Introduction to deep-inelastic scattering

“Selected Topics” Warsaw, Poland

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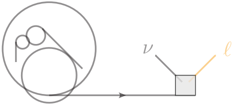
Institute of Nuclear Physics – Polish Academy of Science (IFJ PAN)

18 September 2024



most important: these lectures are low-key; questions are great

I am literally here to tell you what I know

Lecture Plan (Wednesday)

Lecture III:

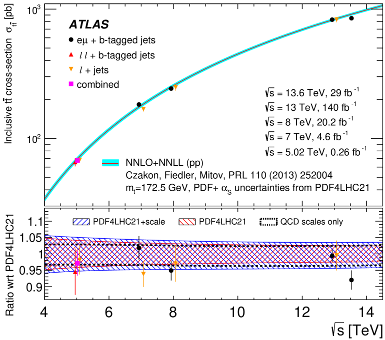
- Pt1: **modern motivation** for deep-inelastic scattering
- Pt2: building a scattering formula for **heavy nuclei**
- Pt3: **intuition and secrets**
- Pt4: **improving** our scattering formula

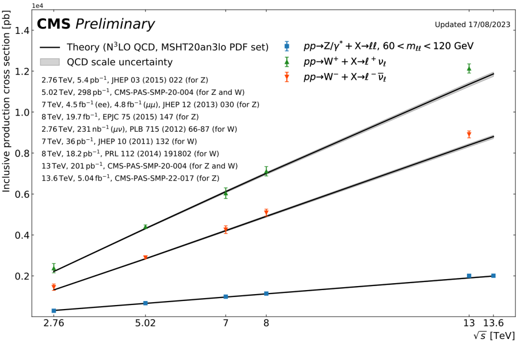
Freedom at 15:30ish

Pt1. a bit of fun

a bit of motivation

for the next 15+ years ('40s) we will be guided by high-energy, high-precision hadron-hadron collision data (ion-ion data, too!)



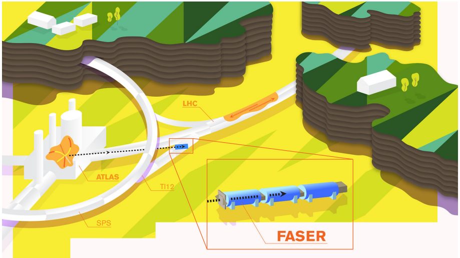


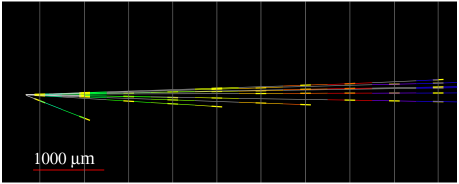
The high-luminosity program has a **robust and exciting portfolio** for Standard Model measurements and new physics searches

ongoing discussions on future programs at CERN (FCC!), e.g., European Strategy Update ('20); Snowmass ('21) [2209.14872];

new! SOI between USA and CERN "*expresses intention by the United States to collaborate on FCC-ee*" (April'24)

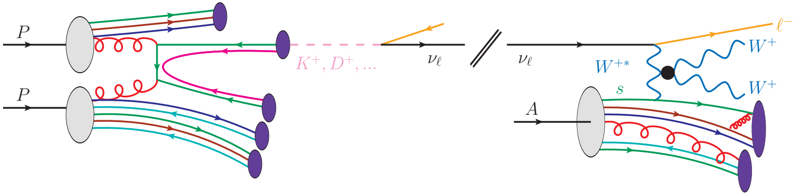
In the past few years, the LHC has been established as an intense (laboratory) source of TeV-scale ν **(a remarkable expt. achievement!)**





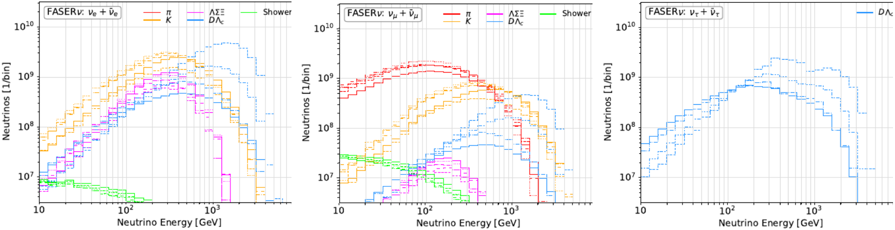
Candidate LHC neutrino event from FASER's pilot run

New programs (FASER, SND@LHC) **now collecting ν-nucleus scattering data**

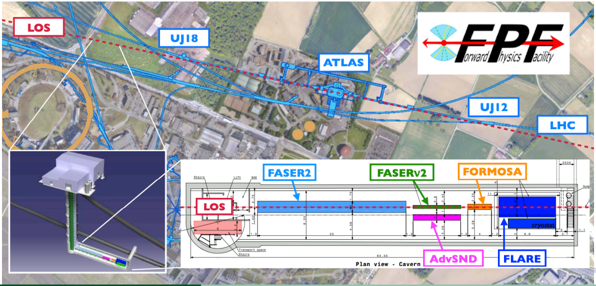


ν **fluxes** from LHC **(a) are large** and **(b) span $1-4$ TeV in energy**

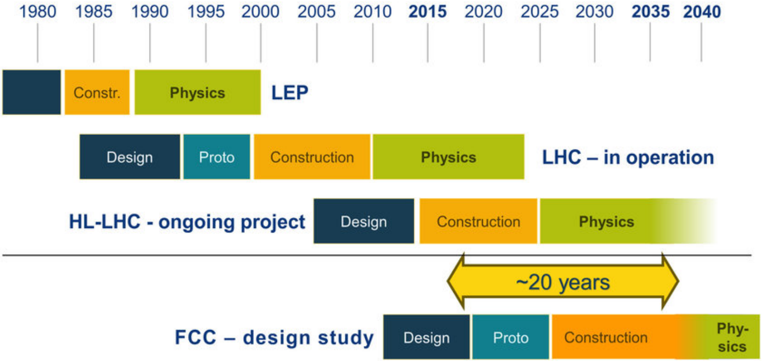
Kling & Nevay (PRD'21)



Detectors at the the Forward Physics Facility, a proposed cavern alongside ATLAS, can see $\mathcal{O}(10^6)$ TeV-scale νDIS events [2203.05090]; Feb'24 meeting



CERN beyond the HL-LHC

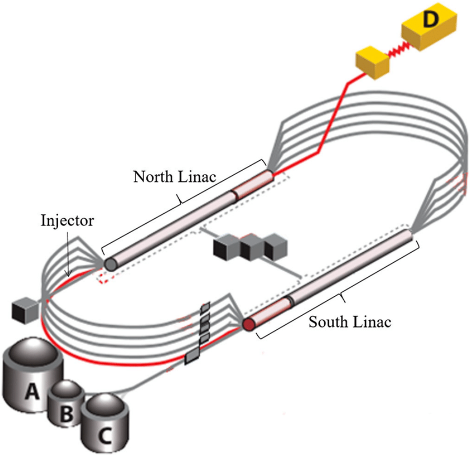


Ongoing discussions on the future of CERN
– leading candidate is Future Circular Collider (FCC) program
– e^+e^- collisions $\rightarrow$ $pp/pA/AA$ European Strategy Update ('20), Snowmass ('21) [2209.14872]

a more immediate future

Continuous Electron Beam Accelerator Facility (CEBAF)

long-running e^- DIS program at Jefferson National Lab. in US



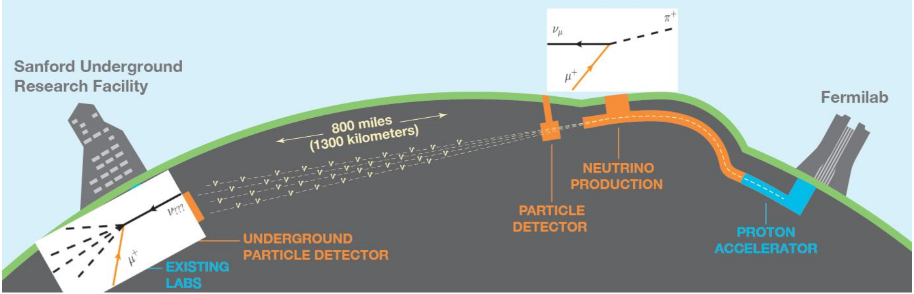
CEBAF outlook into ’30s (possibly beyond)

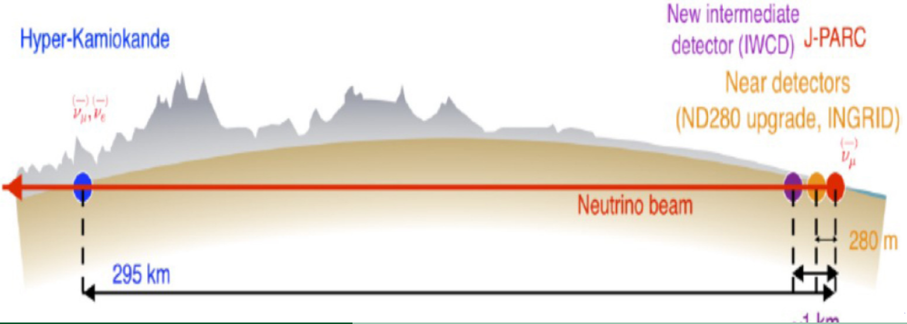
Recap

	Present Design	Possible	Challenges
Luminosity increase	Hall A & C @11GeV Total < 85 μA (< 82 μA Each dump limit)	Hall A & C @11GeV Total < 140 μA (< 82 μA Each dump limit)	▪ RF Beam Loading ▪ Dump Cooling ▪ BBU Instability
Positron option	Not Yet an Option	>100 nA Unpolarized Or >10 nA Polarized e+	▪ Target Design ▪ e+ Collection ▪ Beam dynamics, Injector and Main ▪ High Intensity e- Beam (~1 mA) Need ▪ Production Energy Choice and Design ▪ Gaining Experience
Energy increase	Up to 11 GeV to A, B, or C 12 GeV to D	20 – 24 GeV	▪ Scaling Up FFA Optics to Several GeVs ▪ Dump Cooling & Enviro. Evaluation ▪ Injector Energy increase ~ factor 4. ▪ BBU instability

The Future of CEBAF - Reza Kazimi (*DIS2022 May 3*) 23 Jefferson Lab

DUNE & Hyper-Kamiokande ('30s-ish-)



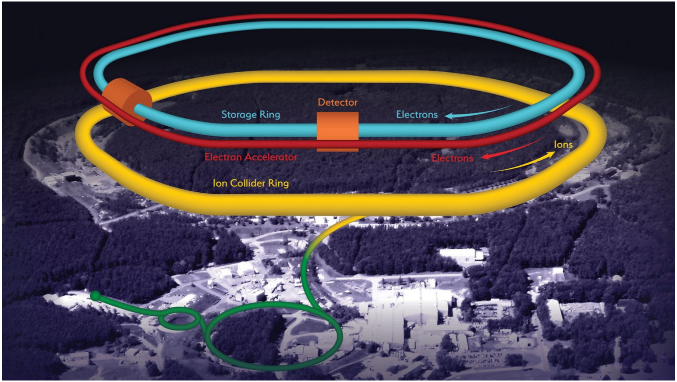


add +4ish years

DUNE and Hyper-K Timeline

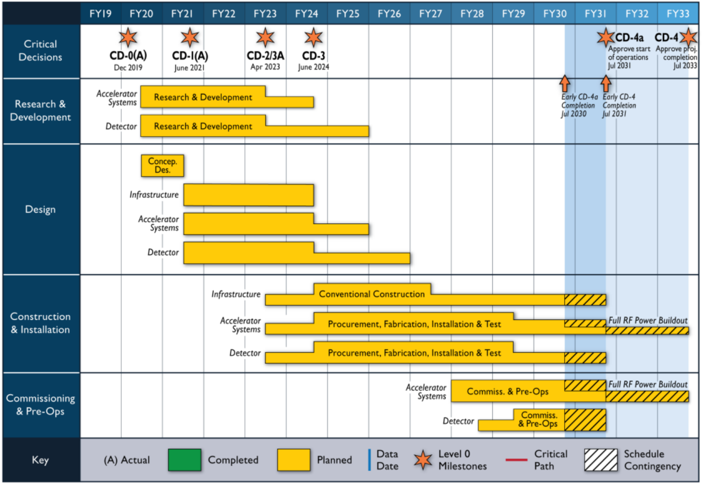
	DUNE	Hyper-K
Cavern Excavation	2018 – 2020	2019 – 2024
FD Construction	2022 – 2024	2024 – 2025
FD Fill	2024 – 2025 (10+10kt V_A)	2025 – 2026 (187kt V_A)
Data Taking	2025 (cosmic) / 2026 (ν beam)	2026
ND Ready	2027	In place

Electron-Ion Collider



The EIC is a next-generation $\ell - \mathcal{A}$ collider with first collisions in '30s

Reference Schedule toward CD-4



the big picture

bonanza of lepton-nucleus scattering data through ’40s/’50s

possibly beyond

Several νDIS and $e^{\pm}$DIS programs already collecting data:

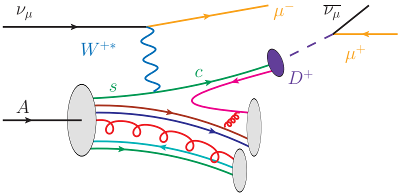
- Fermilab (short-baseline ν)
- JLab (12 GeV CEBAF)
- CERN (FASER/SND experiments)

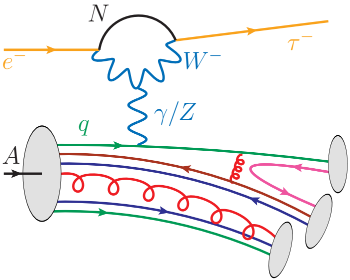
with more planned for '20s-'50s:

- BNL (EIC) ✓
- LBNF (DUNE) ✓
- CERN (Forward Physics Facility)

with various agendas:

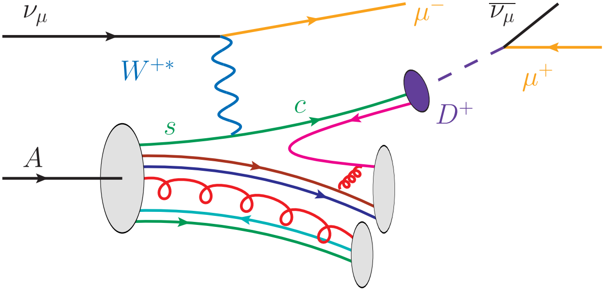
- precision ν oscillations
- precision hadronic structure
- QCD at the extremes
- search for LFV
- search for feably coupled phys.

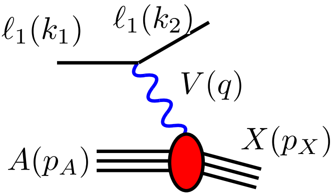


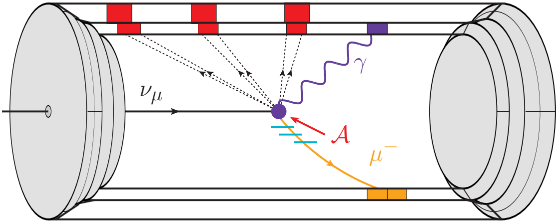


theory developments over next 5 years will impact day-1 physics

deep-inelastic scattering (DIS)

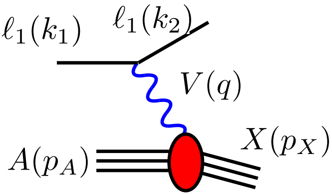


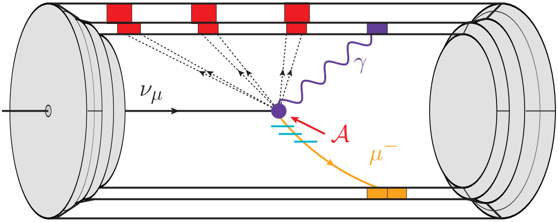




DIS scattering experiments are counting experiments:

- **count** # of candidate signal events, e.g., $1 mu^{\pm} + X$ satisfying criteria
- **estimate** # of background events from data-driven control region
- **calculate** statistical significance





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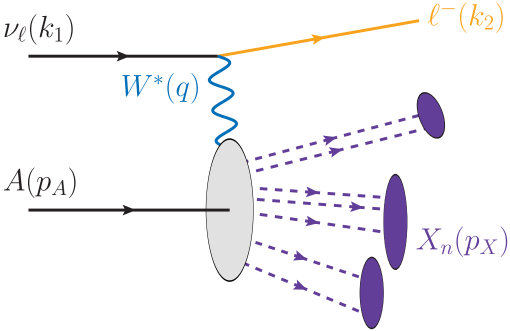
Theory needed to estimate number (and unc.) of signal and bkg events:

$$\underbrace{N_{\text{candidates}}}_{\text{hep/nucl}-\text{ex}} = \underbrace{\mathcal{L}(\text{data!})}_{\text{accelerators}} \times \underbrace{\sigma(\text{scattering likelihood})}_{\text{hep/nucl}-\text{th/ph}}$$

Generically, hard scattering of $\ell \in \{\ell^\pm, \nu, \overline{\nu}\}$ off **nucleons** well-described by **kinematic factor** (lepton bit) and "**structure functions**" (hadron bit)



$$d\sigma(\nu\mathcal{A} \to \ell X) = \sum_i \underbrace{(\text{some function of } p_A, q)_i}_{\text{calculable from first principles}} \times \underbrace{F_i^{\nu\mathcal{A}}(p_A, q)}_{\text{parameterizes response of } \mathcal{A}}$$

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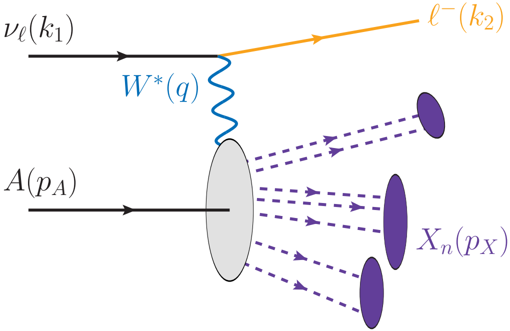

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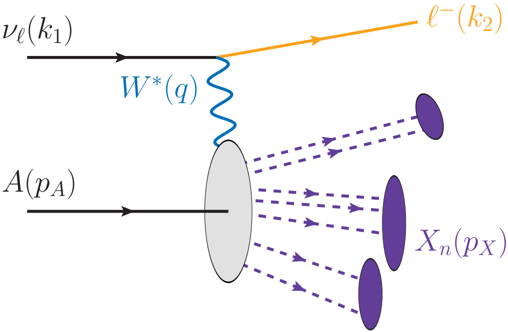
Quark-Parton Model: $F^{\nu\mathcal{A}}(p_A, q) \sim \sum_{k=q,g,\overline{q}} f_k^{\mathcal{A}}(z)$, $\quad x = -q^2/(2p_A \cdot q)$

- $f_k^{\mathcal{A}}$ is the **parton (number) density function (PDF)** of k in $\mathcal{A}$

Pt2. building a scattering formula for nuclei



[1]

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[1] for a more complete & pedagogical treatment, see appendices of RR, et al [Prog.Part.Nucl.Phys. 136 ('24) 104096]

light cone dominance

starting point for DIS on p is stipulating kinematics. typically,

$$Q^2 = -q^2 > 0 \gg m_{\text{proton}}^2 \text{ [proton case]}$$

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naïve application to ^{56}Fe or ^{197}Au would require

$$Q^2 \gg (50 \text{ GeV})^2 \sim \left(\frac{M_Z}{2}\right)^2 \text{ or } (180 \text{ GeV})^2 \sim m_t^2 \text{ [incorrect]}$$

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more precise statement

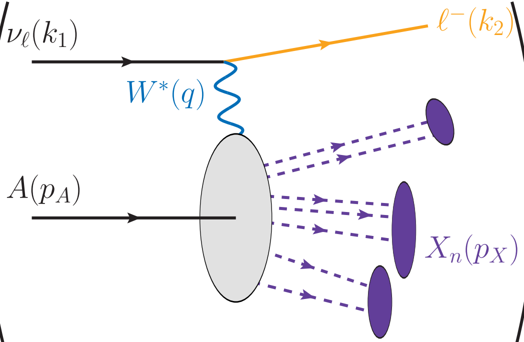
$$Q^2 \gg \Lambda_{\rm non-pert.} \sim \mathcal{O}(1)\ {\rm GeV} \gg \Lambda^2_{\rm QCD} \sim m_q^2 \text{ [general case]}$$

Bjorken scaling still works at moderate energies since $\mathcal{O}(\Lambda^2_{\rm QCD}/Q^2) \ll 1$

Georgi, Politzer ('79); Muta ('98/'10)

draw diagrams, currents, and build the matrix element

$-i\mathcal{M}$ (



) $= \; J^{\alpha}_{\ell_f \ell_i} \cdot \Delta_{\alpha\mu} \cdot J^{\nu}_{X_n A}$

matrix element:

$$
\begin{aligned}
-i\mathcal{M} &= \langle X\ell_2 | J^{\mu}_{\ell_2\ell_1}(0) \cdot \Delta^{V}_{\mu\sigma}(q) \cdot J^{\sigma}_{XA}(0) | A\ell_1 \rangle \\
&= \langle \ell_2 | J^{\mu}_{\ell_2\ell_1}(0) | \ell_1 \rangle \cdot \Delta^{V}_{\mu\sigma}(q) \cdot \langle X | J^{\sigma}_{XA}(0) | A \rangle
\end{aligned}
$$

matrix element:

$$-i\mathcal{M} = \langle X\ell_2|J^{\mu}_{\ell_2\ell_1}(0)\cdot\Delta^{V}_{\mu\sigma}(q)\cdot J^{\sigma}_{XA}(0)|A\ell_1\rangle$$
$$= \langle\ell_2|J^{\mu}_{\ell_2\ell_1}(0)|\ell_1\rangle\cdot\Delta^{V}_{\mu\sigma}(q)\cdot\langle X|J^{\sigma}_{XA}(0)|A\rangle$$

lepton current:

$$L^{\mu} \equiv \langle\ell_2|J^{\mu}_{\ell_2\ell_1}(0)|\ell_1\rangle = -i\tilde{g}\,\overline{u}(k_2,\lambda_2)\left[g^{\ell}_{V}\gamma^{\mu}+g^{\ell}_{A}\gamma^{\mu}\gamma^{5}\right]u(k_1,\lambda_1)$$

lepton and hadronic currents

matrix element:

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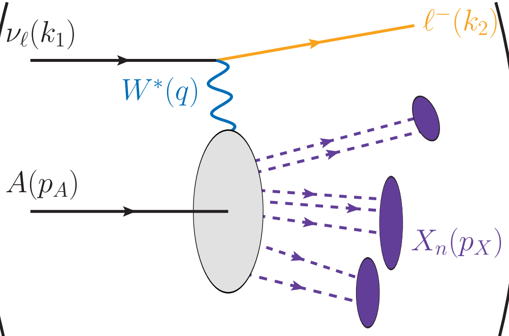
hadronic current:

$$H^{A\sigma} \equiv \langle X|J^{\sigma}_{XA}(0)|A\rangle \equiv \overline{u}_X(p_X,\lambda_X)[\dots]^{\sigma}u(p_A,\lambda_A)$$

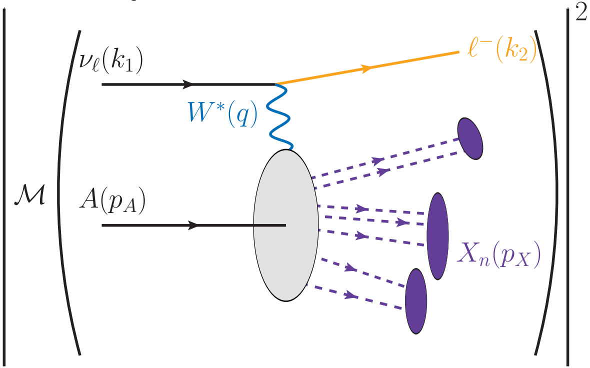
squaring and summing over spins gives us $H_n^{\mu\nu}$ (exclusive, n-body)

$$\Sigma_{\text{spins}} \left| \mathcal{M} \left(\quad \right) \right|^2 \sim L_{\mu\nu} \cdot H_n^{\mu\nu}$$



n-body phase space integral *and* summing over n gives us $W_A^{\mu\nu}$

$\frac{d^3\sigma}{dk_2^3} \sim \int dPS_n \; \Sigma_n \; \Sigma_{\rm spins}$



$\sim \; L_{\mu\nu} \; \cdot \; W_A^{\mu\nu}$

this step sometimes omitted in textbooks, e.g., Halzen & Martin

Summing over X_n ensures "inclusivity" and closure, $1 = \sum_n |X_n\rangle\langle X_n|$

DIS cross section (scattering rate) given by

$$\sigma(\ell_1 + A \to \ell_2 + X_n) = \int dPS_{n+1} \frac{d\sigma}{dPS_{n+1}}\,, \qquad \text{with} \qquad \frac{d\sigma}{dPS_{n+1}} = \frac{1}{\mathcal{F}} \frac{1}{S_{\ell_1} S_A} \sum_{\{\lambda\}} \sum_{\text{dof}} |\mathcal{M}|^2$$

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Inserting known quantities

$$k_2^0 \times \frac{d\sigma}{d^3 k_2} = \frac{1}{(16\pi^2)s} \frac{2}{S_{\ell_1} S_A} \frac{1}{(q^2 - M_V^2)^2} \left(\sum_{\{\lambda\}} L^{\mu\nu} \right) \cdot W^A_{\mu\nu}$$

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Leptonic currents (depend on exchange boson)

$$\sum_{\{\lambda\}} L^{\mu\nu} \Big|_{\text{QED}} = 4e^2 \left\{ k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - (k_1 \cdot k_2) g^{\mu\nu} \right\}$$

$$\sum_{\{\lambda\}} L^{\mu\nu} \Big|_{W} = g_W^2 \left\{ k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - (k_1 \cdot k_2) g^{\mu\nu} + i k_{1\alpha} k_{2\beta} \epsilon^{\mu\nu\alpha\beta} \right\}$$

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Hadronic current is formally defined by[2]:

$$W^A_{\mu\nu} = \frac{1}{4\pi} \underbrace{\sum_{n=1}^{\infty}}_{\text{sum over n}} \int dPS_n \sum_{\text{dof}} \underbrace{H^A_{\mu\nu\ n}}_{\text{matrix for n hadrons}}$$

[2]Same as usual expression $W^A_{\mu\nu} = \frac{1}{4\pi} \int d^4z\ e^{iq\cdot z} \langle A | J^\dagger_{had.\mu}(z)\ J_{had.\nu}(0) | A \rangle$. See Eq. (A.22) and below of [2301.07715]

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$$= -g_{\mu\nu} F_1^A + \frac{p_{A\mu} p_{A\nu}}{Q^2} 2x_A F_2^A - i\epsilon_{\mu\nu\rho\sigma} \frac{p_A^\rho q^\sigma}{Q^2} x_A F_3^A$$

$$+ \frac{q_\mu q_\nu}{Q^2} 2F_4^A + \frac{p_{A\mu} q_\nu + p_{A\nu} q_\mu}{Q^2} 2x_A F_5^A + \frac{p_{A\mu} q_\nu - p_{A\nu} q_\mu}{Q^2} 2x_A F_6^A$$

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point #1: $W^A_{\mu\nu}$ has at most six unknown components $(4 \times 4) = \mathbf{1} + \underline{\mathbf{1}} + \mathbf{4} + \underline{\mathbf{4}} + \mathbf{6}$

[3] parton model says $F_i = \sum f_{j/p}$; see Collins ('11) for nice discussion on this!

[4] see Sterman [hep-ph/9606312] and see Collins ('11) for nice discussions!

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point #3: $F_i(x, Q^2)$ are independent of underlying theory[3]

point #4: $W^A_{\mu\nu}$ is defined in the "DIS" limit:

$$x_A = \frac{Q^2}{2p_A \cdot q} \text{ is fixed and } Q^2 \gg \Lambda^2_{\rm NP}, \text{ where } \Lambda_{\rm NP} \sim \mathcal{O}(1-2)\ {\rm GeV}$$

[3] parton model says $F_i = \sum f_{j/p}$; see Collins ('11) for nice discussion on this!

[4] see Sterman [hep-ph/9606312] and see Collins ('11) for nice discussions!

point #1: $W^A_{\mu\nu}$ has at most six unknown components $(4\times4) = \mathbf{1} + \underline{\mathbf{1}} + \mathbf{4} + \underline{\mathbf{4}} + \mathbf{6}$

point #2: "structure functions" $F_i(x, Q^2)$ are well-defined experimentally

point #3: $F_i(x, Q^2)$ are independent of underlying theory[3]

point #4: $W^A_{\mu\nu}$ is defined in the "DIS" limit:

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point #5: in practice, $F_{4,5,6}$ can be neglected, but not always[4]

$$W^A_{\mu\nu} = -g_{\mu\nu}\ F^A_1(x_A) + \frac{p_{A\mu}p_{A\nu}}{Q^2}\ 2x_A\ F^A_2(x_A) - \mathcal{O}(\mathcal{P})\ x_A F^A_3(x_A)$$
$$+\ \mathcal{O}\left(\frac{m^2_\nu, m^2_\ell}{Q^2}\right)\ 2F^A_4(x_A) + \mathcal{O}\left(\frac{m^2_\nu, m^2_\ell}{Q^2}\right)\ 2x_A\ F^A_5(x_A) + \mathcal{O}(\mathcal{CP})\ 2x_A\ F^A_6(x_A)$$

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what is next?

building structure functions

To build **structure functions**, follow Georgi & Politzer ('76,'76) with guidance from "modern" literature:

1. Get coffee ☺ ✓
2. Build inclusive, had. tensor in DIS: $W_A^{\mu\nu}$ ✓
3. Define forward-scatt. tensor $T_A^{\mu\nu}$ (nice analytic properties)
4. Relate $W_A^{\mu\nu}$ and $T_A^{\mu\nu}$
5. "expand" and "simplify" $T_A^{\mu\nu}$
6. Get coffee ☺

Define the time-ordered ME for (virtual) $AV^* \to AV^*$ scattering

$$T^A_{\mu\nu} = \int \; d^4z \; e^{iq\cdot z} \langle A| \; \mathcal{T} J^\dagger_{had.\mu}(z) \; J_{had.\nu}(0)|A\rangle$$

$$= -g_{\mu\nu} \; \Delta T^A_1 \; + \frac{p_{A\mu} p_{A\nu}}{M_A^2} \; \Delta T^A_1 \; - i\epsilon_{\mu\nu\rho\sigma} \frac{p_A^\rho q^\sigma}{M_A^2} \; \Delta T^A_3$$

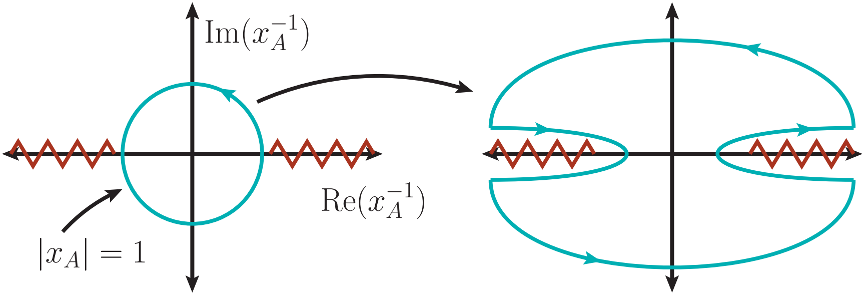
$$+ \frac{q_\mu q_\nu}{M_A^2} \; \Delta T^A_4 \; + \frac{p_{A\mu} q_\nu + p_{A\nu} q_\mu}{M_A^2} \; \Delta T^A_5 \; + \frac{p_{A\mu} q_\nu - p_{A\nu} q_\mu}{M_A^2} \; \Delta T^A_6$$

point #1: related to $W^A_{\mu\nu}$ by Fourier transformations

point #2: $T^A_{\mu\nu}$ is defined in the "short-distance" limit:

$$\frac{x_A}{Q} \text{ is fixed and } (Q^2/M_A^2) \to \infty$$

$W^A_{\mu\nu}$ and $T^A_{\mu\nu}$ are different (DIS vs short-distance limit) but related by a contour



$$\Delta T_i^A(x_A^{-1} + i\varepsilon) - \Delta T_i^A(x_A^{-1} - i\varepsilon) = \text{(some factor)} \times F_i^A(x_A, Q^2)$$

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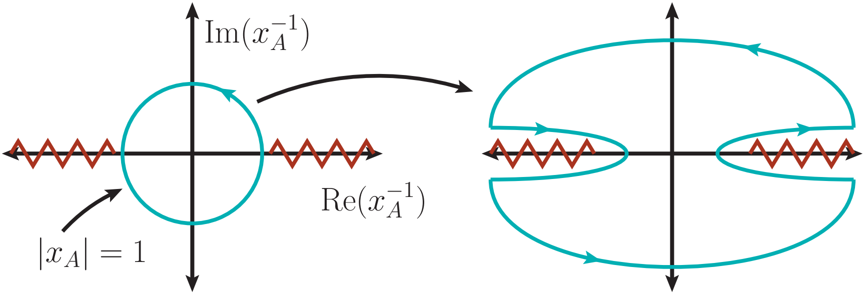

$$\Delta T_i^A(x_A^{-1} + i\varepsilon) - \Delta T_i^A(x_A^{-1} - i\varepsilon) = (\text{some factor}) \times F_i^A(x_A, Q^2)$$

Taylor-expand ΔT_i around $x_A^{-1} \to 0$ + Cauchy's Thm see also Collins ('84)!

$$\Delta T_i^A \; = \; (\text{some other factor}) \; \times \; \sum_N^{\infty} \underbrace{F_i^{AN}(Q^2)}_{N^{th}\ \text{Mellin moment} \, = \int_0^1 dy \;\, y^{(N-1)} F_i(y)} x_A^{-N}$$

the last step

the operator product expansion

the operator product expansion (in a nutshell)

The OPE is a formalism for decomposing products of operators

Wilson ('69); Brandt, Preparata ('71); Christ, et al ('72)

$$\langle \text{some number of operators } \hat{\mathcal{O}} \rangle = \sum_k \underbrace{\mathcal{C}_k}_{\text{Wilson coeff.}} \times \langle \text{fewer operators } \hat{\mathcal{O}} \rangle$$

[5] Power counting is ordered by "twist", τ = (dim. of EFT operator) − (# of Lorentz indices); see also Stermen (TASI'95)

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Assume $T^A_{\mu\nu}$ admits an OPE in the short-distance limit:[5]

$$\lim_{z\to 0} T^A_{\mu\nu} \overset{\text{OPE}}{=} -2i \sum_{\iota,n} \underbrace{c_{\mu\nu\mu_1\dots\mu_n}(q)}_{\text{Wilson coeff.}} \underbrace{\langle A|\hat{\mathcal{O}}^{\mu_1\dots\mu_n}_{\iota,\tau=2}|A\rangle}_{\text{hadronic ME}}$$

$$+ \quad \mathcal{O}(\tau > 2) \leftarrow \text{power corrections}$$

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$$+ \quad \mathcal{O}(\tau > 2) \leftarrow \text{power corrections}$$

"Leading twist" collects entire tower of operators $\hat{\mathcal{O}} \sim D^k / \Lambda^{k+2}$

[5] Power counting is ordered by "twist", τ = (dim. of EFT operator) − (# of Lorentz indices); see also Sterman (TASI'95)

the operator product expansion (in a nutshell)

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$$\begin{aligned}\lim_{z\to 0} T^A_{\mu\nu}(q, p_A) \overset{\text{OPE}}{=} &- 2i \sum_{k=k_{\min}}^{\infty} \Big[-2g_{\mu\nu} q_{\mu_1} q_{\mu_2} C_1^{2k} + g_{\mu\mu_1} g_{\nu\mu_2} Q^2 C_2^{2k} - i\epsilon_{\mu\nu\alpha\beta} g^{\alpha}_{\mu_1} q^{\beta} q_{\mu_2} C_3^{2k} \\ &+ 4\frac{q_\mu q_\nu}{Q^2} q_{\mu_1} q_{\mu_2} C_4^{2k} + 2(g_{\mu\mu_1} q_\nu q_{\mu_2} \pm g_{\nu\mu_1} q_\mu q_{\mu_2}) C_{5,6}^{2k} \Big] \\ &\times \frac{2^{2k}}{(Q^2)^{2k}} \times \left(\prod_{m=3}^{2k} q_{\mu_m} \right) \times A^{2k}_{\tau=2}(p_A^2) \times \tilde{\Pi}^{\mu_1\cdots\mu_{2k}} + \mathcal{O}(\tau > 2)\end{aligned}$$

how to justify this?

turn off QCD

Starting point is the hadronic current:

$$T^{A}_{\mu\nu} = \int \ d^4z \ e^{iq\cdot z}\langle A| \ \mathcal{T} J^{\dagger}_{had.\mu}(z) \ J_{had.\nu}(0)|A\rangle$$

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step #1: Wick's contraction theorem:

$$\begin{aligned} \mathcal{T}\left\{J^{\dagger}_{had.\mu}(z) \ J_{had.\nu}(0)\right\} &= \ :\left[\overline{\psi}(z)\gamma_\mu\psi(z)\right]\left[\overline{\psi}(0)\gamma_\nu\psi(0)\right]: \\ &+ \ (1 \text{ Wick contraction})_{\mu\nu} \ \times \ 2 \\ &+ \ \mathrm{Tr}\left[S_F(0-z)\gamma_\mu S_F(z-0)\gamma_\nu\right] \end{aligned}$$

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step #2: *expand! expand! expand! and trace!*

$$S_F(z) = \frac{-1}{4\pi^2}\frac{2\not{z}+im_q z^2}{(z^2-i\varepsilon)^2} + \text{less singular in } 1/z^2$$

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$$\psi(z) = \psi(z=0) + \left[\partial_\mu\psi(z)\right]_{z=0}\cdot z^\mu + \left[\partial_{\mu_2}\partial_{\mu_1}\psi(z)\right]_{z=0}\cdot z^{\mu_1}z^{\mu_2} + \ldots$$

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$$\Longrightarrow \ \mathrm{Tr}\left[S_F(-z)\gamma_\mu S_F(z)\gamma_\nu\right] \sim \frac{\mathbb{I}_4}{4\pi^4(z^2-i\varepsilon)^4}\left(4z^2 g_{\mu\nu} + 8z_\mu z_\nu + m_q^2 z^4 g_{\mu\nu}\right)$$

plug and chug:

$$T^{A}_{\mu\nu} = \int d^4z\ e^{iq\cdot z}\langle A|\ \mathcal{T} J^{\dagger}_{had.\mu}(z)\ J_{had.\nu}(0)|A\rangle$$
$$= \int d^4z\ e^{iq\cdot z}\langle A|\ \mathrm{Tr}\left[S_F(-z)\gamma_\mu S_F(z)\gamma_\nu\right]|A\rangle + \dots$$

we can explore this

$$
\begin{aligned}
T^A_{\mu\nu} &= \int d^4z \; e^{iq\cdot z} \\
\times & \quad \langle A| \, \frac{\mathbb{I}_4}{4\pi^4 (z^2 - i\varepsilon)^4} \times \left(4z^2 g_{\mu\nu} + 8 z_\mu z_\nu + m^2 z^4 g_{\mu\nu}\right) |A\rangle + \ldots
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after the Fourier integral

- for a scalar interaction $\hat{\mathcal{O}}^{\mathrm{S}} = \mathbb{I}_4$, $\langle A|\, \hat{\mathcal{O}}^{\mathrm{S}}|A\rangle \equiv \mathcal{A}^S$ is the hadronic ME

we can explore this

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- $z^2 g_{\mu\nu} \rightarrow Q^2 g_{\mu\nu}$

we can explore this

$$T^A_{\mu\nu} = \int d^4z\ e^{iq\cdot z}$$

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- for a scalar interaction $\hat{\mathcal{O}}^{\mathrm{S}} = \mathbb{I}_4$, $\langle A|\ \hat{\mathcal{O}}^{\mathrm{S}}|A\rangle \equiv \mathcal{A}^S$ is the hadronic ME
- $z^2 g_{\mu\nu} \to Q^2 g_{\mu\nu}$
- $z_\mu z_\nu \to q_\mu q_\nu$

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- $z^2 g_{\mu\nu} \to Q^2 g_{\mu\nu}$
- $z_\mu z_\nu \to q_\mu q_\nu$
- $\mathcal{O}\left(\# \times (\mathrm{coup})/\pi^4\right) \to C$ Wilson coefficients

we can explore this

$$T^A_{\mu\nu} \;=\; \int d^4z\; e^{iq\cdot z}$$

$$\times \qquad \langle A|\, \frac{\mathbb{I}_4}{4\pi^4(z^2 - i\varepsilon)^4} \times \left(4z^2 g_{\mu\nu} + 8z_\mu z_\nu + m^2 z^4 g_{\mu\nu}\right)|A\rangle + \ldots$$

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$$\lim_{z\to 0} T^A_{\mu\nu}(q,p_A) \overset{\rm OPE}{=} -2i \sum_{k=k_{\rm min}}^{\infty} \Big[-2g_{\mu\nu}q_{\mu_1}q_{\mu_2}C_1^{2k} + g_{\mu\mu_1}g_{\nu\mu_2}Q^2C_2^{2k} - i\epsilon_{\mu\nu\alpha\beta}g^{\alpha}_{\mu_1}q^{\beta}q_{\mu_2}C_3^{2k}$$
$$+4\frac{q_\mu q_\nu}{Q^2}q_{\mu_1}q_{\mu_2}C_4^{2k} + 2(g_{\mu\mu_1}q_\nu q_{\mu_2} \pm g_{\nu\mu_1}q_\mu q_{\mu_2})C_{5,6}^{2k} \Big]$$
$$\times \frac{2^{2k}}{(Q^2)^{2k}} \times \left(\prod_{m=3}^{2k} q_{\mu_m}\right) \times A^{2k}_{\tau=2}(p_A^2) \times \tilde{\Pi}^{\mu_1\cdots\mu_{2k}} + \mathcal{O}(\tau > 2)$$

assume that QCD has same OPE as QCD-less theory

After combinatorics and collecting like-terms, get things like this:

$$\Delta \tilde{T}_1^A = (-4i) \sum_{k=1}^{\infty} \left[C_1^{2k} A_{\tau=2}^{2k} \right] \sum_{j=0}^{k} \frac{(2k-j)!(2k)!}{(2k)!j!(2k-2j)!} \left(\frac{M_A^2}{Q^2} \right)^j x_A^{-(2k-2j)}$$
$$+ (-4i) \sum_{k=1}^{\infty} \left[C_2^{2k} A_{\tau=2}^{2k} \right] \sum_{j=1}^{k} \frac{(2k-j)!(2k-2)!}{(2k)!(j-1)!(2k-2j)!} \left(\frac{M_A^2}{Q^2} \right)^j x_A^{-(2k-2j)}$$
$$+ \mathcal{O}(\tau > 2)$$

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$$+\ \mathcal{O}(\tau > 2)$$

After taking the $(M_A^2/Q^2) \to 0$ **limit, recover something remarkable:**

$$\tilde{F}_i^{AN} \ = \ C_i^N A_{\tau=2}^N \ + \ \mathcal{O}(\tau > 2) \quad \text{for} \quad i = 1, 3-6,$$

$$\tilde{F}_2^{A(N-1)} \ = \ C_2^N A_{\tau=2}^N \ + \ \mathcal{O}(\tau > 2)$$

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After taking the $(M_A^2/Q^2) \to 0$ **limit, recover something remarkable:**

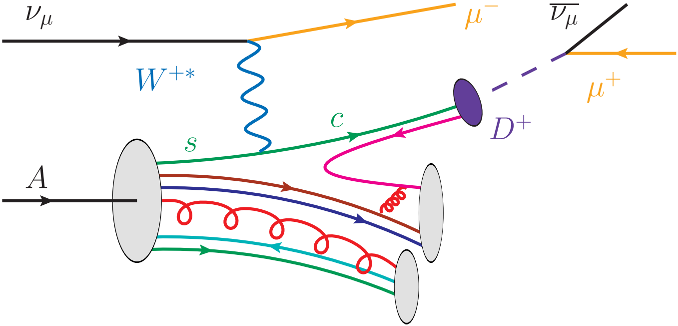
$$\tilde{F}_i^{AN} \ = \ Z(\mu) C_i^N(\mu) A_{\tau=2}^N \ + \ \mathcal{O}(\tau > 2) \quad \text{for} \quad i = 1, 3-6,$$

$$\tilde{F}_2^{A(N-1)} \ = \ Z(\mu) C_2^N(\mu) A_{\tau=2}^N \ + \ \mathcal{O}(\tau > 2)$$

str. fns. = (RGE) × (short-dist. phys.) × (long-dist. phys.)

Collinear Factorization Theorem for inclusive deep-inelastic scattering

Collins, Soper ('87); Collins ('11)

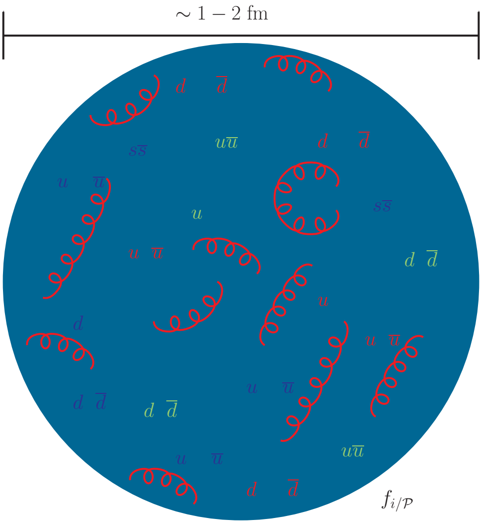


$$d\sigma(\nu\mathcal{A}\to \ell X) = \underbrace{\sum_{k,X_n}}_{inclusive} \underbrace{\Delta_{kk'}}_{\text{shower/RGE}} \otimes \underbrace{f_{k'}}_{\text{PDF}} \otimes \underbrace{d\hat{\sigma}_{\nu k'\to X_n}}_{\text{hard scattering}} + \underbrace{\mathcal{O}\left(\frac{\Lambda^k_{\rm NP}}{Q^{k+2}}\right)}_{\text{fun bit}}$$

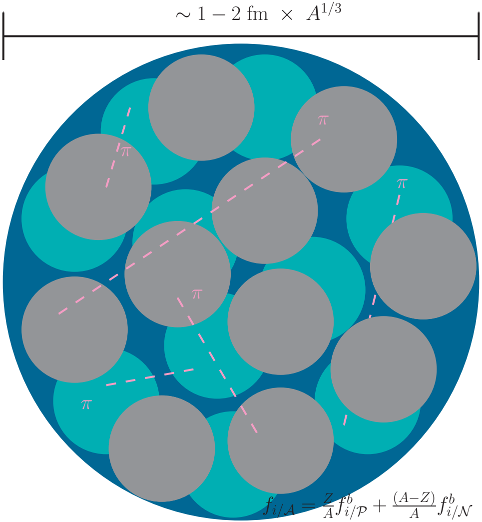
pro tip: str. fn. = $\sum_k$ **(coup)**$_k$ $\times$ **PDF**$_k$

Pt3. intuition

for proton, $F_i^N = C_i^N \times A^N$ + power corrections
$\Longrightarrow$ "PDFs = QCD $\times$ hadronic matrix element"



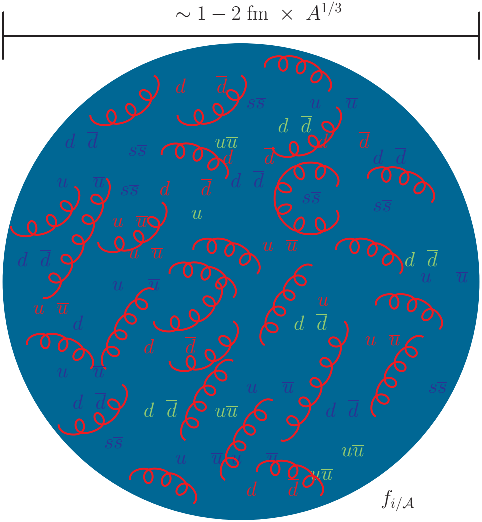
for A, it is common to parameterize PDF as combination of "bound" $\mathcal{P}$ and $\mathcal{N}$ PDFs



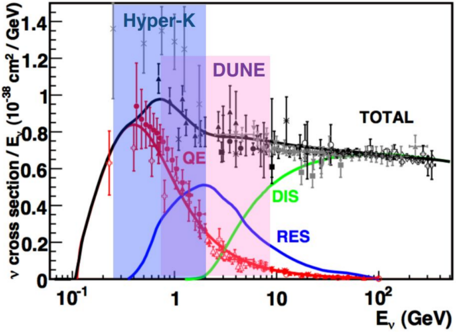
$$f_{i/A} = \frac{Z}{A} f^{b}_{i/\mathcal{P}} + \frac{(A-Z)}{A} f^{b}_{i/\mathcal{N}}$$

for A, $F_i^{AN} = C_i^N \times A^N$ + power corrections

$\Longrightarrow$ "PDFs = QCD $\times$ had. ME" ("nucleon" picture not necessary)



the dark secret of ν scattering experiments

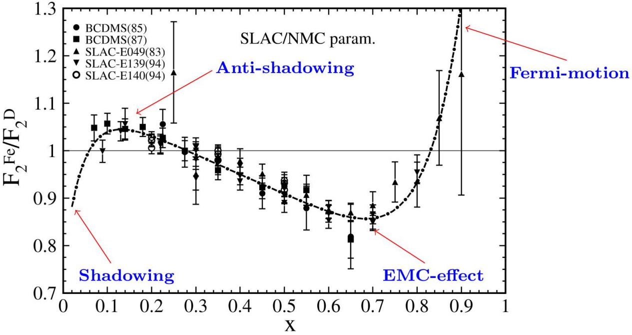


in practice, νDIS needs nuclear targets

1. ν only interact through weak force: targets must be bigger ($\mathcal{O}(10)$tons) and denser (Ar,Fe,Pb) $\implies$ more nuclear

2. fact of life: nuclear dynamics impact hadronic structure

Plotted: $\frac{F_2^{\rm iron}}{F_2^{\rm deuteron}}$ for ℓ-DIS



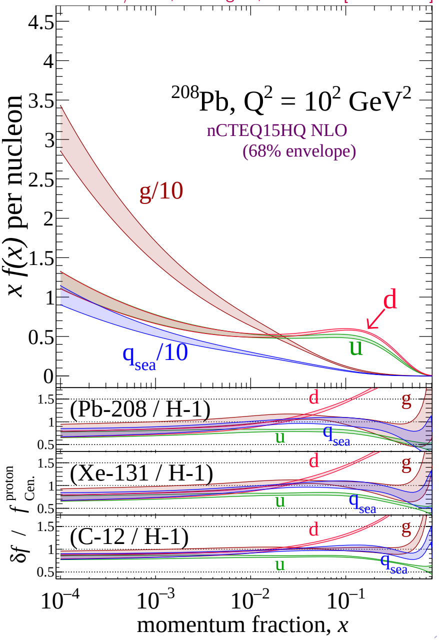
For non-expert, QED (γ) contribution to F_2: $F_2(\xi) \approx \sum_{i\in\{q,\overline{q},g\}} Q_i^2\ \xi\ f_i^A(\xi)$

Schienbein, et al [0710.4897]

Plotted: PDF of avg. nucleon in ^{208}Pb vs (avg) energy fraction carried by parton

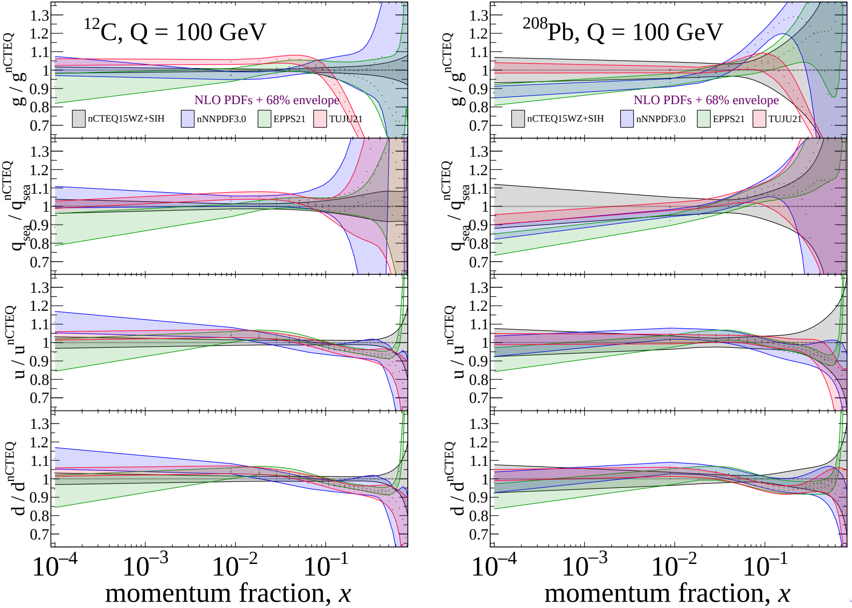
- huge g content (always easy to make more g)
- $q_{\rm sea}$, d, and u content converge for $x \lesssim 10^{-2}$ (dominated by $g^* \to q\overline{q}$ splitting)
- densities smaller (larger) than proton for $x \lesssim 10^{-2} (x \gtrsim 10^{-2})$
- qualitatively different from proton
- smaller $\mathcal{A}$ are more proton-like

w/ Fuks, Marougkas, Sztandera [2405.19399]



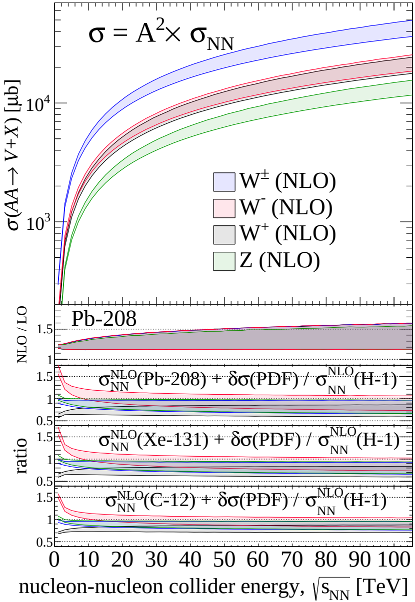
Plotted: Ratios of nuclear PDFs vs (avg) energy fraction carried by parton

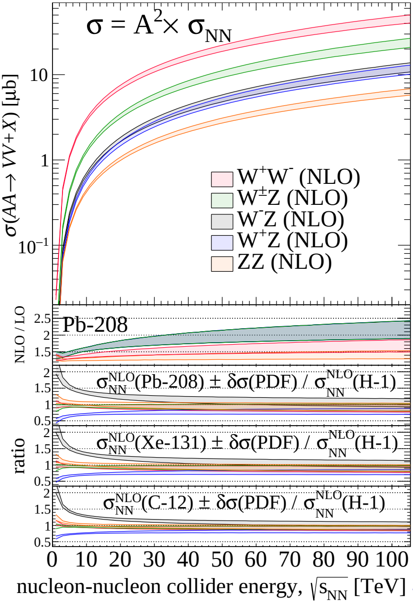
w/ Fuks, Marougkas†, Sztandera† [2405.19399]



Plotted: scattering rate vs nucleon-nucleon collider energy

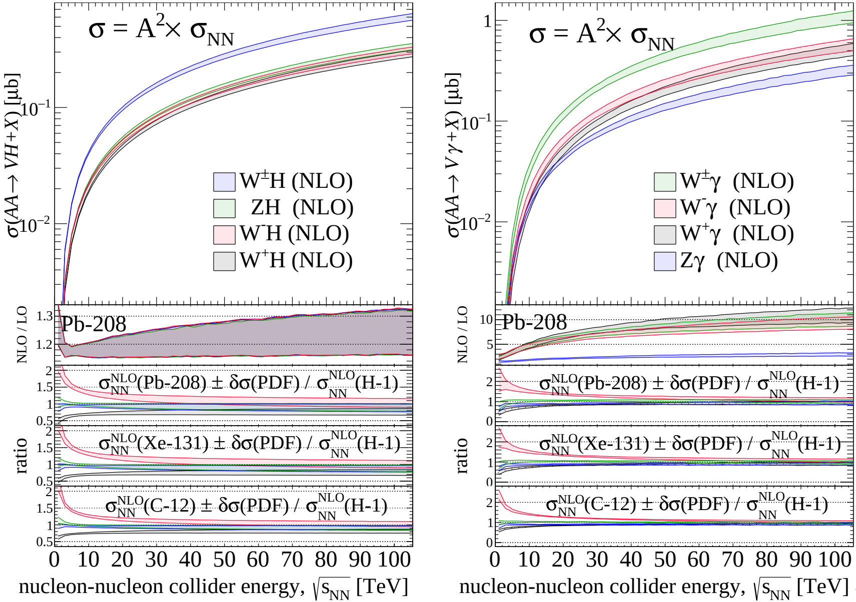
w/ Fuks, Marougkas†, Sztandera† [2405.19399]





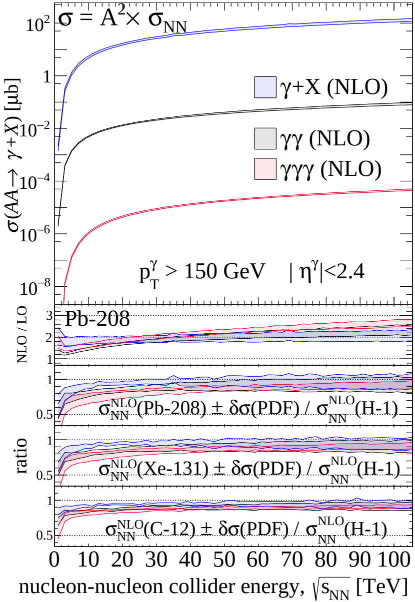
Plotted: scattering rate vs nucleon-nucleon collider energy

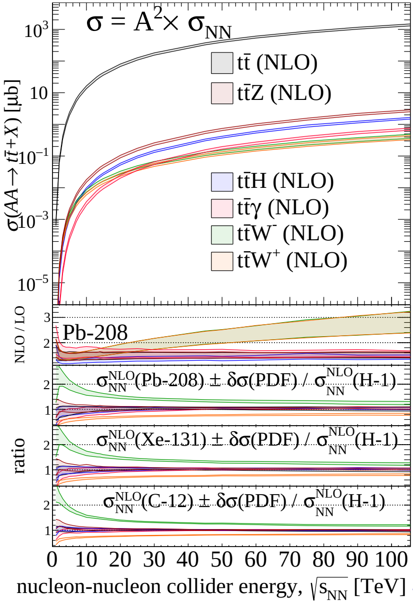
w/ Fuks, Marougkas†, Sztandera† [2405.19399]



Plotted: scattering rate vs nucleon-nucleon collider energy

w/ Fuks, Marougkas†, Sztandera† [2405.19399]

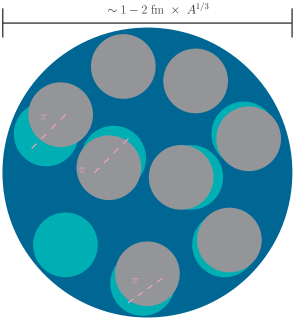




hypothesis-testing models of nuclear structure

hypothesis: **content of $\mathcal{A}$** fixed by free $\mathcal{N} + \mathcal{P}$ and bound $(\mathcal{N}\mathcal{P})$ pairs

$$f_i^{\mathcal{A}}(x,Q) = \frac{Z}{A}\Big[(1 - C_{\mathcal{P}}^{A}) \times f_i^{\mathcal{P}}(x,Q) + C_{\mathcal{P}}^{\mathcal{A}} \times f_i^{\mathrm{SRC}\,\mathcal{P}}(x,Q)\Big] + \frac{(A-Z)}{A}\Big[(1 - C_{\mathcal{N}}^{A}) \times f_i^{\mathcal{N}}(x,Q) + C_{\mathcal{N}}^{A} \times f_i^{\mathrm{SRC}\,\mathcal{N}}(x,Q)\Big]$$



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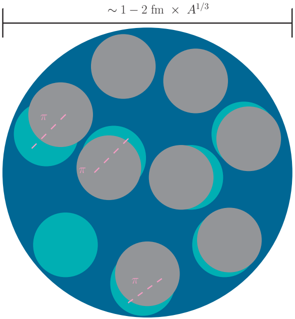
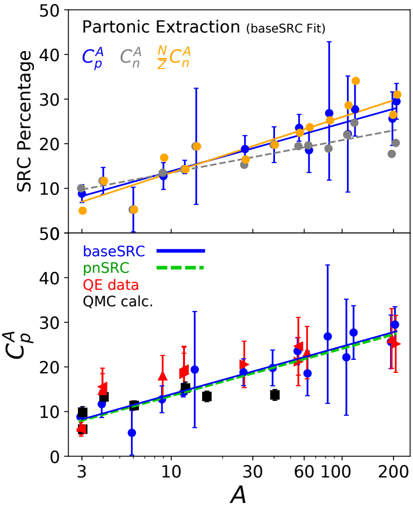




take away: nucl. structure encoded in q/g densities

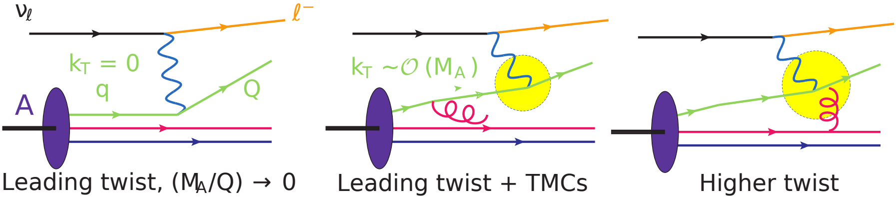
w/ Denniston, Hen, Olness, et al (PRL'24) [2312.16293]

Pt4. going beyond leading twist

$$d\sigma(\nu\mathcal{A}\to\ell X) \;=\; \underbrace{\sum_{k,X_n}}_{\textit{inclusive}} \; \underbrace{\Delta_{kk'}}_{\text{shower/RGE}} \;\otimes\; \underbrace{f_{k'}}_{\text{PDF}} \;\otimes\; \underbrace{d\hat{\sigma}_{\nu k'\to X_n}}_{\text{hard scattering}} \;+\; \underbrace{\mathcal{O}\left(\frac{\Lambda^k_{\rm NP}}{Q^{k+2}}\right)}_{\text{show time!}}$$

$\mathcal{O}\left(\frac{\Lambda_{NP}^k}{Q^{k+2}}\right)$ corrections have several origins (kinematical and dynamical)

Georgi, Politzer ('76,'76); Ellis, Furmanski, Petronzio ('82,'82); lots more



Importance of “subleading” (aka power) corrections

$\mathcal{O}\left(\frac{\Lambda_{\rm NP}^k}{Q^{k+2}}\right)$ corrections increasingly important at **small** Q^2**, large** x!

“target mass corrections” (TM) →

Georgi, Politzer (’76,’76)

“renormalon” corrections” (HT) →

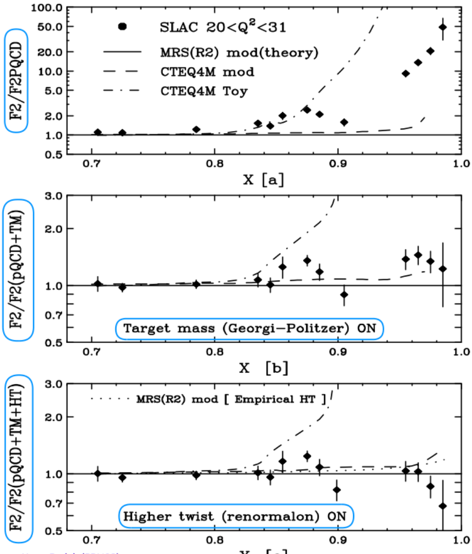
Dasgupta, Webber (’91)

in extreme kinematics, necessary:

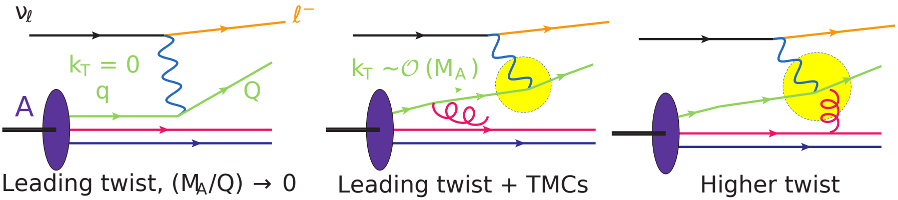
– describe DIS data

– extend validity of Fact. Thm.

– extract PDFs from structure fns.



Yang, Bodek (PRL’98)



kinematical corrections, i.e., **target mass corrections (TMCs)**, can be incorporated in structure functions, $F_i(x, Q^2)$

Georgi, Politzer ('76,'76); Ellis, Furmanski, Petronzio ('82,'82); lots more; Kretzer, Reno ('02,'03); Schienbein, et al [0709.1775]

In practice, replace $F_i^{A\ (\text{No TMC})} \to F_i^{A\ (\text{TMC})}$ in cross sections:

$$\frac{d^2\sigma^{\rm NC}}{dx\,dy} = x(s - M^2)\,\frac{d^2\sigma^{\rm NC}}{dxdQ^2} = \frac{4\pi\alpha^2}{xyQ^2}\left[\frac{Y_+}{2}\,\sigma^{NC}_{\rm Red.}\right],$$

$$\sigma^{NC}_{\rm Red.} = \left(1 + \frac{2y^2\varepsilon^2}{Y_+}\right)F_2^{\rm NC} \mp \frac{Y_-}{Y_+}\,xF_3^{\rm NC} - \frac{y^2}{Y_+}F_L^{\rm NC},$$

back to structure functions![6]

[6] w/ Muzakka, Olness, Schienbein, et al [(JPPNP'24)]

applying the OPE (massive target)

After combinatorics and collecting like-terms, get things like this:

$$\underbrace{\Delta\tilde{T}_1^A}_{\sim\sum F_1^{AN}} = (-4i)\ \sum_{k=1}^{\infty} \underbrace{\left[C_1^{2k} A_{\tau=2}^{2k}\right]}_{F_1^{A(2k)}|_{\text{no TMC}}} \sum_{j=0}^{k} \frac{(2k-j)!(2k)!}{(2k)!j!(2k-2j)!} \left(\frac{M_A^2}{Q^2}\right)^j x_A^{-(2k-2j)}$$

$$+ (-4i)\ \sum_{k=1}^{\infty} \underbrace{\left[C_2^{2k} A_{\tau=2}^{2k}\right]}_{F_2^{A(2k-1)}|_{\text{no TMC}}} \sum_{j=1}^{k} \frac{(2k-j)!(2k-2)!}{(2k)!(j-1)!(2k-2j)!} \left(\frac{M_A^2}{Q^2}\right)^j x_A^{-(2k-2j)}$$

$$+\ \mathcal{O}(\tau > 2)$$

In massive limit, "massless" structure functions mix

after applying several integral identities

Nuclear structure functions with TMCs

Generically, str. fn. with TMCs have the form:

$$F_j^{A,\mathrm{TMC}}\underbrace{(x_A,}_{\textbf{Bjorken}} Q^2) = \sum_{i=1}^{6} A_j^i \underbrace{F_i^{A,(0)}(\xi_A, Q^2)}_{\text{no TMCs}} + B_j^i h_i^A \underbrace{(\xi_A,}_{\xi_A = 2x_A/(1+\sqrt{1+4x_A^2 M_A^2/Q^2})} Q^2) + C_j g_2^A(\xi_A, Q^2)$$

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Lots to unpack!

- for $i \neq j$, structure function mixing!
- $A_i^i \sim \mathcal{O}(1)$; all other coeff. $\sim \mathcal{O}(x_A M_A^2/Q^2)$
- A_j^i, B_j^i, C_j are coefficients $\mathcal{O}\left(x_A, (x_A^2 M_A^2/Q^2)\right)$
- h_i and g_2 are convolutions over $F_i(y)|_{\mathrm{no-TMC}}$

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- h_i and g_2 are convolutions over $F_i(y)|_{\mathrm{no-TMC}}$

Example:

$$F_1^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A}{\xi_A r_A}\right) F_1^{A,(0)}(\xi_A) + \left(\frac{M_A^2 x_A^2}{Q^2 r_A^2}\right) h_2^A(\xi_A) + \left(\frac{2M_A^4 x_A^3}{Q^4 r_A^3}\right) g_2^A(\xi_A)$$

Nuclear structure functions with TMCs

Generically, str. fn. with TMCs have the form (at leading power):

$$F_j^{A,\mathrm{TMC}}(x_A, Q^2) \;=\; \sum_{i=1}^{6} A_j^i \underbrace{F_i^{A,(0)}(\xi_A, Q^2)}_{\text{no TMCs}} + B_j^i h_i^A(\xi_A, Q^2) + C_j g_2^A(\xi_A, Q^2)$$

$$\tilde{F}_1^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A}{\xi_A r_A}\right)\tilde{F}_1^{A,(0)}(\xi_A) + \left(\frac{M_A^2 x_A^2}{Q^2 r_A^2}\right)\tilde{h}_2^A(\xi_A) + \left(\frac{2M_A^4 x_A^3}{Q^4 r_A^3}\right)\tilde{g}_2^A(\xi_A)\,,$$

$$\tilde{F}_2^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A^2}{\xi_A^2 r_A^3}\right)\tilde{F}_2^{A,(0)}(\xi_A) + \left(\frac{6M_A^2 x_A^3}{Q^2 r_A^4}\right)\tilde{h}_2^A(\xi_A) + \left(\frac{12M_A^4 x_A^4}{Q^4 r_A^5}\right)\tilde{g}_2^A(\xi_A)\,,$$

$$\tilde{F}_3^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A}{\xi_A r_A^2}\right)\tilde{F}_3^{A,(0)}(\xi_A) + \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^3}\right)\tilde{h}_3^A(\xi_A)\,,$$

$$\begin{aligned}\tilde{F}_4^{A,\mathrm{TMC}}(x_A) = &\left(\frac{x_A}{\xi_A r_A}\right)\tilde{F}_4^{A,(0)}(\xi_A) - \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^2}\right)\tilde{F}_5^{A,(0)}(\xi_A) + \left(\frac{M_A^4 x_A^3}{Q^4 r_A^3}\right)\tilde{F}_2^{A,(0)}(\xi_A)\\ &+ \left(\frac{M_A^2 x_A^2}{Q^2 r_A^3}\right)\tilde{h}_5^A(\xi_A) - \left(\frac{2M_A^4 x_A^4}{Q^4 r_A^4}\right)\left(2-\xi_A^2 M_A^2/Q^2\right)\tilde{h}_2^A(\xi_A)\\ &+ \left(\frac{2M_A^4 x_A^3}{Q^4 r_A^5}\right)\left(1-2x_A^2 M_A^2/Q^2\right)\tilde{g}_2^A(\xi_A)\,,\end{aligned}$$

$$\begin{aligned}\tilde{F}_5^{A,\mathrm{TMC}}(x_A) = &\left(\frac{x_A}{\xi_A r_A^2}\right)\tilde{F}_5^{A,(0)}(\xi_A) - \left(\frac{M_A^2 x_A^2}{Q^2 r_A^3 \xi_A}\right)\tilde{F}_2^{A,(0)}(\xi_A)\\ &+ \left(\frac{M_A^2 x_A^2}{Q^2 r_A^3}\right)\tilde{h}_5^A(\xi_A) - \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^4}\right)\left(1-x_A\xi_A M_A^2/Q^2\right)\tilde{h}_2^A(\xi_A)\\ &+ \left(\frac{6M_A^4 x_A^3}{Q^4 r_A^5}\right)\tilde{g}_2^A(\xi_A)\,,\end{aligned}$$

$$\tilde{F}_6^{A,\mathrm{TMC}}(x_A) = \left(\frac{x_A}{\xi_A r_A^2}\right)\tilde{F}_6^{A,(0)}(\xi_A) + \left(\frac{2M_A^2 x_A^2}{Q^2 r_A^3}\right)\tilde{h}_6(\xi_A)\,.$$

some numbers

running the numbers

we use NLO PDFs (`nCTEQ15`) to build str. fns. At LO, these are

$$F_1^{\nu A} = (d + s + \bar{u} + \bar{c}), \qquad F_1^{\bar{\nu} A} = (u + c + \bar{d} + \bar{s})$$

$$F_2^{\nu A} = 2x\,(d + s + \bar{u} + \bar{c}), \qquad F_2^{\bar{\nu} A} = 2x\left(u + c + \bar{d} + \bar{s}\right)$$

$$F_3^{\nu A} = +2\,(d + s - \bar{u} - \bar{c}), \qquad F_3^{\bar{\nu} A} = -2\left(u + c - \bar{d} - \bar{s}\right)$$

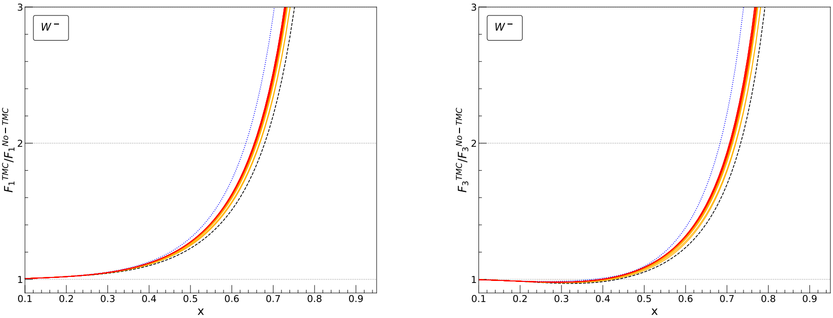
$$F_2^{l^\pm A} = x\frac{1}{9}\left[4(u+\bar{u}) + (d+\bar{d}) + 4(c+\bar{c}) + (s+\bar{s})\right]$$

for many targets

Symbol	A	Z	Symbol	A	Z	Symbol	A	Z	Symbol	A	Z
H	1	1	Be	9	4	Ca	40	20	Xe	131	54
D	2	1	C	12	6	Fe	56	26	W	184	74
^{3}He	3	2	N	14	7	Cu$_{\rm iso}$	64	32	Au	197	79
He	4	2	Ne	20	10	Kr$_{\rm iso}$	84	42	Au$_{\rm iso}$	197	98.5
Li	6	3	Al	27	13	Ag$_{\rm iso}$	108	54	Pb$_{\rm iso}$	207	103.5
Li	7	3	Ar	40	18	Sn$_{\rm iso}$	119	59.5	Pb	208	82

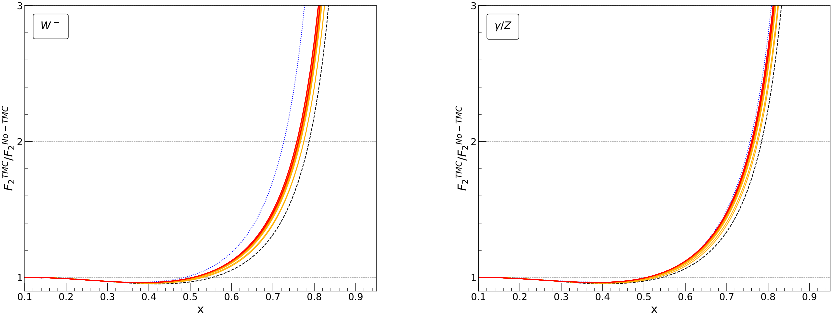
ratio of F_i^{TMC} / $F_i^{\mathrm{no\ TMC}}$

Plotted: ratio for (L) $F_1^{W^-}$ and (R) $F_3^{W^-}$ at $Q = 1.5$ GeV



Can you spot the ^{1}H and ^{2}D curves?

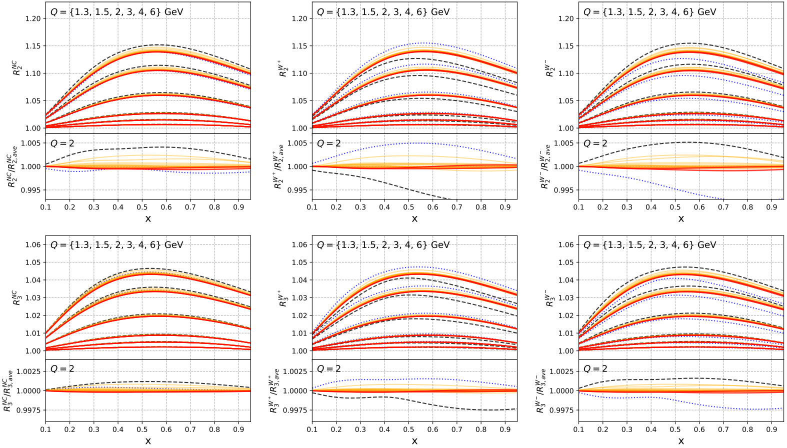
Plotted: ratio for (L) $F_2^{W^-}$ and (R) $F_2^{\gamma/Z}$ at $Q = 1.5$ GeV



Can you spot the ^{1}H and ^{2}D curves?

ratio of F_i^{TMC} / $F_i^{\mathrm{leading\ TMC}}$

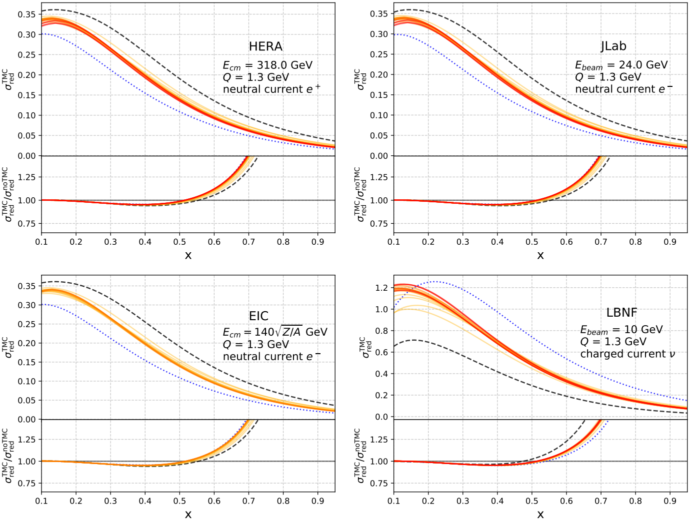
Plotted: ratio for (L) $F_i^{Z/\gamma}$, (C) $F_i^{W^+}$, (R) $F_i^{W^-}$ for $i = 2$ (upper) and $i = 3$ (lower)



remarkable uniformity! **(good enough to fit! ☺)**

reduced cross sections

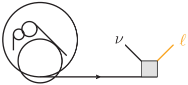
Plotted: (upper) reduced cross sections with nTMCs; (lower) ratio to w/o



summary and conclusion

We are entering an era of precision DIS that strongly complements the ongoing hadron program

- **ongoing efforts** to improve theory predictions
- **theory improvements** applicable to programs at CERN, US labs
- **lots not covered,** so see new (pedagogical) review JPPNP ('24) [2301.07715]



Thank you!