Probing Graviton Mass

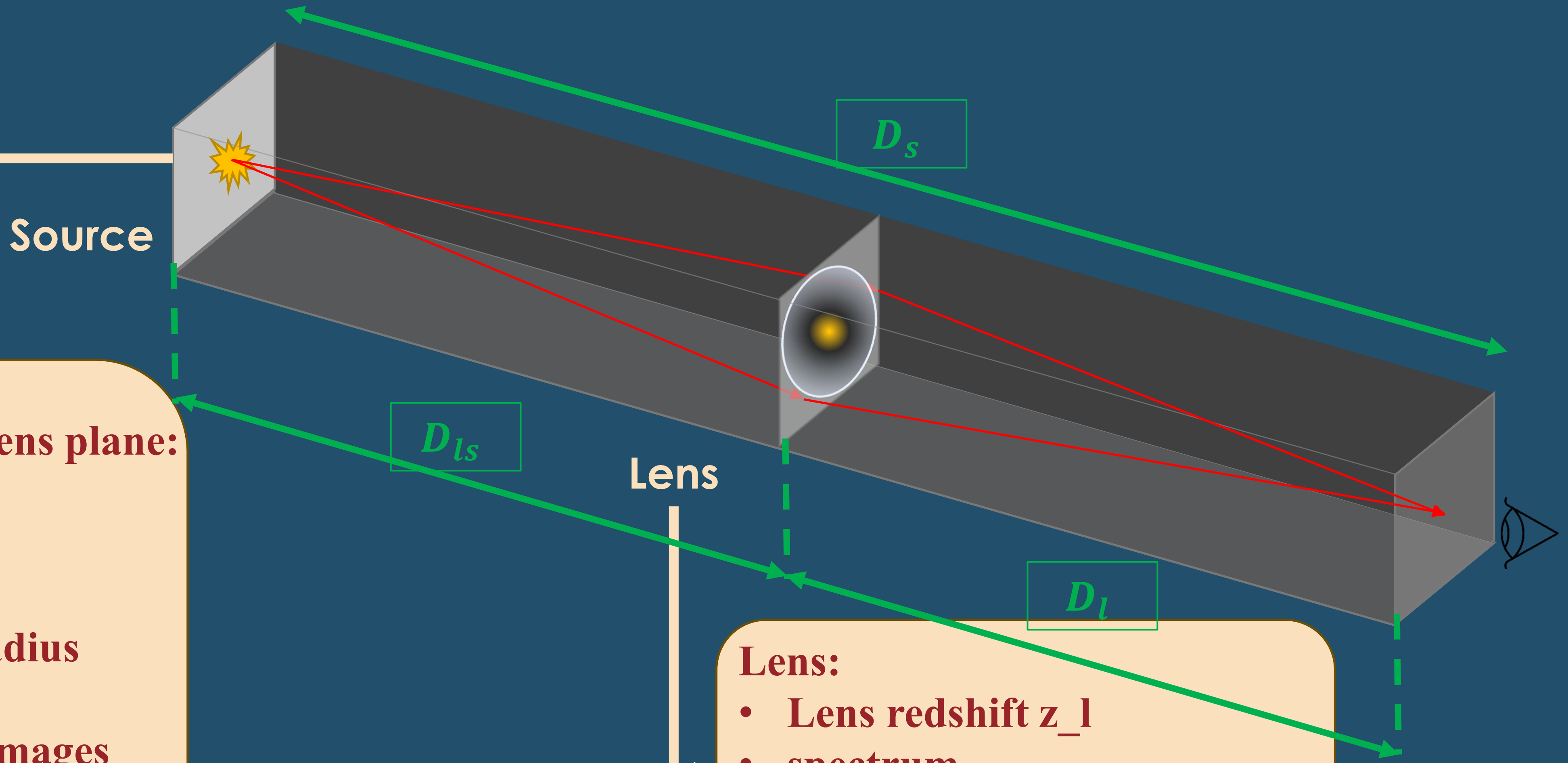
Through Strong Lensed Gravitational waves

Shuaibo Geng

Collaborators : Sreekanth Harikumar, Marek Biesiada


Credit: NASA/ESA/CSA
Webb Telescope

Lensing Structure

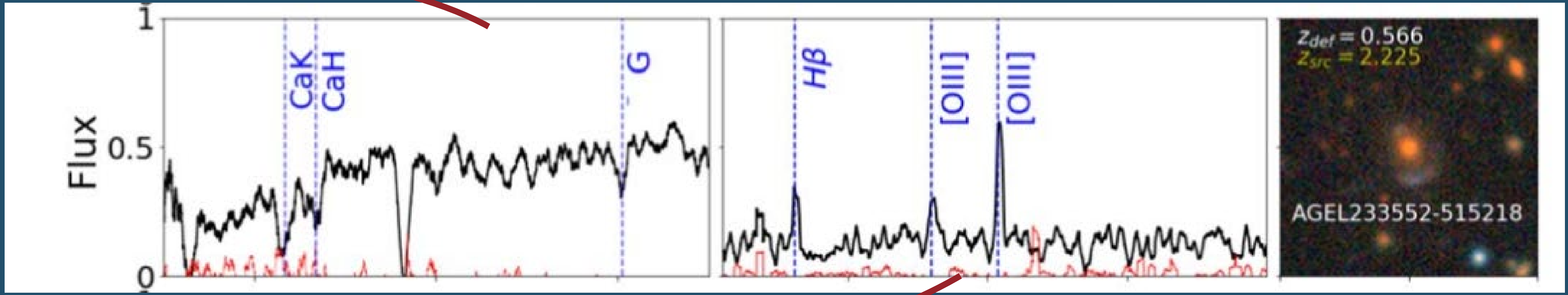


Source Images on the lens plane:
- Source redshift z_s
- Spectrum
- Images' separations
- Effective Einstein radius
- Flux
- Flux ratio between images
- Time-delays
- ……

Lens:
- Lens redshift z_l
- spectrum
- Velocity dispersion
- ……

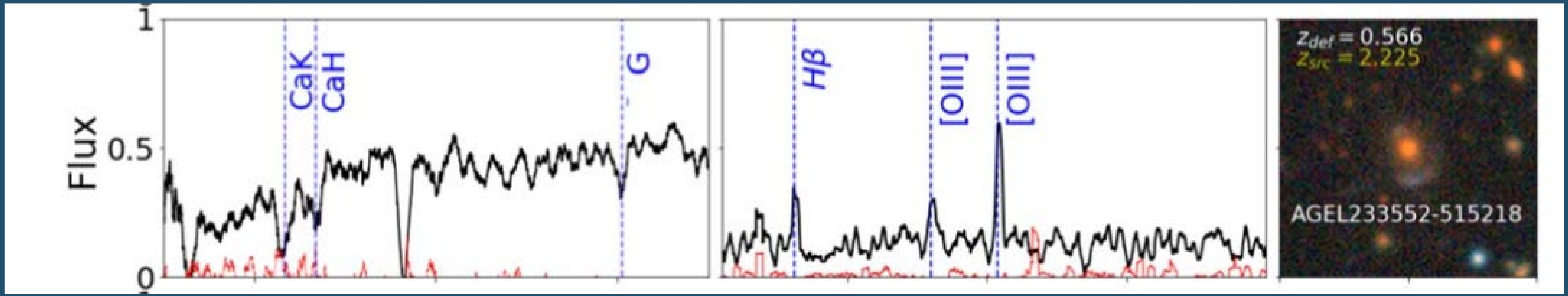
Lensing Observables

Lensing Observables



Flux
Spectrum
Redshifts
Image separations
Einstein radius
Time delays

Lensing Observables



Flux
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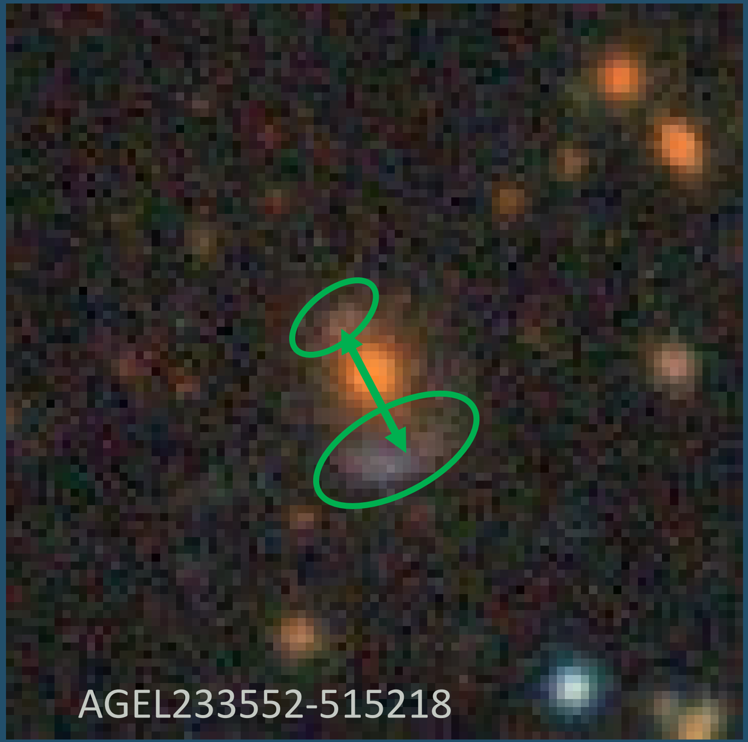

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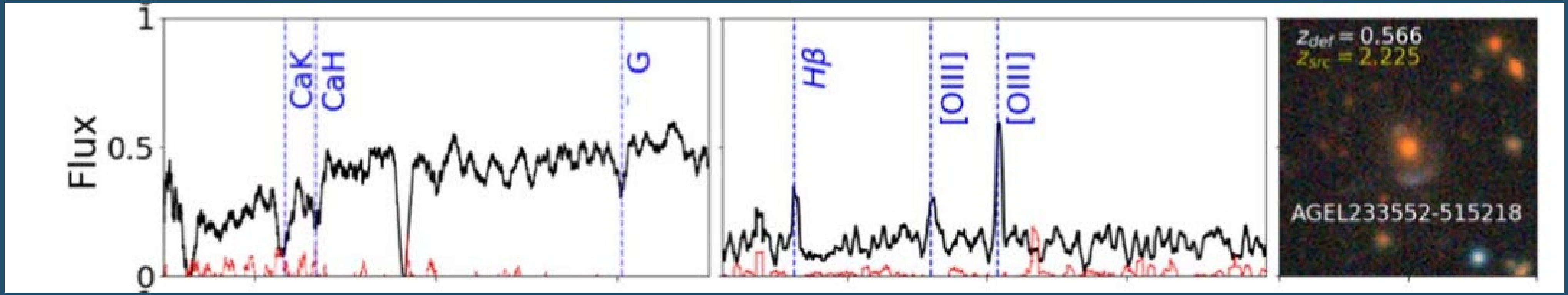

Flux
Spectrum
Redshifts
Image separations
Einstein radius
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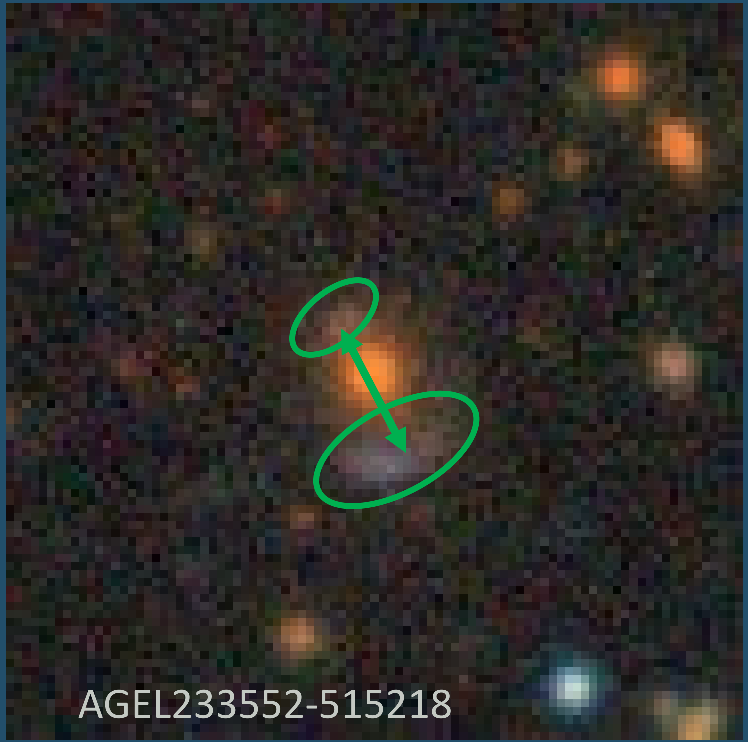
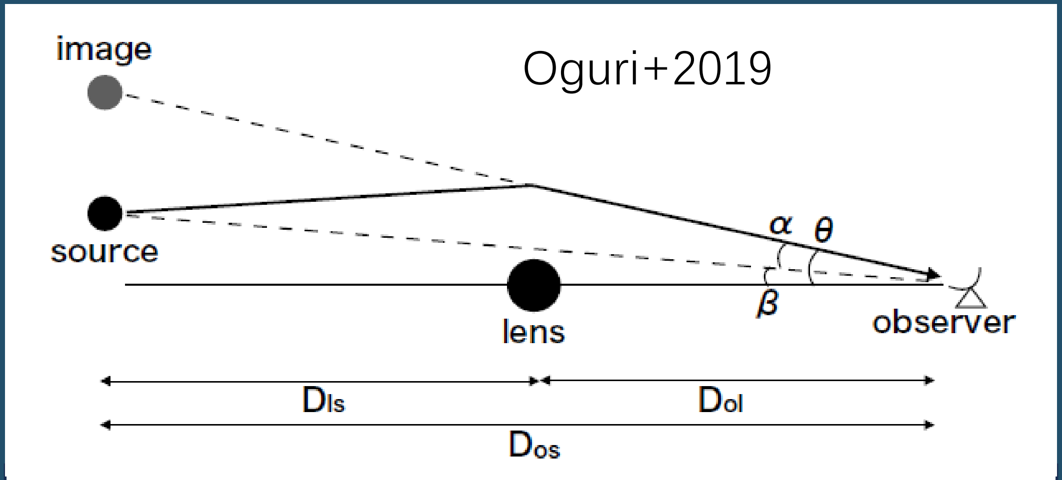
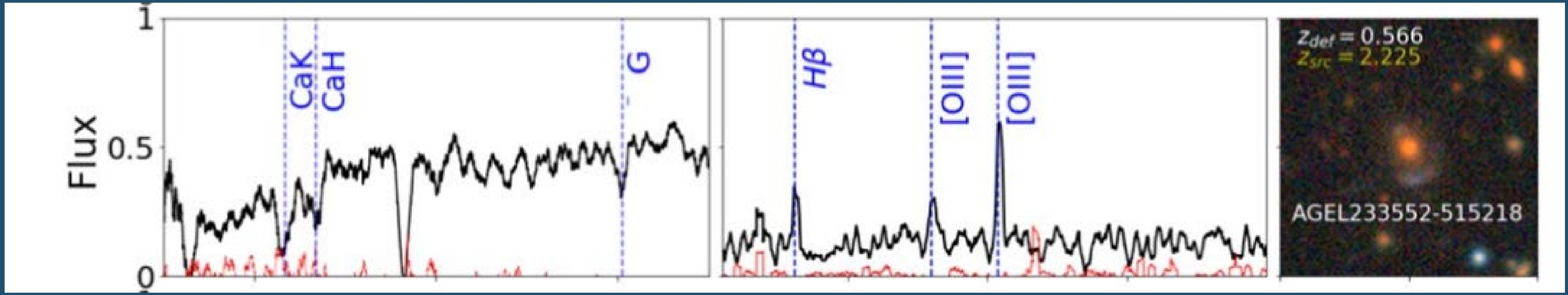




$$\Delta t = \frac{1+z_l}{c}\frac{D_{ol}D_{os}}{D_{ls}}\left[\frac{(\theta-\beta)^2}{2}-\phi(\theta)\right],$$

Lensing Observables



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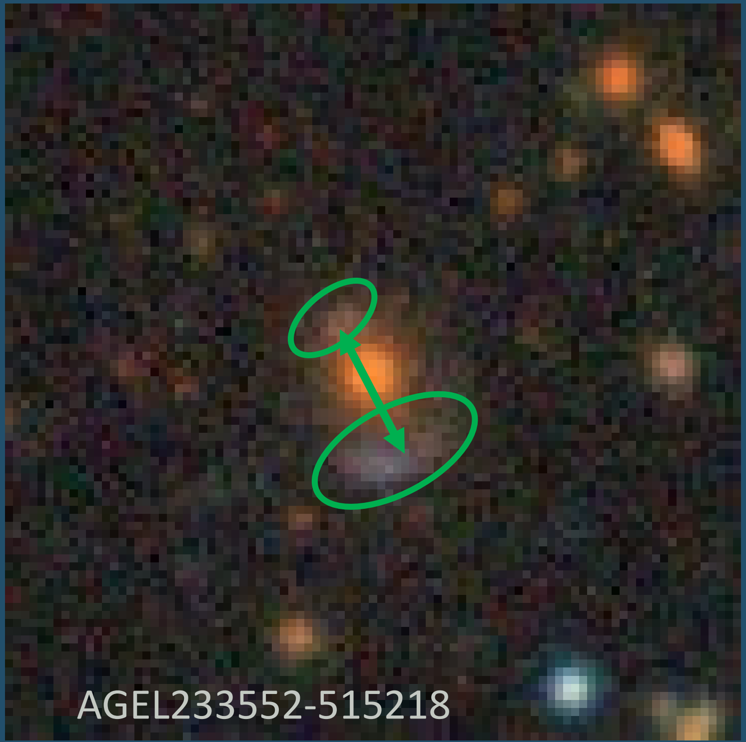

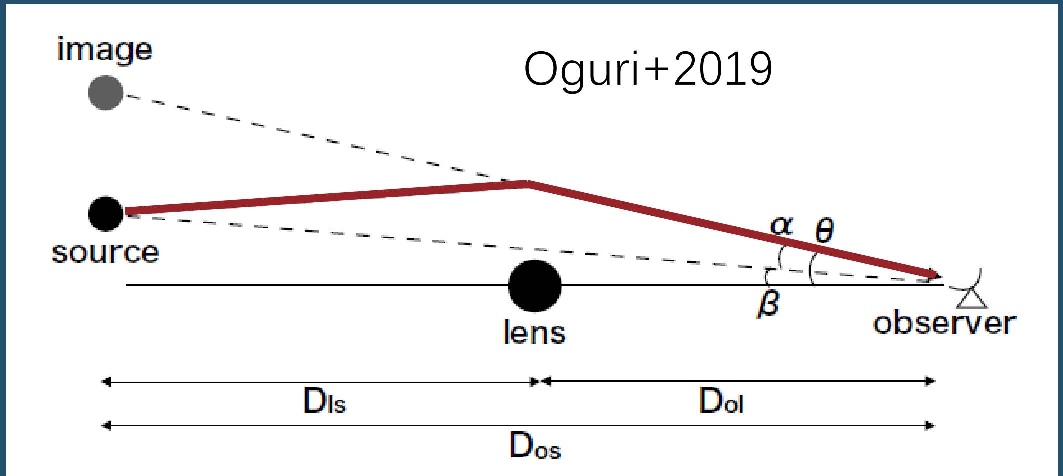

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Lensing Observables

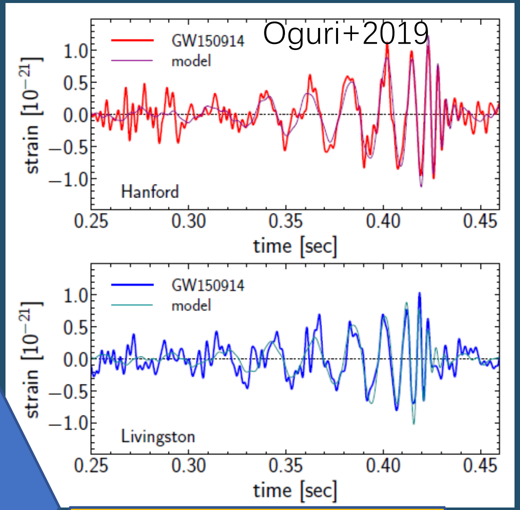
Explosive transients

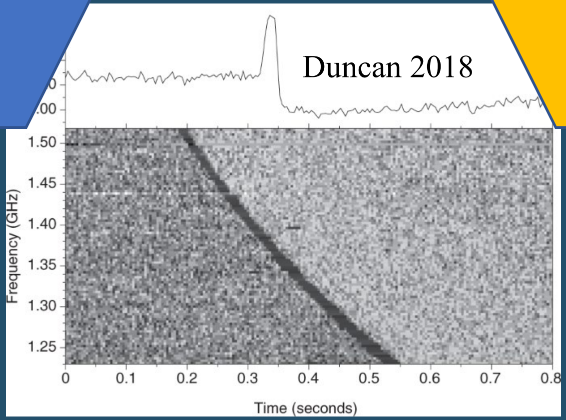
Supernovae

Gamma-ray bursts

Fast radio bursts

Gravitational waves





Supernova

History:

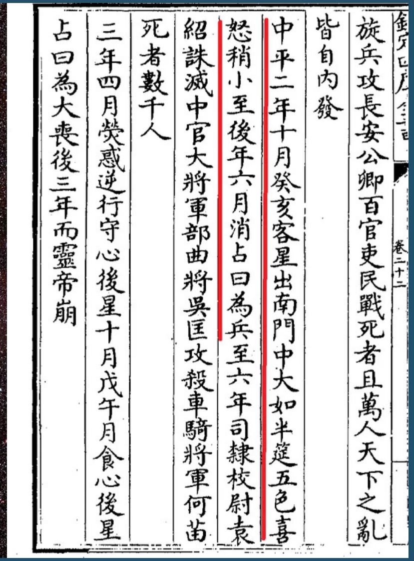
- First documented in 185 A.D. by Chinese astronomer, named with "Guest star"
- Thousands are observed currently

Properties:

- Type I: No Hydrogen (Ia: Si II; Ib: He I; Ic: no He)
  Type II: With Hydrogen (light curve shape, narrow lines)
- High luminous
- Long time scale
- Stable peak luminosity (SN Ia)

Applications:

- Star evolution
- Stellar initial mass function
- Cosmic expansion (standard candle)


旋兵攻長安公卿百官吏民戰死者且萬人天下之亂
皆自內發
中平二年十月癸亥客星出南門中大如半筵五色喜
怒稍小至後年六月消占曰為兵至六年司隸校尉袁
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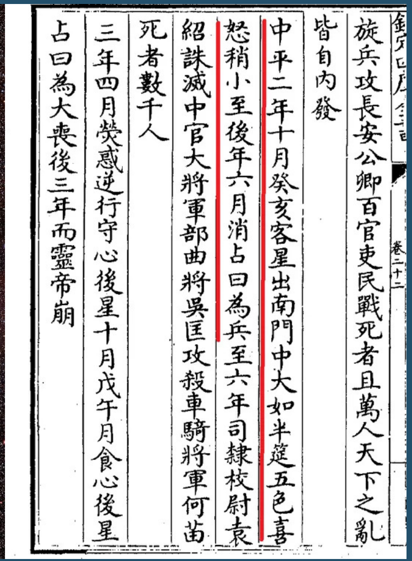
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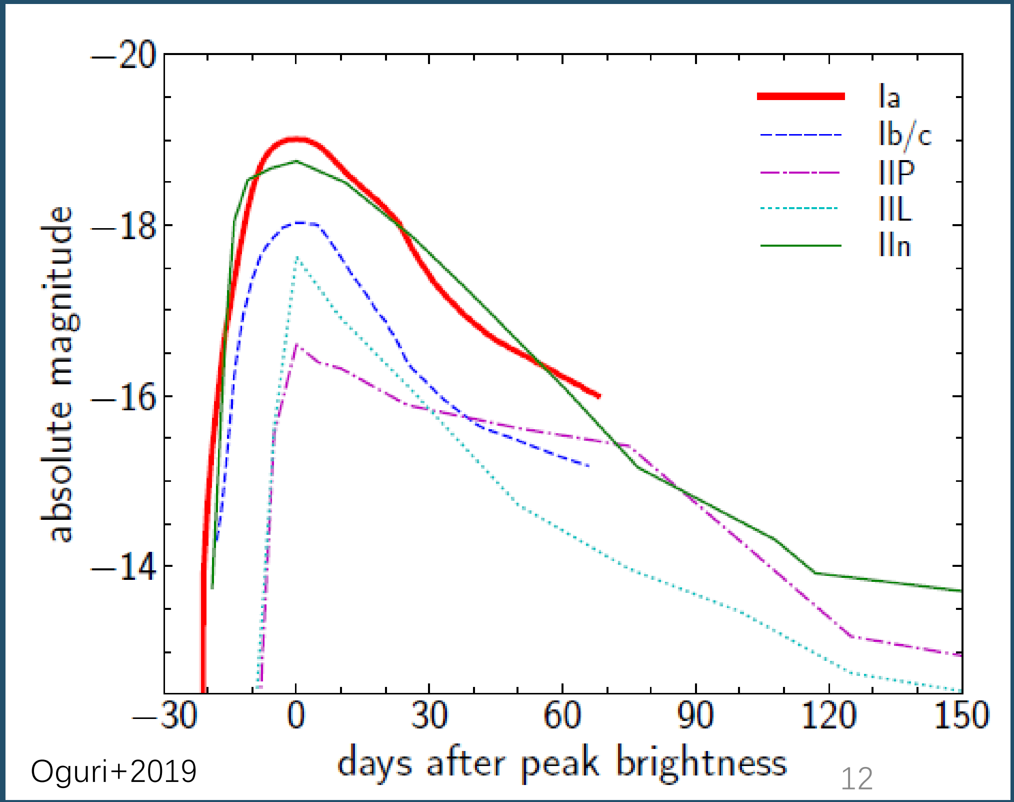

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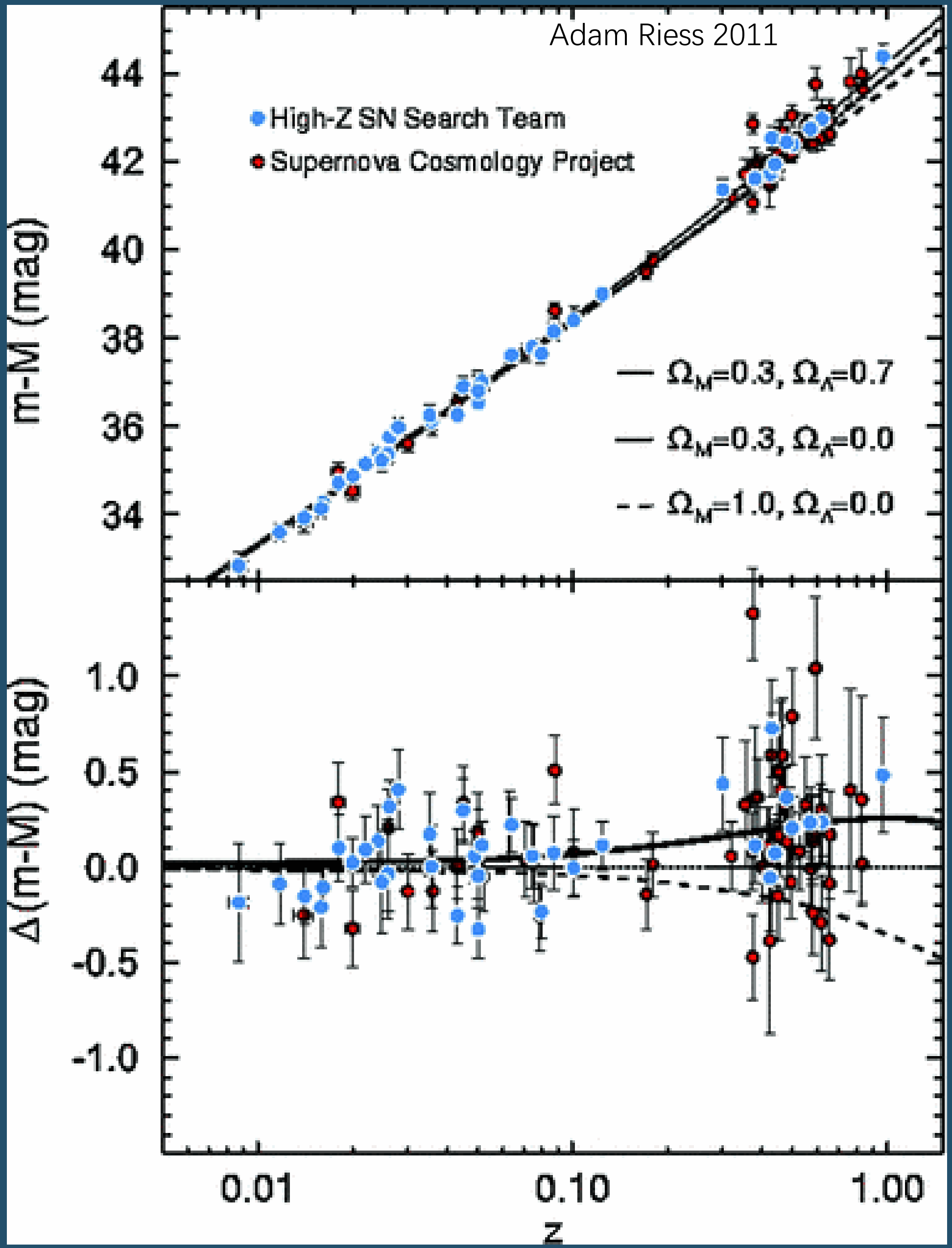

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Lensed SNe Ia
- Flux ratio
- Spectrum
- Redshifts
- Image separations
- Einstein radius
- Time delays

Gamma-ray Burst

History:
- First detected by Vela satellites in 1967
- BEAST: isotropic distribution (extragalactic origin)
- GW170817: GW+Gamma+Xray+NIR+optical

Properties:
- Short burst: < 2s (binary mergers of compact objects)
  Long burst: > 2s (death of massive stars, accompanied by core-collapse SNe)
- Extremely luminous

Applications:
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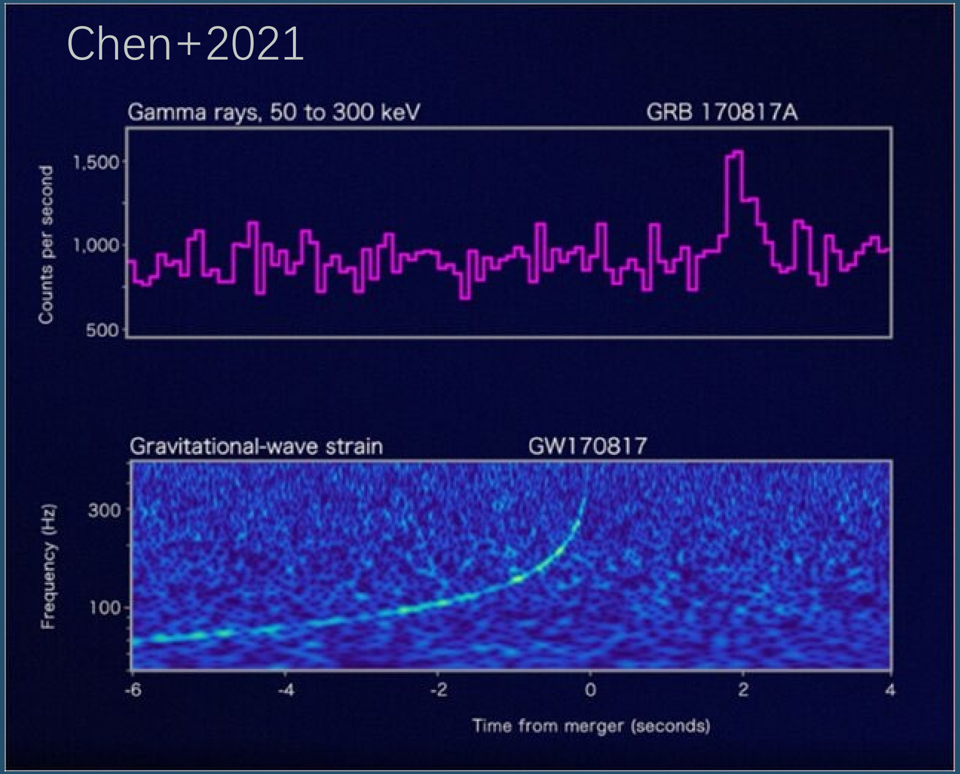
- Potential standard candle?



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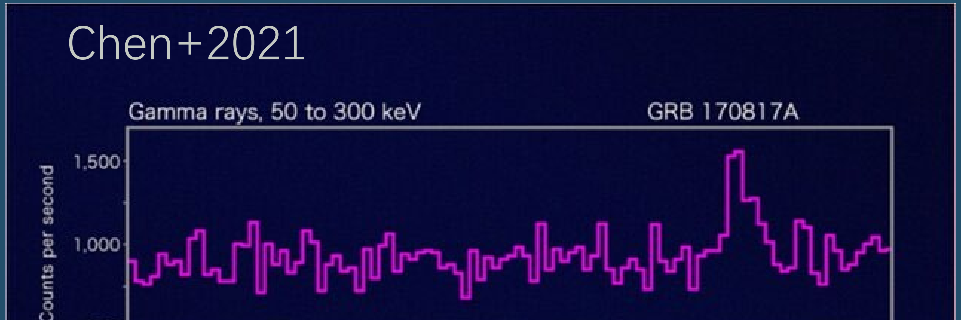

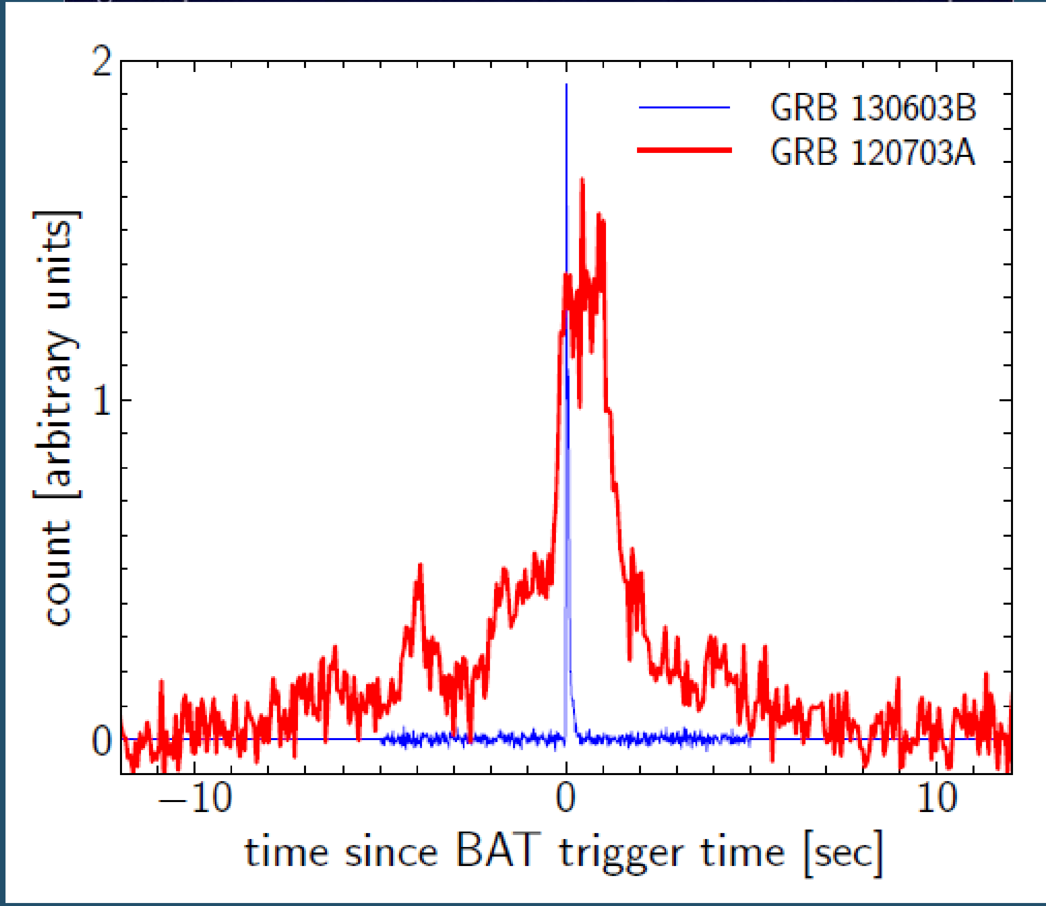

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Lensed Gamma-ray Bursts
- Flux ratio
- Spectrum
- Redshifts
- Image separations
- Einstein radius
- Time delays

Fast radio burst

History:

- First discovered with the Parkes Observatory in 2007 by Lorimer et al.
- Fake signal (from microwave oven)

Properties:

- Short time scale (~msec)
- Dispersive effects (EM waves propagate through a plasma)
- ~10 repeat bursts

Applications:

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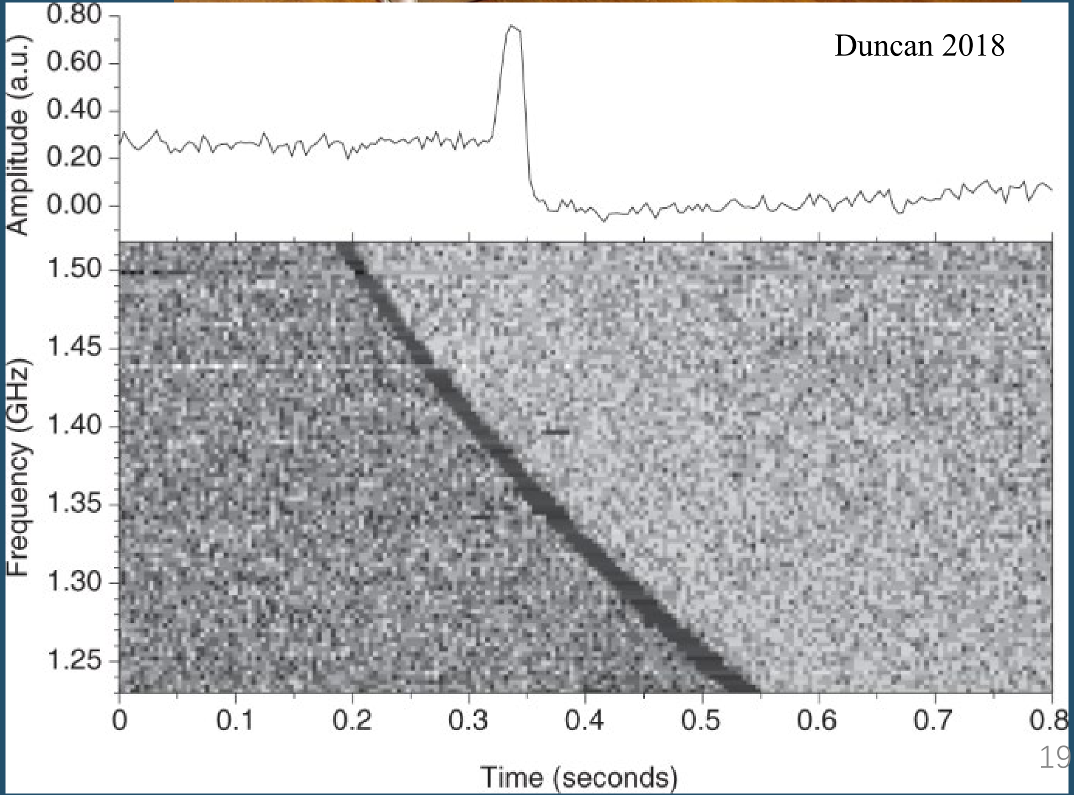

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Lensed FRB

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Gravitational wave

History:

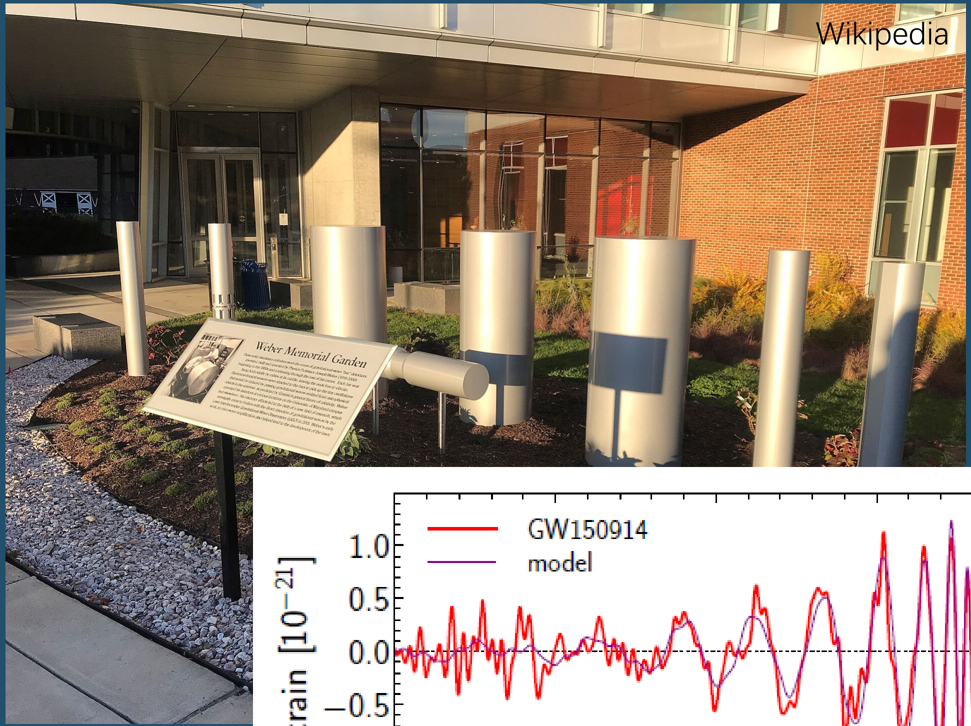
- Predicted by Einstein in 1916
- First attempt to detect GW in 1968 (Weber bar)
- First direct detected in 2015 by LIGO
- GW170817: GW+Gamma+Xray+NIR+optical

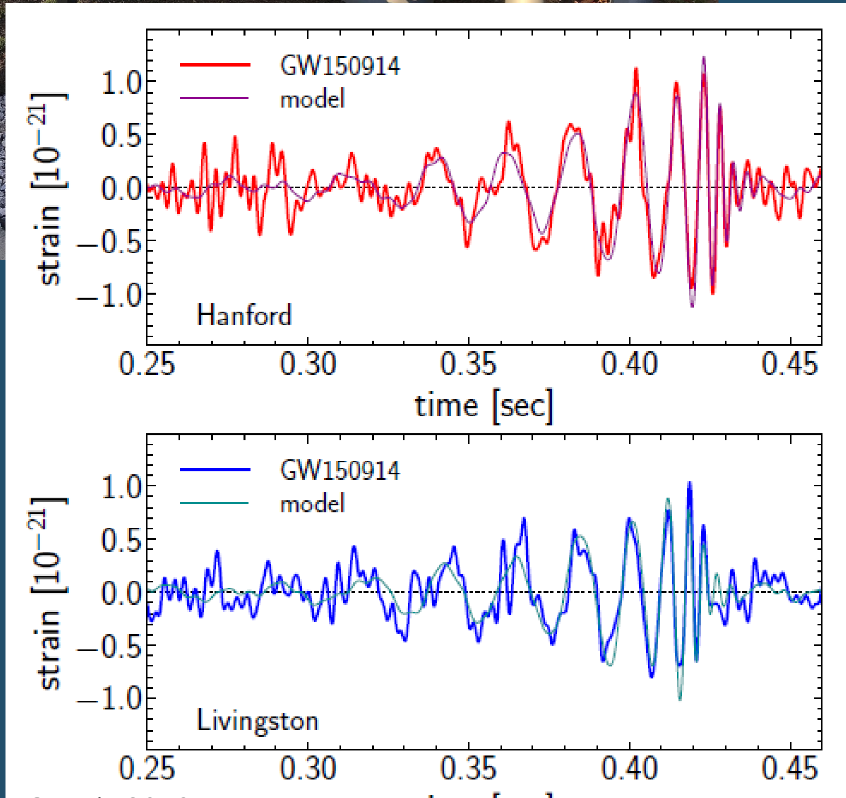
Properties:

- Well-study based on General Relativity
- Unaffected by interstellar medium or intergalactic medium

Applications:

- Mass initial function
- Compact objects (Black holes, neutron stars, white dwarfs)
- Cosmology study (Standard Siren)





Oguri+2019

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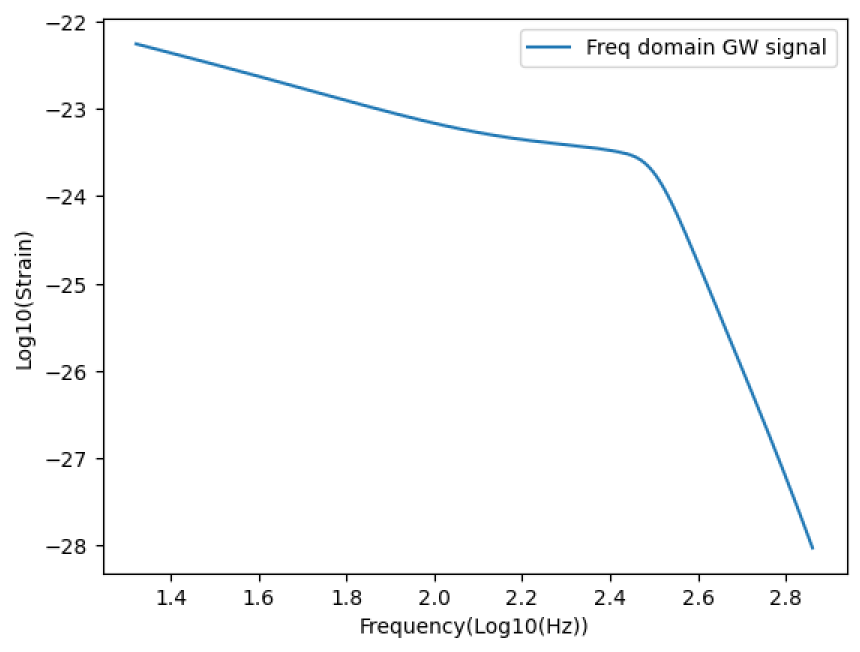
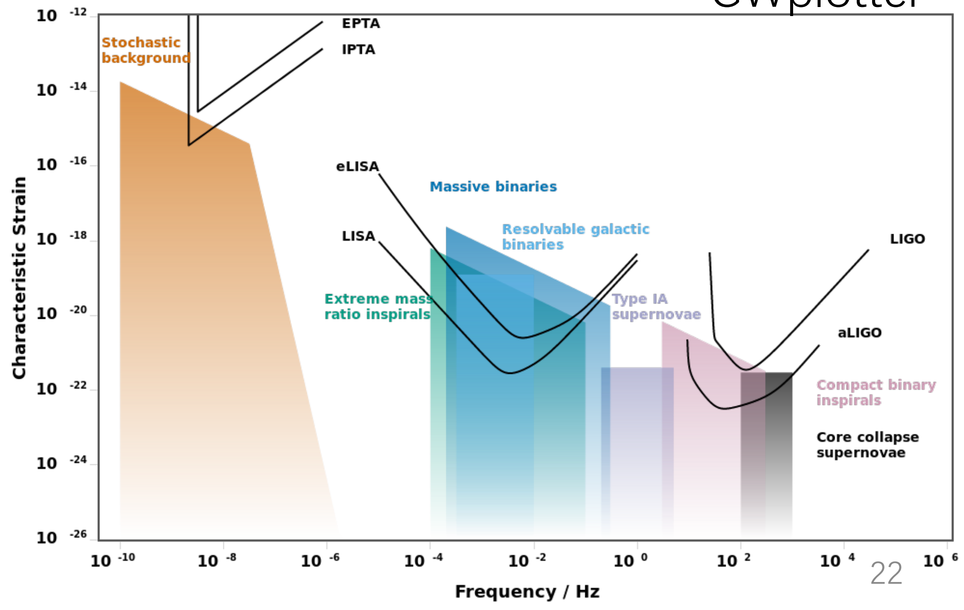


GWplotter



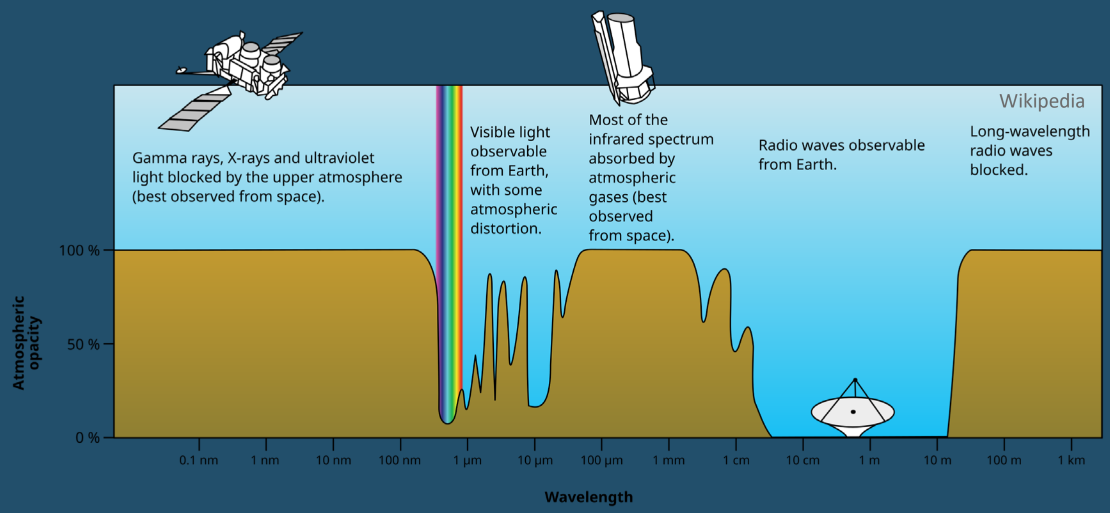
lensed transients

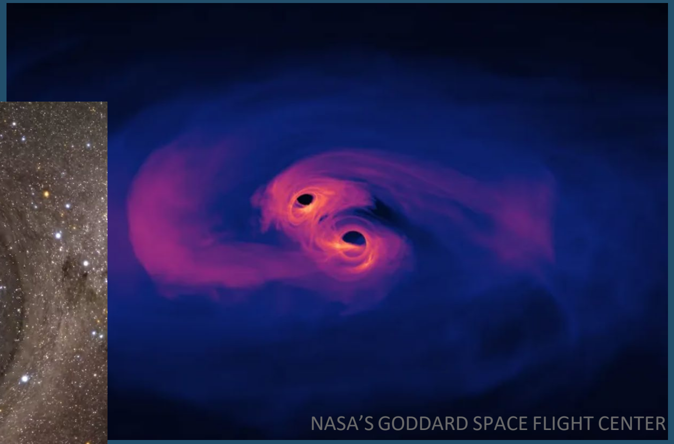
Source event rate in local Universe

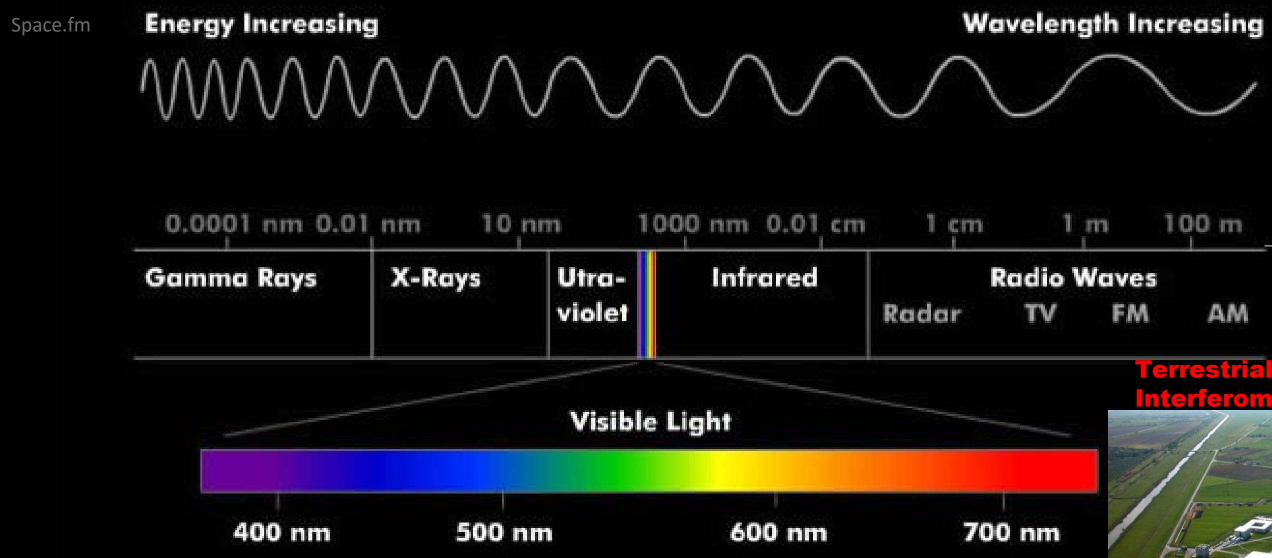
Lensing event rate within different redshift range

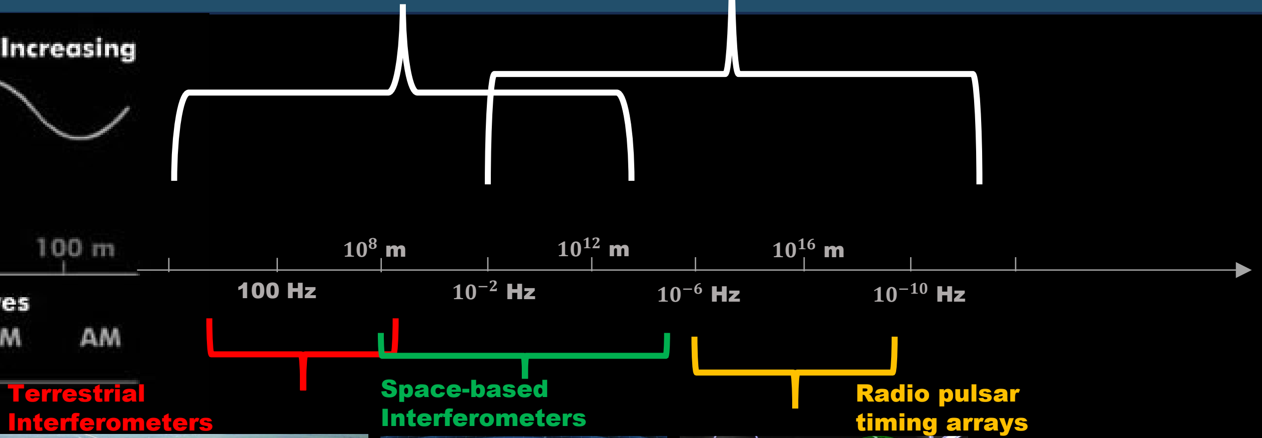
Type	Subclass	$R^{\rm loc}$ [$\rm Gpc^{-3}yr^{-1}$]	α_z	$R_{\rm sl}(<0.5)$ [$\rm sky^{-1}yr^{-1}$]	$R_{\rm sl}(<1)$ [$\rm sky^{-1}yr^{-1}$]	$R_{\rm sl}(<2)$ [$\rm sky^{-1}yr^{-1}$]	$R_{\rm sl}(<3)$ [$\rm sky^{-1}yr^{-1}$]
Supernova	Ia	3×10^4	1	1.6	30	320	1300
	core-collapse	7×10^4	2	5.4	130	2000	10000
	superluminous	200	2	0.02	0.38	5.8	29
Gamma-ray burst	long	1	2	<0.01	<0.01	0.03	0.15
	short	3	1	<0.01	<0.01	0.03	0.13
Fast radio burst	...	10^4	2	0.78	19	290	1500
Gravitational wave	BBH	30	2	<0.01	0.06	0.88	4.4
	BNS	600	1	0.03	0.61	6.5	25
	BHNS	10	1	<0.01	0.01	0.11	0.4

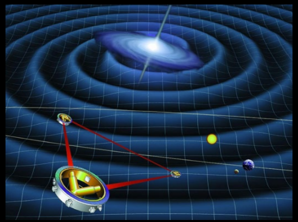
EM+GW spectrum

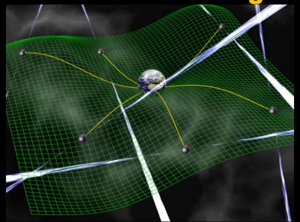




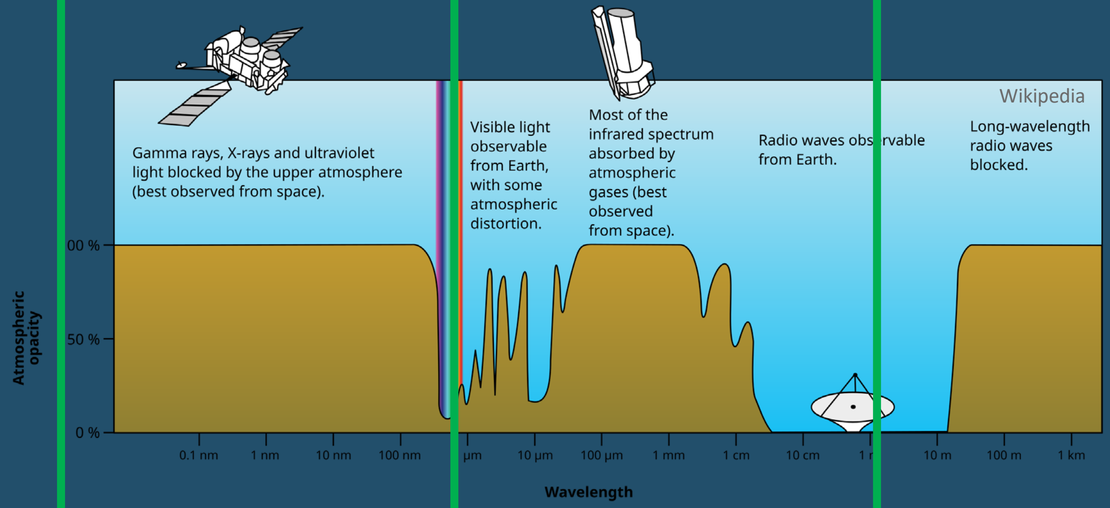


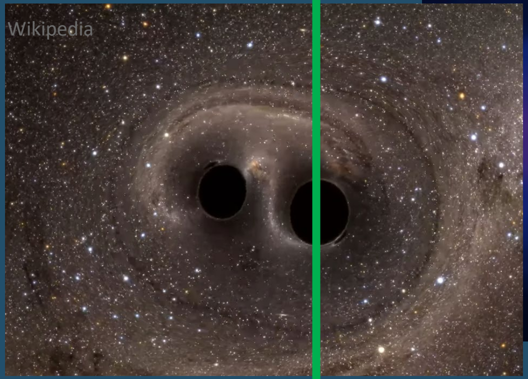


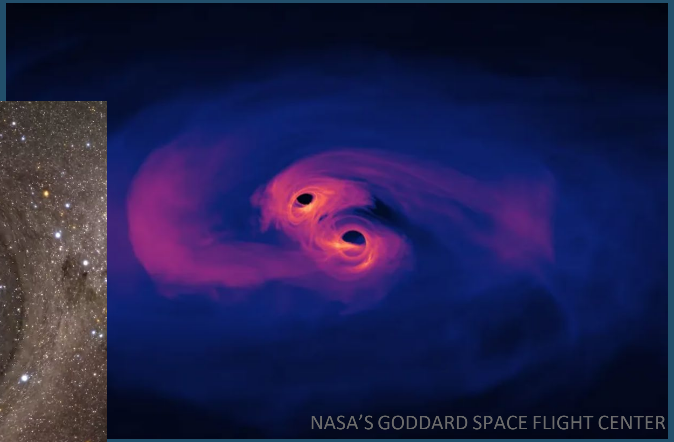


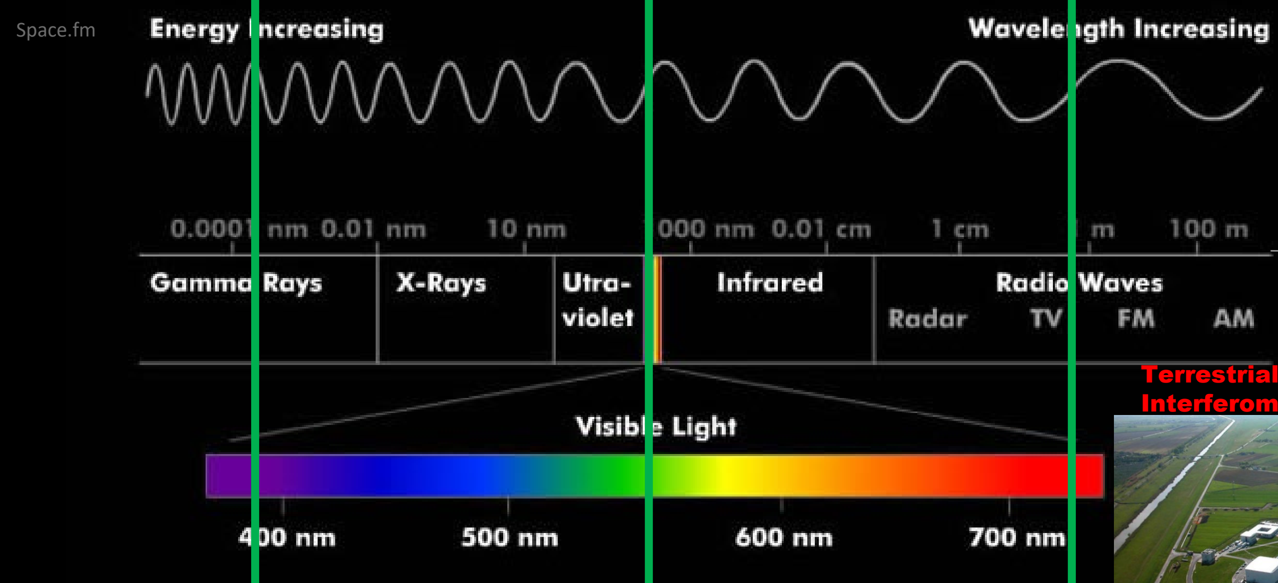


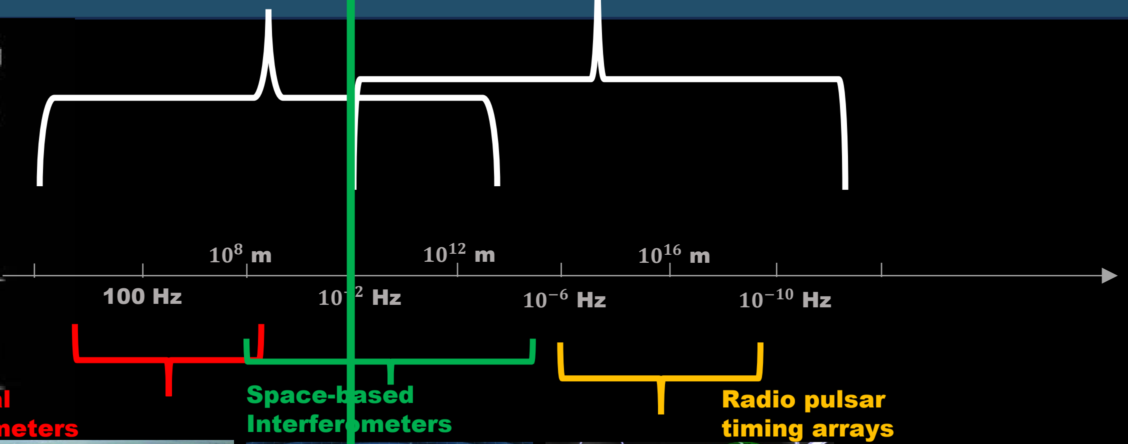
EM+GW spectrum

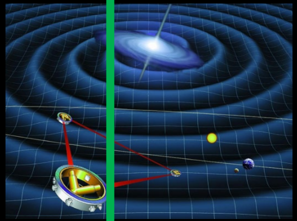


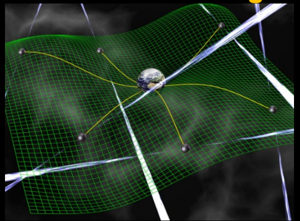




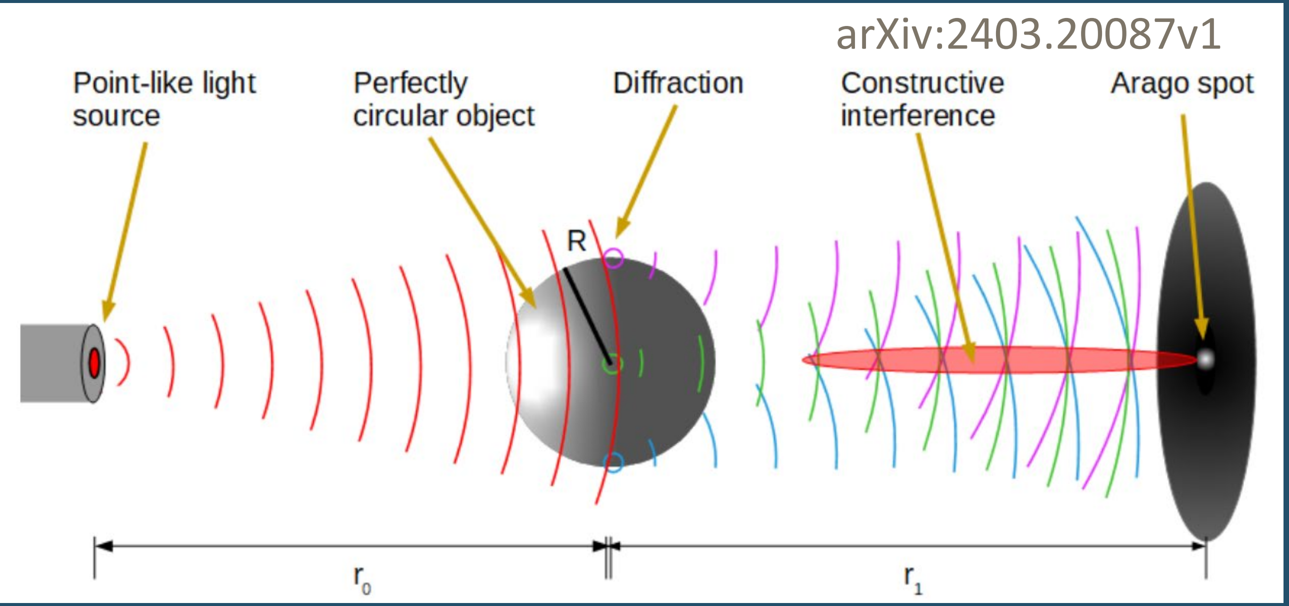


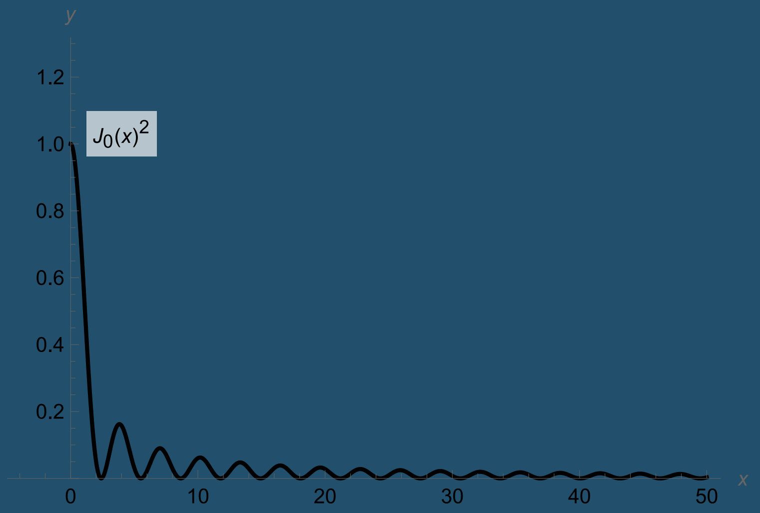


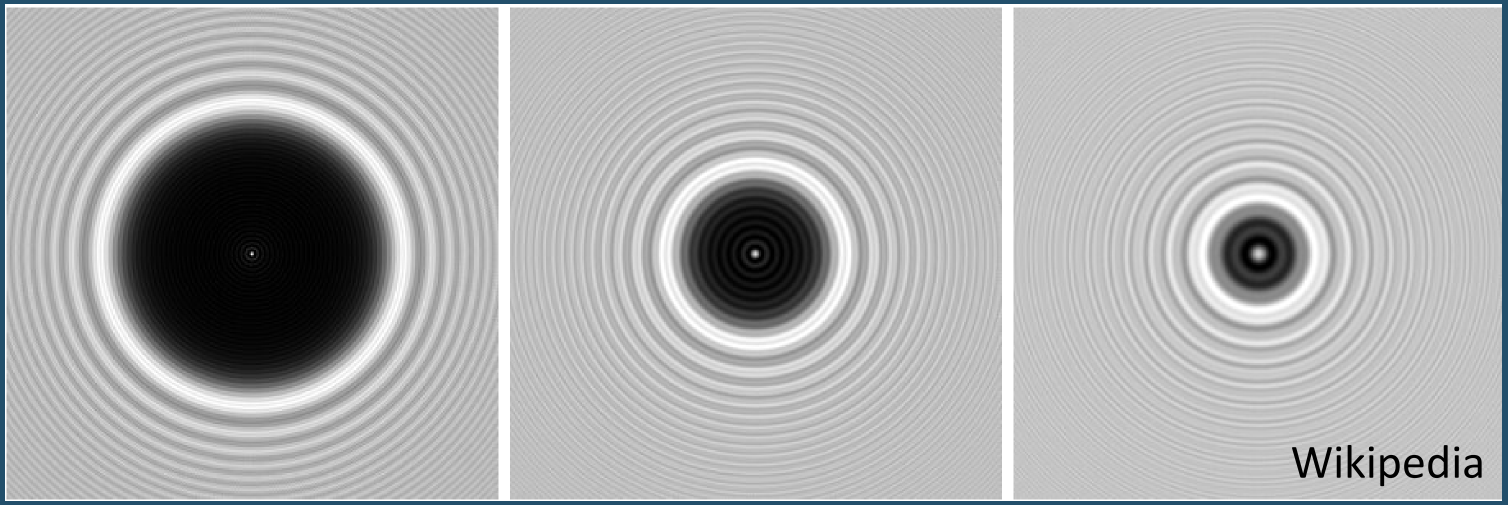




Wave effect

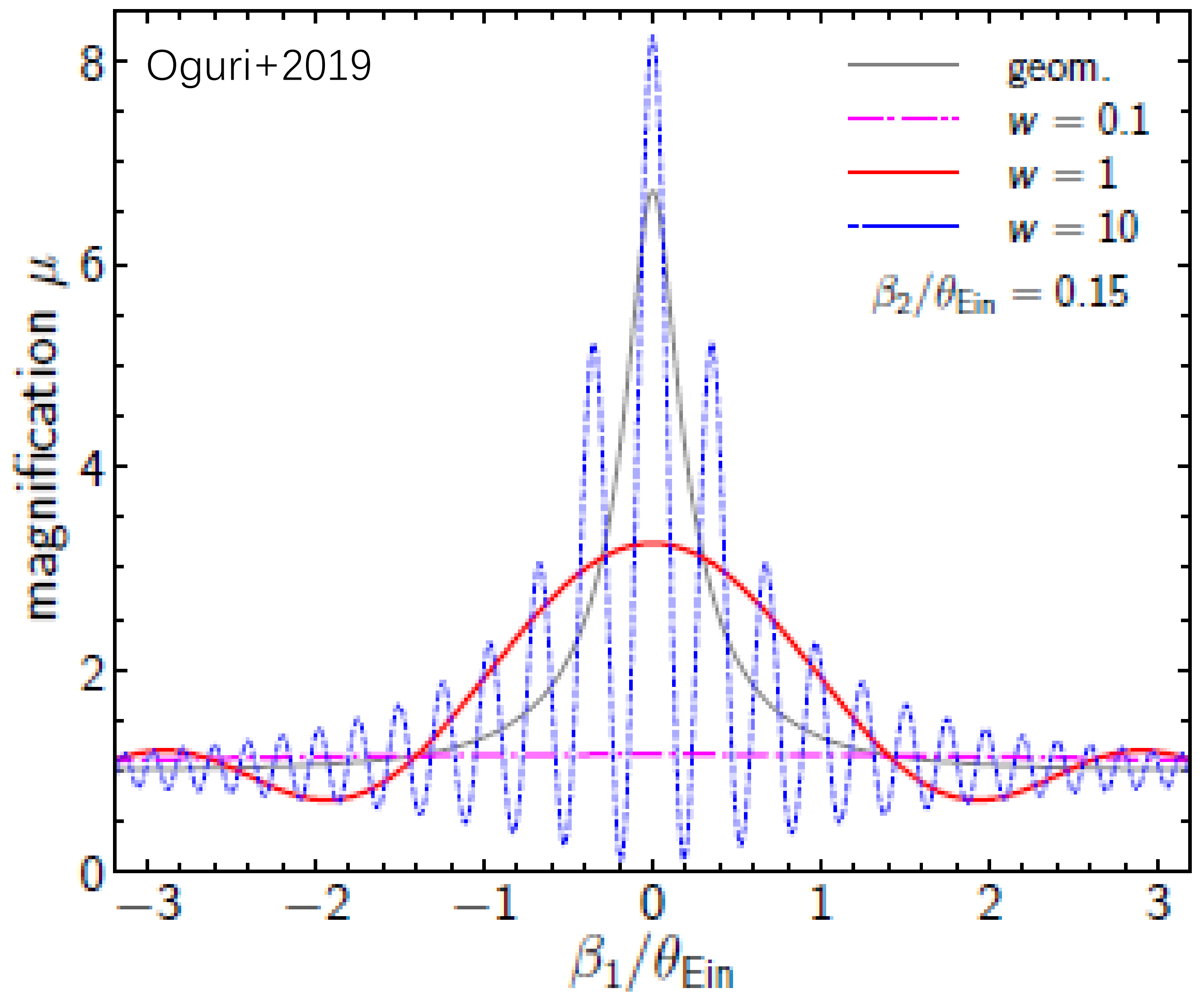




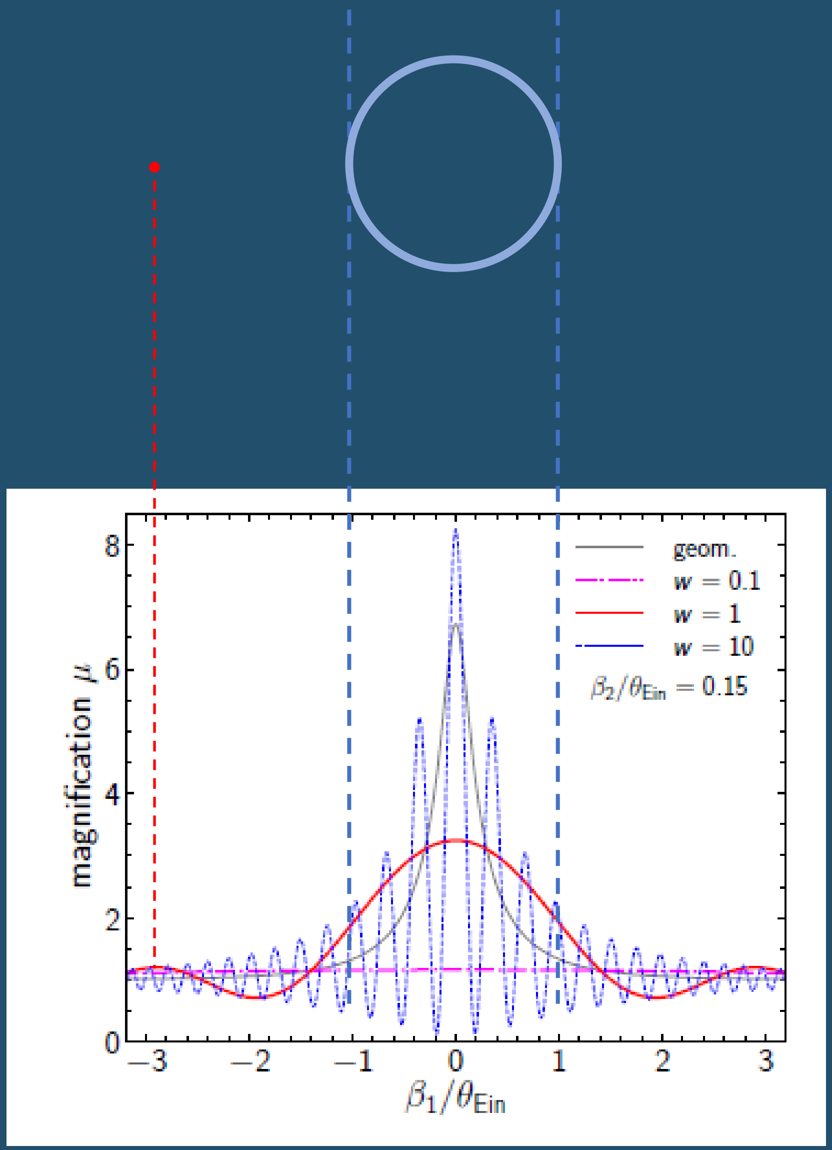


$$U(P_1, r) \propto J_0^2\left(\frac{\pi r d}{\lambda b}\right)$$

Wave optics


Oguri+2019
geom.
w = 0.1
w = 1
w = 10
β2/θEin = 0.15
magnification μ
β1/θEin
8
6
4
2
0
−3
−2
−1
0
1
2
3

Wave optics



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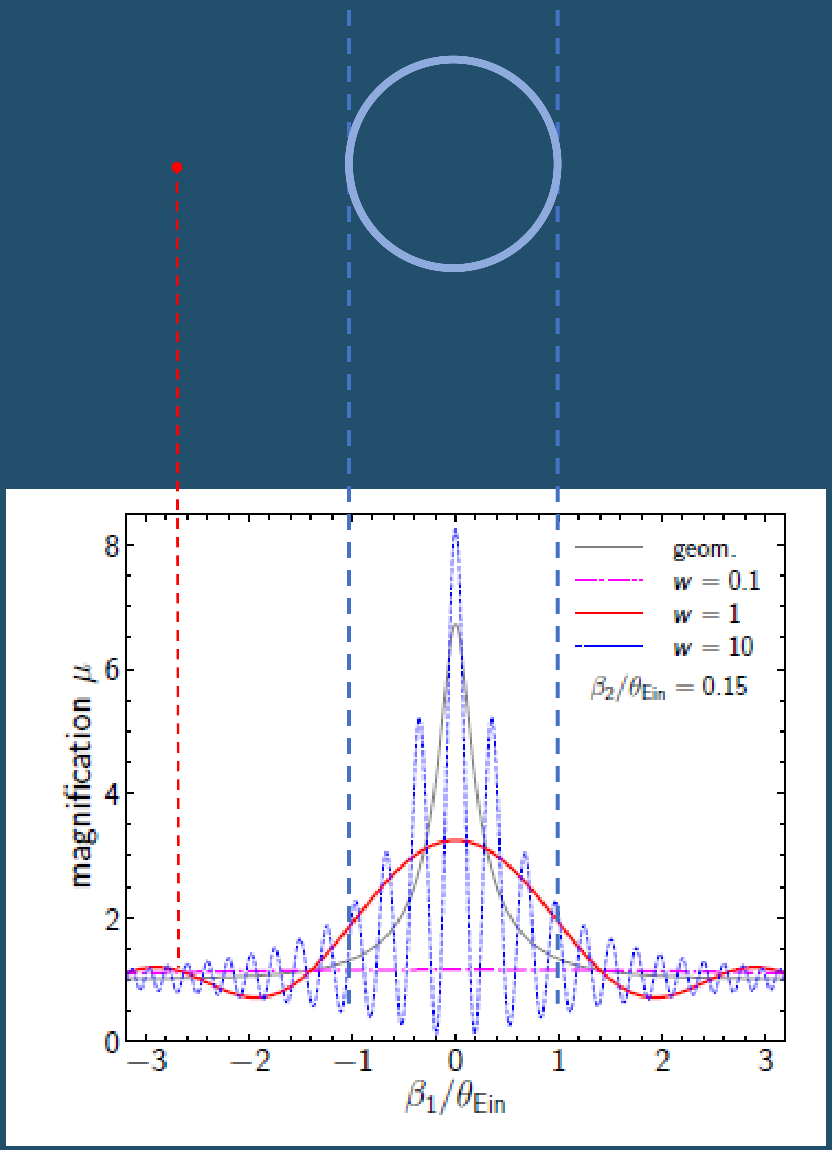

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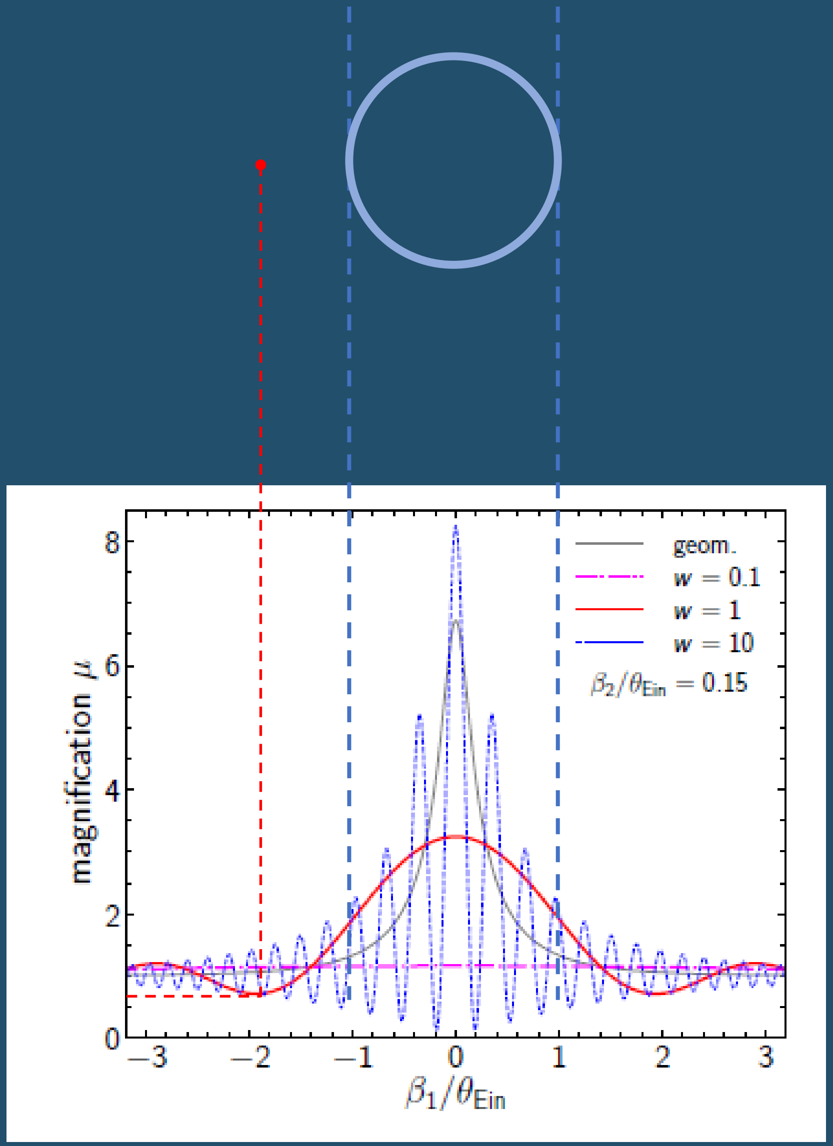

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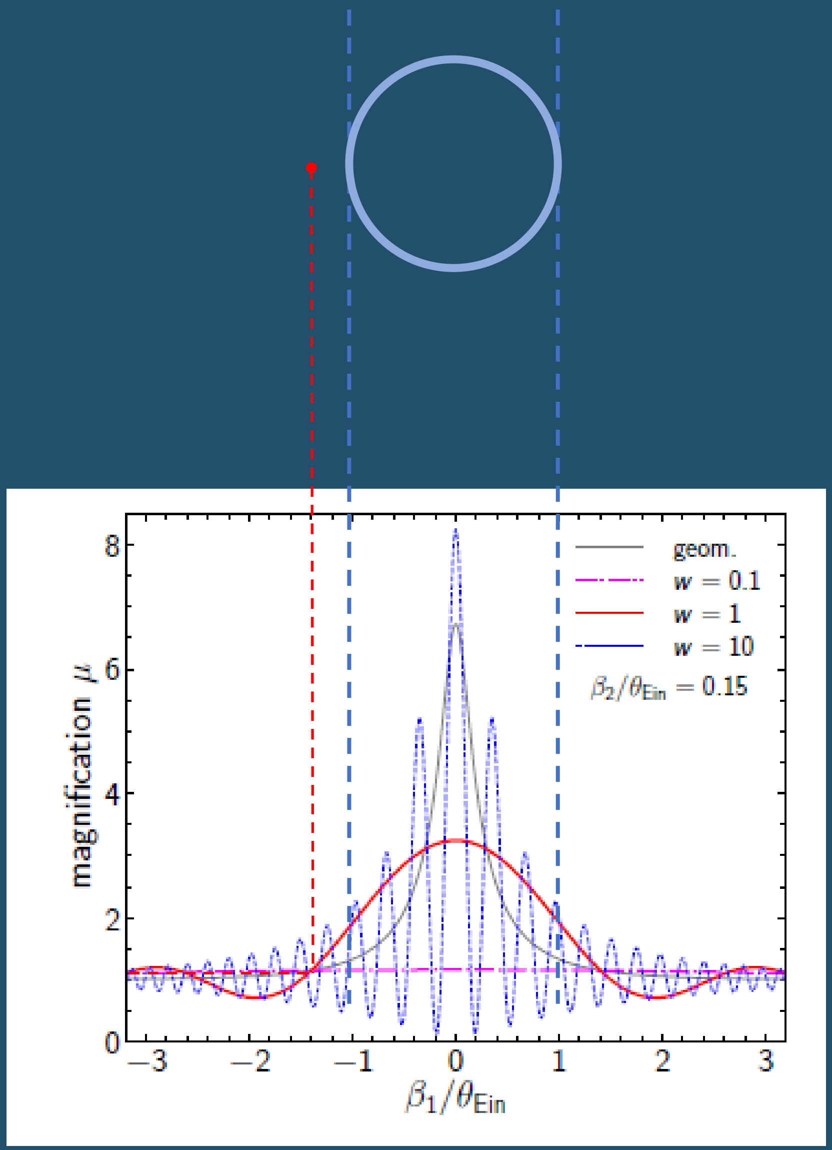

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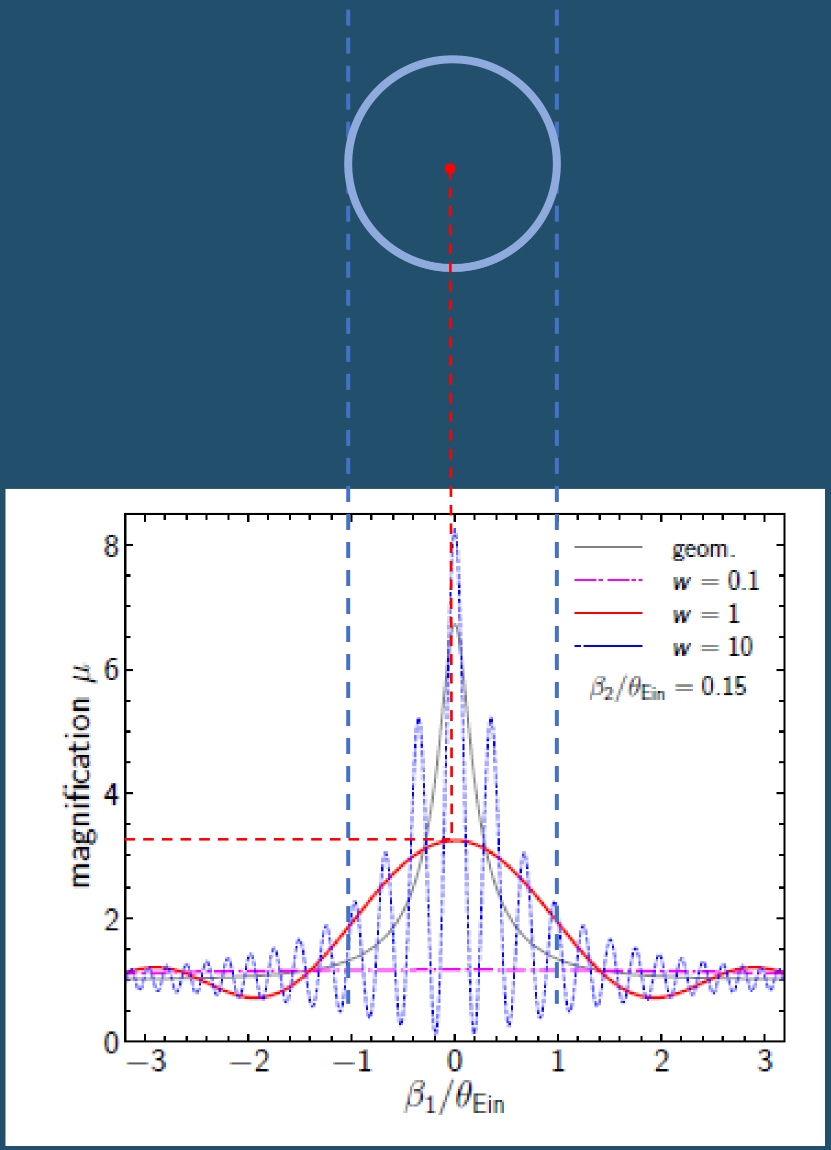

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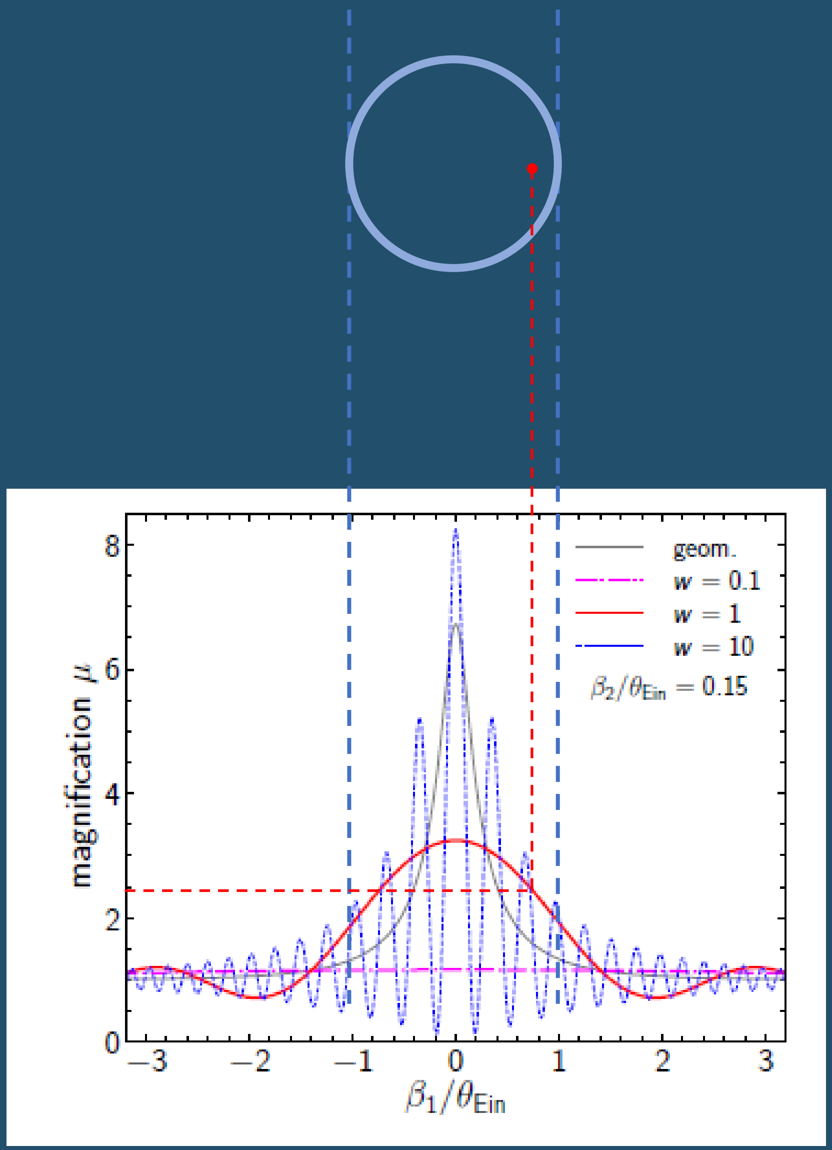

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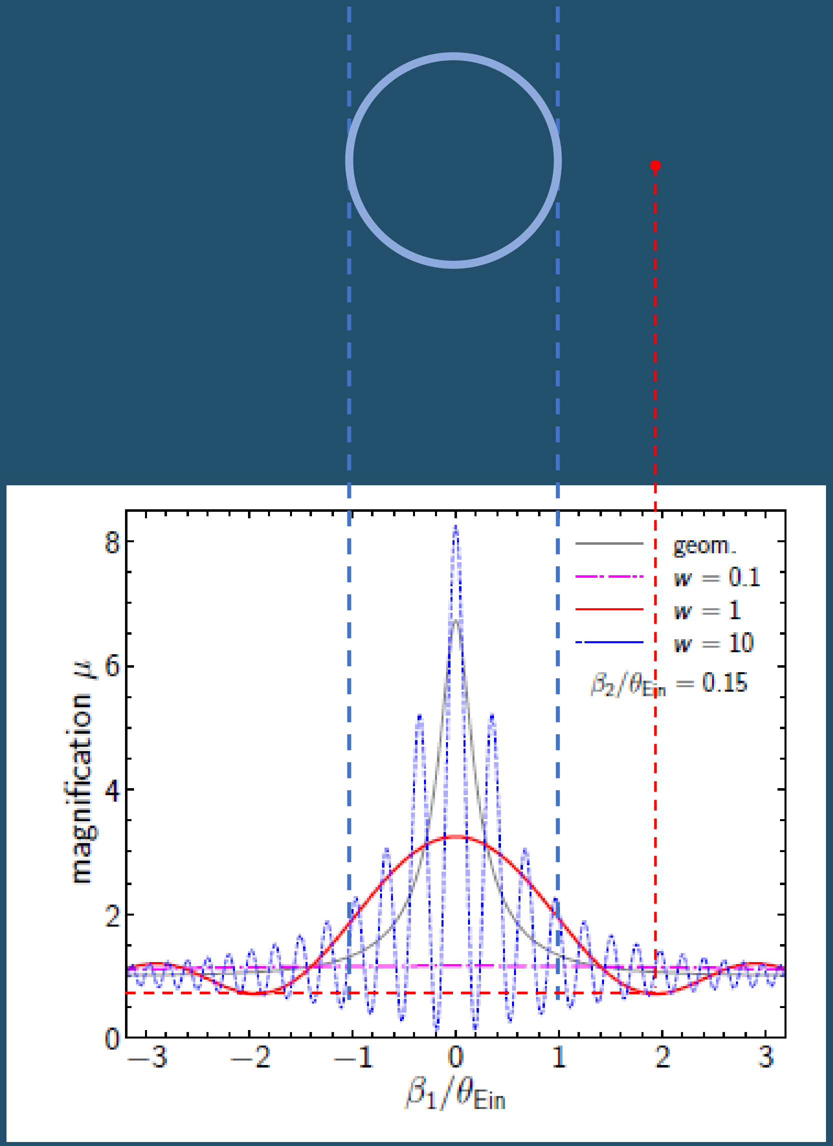
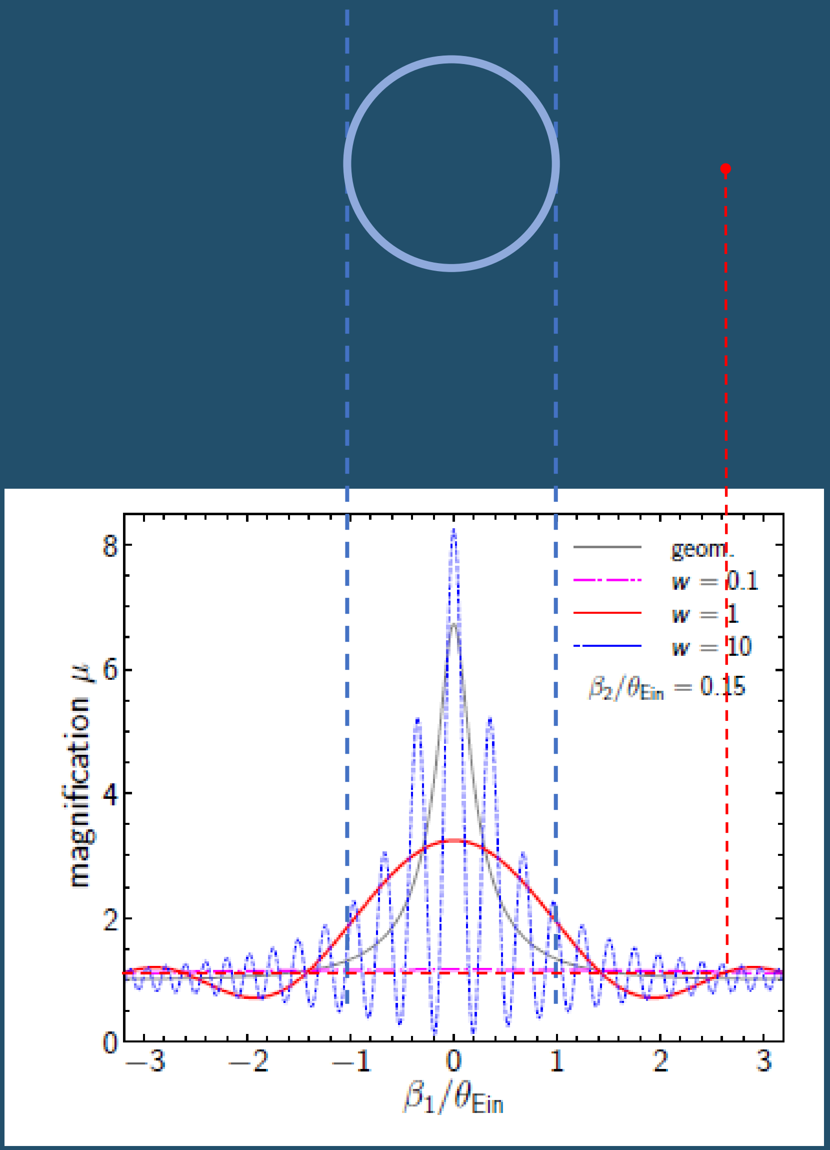
Wave optics

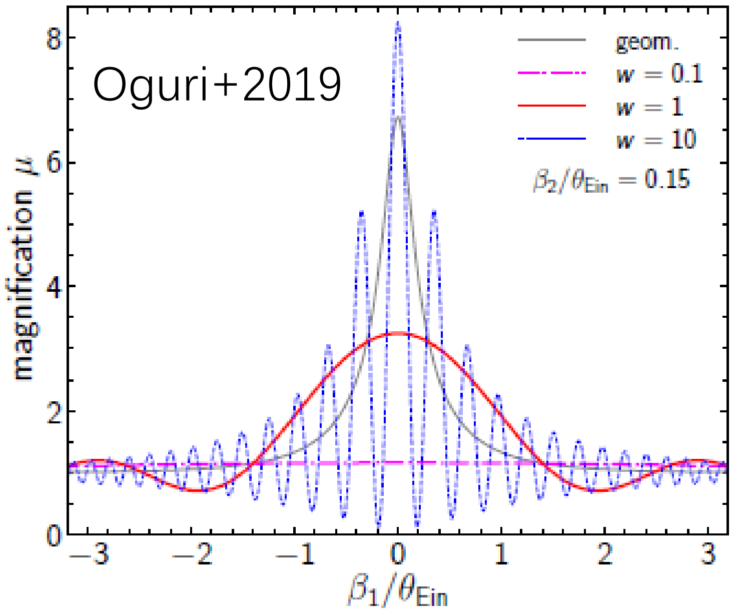


geom.
$w = 0.1$
$w = 1$
$w = 10$
$\beta_2/\theta_{Ein} = 0.15$
magnification μ
β_1/θ_{Ein}

Wave optics


geom.
$w = 0.1$
$w = 1$
$w = 10$
$\beta_2/\theta_{\rm Ein} = 0.15$
magnification μ
$\beta_1/\theta_{\rm Ein}$
0
2
4
6
8
−3
−2
−1
0
1
2
3

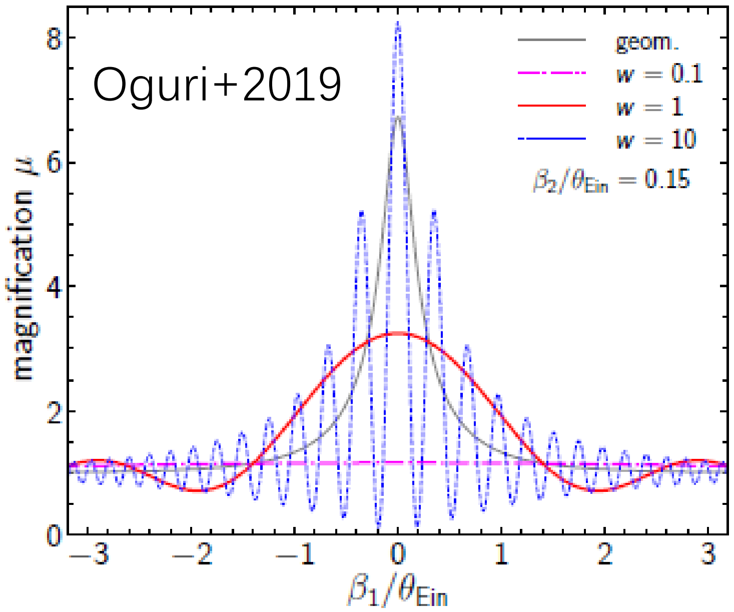
Wave optics to geometric optics



$$F(f,\boldsymbol{\beta}) = \frac{1+z_l}{c}\frac{D_{\rm ol}D_{\rm os}}{D_{\rm ls}}\frac{f}{i}\int d^2\boldsymbol{\theta}\exp\left[2\pi i f\Delta t(\boldsymbol{\theta},\boldsymbol{\beta})\right],$$

$$w = 2\pi f\Delta t_{\rm fid} = 2\pi f\frac{1+z_l}{c}\frac{D_{\rm ol}D_{\rm os}}{D_{\rm ls}}\theta_{\rm Ein}^2.$$

Wave optics to geometric optics



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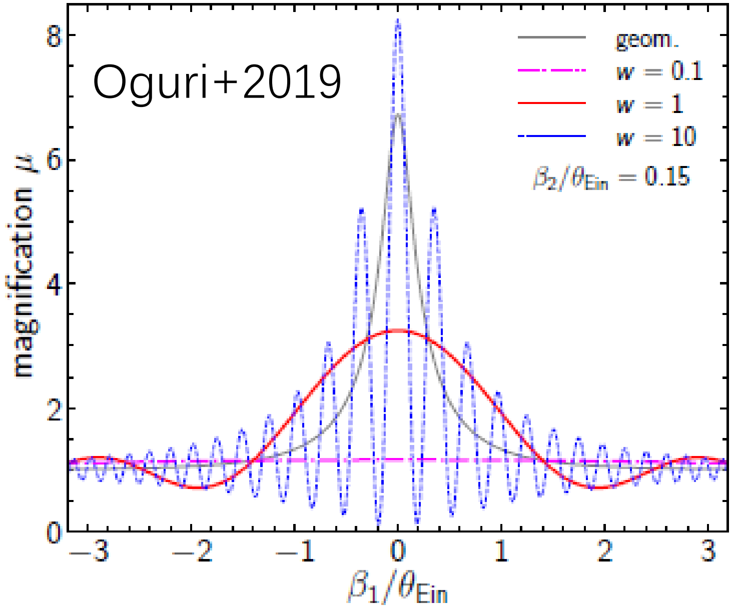
$$T(\hat{\boldsymbol{\theta}},\hat{\boldsymbol{\beta}}) = \frac{(\hat{\boldsymbol{\theta}}-\hat{\boldsymbol{\beta}})^2}{2} - \frac{\phi(\hat{\boldsymbol{\theta}})}{\theta_{\rm Ein}^2},$$

Wave optics to geometric optics



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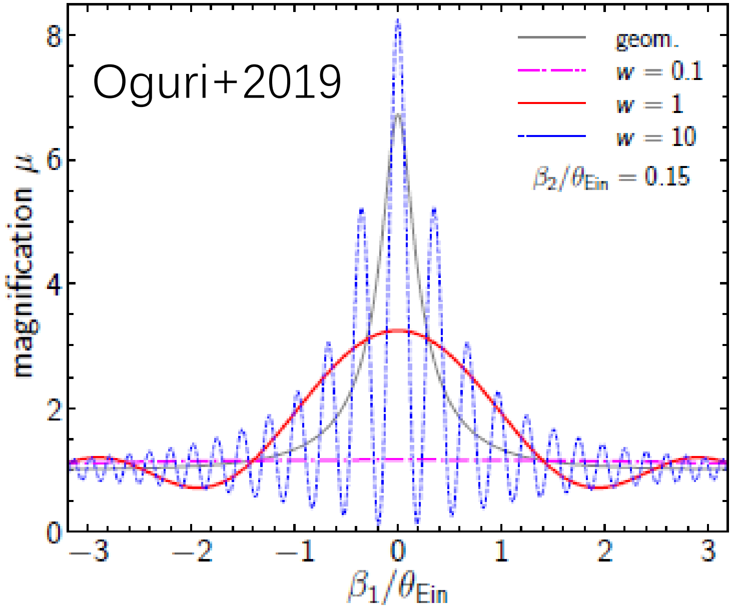
Wave optics to geometric optics



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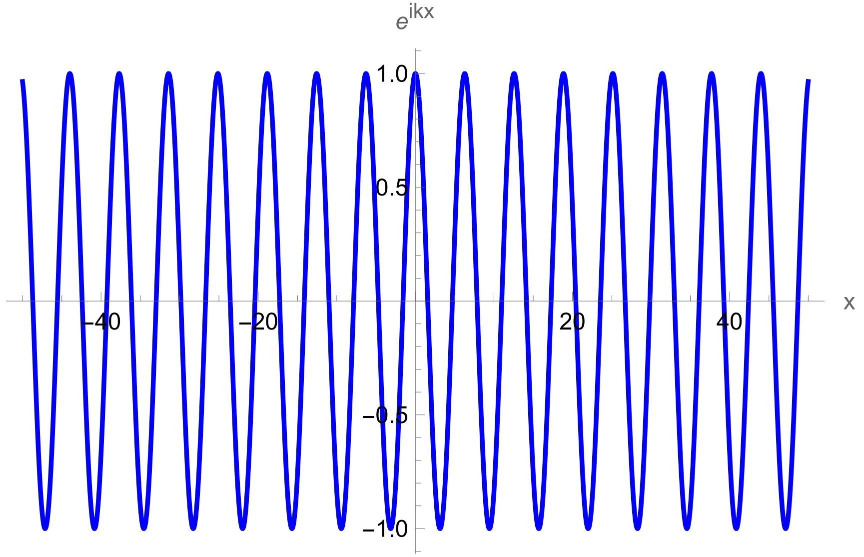
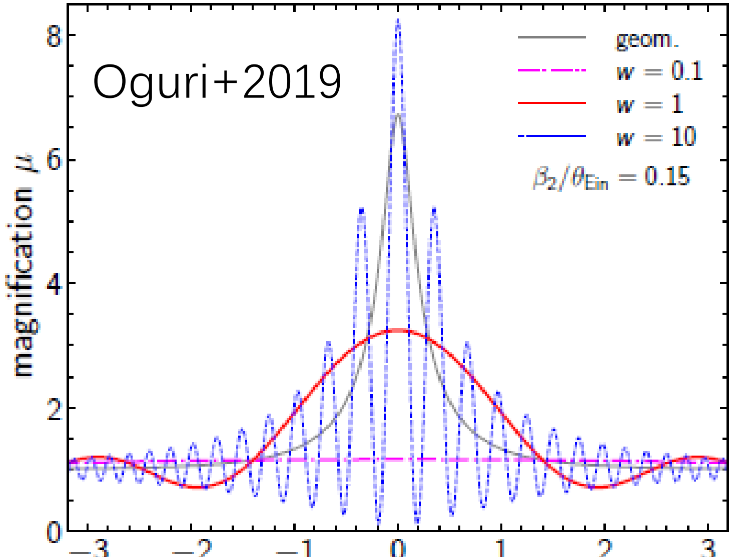


Wave optics to geometric optics



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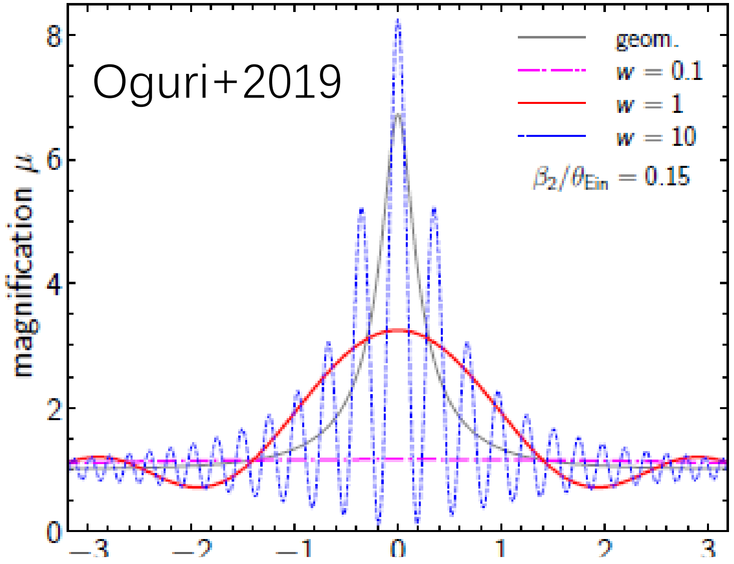
$$T(\hat{\boldsymbol{\theta}},\hat{\boldsymbol{\beta}}) = \frac{(\hat{\boldsymbol{\theta}}-\hat{\boldsymbol{\beta}})^2}{2} - \frac{\phi(\hat{\boldsymbol{\theta}})}{\theta_{\rm Ein}^2},$$

$$\int_{\mathbb{R}^n} g(x)e^{ikf(x)}dx = \sum_{x_0\in\Sigma} e^{ikf(x_0)}|\det(\mathrm{Hess}(f(x_0)))|^{-1/2}e^{\frac{i\pi}{4}\mathrm{sgn}(\mathrm{Hess}(f(x_0)))}(2\pi/k)^{n/2}g(x_0) + o(k^{-n/2})$$

Stationary phase approximation

$$k \to \infty$$

Wave optics to geometric optics



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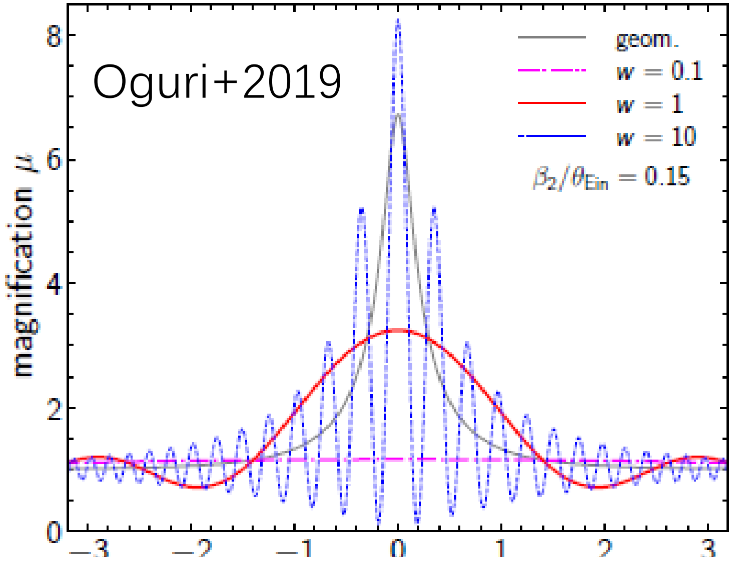
Wave optics to geometric optics



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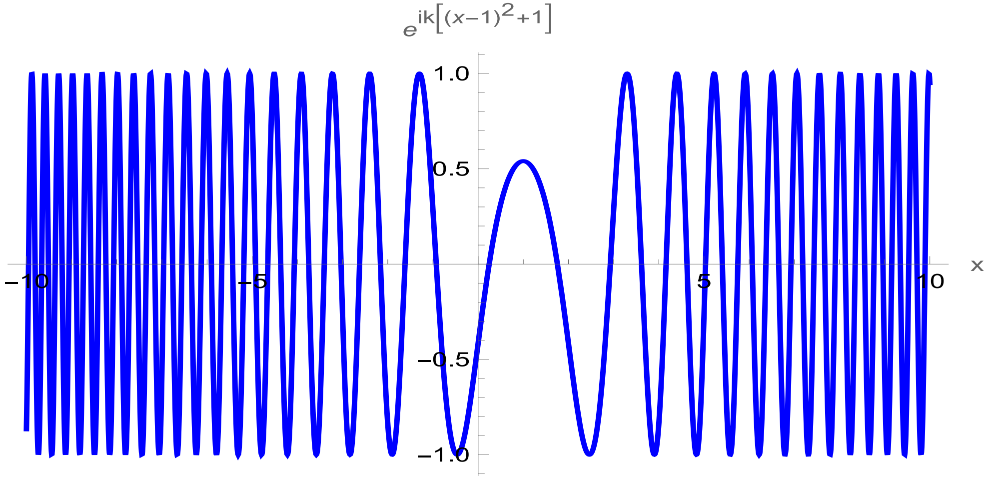
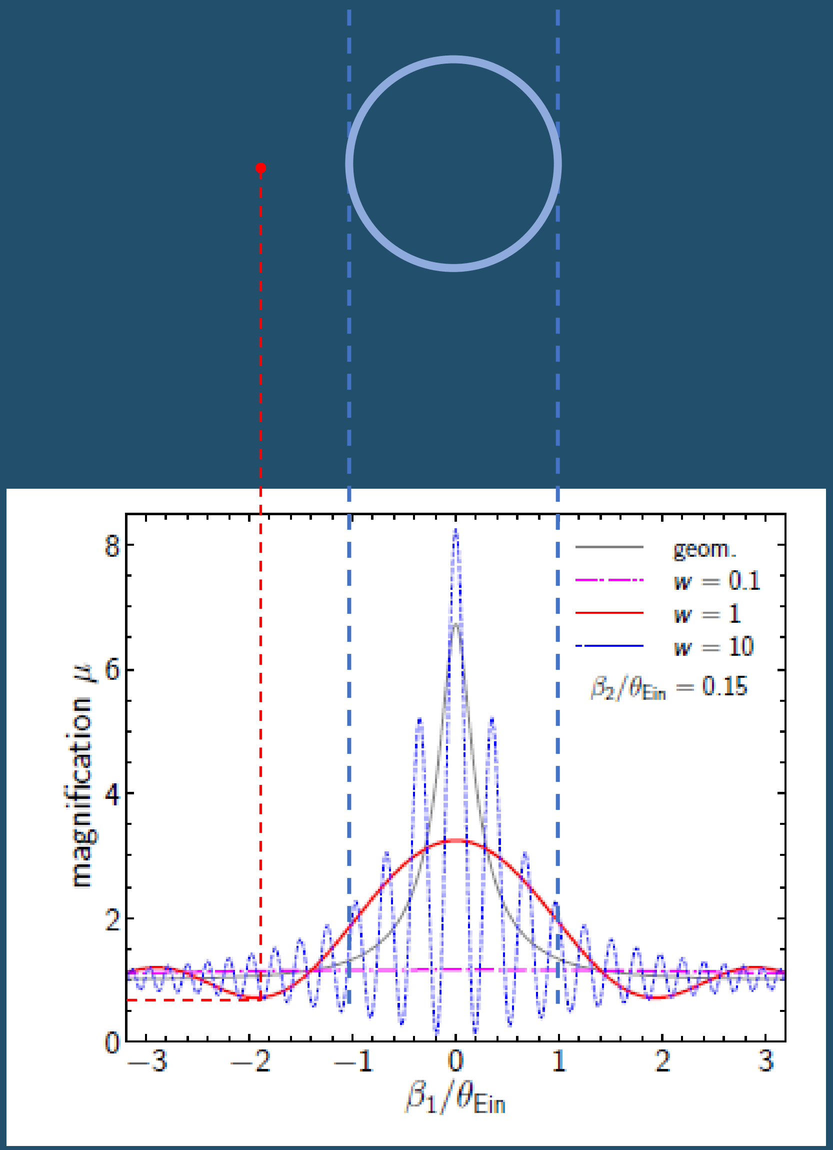
Stationary phase approximation

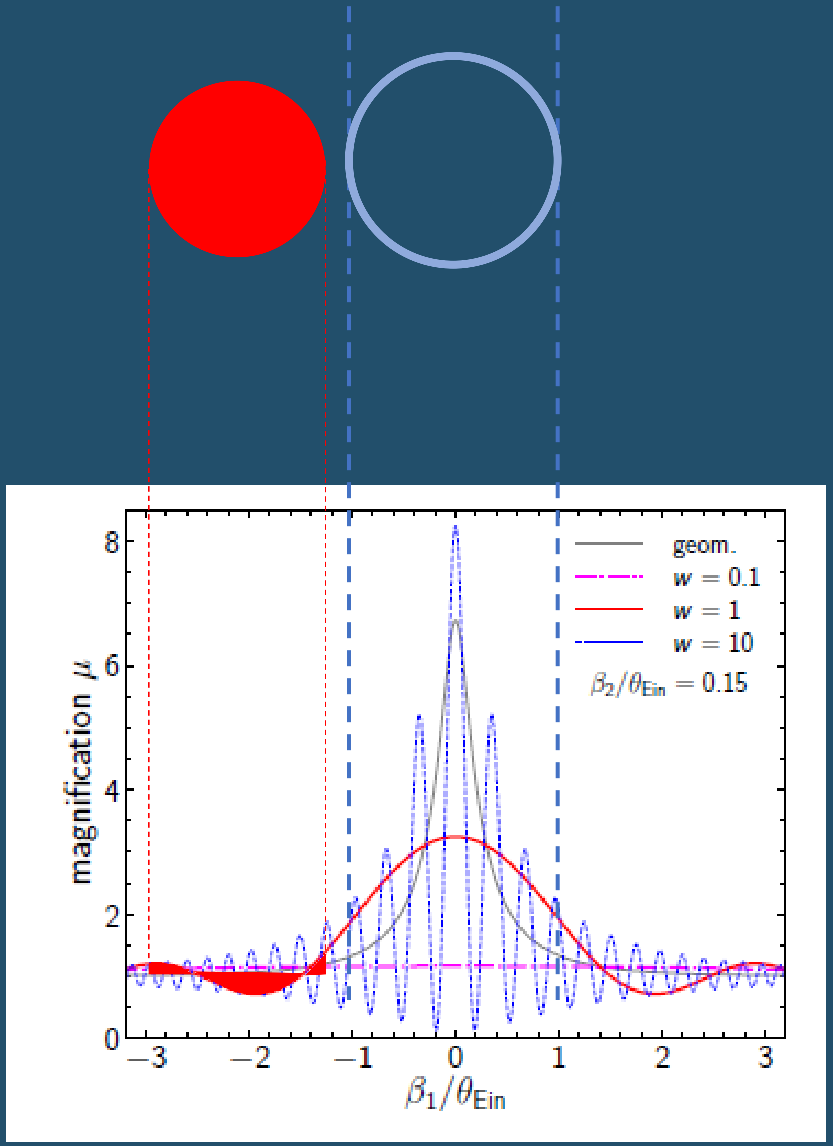
$$k \to \infty$$



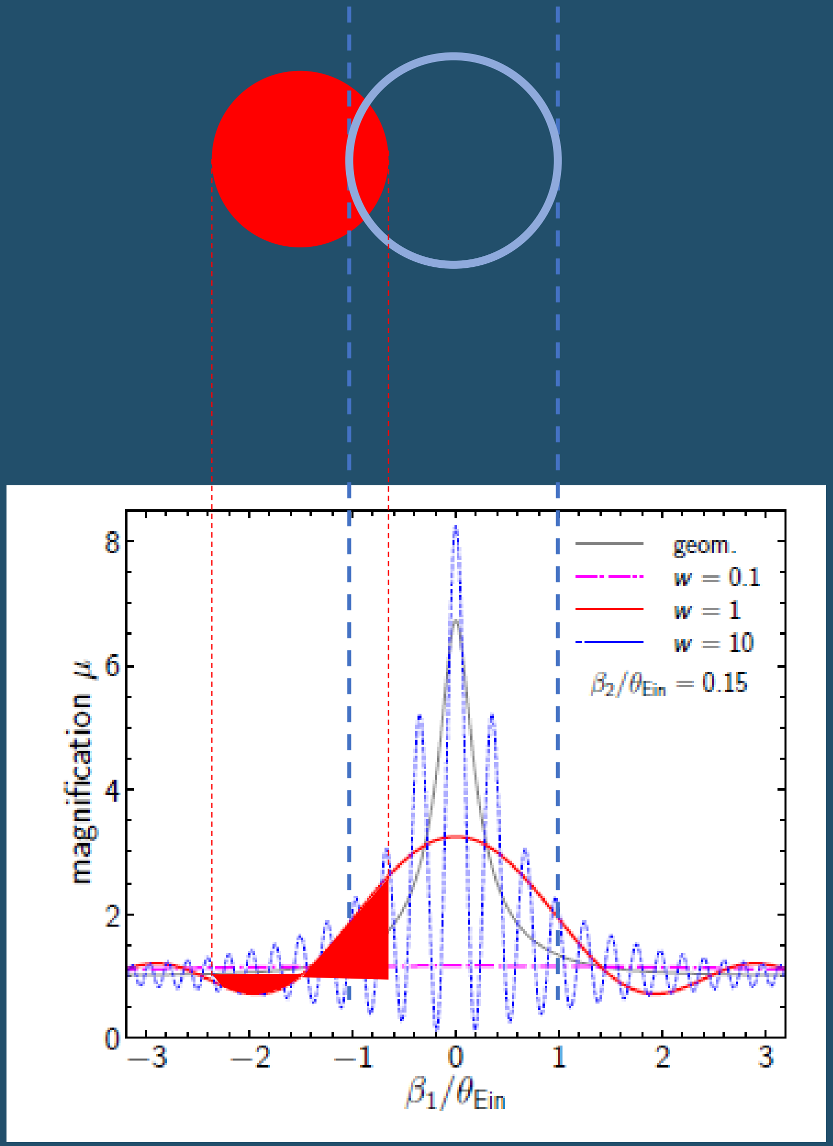
Wave optics


geom.
$w = 0.1$
$w = 1$
$w = 10$
$\beta_2/\theta_{\rm Ein} = 0.15$
magnification μ
$\beta_1/\theta_{\rm Ein}$

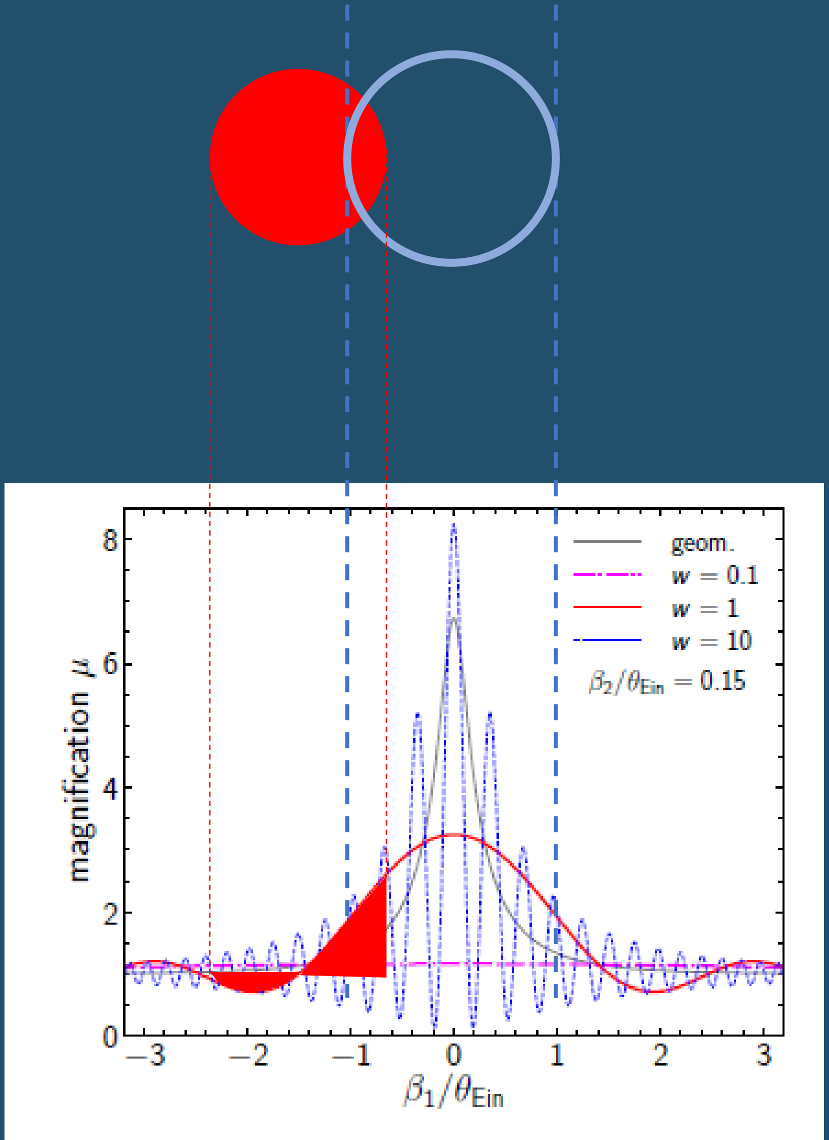
The less the better



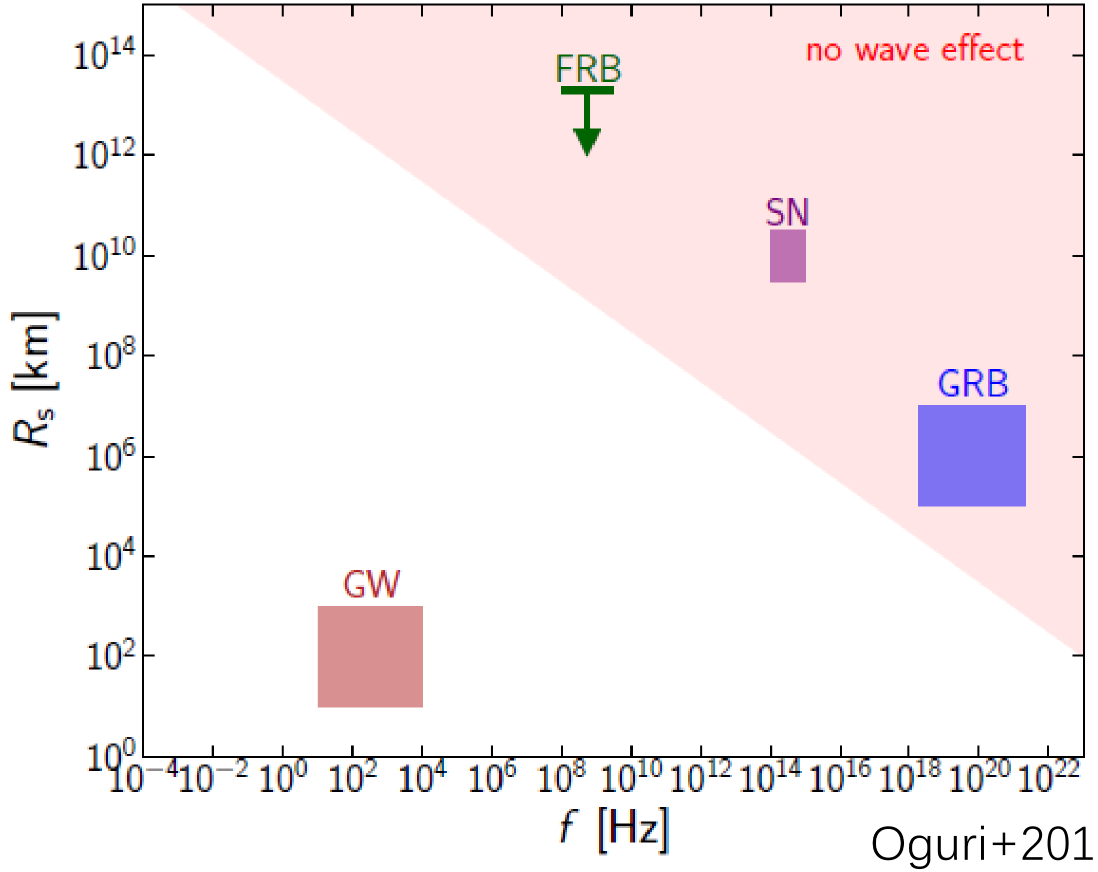
The less the better



The less the better



$$\beta_s \lesssim \theta_{\rm Ein}/w \qquad R_s \lesssim \frac{D_{os}\theta_{\rm Ein}}{w}.$$



Oguri+2019

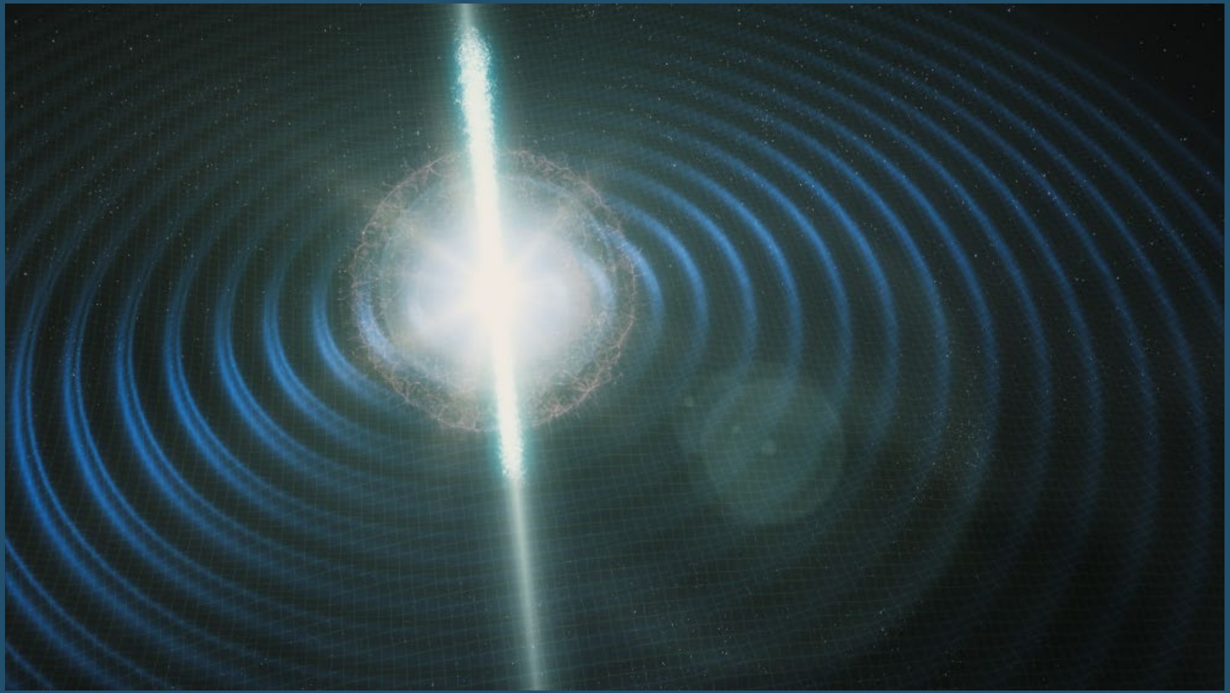
Lensed GW

- Cosmology Study
- Compact Dark Matter Detecting
- Fundamental physics Test

Cosmology Study

Lensed Standard Sirens:

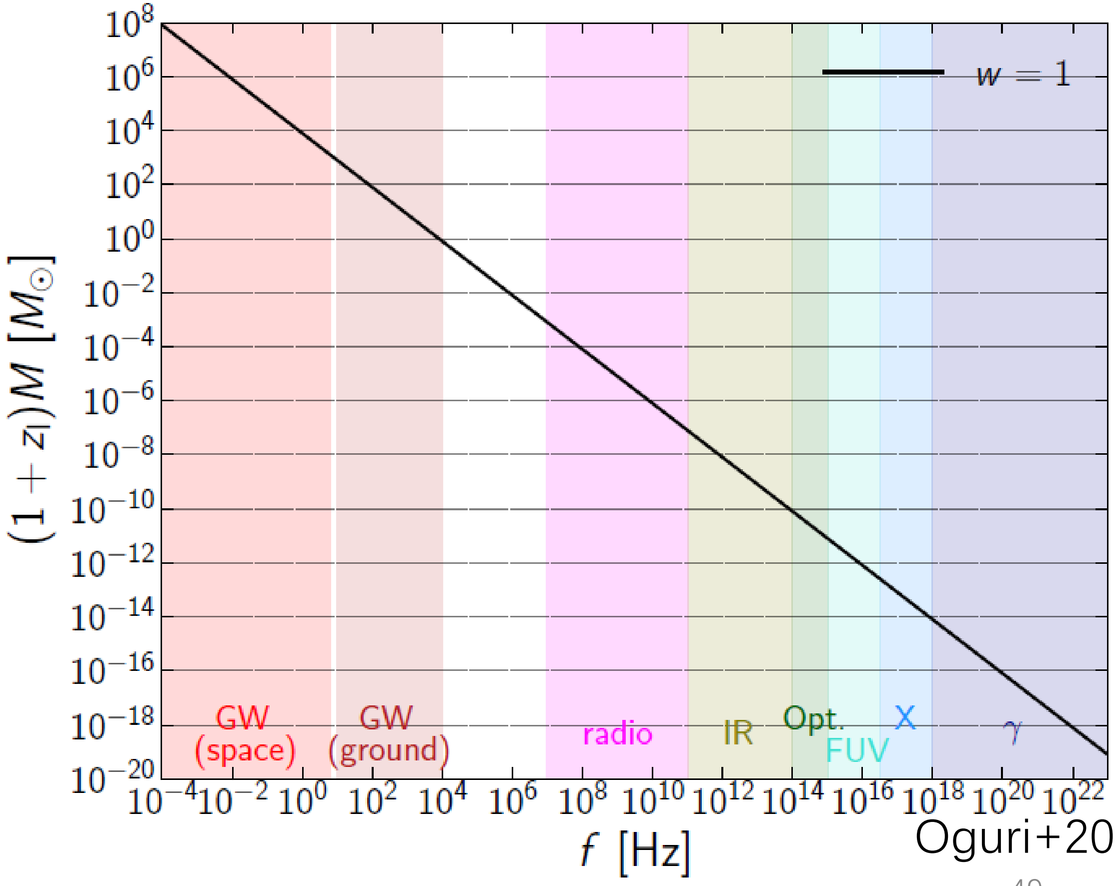
- Short time duration
- Known peak power
- EM counterparts



Dark Matter

Different bands have corresponding most sensitive mass of dark matters for wave optics behaviors.

$$w = 2\pi f \frac{4GM(1+z_l)}{c^3}$$
$$\approx 1.24 \times 10^{-4}(1+z_l)\left(\frac{M}{M_\odot}\right)\left(\frac{f}{\mathrm{Hz}}\right).$$



Oguri+2019

Fundamental Physics Test

Do gravitational waves propagate at the speed of light?
What if the graviton has mass?

Composition: Elementary particle
Statistics: Bose-Einstein statistics
Family: spin-2 boson
Interaction: Gravitation
Mean lifetime: Stable
Electric charge: 0

Fundamental Physics Test

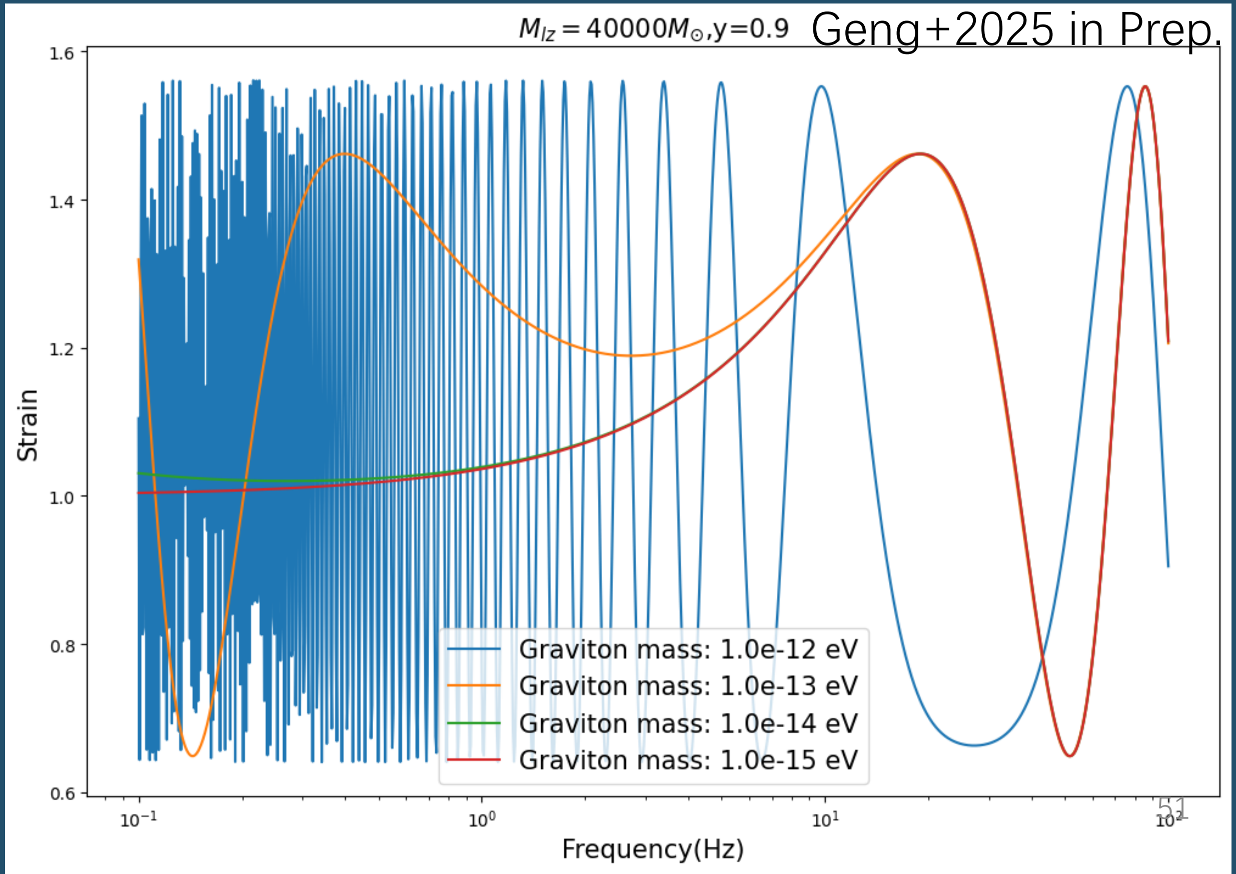
Do gravitational waves propagate at the speed of light?
What if the graviton has mass?

$$E^2 = p^2c^2 + m^2c^4$$

$$\frac{v^2}{c^2} = 1 - \frac{m^2c^4}{E^2}$$

$$\frac{v}{c} \simeq 1 - \frac{1}{2}\frac{m^2c^4}{E^2} = 1 - \frac{1}{2}\frac{m^2c^4}{h^2f^2}$$

$$\beta = \frac{c}{v} = \frac{1}{1 - \frac{1}{8\pi^2}\frac{m_g^2}{f^2}} \simeq 1 + \frac{1}{8\pi^2}\frac{m_g^2}{f^2}$$



Fundamental Physics Test

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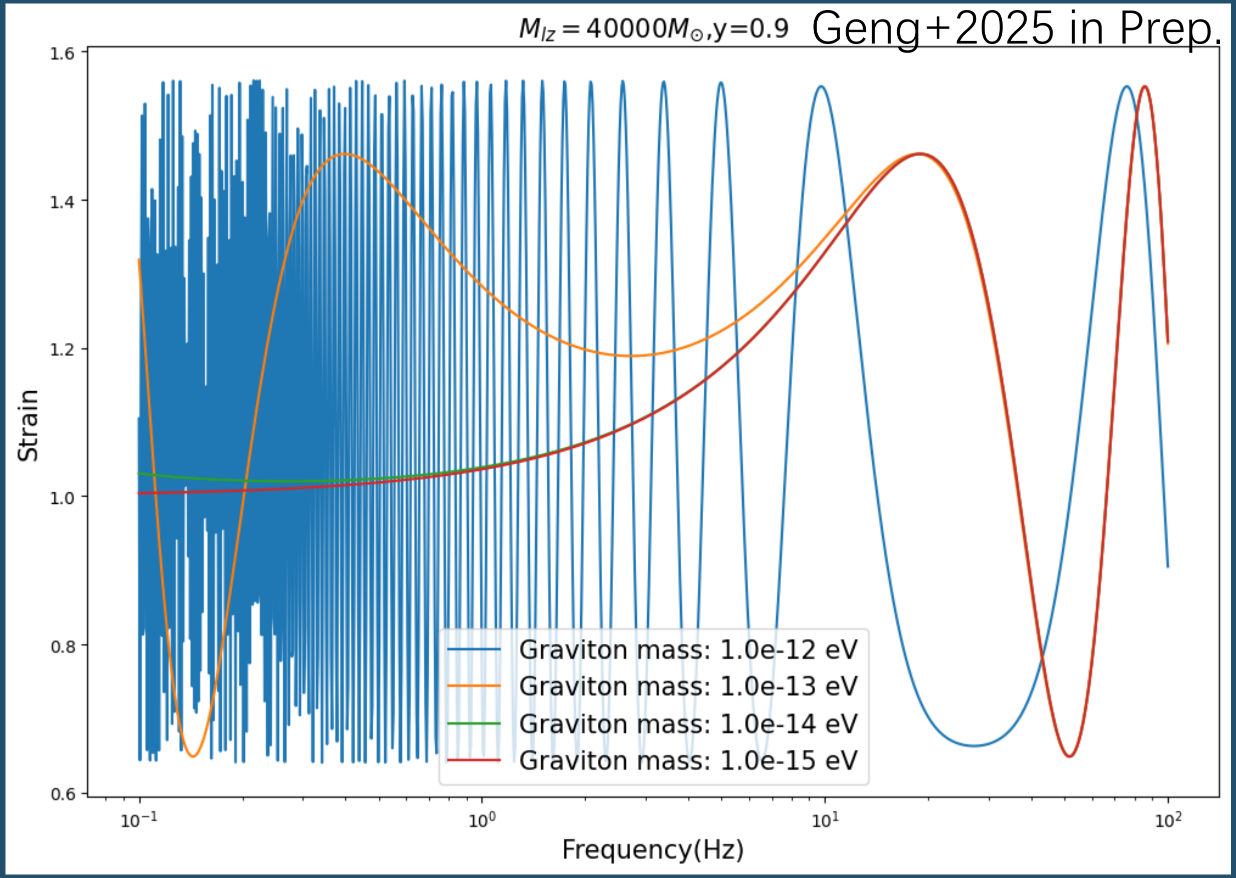
Lensed massive GW

$$F(f) = \exp\left[\frac{\pi w\beta}{4} + i\frac{w\beta}{2}\left(\ln\left(\frac{w\beta}{2}\right) - 2\phi_m(y)\right)\right] \times \Gamma\left(1 - \frac{i}{2}w\beta\right) {}_1F_1\left(\frac{i}{2}w\beta, 1; \frac{i}{2}w\beta y^2\right)$$

Amplification factor under point-mass lens model



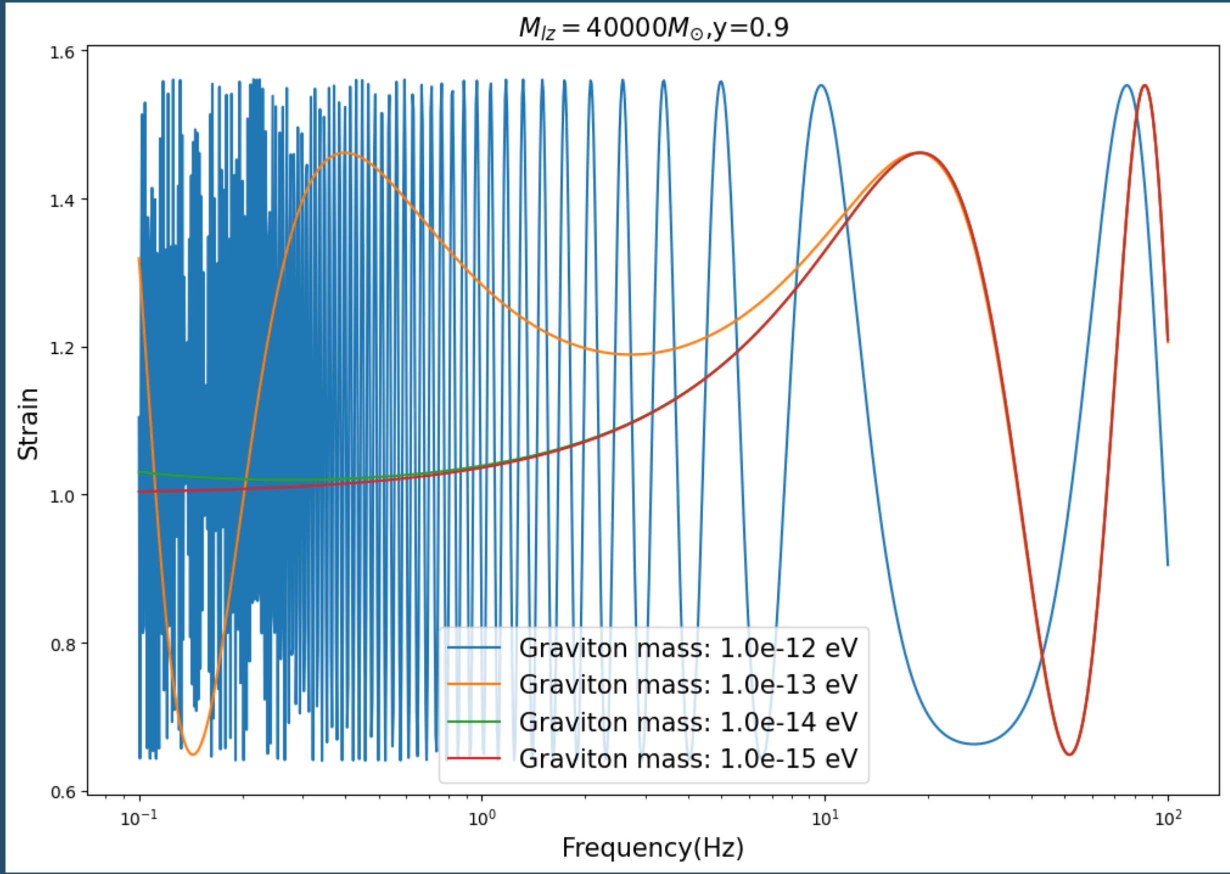
Oscillating

Symmetric

Lensed massive GW

$$F(f) = \exp\left[\frac{\pi w\beta}{4} + i\frac{w\beta}{2}\left(\ln\left(\frac{w\beta}{2}\right) - 2\phi_m(y)\right)\right] \times \Gamma\left(1 - \frac{i}{2}w\beta\right) {}_1F_1\left(\frac{i}{2}w\beta, 1; \frac{i}{2}w\beta y^2\right)$$

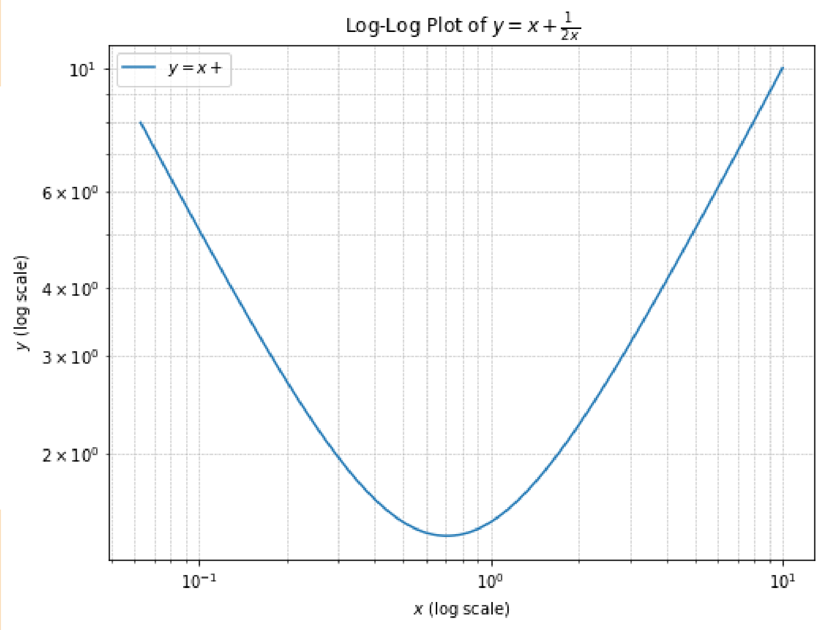
Amplification factor under point-mass lens model



$$w \propto f$$

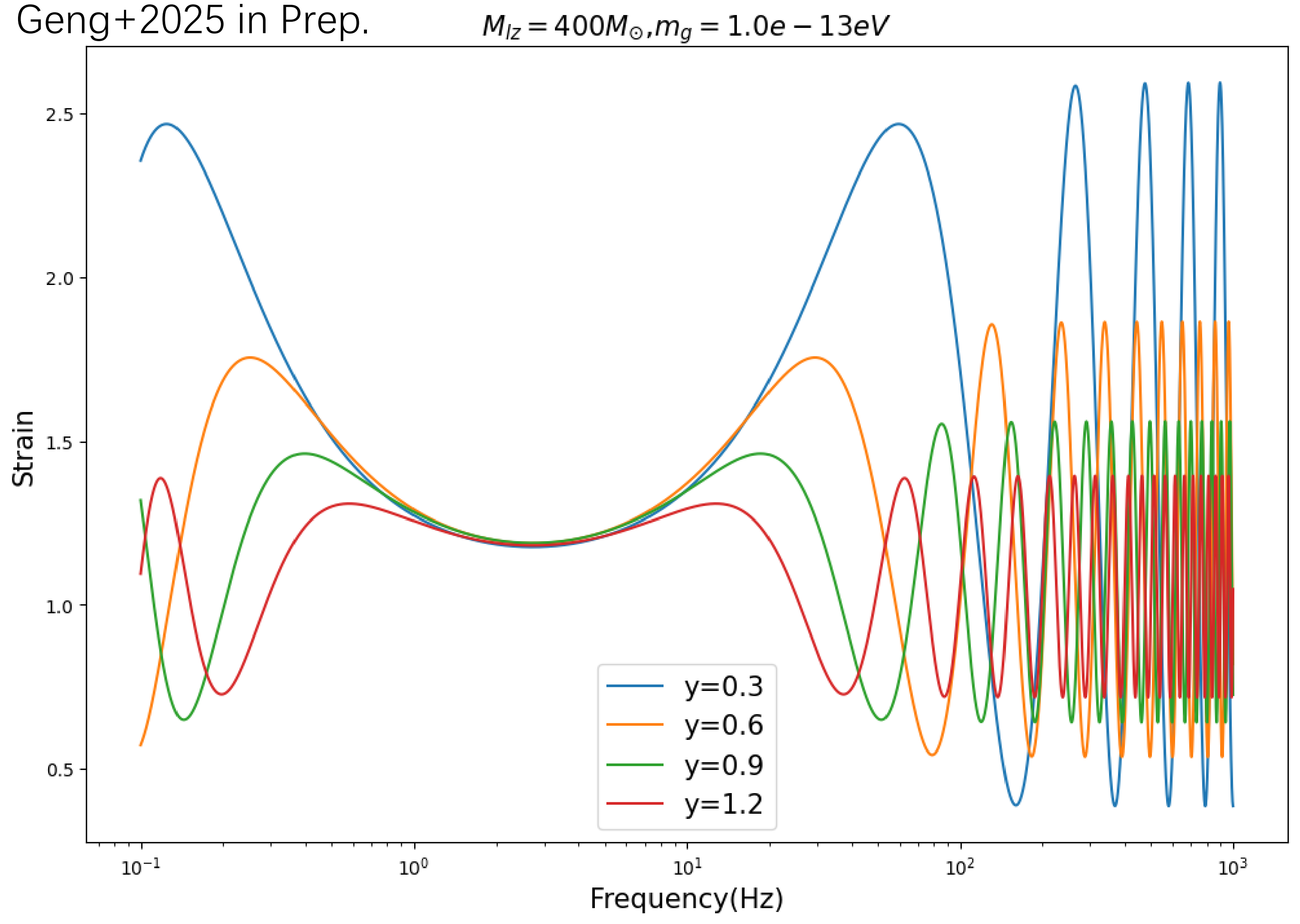
$$\beta \sim 1 + \frac{m_g^2}{2f^2}$$

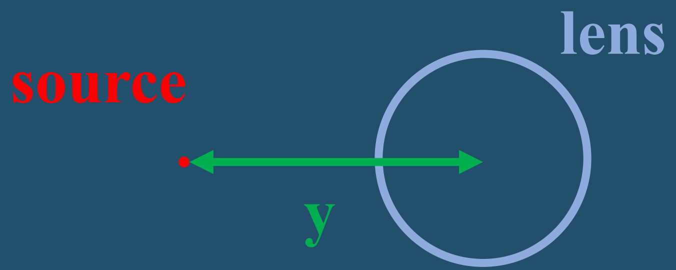
$$w\beta \sim f + \frac{m_g^2}{2f}$$



$$f_0 \propto \frac{m_g}{2}$$

Lensed massive GW



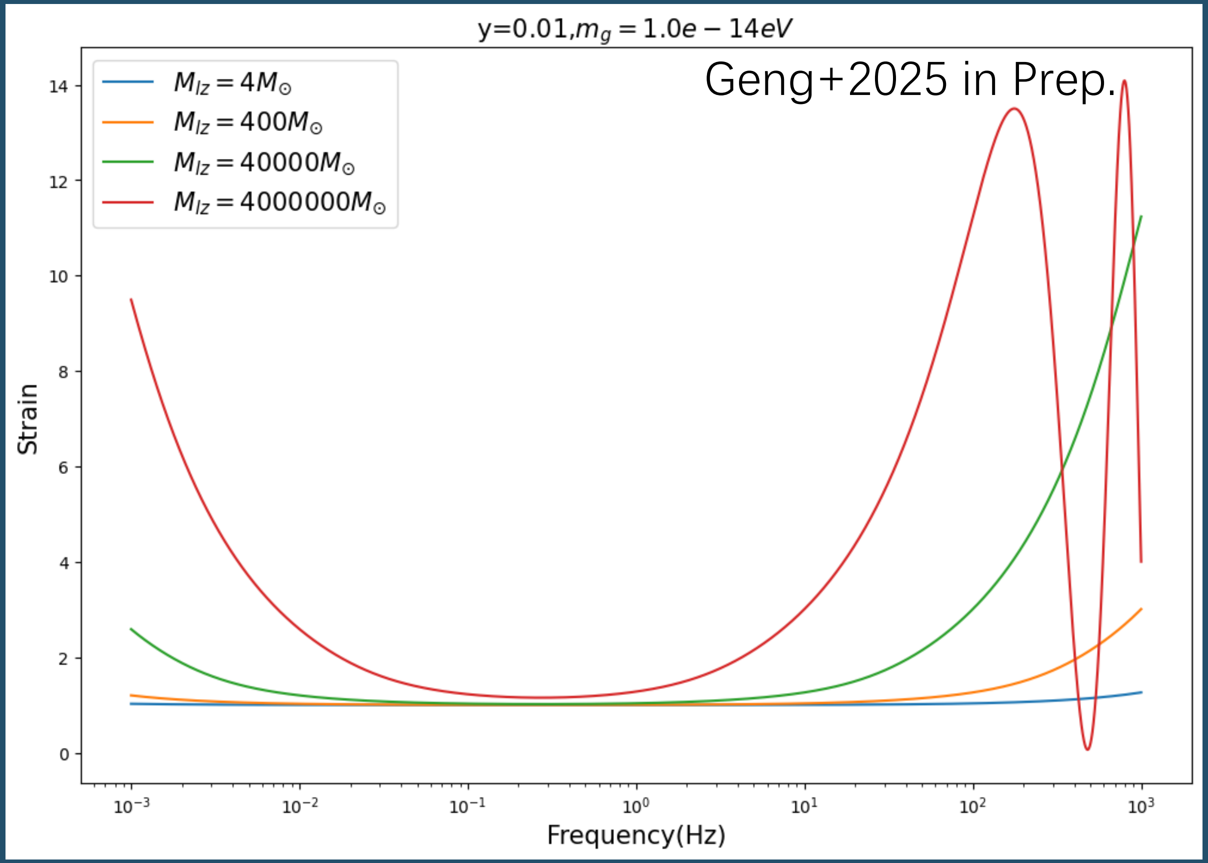


Smaller impact factor

- Higher amplification
- Wider full width at half maxima (FWHM)

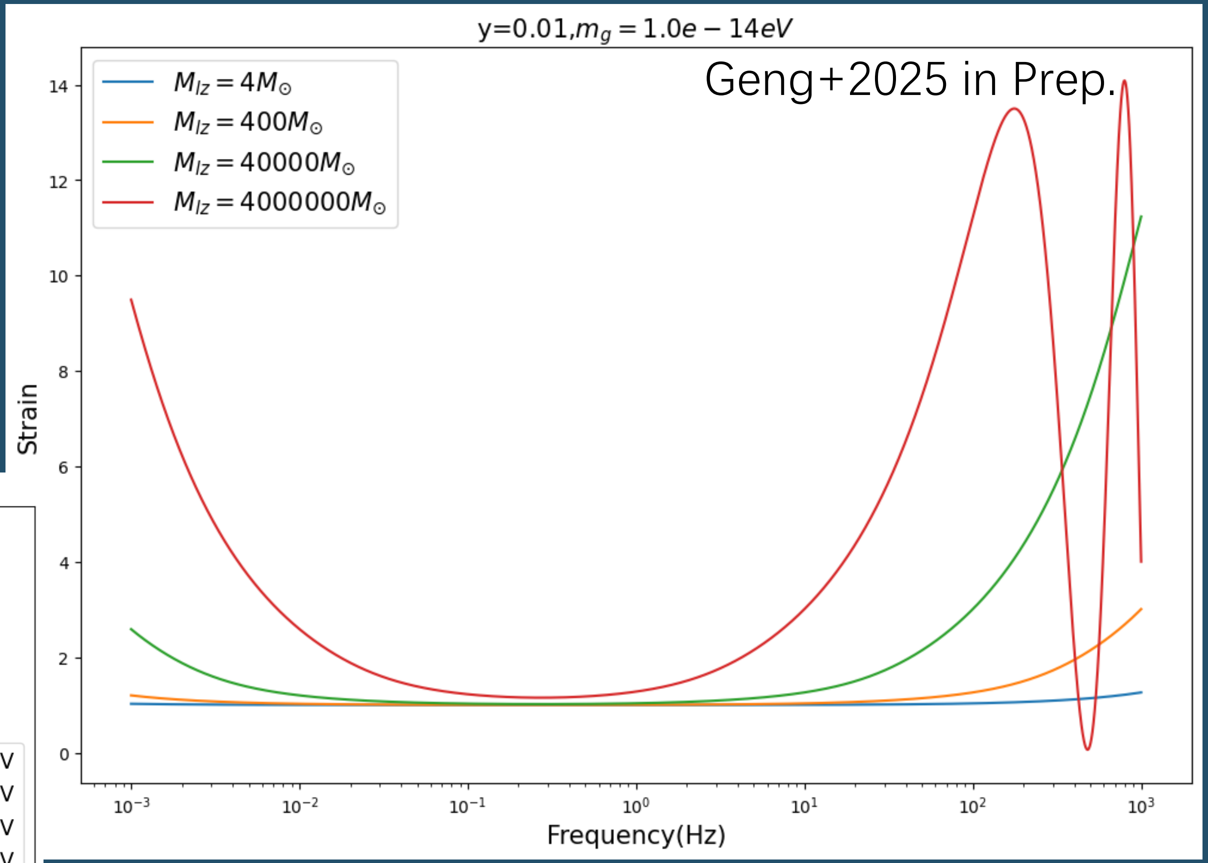
Lensed massive GW

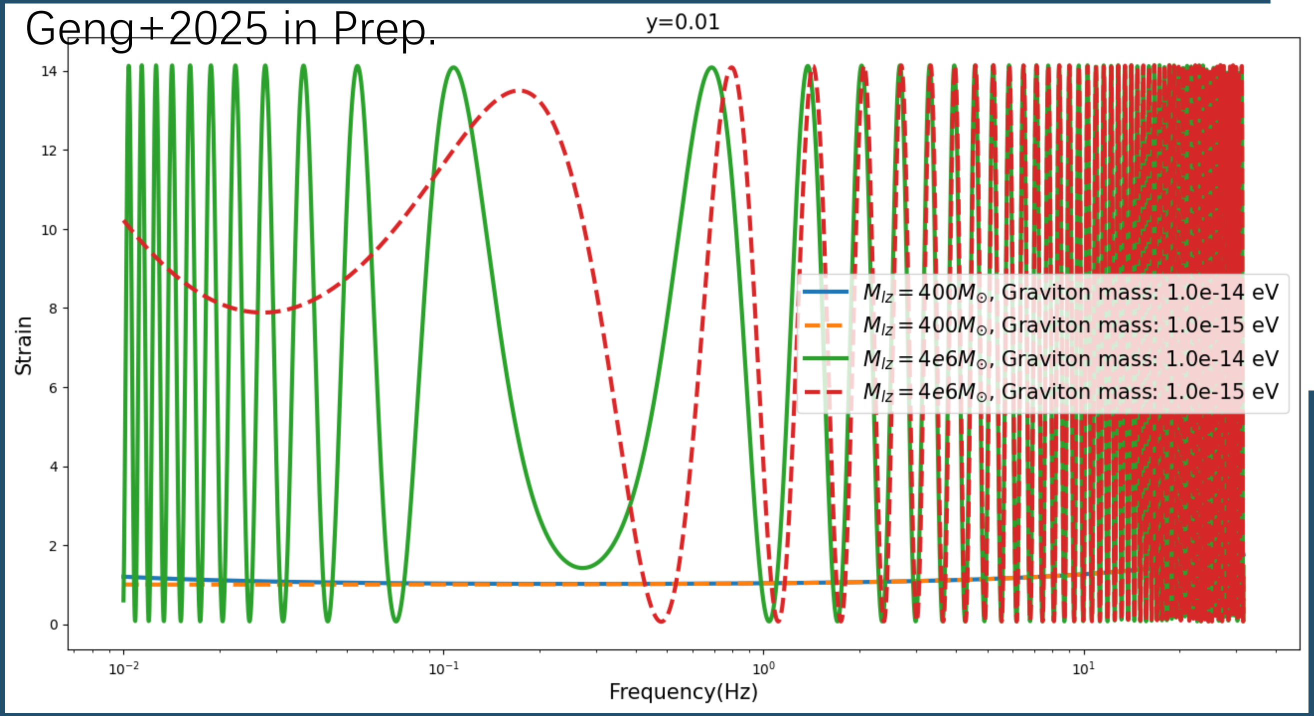
More massive lens
More significant oscillating pattern



Lensed massive GW

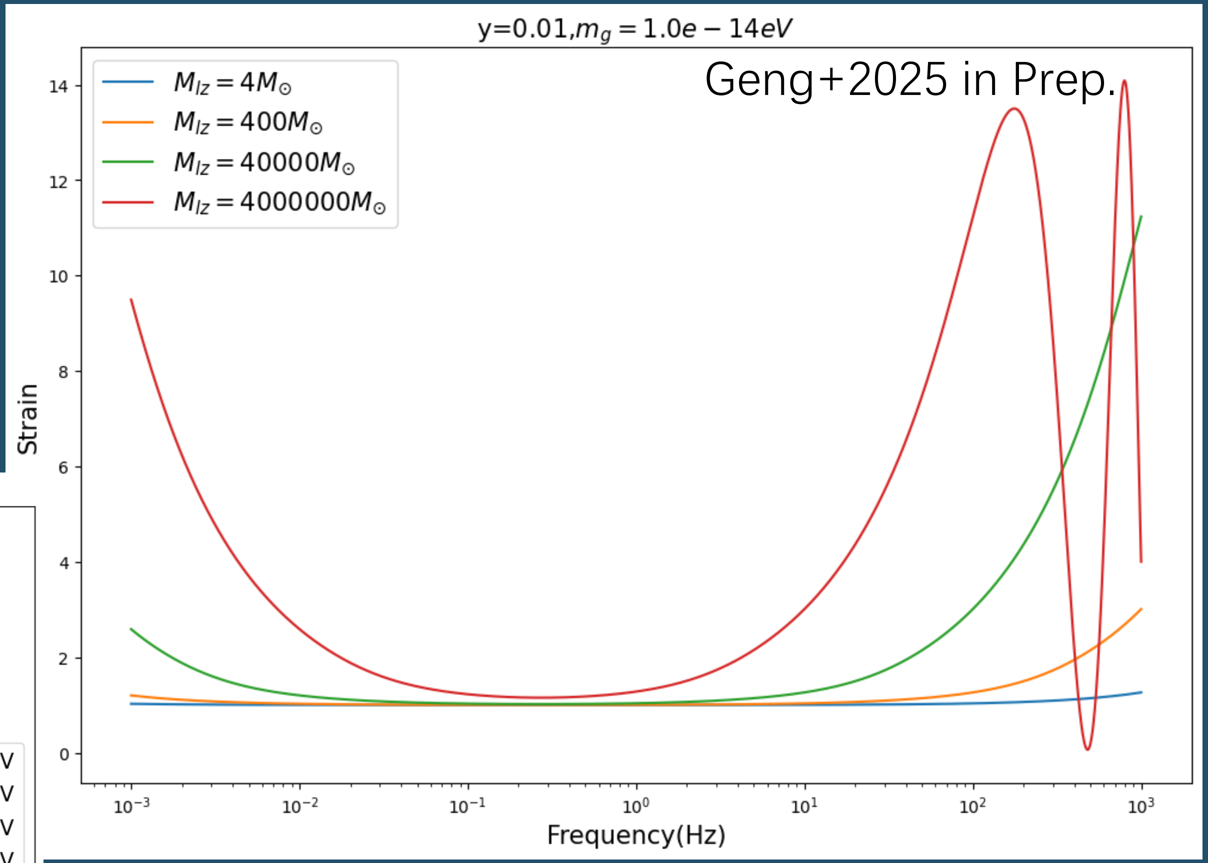
More massive lens
More significant oscillating pattern

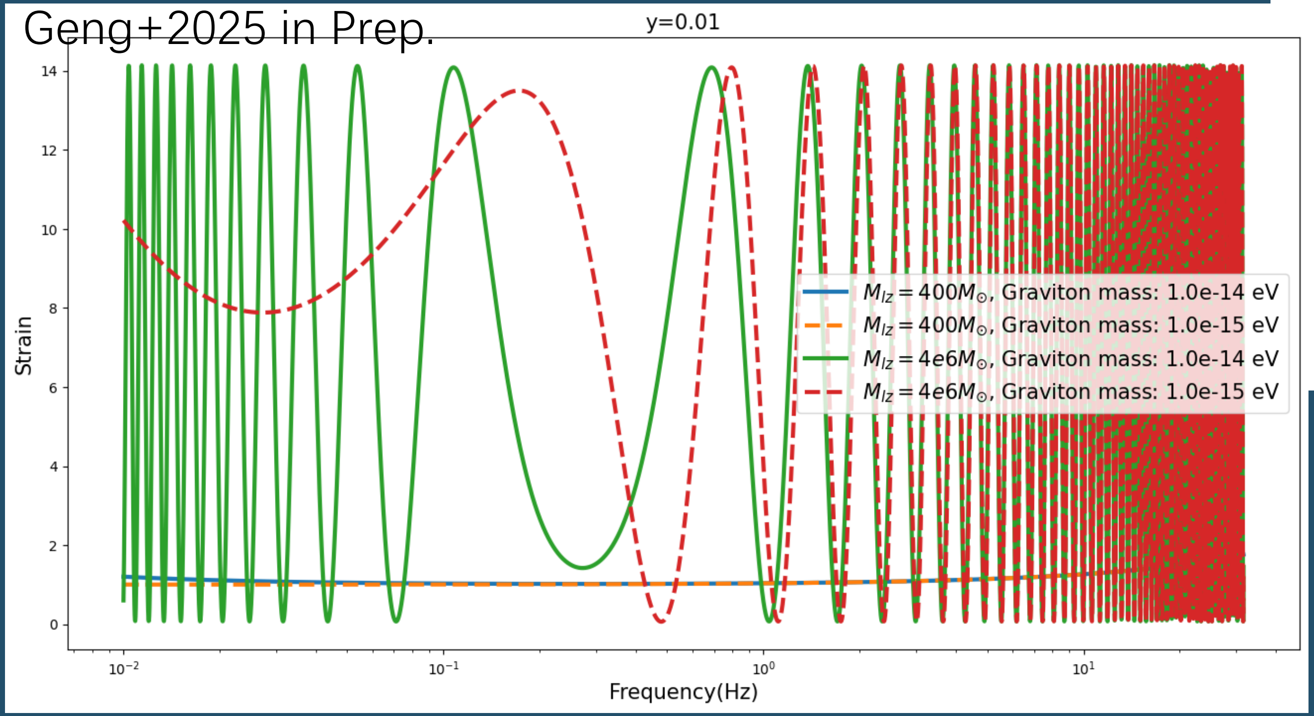




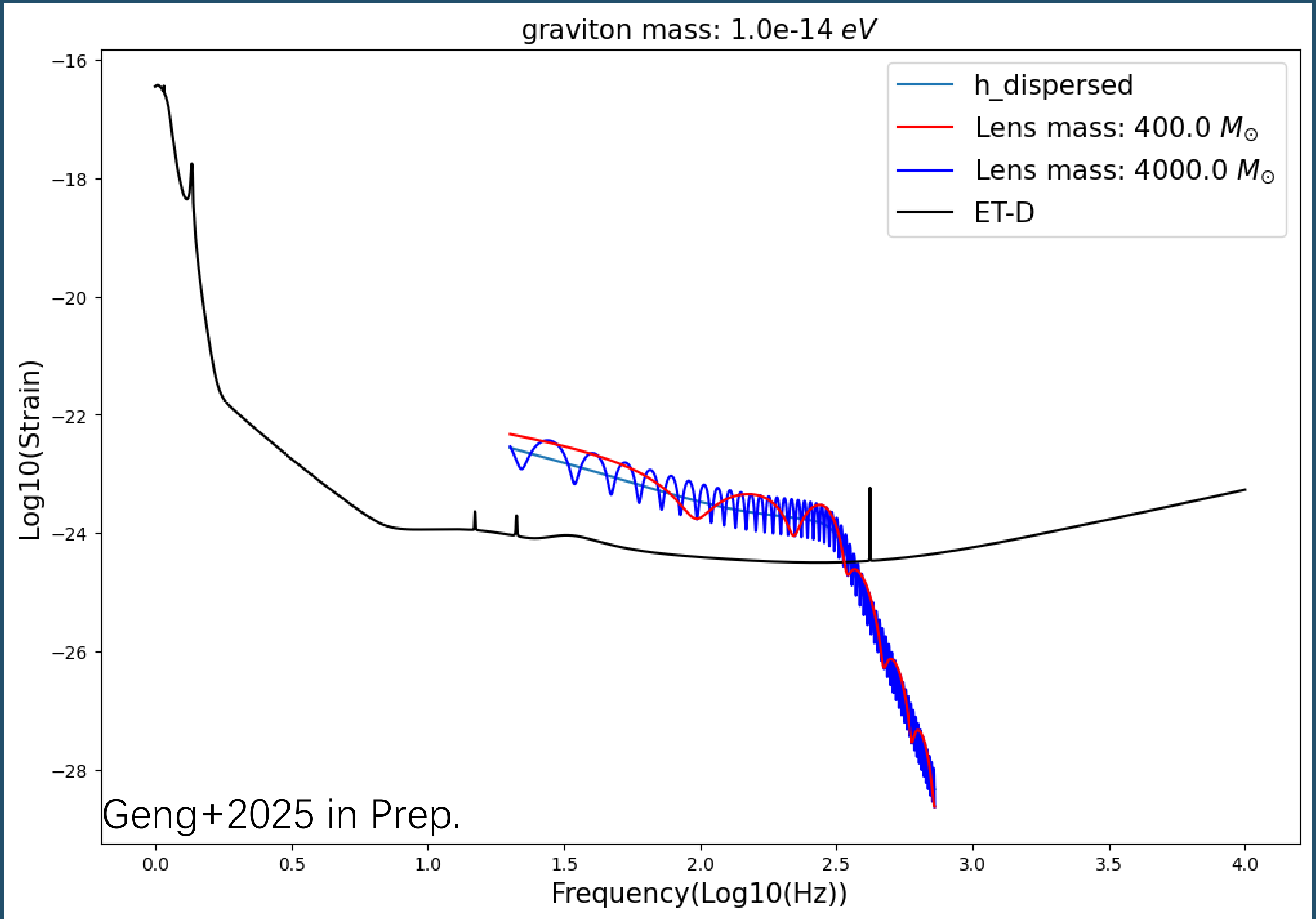
Lensed massive GW

More massive lens
More significant oscillating pattern





Lensed massive GW



GW150914-like source

$$M_1 = 36M_\odot$$
$$M_2 = 20M_\odot$$
$$D_L = 500Mpc$$
$$y = 0.9$$

ET

Unlensed SNR: **315.3**

Single detector lensed SNR: **380.1**

Networks lensed SNR: **608.7**

ET+LIGO networks SNR: 622.5

Conclusions & perspectives

- Lensed transients will bring new opportunities to explore our Universe
- Wave effects of lensing GW will help us to explore the possible mass of the graviton
- With next generation of GW detector (e.g. ET), we can achieve higher precision on the mass of graviton through lensed GW signals
- Next step: apply Fisher matrix on lensing and graviton parameters