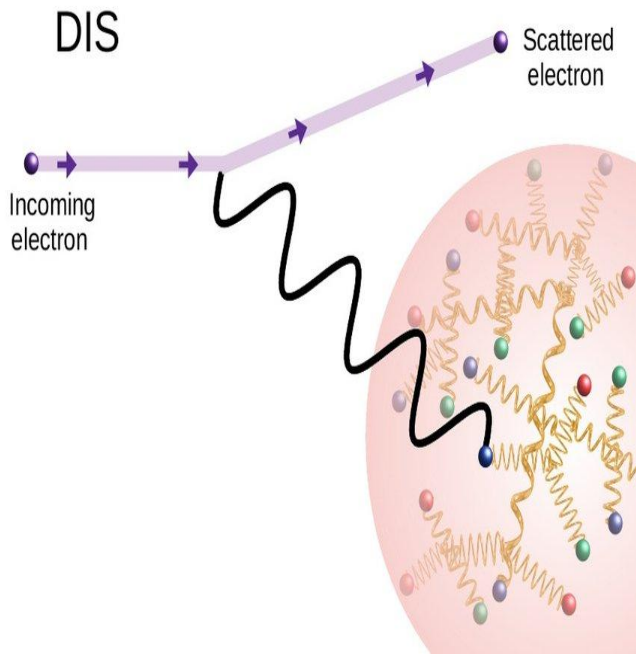
E_p = 920 GeV

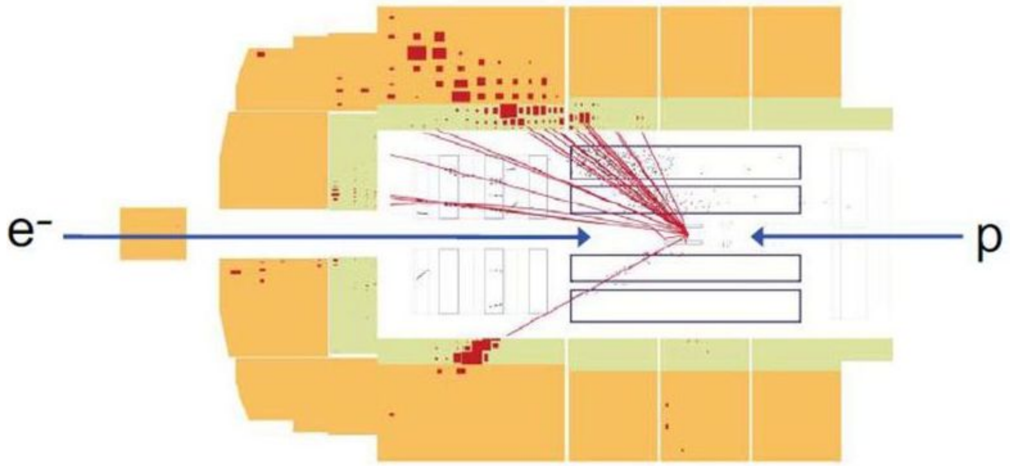




Credit: HERA

Deep Inelastic Scattering at HERA





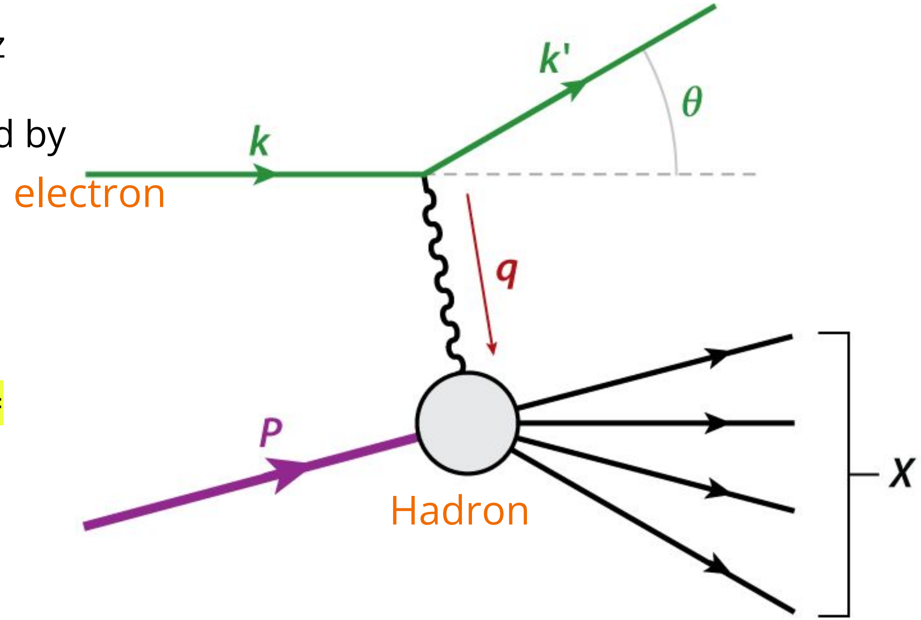
Deep: very high energy of leptons= very short wavelength= ability to probe the distances that are small compared to size of target hadron

Credit: HERA

Deep Inelastic Scattering : Kinematics

Inclusion cross-section in DIS in terms of Lorentz invariants:

- $x \equiv$ longitudinal momentum fraction carried by parton in the hadron;
  $x = p_i/P$
  p_i = initial momentum of struck parton
  P = momentum transfer of target
- Photon Virtuality: $q^2 = -Q^2 < 0$
- s= the square of center of mass energy; $s = (p+k)^2$
- $W^2 = (p+q)^2 = Q^2/x$



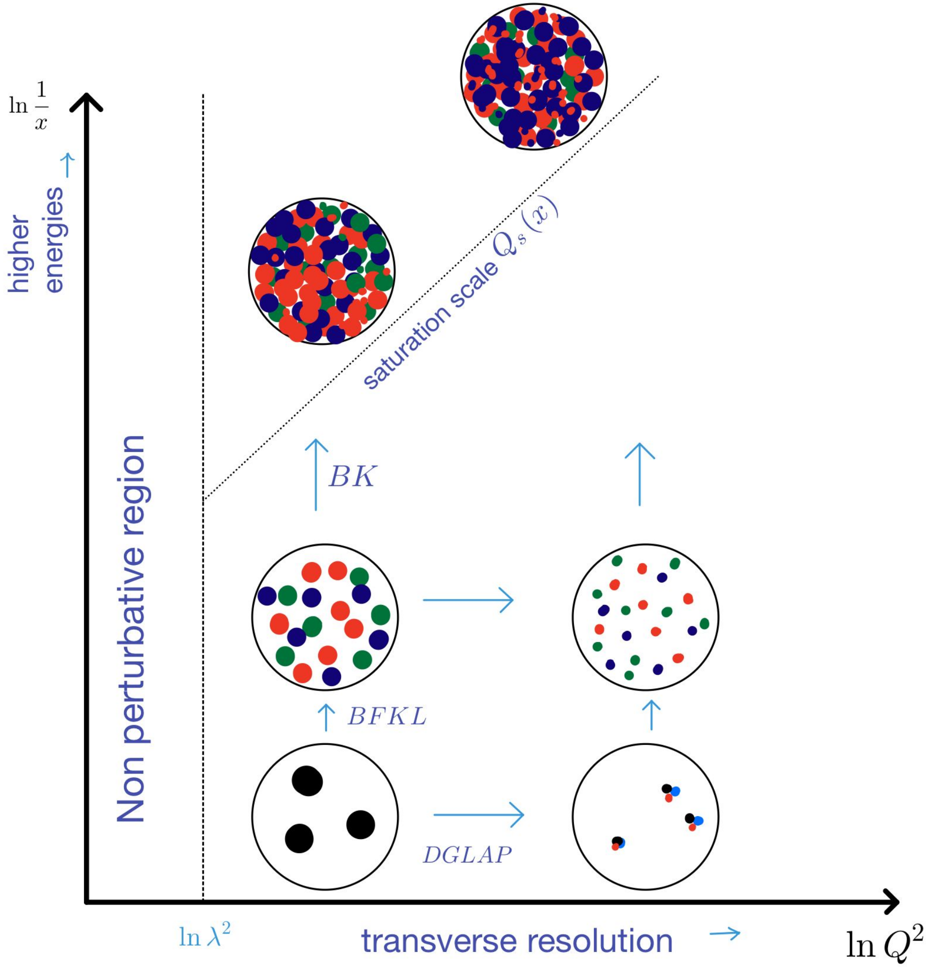
Gelis F, et al. 2010.
Annu. Rev. Nucl. Part. Sci. 60:463–89

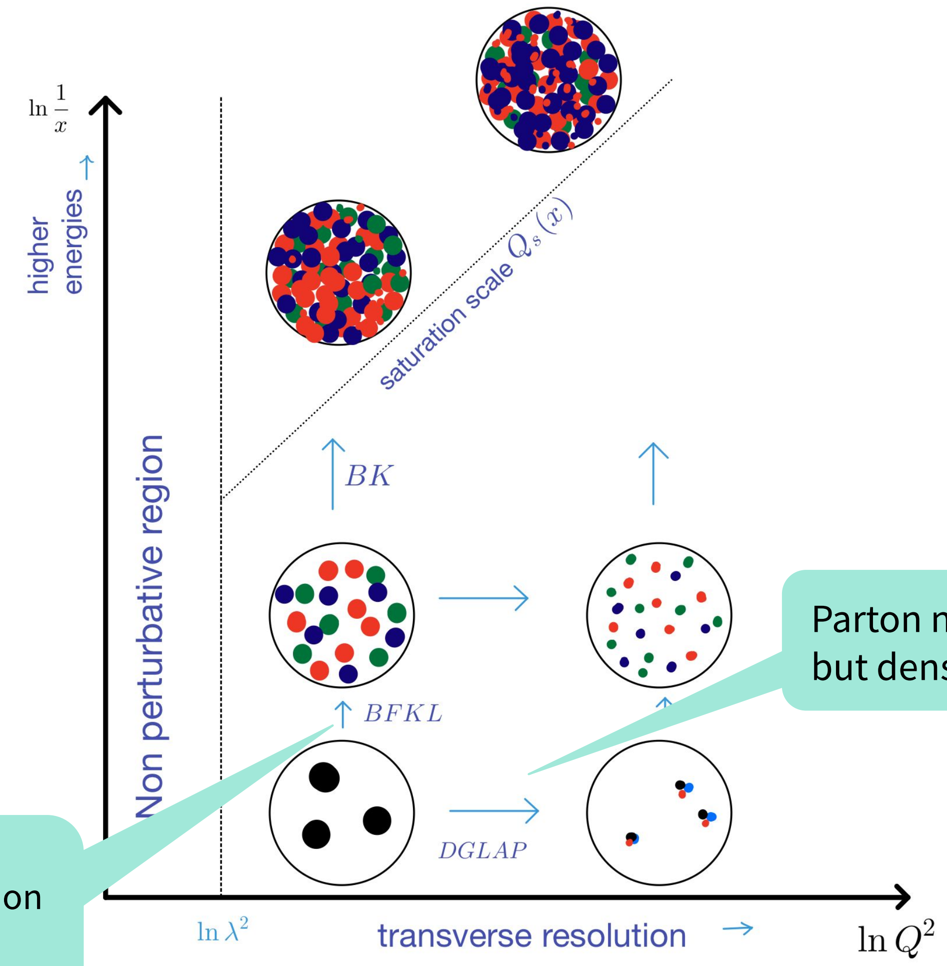
Saturation Physics:

Phase Diagram of Gluon Saturation

Two ways to increase the energy
s= Q^2/x:

1. $Q^2 \longrightarrow$ infinity, fixed x
2. $x \longrightarrow 0$, fixed Q^2

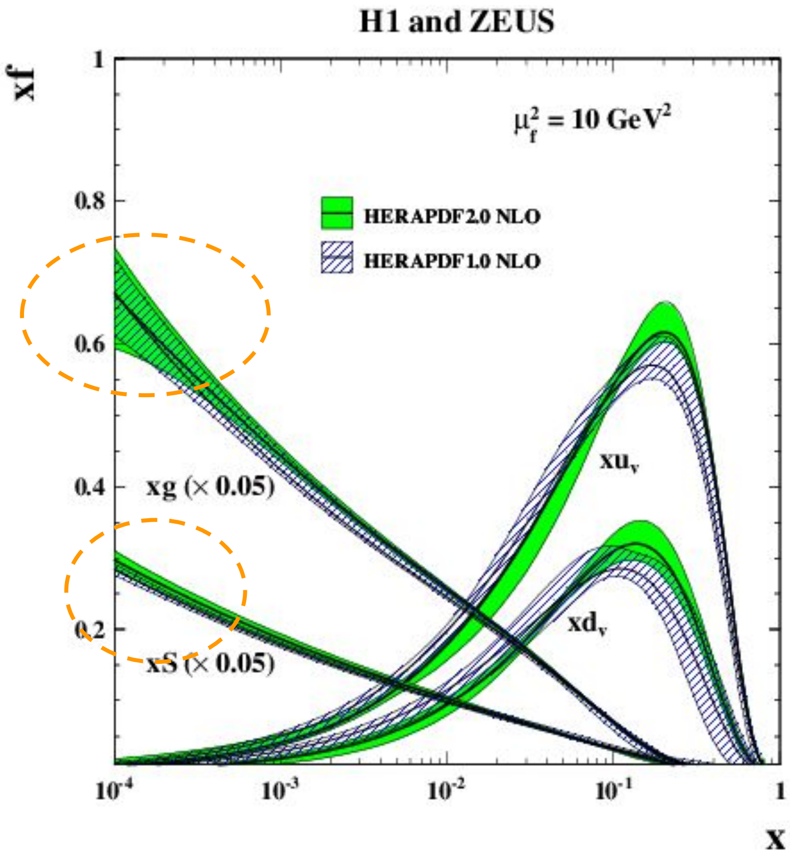




Parton no. increases, but density decreases

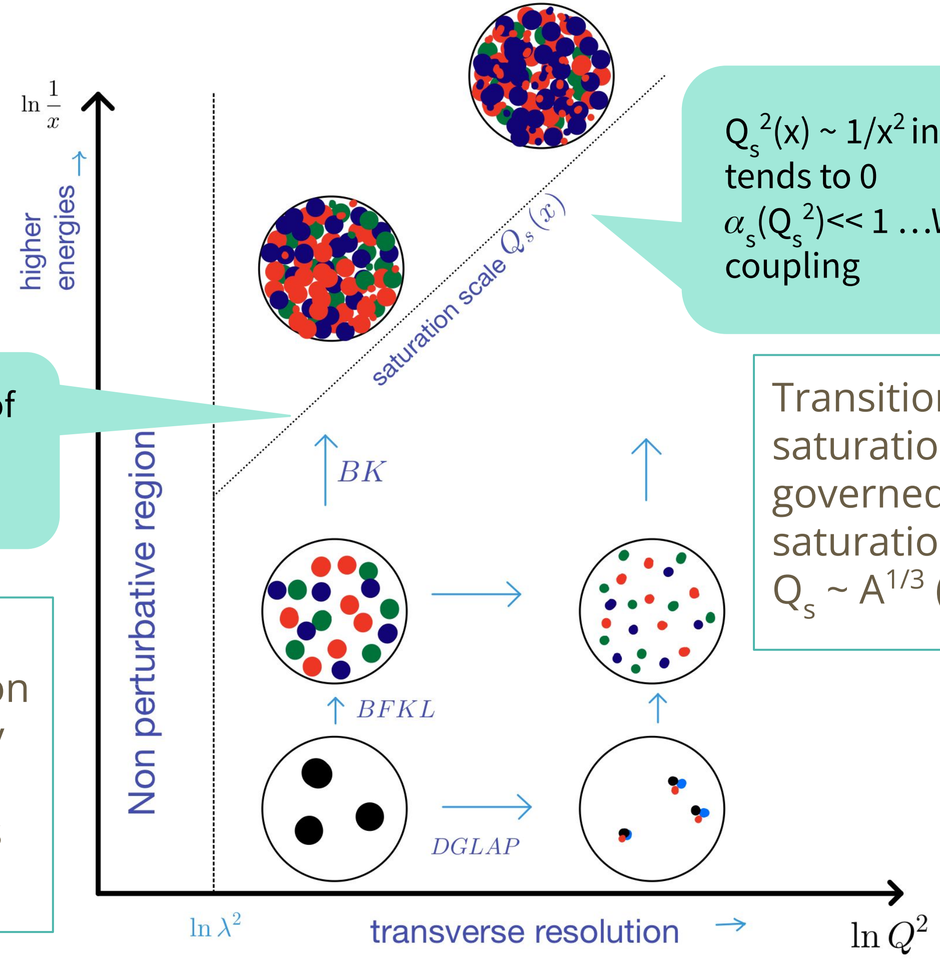
Multiple Gluon emissions
$N_g \sim e^{\square \ln(1/x)}$

- At very high energy: small-x, gluons and sea quarks dominate
- HERA Data shows signatures of saturation at small-x



Credit: DOI:10.5506/APHYSPOLBSUPP.8.957

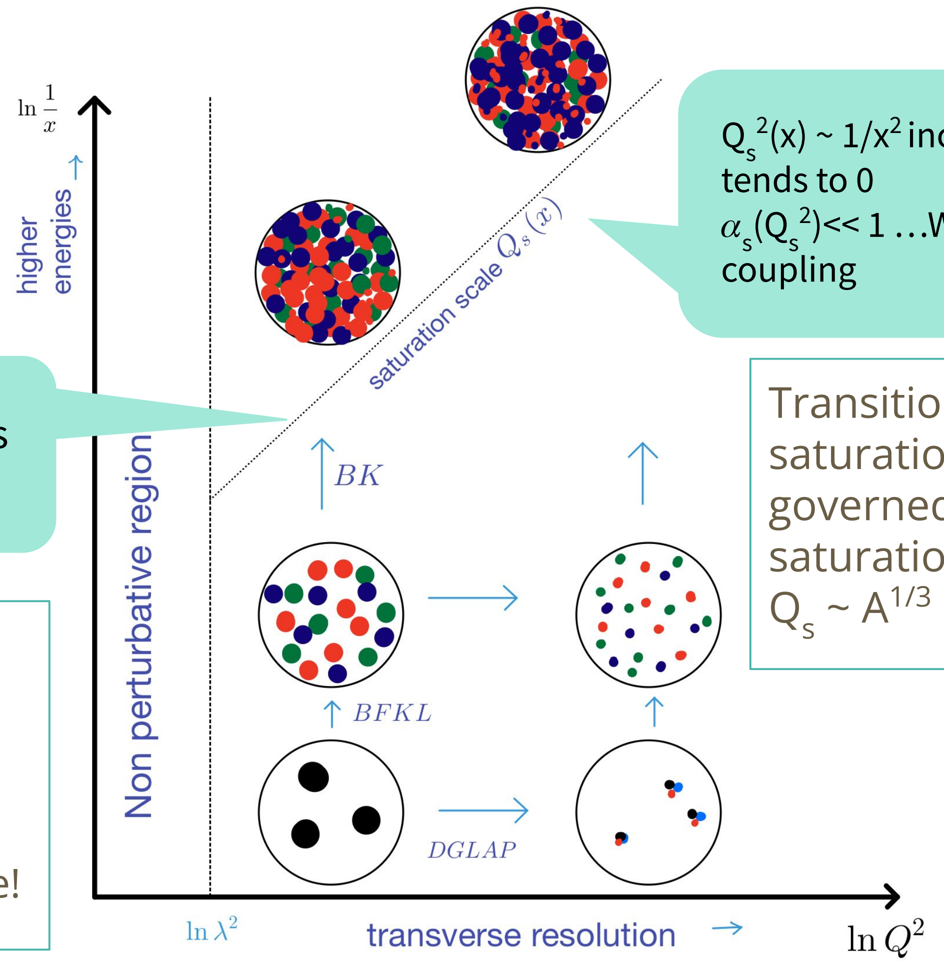
Non linear interaction of gluons
unitarity



$Q_s^2(x) \sim 1/x^2$ increases x tends to 0
$\alpha_s(Q_s^2) << 1$ …Weak coupling

Transition to saturation region is governed by saturation scale Q_s.
$Q_s \sim A^{1/3} (1/x)^{\lambda}$

Saturation physics allows us to study region of high parton density with weak coupling methods, calculations are doable!



Non-linear interactions of gluons
$N_g \leq 1$
unitarity

Saturation physics allows us to study region of high parton density with weak coupling methods, calculations are doable!

$Q_s^2(x) \sim 1/x^2$ increases x tends to 0
$\alpha_s(Q_s^2) \ll 1$ …Weak coupling

Transition to saturation region is governed by saturation scale Q_s.
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Color Glass Condensate

Parton Density

Parton gas

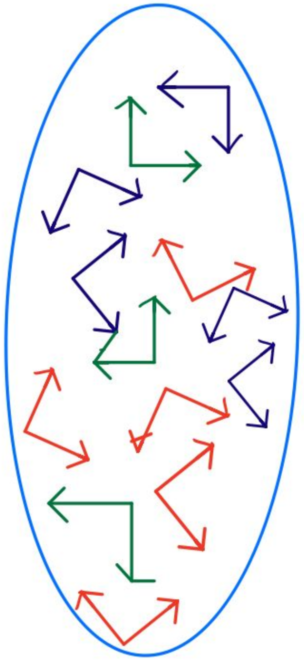
Color Glass Condensate (CGC)

Color : Gluons have "Color"

Glass: the small-x gluons are created by slowly moving patrons(with large x) which are randomly distributed over transverse plane $\longrightarrow$ it looks like almost frozen over natural time scale of scattering(This is very similar to **spin glass**, where spins are distributed randomly and move very slowly)

Condensate: It is dense matter of gluons. Can be better described as fields rather than point particles!

Can be studied by weak coupling methods!



Generally in saturation physics in Color Glass Condensate (CGC) framework 2 approximations:

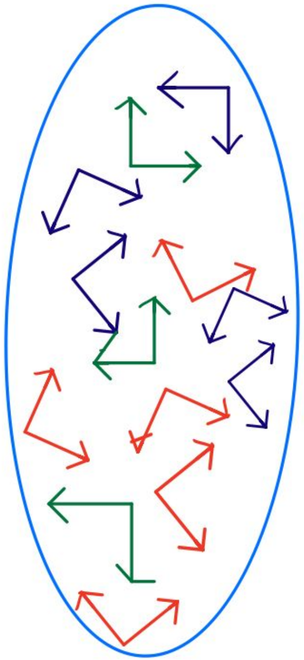
- **Semi-classical approximation:**
  Dense target given by Strong semi-classical gluon field $A_\mu(x)$

- **Eikonal approximation :**
  Limit of infinite boost of $A_\mu(x)$

Generally in saturation physics in Color Glass Condensate (CGC) framework 2 approximations:

- **Semi-classical approximation:**

  Dense target given by **Strong semi-classical gluon field $A_\mu(x) \sim 1/g \gg 1$**

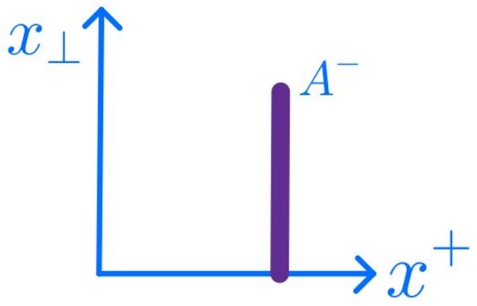


Generally in saturation physics in Color Glass Condensate (CGC) framework 2 approximations:

- **Eikonal approximation :**

  Limit of infinite boost of $A_\mu(x)$

- Taking into account **only leading power in** terms of high energy : (here, leading order component w.r.t. γ_t)
- Good enough approximation to describe physics at very high energy accelerators.



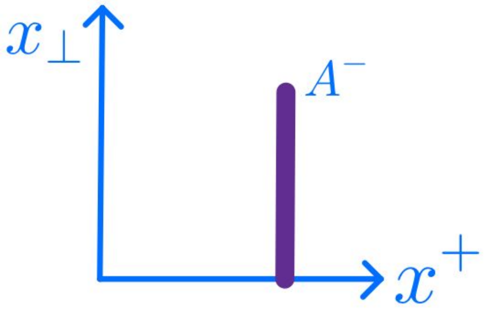
Eikonal Order: For $A_\mu(x)$

Eikonal Order

1. Shockwave approx.: target is localised in the longitudinal direction $x^+ = 0$ (**zero width**).
2. **Only leading - component of target considered**, subleading components are neglected (suppressed by γ_t)
3. Time dilation and static approximation: **x^- dependence of target neglected**

In light-cone coordinate, w.r.t. Lorentz boost factor of target (γ_t)

$$A^- = \mathcal{O}(\gamma_t) >> A^j = \mathcal{O}(1) >> A^+ = \mathcal{O}(1/\gamma_t)$$



Possible Applications:

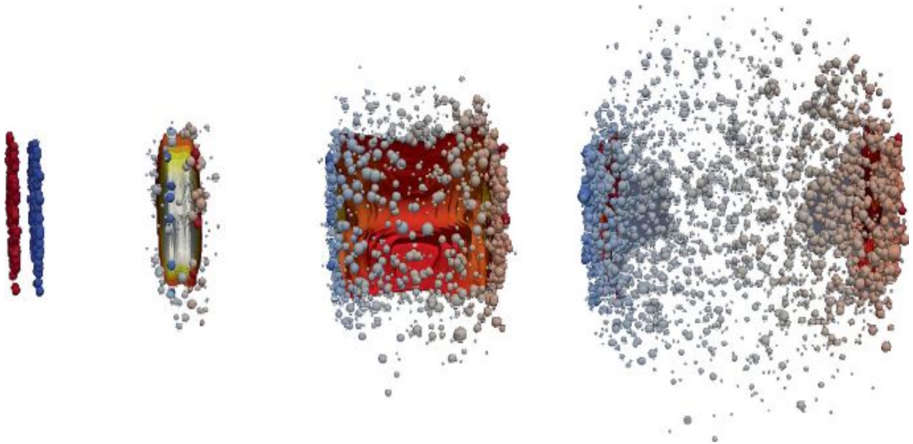
- Dilute-Dilute Scattering:
- Dense-Dense Scattering:
- Dilute-Dense Scattering:

Possible Applications:

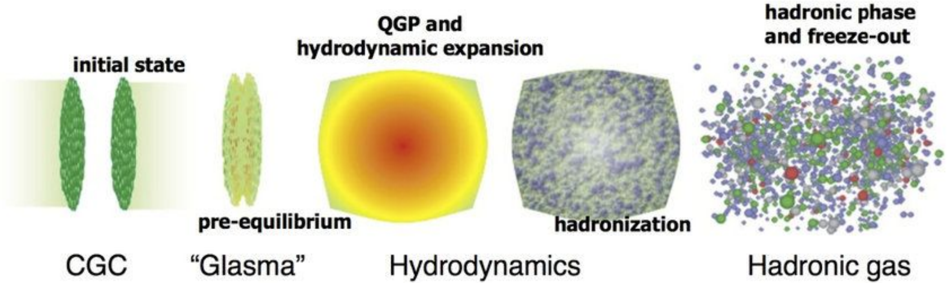
- **Dilute-Dilute Scattering:**
    - No saturation effect, BFKL formalism
- Dense-Dense Scattering:
- Dilute-Dense Scattering:

Possible Applications:

- **Dense-Dense Scattering:**
  - Target and projectile **both are saturated** (Non-linear dynamics of Yang-Mills fields)
  - Applied to heavy ion collision, pp at very high energies



Credit: Strongly Interacting Matter under Rotation, 2021, Volume 987,Karpenko



Credit: arXiv:1309.7616

Possible Applications:

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  - Target and projectile **both are saturated** (Non-linear dynamics of Yang-Mills fields)
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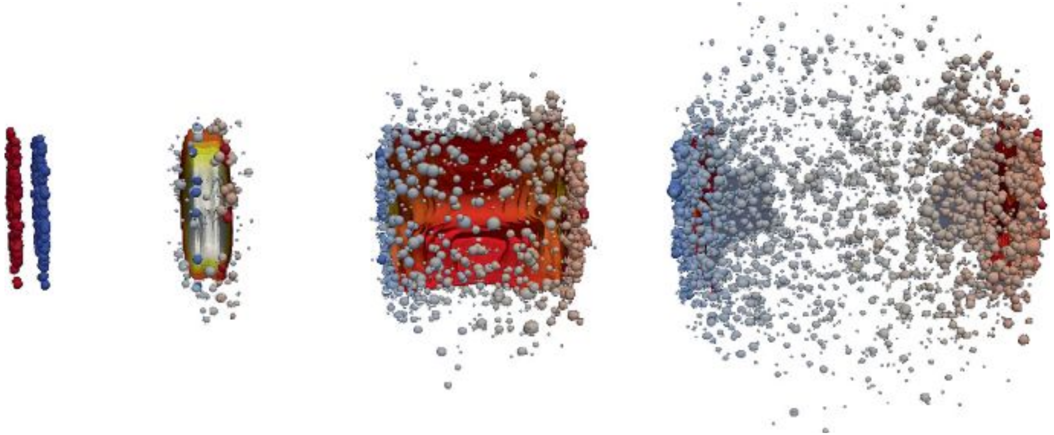

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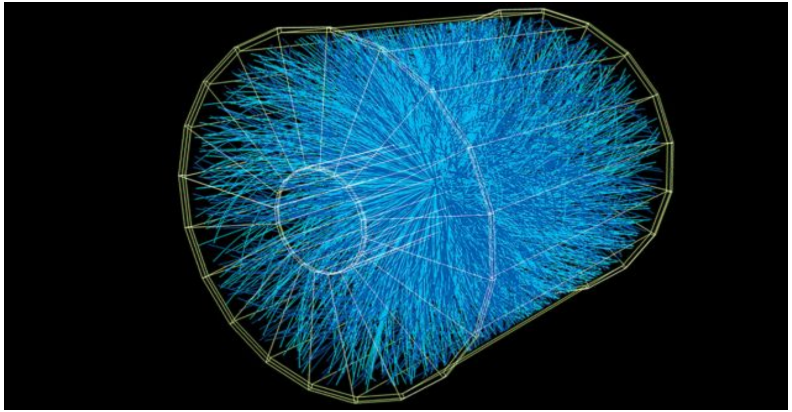
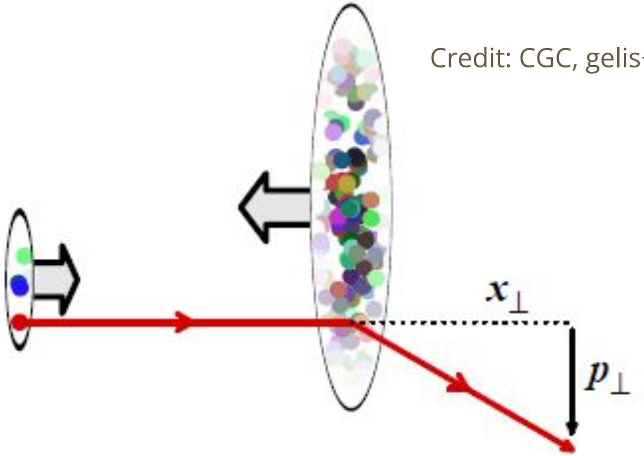


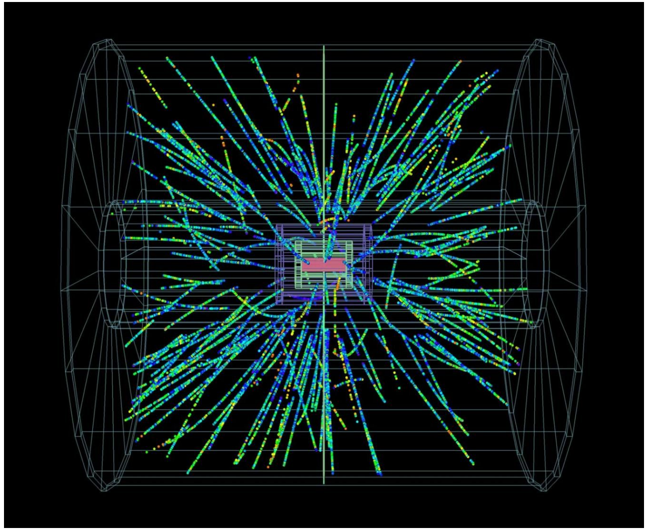
Credit: ALICE, pb-pb collision

Possible Applications:

- **Dilute-Dense Scattering:**
  - Target is saturated (CGC formalism)
  - Can be applied to: DIS on A, pA collisions



Credit: CGC, gelis+



The first proton-lead collisions of 2013 send showers of particles through the ALICE detector (Image: CERN)

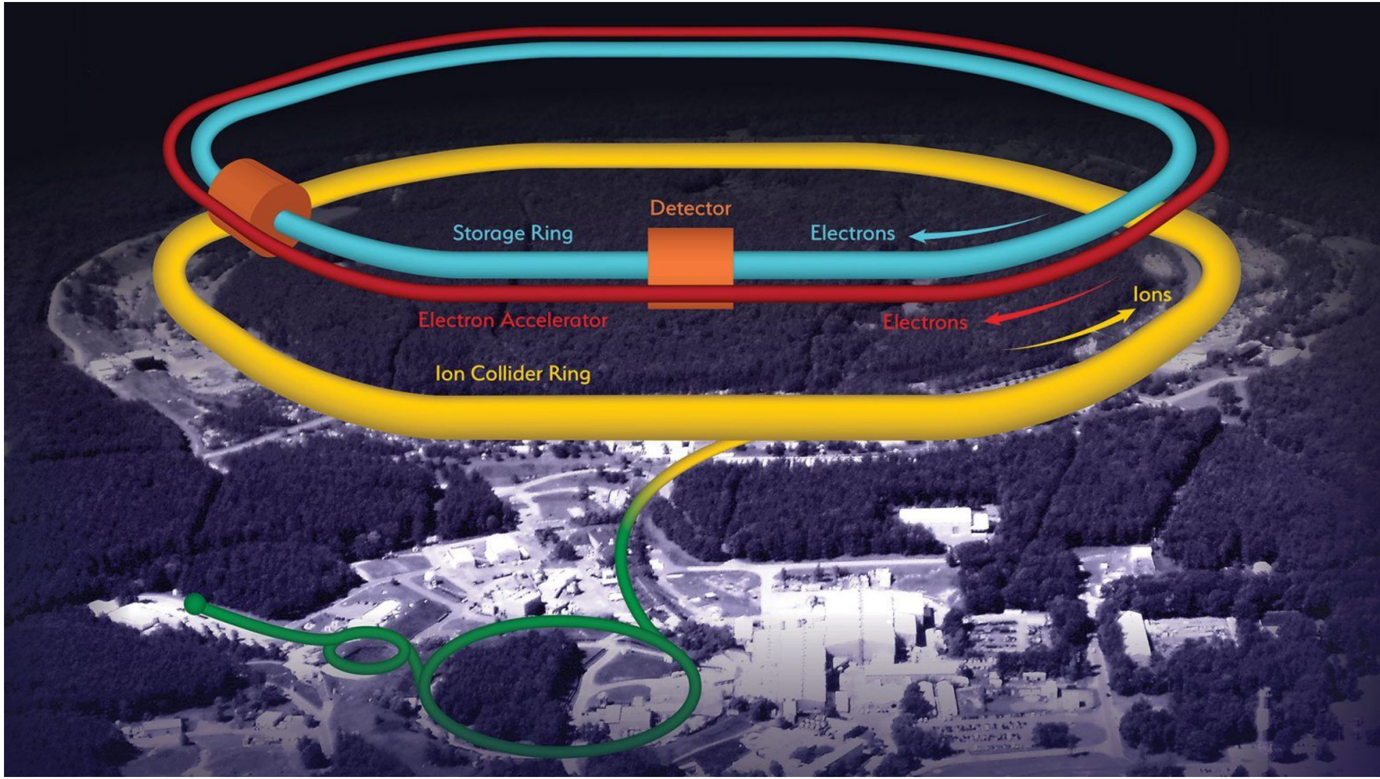
Electron Ion Collider (Upcoming)

Lower centre of mass energy than HERA but Higher Luminosity (100-1000 times HERA)

↓

Cover wide Kinematic Range

Can probe proton as well as Ions(no. nucleus)



Credit: EIC

Eikonal Order and Going beyond Eikonal Order

Eikonal Approximation:

- Taking into account only leading power in terms of high energy
- Good enough approximation to describe physics at very high energy accelerators.

Going Beyond Eikonal order:

- Taking into account terms suppressed in energy.
- In comparatively moderate energy accelerators (EIC and RHIC) sub-eikonal corrections might be sizable.

Main Objective:

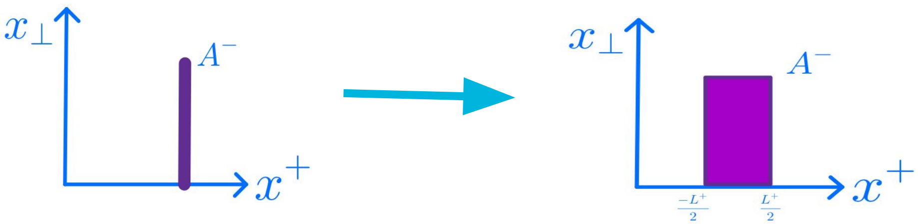
Providing sub-eikonal corrections to the various observable in CGC Framework

Going Beyond Eikonal Order.................

w.r.t. Lorentz boost factor of target (γ_t)

$$A^- = \mathcal{O}(\gamma_t) >> A^j = \mathcal{O}(1) >> A^+ = \mathcal{O}(1/\gamma_t)$$

Going Beyond Eikonal Order: For $A_\mu(x)$



Eikonal Order

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2. **Only leading - component of target considered**, subleading components are neglected (suppressed by γ_t)
3. Time dilation and static approximation: **x^- dependence of target neglected**

Next-to-eikonal Order

1. Instead of infinite thin shockwave as a target, we consider **finite width** of a target.
2. Include **transverse component** of background field(target).
3. Consider background field is x^- dependent: **dynamics of the target are considered**.

Going Beyond Eikonal Order: Quark Background Field

- Due to large boost of the target along x^-: its **localized in longitudinal x^+** direction around small support.
- If we consider projections on quark background field then,

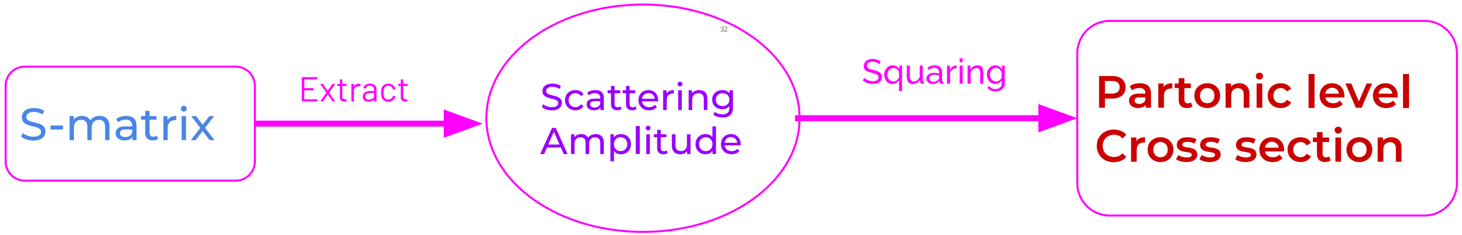
$$\Psi(z) = \frac{\gamma^+\gamma^-}{2}\Psi(z) + \frac{\gamma^-\gamma^+}{2}\Psi(z) = \Psi^-(z) + \Psi^+(z)$$

$$\boxed{\mathcal{O}(\sqrt{\gamma_t})} \qquad \boxed{\mathcal{O}(1/\sqrt{\gamma_t})}$$

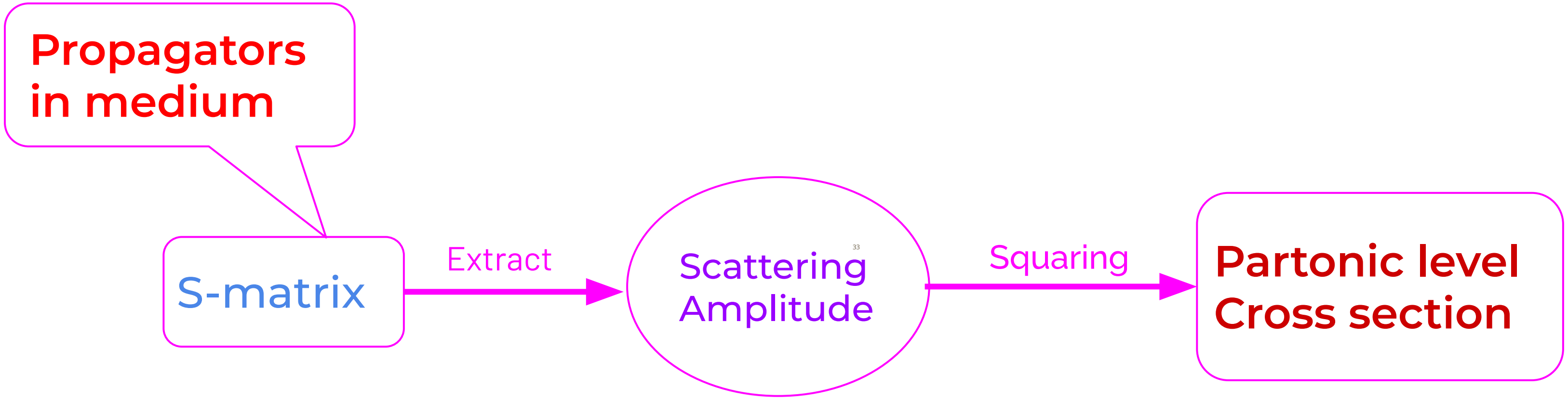
- For **Next-to-eikonal (NEik) corrections, only - component** considered and + component is neglected (contribute at NNEik only).

How do we compute the cross-section?

Cross-section



Cross-section



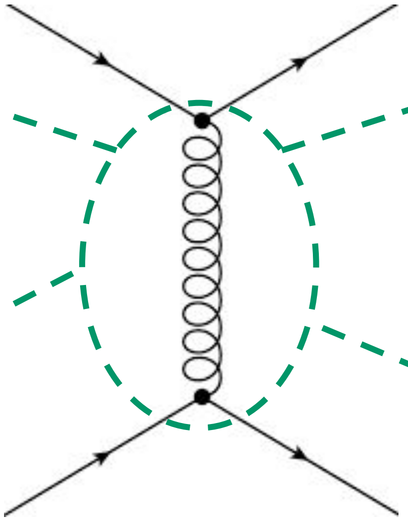
Propagators??

At NEik order: in finite width medium

Propagators??

associated with specific particles or fields

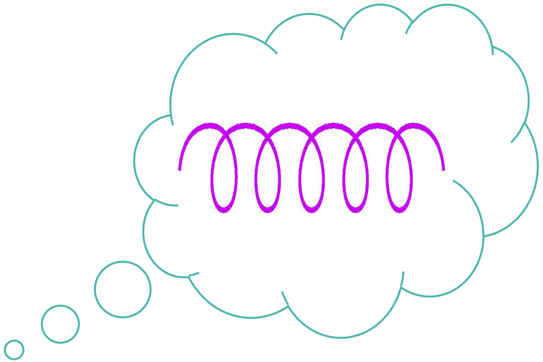
provides information about **propagation** of particle/field.



depends on 1.the momentum, mass, and energy of the particle or field involved 2. the interactions and forces present in the system.

fundamental in computing probabilities of particle interactions, scattering processes, and other quantum phenomena.

Gluon Propagator: in Vacuum



In vacuum Gluon propagator (without presence of medium) in momentum space is give as:

$$G^{\mu\nu}_{0,F}(p) = \frac{i}{p^2 + i\varepsilon}\left[-g^{\mu\nu} + \frac{p^\mu \eta^\nu + \eta^\mu p^\nu}{p \cdot \eta}\right]$$

This is in Light-cone gauge, $A^+ = 0$ and $\eta^2=0$
Where, $\eta^\mu = g^{\mu+}$

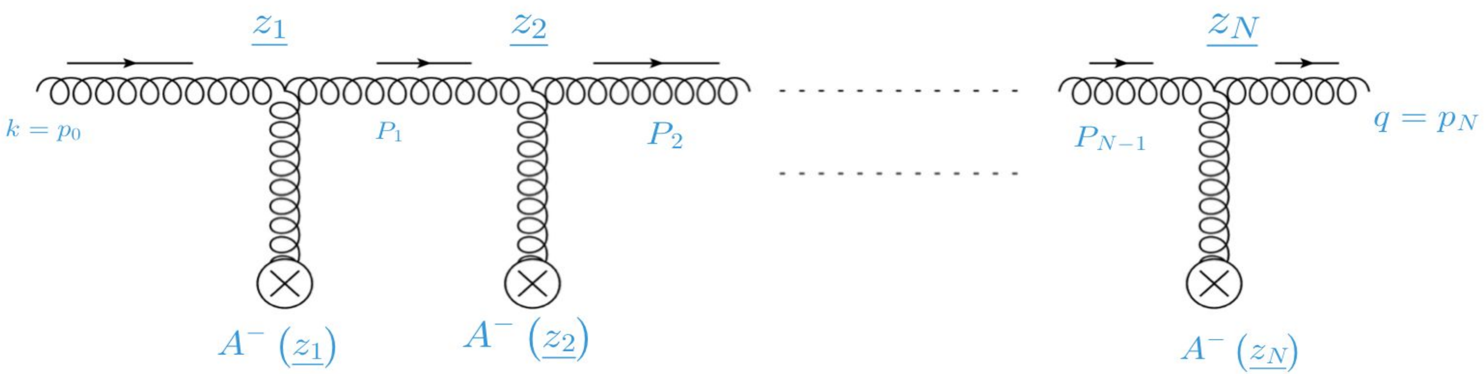
How to obtain these propagators?

At NEik order: in finite width medium
INCLUDING INTERACTION WITH MEDIUM

*Based on Altinoluk, Beuf, **SM** (2411.15047)*

Gluon Propagator at Eikonal order

- To calculate it, we **re-sum multiple interaction diagrams** of Gluon background field.



Similar for quarks in Altinoluk, Beuf, Czajka, Tymowska [2012.03886] , Altinoluk, Beuf [arXiv:2109.01620]

*Based on Altinoluk, Beuf, **SM** (2411.15047)*

Gluon Propagator at NEik order

Recipe: For General gluon propagator at next-to-eikonal order travelling through entire medium

a. First compute Eikonal order Gluon Propagator in gluon background field.

   i. Only "-" leading component of classical gluon background field is considered.

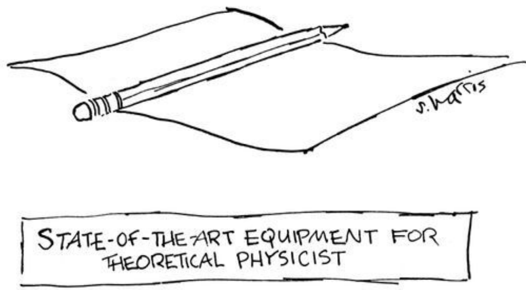
b. Use this computed gluon propagator to obtain next-to-eikonal (NEik) contributions

   i. Due to considering finite width of the target

   ii. Due to interaction with transverse components of medium

   iii. Due to dynamics of target (including x- dependence)

Total Gluon propagator at NEik order



Total gluon propagator upto NEik order travelling through the entire medium (dynamic gluon background field) for the case $x^+ > L^+/2$ and $y^+ < -L^+/2$ with $x^+ > y^+$ is:

$$
\begin{aligned}
G_F^{\mu\nu}(x,y) = & \int \frac{d^3\underline{q}}{(2\pi)^3}\; e^{-ix\cdot\check{q}}\;\theta(q^+) \int \frac{d^3\underline{k}}{(2\pi)^3}\; e^{iy\cdot\check{k}}\;\theta(k^+)\;\frac{1}{q^++k^+} \\
& \times \Big[-g^{\mu\nu} + \frac{\check{k}^\mu\eta^\nu}{k^+} + \frac{\eta^\mu\check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k})\Big] \int d^2z_\perp e^{-i(q_\perp-k_\perp)z_\perp} \\
& \times \int dz^-\; e^{i(q^+-k^+)z^-}\; \boxed{\mathcal{U}_A\Big(\frac{L^+}{2},\frac{-L^+}{2},z_\perp,z^-\Big)} \\
& + \int \frac{d^3\underline{q}}{(2\pi)^3}\frac{e^{-ix\cdot\check{q}}}{2q^+}\theta(q^+) \int \frac{d^3\underline{k}}{(2\pi)^3}\frac{e^{iy\cdot\check{k}}}{2k^+}\theta(k^+) \int dz^-\; e^{iz^-(q^+-k^+)} \\
& \times \int d^2z_\perp\; e^{-iz_\perp(q_\perp-k_\perp)}\Big\{\Big(-g^{\mu\nu} + \frac{\check{k}^\mu\eta^\nu}{k^+} + \frac{\eta^\mu\check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k})\Big) \\
& \times \Big(-\frac{q^j+k^j}{2}\int_{\frac{-L^+}{2}}^{\frac{L^+}{2}} dz^+\Big[\mathcal{U}_A\Big(\frac{L^+}{2},z^+;z_\perp,z^-\Big)\Big(\overrightarrow{D}_{z^j}-\overleftarrow{D}_{z^j}\Big)\mathcal{U}_A\Big(z^+,-\frac{L^+}{2};z_\perp,z^-\Big)\Big] \\
& - i\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+\Big[\mathcal{U}_A\Big(\frac{L^+}{2},z^+;z_\perp,z^-\Big)\Big(\overleftarrow{D}_{z^j}\overrightarrow{D}_{z^j}\Big)\;\mathcal{U}_A\Big(z^+,-\frac{L^+}{2};z_\perp,z^-\Big)\Big]\Big) \\
& + \Big(g^{\mu j}g^{\nu i} - \frac{\eta^\mu g^{\nu i}q^j}{q^+} - \frac{g^{\mu j}k^i\eta^\nu}{k^+} + \frac{\eta^\mu\eta^\nu k^i q^j}{q^+k^+}\Big) \\
& \times \Big(\int dz^+\;\mathcal{U}_A\Big(\frac{L^+}{2},z^+;z_\perp,z^-\Big)\; gT\cdot F_{ij}\;\mathcal{U}_A\Big(z^+,-\frac{L^+}{2};z_\perp,z^-\Big)\Big)\Big\}
\end{aligned}
$$

*Based on Altinoluk, Beuf, **SM** (2411.15047)*

Applications:

1. SIDIS
2. Inclusive DIS

Based on Altinoluk, Beuf, **SM**
(10.22323/1.469.0077)

Dipole Approximation: SIDIS

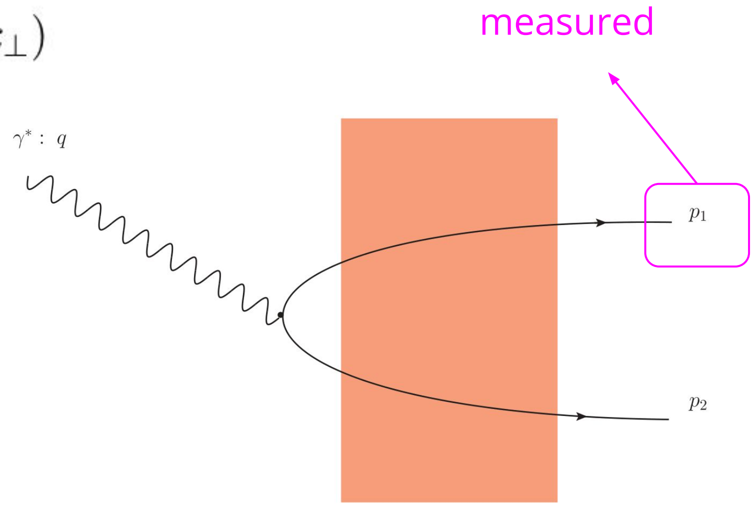
- In TMD factorization, ,

$$\frac{d\sigma}{d\mathcal{P}} \propto \int \frac{dz}{z_f} \frac{D(z)}{z_f^2} f(q_\perp, x) \times H(\xi, k_\perp)$$

- In Dipole factorization sea quark TMD is recovered.
(Marquet, Xiao, Yuan [arXiv:0906.1454])

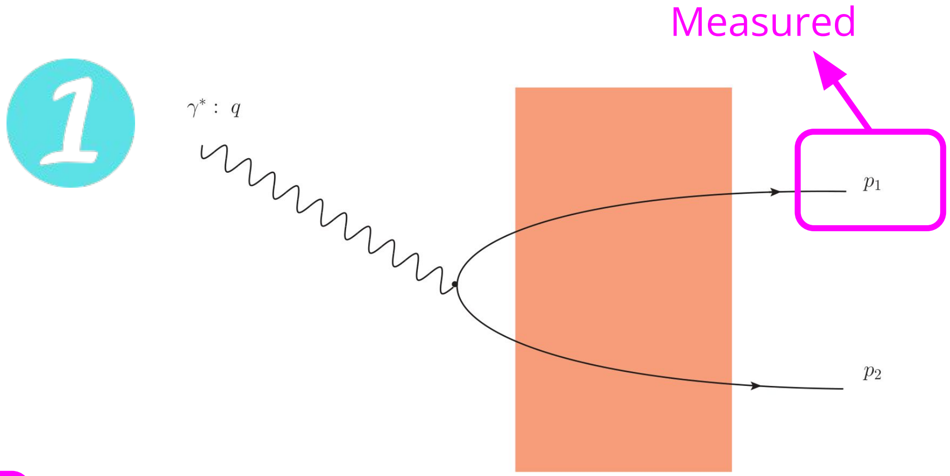
$$f(q_\perp, x) \propto \mathcal{C} \otimes S(r_\perp, b_\perp)$$

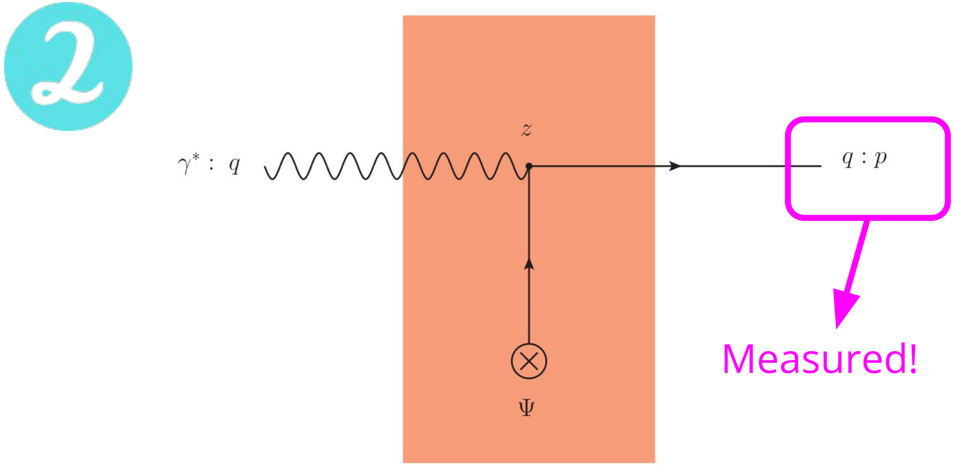
- But in this model contribution coming due to valence quarks are not included.



Semi Inclusive Deep Inelastic Scattering (SIDIS):

- At low-x, for this process: two kinds of contributions!
- Each of them are expected to be dominant in different kinematic regions.



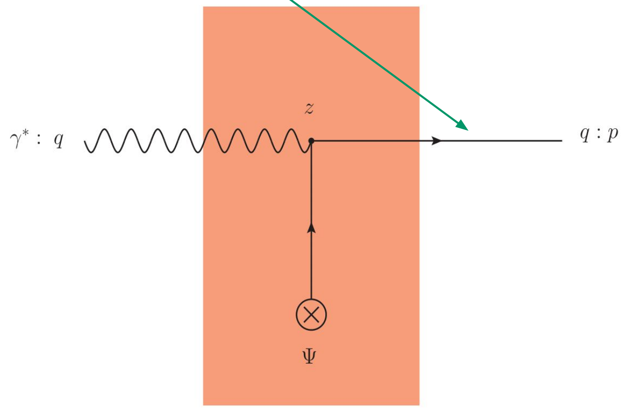


- Contribution (1) is studied by Marquet, Xiao, Yuan[arXiv:0906.1454]. There is contribution at eikonal order.
- In this talk contribution coming due to (2) is discussed. No contribution at eikonal order.

SIDIS: S-matrix computation

S-matrix at NEik order calculated : only Ψ^-(z) of component considered

$$S_{\gamma*\to q} = \lim_{x^+\to\infty} \int d^2x_\perp \int dx^- \; e^{i\check{p}\cdot x} \int d^4z \; \epsilon^\lambda_\mu(q) \; e^{-iq\cdot z} \; \overline{u}(p,h) \; \gamma^+ \boxed{S_F(x,z)|^{IA}_{Eik}} (-iee_f\gamma^\mu)\Psi^-(z)$$



Based on Altinoluk, Beuf, ***SM*** *(10.22323/1.469.0077)*

Similar calculations in case of q-g dijets are done by Altinoluk, Armesto, & Beuf (arXiv:2303.12691)

SIDIS: S-matrix computation

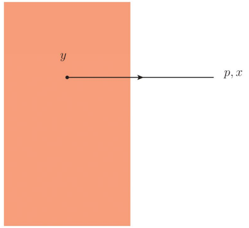
*Based on Altinoluk, Beuf, **SM** (10.22323/1.469.0077)*

S-matrix at NEik order calculated :

$$S_{\gamma*\to q} = \lim_{x^+\to\infty} \int d^2x_\perp \int dx^- \; e^{i\check{p}\cdot x} \int d^4z \; \epsilon_\mu^\lambda(q) \; e^{-iq\cdot z} \; \overline{u}(p,h) \; \gamma^+ \boxed{S_F(x,z)|_{Eik}^{IA}} (-iee_f\gamma^\mu)\Psi^-(z)$$

Quark Propagator from inside to after the medium

$$S_F(x,y)_{\beta\alpha}|^{\text{IA,q}}_{\text{Eik}} = \delta S_F(x,y)_{\beta\alpha}|^{\text{IA,q}}_{\text{Eik},A^-} + \delta S_F(x,y)_{\beta\alpha}|^{\text{IA,q}}_{\text{Eik,Single}A_\perp}$$



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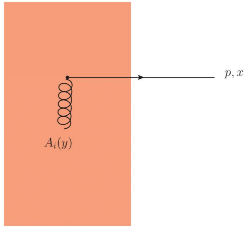

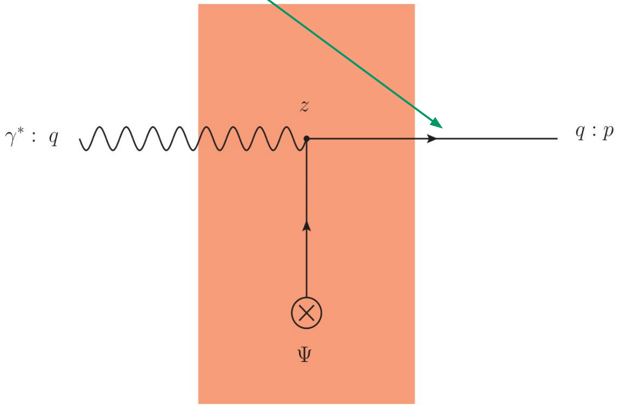

SIDIS: S-matrix computation

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Quark Propagator from inside to after the medium

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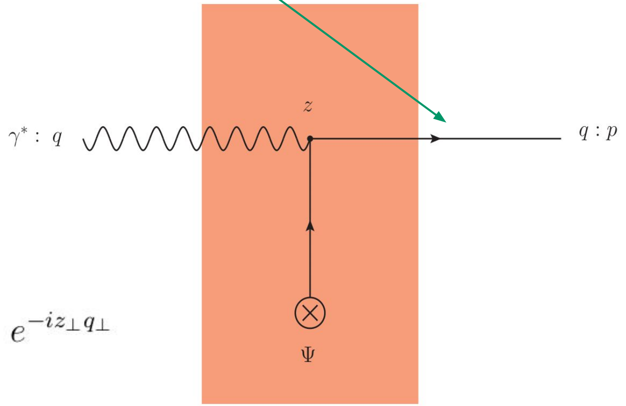
$$S_F(x,z)|^{IA,q}_{Eik} = \int \frac{d^3q}{(2\pi)^3}\frac{\theta(q^+)}{2q^+}\; e^{-ix\check{q}}\;(\check{\not{q}}+m)U_F(x^+,z^+,z_\perp)\;\left[1-\frac{\gamma^+\gamma^i}{2q^+}i\overleftarrow{D}^F_{z^i}\right]\; e^{iz^-q^+}\; e^{-iz_\perp q_\perp}$$



SIDIS: S-matrix computation

S-matrix at NEik order calculated :

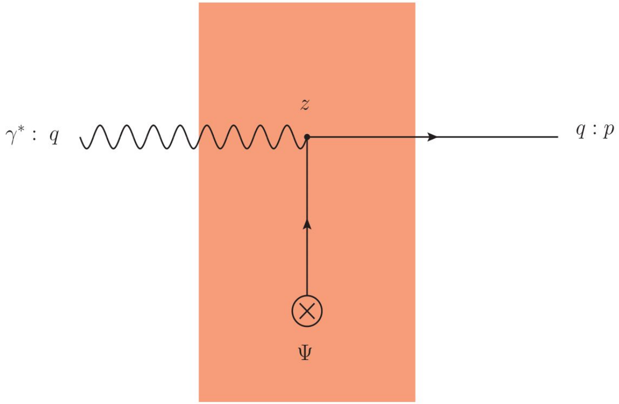
$$S_{\gamma * \to q} = \lim_{x^+ \to \infty} \int d^2 x_\perp \int dx^- \; e^{i\check{p}\cdot x} \int d^4 z \; \boxed{\epsilon_\mu^\lambda(q)} \; e^{-iq\cdot z} \; \overline{u}(p,h) \; \gamma^+ \; S_F(x,z)|_{Eik}^{IA}(-iee_f\gamma^\mu)\Psi^-(z)$$

- Two polarizations of photons are considered:
  - Longitudinal Polarization:

    No contribution at NEik order

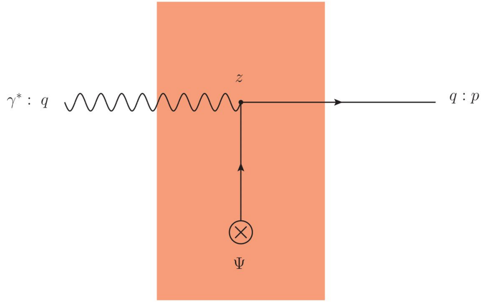
  - Transverse Polarization:

    Contribution at NEik order



Similar calculations in case of q-g dijets are done by Altinoluk, Armesto, & Beuf (arXiv:2303.12691)

SIDIS: S-matrix computation



Finally, S-matrix for SIDIS process:

$\mathcal{O}(1/\gamma_t)$

$$S_{\gamma_T^*\to q} = 2\pi\delta(q^+ - p^+) \int dz^+ \int d^2 z_\perp \; e^{i(q_\perp - p_\perp)z_\perp} \; \overline{u}(p,h)$$
$$\times\, \epsilon_\lambda^j (iee_f) U_F(\infty, z^+, z_\perp) \; \underline{\Big(\frac{\gamma^j\gamma^+\gamma^-}{2}\Big)\Psi(z)}$$

$\mathcal{O}(\sqrt{\gamma_t})$

- Cross-section:

$$\frac{d^2\sigma^{\gamma_T^*\to q}}{d^2 p_\perp} = \frac{e^2 e_f^2}{(2\pi)^2}\frac{1}{2}\frac{1}{2q^+}\int d^2 z'_\perp \int d^2 z_\perp \; e^{i(q_\perp - p_\perp)(z_\perp - z'_\perp)} \int dz'^+ \int dz^+$$
$$\times \left\langle \overline{\Psi}(z')\gamma^- \,\mathcal{U}_F^\dagger(\infty, z'^+, z'_\perp) \;\; \mathcal{U}_F(\infty, z^+, z_\perp) \;\Psi(z)\right\rangle$$

Over all suppression of $\mathcal{O}(1/\gamma_t)$: NEik order

SIDIS: Relation at small-x between CGC and TMD calculations

- In Unpolarized target, the CGC-like target average $\langle \mathcal{O} \rangle$ is proportional to the quantum expectation value in the momentum state of target.

$$\langle \mathcal{O} \rangle = \lim_{P'_{tar} \to P_{tar}} \frac{\langle P'_{tar} \,|\, \mathcal{O} \,|\, P_{tar} \rangle}{\langle P'_{tar} \,|\, P_{tar} \rangle}$$

where,

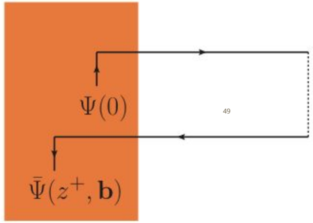
$$\langle P'_{tar} | P_{tar} \rangle = 2P^{-}_{tar} (2\pi)^3 \delta(P'^{-}_{tar} - P^{-}_{tar}) \delta^{(2)}(P'_{tar\perp} - P_{tar\perp})$$

- Using this relation, we can relate obtained cross-section with unpolarized transverse momentum dependent (TMD) quark distribution.

SIDIS: Relation at small-x between CGC and TMD calculations

- unpolarized transverse momentum dependent (TMD) quark distribution:

$$f_1^q(x,k_\perp) = \frac{1}{(2\pi)^3}\int_{b_\perp} e^{ik_\perp b_\perp}\int_{z^+} e^{-iz^+xP^-_{tar}}\left\langle P_{tar}\left|\overline{\Psi}(z^+,b_\perp)\frac{\gamma^-}{2}\mathcal{U}_F^\dagger(\infty,z^+;b_\perp)\mathcal{U}_F(\infty,0;0)\Psi(0,0)\right|P_{tar}\right\rangle$$



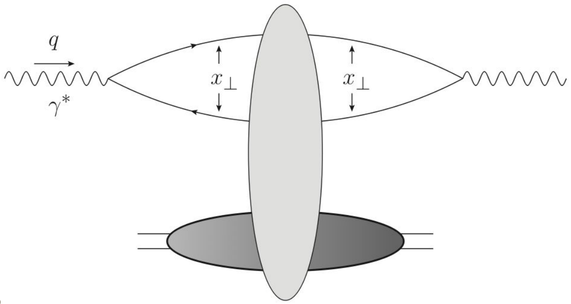
- By comparing with quark TMD function, we get cross section:

$$\boxed{\frac{d^2\sigma^{\gamma_T^*\to q}}{d^2p_\perp} = \frac{\pi e^2 e_f^2}{W^2}\, f_1^q(x=0,p_\perp - q_\perp)}$$

Suppression by centre of mass energy $1/W^2$ characterizes NEik contribution in terms of exchange t channel quark!

TMD-[arXiv:2303.12691]

Dipole Approximation: Inclusive DIS



From Dipole approximation, we can write total cross-section for DIS as

$$\sigma_{L,T}^{\gamma^* p}(x, Q^2) = \sum_f \int d^2\mathbf{r} \int_0^1 \frac{dz}{4\pi} \left|\Psi_{\gamma^*_{L,T}\to q\bar{q}}\right|^2 \sigma_{q\bar{q}}(x, r)$$

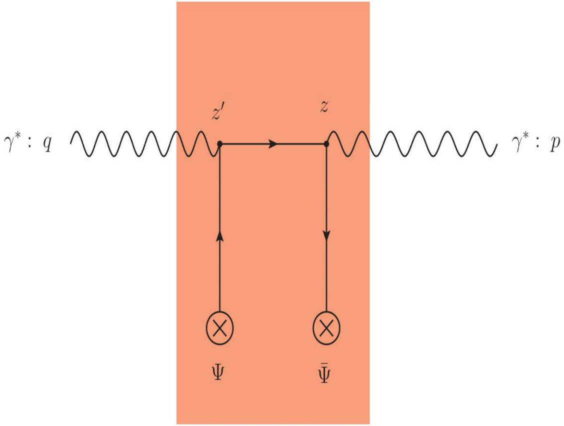
- This model is sufficient and explains contribution coming due to ***sea quarks*** at low-x.
- But in limit $z \longrightarrow 1$ or $z \longrightarrow 0$, dipole approximation is not justified.
- We have to include corrections to explain kinematics near limits 1 and 0.
- Also, in this model contribution coming due to *valence quarks are not* included.

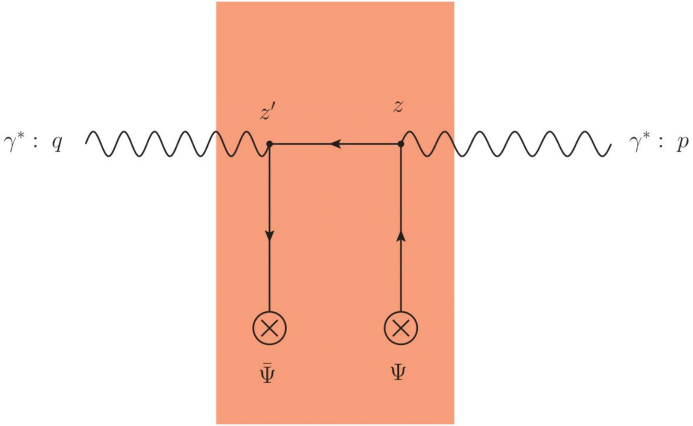
Diagram:
DOI: 10.1017/9781009291446

Inclusive DIS:

*Based on Altinoluk, Beuf, **SM** (10.22323/1.469.0077)*

- Two contributions added together:
  - From quark propagator
  - From antiquark propagator
- No contribution at Eikonal order due to quark background field.





Inclusive DIS: Due to quark propagator from inside to inside the medium

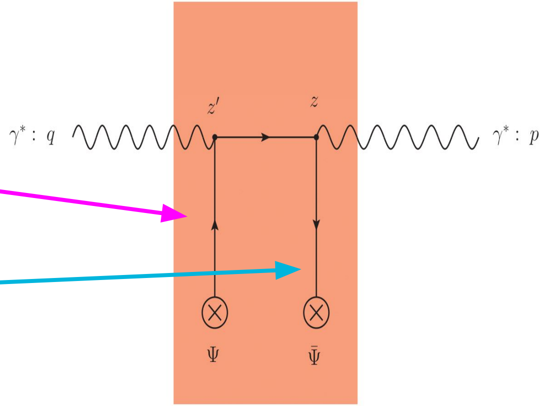
- Two contributions added together:
  - From quark propagator
  - From antiquark propagator
- No contribution at Eikonal order due to quark background field.
- S-matrix for contribution with quark propagator is

$$S^{q}_{\gamma\to\gamma} = \int d^4z \int d^4z' \, \theta(z^+ - z'^+) \, \epsilon^{\lambda_2}_{\mu}(p)^* \, e^{ip\cdot z} \, \boxed{\overline{\Psi}^{-}(z)} (-iee_f\gamma^\mu) \, S_F(z,z')\big|^{II,q}_{Eik} \, (-iee_f\gamma^\nu) \, \boxed{\Psi^-(z')}$$
$$\times \, \epsilon^{\lambda_1}_{\nu}(q) \, e^{-iq\cdot z'}$$

Two insertions of quark background field

$$\Psi^-(z') = \frac{\gamma^+\gamma^-}{2}\Psi(z')$$

$$\overline{\Psi}^{-}(z) = \overline{\Psi}(z)\frac{\gamma^-\gamma^+}{2}$$



Inclusive DIS: Due to quark propagator from inside to inside the medium

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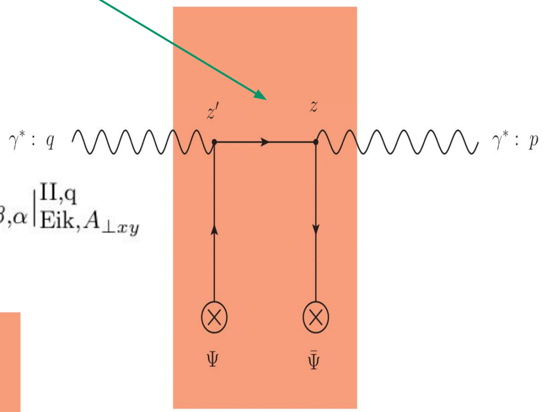
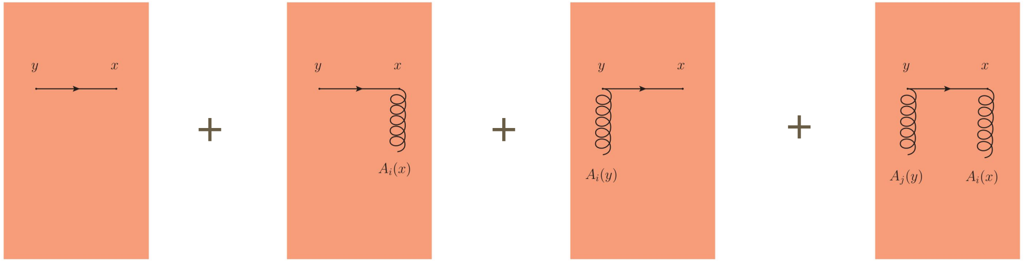
$$S^{q}_{\gamma\to\gamma} = \int d^4z \int d^4z'\; \theta(z^+ - z'^+)\; \epsilon^{\lambda_2}_{\mu}(p)^*\; e^{ip\cdot z}\; \overline{\Psi}^{-}(z)(-iee_f\gamma^{\mu})\, \boxed{S_F(z,z')\big|^{II,q}_{Eik}}\, (-iee_f\gamma^{\nu})\; \Psi^{-}(z')$$
$$\times\, \epsilon^{\lambda_1}_{\nu}(q)\; e^{-iq\cdot z'}$$

We have to compute quark propagator from inside to inside the medium at eikonal order



$$S_F(x,y)_{\beta,\alpha}\big|^{\mathrm{II,q}}_{\mathrm{Eik}} = S_F(x,y)_{\beta,\alpha}\big|^{\mathrm{II,q}}_{\mathrm{Eik},A^-} + S_F(x,y)_{\beta,\alpha}\big|^{\mathrm{II,q}}_{\mathrm{Eik},A_{\perp x}} + S_F(x,y)_{\beta,\alpha}\big|^{\mathrm{II,q}}_{\mathrm{Eik},A_{\perp y}} + S_F(x,y)_{\beta,\alpha}\big|^{\mathrm{II,q}}_{\mathrm{Eik},A_{\perp xy}}$$

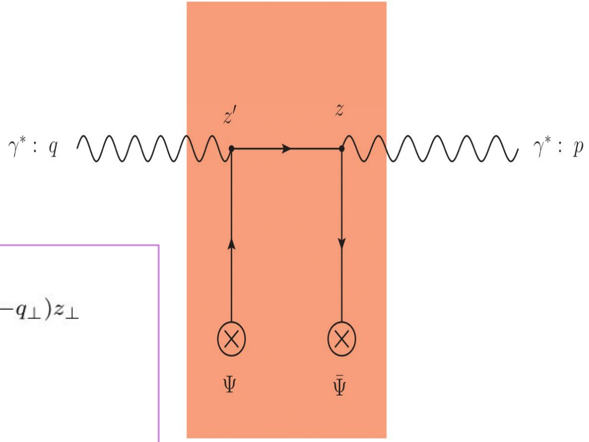


Inclusive DIS: Due to quark propagator from inside to inside the medium

- S-matrix for contribution with quark propagator is

$$S^q_{\gamma\to\gamma} = \int d^4z \int d^4z' \, \theta(z^+ - z'^+) \, \epsilon^{\lambda_2}_\mu(p)^* \, e^{ip\cdot z} \, \overline{\Psi}^-(z)(-iee_f\gamma^\mu) \, S_F(z,z')\big|^{II,q}_{Eik} \, (-iee_f\gamma^\nu) \, \Psi^-(z')$$
$$\times \, \epsilon^{\lambda_1}_\nu(q) \, e^{-iq\cdot z'}$$

- Two **polarizations of photons** are considered:
  - Longitudinal Polarization: no contribution at NEik order
  - Transverse Polarization: Contribution at NEik order



$$S^q_{\gamma^*\to\gamma^*} = 2\pi\delta(p^+ - q^+) \, (e^2 e_f^2) \, \epsilon^{i\,*}_{\lambda_2} \, \epsilon^j_{\lambda_1} \int d^2z_\perp \int dz^+ \int dz'^+ \, \theta(z^+ - z'^+) \, e^{-i(p_\perp - q_\perp)z_\perp}$$
$$\times \, \overline{\Psi}_\beta(z^+, z_\perp) \, U_F(z^+, z'^+, z_\perp)_{\beta\alpha} \, \left(\frac{\gamma^i\gamma^j\gamma^-}{2}\right) \, \Psi_\alpha(z'^+, z_\perp)$$

Inclusive DIS: Cross-section Computation

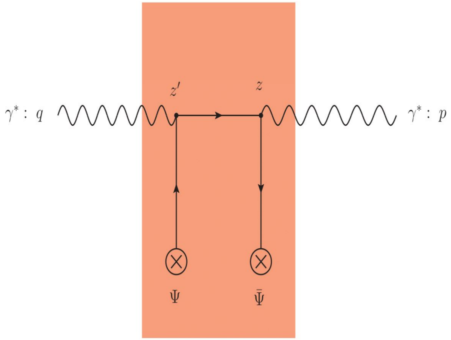
For contribution due to from inside to inside the medium quark propagator

- S-matrix:

$$S^{q}_{\gamma^*\to\gamma^*} = 2\pi\delta(p^+ - q^+)\ (e^2 e_f^2)\ {\epsilon^{i}_{\lambda_2}}^*\ \epsilon^{j}_{\lambda_1} \int d^2 z_\perp \int dz^+ \int dz'^+\ \theta(z^+ - z'^+)\ e^{-i(p_\perp - q_\perp)z_\perp}$$

$$\times\ \overline{\Psi}_\beta(z^+, z_\perp)\ U_F(z^+, z'^+, z_\perp)_{\beta\alpha}\ (\frac{\gamma^i\gamma^j\gamma^-}{2})\ \Psi_\alpha(z'^+, z_\perp)$$

- From Optical theorem:



$$\sigma^{\gamma*}_{\lambda} = 2\mathrm{Im}\mathcal{M}_{\gamma^*_\lambda\to\gamma^*_\lambda} = 2\mathrm{Re}(-i)\mathcal{M}_{\gamma^*_\lambda\to\gamma^*_\lambda}$$

- Cross-section:

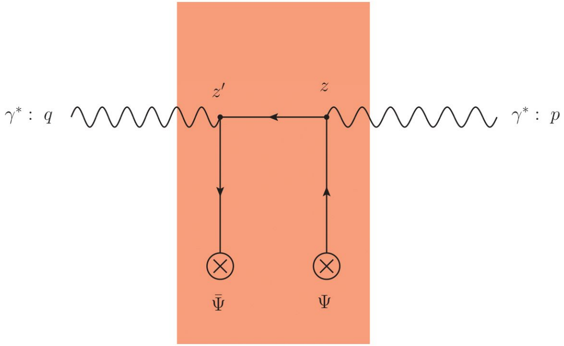
$$\sigma^{\gamma^*}_{\mathrm{T}}|^q = \mathrm{Re}\Big\{\frac{(e^2 e_f^2)}{2q^+}\int d^2 z_\perp \int dz^+ \int dz'^+\ \theta(z^+ - z'^+)\ \overline{\Psi}(z^+, z_\perp)\ U_F(z^+, z'^+, z_\perp)\gamma^-\ \Psi(z'^+, z_\perp)\Big\}$$

Inclusive DIS: Due to antiquark propagator from inside to inside the medium

- No contribution at Eikonal order due to quark background field.
- S-matrix:

$$S^{\overline{q}}_{\gamma\to\gamma} = \int d^4z \int d^4z'\ \theta(z^+ - z'^+)\ \epsilon^{\lambda_2}_{\mu}(p)^*\ e^{ip\cdot z}\ \overline{\Psi}^-(z')(-iee_f\gamma^\nu)\ S_F(z',z)\big|^{II,\overline{q}}_{Eik}\ (-iee_f\gamma^\mu)$$
$$\times\ \Psi^-(z)\ \epsilon^{\lambda_1}_{\nu}(q)\ e^{-iq\cdot z'}$$

$$S^{\overline{q}}_{\gamma^*\to\gamma^*} = 2\pi\delta(p^+ - q^+)\int d^2z_\perp\ e^{-i(p_\perp - q_\perp)z_\perp}\int dz^+\int dz'^+\ \theta(z^+ - z'^+)\ \epsilon^j_{\lambda_1}\epsilon^i_{\lambda_2}{}^*$$
$$\times\ (e^2e_f^2)\ \overline{\Psi}_\beta(z'^+, z_\perp)\frac{\gamma^j\gamma^-\gamma^i}{2}\ U_F^\dagger(z^+, z'^+, z_\perp)_{\beta\alpha}\ \Psi_\alpha(\underline{z})$$



- Cross-section:

$$\sigma_T^{\gamma^*}|^{\overline{q}} = \mathrm{Re}\left\{\frac{-(e^2e_f^2)}{2q^+}\int d^2z_\perp\int dz^+\int dz'^+\ \theta(z^+ - z'^+)\ \overline{\Psi}_\beta(z'^+, z_\perp)\gamma^-\ U_F^\dagger(z^+, z'^+, z_\perp)_{\beta\alpha}\ \Psi_\alpha(\underline{z})\right\}$$

Inclusive DIS: Relation at small-x between CGC and Integrated PDF calculations

In general, $F_1 = \frac{1}{2x} F_T$

$$F_2 = F_T + F_L$$

&

$$\sigma_{T,L} = \frac{(2\pi)^2 \alpha_{em}}{Q^2} F_{T,L} = \frac{\pi e^2}{Q^2} F_{T,L}$$

Structure function can be written in terms of cross section

Inclusive DIS: Relation at small-x between CGC and Integrated PDF calculations

$$\langle \mathcal{O} \rangle = \lim_{P'_{tar} \to P_{tar}} \frac{\langle P'_{tar} \,|\, \mathcal{O} \,|\, P_{tar} \rangle}{\langle P'_{tar} \,|\, P_{tar} \rangle}$$

Similar to SIDIS, we will use relation of operators

$$\sigma_{T,L} = \frac{(2\pi)^2 \alpha_{em}}{Q^2} F_{T,L} = \frac{\pi e^2}{Q^2} F_{T,L}$$

Structure function can be written in terms of cross section

$$F_T^q = \frac{Q^2}{2q^+ P^-_{tar}} \sum_f e_f^2 \,\mathrm{Re} \int \frac{dz^+}{\pi} \theta(z^+) \;\langle P_{tar} | \overline{\Psi}(z^+, 0) \;\frac{\gamma^-}{2}\; U_F(z^+, 0, 0) \;\Psi(0, 0) \,| P_{tar} \rangle$$

$$\mathrm{Re} \int \frac{dz^+}{\pi} \theta(z^+) \;\langle P_{tar} | \overline{\Psi}(z^+, 0) \;\frac{\gamma^-}{2}\; U_F(z^+, 0, 0) \;\Psi(0, 0) \,| P_{tar} \rangle = q(x)|_{x=0}$$

Inclusive DIS: Relation at small-x between CGC and Integrated PDF calculations

$$\langle \mathcal{O} \rangle = \lim_{P'_{tar} \to P_{tar}} \frac{\langle P'_{tar} \,|\, \mathcal{O} \,|\, P_{tar} \rangle}{\langle P'_{tar} \,|\, P_{tar} \rangle}$$

Similar to SIDIS, we will use relation of operators

$$\sigma_{T,L} = \frac{(2\pi)^2 \alpha_{em}}{Q^2} F_{T,L} = \frac{\pi e^2}{Q^2} F_{T,L}$$

Structure function can be written in terms of cross section

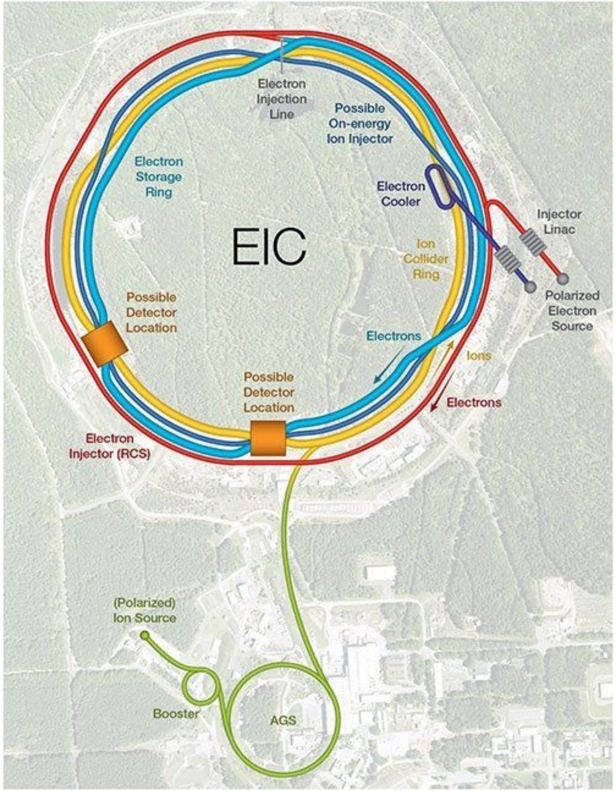
$$F_L^q = 0 \qquad F_2^q = F_T^q = \frac{Q^2}{W^2} \sum_f e_f^2 \; q_f(x)|_{x=0}$$

we recover quark contribution to structure function in terms of quark PDF

We recover naive parton model expression at NEik order!

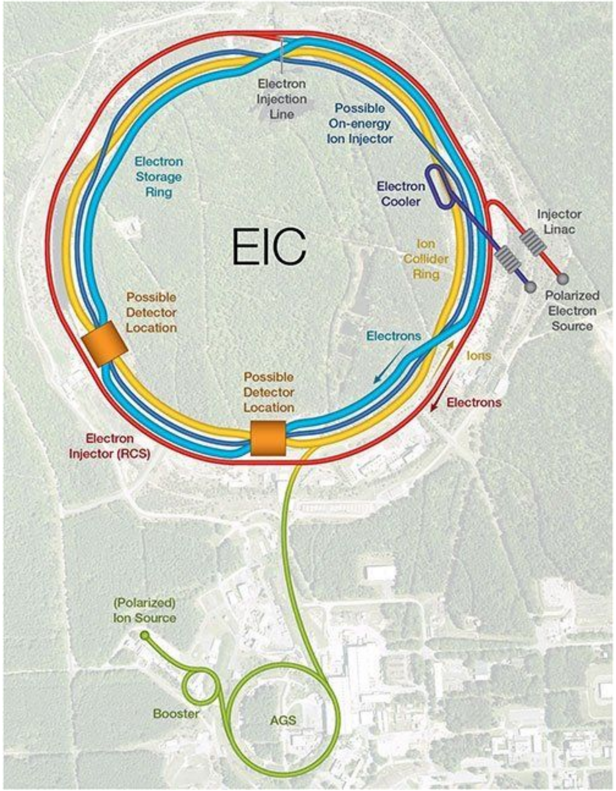
We can also obtain similar contribution in the case of antiquark

Summary



- Saturation Physics in relatively new physics
  - Lot of interesting things and physics yet to be explored
- Upcoming electron-ion collider brings prospect of precision era
  - So sub eikonal corrections might be sizable in upcoming future!

Summary



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Thank You!