Introduction to QCD Jets

“Selected Topics” Warsaw, Poland

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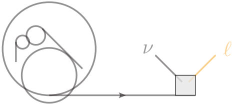
Institute of Nuclear Physics – Polish Academy of Science (IFJ PAN)

17 September 2024



most important: these lectures are low-key; questions are great

I am literally here to tell you what I know

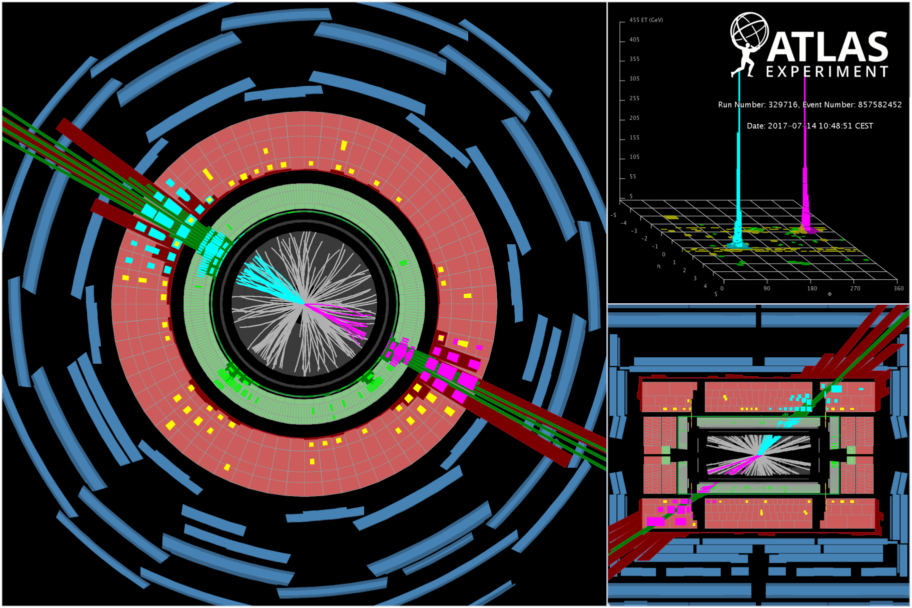
Lecture Plan (Tuesday)

Lecture II:

- Pt1: the **parton model** as a phenomenological model for QCD
- Pt2: **soft factorization** in massless gauge theories
- Pt3: **collinear factorization** in massless gauge theories
- Pt4: **parton showers**
- Pt5: what are **jets**?

Lunch at 12:30ish

the big picture

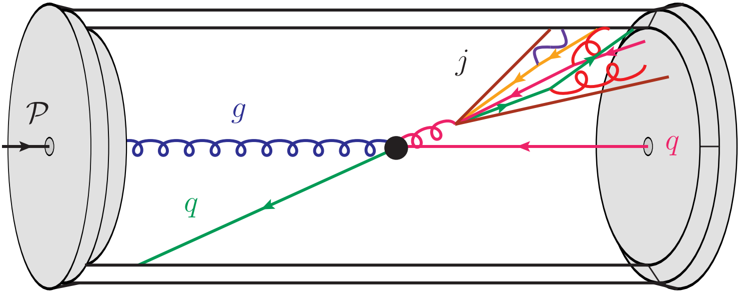


Display of a dijet event (Run=329716, Event=8575822452) with m_{jj}=9.5 TeV, produced in pp collisions at $\sqrt{s}$=13 TeV data collected in 2017. The two jets with highest p_T have p_T=3.0 and 2.9 TeV, one is at η=-1.2 (magenta) and the other at η=0.9 (cyan). The view of the event in the plane transverse to the beam direction is shown on the left side. The top-right figure represents the calorimeter clusters transverse energies in the η-φ plane. The bottom-right figure present the event in the longitudinal view (Z-Y plane).

Big Picture of Today's Lectures

Today is about introducing **jets** at **hadron** colliders

- what **are** jets?
- how do jets **form**?
- how are jets **defined**?



part 1: the parton model

subtle question: how to get $\mu^+\mu^-$ in proton-proton collisions?

(protons do not carry lepton number)

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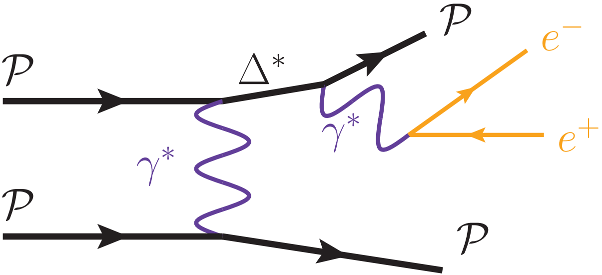

The parton model is the idea that the hadrons, e.g., the proton, are **bound states** of **wee bits of "valence"** in a **"sea" of more wee bits**

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- **valence quanta** carry net quantum charges of bound state
- **sea quanta** carry various quantum charges and opposite charges
- **prediction:** "parton-antiparton annihilation" into **massive leptons**

(Drell and Yan [PRL '70])

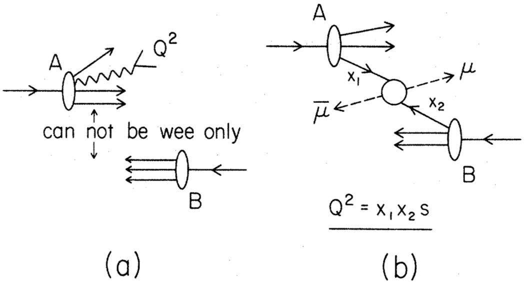

FIG. 1. (a) Production of a massive pair Q^2 from one of the hadrons in a high-energy collision. In this case it is kinematically impossible to exchange "wee" partons only. (b) Production of a massive pair by parton-antiparton annihilation.

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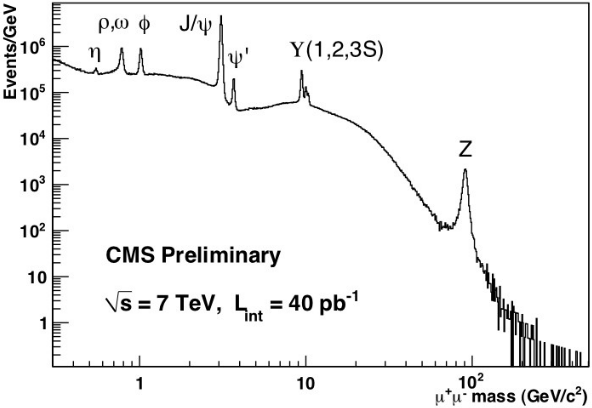

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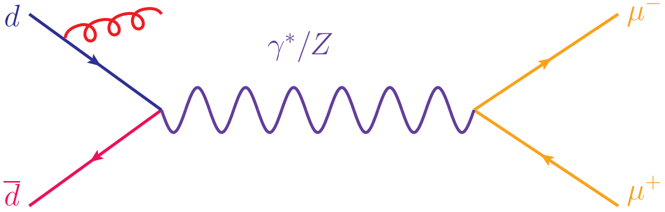

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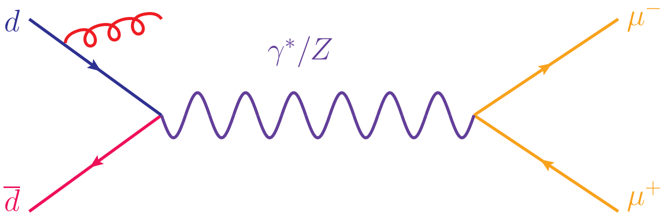
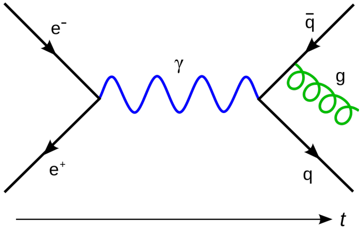


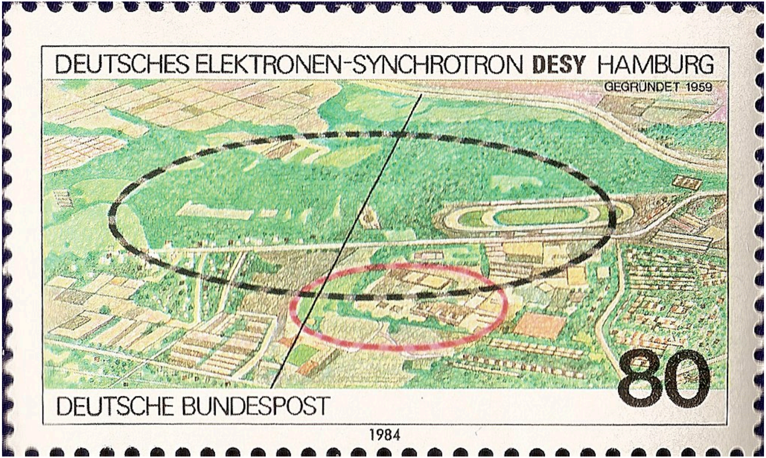
- **prediction:** "Drell-Yan" in reverse $e^- e^+ \to q\overline{q}g$ and measure spin!



Running the Experiment: PETRA at DESY

PETRA: an e^+e^- collider with $\sqrt{s} = 13 - 32$ GeV

- Experiments: **TASSO** (below!), JADE, MARK J, PLUTO
- Collider signature: $e^+e^- \to$ 3 prongs with $\frac{1}{2} - \frac{1}{2} - 1$ angular dist.



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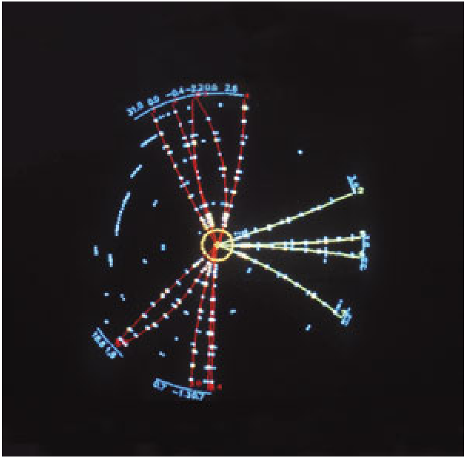

Evidence for direct production of gluons! ('79)

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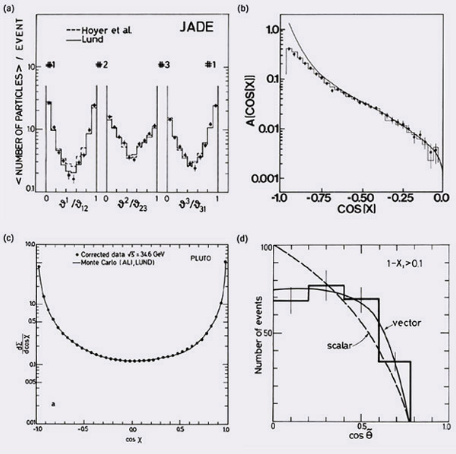
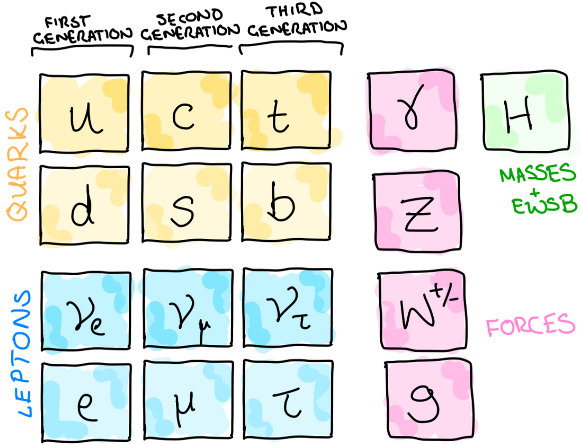
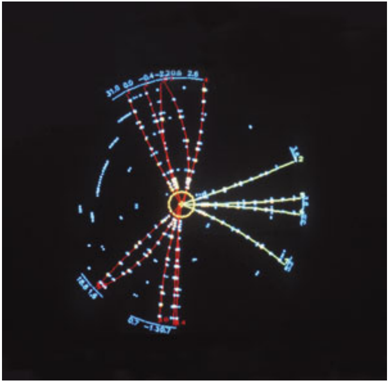


(CERN Courier [Nov '04])

Discovery of a spin-1 gluon established **QCD** as the **standard description** (model!) of strong nuclear interactions



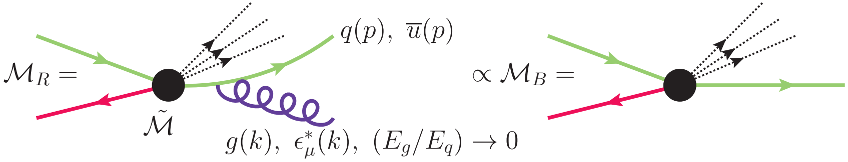
how does the parton model lead to this?



Pt2: soft factorization in massless gauge theories

Soft Factorization in Massless Gauge Theories

Factorization in gauge theories is when a radiation amplitude $\mathcal{M}_R$ in certain kinematic limits simplifies to the product of (a) a no-radiation amplitude $\mathcal{M}_B$ (Born!) term and (b) a **universal** (process-independent!) term:



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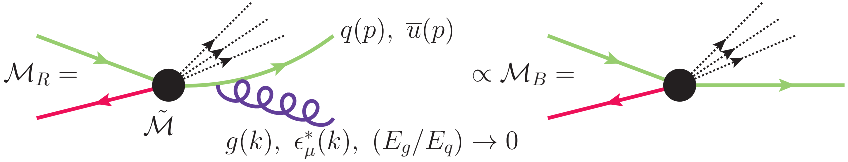

Intuition:

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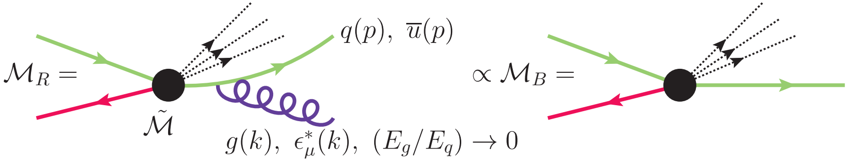

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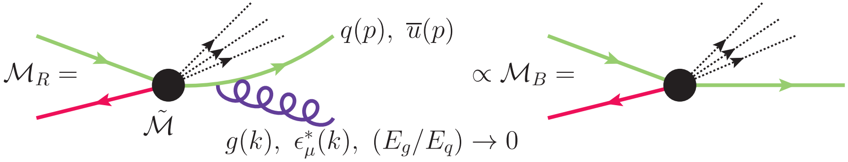

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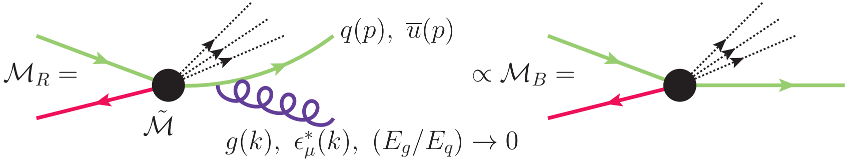
- **Separation of scales:**

$$\textbf{Prob}(\text{hard+rad})\ \text{``="}\ \textbf{Prob}(\text{rad}) \otimes \textbf{Prob}(\text{hard}) + \mathcal{O}(\mu/Q)$$

Soft Factorization in Massless Gauge Theories



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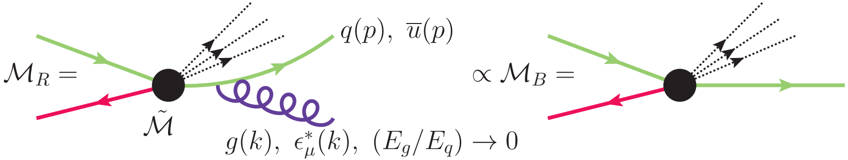

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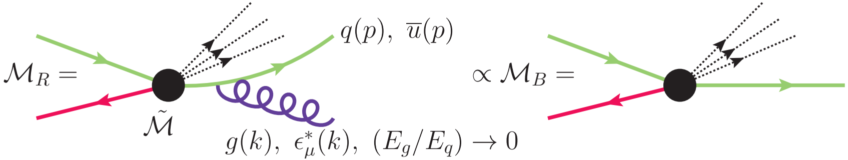

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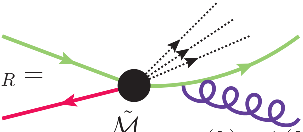
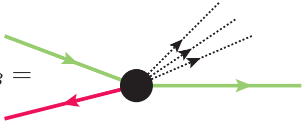
Soft Factorization in Massless Gauge Theories

$\mathcal{M}_R =$



$\propto \mathcal{M}_B =$



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After anti-commuting and applying Dirac equation $(\overline{u}\ \not{p}_u = 0)$:

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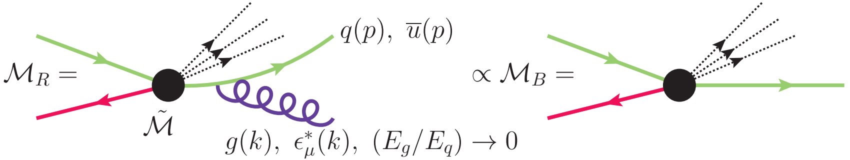

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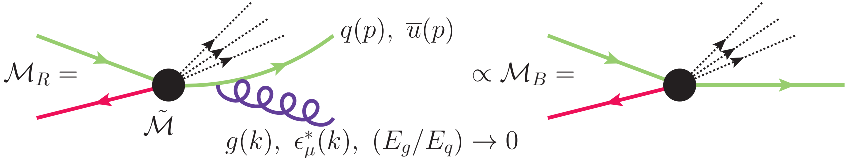

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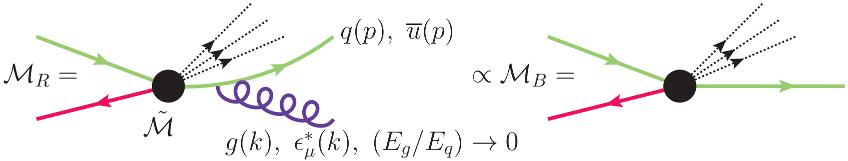
Soft Factorization in Massless Gauge Theories



Generically, for radiation $E_g \ll Q$

$$\left|\mathcal{M}_{\mathcal{R}}\right|^2\Big|_{E_g \ll E_q} \approx g_s^2 \sum_{\text{i,j}\in\{\text{external}\}} \text{(color factor)} \times \frac{(p_i \cdot p_j)}{(p_i \cdot k)(p_j \cdot k)} \times |\mathcal{M}_B|^2$$

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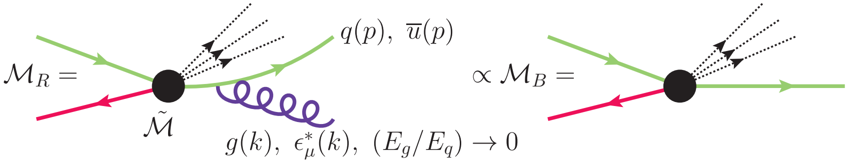

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A remarkably complicated situation is remarkably simple to write

simplifies further for QED

another perspective on factorization in massless gauge theories:

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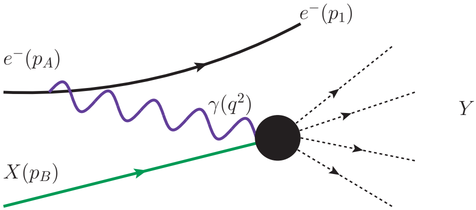
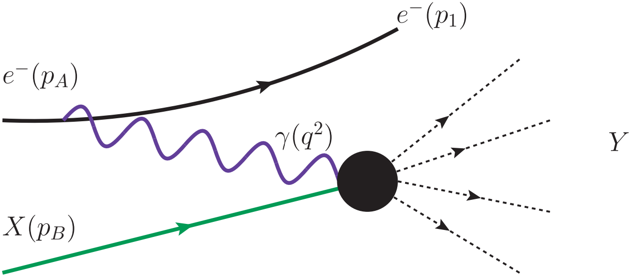
the Weizsäcker-Williams approximation

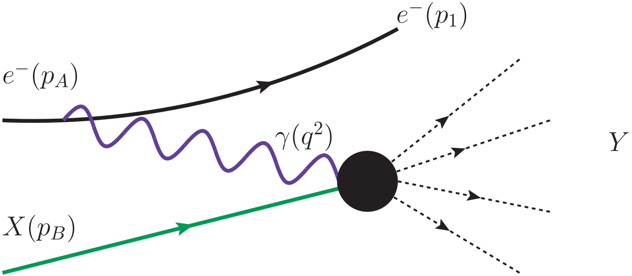


Pt3: collinear factorization in massless gauge theories



Consider the scattering process $eX \to e'Y$. Generically, $\mathcal{M}$ is

$$\mathcal{M} \quad = \quad \left[\overline{u}(p_1)(-ieq_e)\gamma^\mu u(p_A)\right] \frac{(-i)\left(g_{\mu\nu} - (\xi - 1)q_\mu q_\nu/q^2\right)}{q^2} \mathcal{M}^\nu(\gamma^* X \to Y)$$



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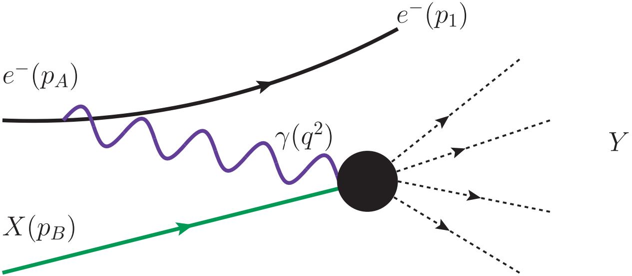
idea: when $\theta_e \ll 1$, γ^* goes on-shell and becomes an asymptotic state



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$$\Longrightarrow \ \mathcal{M}(eX \to eY) \ = \ \mathcal{M}(e \to e\gamma) \ \times \ \mathcal{M}(\gamma X \to eY) \ + \ \mathcal{O}(\theta_e)$$

a brief digression on polarization

Decomposing Propagators

Completeness relationships between **propagators** & **polarization vectors** in gauge theories are subtle. Consider **QED** in Feynman gauge

$\implies \xi = 1$ so $(1-\xi)q_\mu q_\nu/q^2 \to 0$:

$$-g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & +1 & & \\ & & +1 & \\ & & & +1 \end{pmatrix} = \sum_{\lambda=\pm,0,S} \varepsilon_\mu(q,\lambda)\varepsilon^*_\nu(q,\lambda)$$

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For $q = (q^0, 0, 0, q^3)$ and **transverse** pols $\varepsilon_\mu(\lambda = \pm) = (0, \mp 1, -i, 0)/\sqrt{2}$

$$\sum_{\lambda=\pm} \varepsilon_\mu(q,\lambda)\varepsilon^*_\nu(q,\lambda) = \begin{pmatrix} 0 & & & \\ & +1 & 0 & \\ & 0 & +1 & \\ & & & 0 \end{pmatrix}$$

Decomposing Propagators

For $q = (q^0, 0, 0, q^3)$ and **longitudinal** $\varepsilon_\mu(\lambda = 0) = (q^3, 0, 0, q^0)/\sqrt{q^2}$

$$\sum_{\lambda=0} \varepsilon_\mu(q,\lambda)\varepsilon_\nu(q,\lambda) = \frac{q^2}{q^2}\begin{pmatrix} -1 & & & \\ & 0 & & \\ & & 0 & \\ & & & +1 \end{pmatrix} + \frac{q_\mu q_\nu}{q^2}$$

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For "**auxiliary**" (A) or "**scalar**" (S) polarization $\varepsilon_\mu(\lambda = S) = q_\mu/\sqrt{-q^2}$

$$\sum_{\lambda=S} \varepsilon_\mu(q,\lambda)\varepsilon_\nu(q,\lambda) \;=\; -\frac{q_\mu q_\nu}{q^2}$$

Decomposing Propagators

For $q = (q^0, 0, 0, q^3)$ and **longitudinal** $\varepsilon_\mu(\lambda = 0) = (q^3, 0, 0, q^0)/\sqrt{q^2}$

$$\sum_{\lambda=0} \varepsilon_\mu(q,\lambda)\varepsilon_\nu(q,\lambda) = \frac{q^2}{q^2}\begin{pmatrix} -1 & & & \\ & 0 & & \\ & & 0 & \\ & & & +1 \end{pmatrix} + \frac{q_\mu q_\nu}{q^2}$$

For "**auxiliary**" (A) or "**scalar**" (S) polarization $\varepsilon_\mu(\lambda = S) = q_\mu/\sqrt{-q^2}$

$$\sum_{\lambda=S} \varepsilon_\mu(q,\lambda)\varepsilon_\nu(q,\lambda) = -\frac{q_\mu q_\nu}{q^2}$$

Precise form for $\lambda = 0, S$ depends on several factors:

- broken (massive) or unbroken (massless) gauge symmetry
- gauge (Feynman vs Landau vs Unitary vs Axial)
- gauge fixing ($\xi = 1$ or $n^2 = -1$)

Decomposing Propagators

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Example: for W/Z in Unitary gauge, $\varepsilon_\mu^{W/Z}(\lambda = S) = q_\mu\sqrt{\frac{1}{M_V^2} - \frac{1}{q^2}}$

$$\sum_{\lambda=S} \varepsilon_\mu(q,\lambda)\varepsilon_\nu(q,\lambda) = -\frac{q_\mu q_\nu}{q^2} + \frac{q_\mu q_\nu}{M_V^2}$$

Decomposing Propagators

For $q = (q^0, 0, 0, q^3)$ and **longitudinal** $\varepsilon_\mu(\lambda = 0) = (q^3, 0, 0, q^0)/\sqrt{q^2}$

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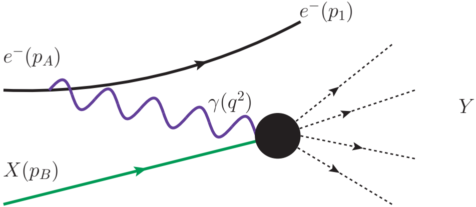
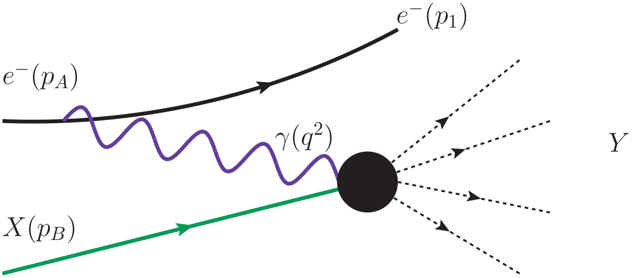
Bonus, longitudinal polarization vectors can be written as Dawson ('85)

$$\varepsilon_\mu(\lambda = 0) \ = \ \frac{q_\mu}{\sqrt{q^2}} + \mathcal{O}\left(\frac{\sqrt{q^2}}{q^0}\right)$$ importance of this soon!

the Weizsäcker-Williams approximation





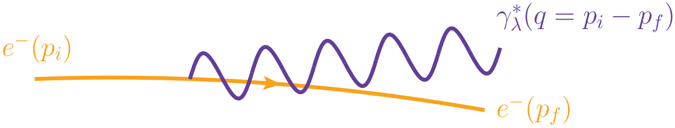
Consider the scattering process $eX \to e'Y$. Generically, $\mathcal{M}$ is

$$\begin{aligned} \mathcal{M} &= [\overline{u}(p_1)(-ieq_e)\gamma^\mu u(p_A)] \frac{(-i)\left(g_{\mu\nu} - (\xi-1)q_\mu q_\nu/q^2\right)}{q^2} \mathcal{M}^\nu(\gamma^* X \to Y) \\ &= \frac{(-ieq_e)}{q^2}[\overline{u}(p_1)\gamma^\mu u(p_A)] \left(\sum_{\lambda\lambda'} \varepsilon^*_\mu(q,\lambda)\varepsilon_\nu(q,\lambda')\right) \mathcal{M}^\nu(\gamma^* X \to Y). \end{aligned}$$

First focus on matrix element for $\ell \to \ell\gamma_\lambda$ splitting

Calculating $\mathcal{M}(\ell \to \ell V)$

Many ways to evaluate $\mathcal{M}$. Helicity amplitudes are <u>super</u> fast:



$$-i\mathcal{M}(\ell_{\lambda_i} \to \ell_{\lambda_f} V_{\lambda_V}) \ = \ J^\mu_{\ell_{\lambda_f}\ell_{\lambda_i}} \ \cdot \ \varepsilon^*_\mu(q, \lambda_V)$$

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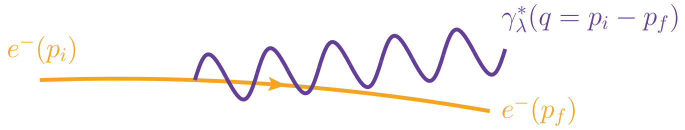

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lepton current (only two helicity configurations!)

$$J^\mu_{\ell_L \ell_L} = (-ieQ_e) \left[\underbrace{\overline{u}_L(p_f)}_{LH\ \ helicty} \gamma^\mu \underbrace{u_L(p_i)}_{LH\ \ helicty} \right]$$

$$J^\mu_{\ell_R \ell_R} = (-ieQ_e) \left[\underbrace{\overline{u}_R(p_f)}_{RH\ \ helicty} \gamma^\mu \underbrace{u_R(p_i)}_{RH\ \ helicty} \right]$$

Calculating $\mathcal{M}(\ell \to \ell V)$

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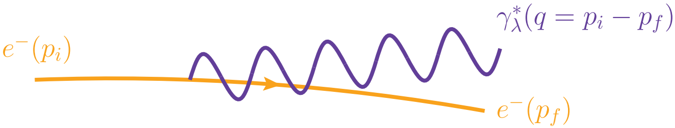

$$-i\mathcal{M}(\ell_{\lambda_i} \to \ell_{\lambda_f} V_{\lambda_V}) \;=\; J^{\mu}_{\ell_{\lambda_f}\ell_{\lambda_i}} \cdot \varepsilon^*_\mu(q, \lambda_V)$$

outgoing photon (four helicity polarizations!)

- $\lambda = \pm \implies \varepsilon_\mu = (0, \pm\cos\theta_\gamma, -i, \pm\sin\theta_\gamma)/\sqrt{2}$
- $\lambda = 0 \implies \varepsilon_\mu \sim q_\mu/\sqrt{q^2} + \mathcal{O}(\sqrt{q^2}/q^0)$
- $\lambda = S \implies \varepsilon_\mu \sim q_\mu/\sqrt{q^2}$

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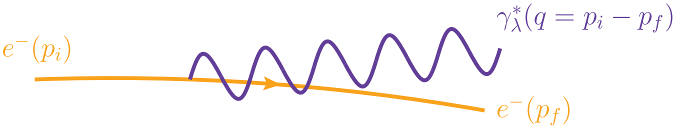

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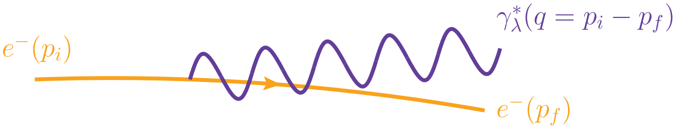
Dirac equation for massless fermions

$$J^\mu_{\ell_f \ell_i} \cdot \ q_\mu \quad = \quad \text{(coupling factors)} \ \times \ [\overline{u}(p_f)\, \gamma^\mu \, (p_i - p_f)_\mu \, u(p_i)]$$

Calculating $\mathcal{M}(\ell \to \ell V)$

Many ways to evaluate $\mathcal{M}$. Helicity amplitudes are super fast:



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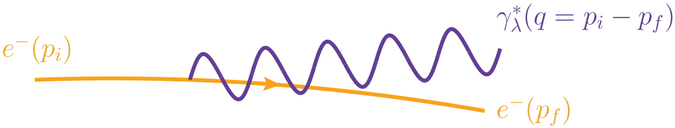
Dirac equation for massless fermions

$$\begin{aligned} J^\mu_{\ell_f \ell_i} \cdot q_\mu &= \text{(coupling factors)} \times [\overline{u}(p_f)\, \gamma^\mu\, (p_i - p_f)_\mu\, u(p_i)] \\ &= \left[\overline{u}(p_f)\, \underbrace{\not{p}_i u(p_u)}_{=\cancel{m_\ell} u(p_i)}\right] - \left[\underbrace{\overline{u}(p_f)\, \not{p}_f}_{=\overline{u}(p_f)\cancel{m_\ell}^{\,0}}\, u(p_i)\right] = 0 \end{aligned}$$

Calculating $\mathcal{M}(\ell \to \ell V)$

Many ways to evaluate $\mathcal{M}$. Helicity amplitudes are super fast:



$$-i\mathcal{M}(\ell_{\lambda_i} \to \ell_{\lambda_f} V_{\lambda_V}) = J^\mu_{\ell_{\lambda_f}\ell_{\lambda_i}} \cdot \varepsilon^*_\mu(q, \lambda_V)$$

outgoing photon (~~four~~ 2.5 helicity polarizations!)

- $\lambda = \pm \implies \varepsilon_\mu = (0, \pm\cos\theta_\gamma, -i, \pm\sin\theta_\gamma)/\sqrt{2}$
- $\lambda = 0 \implies \varepsilon_\mu \sim \cancel{q_\mu/\sqrt{q^2}} + \mathcal{O}(\sqrt{q^2}/q^0)$
- $\lambda = S \implies \cancel{\varepsilon_\mu \sim q_\mu/\sqrt{q^2}}$

Spinors and Spinor (Current) Algebra

To evaluate $J^{\mu}_{\ell_{\lambda_f}\ell_{\lambda_i}}$, we need spinors and spinor algebra (simply matrix algebra!)

$$p^{\mu} = (E, |\vec{p}|\sin\theta\cos\phi, |\vec{p}|\sin\theta\sin\phi, |\vec{p}|\cos\theta), \quad E^2 = |\vec{p}|^2 + m^2$$

2-component spinors:

$$\begin{aligned}
\chi_{\lambda=+1}(\hat{p}) &= \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}}\begin{pmatrix} |\vec{p}|+p_z \\ p_x+ip_y \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} \end{pmatrix} \\
\chi_{\lambda=-1}(\hat{p}) &= \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}}\begin{pmatrix} -p_x+ip_y \\ |\vec{p}|+p_z \end{pmatrix} = \begin{pmatrix} -e^{-i\phi}\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{pmatrix} \\
\chi_{\lambda}(-\hat{p}) &= -\lambda e^{i\lambda\phi}\chi_{-\lambda}(\hat{p}).
\end{aligned}$$

4-component spinors:

$$u_{\lambda}(p) = \begin{pmatrix} \sqrt{E-\lambda|\vec{p}|}\,\chi_{\lambda}(\hat{p}) \\ \sqrt{E+\lambda|\vec{p}|}\,\chi_{\lambda}(\hat{p}) \end{pmatrix}, \quad v_{\lambda}(p) = \begin{pmatrix} -\lambda\sqrt{E+\lambda|\vec{p}|}\,\chi_{-\lambda}(\hat{p}) \\ \lambda\sqrt{E-\lambda|\vec{p}|}\,\chi_{-\lambda}(\hat{p}) \end{pmatrix}$$

Spinors and Spinor (Current) Algebra

Spinor algebra ***appears complicated,*** but simplifies a lot

In the **lab frame**, $p_i = E_i\,(1, 0, 0, +1)$ and this gives

$$u_L(p_i) = \sqrt{2E_i}\begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad \text{and} \quad u_R(p_i) = \sqrt{2E_i}\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

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In the **same frame**, $p_f = E_f\,(1, \sin\theta_\ell, 0, \cos\theta_\ell)$ and this gives

$$\overline{u_L}(p_f) = \sqrt{2E_f}\begin{pmatrix}0\\0\\-\sin\frac{\theta_\ell}{2}\\\cos\frac{\theta_\ell}{2}\end{pmatrix}^T \quad \text{and} \quad \overline{u_R}(p_f) = \sqrt{2E_f}\begin{pmatrix}\cos\frac{\theta_\ell}{2}\\\sin\frac{\theta_\ell}{2}\\0\\0\end{pmatrix}^T$$

Spinors and Spinor (Current) Algebra

Contracting Dirac/spinor indices gives:

pro tip: this entails $(1 \times 4) \cdot (4 \times 4) \cdot (4 \times 1)$ matrix multiplication for each $\gamma^0, \gamma^1, \ldots$.

$$J^\mu_{\ell_L \ell_L} = (-ieQ_e)\ \sqrt{2E_i E_f}\ \left(\cos\frac{\theta_\ell}{2}, \sin\frac{\theta_\ell}{2}, -i\sin\frac{\theta_\ell}{2}, \cos\frac{\theta_\ell}{2}\right)$$

$$J^\mu_{\ell_R \ell_R} = (-ieQ_e)\ \sqrt{2E_i E_f}\ \left(\cos\frac{\theta_\ell}{2}, \sin\frac{\theta_\ell}{2}, +i\sin\frac{\theta_\ell}{2}, \cos\frac{\theta_\ell}{2}\right)$$

Splitting Amplitudes

Contracting Lorentz indices gives:

$$\begin{aligned}
\mathcal{M}(e^-_L \to e^-_L \gamma_+) &= (+2ieQ_e)\sqrt{2E_1 E_A}\cos\left(\frac{\theta_l + \theta_\gamma}{2}\right)\sin\left(\frac{\theta_\gamma}{2}\right) \\
\mathcal{M}(e^-_L \to e^-_L \gamma_-) &= (-2ieQ_e)\sqrt{2E_1 E_A}\cos\left(\frac{\theta_\gamma}{2}\right)\sin\left(\frac{\theta_l + \theta_\gamma}{2}\right) \\
\mathcal{M}(e^-_L \to e^-_L \gamma_0) &\sim \mathcal{O}\left(\sqrt{q^2}/E_A\right)
\end{aligned}$$

A few observations:

Splitting Amplitudes

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A few observations:

- $\mathcal{M}(e_R^- \to e_R^- \gamma_\lambda)$ related by parity inversion
-
-

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- $\mathcal{M}(e^-_R \to e^-_R \gamma_\lambda)$ related by parity inversion
- we want to see what happens when γ goes on shell, i.e., $\gamma^* \to \gamma$

(in practice, neglect $\mathcal{O}(\sqrt{q^2}/E_A)$ terms

-

Splitting Amplitudes

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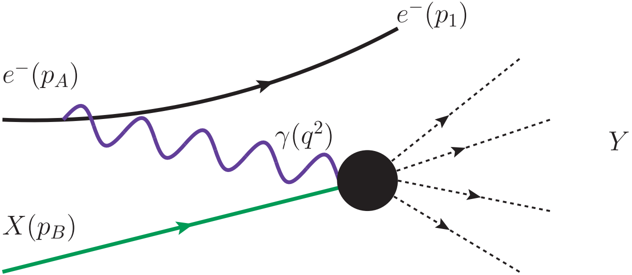
- we want to see what happens when γ goes on shell, i.e., $\gamma^* \to \gamma$

(in practice, neglect $\mathcal{O}(\sqrt{q^2}/E_A)$ terms)

- γ goes on shell, i.e., $\gamma^* \to \gamma$ also means $p_T^\ell \to 0$ since $p_T^2 \propto q^2$

(in practice, expand to lowest order in θ_l, θ_γ)

Collinear Factorization in Massless Gauge Theories



In high-energy collinear $e \to e\gamma$ splitting, $\gamma^* \to \gamma$ and **unphysical polarizations decouple** and **matrix elements factor out** **("factorize")**

$$\mathcal{M}(eX \to eY)$$
$$= \sum_{\lambda=\pm} \mathcal{M}(e \to e\gamma_\lambda) \times \frac{1}{q^2} \times \mathcal{M}(\gamma_\lambda X \to eY) + \mathcal{O}\left(\theta_\ell, \theta_\gamma, \frac{\sqrt{q^2}}{E_A}\right)$$

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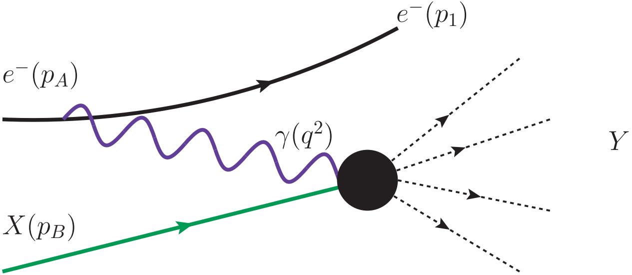

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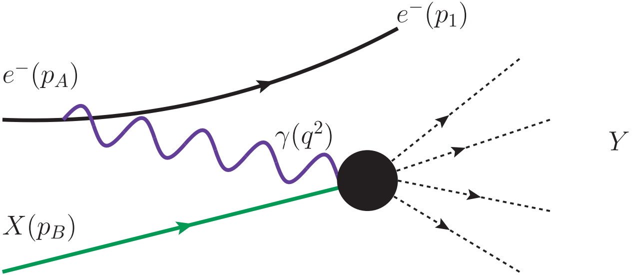
$$\mathcal{M}(eX \to eY)$$

$$= \sum_{\lambda=\pm} \underbrace{\mathcal{M}(e \to e\gamma_\lambda)}_{\text{proc. independent}} \times \frac{1}{q^2} \times \underbrace{\mathcal{M}(\gamma_\lambda X \to eY)}_{\text{proc. dependent}} + \mathcal{O}\left(\theta_\ell, \theta_\gamma, \frac{\sqrt{q^2}}{E_A}\right)$$

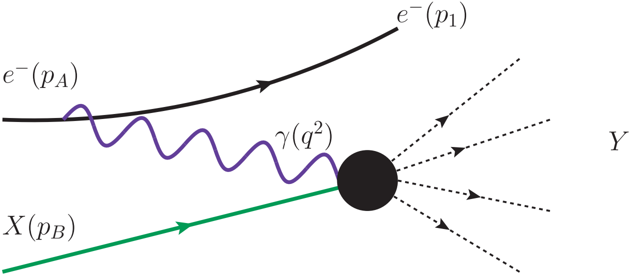
Collinear Factorization in Massless Gauge Theories



In high-energy collinear $e \to e\gamma$ splitting, $\gamma^* \to \gamma$ and **unphysical polarizations decouple** and **(matrix element)^2 factorize**

$$|\mathcal{M}(eX \to eY)|^2$$
$$= \frac{1}{q^4} \sum_{\lambda=\pm} |\mathcal{M}(e \to e\gamma_\lambda)|^2 \times |\mathcal{M}(\gamma_\lambda X \to eY)|^2 + \mathcal{O}\left(\theta_\ell, \theta_\gamma, \frac{\sqrt{q^2}}{E_A}\right)$$

Collinear Factorization in Massless Gauge Theories



Defining $z = E_\gamma / E_A$, then for $\theta_e \ll 1$ we get something nice

(see also Peskin and Schroeder to get here)

$$\sum_{\lambda,\text{d.o.f.}} |\mathcal{M}(e \to e\gamma)|^2 \;=\; (\text{coup. factors})^2 \frac{4q^2}{z(1-z)} \left[\frac{1+(1-z)^2}{z}\right]$$

now to put everything together

Phase Space and Fermi's Golden Rule

1. Phase Space: in general, dPS_{n_Y+1} can be split into dPS_{n_Y} and dPS_1

$$
\begin{aligned}
dPS_{n_Y+1}(p_X, p_f; p_{f'}, \{p_k\}) &= (2\pi)^4 \delta\left(p_X + p_f - p_{f'} - \sum_k^{n_Y} p_k\right) \prod_k^{n_Y+1} \frac{d^3 p_k}{(2\pi)^3 2E_k} \\
&= (2\pi)^4 \delta\left(p_X + q - \sum_k^{n_Y} p_k\right) \frac{d^3 p_{f'}}{(2\pi)^3 2E_{f'}} \prod_k^{n_Y} \frac{d^3 p_k}{(2\pi)^3 2E_k} \\
&= dPS_{n_Y}(p_X, q; p_{f'}, \{p_k\}) \times \frac{d\phi_{f'}\ dz\ dq^2}{4(2\pi)^3}
\end{aligned}
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Phase Space and Fermi's Golden Rule

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$$dPS_{n_Y+1}(p_X, p_f; p_{f'}, \{p_k\}) = (2\pi)^4 \delta\left(p_X + p_f - p_{f'} - \sum_k^{n_Y} p_k\right) \prod_k^{n_Y+1} \frac{d^3 p_k}{(2\pi)^3 2E_k}$$
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$$= dPS_{n_Y}(p_X, q; p_{f'}, \{p_k\}) \times \frac{d\phi_{f'}\ dz\ dq^2}{4(2\pi)^3}$$

2. **Fermi's Golden Rule:** for scattering rates (split according to dPS!)

$$\sigma = \int dPS_{n_y+1} \times \frac{d\hat{\sigma}}{dPS_{n_Y+1}} = \int dPS_{n_y} \int dPS_1 \times \frac{d\hat{\sigma}}{dPS_{n_Y+1}},$$

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&= (2\pi)^4 \delta\left(p_X + q - \sum_k^{n_Y} p_k\right) \frac{d^3 p_{f'}}{(2\pi)^3 2E_{f'}} \prod_k^{n_Y} \frac{d^3 p_k}{(2\pi)^3 2E_k} \\
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where $$\frac{d\sigma}{dPS_{n_Y+1}} = \underbrace{\frac{1}{2Q^2}}_{\text{flux}} \underbrace{\frac{1}{(2s_a+1)(2s_b+1)N_c^a N_c^b}}_{\text{spin/color averaging}} \sum_{d.o.f.} |\mathcal{M}_{n_Y+1}|^2$$

Phase Space and Fermi's Golden Rule

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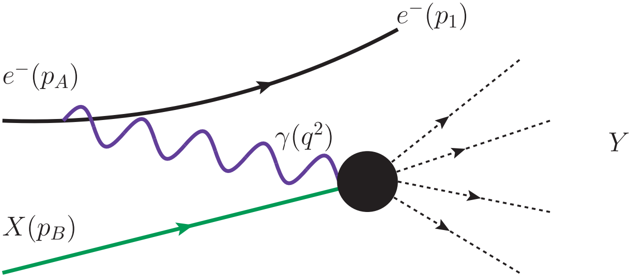
$$
\begin{aligned}
dPS_{n_Y+1}(p_X, p_f; p_{f'}, \{p_k\}) &= (2\pi)^4 \delta\left(p_X + p_f - p_{f'} - \sum_k^{n_Y} p_k\right) \prod_k^{n_Y+1} \frac{d^3 p_k}{(2\pi)^3 2E_k} \\
&= (2\pi)^4 \delta\left(p_X + q - \sum_k^{n_Y} p_k\right) \frac{d^3 p_{f'}}{(2\pi)^3 2E_{f'}} \prod_k^{n_Y} \frac{d^3 p_k}{(2\pi)^3 2E_k} \\
&= dPS_{n_Y}(p_X, q; p_{f'}, \{p_k\}) \times \frac{d\phi_{f'}\ dz\ dq^2}{4(2\pi)^3}
\end{aligned}
$$

2. **Fermi's Golden Rule:** for scattering rates (split according to dPS!)

$$
\sigma = \int dPS_{n_y+1} \times \frac{d\hat{\sigma}}{dPS_{n_Y+1}} = \int dPS_{n_y} \int dPS_1 \times \frac{d\hat{\sigma}}{dPS_{n_Y+1}},
$$

where
$$
\frac{d\sigma}{dPS_{n_Y+1}} = \underbrace{\frac{1}{2Q^2}}_{\text{flux}} \underbrace{\frac{1}{(2s_a+1)(2s_b+1)N_c^a N_c^b}}_{\text{spin/color averaging}} \sum_{d.o.f.} \underbrace{\left|\mathcal{M}_{n_Y+1}\right|^2}_{\text{use factorized ME here!}}
$$

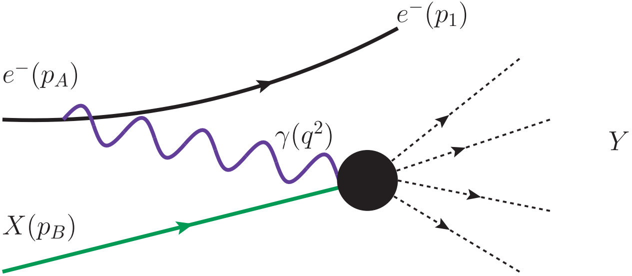


3. Combine and reorganize

$Q^2 = (q + p_B)^2 = 2q \cdot p_B \equiv z \times (p_A \cdot p_B) = z \times s$

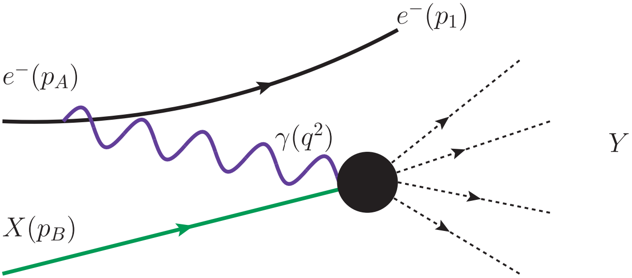
$$\sigma(e^- X \to e^- Y)\Big|_{\theta_e \ll 1} = \int \frac{d\phi_1 dz dq^2}{4(2\pi)^3}(1-z)$$

$$\sum_{\lambda,\text{d.o.f.}} |\mathcal{M}(e \to e\gamma^*)|^2 \quad \times \quad \frac{1}{q^4} \quad \times \frac{Q^2}{2s} \; \hat{\sigma}(\gamma X \to Y)$$



4. Integrate! Integrate! Integrate!

$$\sigma\Big|_{\theta_e \ll 1} = \int dz \left(\frac{\alpha q_e^2}{2\pi}\right) P_{\gamma e}(z) \times \int \frac{dq^2}{q^2} \times \hat{\sigma}(\gamma X \to Y)$$



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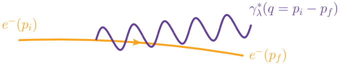
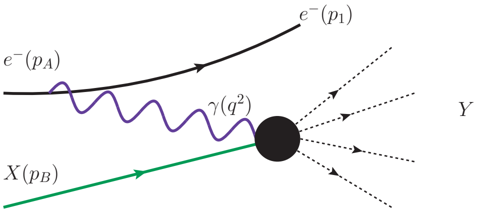
The function $P_{\gamma e}(z)$ is one of the **Altarelli-Parisi splitting functions**

gives splitting probability of $e \to \gamma$ for $E_\gamma = z \times E_A$



$$P_{\gamma e}(z) \equiv P_{\gamma \leftarrow e}(z) = \frac{1 + (1-z)^2}{z}$$

Weizsäcker-Williams Approximation



5. After integrating over virtuality of γ

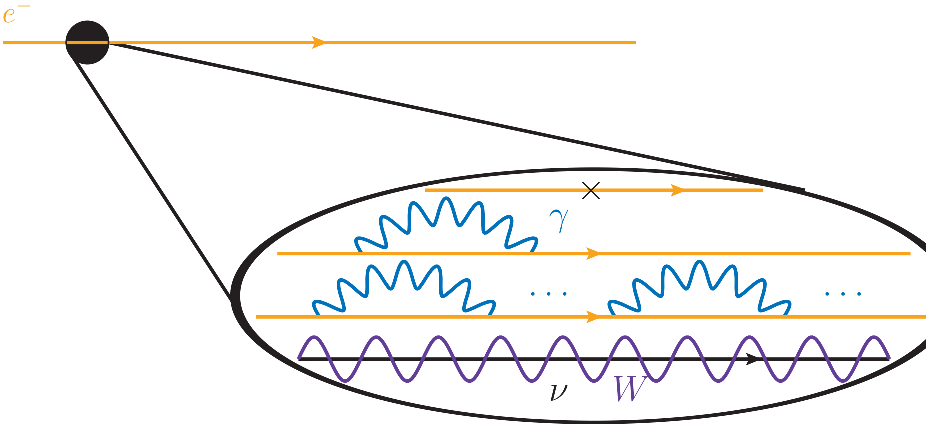
$$\sigma(e^- X \to e^- Y)\Big|_{\theta_e \ll 1} = \int dz \underbrace{\frac{\alpha q_e^2}{2\pi} P_{\gamma e}(z) \log\left(\frac{Q^2}{m_e^2}\right)}_{f_{\gamma/e}(z,Q)} \hat{\sigma}(\gamma X \to Y)$$

$f_{\gamma e}$ can be identified as the γ (number) density within e

also known as the parton (number) density function of parton distribution function

intuition?

bare vs dressed particles in QFT



physical (dressed) particles contain a sea of partons

Weizsäcker-Williams Approximation

Beyond leading order (and more systematically!), $f_{\gamma e}$ picks up additional contibutions:

$$f_{\gamma/e}(z,\mu_r^2) = \underbrace{f_{\gamma/e}(z)}_{\text{“first emission”}} + \underbrace{\frac{\alpha}{2\pi}\int_z^1 \frac{dx}{x}\, P_{\gamma e}(x)\, f_{e/e}\left(\frac{z}{x}\right)\, \log\frac{\mu_r^2}{\mu_0^2} + C(x,z)}_{\text{add'l } \gamma \text{ emissions}}$$

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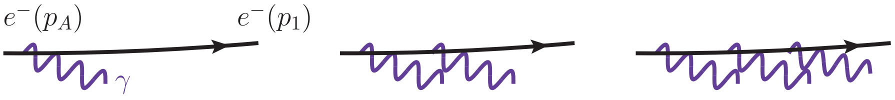

$\implies$ a scale evolution: $\delta f_{\gamma/e}(z,\mu_r^2) \approx \frac{\alpha}{2\pi}\int_z^1 \frac{dx}{x}\, P_{\gamma e}(x)\, f\left(\frac{z}{x},\mu_r^2\right)\delta\log\mu_r^2$

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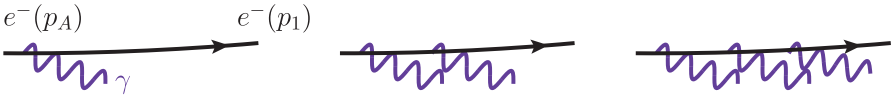

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$\implies$ “dressing” / renormalization of beams from $\mu_0^2 = q^2 \ll E_A^2$ to μ_r^2

$$\frac{\partial f_{\gamma/e}(z,\mu_r^2)}{\partial \log \mu^2} = \frac{\alpha}{2\pi}\int_z^1 \frac{dx}{x}\, P_{\gamma e}(x)\, f\left(\frac{z}{x},\mu_r^2\right)$$

WWA aka Collinear Factorization

renormalization / "dressing" accounts for all collinear rad. with $k_T < \mu_r$

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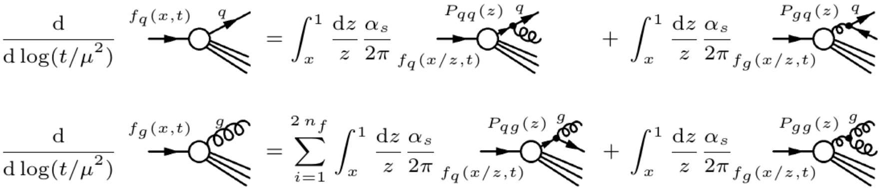
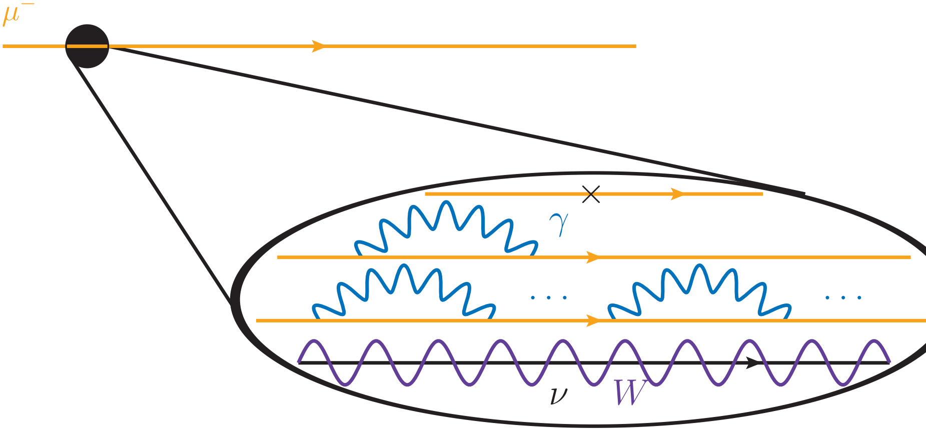
Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations



[S. Hoeche]

this holds for QED and QCD

possibly also for EW theory

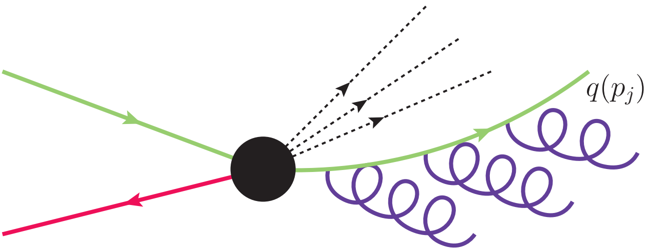


physical (dressed) particles contain a sea of partons

Pt4: parton showers

The idea of a parton shower is to capture soft/collinear ISR/FSR

(which are near the singular regions of FO matrix elements!)



$$\sim \alpha_s^n(k_T^2)\log^n\left(\frac{t}{k_T^2}\right)$$

Such ISR/FSR are not naively $\mathcal{O}(\alpha_s)$ suppressed, i.e., power suppressed, since momentum scale of emission is small compared to hard scale Q

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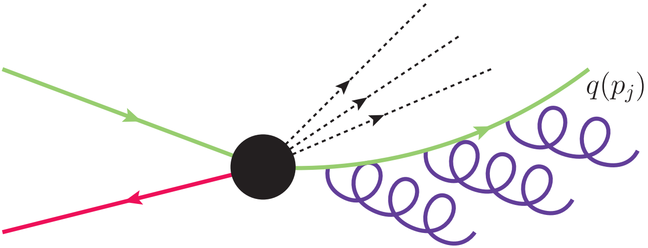

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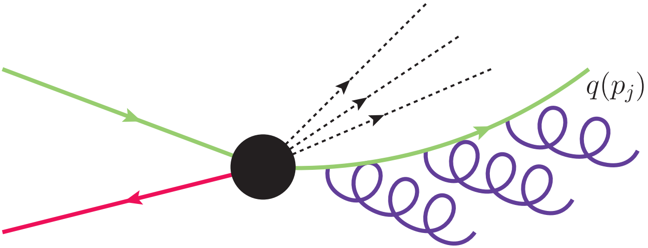

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Using **collinear factorization and unitarity (prob conservation!),** we can account for such radiation to all orders in perturbation theory!

For $1 \to 2$ splitting at virtuality t, coll. fact. in QCD / QCD are same:

(just replace $\alpha \to \alpha_s C_{F(A)}$ for $q(g)$)

$$\sigma_{2\to(n+1)} \sim \sigma_{2\to n} \int dz \ \frac{dt}{t} \ \frac{\alpha_s C_i}{2\pi} P_{ji}(z)$$

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On event-by-event basis, throw random number $y \in [0,1]$. Check if splittings occur. If yes, add $1 \to 2$ splitting. Restart, until no splitting.

$$d\sigma(pp \to \mathcal{B} + X) = \sum_{a,b} f_a \otimes f_b \otimes \underbrace{\Delta_{ab}}_{=\mathcal{P}(z)\ \text{(survival rate)}} \otimes d\hat{\sigma}(ab \to \mathcal{B}) + \mathcal{O}\left(\Lambda_{\rm NP}^p / Q^{p+2}\right)$$

how accurate is all of this?

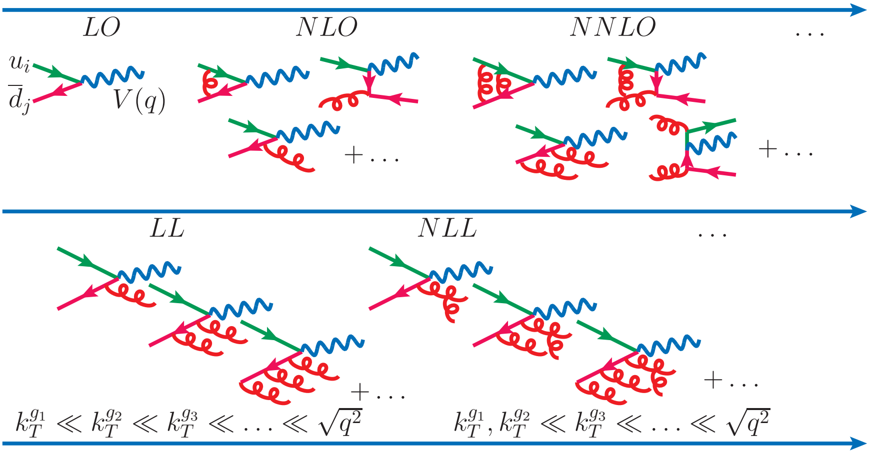
how accurate is all of this? leading log (LL), usually

Fixed Order vs Resummation

Resummation: REnormalization-grouped improved SUMMATION

- a technique for accounting for a particular type if radiation,

e.g., strongly ordered collinear gluons to all orders in pert. theory



Sketch of Factorization and Exponentiation

Combine our factorized results:

$$\mathcal{M}_{\rm V+1\ soft/collinear\ radiation} = \Big(\underbrace{\text{rad. pole} + \text{loop pole}}_{\text{universal factor}}\Big) \times \mathcal{M}_{\rm V}^{\rm FO}$$

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Summing over all such emissions gives us a closed result:

$$d\sigma_{\rm V+any\ soft/collinear} = \sum_k \frac{1}{k!}[\mathcal{S}]^k \times \sigma_{\rm DY}^{\rm FO} = \exp[\mathcal{S}] \times \sigma_{\rm V}^{\rm FO}$$

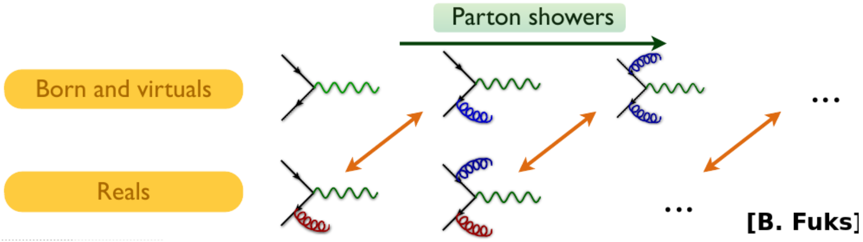
is there double counting?

is there double counting? yes, usually!

MC@NLO

Phase space double counting is now a concern when adding PS to NLO:

- At NLO, is the first/hardest emission from the PS or ME?



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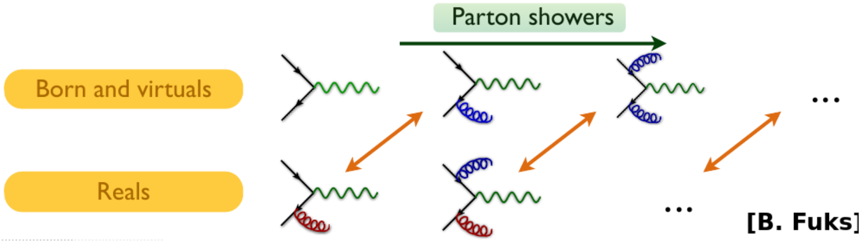

Solution: MC@NLO (Frixione, Webber [hep-ph/0204244]; other solutions, too! e.g., POWHEG)

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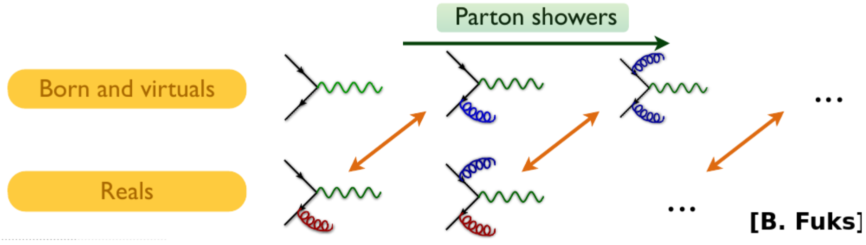

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- PS and ME describe these regions well, respectively.

Importantly,

- The PS assumptions are invalidated at wide angles
- The ME becomes perturbatively instable in soft/collinear limits.

MC@NLO in practice:

(Frixione, Webber [hep-ph/0204244])

- introduce auxiliary term $\pm d\sigma^{A}$ to move $1/\varepsilon^{\mathrm{IR}}$ cancellation
- introduce auxiliary term $\pm d\sigma^{MC}$ to move first $\mathcal{O}(\alpha_s)$ emission
- **result:** moves "parton shower" out of $dPS_{(n+1)}$ and into dPS_n

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Main Monte Carlo formula at NLO in QCD:

$$\sigma^{\mathrm{NLO}} = \int dPS_n \left[d\sigma^{B} + d\sigma^{CT} + \left(d\sigma^{V} + \int dPS_1 d\sigma^{A} \right) \right] \qquad \text{(Born-like)}$$

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$$+ \int dPS_{(n+1)} \ \left[d\sigma^{R} - d\sigma^{MC}\right] \quad \text{(real events)}$$

normalization okay but dists. wrong without parton shower!

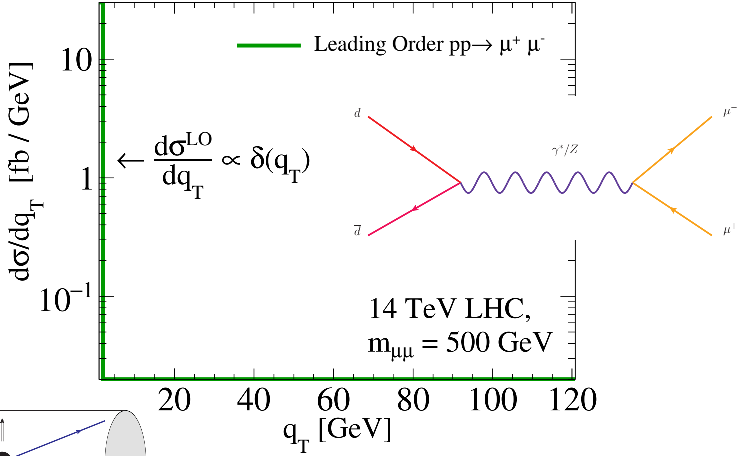
language of precision:

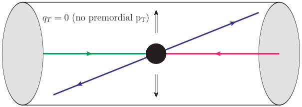
precision of normalization $\neq$ precision of distribution

Not all observables are well-defined when σ is known only at LO

Example: transverse momentum (q_T) of W/Z system in $pp \to V + X$

- $q_T = 0$ at Born-level (no recoil!) and singular at $\mathcal{O}(\alpha_s)$





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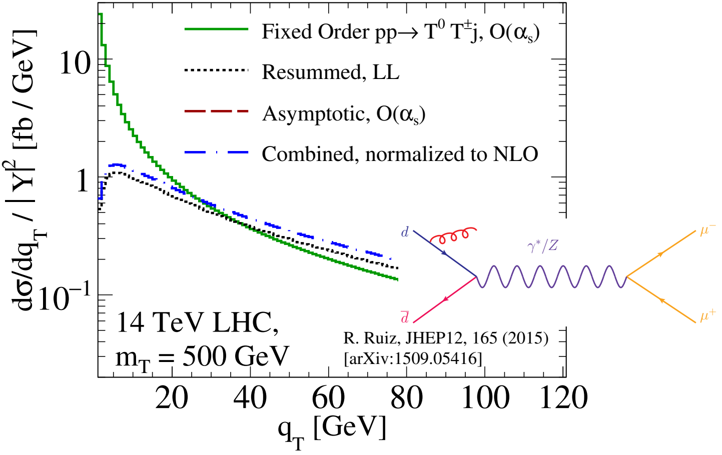
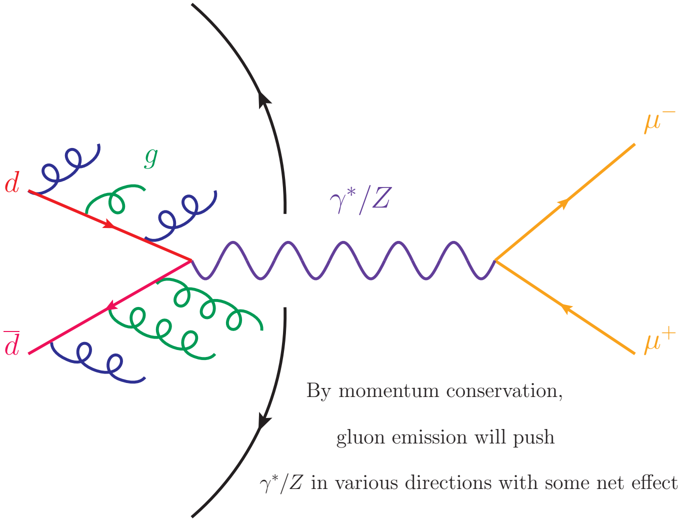
- Lowest order q_T physical is when σ is known at NLO w/ leading log. (LL) q_T-resummation (or +PS) $\implies$ $d\sigma/dq_T$ is LO+LL(q_T)



intuition



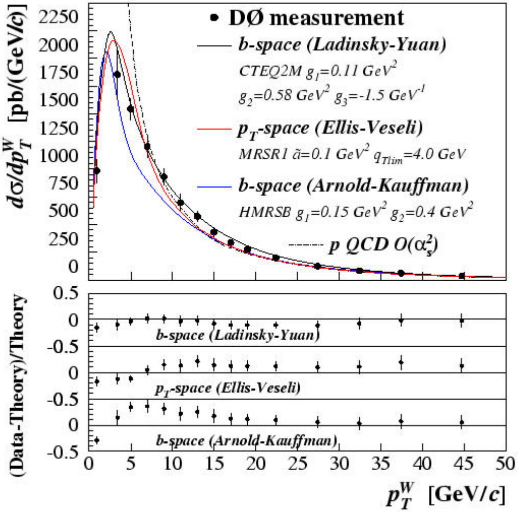
By momentum conservation,

gluon emission will push

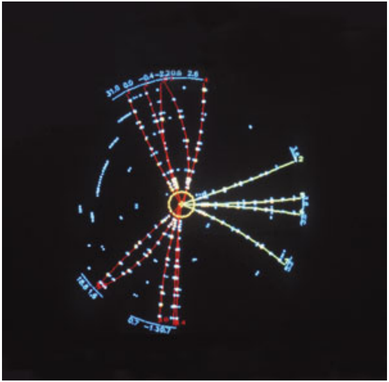
γ^*/Z in various directions with some net effect

Example: transverse momentum (q_T) of W/Z system in $pp \to V + X$

- Lowest order q_T physical is when σ is known at NLO w/ leading log. (LL) q_T-resummation (or +PS) $\implies$ $d\sigma/dq_T$ is LO+LL(q_T)



Pt5: what are jets?[1]



[1] from an RGE perspective: Contopanagos, Laenen, Sterman ('96); Becher, Broggio, Ferroglia ('14); etc

Collinear Factorization Theorem

Collins, Soper, Sterman ('85,'88,'89); Collins, Foundations of pQCD (2011)

$$d\sigma(pp \to \mathcal{B} + X) = \sum_{a,b} f_a \otimes f_b \otimes \Delta_{ab} \otimes d\hat{\sigma}(ab \to \mathcal{B}) + \mathcal{O}\left(\Lambda_{\mathrm{NP}}^{p} / Q^{p+2}\right)$$

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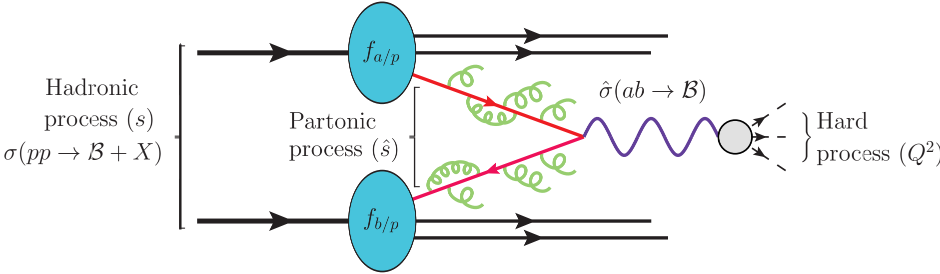
hadron-level scattering prob. is the product (convolution) of parton -distrib. (PDFs), -emission (Sudakov), and -scattering probs. ($|\mathcal{M}|^2$)



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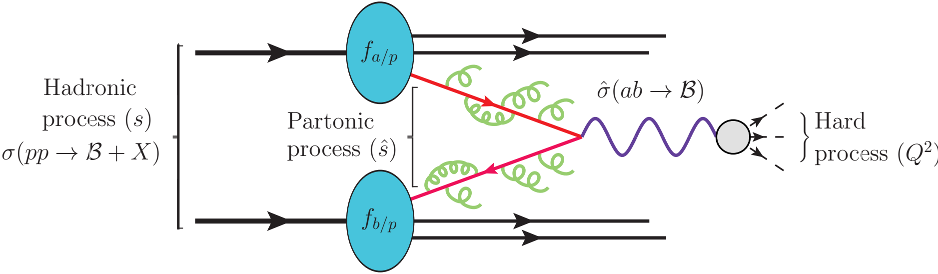

Now, ignore parton shower, and simply write:

$$\sigma(pp \to \mathcal{B} + X) = \sum_{a,b} f_a \otimes f_b \otimes \hat{\sigma}(ab \to \mathcal{B}) + \mathcal{O}\left(\Lambda^p_{\rm NP}/Q^{p+2}\right)$$

Rewrite the **CFT** in Mellin ("probability") space: $\tilde{f}^N = \int_0^1 dx\; x^{N-1} f(x)$

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note: $U(\mu_2,\mu_1)$ takes f from (small) $\mu_r = \mu_1$ to (large) $\mu_r = \mu_1$

Remember: μ are *put in by hand* to make the math easier

(nature does not care about μ!)

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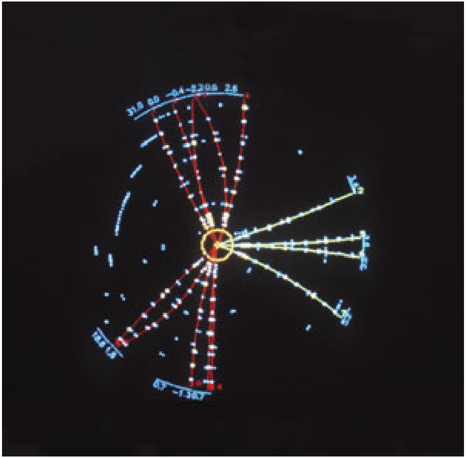
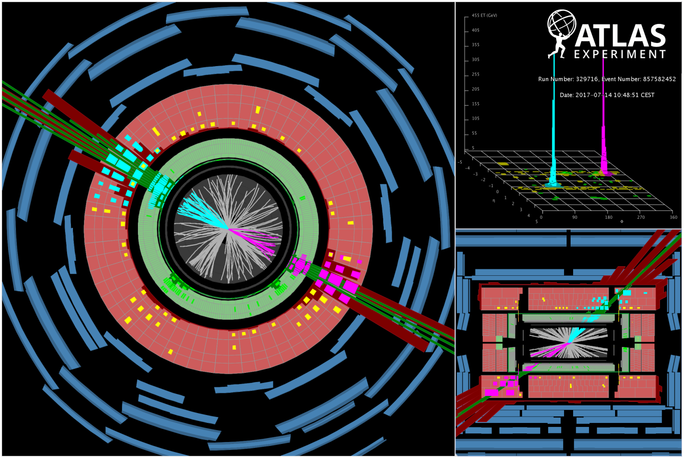
what does this mean?

how are QCD partons geometrically distributed at $Q = 13 - 32$ GeV?



how are QCD partons geometrically distributed at $Q = 9.5$ TeV?



QCD partons become collimated at high energies

Even if the parton shower Δ is ignored, scale invariance gives it back:

$$\tilde{\sigma}^N = \tilde{f}_a^N(\mu_{f1}) \times \tilde{f}_b^N(\mu_{f2}) \times \underbrace{\left[U_1(\mu_{f1})U_2(\mu_{f2})E^\dagger(\mu_h)\right]}_{=\Delta} \times \tilde{\hat{\sigma}}_{ab}^N(\mu_h)$$

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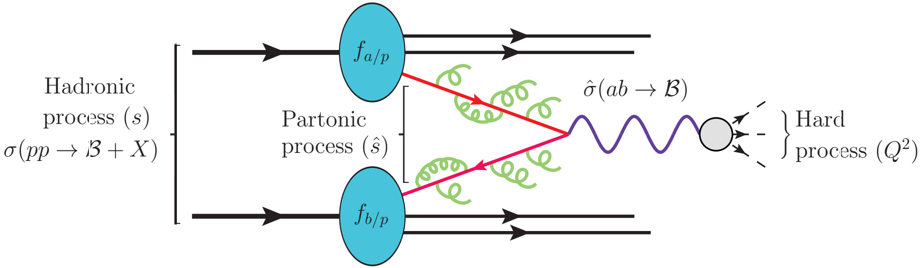
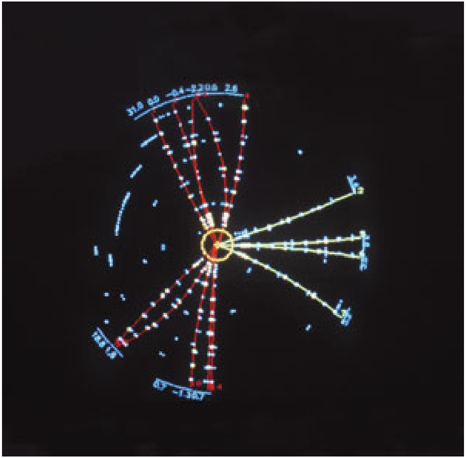
1. **The parton shower (as a Sudakov factor) is physical!**

2. **The parton shower (as an RG evo. factor) dresses bare partons!**



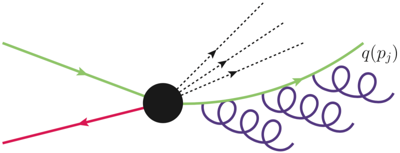
pronged events are macroscopic manifestations

of "particle dressing" (QFT!)

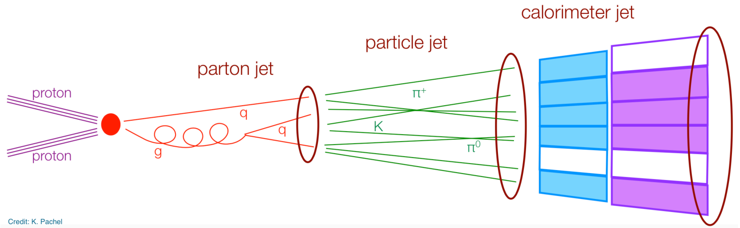


physical (dressed) particles contain a sea of partons

External quarks and gluons typically carry small virtuality (off mass-shell!)
$\Longrightarrow$ forced by uncertainty principle to radiate gluons just like EM!



Nature favors many, low E emissions vs few, high E emissions



$\Longrightarrow$ strong QCD coupling leads to formation of a collimated streams of energetic bound states (hadrons!), which we call ***jets***

bonus: jet identification

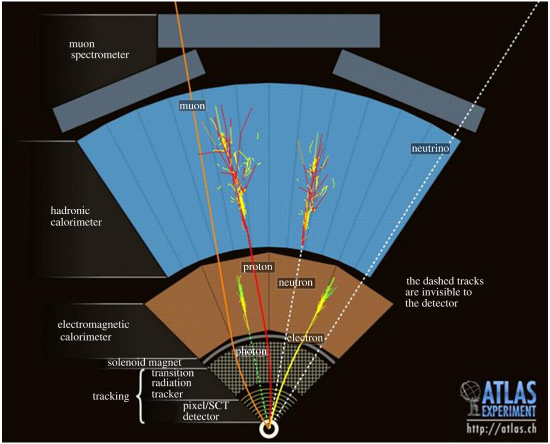
LHC detector experiments are masterpieces

- **MASSIVE** (5-8 stories; 7-14kt)
- 4π coverage (LHCb$\sim 2\pi$)
- Si tracking (giant CCD camera!)
- concentric calorimeters
- spectrometer or similar

neutrino ID: ☹

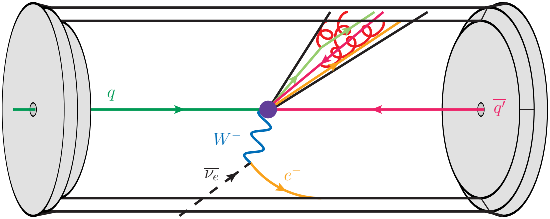
electron ID:
- tracker, ECAL
- kinematic + isolation req.

quark ID:
- tracker, ECAL, HCAL
- jet clustering

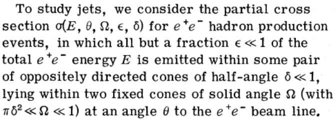


what is a jet?

Suggestions? Intuitively: boosted, collimated hadronic activity.



Sterman-Weinberg jets: Cones with opening angle $\delta \ll 1$ containing up to $(E_{Tot.} - \epsilon)/N$ energy from hadrons.



To study jets, we consider the partial cross section $\sigma(E, \theta, \Omega, \epsilon, \delta)$ for e^+e^- hadron production events, in which all but a fraction $\epsilon \ll 1$ of the total e^+e^- energy E is emitted within some pair of oppositely directed cones of half-angle $\delta \ll 1$, lying within two fixed cones of solid angle Ω (with $\pi\delta^2 \ll \Omega \ll 1$) at an angle θ to the e^+e^- beam line.

Sterman, Weinberg ('77)

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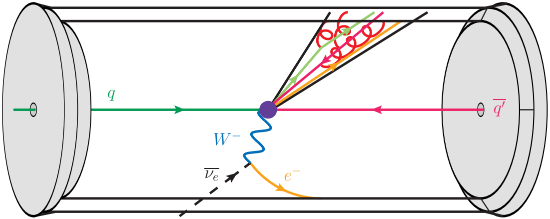

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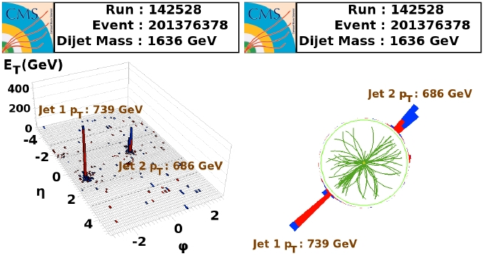
Since 1977, our understanding of jets and their constituents has evolved.

In particular,

- Application of **infrared and collinear (IRC) safety**
- Invention of **sequential jet clustering algorithms**

Idea of IRC: physical quantities do not change under the emission of a single soft ($E_{rad} \to 0$) and/or collinear ($\theta_{ij} \to 0$) radiation



In TH and EXP, **soft/collinear parton splitting should not change the number of jets**

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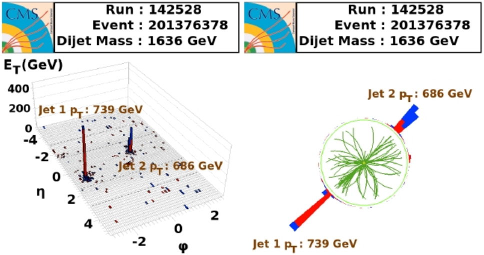

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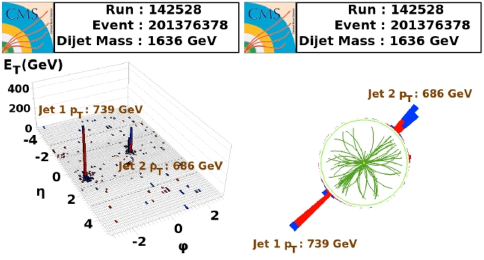

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Formally, for momenta $p_i = p_j + p_k$, an observable $\mathcal{O}$ is **IRC-safe** if

$$\mathcal{O}_{n+1}(p_1, \ldots, p_j, p_k, \ldots, p_{n+1}) = \mathcal{O}_n(p_1, \ldots, p_j + p_k, \ldots, p_{n+1})$$

when $E_j \to 0$, $E_k \to 0$, or $\hat{p}_i \cdot \hat{p}_j = \cos\theta_{ij} \to 1$

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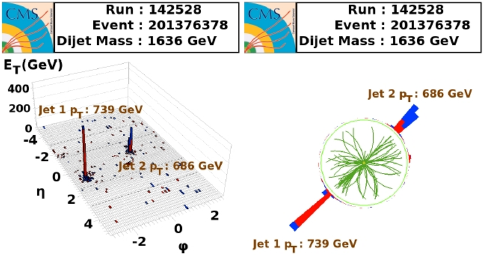

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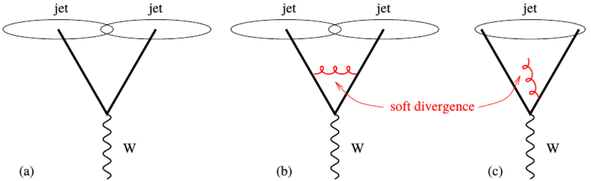
Summary: collimated/soft objects are unresolvable

unsafe jets

(cone clustering algorithm)

Consider two SW jets of radius R with their centers separated by R



G. Salam [0906.1833]

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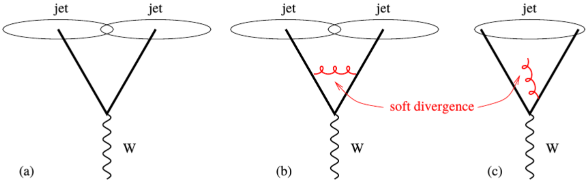

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Beyond LO, a single soft radiation can cause the two jets to merge, or the appearance of a third jet

SW jets do not require a min. energy threshold!

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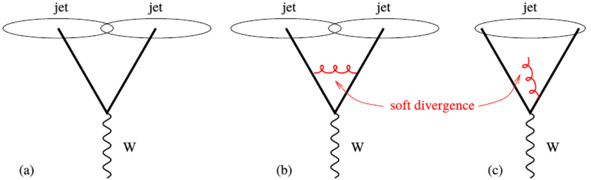

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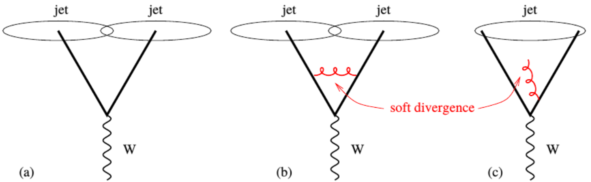

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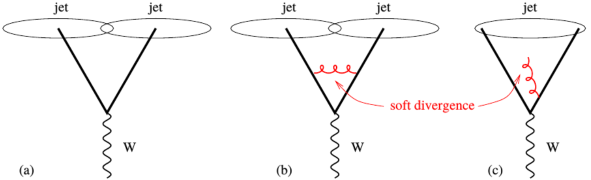

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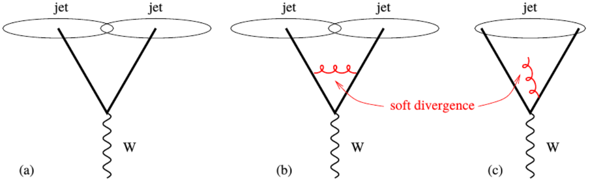

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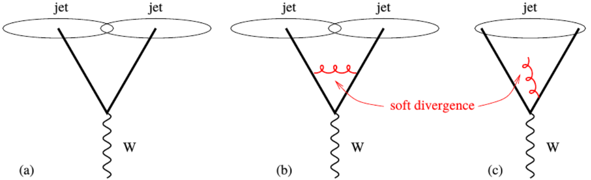

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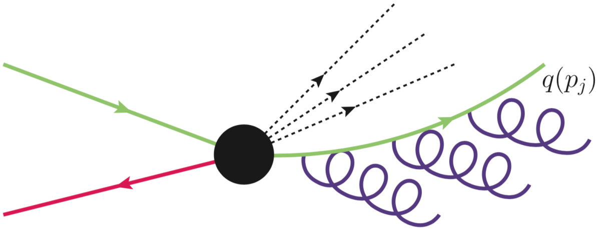
SW jets do not require a min. energy threshold!

In soft/collinear limit, $(n+1)$-body kinematics map to the n-body configuration $\implies$ cancellation of real and virt. $1/\varepsilon^{\rm IR}$ [KLN Thm]

ALERT: If the number of jets changes in the soft limit, then phase spaces are different $\implies$ $1/\varepsilon^{\rm IR}$ do not cancel $\implies$ **violation of KLN Thm**

IRC-safe jets

Aim of jet-building is to "identify" hard partons by "undoing" splitting



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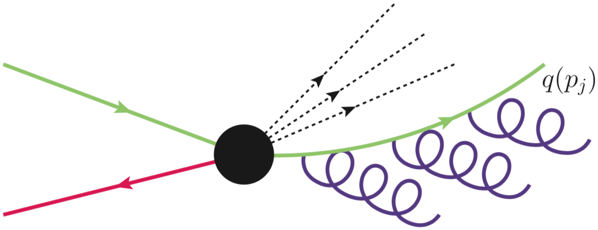
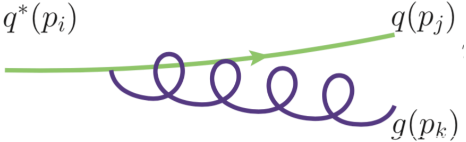


Consider a very energetic $1 \to 2$ parton splitting



Since $\mathcal{M} \propto 1/m_{jk}^2$, where $m_{jk}^2 \sim E_j E_k \theta_{jk}$

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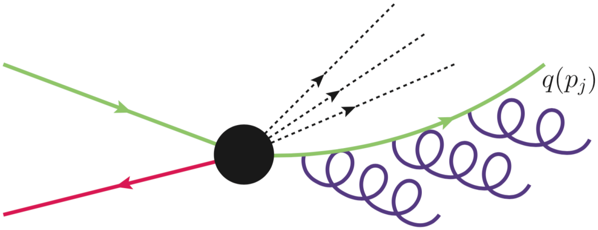

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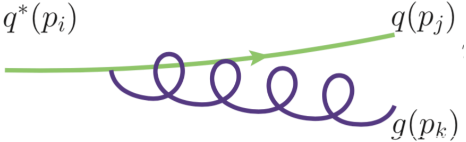
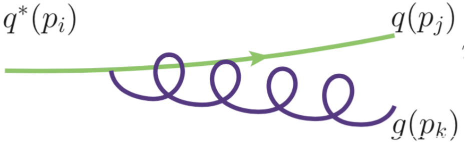


Since $\mathcal{M} \propto 1/m_{jk}^2$, where $m_{jk}^2 \sim E_j E_k \theta_{jk}$

$\Longrightarrow$ preference for low energy $(E_j, E_k \to 0)$/small angles $(\theta_{jk} \to 0)$!

Aim of modern jet-building is to *undo nature*



and cluster according to smallest "distance" measure:

$$d_{jk} = \min(p_T^j, p_T^k)^p \Delta R_{jk}^2 / R^2, \qquad \Delta R_{jk} = \text{(angular opening)}$$

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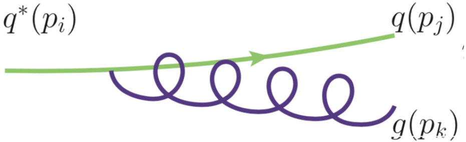

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- For $p = 2$, $d_{jk} \propto m_{jk}^2 \implies$ distance in virtuality space

  (best describes matrix element)

- For $p = 0$, $d_{jk} \propto \theta_{jk}^2 \implies$ distance to geometric neighbors

  (best for studying jet substructure)

- For $p = -2 \implies$ distance from hardest object to geometric neighbor

  (possesses "ideal" properties)

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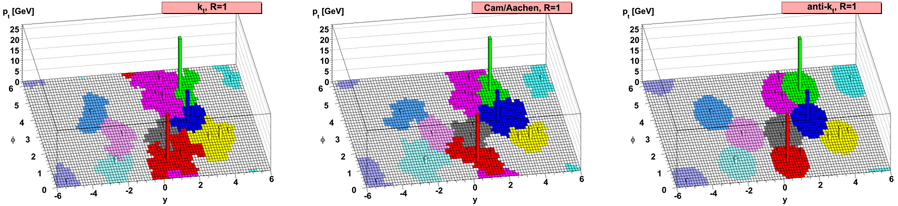
- $p = 0$ is the Cambridge/Aachen algo. and clusters closest neighbors
  - Useful for studying jet substructure since no momentum scale bias
- $p = -2$ is the anti-k_T-algo. and clusters hardest object to neighbors
  - "Ideal" properties and reproduces cone-like jet structure



By construction, all k_T-style algorithms are IRC-safe. [0802.1189]

summary

Jets are powerful tools for studying physics

- **what is the origin of jets?** RG evolution / “dressing” of bare partons
- **how are jets useful?** they quantify particle splitting (not just QCD partons!)
- **how are jets identified?** recombining their splitting histories
- **where can jets be used?** everywhere: from SM to BSM!

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Jets are incredibly rich structures and tell us much about gauge theories

we should respect the physics of jets but not be scared of it!

thank you! questions?