Intro. to (SM & BSM) Neutrinos

"Selected Topics" Warsaw, Poland

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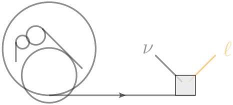
16 September 2024



a few plesantries

most important: these lectures are low-key; questions are great

I am literally here to tell you what I know

apologies and disclaimers

Lectures are "Summer School" style

- More material/slides than allowed by time
- Some slides will be skipped (kept for completeness)
- NOT an historical summary

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Goal: fill in some gaps between courses and research

- Explain what goes into plots often shown in seminars & conferences
- Healthy mixture of math and plots (ν physics is rigorous physics)
- Personally, I have never seen some of the following in a lecture

(sorry also for the typos!)

Lecture Plan (Monday)

Lecture I:

- Pt1: The **Standard Model (SM)** neutrino ν
- Pt2: The ν that nature gave us: intro. to ν oscillations
- Pt3: Theory consequences of ν masses
- Pt4: Introduction to ν mass models (backup!)

Lunch at 12:30ish

Pt1. the Standard Model neutrino

Particle Physics: Then and Now

Throughout the 20th century, a chief goal of particle physics was to establish the **particle spectrum**, their **structures**, and their **properties**

possible with many tools, e.g., production at colliders, tabletop measurements of fundamental symm., and rare decays

credit: I. Bigaran

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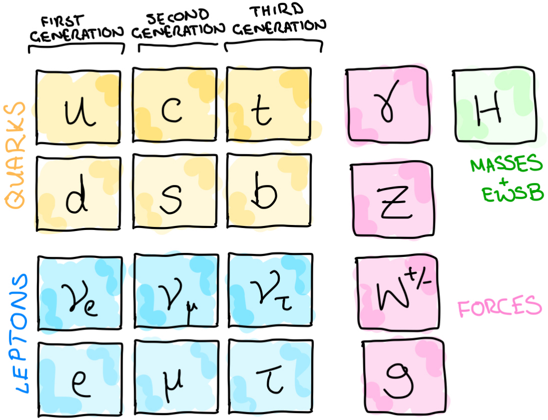
The Standard Model (SM) of particle physics

– position indicates quantum numbers/ charges

(just like in chemistry!)

– e.g., spin, flavor, color, electromagnetic, weak hyper charge



credit: I. Bigaran

Position makes quantum numbers, e.g., gauge charges, manifest

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = \overbrace{\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}}^{T_L^3}$$

$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$$

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$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$$

$$Q_f = T^3_{Lf} + \frac{1}{2} Y_f \implies Y_{Q_L} = +1/3$$
$$Q_f = T^3_{Lf} + \frac{1}{2} Y_f \implies Y_{L_L} = -1$$

Position makes quantum numbers, e.g., gauge charges, manifest

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$$Q_f = T^3_{Lf} + \frac{1}{2} Y_f \quad \Longrightarrow \quad Y_{Q_L} = +1/3$$

$$Q_f = T^3_{Lf} + \frac{1}{2} Y_f \quad \Longrightarrow \quad Y_{L_L} = -1$$

Sanity: $(Q^{\text{upper}} - Q^{\text{lower}}) = (T_L^{\text{upper}} - T_L^{\text{lower}}) = +1$

Sanity: $(2Nc) \cdot Y_{Q_L} + 2Y_{L_L} = 0$

Exercise: show that $N_c \cdot Y_{u_R} + N_c \cdot Y_{d_R} + Y_{e_R} = 0$

Position makes quantum numbers, e.g., gauge charges, manifest

Species	Symbol	$SU(3)_C \times SU(2)_L \times U(1)_Y$ Rep.	$U(1)_{EM}$ Charge [Units of $e > 0$]
Quark	$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(\mathbf{3}, \mathbf{2}, +\frac{1}{3})$	$\begin{pmatrix} +2/3 \\ -1/3 \end{pmatrix}$
Quark	u_R	$(\mathbf{3}, \mathbf{1}, +\frac{4}{3})$	$+2/3$
Quark	d_R	$(\mathbf{3}, \mathbf{1}, -\frac{2}{3})$	$-1/3$
Lepton	$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, -1)$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
Lepton	e_R	$(\mathbf{1}, \mathbf{2}, -2)$	-1

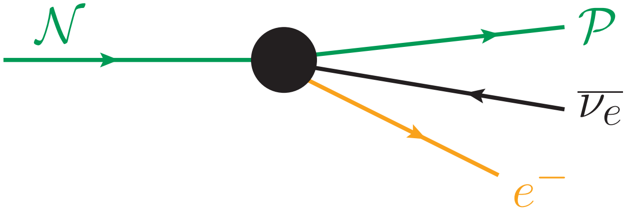
technical note: here, fermions are states in the **gauge/interaction basis** (≠ mass basis)

– not consistent to assign masses need to rotate into mass basis!

question: how do we know that ν carries weak charges?

a few steps back

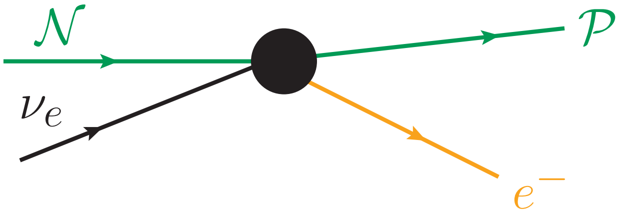
Nuclear β decay is governed by Fermi Theory



$$\mathcal{L}_{\rm Fermi} = G_F \left[\overline{\mathcal{N}}\gamma^\mu P_L \mathcal{P}\right] \cdot \left[\overline{\nu_e}\gamma_\mu P_L e\right]$$

Fermi('31)

Inverting ν_e leg $\Longrightarrow$ inverse β decay **("elastic" ν-nucleus scattering)**



$$-i\mathcal{M}(\nu_e\mathcal{N} \to e^-\mathcal{P}) \sim G_F \ [\overline{u}(k_{\mathcal{P}})\gamma^\mu P_L u(k_{\mathcal{N}})] \cdot [\overline{u}(k_e)\gamma_\mu P_L u(k_{\nu_e})] \sim G_F \ E_\nu^2$$

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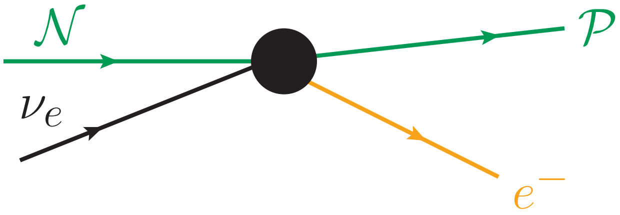

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$$\implies \sigma(\nu_e\mathcal{N} \to e^-\mathcal{P}) \sim \frac{1}{(\text{flux})}\ \not{\Sigma}_{\text{dof}}\ (\text{phase space}) \times |\mathcal{M}|^2 \sim G_F^2\frac{E_\nu^4}{\pi E_\nu^2}$$

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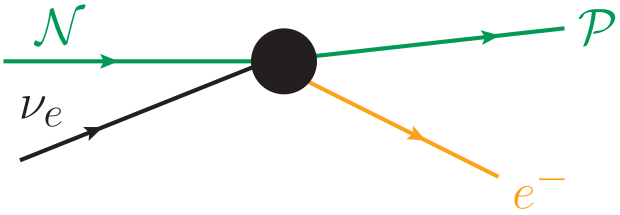

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$\Longrightarrow$ **scatt. rate (σ) grows with scatt. energy without bound**

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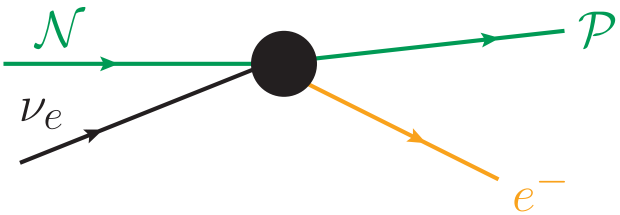

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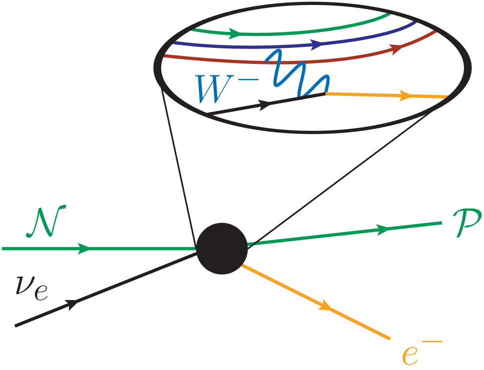
$\Longrightarrow$ **scatt. rate (σ) grows with scatt. energy without bound**

$\Longrightarrow$ **violation of unitarity in scattering theory, i.e., $\sum$(prob)≤ 1**

Inverse β decay is a charged-current interaction!



Fermi thry is the low-energy manifestation of the electroweak thry

$$\left(\frac{g_W}{\sqrt{2}}\right)^2 \times \left(\frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2 + i\Gamma_W M_W}\right) \xrightarrow{|q^2| \ll M_W^2} \frac{-g_W^2}{2M_W^2} = -2\sqrt{2}G_F$$

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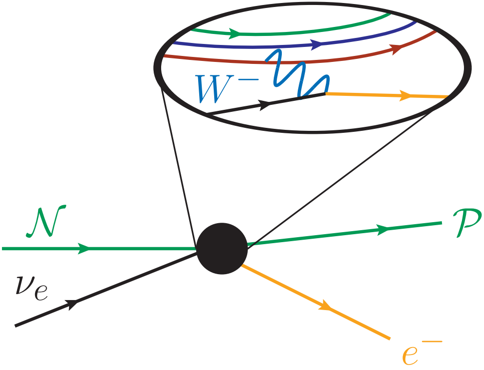

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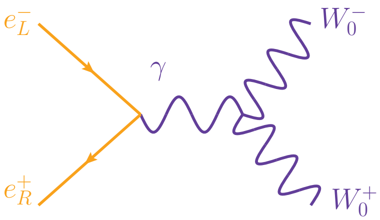
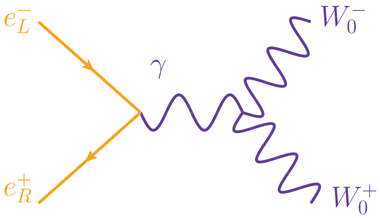
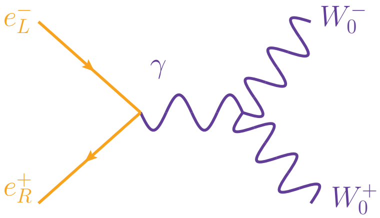
$$\implies \sigma(\nu_e \mathcal{N} \to e^- \mathcal{P}) \sim \frac{g_W^4}{\pi} \frac{E_\nu^2}{(E_\nu^2 - M_W^2)^2}$$ ← high-E behavior is regulated (finite at large E_ν)

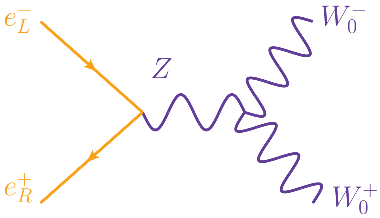
question: how do we know that ν carries weak charges?


e_L^-
e_R^+
γ
W_0^-
W_0^+
e_L^-
e_R^+
Z
W_0^-
W_0^+



$$-i\mathcal{M}(e_L^- e_R^+ \xrightarrow{\gamma} W_0^+ W_0^-) \sim \sqrt{E}\sqrt{E}(Qe)\cdot\frac{1}{E^2}\cdot(g\sin\theta_W E)\cdot E^2 = Qg^2\sin^2\theta_W E^2$$





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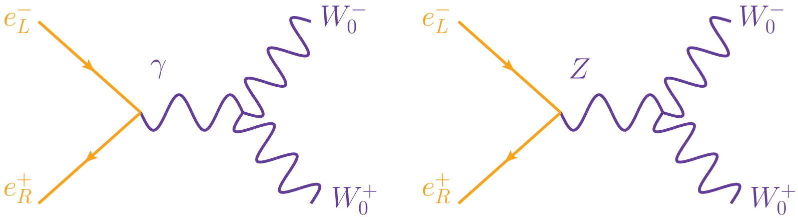
$$= T_{Le}^3 g^2 E^2 - Qg^2\sin^2\theta_W E^2$$



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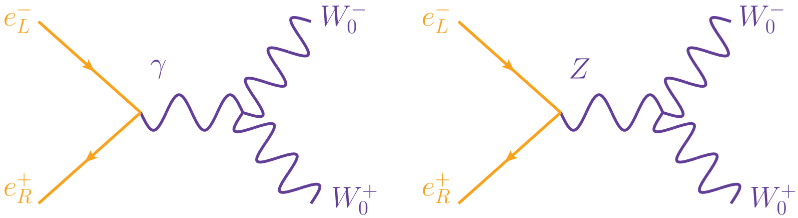
$\Longrightarrow$ **scatt. amplitude** $(\mathcal{M}_{\gamma+Z} \sim E^2)$ **grows without w/o bound!**



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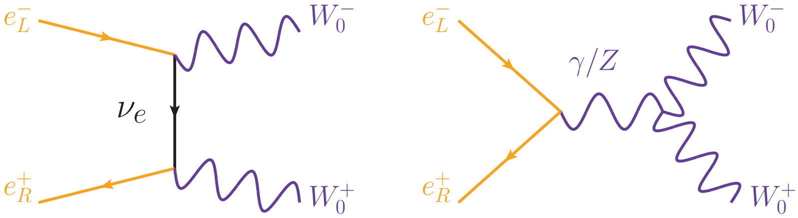
$\Longrightarrow$ **scatt. amplitude ($\mathcal{M}_{\gamma+Z} \sim E^2$) grows without w/o bound!**

$\Longrightarrow$ **violation of unitarity!**



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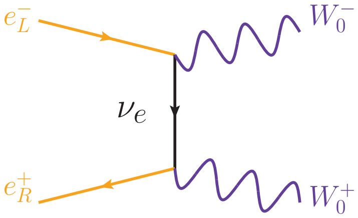

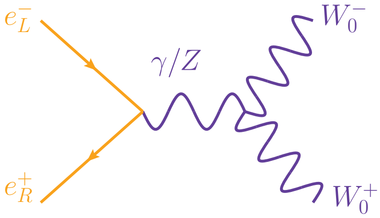

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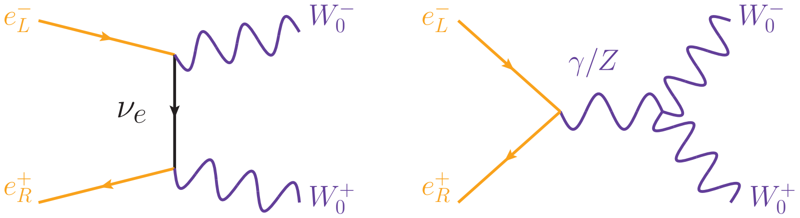
$$-i\mathcal{M}(e_L^- e_R^+ \xrightarrow{\nu} W_0^+ W_0^-) \sim \sqrt{E}\sqrt{E} \cdot \left(\frac{g}{\sqrt{2}}\right)^2 \cdot \frac{E}{E^2} \cdot E^2 \sim +\frac{1}{2} g^2 E^2$$



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$$\implies (T^3_{Le} + 1/2) = 0 \text{ or } T^3_{L\nu} = +1/2 \text{ since } T^3_{L\nu} = -T^3_{Le}.$$

Delicate (structural) cancellations when all particles are included!

Position makes quantum numbers, e.g., gauge charges, manifest

Species	Symbol	$SU(3)_C \times SU(2)_L \times U(1)_Y$ Rep.	$U(1)_{EM}$ Charge [Units of $e > 0$]
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Quark	u_R	$(\mathbf{3}, \mathbf{1}, +\frac{4}{3})$	$+2/3$
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Lepton	$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, -1)$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
Lepton	e_R	$(\mathbf{1}, \mathbf{2}, -2)$	-1

how many ν are there?

In the SM **(and nature)** $m_\nu \ll M_Z \implies Z \to \nu_\ell \overline{\nu_\ell}$ possible for all ℓ

$\implies \Gamma_\nu \equiv \Gamma(Z \to \nu_e \overline{\nu_e}) = \Gamma(Z \to \nu_\mu \overline{\nu_\mu}) = \Gamma(Z \to \nu_\tau \overline{\nu_\tau})$ "=" in SM and "≈" in nature

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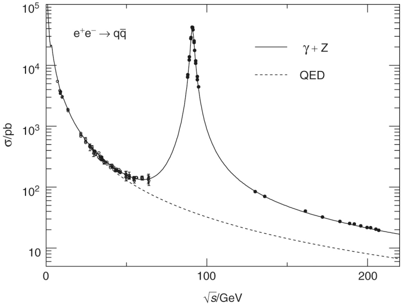
- Paramertize total width of Z as $\Gamma_Z^{\rm Tot.} = \Gamma_{\ell\ell} + \Gamma_{\rm Had.} + N_\nu^{\rm Active} \times \Gamma_\nu$
- Number of light, active ν ($N_\nu^{\rm Active}$) can be determined from $e^+e^- \to Z \to$ had. line shape



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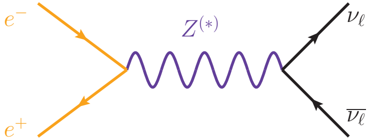
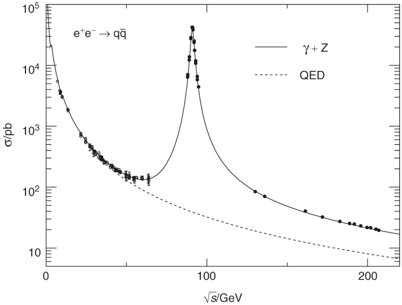
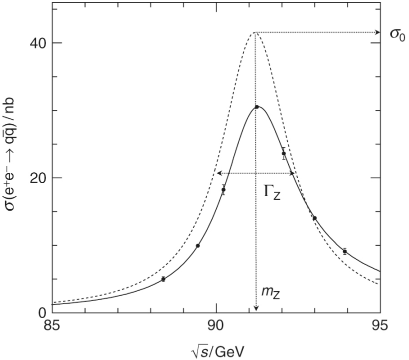


- Number of light, active ν (N_ν^{Active}) can be determined from $e^+e^- \to Z \to$ had. line shape

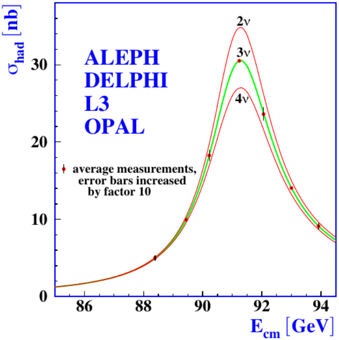


Thomson ('13)



One of the most important (and neatest!) LEP results:

- From line shape, $N_\nu^{\rm Active} = 2.9840 \pm 0.0082$
- From inv. Z decays, $N_\nu^{\rm Active} = 2.92 \pm 0.05$
- 2σ deviations consistent with $Z \to N\nu$ decays [Jarlskog, ('91)]



- Helps drive (mild) preference for non-unitarity of 3×3 mixing

See, e.g., Fernandez-Martinez, et al [1605.08774]

Important: e^+e^- colliders under discussion can resolve this tension [1411.5230]

the massless ν hypothesis

The Massless ν Hypothesis

In quantum field theory: we learn about three types of fermions

$$\mathcal{L}_{\text{Kin.}} = \overline{\psi} i \not{\partial} \psi \qquad \mathcal{L}_{\text{Kin.}} = \overline{\psi} \left(i \not{\partial} - m \right) \psi \qquad \mathcal{L}_{\text{Kin.}} = \frac{1}{2} \overline{\psi} \left(i \not{\partial} - m \right) \psi$$

Weyl fermion $(m = 0)$ **Dirac fermion** $(m \neq 0)$ **Majorana fermion** $(m \neq 0)$

The Massless ν Hypothesis

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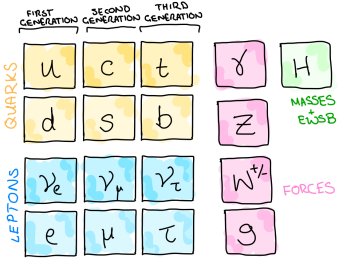
Weyl fermion $(m = 0)$ **Dirac fermion** $(m \neq 0)$ **Majorana fermion** $(m \neq 0)$

- **History:** *Model of Leptons* (Weinberg'67) hypothesizes massless ν (no evidence for $m_\nu \neq 0$)
- **Data:** evidence only for $m_\nu \neq 0$, not whether ν is Dirac or Majorana (more soon!)
- **The** 1/2 **Problem**: What is the Kinetic Lagrangian of the ν realized in nature?



ν masses is physics beyond the SM!!!

Pt2. the ν that nature gave us

ν oscillations

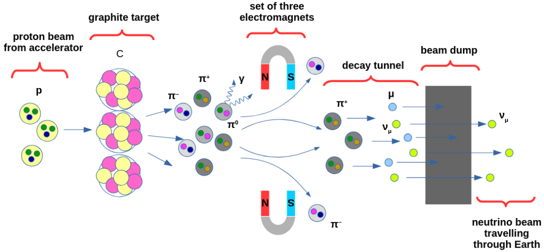
start with a ν beam[1]

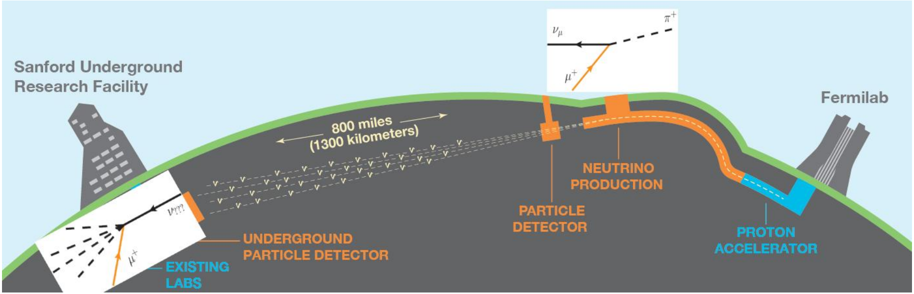
[1] this is not the order in which ν oscillations were discovered

"accelerator ν's," i.e., high-energy ν's beams, are tertiary/3rd-stage beams:

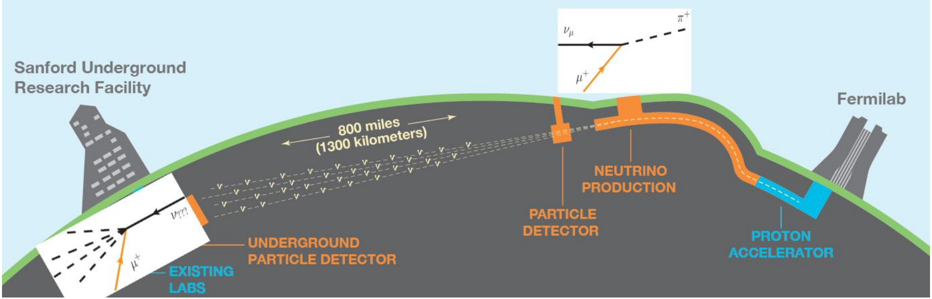
$pA \to \pi^\pm, K^\pm, \ldots \to \ell^\pm \nu$

Lederman, Schwartz, Steinberger (PRL'62) ('88)



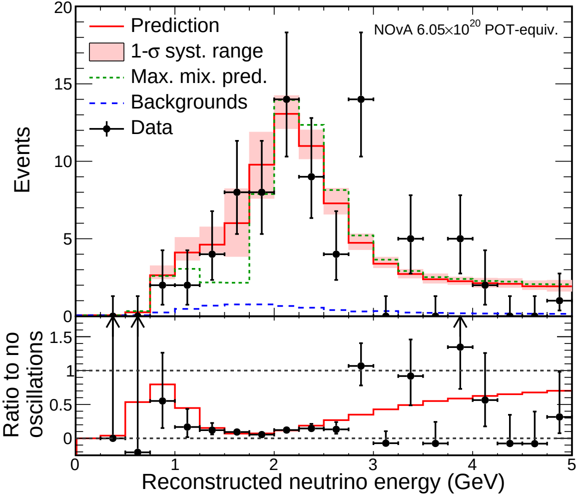


Idea: **count** ν_μ at **near detector** and **compare** to # at **far detector**



Result: far detector reports ν_μ deficit + unexpected apperance of ν_e/ν_τ

(focus on the lower panel!)



Interpretation: neutrinos are transitioning between **flavor eigenstates** and **mass eigenstates**:

$$\nu_{\ell_1} \to \nu_{\text{mass}} \to \nu_{\ell_2}$$

NOνA ν_μ disappearance [1701.05891]

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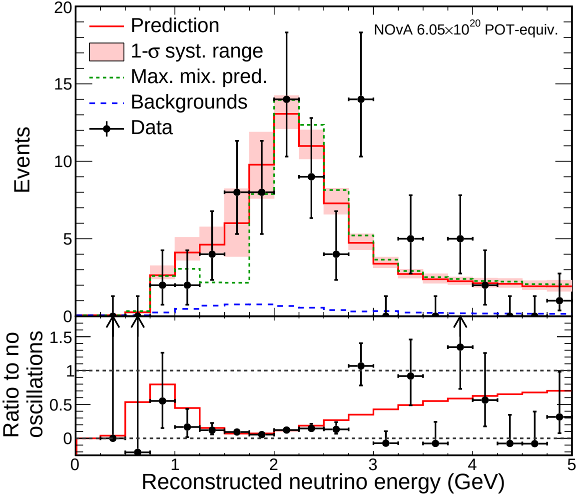

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evidence for ν masses!

('15) SNO, Super-K

NOνA ν_μ disappearance [1701.05891]

the massive ν hypothesis

Consider left-handed (LH), $SU(2)_L$ lepton doublets (**gauge eigenbasis**):

$$L_{aL} = \begin{pmatrix} \nu_a \\ l_a \end{pmatrix}_L, \quad a = 1, 2, 3.$$

The SM $W^{\pm}$ boson coupling to **leptons** in the **flavor eigenbasis** is

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} W_{\mu}^{+} \sum_{l=1}^{3} \left[\overline{\nu_{lL}} \gamma^{\mu} P_L l^{-} \right] \ + \ \text{H.c.}$$

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Supposing $m_\nu \neq 0$, we can rotate ν_l and l into the **mass eigenbasis**:

$$\nu_l = \sum_{m=1}^{3} \Omega_{lm} \nu_m \quad \text{and} \quad l = \sum_{\ell=3}^{\tau} \Omega_{l\ell} \ell$$

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This allows us to describe SM $W^\pm$ boson coupling to ν *with* $m_\nu \neq 0$:

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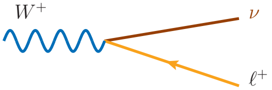
Like the CKM, SM Feynman rules are modified by **PMNS** mixing factor:

$$\Gamma^\mu = \frac{-ig}{\sqrt{2}} \gamma^\mu P_L \to \tilde{\Gamma}^\mu = \frac{-ig}{\sqrt{2}} U^*_{m\ell} \gamma^\mu P_L$$



2-State Neutrino Mixing

Generically, mixing between **flavor eigenstates** and **mass eigenstates** is given by unitary transformation/rotation

$$\underbrace{\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}}_{\text{flavor basis}} = \underbrace{\begin{pmatrix} U_{e1} & U_{e2} \\ U_{\mu 1} & U_{\mu 2} \end{pmatrix}}_{\text{mixing}} \underbrace{\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}}_{\text{mass basis}} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

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For a **two-state system**, the state vector for ν_ℓ $(\ell = e, \mu)$ is simply

$$\underbrace{|\nu_e\rangle}_{\text{flavor basis}} = U_{e1} \underbrace{|\nu_1\rangle}_{\textit{light}} + U_{e2} \underbrace{|\nu_2\rangle}_{\textit{heavy}} = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

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If we treat the spacetime propagation of ν_m $(m = 1, 2)$ as a plane wave, then the evolution from $x^\mu = x_a^\mu$ to $x^\mu = x_b^\mu$ is

$$|\nu_\ell(x_b, x_a)\rangle = U_\ell(x_b, x_a)|\nu_\ell\rangle = U_{\ell 1} U_1(x_b, x_a)|\nu_1\rangle + U_{\ell 2} U_2(x_b, x_a)|\nu_2\rangle$$

Evolution through space and time

Assuming $\hat{p}_\nu = \Delta\hat{x}$, the plane wave evolution over $L = |\vec{x}_b - \vec{x}_a]|$ is

$$U_m(x_b, x_a) = e^{-ip_m\cdot(x_b - x_a)}$$

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$$(E_m\Delta t_m - |\vec{p}_m|L) \approx (E_m - |\vec{p}_m|)\,L$$

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Since $m_2, m_1 \ll E_1, E_2$, the E_m can be approximated as the same:

$$|\nu_e(E, L)\rangle = U_{e1}e^{-im_1^2L/2E}|\nu_1\rangle + U_{e2}e^{-im_2^2L/2E}|\nu_2\rangle$$

$$|\nu_\mu(E, L)\rangle = U_{\mu1}e^{-im_1^2L/2E}|\nu_1\rangle + U_{\mu2}e^{-im_2^2L/2E}|\nu_2\rangle$$

We are now ready to compute oscillation transtions!

Neutrino Oscillation Transitions

To reproduce the ν_μ deficit, consider the $\nu_\mu \to \nu_\mu$ *transition amplitude*:

$$\mathcal{M}(\nu_\mu \to \nu_\mu) \equiv \langle \nu_\mu | \nu_\mu(E, L) \rangle$$

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Since $|\nu_m\rangle$ are mass **eigenstates**, $\langle \nu_{m'} | \nu_m \rangle = \delta_{m'm}$. This implies

$$\mathcal{M}(\nu_\mu \to \nu_\mu) = e^{-i m_1^2 L/2E} |U_{\mu 1}|^2 \langle \nu_1 | \nu_1 \rangle + e^{-i m_2^2 L/2E} |U_{\mu 2}|^2 \langle \nu_2 | \nu_2 \rangle$$

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The $\nu_\mu \to \nu_\mu$ *transition probability* is

$$\Pr(\nu_\mu \to \nu_\mu) = |\mathcal{M}(\nu_\mu \to \nu_\mu)|^2 = |U_{\mu 1}|^4 + |U_{\mu 2}|^4$$

$$+ e^{-i \Delta m_{21}^2 L/2E} |U_{\mu 1}|^2 |U_{\mu 2}|^2 + e^{+i \Delta m_{21}^2 L/2E} |U_{\mu 1}|^2 |U_{\mu 2}|^2$$

note: $\Delta m_{21}^2 \equiv (m_2^2 - m_1^2)$

Some Quick Algebra

Recalling that $U_{e1} = U_{\mu 2} = \cos\theta$ and $U_{e2} = -U_{\mu 1} = \sin\theta$,

$$\Pr(\nu_\mu \to \nu_\mu) = \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta\cos\left[\frac{\Delta m^2_{21} L}{2E}\right]$$

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Lots to unpack:

$$\Pr(\nu_\mu \to \nu_\mu) = \underbrace{1}_{\text{unitarity}} - \underbrace{\sin^2(2\theta)}_{\text{amplitude of dip}} \quad \underbrace{\sin^2\left[\frac{\Delta m^2_{21} L}{4E}\right]}_{\text{spacing between beats}}$$

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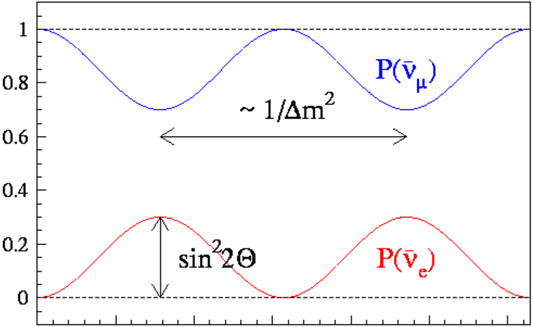
By conservation of probability $1 = \Pr(\nu_\mu \to \nu_\mu) + \Pr(\nu_\mu \to \nu_e)$, so the $\nu_\mu \to \nu_e$ *appearance probability* is

$$\Pr(\nu_\mu \to \nu_e) = 1 - \Pr(\nu_\mu \to \nu_\mu) = \sin^2(2\theta)\sin^2\left[\frac{\Delta m_{21}^2 L}{4E}\right]$$

Understanding Neutrino Oscillation Plots

$$\Pr(\nu_\mu \to \nu_\mu) = \underbrace{1}_{\text{unitarity}} - \underbrace{\sin^2(2\theta)}_{\text{minimum of dip}} \quad \underbrace{\sin^2\left[\frac{\Delta m^2_{21} L}{4E}\right]}_{\text{spacing between beats}}$$

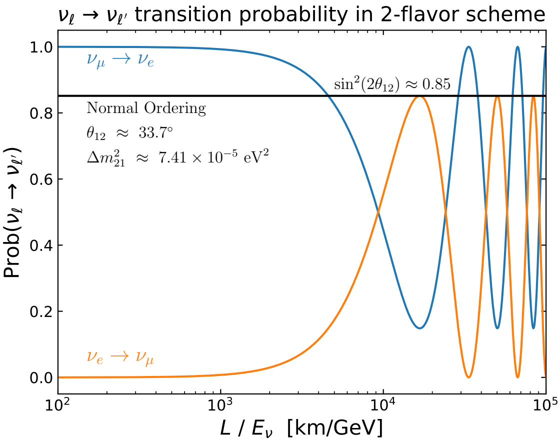
$$\Pr(\nu_\mu \to \nu_e) = \underbrace{\sin^2(2\theta)}_{\text{maximum of peak}}$$



Understanding Neutrino Oscillation Plots

With updated inputs: gitlab.cern.ch/riruiz/public-projects/-/tree/master/NuPhysSandbox

$$\Pr(\nu_\mu \to \nu_\mu) = 1 - \sin^2(2\theta)\sin^2\left[\frac{\Delta m^2_{21} L}{4E}\right] \qquad \Pr(\nu_\mu \to \nu_e) = \sin^2(2\theta)$$



The 3×3 Paradigm

To date, all oscillation data can be described within the 3×3 Paradigm

- 3 ν_ℓ (flavor states) $\implies$ 3 mixing angles
- 3 ν_k (mass states) $\implies$ 2 mass splittings **one may be massless!**
- 1 CP phase (if Dirac); +2 CP phases (if Majorana)

$$U_{\rm PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{\rm CP}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{\rm CP}} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$s_{ij} = \sin\theta_{ij}, \ \ c_{ij} = \cos\theta_{ij}$

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NuFIT 5.3 (2024)

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$
	$\theta_{12}/^\circ$	$33.66^{+0.73}_{-0.70}$	$31.60 \to 35.94$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$
	$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.407 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
	$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
	$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00058}$	$0.02029 \to 0.02391$	$0.02219^{+0.00059}_{-0.00057}$	$0.02047 \to 0.02396$
	$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.90$
	$\delta_{CP}/^\circ$	197^{+41}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.511^{+0.027}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.024}$	$-2.581 \to -2.409$
		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 9.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$
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	$\sin^2\theta_{23}$	$0.454^{+0.019}_{-0.016}$	$0.411 \to 0.606$	$0.568^{+0.016}_{-0.021}$	$0.412 \to 0.611$
	$\theta_{23}/^\circ$	$42.3^{+1.1}_{-0.9}$	$39.9 \to 51.1$	$48.9^{+0.9}_{-1.2}$	$39.9 \to 51.4$
	$\sin^2\theta_{13}$	$0.02224^{+0.00056}_{-0.00057}$	$0.02047 \to 0.02397$	$0.02222^{+0.00069}_{-0.00057}$	$0.02049 \to 0.02420$
	$\theta_{13}/^\circ$	$8.58^{+0.11}_{-0.11}$	$8.23 \to 8.91$	$8.57^{+0.13}_{-0.11}$	$8.23 \to 8.95$
	$\delta_{CP}/^\circ$	232^{+39}_{-25}	$139 \to 350$	273^{+24}_{-26}	$195 \to 342$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.505^{+0.024}_{-0.026}$	$+2.426 \to +2.586$	$-2.487^{+0.027}_{-0.024}$	$-2.566 \to -2.407$

Pt3. Theory consequences of ν masses

“The $1/2$ Problem” from a different perspective

Fermion masses and chirality

For fermions chirality and masses are linked

[2] friendly reminder: $\overline{\psi} = \psi^{\dagger}\gamma^{0}$ and $P_L\gamma^0 = \gamma^0 P_R$.
we also have $P_L P_L \psi_L = P_L \psi_L = \psi_L$ (also true for R).

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$$\mathcal{L} = m\,\overline{\psi}\,\psi$$

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Conclusion: only (LR) and (RL) survive since $P_L \cdot P_R = P_R \cdot P_L = 0$

- if $\psi_R = (\psi_L)^c$, then ψ is a Majorana fermion
- if $\psi_R \neq (\psi_L)^c$, then ψ is a Dirac fermion

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In SM: Higgs field $(\Phi_{\rm SM})$ couples LH and RH chiral fermions

- Yukawa couple **opposite chirality**, e.g., $\mathcal{L}_{\rm Yuk.} = y_e^{ij}\,\overline{L_L^i}\,\Phi\,e_R^j + {\rm H.c.}$
- Covariant derivatives couple **same chirality**, e.g., $\mathcal{L}_{\rm Kin.} = \overline{L_L}\,i\,\not{D}\,L_L$

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accommodating Dirac masses in the SM (1/2)

To generate Dirac masses for ν like other SM fermions, we need ν_R

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$$= \underbrace{-y_\nu\langle\Phi\rangle}_{=m_D}\overline{\nu_L}\nu_R \ + \ \text{H.c.} \ + \ \dots$$

accommodating Dirac masses in the SM (2/2)

Adding ν_R's to SM seems trivial but...

- ν_R's are neutral under all SM gauge interactions (before and after EWSB)
- If ν_R's are **Majorana fermions**, must **include** RH Majorana masses

$$\mathcal{L}_M = \frac{1}{2}\mu_R\overline{\nu_R^c}\nu_R + \mathrm{H.c.}$$

- If ν_R's are **Dirac fermions**, must **forbid** RH Majorana masses by imposing some new symmetry/conservation law

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Adding ν_R's to the **SM** means:

- a new scale μ_R that **breaks lepton number** symmetry
- a new symmetry that **conserves lepton number** symmetry
- both e.g., spontaneous $B-L$ breaking

However, the origin of $m_\nu \neq 0$ might not even involve ν_R

to date, data gives no preference for Dirac or Majorana nature

the SM does provide some theoretical guidance!

$m_\nu \neq 0 \implies$ **new physics must exist**

Ma('98) + others

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$$m_\nu \neq 0 + \text{left}-\text{handed (LH) weak currents}$$

$(renormalizability)$

LH Majorana mass : $\frac{1}{2} m_\nu^L \overline{\nu_L} \nu_L^c$

Dirac mass : $m_\nu^D \overline{\nu_L} \nu_R$

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$m_\nu^L = y\langle\Delta\rangle$ or new dynamics

$m_\nu^D = y\langle\Phi_{\text{SM}}\rangle$

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$m_\nu^D = y\langle\Phi_{\rm SM}\rangle$

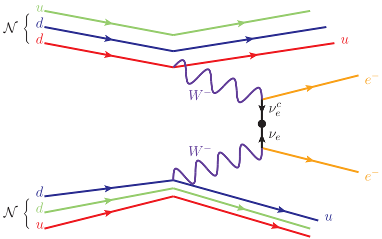
$m_\nu \neq 0$ **+ renormalizability + gauge inv.** $\implies$ **new particles**

New particles must couple to $\Phi_{\rm SM}$ and L, often inducing non-conservation of **lepton number** and/or **lepton flavor**

friendly reminder of lepton symmetries

Lepton Number Violation (LNV) =
(#leptons – #antileptons) not conserved

e.g, nuclear $0\nu\beta\beta$ decay of heavy isotopes $(A, Z) \rightarrow (A, Z+2) + e^- e^-$



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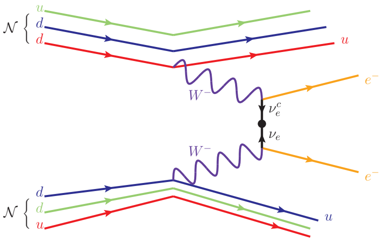
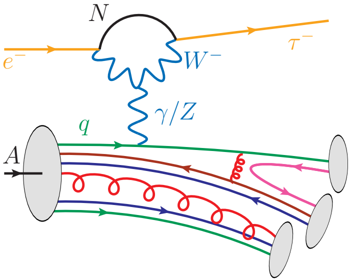




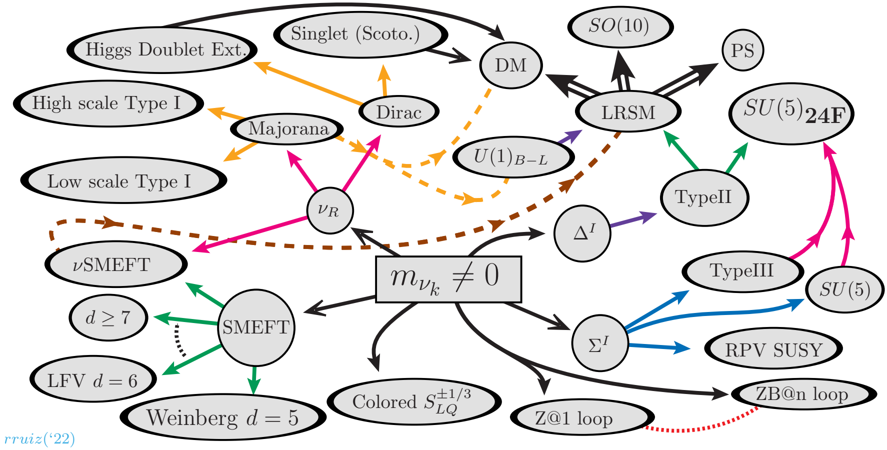
Lepton Flavor Violation (LFV) $=$
(#lepton species – #antilepton species) not conserved,

e.g, $e^- \to \tau^-$ conversion in deeply inelastic scattering (DIS)

lepton number and **lepton flavor** are **accidentally** conserved **in the SM**

Solution to $m_\nu \neq 0$ can be realized in ***many*** ways!

Minkowski ('77); Yanagida ('79); Glashow & Levy ('80); Gell-Mann et al., ('80); Mohapatra & Senjanović ('82); + *many* others



why the obsession with LNV?

The Black Box Theorem

In '82, Schechter & Valle published (PRD'82) a seminal finding:

- Suppose $0\nu\beta\beta$ is mediated within "a 'natural' gauge theory" a $\Delta L = -2$ process →
- u, d and e^- all carry weak charges

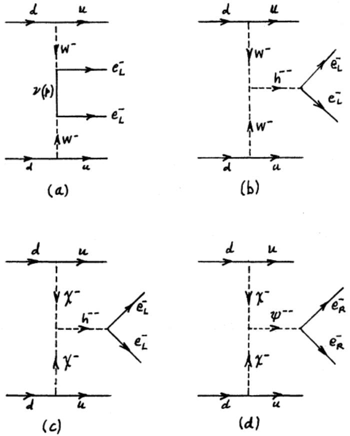


FIG. 1. Diagrams for neutrinoless double-β decay in an SU(2)×U(1) gauge theory. The standard diagram is Fig. 1(a). It is the only one which contains a virtual neutrino (of four-momentum p). d and u are the down and up quarks.

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- Suppose $0\nu\beta\beta$ is mediated within "a 'natural' gauge theory" a $\Delta L = -2$ process →
- u, d and e^- all carry weak charges
- always possible to build a many-loop, 2-point graph with external ν_L, ν_L^c
- $0\nu\beta\beta$ generates a **Majorana mass** for ν
- holds generally for other $\Delta L \neq 0$ process

for further discussions, see:

Hirsch, et al [hep-ph/0608207] and Pascoli, et al [1712.07611]

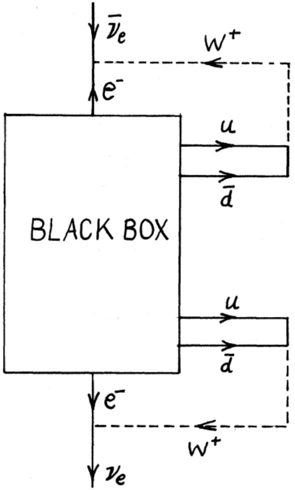


FIG. 2. Diagram showing how any neutrinoless double-β decay process induces a $\bar{\nu}_e$-to-ν_e transition, that is, an effective Majorana mass term.

LNV $\Longleftrightarrow$ Majorana nature of ν

well, why not look for $0\nu\beta\beta$?

... is it hard? ☺

☺☺☺

quick review (1 slide)

The SM $W^\pm$ boson coupling to **leptons** in the **flavor eigenbasis** is

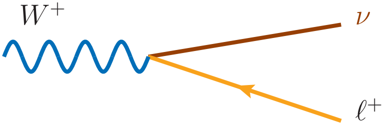
$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} W^+_\mu \sum_{l=1}^{3} \left[\overline{\nu_{lL}} \gamma^\mu P_L l^-\right] + \text{H.c.}$$

The SM $W^\pm$ boson coupling to **leptons** in the **mass eigenbasis** is

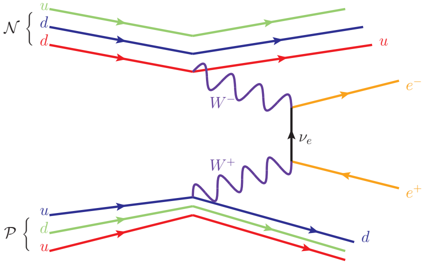
$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} W^+_\mu \sum_{\ell=e}^{\tau} \sum_{m=1}^{3} \left[\overline{\nu_m} \underbrace{U^*_{m\ell}}_{U^*_{m\ell} \equiv \sum_l \Omega^*_{ml} \Omega_{l\ell}} \gamma^\mu P_L \ell^-\right] + \text{H.c.}$$

Like the CKM, SM Feynman rules are modified by **PMNS** mixing factor:

$$\Gamma^\mu = \frac{-ig}{\sqrt{2}} \gamma^\mu P_L \to \tilde{\Gamma}^\mu = \frac{-ig}{\sqrt{2}} U^*_{m\ell} \gamma^\mu P_L$$



Consider the LNC process $\mathcal{N}\mathcal{P} \to \mathcal{P}'\mathcal{N}' e^+ e^-$ as governed by the SM



The helicity amplitude for the LNC subprocess $q_1 q_2 \to \ell_1^- \ell_2^+ q_1' q_2'$ is

$$\mathcal{M}_{LNC} = J^{\mu}_{q_1 q_1'} J^{\nu}_{q_2 q_2'} \Delta^{W}_{\mu\rho} \Delta^{W}_{\nu\sigma} T^{\rho\sigma}_{LNC} \mathcal{D}(p_N)$$

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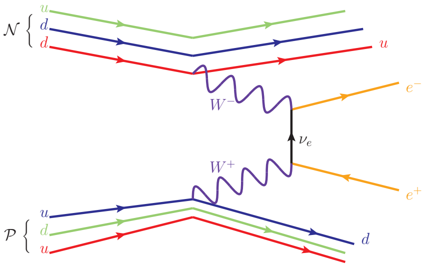

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$$T^{\rho\sigma}_{LNC} = \overline{u_L}(p_1) U_{ek} \gamma^{\rho} P_L \times \Big(\underbrace{\not{p}_k}_{\text{LH helicity state}} + \underbrace{m_k}_{P_L m_k P_R = 0} \Big) \times U_{ek} \gamma^{\sigma} P_L v_R(p_2)$$

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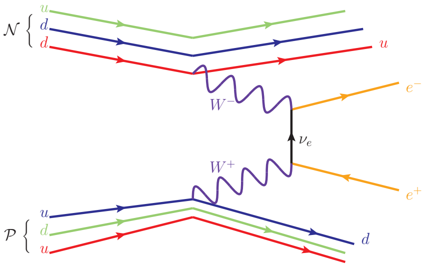

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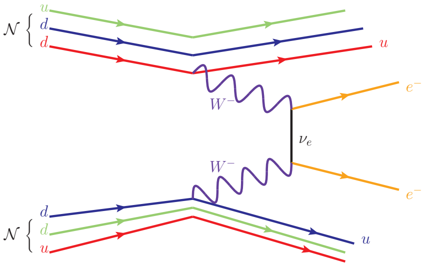
$$\implies \mathcal{M}_{LNC} \sim \frac{p_k}{(p_k^2 - m_k^2)} U_{ek}^2$$

scales with momentum transfer!

Consider the LNV process $\mathcal{N}\mathcal{N} \to \mathcal{P}'\mathcal{P}'e^-e^-$ in minimal SM+m_ν



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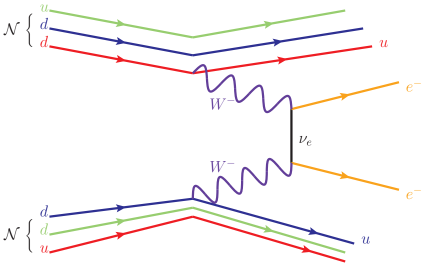

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Intuition: CPT Theorem $\implies$ CT-inversion = P-inversion

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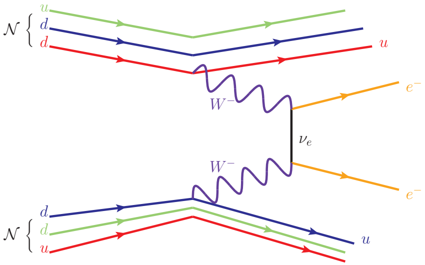

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$$\implies \mathcal{M}_{LNV} \sim \frac{m_k}{(p_k^2 - m_k^2)} U_{ek}^2$$

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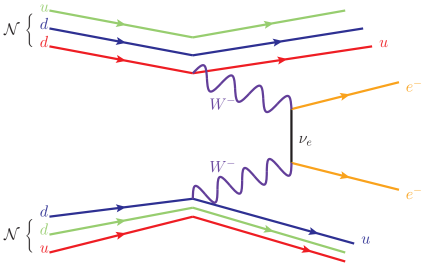

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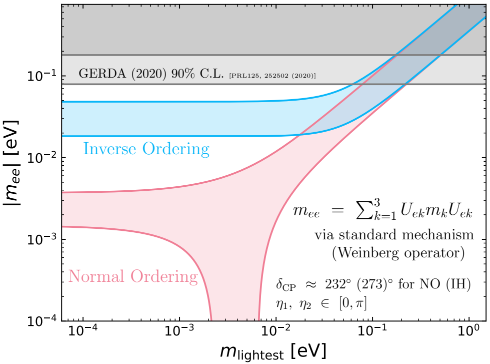
$$\implies \mathcal{M}_{LNV} \sim \frac{m_k}{(p_k^2 - m_k^2)} U_{ek}^2 \approx \frac{m_k}{p_k^2} U_{ek}^2 \times \left[1 + \mathcal{O}\left(\frac{m_k^2}{p_k^2} \right) \right]$$

scales with mass!

Plotted: Excluded/allowed "effective $\beta\beta$ Majorana mass" vs lightes m_ν

$$1/T_{1/2}^{0\nu\beta\beta} = \underbrace{G_{0\nu\beta\beta}}_{\text{phase space}} \quad m_p^2 \quad \underbrace{|\mathcal{A}|^2 \; |m_{ee}|^2}_{\text{matrix element}}, \qquad m_{ee} = \sum_{k=1}^{3} U_{ek} m_k U_{ek}$$



gitlab.cern.ch/riruiz/public-projects/-/tree/master/NuPhysSandbox

Plotted: Excluded/allowed "effective $\beta\beta$ Majorana mass" vs lightes m_ν

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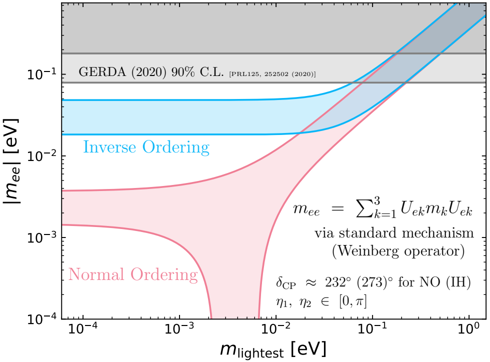

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Weinberg operator only SMEFT operator at $d = 5$:

$$\mathcal{L} = \frac{C_5^{\ell\ell'}}{\Lambda}[\Phi \cdot \overline{L_\ell^c}][L_{\ell'} \cdot \Phi]$$

generates ν mass matrix:

$$m_{\ell\ell'} = C_5^{\ell\ell'} \langle\Phi\rangle^2 / 2\Lambda$$

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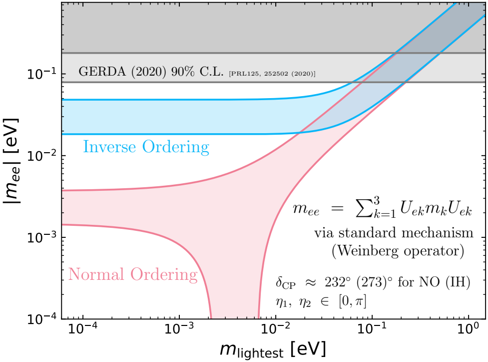

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Searches for nuclear $0\nu\beta\beta$ decay set stringent constraints, e.g.,

GERDA [2009.06079]

$$C_5^{ee}/\Lambda \gtrsim (3.3 - 7.6) \times 10^{14} \ \text{GeV}$$

Important: sensitivity is model dependent!!!

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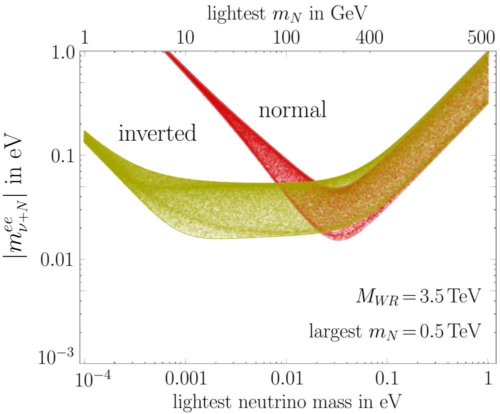
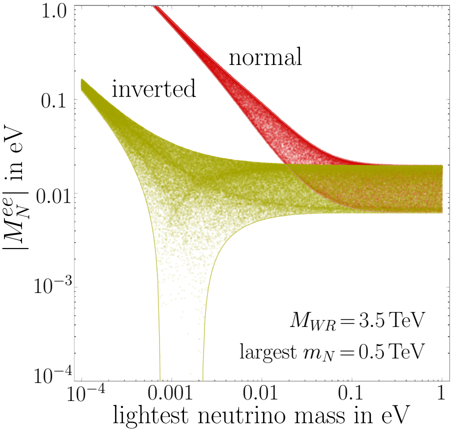




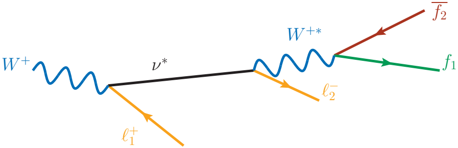
eg., Left-Right Symm. Model, Tello, et al [1011.3522]

how about looking for LNV elsewhere?

The Dirac-Majorana Confusion Theorem

In '82, Kayser also published (PRD'82) a seminal finding:

refined later by Mohapatra & Pal ('98)



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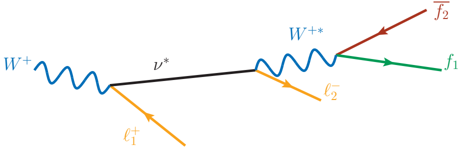

The helicity amplitude for the LNC process $W^+ \to \ell_1^+ \ell_2^- f \overline{f'}$ is

$$\mathcal{M}_{LNC} = \varepsilon_\mu \; T^{\rho\mu}_{LNC} \; \Delta^W_{\nu\rho} \; J^\nu_{f_1 f_2} \mathcal{D}(p_\nu)$$

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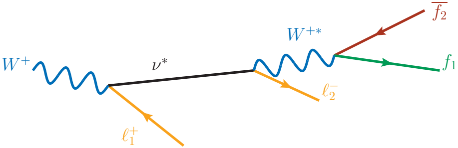

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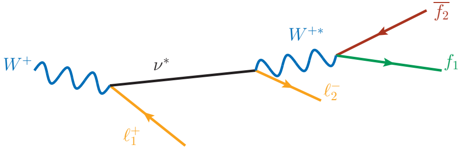

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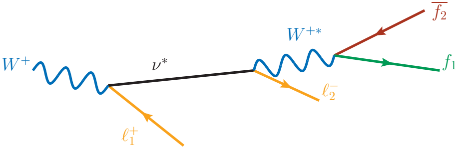

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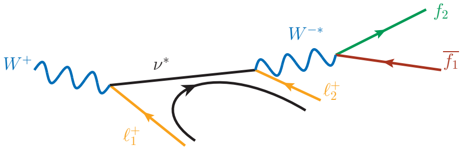
$$\implies \mathcal{M}_{LNC} \sim \frac{p_\nu}{p_\nu^2 - m_\nu^2}$$



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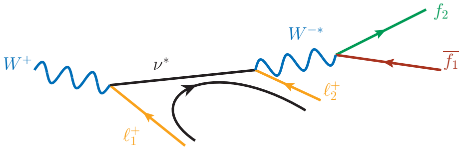

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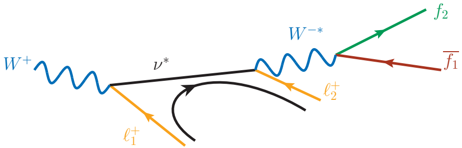

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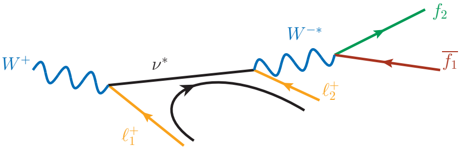

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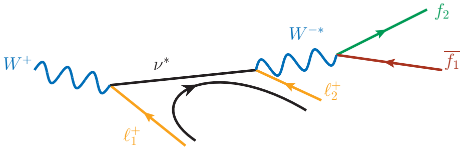

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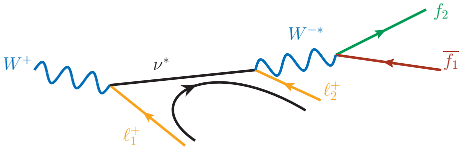

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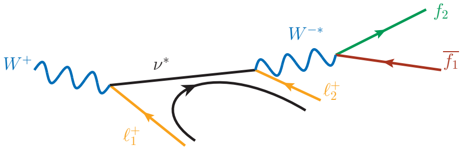
Confusion Theorem: In SM + Majorana ν, the rate of **LNV** $\sim \mathcal{O}(m_\nu)$; in the limit where $(m_\nu^2/M_W^2) \to 0$, Dirac behavior recovered



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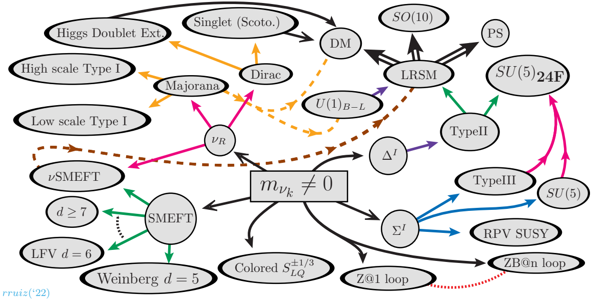
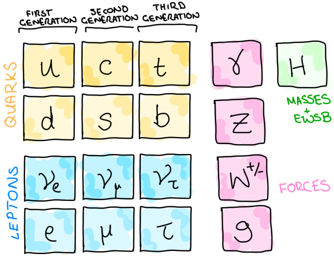
holds for other gauge theories with Majorana fermions Han, RR, et al [1211.6447]; RR [2008.01092]

so much not covered



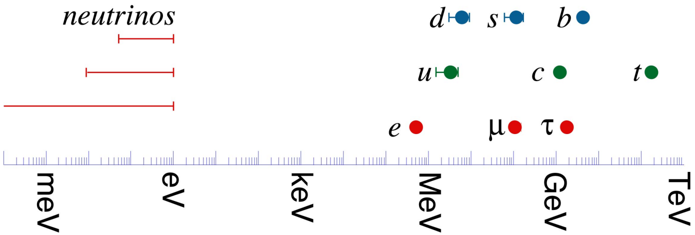
Summary and Outlook

ν are an integral part of the SM



- only color- and electrically neutral elementary fermions
- necessary to ensure gauge cancellations in scattering amplitudes

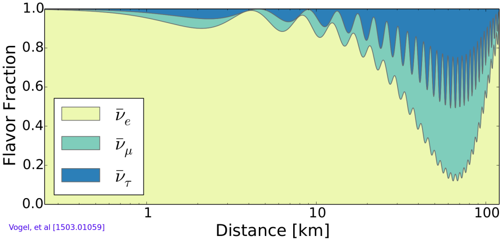
Unambiguously, data show that ν have nonzero masses



– contrary to SM ('15)

– **success** of the ν oscillation paradigm opens **many** questions

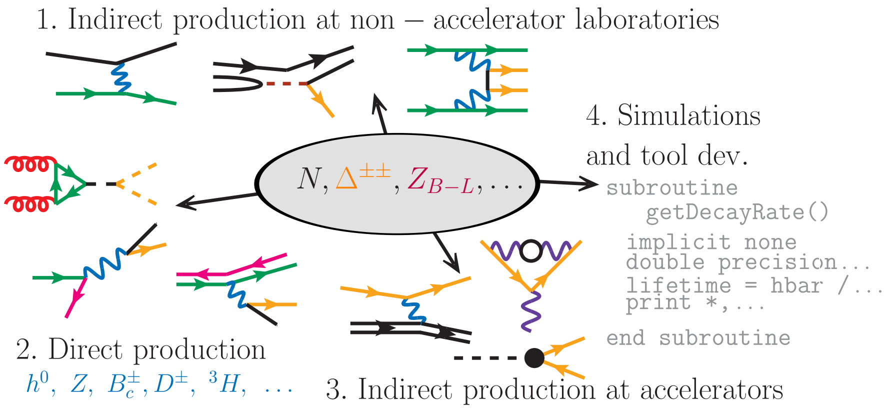
– with general arguments, new particles must exist **(unclear what kind)**



Vogel, et al [1503.01059]

one consequence?

broad implications for experimental physics



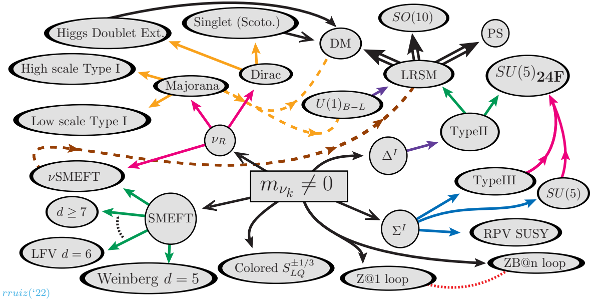
Many complementary ways to explore consequences of m_ν

- colliders and ℓ-DIS facilities $\ell\ell$, ℓh, hh ☺
- short and long baseline experiments and νDIS facilities ☺
- space! (underground-, ground-, water-, ice-based telescopes) ☺

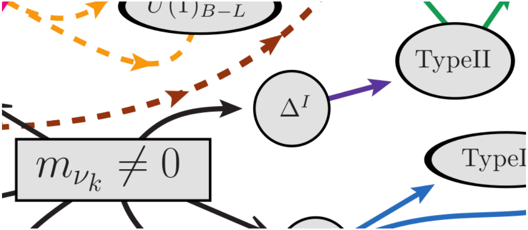
Thank you for your time.

backup

Pt4. ν mass models



Type II Seesaw[4]



[4] Konetschny and Kummer ('77); Schechter and Valle ('80); Cheng and Li ('80); Lazarides, et al ('81); Mohapatra and Senjanovic ('81)

The Type II Seesaw is special: generates m_ν ***without*** hypothesizing ν_R

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Hypothesize a **scalar** $SU(2)_L$ triplet with **lepton number** $L = -2$

$$\hat{\Delta} = \frac{1}{\sqrt{2}}\begin{pmatrix} \Delta^+ & \sqrt{2}\Delta^{++} \\ \sqrt{2}\Delta^0 & -\Delta^+ \end{pmatrix}, \quad \text{with} \quad \mathcal{L}_{\Delta\Phi} \ni \mu_{h\Delta}\left(\Phi_{\rm SM}{}^\dagger \hat{\Delta} \cdot \Phi_{\rm SM}{}^\dagger + \text{H.c.}\right)$$

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The mass scale $\mu_{h\Delta}$ breaks lepton number, and induces $\langle\Delta\rangle \neq 0$:

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$\Longrightarrow$ **left-handed** Majorana masses for ν

$$\Delta\mathcal{L} = -\frac{y_\Delta^{ij}}{\sqrt{2}} \overline{L^c} \hat{\Delta} L = -\frac{y_\Delta^{ij}}{\sqrt{2}} \left(\overline{\nu^{jc}} \quad \overline{\ell^{jc}}\right) \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix} \begin{pmatrix} \nu^i \\ \ell^i \end{pmatrix}$$

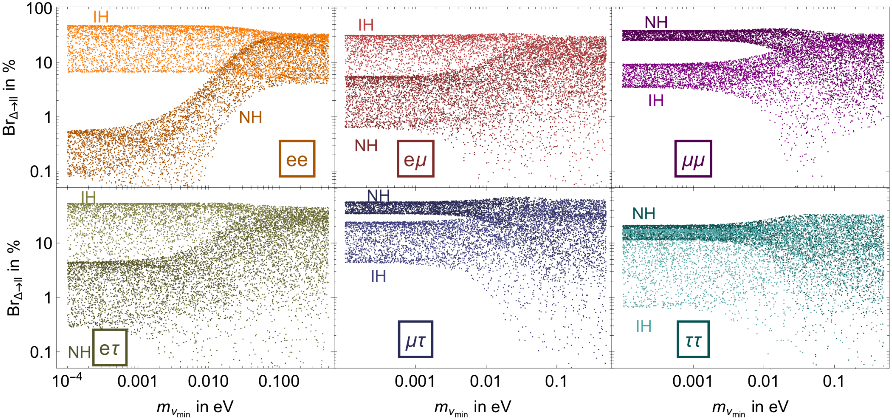
$$\ni -\frac{1}{2} \underbrace{\left(\sqrt{2} y_\Delta^{ij} v_\Delta\right)}_{= m_\nu^{ij}} \overline{\nu^{jc}} \nu^i$$

Fewer free parameters $\implies$ richer experimental predictions

Fileviez Perez, Han, Li, et al, [0805.3536], Crivellin, et al [1807.10224], Fuks, Nemevšek, RR [1912.08975] + others

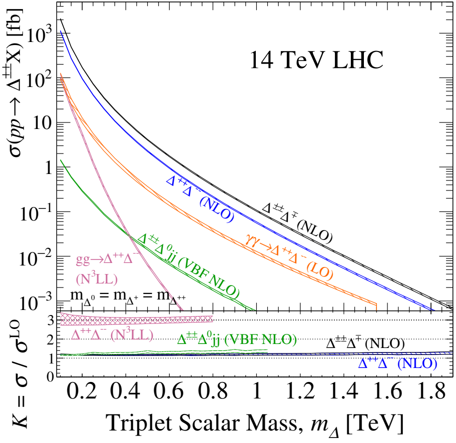
- **Example:** Δ decay rates encode inverse (IH) vs normal (NH) ordering of light neutrino masses

$$\Gamma(\Delta^{\pm\pm} \to \ell_i^\pm \ell_j^\pm) \sim y_\Delta^{ij} \sim (U^*_{\rm PMNS}\, \tilde{m}_\nu^{\rm diag}\, U^\dagger_{\rm PMNS})_{ij}$$

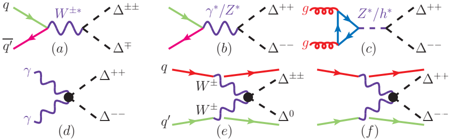


$\Delta^{\pm\pm}$, $\Delta^{\pm}$, Δ^{0}, ξ^{0} all couple to W, Z, γ via gauge couplings

($\Longrightarrow$ unambiguous xsec prediction!)



Fuks, Nemevšek, RR [1912.08975]



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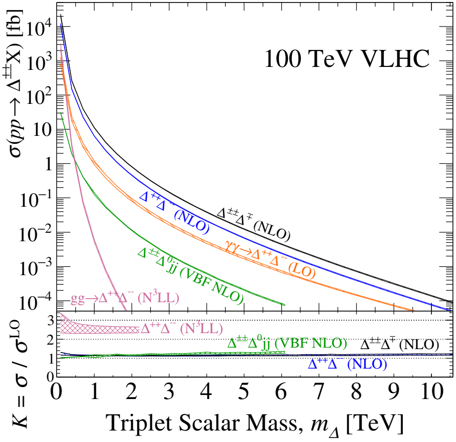

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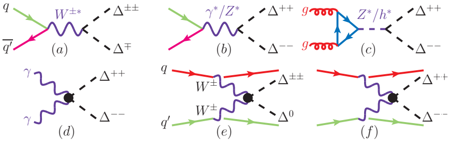

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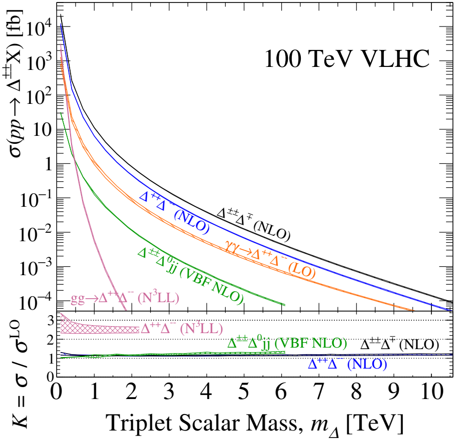

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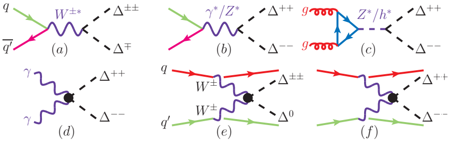
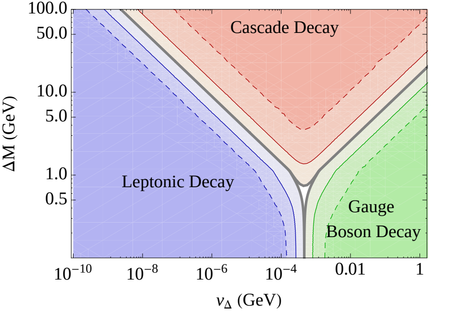


Preferred decay modes of $\Delta^{\pm\pm}$

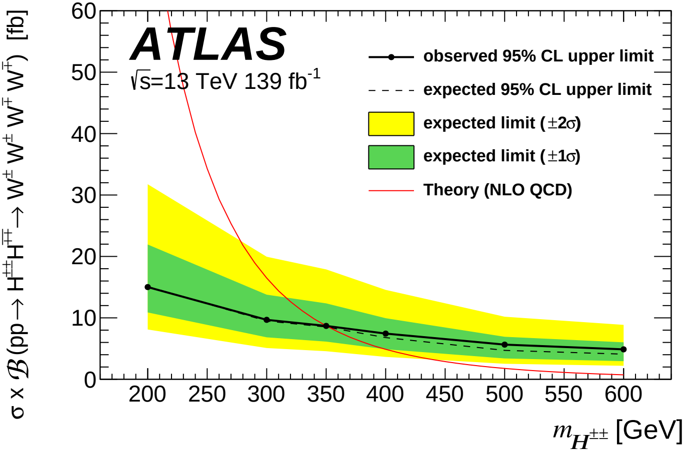
$(\Delta M = m_{++} - m_{+})$



Melfo, Nemevšek, Nesti, Senjanovic, Zhang [1108.4416]

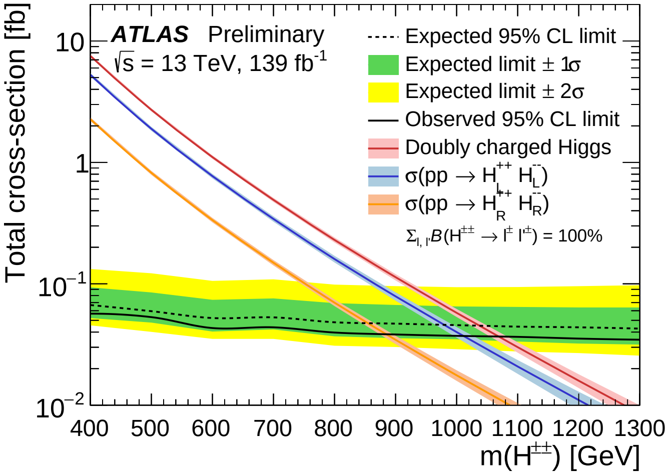
LHC limits on pair production

$$pp \rightarrow \Delta^{++}\Delta^{--} \rightarrow 4W^{\pm} \rightarrow 2-4\ell^{\pm} + / E_T + X \quad (\ell = e, \mu) \text{ [2101.11961]}$$



LHC limits on pair production

$$pp \rightarrow \Delta^{++}\Delta^{--} \rightarrow 4\ell^{\pm} + X \qquad (\ell = e, \mu)$$ [2211.07505]



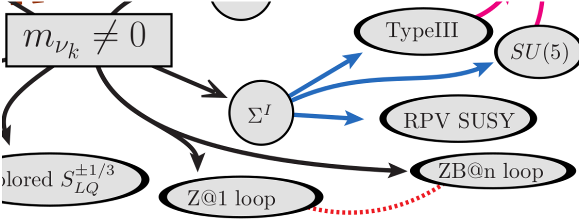
What if $\Delta^{\pm\pm}$, $\Delta^{\pm}$ are discovered?

celebrate! ☺

except... ☹

$\Delta^{\pm\pm}$, $\Delta^{\pm}$ are not unique in new physics models

Zee-Babu Model[5]



[5] Zee ('85×2), Babu ('88)

Zee-Babu model generates m_ν radiatively ***without*** hypothesizing ν_R

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Hypothesize two **scalar** $\mathrm{SU}(2)_L$ singlets k, h with weak hypercharge $Y = -2, -1$ ($\implies Q_k = -2, Q_h = -1$) with **lepton number** $L = -2$

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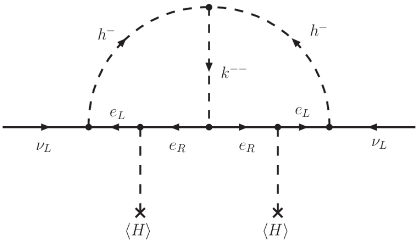
$$\mathcal{L}_{\rm ZB} = \mathcal{L}_{\rm SM} + (D_\mu k)^\dagger (D^\mu k) + (D_\mu h)^\dagger (D^\mu h) + + \left(\mu_{\not{L}}\ hhk^\dagger + \text{H.c.}\right)$$
$$\left[f_{ij}\ \overline{\tilde{L}^i} L^j h^\dagger + g_{ij}\ \overline{(e_R^c)^i} e_R{}^j k^\dagger + \text{H.c.}\right] + \dots$$



[1402.4491]

The mass scale $\mu_{\not{L}}$ breaks lepton number, and induces $m_\nu \neq 0$:

$$\left(\mathcal{M}_\nu^{\rm flavor}\right)_{ij} = 16\mu_{\not{L}}\ f_{ia}\ m_a\ g^*_{ab}\ \mathcal{I}_{ab}(r)\ m_b\ f_{jb}.$$

Few free parameters $\Longrightarrow$ ric experimental predictions

Nebot,et al [0711.0483]; Ohlsson, Schwetz, Zhang [0909.0455]; Herrero-Garcia, Nebot, Rius, et al [1402.4491]; + others

- E.g., $k^{\pm\pm}$, $h^{\pm}$ couplings to leptons encode oscillation physics

Normal ordering:

$$\frac{f_{e\tau}}{f_{\mu\tau}} = \tan\theta_{12}\frac{\cos\theta_{23}}{\cos\theta_{13}} + \tan\theta_{13}\sin\theta_{23}e^{-i\delta}$$

$$\frac{f_{e\mu}}{f_{\mu\tau}} = \tan\theta_{12}\frac{\cos\theta_{23}}{\cos\theta_{13}} - \tan\theta_{13}\sin\theta_{23}e^{-i\delta}$$

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Inverse ordering:

$$\frac{f_{e\tau}}{f_{\mu\tau}} = -\frac{\sin\theta_{23}}{\tan\theta_{13}}e^{-i\delta},$$
$$\frac{f_{e\mu}}{f_{\mu\tau}} = \frac{\cos\theta_{23}}{\tan\theta_{13}}e^{-i\delta},$$
$$\frac{f_{e\tau}}{f_{e\mu}} = -\tan\theta_{23}.$$

Few free parameters $\implies$ ric experimental predictions

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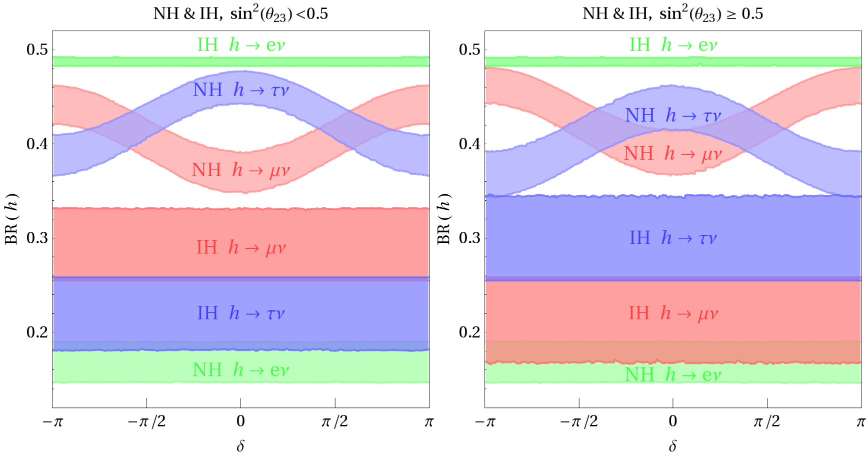
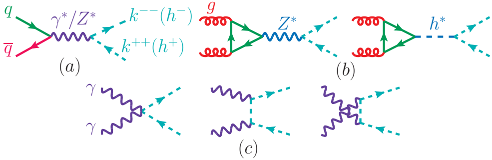
- E.g., $k^{\pm\pm}$, $h^{\pm}$ decay rates encode IH vs NO



$k^{\pm\pm}$, $h^{\pm}$ couple directly to Z, γ via gauge couplings ($\Longrightarrow$ unambiguous xsec prediction!)

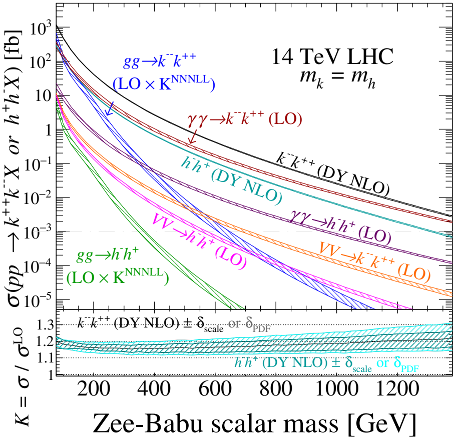


Many production channels but most studies focus on $pp \to k^{++}k^{--}$

If $k^{\pm\pm}$ is the lightest state, then decay rates set by oscillation parameters

(I find this really, really cool ☺)

Discerning from Type II Seesaw is actually difficult



RR [2206.14833]

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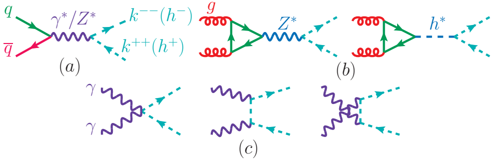

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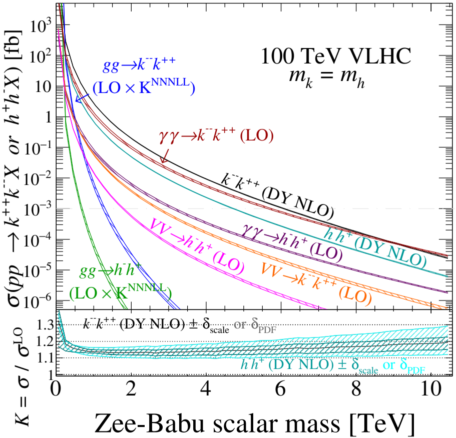

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Guidance from oscillation data

The ratios of $h^{\pm} \to \ell\nu$ couplings are fixed by oscillation data

- ν cannot be tagged at the LHC
- LHC only sensitive to sum over $\nu \implies$ inclusive w.r.t. ν

From flavor-exclusive decay rates:

$$\Gamma(h^{\pm} \to \ell\nu'_{\ell}) = \frac{|f_{\ell\ell'}|^2}{4\pi}\ m_h \left(1 - \frac{m_\ell^2}{m_h^2}\right)$$

define flavor-inclusive decay rates:

$$\Gamma(h^{\pm} \to e^{\pm}\nu_X) = \sum_{\ell=e}^{\tau}\ \Gamma(h^{\pm} \to e^{\pm}\nu_\ell)$$

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$$\mathcal{R}^h_{e\mu} = \frac{\mathrm{BR}(h^\pm \to e^\pm\nu_X)}{\mathrm{BR}(h^\pm \to \mu^\pm\nu_X)} = \frac{|f_{e\mu}|^2 + |f_{e\tau}|^2}{|f_{e\mu}|^2 + |f_{\mu\tau}|^2} = \frac{|\frac{f_{e\mu}}{f_{\mu\tau}}|^2 + |\frac{f_{e\tau}}{f_{\mu\tau}}|^2}{|\frac{f_{e\mu}}{f_{\mu\tau}}|^2 + 1}$$

(equivalent to measuring cross section ratio!)

Using NuFit(v5.1)

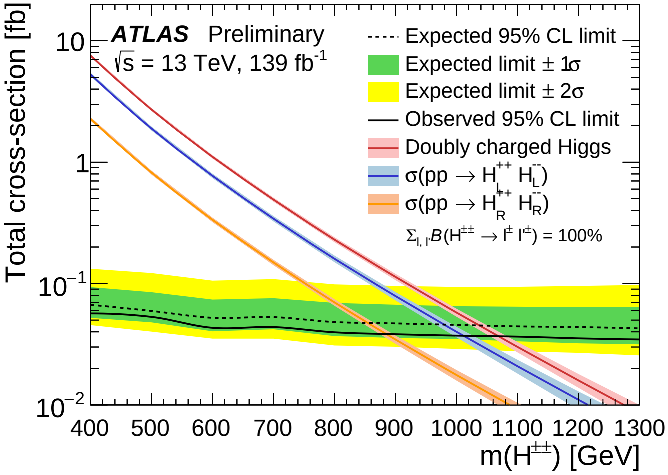
$\mathcal{R}^h_{e\mu}\Big|_{\mathrm{NO}} \approx 0.313^{+55\%}_{-20\%}$ at 3σ

$\mathcal{R}^h_{e\mu}\Big|_{\mathrm{IO}} \approx 0.715^{+3\%}_{-11\%}$ at 3σ

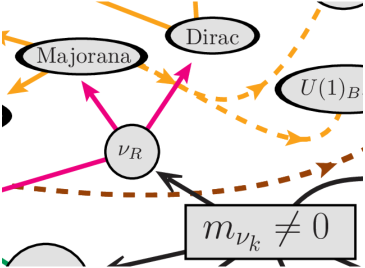
LHC limits on pair production

first direct search for ZB scalars at colliders

$$pp \rightarrow \Delta^{++}\Delta^{--} \rightarrow 4\ell^{\pm} + X \qquad (\ell = e, \mu) \text{ [2211.07505]}$$



right-handed neutrinos[6]



[6] For reviews at colliders, see Cai, Han, Li, RR [1711.02180] and Pascoli, RR, Weiland [1812.08750]

To generate Dirac masses for ν like other SM fermions, we need ν_R

$$\mathcal{L}_{\nu\ \text{Yuk.}} = -y_\nu \overline{L}\tilde{\Phi}\nu_R + H.c. = -y_\nu \begin{pmatrix}\overline{\nu_L} & \overline{\ell_L}\end{pmatrix} \begin{pmatrix}\langle\Phi\rangle + h \\ 0\end{pmatrix} \nu_R + H.c.$$

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ν_R do not exist in the SM, so **hypothesize** that they do and $\nu_R = \nu_R^c$:

$$\implies \mathcal{L}_{\text{mass}} = \frac{-1}{2} \underbrace{\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix}}_{\text{chiral state}} \underbrace{\begin{pmatrix} 0 & m_D \\ m_D & \mu_{\not{L}} \end{pmatrix}}_{\text{mass matrix (chiral basis)}} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$

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After diagonalizing the mass matrix, identify ν_L (chiral eigenstate) in the SM as a linear combination of mass eigenstates:

$$\underbrace{|\nu_L\rangle}_{\text{chiral state}} = \cos\theta \underbrace{|\nu\rangle}_{\text{light mass state}} + \sin\theta \underbrace{|N\rangle}_{\text{heavy mass state (this is a prediction!)}}$$

For ***discovery purposes***, paramerize active-sterile neutrino mixing :

Atre, Han, et al [0901.3589]

$$\underbrace{\nu_{\ell L}}_{\text{flavor basis}} \approx \underbrace{\sum_{m=1}^{3} U_{\ell m}\nu_m + V_{\ell m'=4} N_{m'=4}}_{\text{mass basis. can be Dirac or Maj.}} \quad \text{(neglect heavier } N_{m'}\text{)}$$

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The SM W couplings to **leptons** in the **flavor basis** are

$$\mathcal{L}_{\text{Int.}} = -\frac{g_W}{\sqrt{2}} W_\mu^- \sum_{\ell=e}^{\tau} \left[\overline{\ell}\gamma^\mu P_L \nu_\ell\right] + \text{H.c.}, \qquad \text{where } P_L = \frac{1}{2}(1-\gamma^5)$$

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$\implies$ W couplings to ν and N in the **mass basis** are

$$\mathcal{L}_{\rm Int.} = -\frac{g_W}{\sqrt{2}} W_\mu^- \sum_{\ell=e}^{\tau} \left[\overline{\ell}\gamma^\mu P_L \left(\sum_{m=1}^{3} U_{\ell m}\nu_m + V_{\ell N} N\right)\right] + \text{H.c.}$$

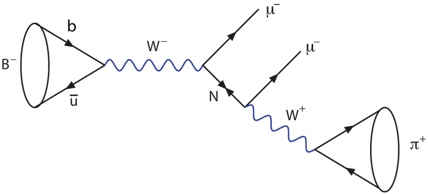
$\implies$ N is **accessible through** $W/Z/h$ bosons

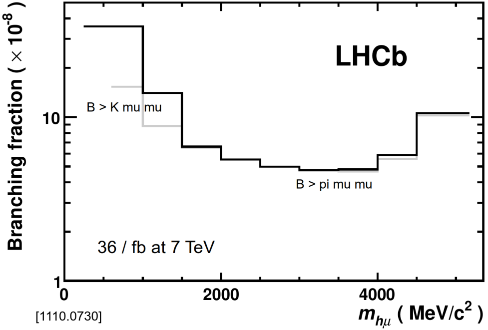
searches for low-mass

heavy neutrinos (N)

For $m_N \ll M_W$, N can appear in decays of baryons, mesons, and $\tau^\pm$!

Atre, Han, Pascoli, & Zhang [0901.3589]; Castro & Quintero [1302.1504]; Yuan, Wang ×2 , Ju, & Zhang [1304.3810]; + others





Production rate of mesons ($\pi^\pm, \mathcal{D}, \mathcal{B}$) at colliders is ***big*** ($\sigma_{bX}^{\rm LHC} \sim 0.1$ mb)

- sufficient to probe ***tiny*** rates of LNV
- sufficient to probe LFV

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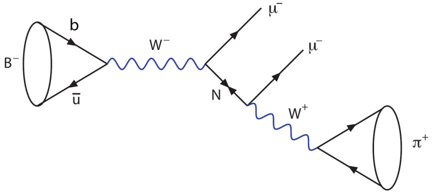

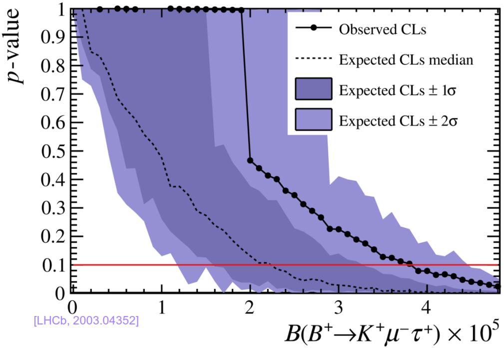

[LHCb, 2003.04352]

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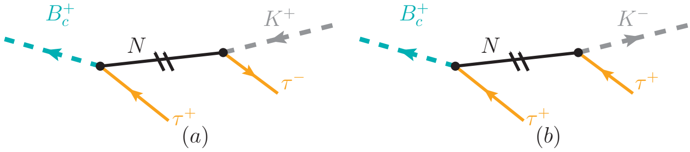
Confusion Theorem $\Longrightarrow$ relative helicity inversion of *N*

Kayser ('82) , Mohapatra & Pal ('98), Denner, et al (NPB'92, PLB'92)

$\Longrightarrow$ shifts in kinematic distributions

Many dedicated works, e.g., Han, RR, et al [1211.6447]; RR [2008.01092]

Shifts can occur at all scales, e.g., meson decays



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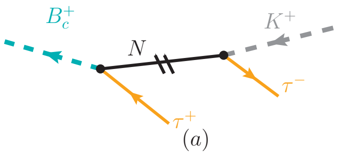
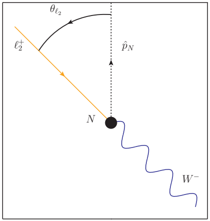
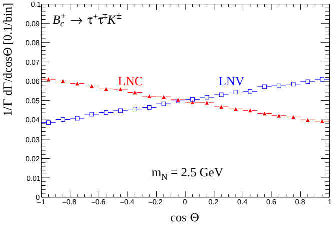


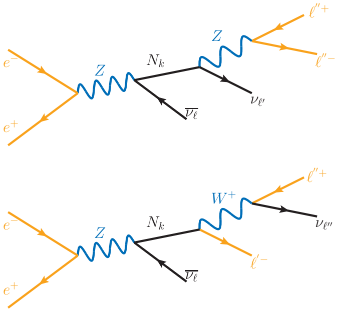


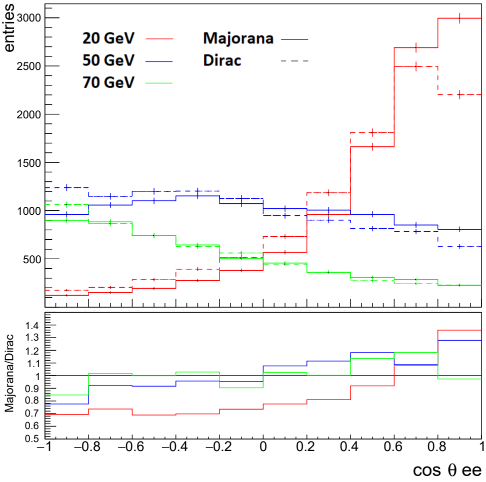


w/ Jeon, Fernandez-Martinez, Kulkarni, et al [(*to appear*)]

Shifts in kinematic distributions also appear when event is not fully reconstructable, e.g. $e^+e^- \to Z \to N\nu \to e^+e^-\nu\nu$

lots of recent activity! E.g., de Gouvea, et al [1808.10518, 2104.05719, 2105.06576 (FCC-ee), 2109.10358]





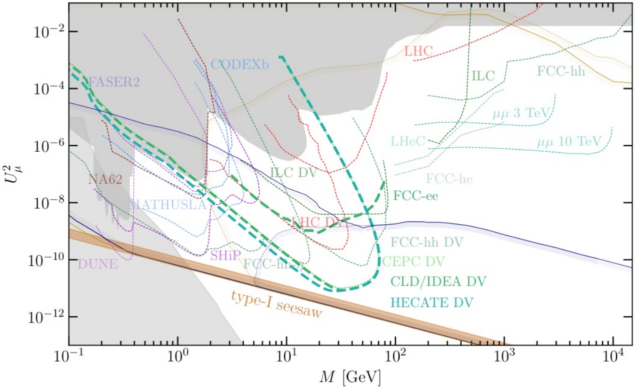
θ_{ee} = opening between final-state e^+e^-

– Dirac = LNC
– Majorana = LNC+LNV

w/ Alimena, Gonzalez Suarez, Sfyrla, Sharma, et al [2203.05502]

Outlook for Current and Future Machines

Community Message: Current + next-gen. facilities can probe *simplest* ($m_{\nu_1} = 0$) leptogenesis scenario w/ ν_R Abdullahi, et al [2203.08039]; w/ Alimena, et al [2203.05502]



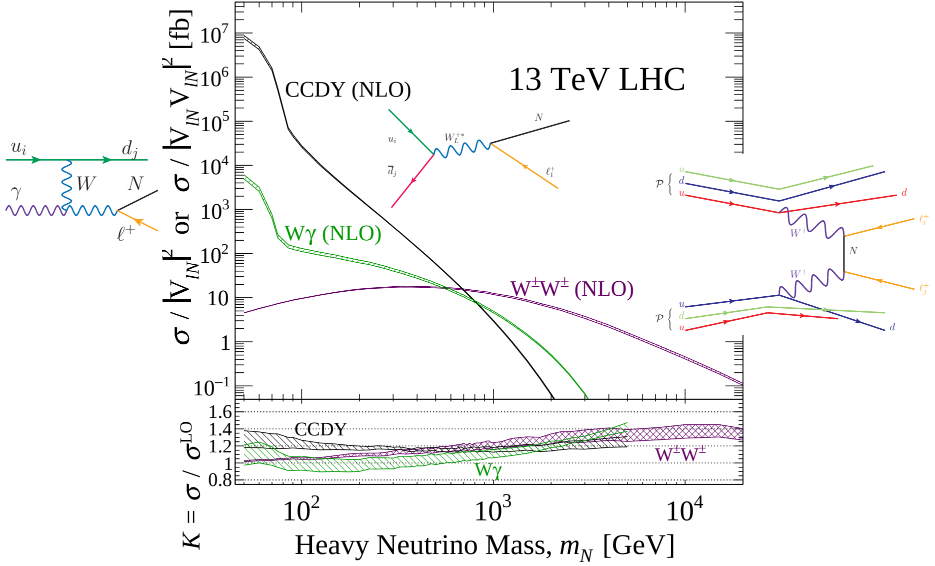
Note: LHC picture evolving with new strategies and channels

Cottin, Helo, Hirsch [1806.05191]; Abada, Bernal, Losada, Marcano [1812.01720]; K. Cheung, H. Ishida, et al [2004.11537]

searches for high-mass

heavy neutrinos (N)

Plotted: Normalized production rate ($\sigma/|V|^{2\ (4)}$) vs m_N

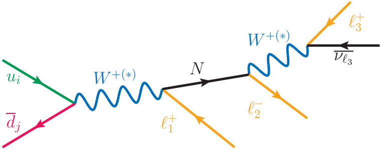


$\gamma W^{\pm}$ and $W^{\pm}W^{\pm}$ scattering drive high-mass scattering rates!

w/ Fuks, Neundorf, Peters, Saimpert [2011.02547; 2012.09882]

what do ATLAS and CMS say?

ATLAS experiment's search for light N with full Run II data



Plotted: Limits on $|V_{\ell N}|^2$ in search for $pp \to 3\ell$+MET

MET $= -|\sum_k \vec{p}_T^{\,k}|$, k=anything

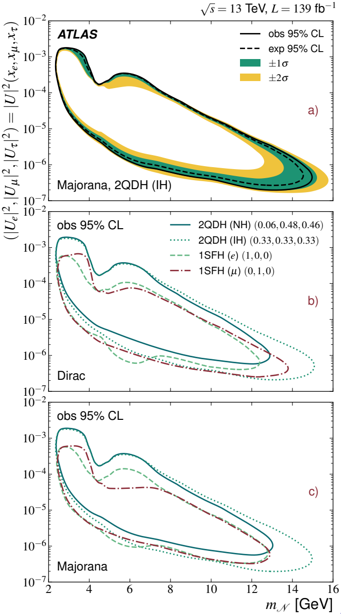
(top) 2 Majorana N

(mid) 1 Dirac N

(btm) 1 Majorana N

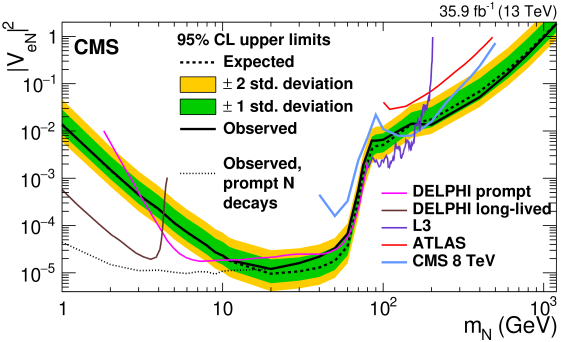
No discovery ☹

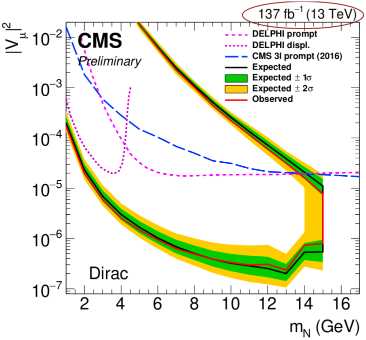
[2204.11988]



CMS experiment's search for light N with Run II data

Plotted: Limits on $|V_{\ell N}|^2$ in search for $pp \to 3\ell + \text{MET}$ $(\ell = e, \mu)$

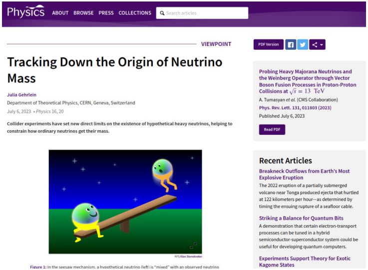


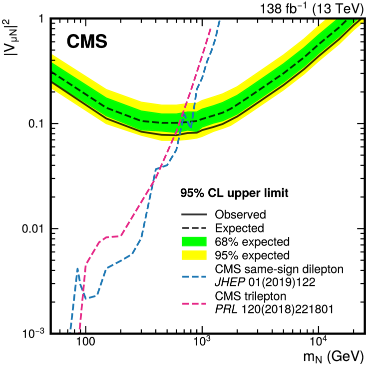


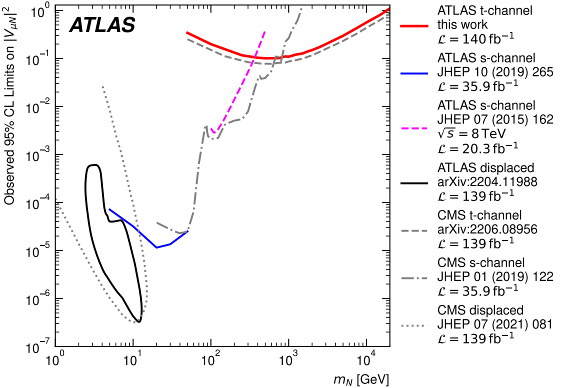
No discovery ☹ but there is hope with 20 – 30× more data! ☺

- (L)CMS experiments's trilepton search for short-lived N [1802.02965]
- (R)CMS search for long-lived N [2201.05578]
- (not shown) same-sign dilepton searches [1806.10905]

Search for $W^\pm W^\pm \to \ell^\pm \ell'^\pm$ novel search strategy by **ATLAS** and **CMS** experiments!







ATLAS (EPJC'23) [2305.14931]

$ee/e\mu$ (PLB'24) [2403.15016]

← CMS (PRL'22) [2206.08956]