# Spinning Gluon and Long Range Correlations at RHIC, LHC and EIC

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Guo, Liu, Yuan, Zhu, arXiv: 2406.05880
Guo, Liu, Yuan, arXiv: 2408.14693

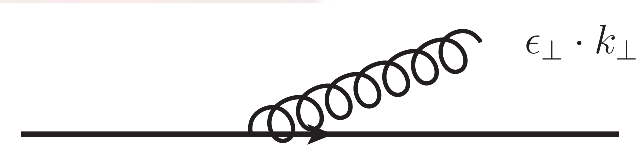
# EIC Science: from quark/gluon to cosmo

- How do the nucleonic properties such as mass and spin emerge from partons and their underlying interactions?
- How are partons inside the nucleon distributed in both momentum and position space?
- What happens to the gluon density in nucleons and nuclei at small *x*? Does it saturate at high energy, giving rise to gluonic matter with universal properties in all nuclei (and perhaps even in nucleons)?
- How do color-charged quarks and gluons, and jets, interact with a nuclear medium? How do confined hadronic states emerge from these quarks and gluons? How do the quark-gluon interactions generate nuclear binding?
- Do signals from beyond-the-standard-model physics manifest in electron-proton/ion collisions? If so, what can we learn about the nature of these new particles and forces?

EIC Whitepaper for LRP
QCD Whitepaper, 2303.02579, NPA 2024

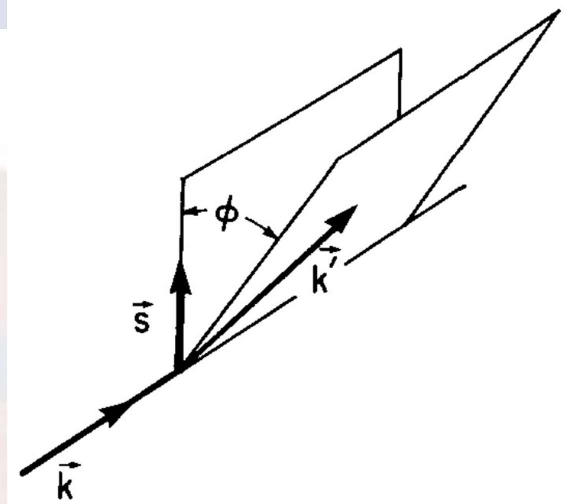
# Spinning gluon: nontrivial part of nucleon tomography

- Gluon is always spinning in high energy/small-x
  - Spinning gluon in inclusive DIS
  - Spinning gluon (helicity-flip) in GPD
  - Spinning gluon (linearly polarized) in TMD
- Spinning gluon in Nucleon EEC
  - Two particle correlations in DIS
  - Long range correlation in pp collisions at RHIC and LHC



# In the context of inclusive DIS: nuclear gluonometry for J>1

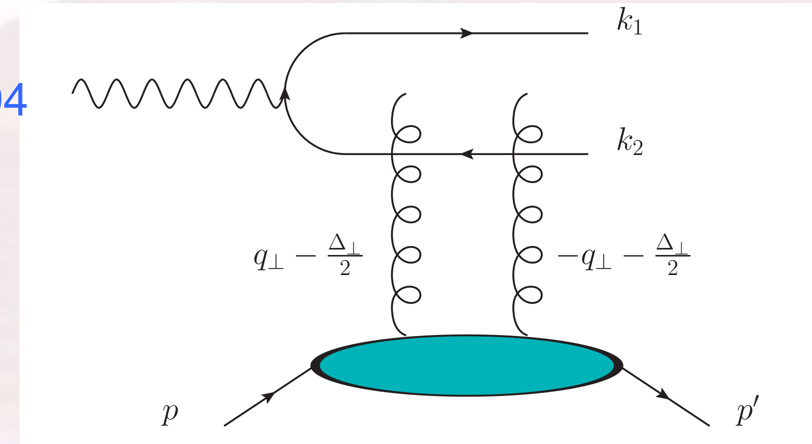
Jaffe, Manohar, 1989



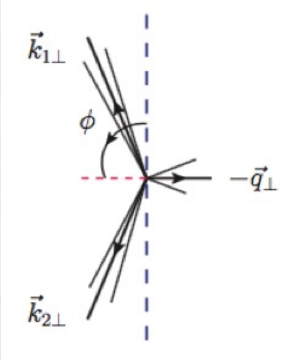

- Structure function difference between polarization along x and y directions, i.e., $\cos(2\phi_S)$ asymmetry
- For nucleons, the asymmetry vanishes in QCD
- Nontrivial asymmetry for nucleus with J>1, e.g., deuteron, only receives contributions from the spinning gluons

# Semi-exclusive process: Diffractive Dijet to Probe the Gluon Tomoraphy

Earlier studies:
Nikolaev, Zakharov 1994
Bartles, Ewerz, Lotter, Wusthoff 1996
Diehl 1996
Braun, Ivanov 2005
…




Hatta-Xiao-Yuan,1601.01585




cos(2ϕ) anisotropy

- The diffractive dijet cross section is proportional to the square of the Wigner distribution → nucleon/nucleus tomography

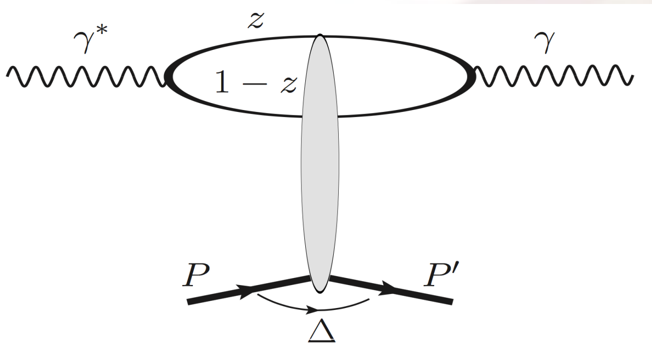
$$x\mathcal{W}_g^T(x,|\vec{q}_\perp|,|\vec{b}_\perp|) + 2\cos(2\phi)x\mathcal{W}_g^\epsilon(x,|\vec{q}_\perp|,|\vec{b}_\perp|)$$

More correlations to study OAM, Spin-Orbital Correlations, …

Boussarie-Hatta-Bhattacharya, 2022,2024

# Spinning gluon in exclusive processes: GPD framework

Hatta-Xiao-Yuan, 1703.02085




Gluon Tomography and Wigner distribution:

$$x\mathcal{W}_g^T(x,|\vec{q}_\perp|,|\vec{b}_\perp|)+2\cos(2\phi)x\mathcal{W}_g^\epsilon(x,|\vec{q}_\perp|,|\vec{b}_\perp|)$$

$$\frac{1}{P^+}\int\frac{d\zeta^-}{2\pi}e^{ixP^+\zeta^-}\langle p'|F^{+i}(-\zeta/2)F^{+j}(\zeta/2)|p\rangle$$
$$=\frac{\delta^{ij}}{2}xH_g(x,\Delta_\perp)+\frac{xE_{Tg}(x,\Delta_\perp)}{2M^2}\left(\Delta_\perp^i\Delta_\perp^j-\frac{\delta^{ij}\Delta_\perp^2}{2}\right)+$$

Hoodbhoy-Ji 98
Diehl 01

- A nontrivial tomography distribution of gluon inside the nucleon
- It contributes to a $\cos(2\phi)$ in photon production (DVCS)

# Spinning gluon in semi-inclusive process: TMD framework

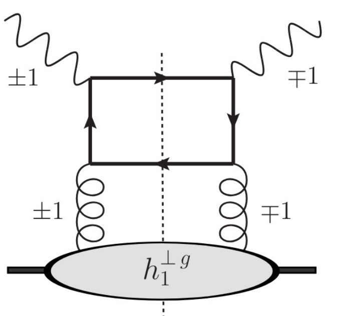
- Linearly polarized gluon distribution

Mulders, Rodrigues, 20021

$$\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle P,S|F^{+i}(0)\,F^{+j}(\xi)|P,S\rangle \qquad \left[-g_T^{ij}G(x,\boldsymbol{k}_T^2)+\left(\frac{k_T^i k_T^j}{M^2}+g_T^{ij}\frac{\boldsymbol{k}_T^2}{2M^2}\right)H^\perp(x,\boldsymbol{k}_T^2)\right]$$

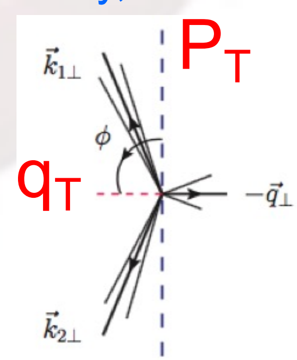
- Can be measured through TMD processes, such as heavy quark pair production in DIS

Boer, Brodsky, Mulders, Pisano, 2011

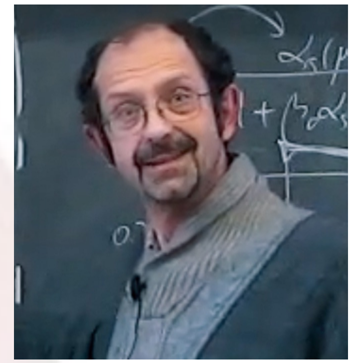



$$\left[A+\frac{\boldsymbol{q}_T^2}{M^2}\,B\,\cos 2(\phi_T-\phi_\perp)\right]$$

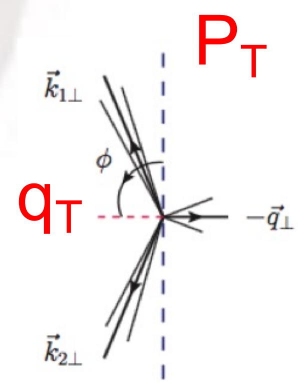
Cos(2$\phi$) between the total and difference of the two leading transverse momenta



# Soft gluon radiations can generate and mixes the azimuthal asymmetry



- Azimuthal angular asymmetries arise from soft gluon radiations
  - $\phi$ is defined as angle between total and different transverse momenta of the two final state particles
- Infrared safe but divergent
  - $<\cos(\phi)>$, $<\cos(2\phi)>$, … divergent, $\sim 1/q_T^2$
  - Examples discussed include Vj, top quark pair production



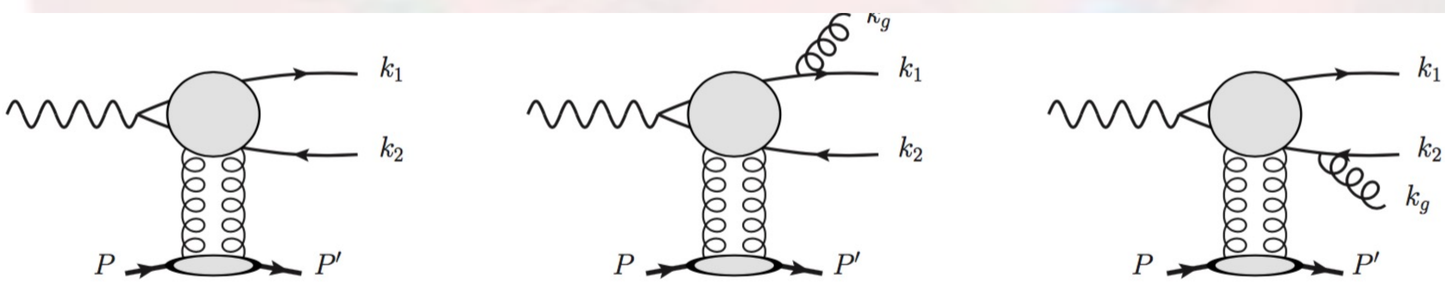

Catani-Grazzini-Sargsyan 2017

# Diffractive dijet production

- Gluon radiation tends to be aligned with the jet direction

$$S_J(q_\perp) = \delta(q_\perp) + \frac{\alpha_s}{2\pi^2}\int dy_g \left(\frac{k_1 \cdot k_2}{k_1 \cdot k_g k_2 \cdot k_g}\right)_{\vec{q}_\perp = -\vec{k}_{g\perp}}$$

$$S_{J0}(|q_\perp|) + 2\cos(2\phi) S_{J2}(|q_\perp|) + \cdots$$




Hatta-Xiao-Yuan-Zhou, 2010.10774, 2106.05307

# Leading power contributions, explicit result at $\alpha_s$

$$S_J(q_\perp) = S_{J0}(|q_\perp|) + 2\cos(2\phi) S_{J2}(|q_\perp|)$$

$$S_{J0}(q_\perp) = \delta(q_\perp) + \frac{\alpha_0}{\pi}\frac{1}{q_\perp^2}\,, \quad S_{J2}(q_\perp) = \frac{\alpha_2}{\pi}\frac{1}{q_\perp^2}\,,$$

where

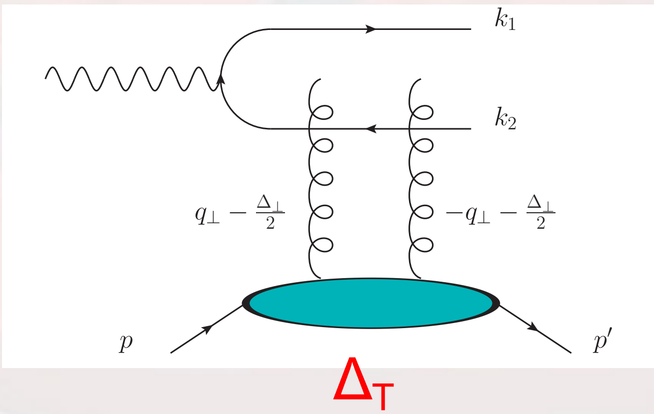
$$\alpha_0 = \frac{\alpha_s C_F}{2\pi} 2\ln\frac{a_0}{R^2}\,, \quad \alpha_2 = \frac{\alpha_s C_F}{2\pi} 2\ln\frac{a_2}{R^2}\,.$$

$a_0$,$a_2$ are order 1 constants, so,

in the small-R limit, <cos(2ϕ)> goes to 1

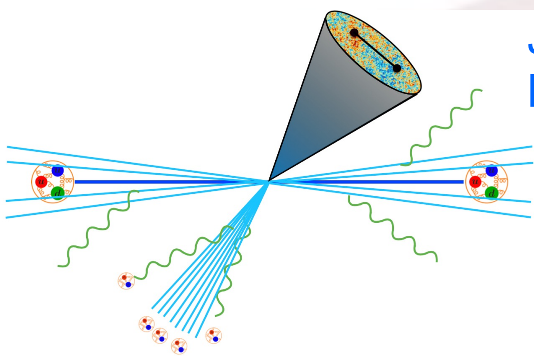
# Comments

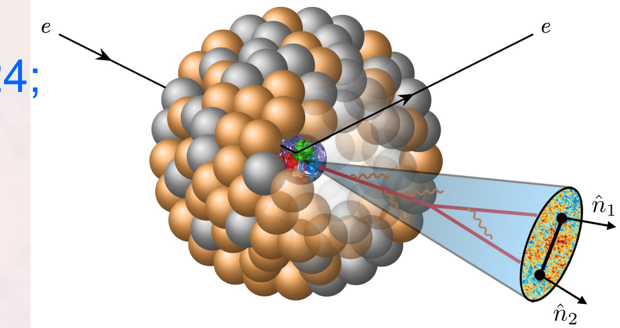
- To avoid the soft gluon radiation contribution, we need to reconstruct nucleon/nucleus recoil momentum to study the tomography

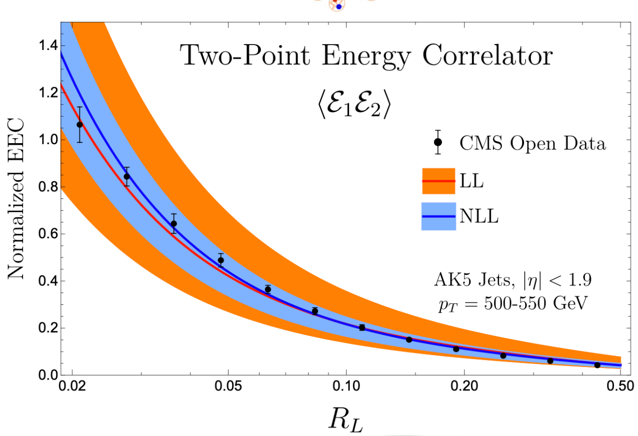


# Energy-Energy Correlators at Colliders

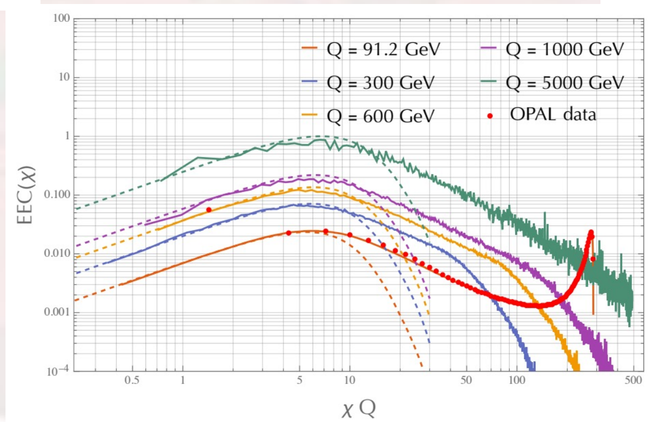
Major focus of recent studies:
Jet substructure
Moult, Zhu, Lee, et al, 20-24;



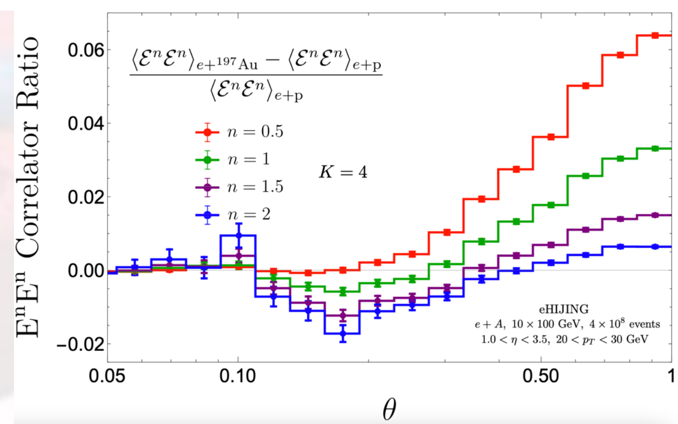



Lee, Mecaj, Moult
2205.03414




Liu, Vogelsang, Yuan, Zhu
2410.16371




Devereaux, et al., 2303.08143
Fu et al., 2411.04866

# Parton Tomography: Nucleon EEC and DIS

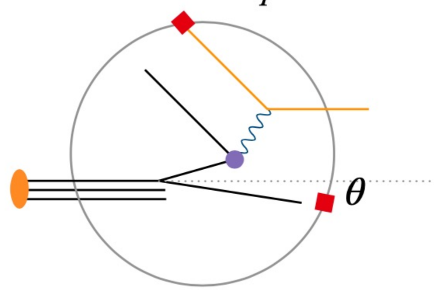
$$f_{q,EEC}(x,\theta) = \int_{-\infty}^{\infty} \frac{dy^-}{2\pi E_P} e^{ixp^+y^-} \frac{\gamma^+}{2} \langle p | \bar{\psi}(0) \mathcal{E}(\theta) \mathcal{L} \psi(y^-) | p \rangle$$

$$= \sum_X \sum_{i\in X} \frac{E_i}{E_P} \delta(\theta_i^2 - \theta^2) \delta((1-x)p^+ - p_X^+) \frac{\gamma^+}{2} \langle p | \bar{\psi}(0) | X \rangle \langle X | \mathcal{L} \psi(0) | p \rangle$$

Liu, Zhu, 2209.02080

Cao, Liu, Zhu, 2303.01530

$$x_B = \frac{Q^2}{2P\cdot q}$$




$$\Lambda_{\rm QCD} \ll Q\theta \ll Q$$

$$\Sigma(x_B, Q^2, \theta) = \int \frac{dz}{z} \hat{\sigma}\left(\frac{x_B}{z}, Q^2, \mu\right) f_{\rm EEC}(z, \theta, \mu)$$

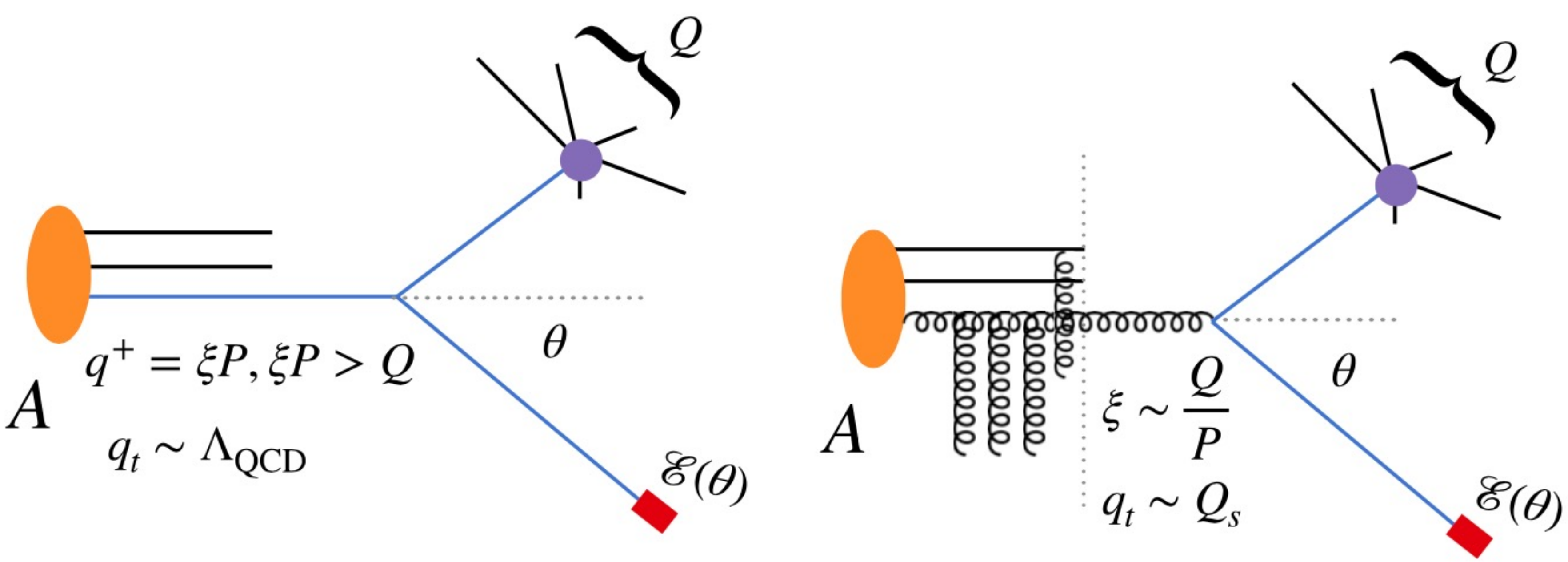
$$\propto \int \frac{dz}{z} \hat{\sigma}\left(\frac{x_B}{z}\right) \frac{1}{\theta^2} \int \frac{d\xi}{\xi} (1 - \frac{z}{\xi}) P(\frac{z}{\xi}) [\xi f(\xi)]$$

Perturbative scaling

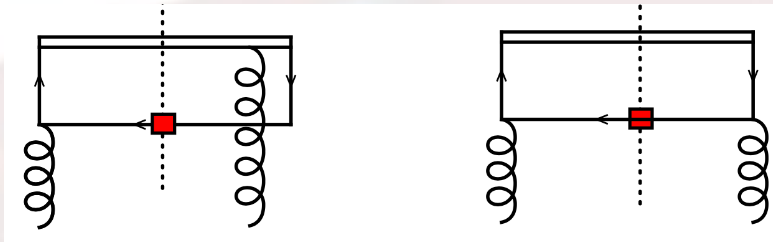
- $\theta$-distribution solely determined by $f_{\rm EEC}$
- In the collinear factorization:
  - $d\Sigma / d\ln\mu = P \otimes \Sigma$, solely determined by the vacuum splitting function
  - $\Sigma \sim \theta^{-2}$ at LO, $\Sigma \sim \theta^{-2+\gamma[\alpha_s]}$ to all orders

All order resummation

# What happens at small-x



# Collinear vs CGC



Collinear: $$f_{q,\mathrm{EEC}}(x,\theta)=\frac{\alpha_s T_R}{2\pi\theta^2}\int_x^1\frac{d\xi}{\xi}(1-\xi)(\xi^2+(1-\xi)^2)\left[\frac{x}{\xi}f_g\left(\frac{x}{\xi}\right)\right]$$

CGC: $$f_{q,\mathrm{EEC}}(x_B,\theta)=\frac{N_C S_\perp}{8\pi^4}\int d^2\vec{g}_t\int_{\xi_{\mathrm{cut}}}^1\frac{d\xi}{\xi}\mathcal{A}_{qg}(\xi,\theta,\vec{g}_t)F_{g,x_B}(\vec{g}_t)$$

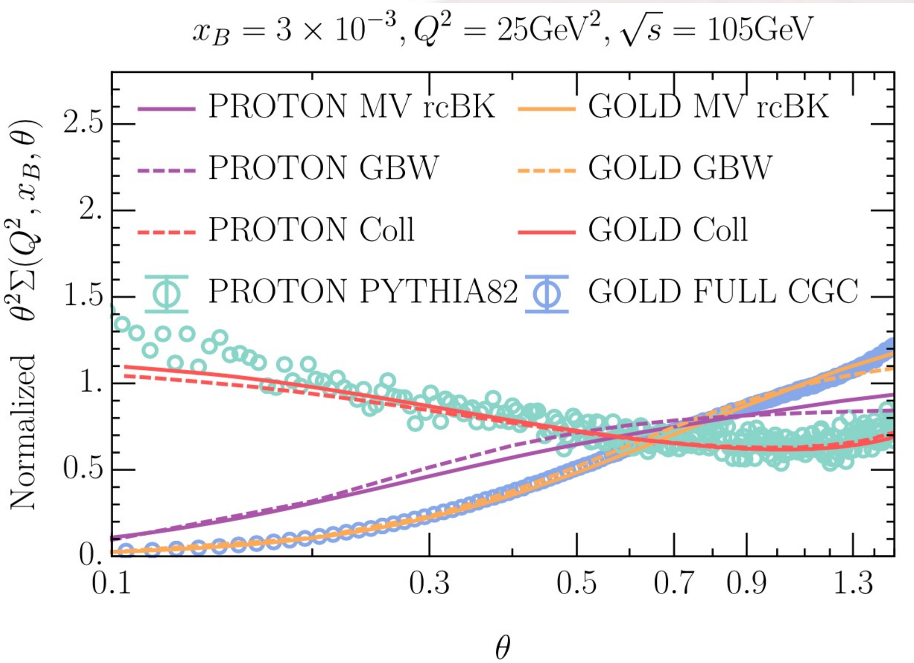
$$\mathcal{A}_{qg}(\xi,\theta,\vec{g}_t)=\frac{1}{\theta^2}(1-\xi)\vec{k}_t^{\,2}(\vec{k}_t-\vec{g}_t)^2\left|\frac{\vec{k}_t}{\xi\vec{k}_t^{\,2}+(1-\xi)(\vec{k}_t-\vec{g}_t)^2}-\frac{\vec{k}_t-\vec{g}_t}{(\vec{k}_t-\vec{g}_t)^2}\right|^2$$

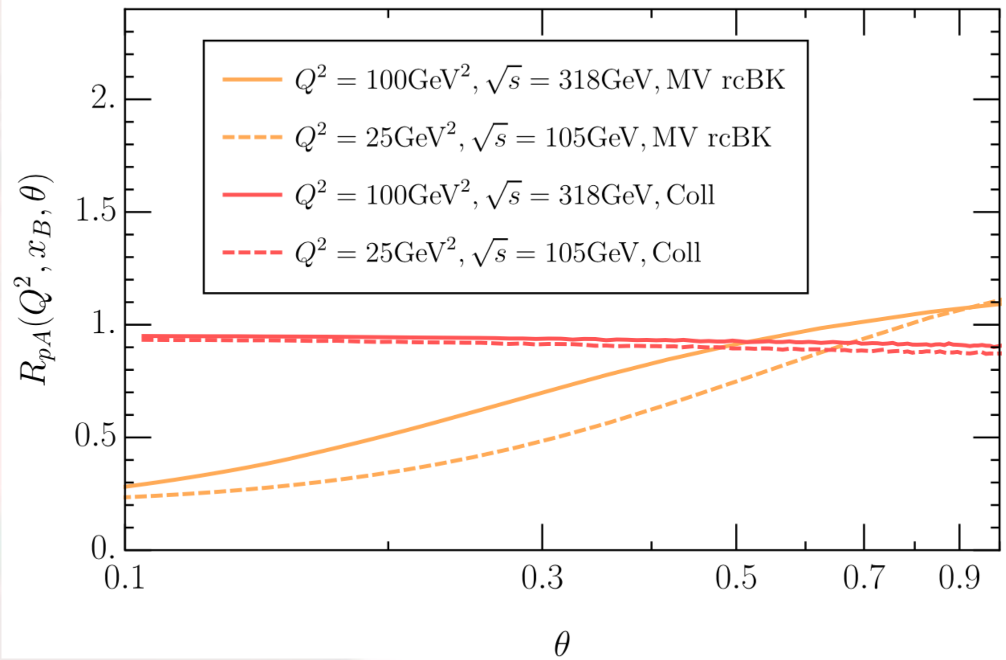
$$k_t=[(1-\xi)/2](Q/2)\theta,$$



See also, NLO: Caucal-Salazar, 2502.02634

# Gluon saturation modify small- $\theta$ behavior



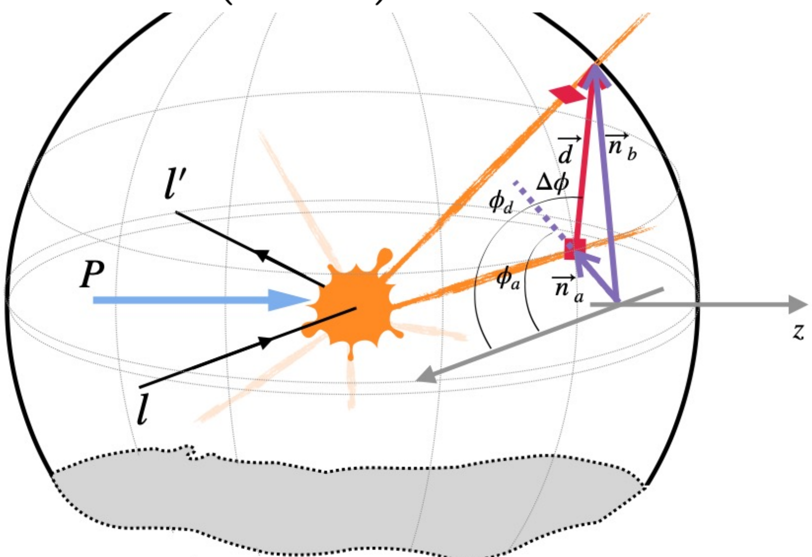



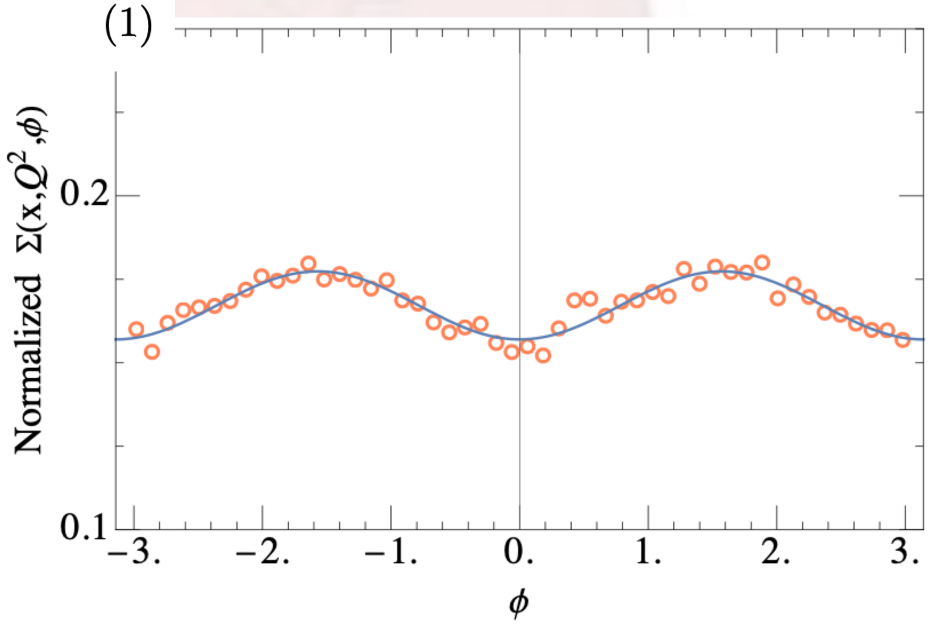

Liu, Liu, Pan, Yuan, Zhu, 2301.01788

# Spinning gluon in NEEC and two particle cos(2$\phi$) correlation in DIS

$$f_{g,\mathrm{EEC}}^{\alpha\beta}(x,\vec{n}_a) = \int \frac{dy^-}{2\pi x P^+} e^{-ixP^+ \frac{y^-}{2}} \times \langle P|\mathcal{F}^{+\alpha}(y^-)\,\mathcal{L}^\dagger[\infty,y^-]\hat{\mathcal{E}}(\vec{n}_a)\mathcal{L}[\infty,0]\mathcal{F}^{+\beta}(0)|P\rangle = \left(-g_T^{\alpha\beta}/2\right) f_{g,\mathrm{EEC}} + h_T^{\alpha\beta} d_{g,\mathrm{EEC}}\,, \quad (1)$$

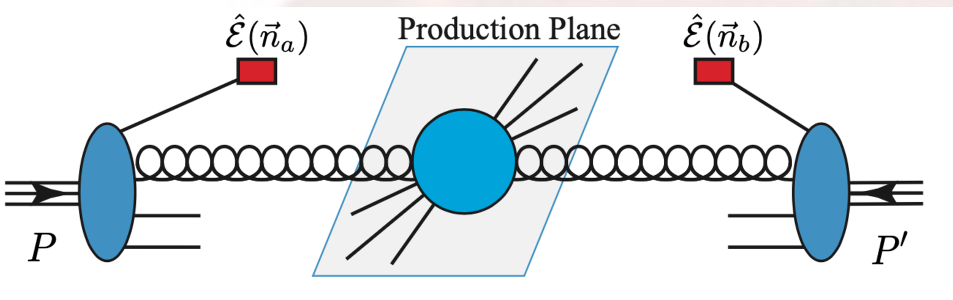
$$h_T^{\alpha\beta} = n_{a,T}^\alpha n_{a,T}^\beta/|n_{a,T}^2| + g_T^{\alpha\beta}/2$$



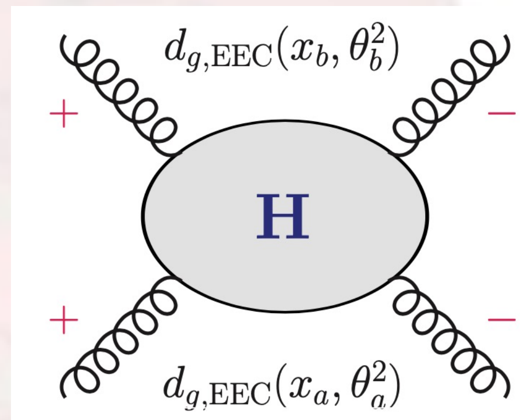



# Spinning gluon and long range azimuthal correlation at RHIC and LHC

Guo, Liu, Yuan, Zhu, arXiv: 2406.05880




$$
\begin{aligned}
&\Sigma(Q^2;\theta_{a,b},\phi) \\
&= \int d\Omega \left\{ x_a f_{g,\mathrm{EEC}}\left(x_a,\theta_a^2\right) x_b f_{g,\mathrm{EEC}}\left(x_b,\theta_b^2\right) \hat{\sigma}_0 \right. \qquad (3) \\
&\left. + x_a d_{g,\mathrm{EEC}}\left(x_a,\theta_a^2\right) x_b d_{g,\mathrm{EEC}}\left(x_b,\theta_b^2\right) \hat{\sigma}_2(Q^2)\cos(2\phi) \right\} \ ,
\end{aligned}
$$



# Two examples at the LHC: Higgs, Top quark pair

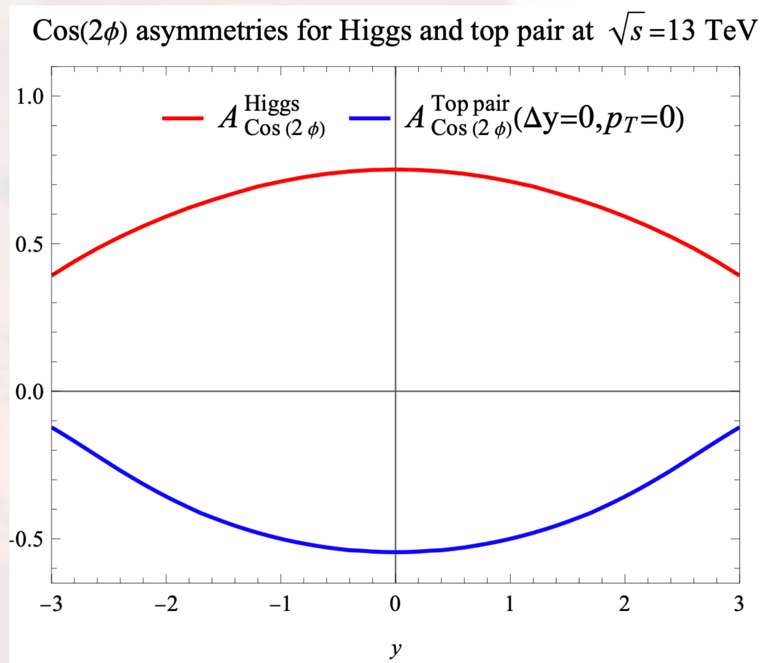
Higgs couples to the spinning gluon directly

$$\hat{\sigma}_2 = \hat{\sigma}_0 = \pi g_\phi^2/64$$

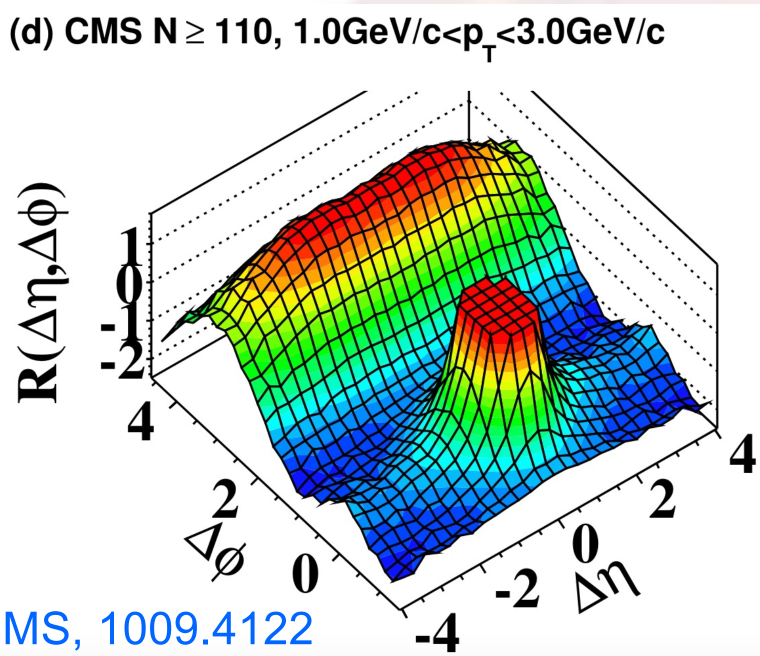
Top quark pair is different

$$\hat{\sigma}_0 = \frac{\alpha_s^2\pi}{\hat{s}^2}\left[\frac{1}{6}\frac{1}{\hat{t}_1\hat{u}_1} - \frac{3}{8}\frac{1}{\hat{s}^2}\right]\left[\hat{t}_1^2 + \hat{u}_1^2 + 4m_t^2\hat{s} - \frac{4m_t^4\hat{s}^2}{\hat{t}_1\hat{u}_1}\right]$$

$$\hat{\sigma}_2 = \frac{\alpha_s^2\pi}{\hat{s}^2}\left[\frac{3}{8}\frac{1}{\hat{s}^2} - \frac{1}{6}\frac{1}{\hat{t}_1\hat{u}_1}\right]\frac{2m_t^4\hat{s}^2}{\hat{t}_1\hat{u}_1}\,, \tag{7}$$



# Extension to multi-jet production: ridge phenomena in pp collisions




CMS, 1009.4122

First step, understand EEC:

$$\Sigma^{jet}(Q^2;\theta_{a,b},\phi) = \sum_{ij}\int d\sigma^{jet}(Q^2)\frac{E_i}{E_P}\frac{E_j}{E_P}\mathcal{F}(\phi;\vec{n}_{a,b}) \times \delta(\vec{n}_a-\vec{n}_i)\delta(\vec{n}_b-\vec{n}_j)\,,$$

$$= \int d\Omega\,\{x_a f_{g,\mathrm{EEC}}\left(x_a,\theta_a^2\right) x_b f_{g,\mathrm{EEC}}\left(x_b,\theta_b^2\right)\hat{\sigma}_0 + x_a d_{g,\mathrm{EEC}}\left(x_a,\theta_a^2\right) x_b d_{g,\mathrm{EEC}}\left(x_b,\theta_b^2\right)\hat{\sigma}_2(Q^2)\cos(2\phi)\}$$

# Cos($2\phi$) and helicity amplitudes in QCD

$$\hat{\sigma}_2 \propto \sum_{\lambda_3\lambda_4} \mathcal{A}(\pm,\pm,\lambda_3,\lambda_4)\mathcal{A}^*(\mp,\mp,\lambda_3,\lambda_4)$$

- Cos($2\phi$) comes from interference between double helicity-flip with the same helicity for the incoming gluons
- QCD amplitude vanishes for same helicity for all external partons or only one has different helicity

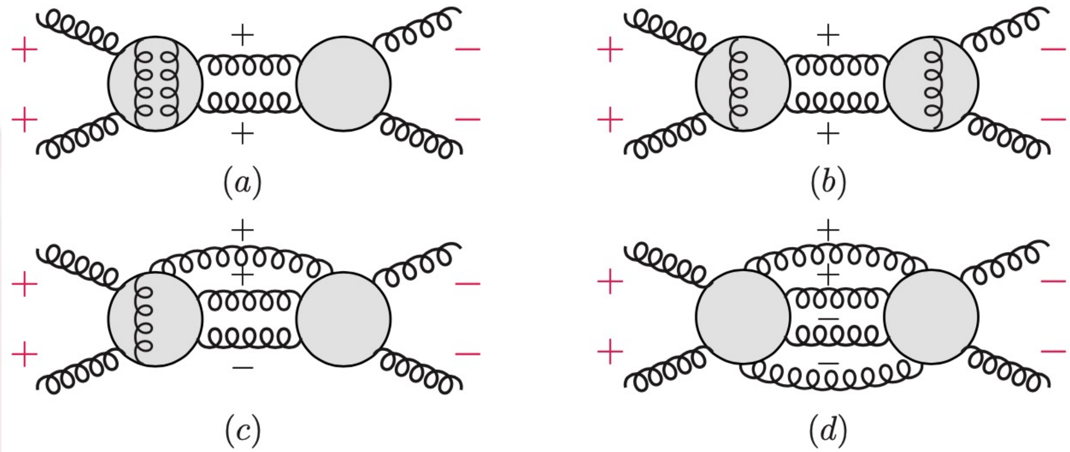
Parke, Taylor, 1986
Berends, Giele, 1988

- Any combinations of $\lambda_3$ and $\lambda_4$ the above vanishes, and one-loop amplitude contribution also vanishes

Bern, Kosower, 1992; …

- Nonvanishing contribution only comes from two-loop amplitudes

# A power counting rule




- Similar conclusion holds for three jet final state, vanishing at the leading order, but survives at NLO
- Four jet final state, $\cos(2\phi)$ is leading order

| Number of Jets | 2 | 3 | $\geq 4$ |
|---|---|---|---|
| $\langle\cos(2\phi)\rangle$ asymmetry | $\mathcal{O}(\alpha_s^2)$ | $\mathcal{O}(\alpha_s)$ | $\mathcal{O}(1)$ |

# Dijet cos(2$\phi$) at NNLO

- Applying the two-loop amplitudes Ahmed, Henn, Mistlberger, 1910.06684; Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi, 2112.11097

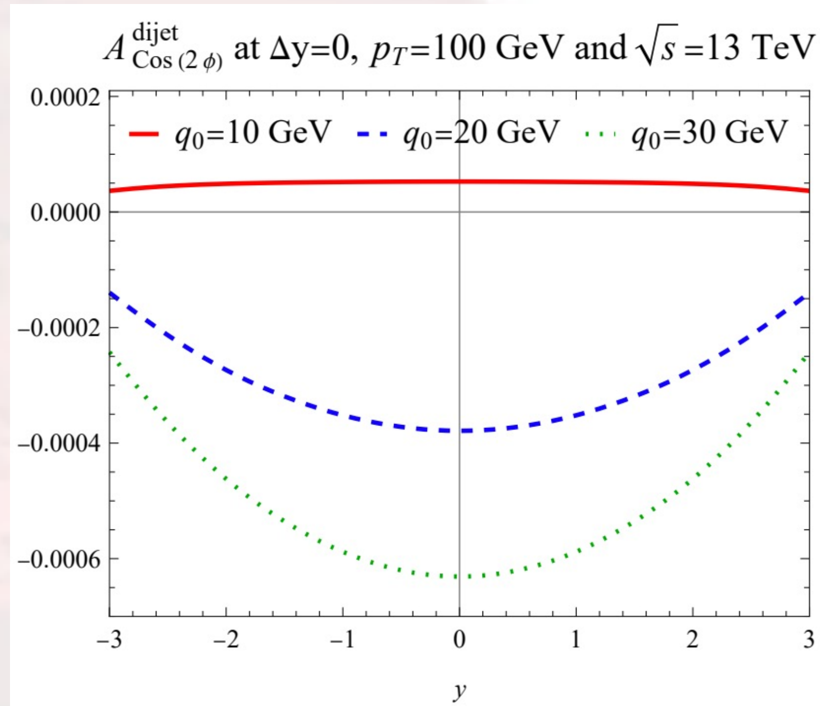
$$\hat{\sigma}_2^{(2)} = A^{(1)}(+++-)A^{(1)*}(--+-) + A^{(1)}(++++)A^{(1)*}(--++) + A^{(2)}(++++)A^{(0)*}(--++) + h.c.$$

- There are IR divergences, which will be cancelled by soft real gluon radiation (with a jet veto $q_0$)

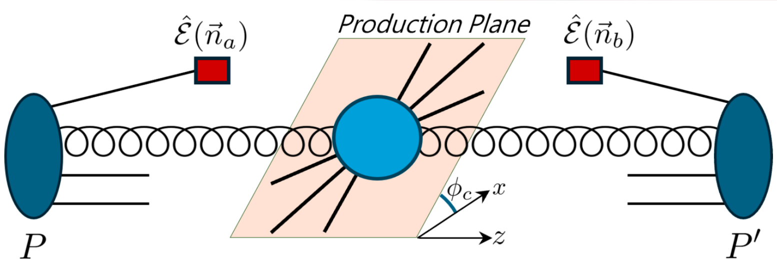
$$\hat{\sigma}_2^{(2)} = \hat{\sigma}_{2,0}^{(2)}(x)\ln\left(\frac{\hat{s}}{q_0^2}\right) + \hat{\sigma}_{2,2}^{(2)}(x) - \hat{\sigma}_{2,1}^{(2)}(x)\left[\mathcal{P}_{gg}^d \otimes d_g(x_a) + \mathcal{P}_{gg}^d \otimes d_g(x_b)\right]$$

# Phenomenological results

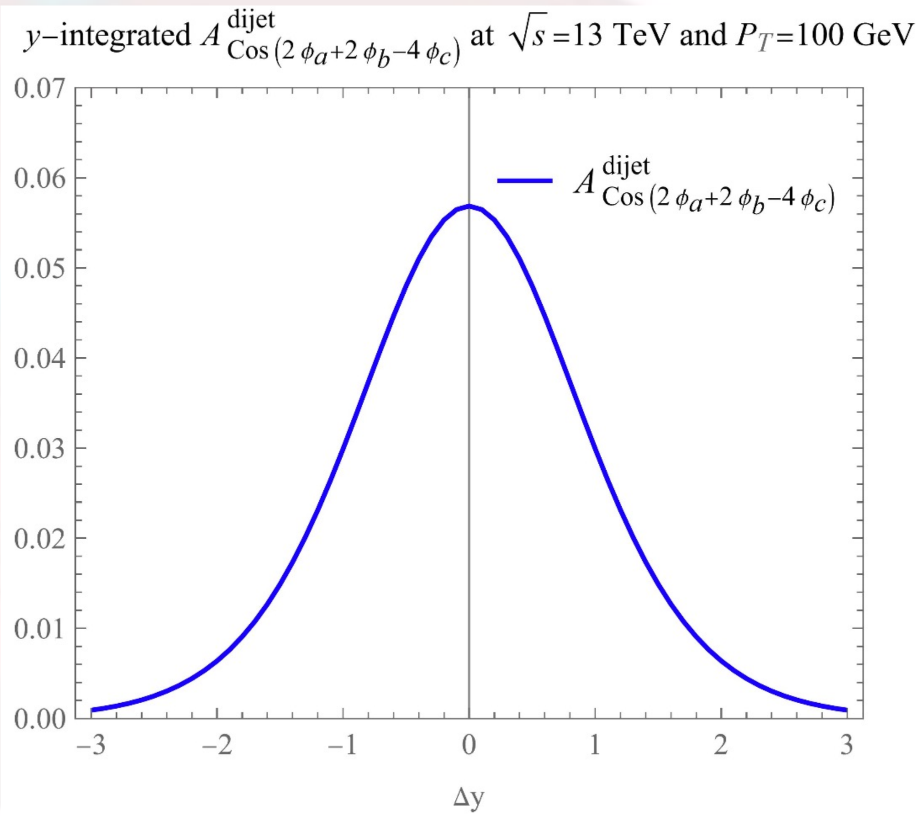
- Suppressed by $(\alpha_s/4\pi)^2$, order $10^{-4}$- $10^{-3}$
  - Roughly same order as that found in the long range ridge for lower end of multiplicity events
- Strongly depends on the jet veto $q_0$



# Cross check with semi-long range correlation




$$\langle \cos(2\phi_a + 2\phi_b - 4\phi_c)\rangle_{\text{dijet}} = \frac{\int d\Omega d_{g,\text{EEC}}(x_a) d_{g,\text{EEC}}(x_b) \frac{9}{2} \frac{\hat{s}^2+\hat{t}^2+\hat{u}^2}{2\hat{s}^2}}{\int d\Omega f_{g,\text{EEC}}(x_a) f_{g,\text{EEC}}(x_b) \frac{9}{2} \frac{(\hat{s}^2+\hat{t}^2+\hat{u}^2)^3}{8\hat{s}^2\hat{t}^2\hat{u}^2}}$$



# Next step

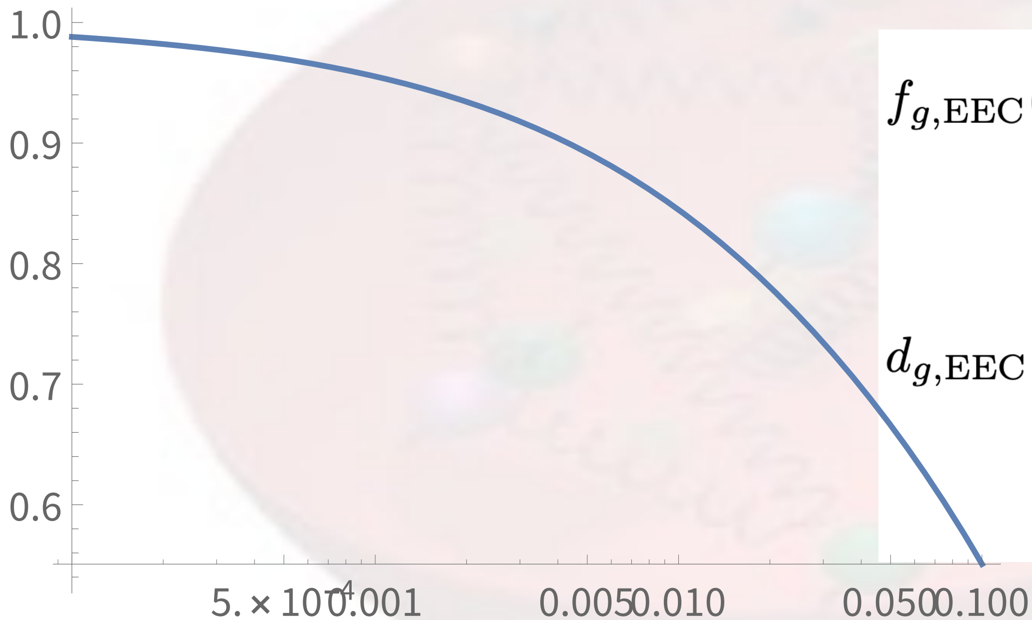
- Compute three-jet, four-jet asymmetry
  - one-loop amplitudes for three-jet final state
  - tree amplitudes for four-jet final state
- Build simulation for the ridge measurements
  - Keep interference and spin information in the parton shower simulations
  - And/or include high number of jets in the final states

# Conclusion

- Spinning gluon is an important aspect of nucleon tomography, in particular, at small-x
- We should be able to observe the long range azimuthal angular correlations due to the spinning gluon effects
- Extension to the spinning quark should be pursued as well

# Back-up

# Gluon is spinning at small-x




$$f_{g,\mathrm{EEC}}(x,\theta_a^2) = \frac{\alpha_s}{2\pi}\frac{1}{\theta_a^2}\int_x^1 \frac{dz}{z}\frac{x(1-z)}{z} \times \left[\mathcal{P}_{g/q}(z) f_q\left(\frac{x}{z}\right) + \mathcal{P}_{g/g}(z) f_g\left(\frac{x}{z}\right)\right]$$

$$d_{g,\mathrm{EEC}}(x,\theta_a^2) = \frac{\alpha_s}{2\pi}\frac{1}{\theta_a^2}\int_x^1 \frac{dz}{z}\frac{x(1-z)}{z} \times \frac{2(1-z)}{z}\left[C_F f_q\left(\frac{x}{z}\right) + C_A f_g\left(\frac{x}{z}\right)\right]$$

# Universal IR Structure in QCD Amplitudes

Catani 1998; Sterman-Tejeda-Yeomans 2003

$$\boldsymbol{I}^{(1)} \equiv \left[ -\sum_i \left( \frac{\gamma_K^{[i](1)}}{2\epsilon^2} + \frac{\mathcal{G}_0^{[i](1)}}{\epsilon} \right) \mathbf{1} + \frac{\mathbf{\Gamma}^{(1)}}{\epsilon} \right] \left( -\frac{\mu^2}{s} \right)^{\epsilon} \qquad \mathbf{\Gamma}^{(1)} = \frac{1}{2} \sum_i \sum_{j \neq i} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \ln \left( \frac{-\mu^2}{s_{ij}} \right)$$

- Applying in our case

$$\hat{\sigma}_2^{(2)v} = \frac{1}{\epsilon} \left( \hat{\sigma}_{2,J}^{(2)v} + \hat{\sigma}_{2,S}^{(2)v} \right)$$

$$\hat{\sigma}_{2,S}^{(2)v} = \frac{2}{\mathcal{V}} \left\langle A^{(1)}(++++) | 2\mathrm{Re} \left[ \mathbf{\Gamma}^{(1)} \right] | A^{(0)}(--++) \right\rangle$$

$$\hat{\sigma}_{2,J}^{(2)v} = -\sum_i \gamma_K^{[i](1)} \hat{\sigma}_2^{(1)\epsilon} = -8 C_A \hat{\sigma}_2^{(1)\epsilon}$$

Helicity amplitudes can be found in Refs. for two loop results:

Ahmed, Henn, Mistlberger, 1910.06684;

Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi, 2112.11097