Influence of dark photon on magnetized and charged particle orbits around static spherically symmetric black hole

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September 20, 2024

The 10th Conference of Polish Society on Relativity

Influence of dark photon on magnetized and charged particle orbits around static spherically symmetric black hole

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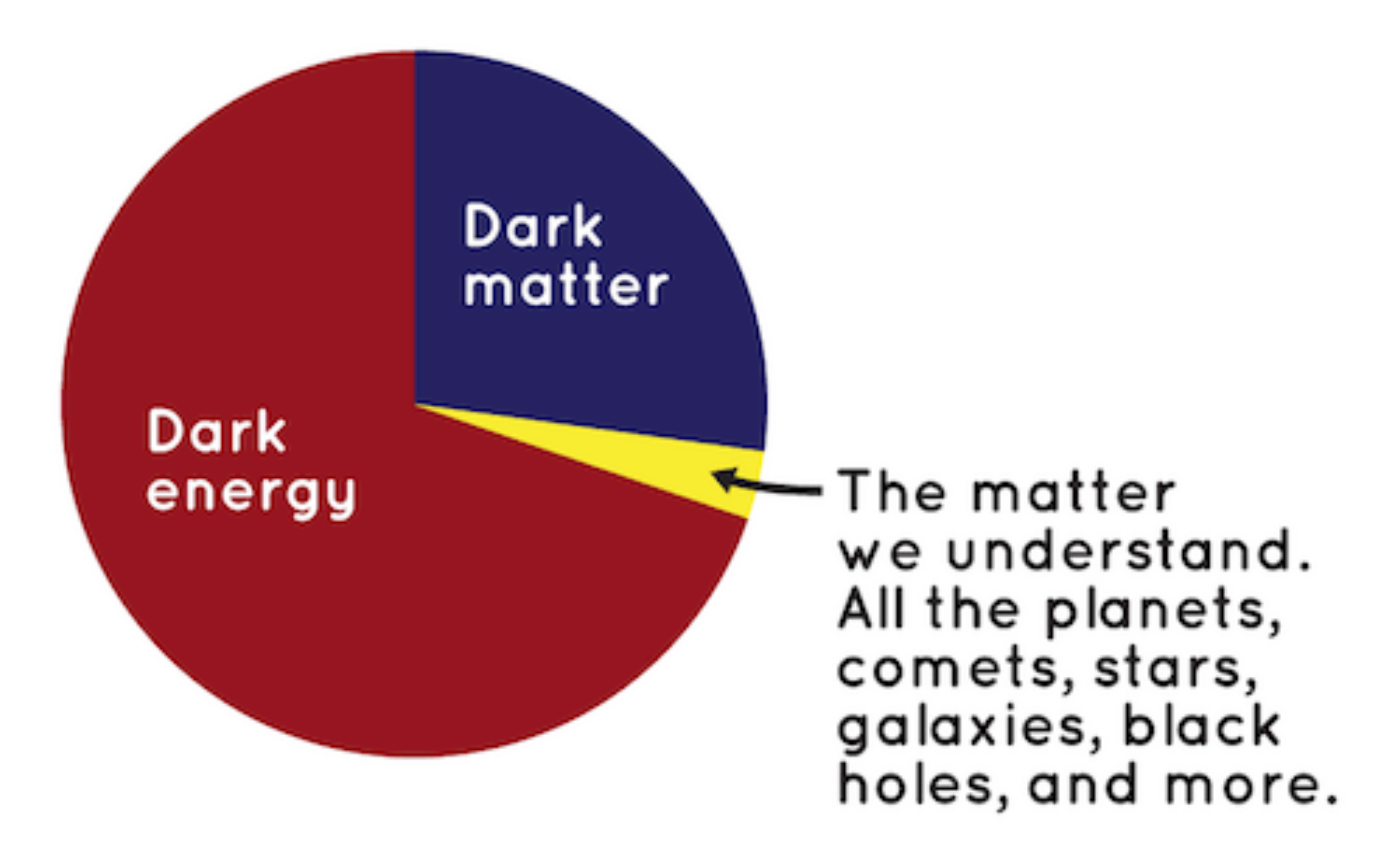
(Dated: August 19, 2024)

Abstract

We elaborate the problem of magnetized particle motion in the spacetime of a static, spherically symmetric black hole influenced by weak magnetic fields stemming from visible and dark matter sectors. The Wald's procedure for obtaining the weakly magnetized solution, generalized to the case of dark photon - Einstein-Maxwell gravity was implemented. The collision process analysis of two particles in the background of the black hole has been studied in order to find the signature of dark matter presence in the nearby of the object in question.

1. Introduction

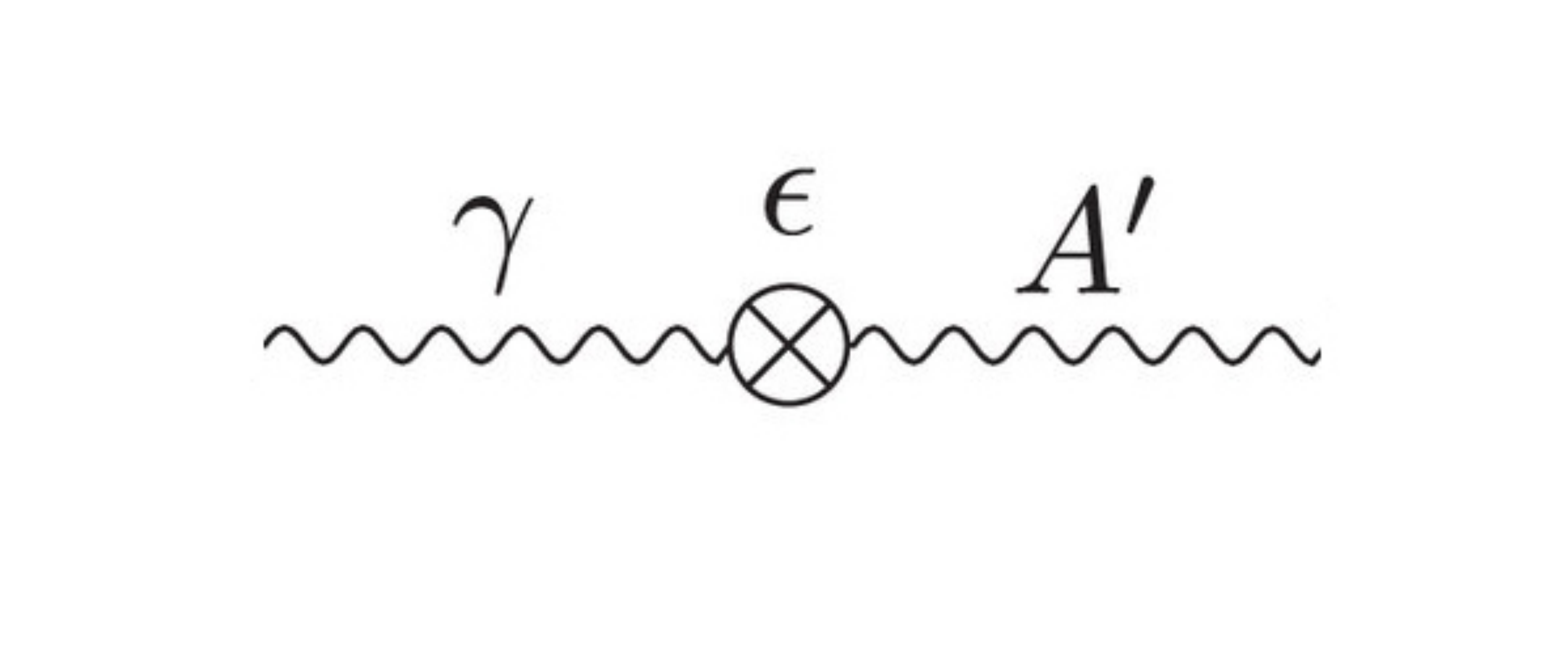


Credit: NASA/JPL-Caltech

Visible matter: 4.9%
Dark matter (DM): 26.6%
Dark energy (DE): 68.5%

- Standard model (SM) explains three fundamental forces and smaller-than-atom particles.
- But SM unable to explain gravity and the nature of DE & DM

- An extension to SM known as Dark sector
- The goal is to explain dark matter
- DM particles interact with each other by exchanging Dark photons



Credit: https://dx.doi.org/10.1088/1402-4896/abfef2

Dark photons

- Carriers of force
- Can switch back and forth with ordinary photons
- Abelian gauge boson coupled to the ordinary Maxwell gauge field

2. Experimental searches

- Inspection of dilation-like coupling to photons caused by ultra-light DM
- Fine structure constant oscillations
- Dark photon emission during supernovae event
- Electron excitation measurements in CCD-like detector
- Search for dark photons in e+ and e- collisions at BABAR experiment at SLAC
- Boosted decision tree technique to distinguish imbalance due to dark photons in ATLAS experiment
- Dark photon creation at LHC by meson decay, quark-anti quark annihilation

3. Model of Dark Photon

The action describing two coupled, massless gauge field is given by:

$$S_{M-\text{dark photon}} = \int d^4x \left(-F_{\mu\nu}F^{\mu\nu} - B_{\mu\nu}B^{\mu\nu} - \alpha F_{\mu\nu}B^{\mu\nu} \right)$$

To get rid of the kinetic mixing term, we define new gauge fields as

$$\tilde{A}_\mu = \frac{\sqrt{2-\alpha}}{2}\left(A_\mu - B_\mu\right) \quad \text{and} \quad \tilde{B}_\mu = \frac{\sqrt{2+\alpha}}{2}\left(A_\mu + B_\mu\right)$$

This gives;

$$F_{\mu\nu}F^{\mu\nu} + B_{\mu\nu}B^{\mu\nu} + \alpha F_{\mu\nu}B^{\mu\nu} \Longrightarrow \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} + \tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu}$$

We set: $\tilde{F}_{\mu\nu} = 2\nabla_{[\mu}\tilde{A}_{\nu]}$ and $\tilde{B}_{\mu\nu} = 2\nabla_{[\mu}\tilde{B}_{\nu]}$

Now, the action can be written as

$$S_{M-dark\ photon} = \int d^4x\Big(- \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu}\Big)$$

Variation of the action wrt $g_{\mu\nu}$, $\tilde{A}_\mu$ and $\tilde{B}_\mu$ gives

$$\nabla_\mu \tilde{F}^{\mu\nu} = 0 \qquad \text{and} \qquad \nabla_\mu \tilde{B}^{\mu\nu} = 0$$

For consistency, we also redefine charges as

$$\tilde{e}_A = \frac{\sqrt{2-\alpha}}{2}\big(e - e_d\big) \qquad \text{and} \qquad \tilde{e}_B = \frac{\sqrt{2+\alpha}}{2}\big(e + e_d\big)$$

The resulting action of massive charged particle influenced by both visible and dark matter sectors is assumed to be:

$$S = -\int m\sqrt{-ds^2} + \tilde{e}_A \int \tilde{A}_\mu dx^\mu + \tilde{e}_B \int \tilde{B}_\mu dx^\mu$$

The standard calculation leads to the following equation of motion:

$$m\,\frac{Du^{\mu}}{d\tau} = \left(\tilde{e}_A\tilde{F}^{\mu\nu} + \tilde{e}_B\tilde{B}^{\mu\nu}\right)u_{\nu},$$

As a result, the four-momentum of the massive particle subject to 2 gauge fields can be written as

$$p_{\mu} = mu_{\mu} + \tilde{e}_A\tilde{A}_{\mu} + \tilde{e}_B\tilde{B}_{\mu}\,.$$

Transforming the charges and fields back, we get

$$p_{\mu} = mu_{\mu} + eA_{\mu} + e_dB_{\mu} + \frac{\alpha}{2}\left(e_dA_{\mu} + eB_{\mu}\right).$$

And this becomes

$$p_{\mu} = mu_{\mu} + e\left(A_{\mu} + \frac{\alpha}{2}B_{\mu}\right)$$ When $e_d = 0$

4. Weakly Magnetized Black Holes

$\nabla_\mu \tilde{F}^{\mu\nu} = 0$ and $\nabla_\mu \tilde{B}^{\mu\nu} = 0$ In Lorenz gauge, written for spherically symmetric EE with vanishing Ricci tensor gives:

$\Box \tilde{A}^\mu = 0$ and $\Box \tilde{B}^\mu = 0$ Where $\Box = g^{\mu\nu} \nabla_\mu \nabla_\nu$

$\tilde{A}_\mu$ and $\tilde{B}_\mu$ can be defined as a linear combination of Killing fields. Therefore,

$\tilde{A}^\mu = \gamma_1 \, \xi^\mu_{(t)} + \gamma_2 \, \xi^\mu_{(\phi)}$ and $\tilde{B}^\mu = \delta_1 \, \xi^\mu_{(t)} + \delta_2 \, \xi^\mu_{(\phi)}$

We make use of following definitions:

$$8\pi M = -\int \epsilon_{\alpha\beta\gamma\delta} \nabla^\gamma \xi^\delta_{(t)} \qquad 16\pi J = \int \epsilon_{\alpha\beta\gamma\delta} \nabla^\gamma \xi^\delta_{(\phi)}$$

$$8\pi Q(\tilde{F}) = \int \epsilon_{\alpha\beta\gamma\delta} \tilde{F}^{\gamma\delta} \qquad 8\pi Q(\tilde{B}) = \int \epsilon_{\alpha\beta\gamma\delta} \tilde{B}^{\gamma\delta}$$

Consider the Maxwell field: $\tilde{F}^{\mu\nu} = \nabla^{\mu}\tilde{A}^{\nu} - \nabla^{\nu}\tilde{A}^{\mu}$

$$\tilde{F}^{\mu\nu} = \nabla^{\mu}\left[\gamma_1\, \xi^{\nu}_{(t)} + \gamma_2\, \xi^{\nu}_{(\phi)}\right] - \nabla^{\nu}\left[\gamma_1\, \xi^{\mu}_{(t)} + \gamma_2\, \xi^{\mu}_{(\phi)}\right]$$

But $\nabla_{\nu}K_{\mu} = -\nabla_{\mu}K_{\nu}$ Where K is Killing vector

Using the above equation $\tilde{F}^{\mu\nu} = 2\gamma_1\nabla^{\mu}\,\xi^{\nu}_{(t)} + 2\gamma_2\nabla^{\mu}\,\xi^{\nu}_{(\phi)}$

Using the relation $8\pi Q(\tilde{F}) = \int \epsilon_{\alpha\beta\gamma\delta}\tilde{F}^{\gamma\delta}$ we get, $Q(\tilde{F}) = 2\gamma_1 M + 4\gamma_2 J$

Assuming $\gamma_2 = \dfrac{B_0^{(\tilde{F})}}{2}$ gives $\gamma_1 = -\dfrac{Q(\tilde{F})}{2M} + aB_0^{(\tilde{F})}$ where $a \equiv \dfrac{J}{M}$

Hence, $$\tilde{A}^{\mu} = \frac{B_0^{(\tilde{F})}}{2}\left[\xi^{\mu}_{(\phi)} + 2\,a\xi^{\mu}_{(t)}\right] - \frac{Q(\tilde{F})}{2M}\xi^{\mu}_{(t)}$$

Similarly $$\tilde{B}^{\mu} = \frac{B_0^{(\tilde{B})}}{2}\left[\xi^{\mu}_{(\phi)} + 2\,a\xi^{\mu}_{(t)}\right] - \frac{Q(\tilde{B})}{2M}\xi^{\mu}_{(t)}$$

$$B_0^{(\tilde{F})} = \frac{\sqrt{2-\alpha}}{2}\left(B_0^{(F)} - B_0^{(B)}\right) \quad \text{and} \quad B_0^{(\tilde{B})} = \frac{\sqrt{2+\alpha}}{2}\left(B_0^{(F)} + B_0^{(B)}\right)$$

$$Q(\tilde{F}) = \frac{\sqrt{2-\alpha}}{2}\left(Q(F) - Q(B)\right) \quad \text{and} \quad Q(\tilde{B}) = \frac{\sqrt{2+\alpha}}{2}\left(Q(F) + Q(B)\right)$$

When a=0 and Q =0, we obtain

$$A^{\mu} = \frac{B_0^{(F)}}{2}\xi^{\mu}_{(\phi)}$$

$$B^{\mu} = \frac{B_0^{(B)}}{2}\xi^{\mu}_{(\phi)}$$

5 Collissions of particles in the vicinity Black Hole

We solve the Hamilton-Jacobi equation and obtain the four-velocity components of a particle in the vicinity of BH.

$$g^{\alpha\beta}\left(\frac{\partial S}{\partial x^{\alpha}}-\tilde{e}_A\tilde{A}_{\alpha}-\tilde{e}_B\tilde{B}_{\alpha}\right)\left(\frac{\partial S}{\partial x^{\beta}}-\tilde{e}_A\tilde{A}_{\beta}-\tilde{e}_B\tilde{B}_{\beta}\right)=-m^2+m\,\tilde{D}^{\mu\nu}_{\tilde{F}}\tilde{F}_{\mu\nu}+m\,\tilde{D}^{\mu\nu}_{\tilde{B}}\tilde{B}_{\mu\nu}$$

$\tilde{D}^{\mu\nu}_{(\tilde{F})(\tilde{B})}$:descibes properties of the particle immersed in both U(1)-gauge field

Their explicit forms are

$$\tilde{D}^{\mu\nu}_{(\tilde{F})}\,\tilde{F}_{\mu\nu}=\frac{\sqrt{2-\alpha}}{2}\left(\mu_{\delta}^{(F)}-\mu_{\delta}^{(B)}\right)\,\tilde{\epsilon}^{\mu\nu\rho\delta}u_{\rho}\,\tilde{F}_{\mu\nu}$$

$$\tilde{D}^{\mu\nu}_{(\tilde{B})}\,\tilde{B}_{\mu\nu}=\frac{\sqrt{2+\alpha}}{2}\left(\mu_{\delta}^{(F)}+\mu_{\delta}^{(B)}\right)\,\tilde{\epsilon}^{\mu\nu\rho\delta}u_{\rho}\,\tilde{B}_{\mu\nu}$$

$\mu_{\delta}^{(F)},\mu_{\delta}^{(B)}$: stand for the four-vectors of dipole moments of magnetic particle bounded by Maxwell and dark photon

$$S = -E\,t + L\,\phi + \int dr \sqrt{g(r)\left[\frac{E^2}{f(r)} - \frac{1}{r^2}\left(L - \tilde{e}_A\tilde{A}_\phi - \tilde{e}_B\tilde{B}_\phi\right)^2 - m^2 + m\left(\tilde{D}^{\mu\nu}_{(\tilde{F})}\,\tilde{F}_{\mu\nu} + \tilde{D}^{\mu\nu}_{(\tilde{B})}\,\tilde{B}_{\mu\nu}\right)\right]}$$

We use this form of action in the Hamilton-Jacobi equation to obtain four-velocity.

The centre of mass (COM) energy of the particles is $\mathscr{E}^2_{CM} = 1 - g_{\mu\nu}u_1^\mu\,u_2^\nu$

The explicit form of COM energy in our case is

$$\mathscr{E}^2_{CM} = 1 + \frac{\mathscr{E}_1\,\mathscr{E}_2}{f(r)} - \frac{1}{r^2}\left(l_1 - \frac{\tilde{e}_A\tilde{A}_\phi}{m} - \frac{\tilde{e}_B\tilde{B}_\phi}{m}\right)\left(l_2 - \frac{\tilde{e}_A\tilde{A}_\phi}{m} - \frac{\tilde{e}_B\tilde{B}_\phi}{m}\right) -$$

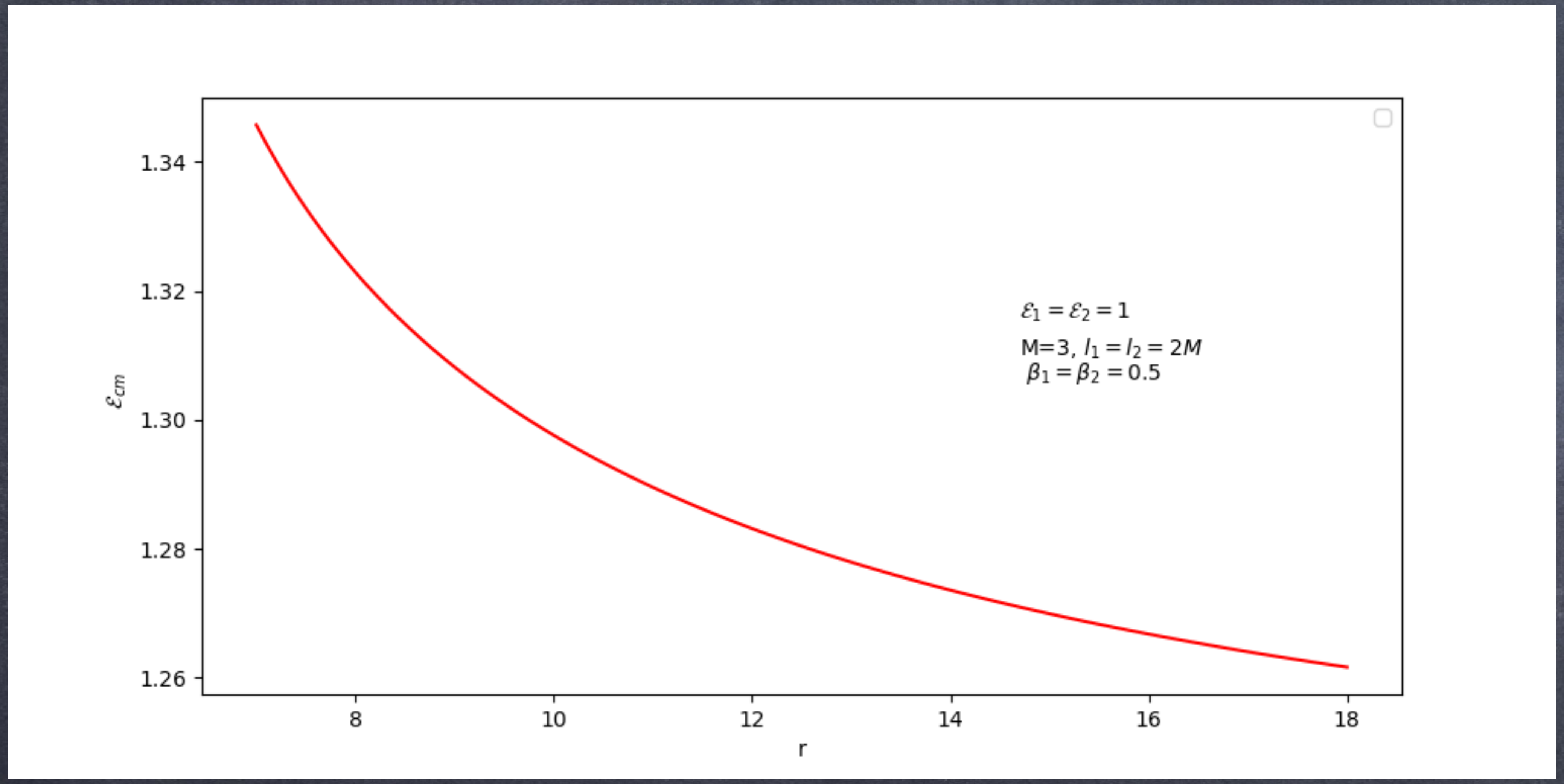
$$\frac{1}{f(r)}\sqrt{\left[\mathscr{E}_1^2 - \frac{f(r)}{r^2}\left(l_1 - \frac{\tilde{e}_A\tilde{A}_\phi}{m} - \frac{\tilde{e}_B\tilde{B}_\phi}{m}\right)^2 - 1 + \frac{f(r)\left(D^{\mu\nu}_{(\tilde{F})}\tilde{F}_{\mu\nu} + D^{\mu\nu}_{(\tilde{B})}\tilde{B}_{\mu\nu}\right)_{(1)}}{m}\right]} \times$$

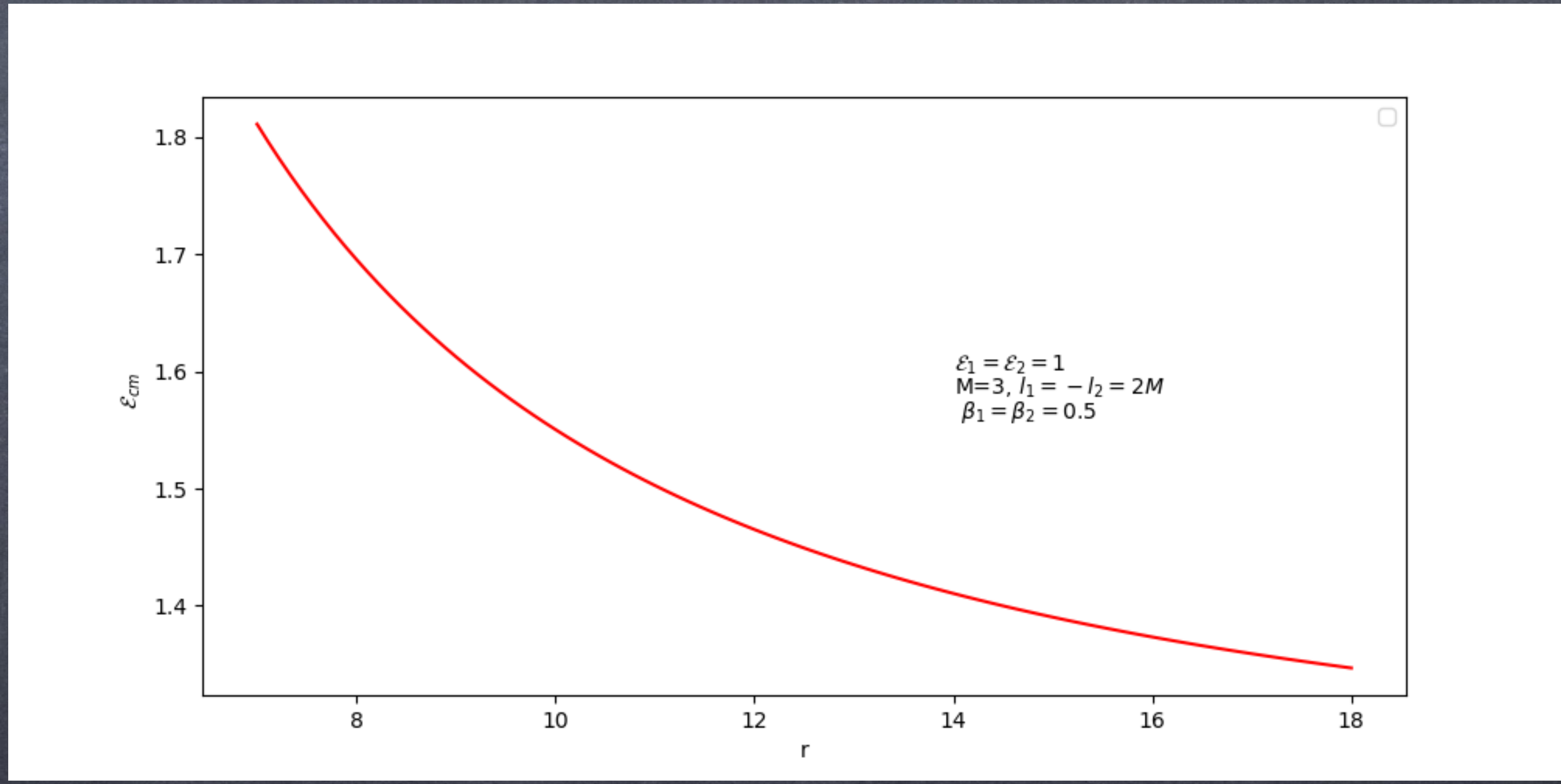
$$\sqrt{\left[\mathscr{E}_2^2 - \frac{f(r)}{r^2}\left(l_2 - \frac{\tilde{e}_A\tilde{A}_\phi}{m} - \frac{\tilde{e}_B\tilde{B}_\phi}{m}\right)^2 - 1 + \frac{f(r)\left(D^{\mu\nu}_{(\tilde{F})}\tilde{F}_{\mu\nu} + D^{\mu\nu}_{(\tilde{B})}\tilde{B}_{\mu\nu}\right)_{(2)}}{m}\right]} \qquad (12)$$

5.1 Two magnetized particles

$$\mathcal{E}_{CM}^2 = 1 + \frac{\mathcal{E}_1\,\mathcal{E}_2}{f(r)} - \frac{1}{f(r)}\left[\mathcal{E}_1^2 - f(r)\left(\frac{l_1^2}{r^2} + 1 - \sqrt{f(r)}\tilde{\beta}_1(F,B,\alpha)\right)\right]^{\frac{1}{2}} \times \left[\mathcal{E}_2^2 - f(r)\left(\frac{l_2^2}{r^2} + 1 - \sqrt{f(r)}\tilde{\beta}_2(F,B,\alpha)\right)\right]^{\frac{1}{2}} - \frac{l_1\,l_2}{r^2}$$

l>0: Repulsive Lorentz force (away from BH)
L<0: Attractive Lorentz force (towards BH)





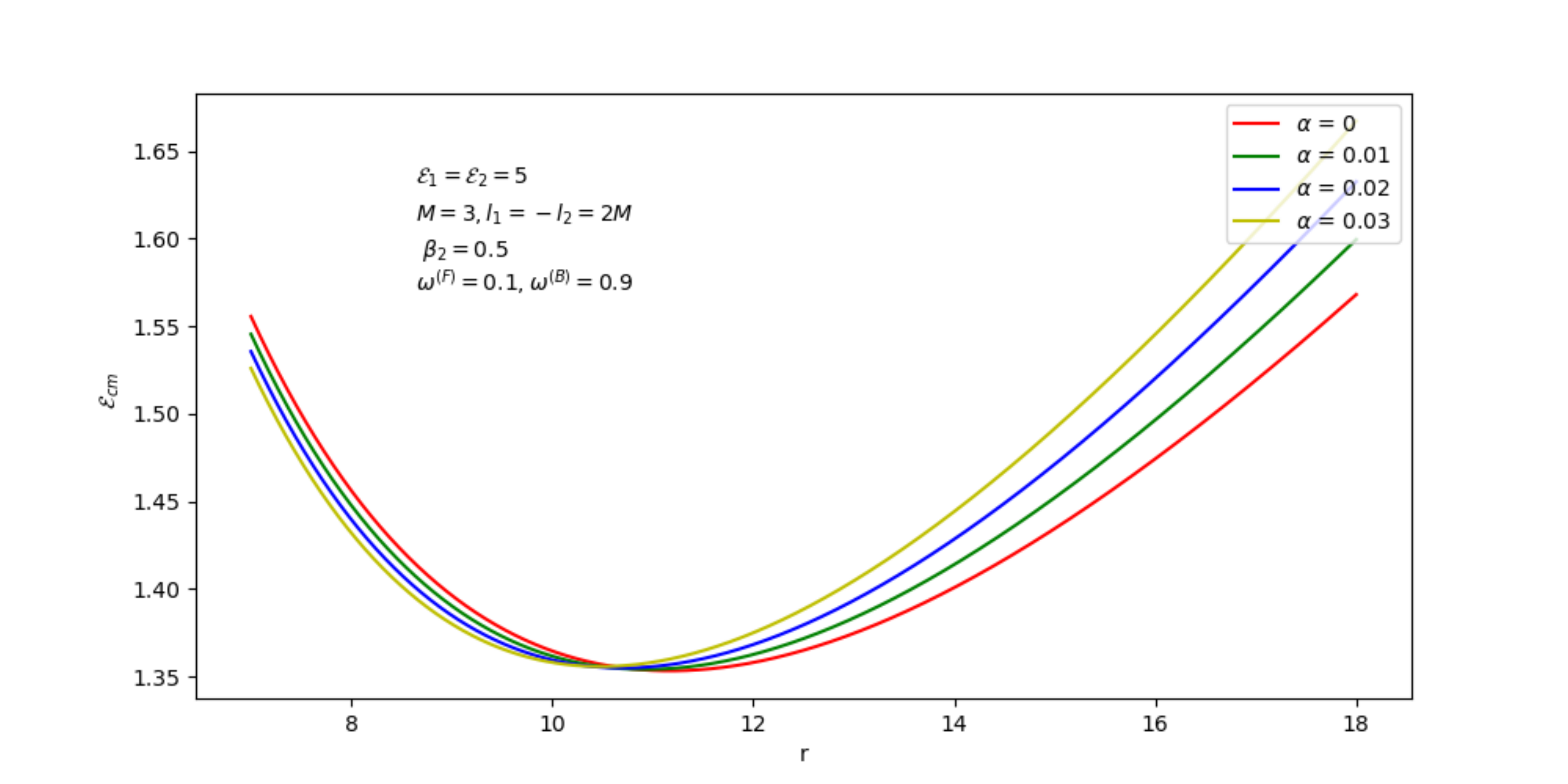
COM energy increases as one moves closer to the event horizon

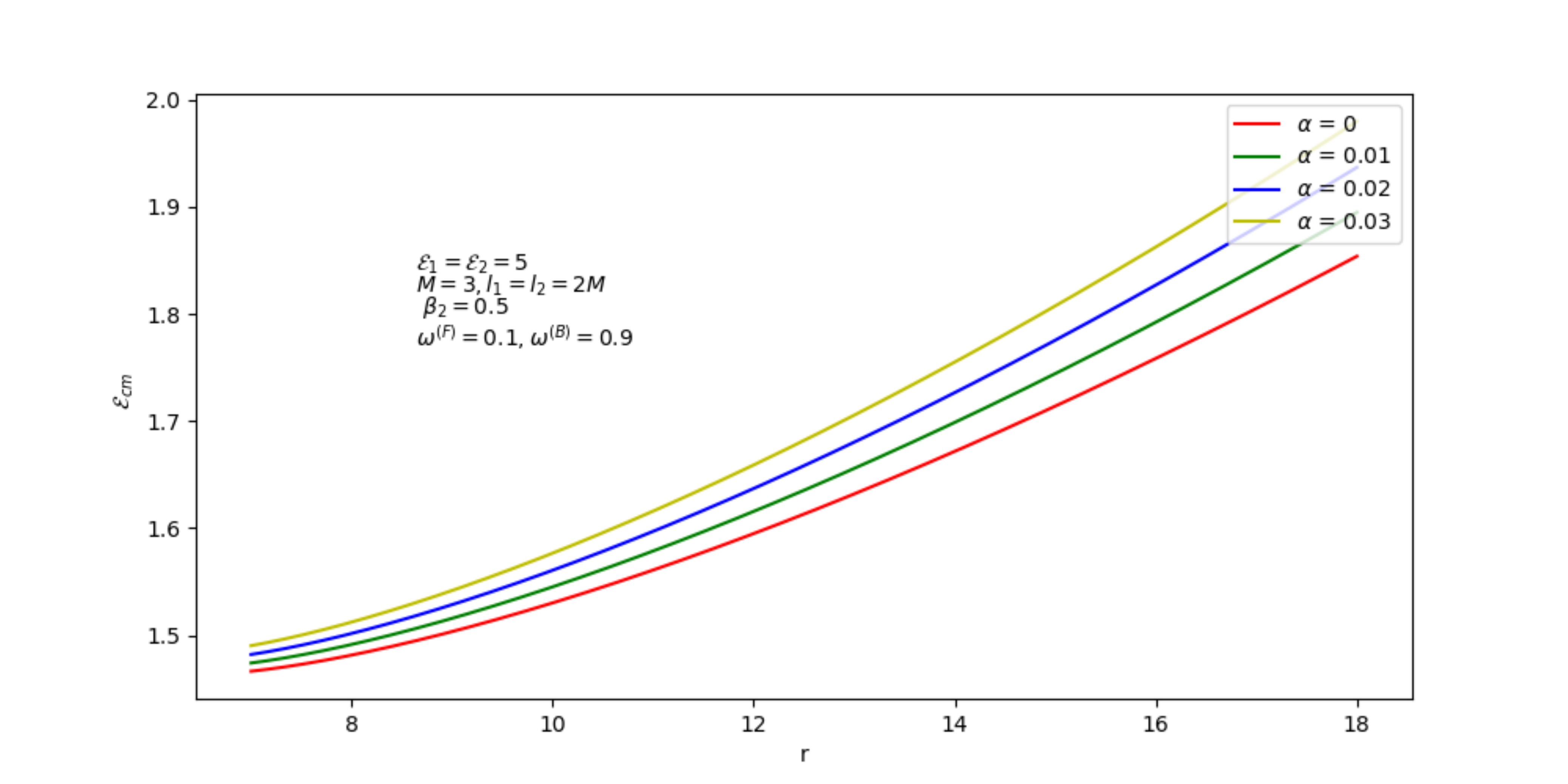
5.2 Magnetized and charged particles

$$\mathcal{E}_{CM}^2 = 1 + \frac{\mathcal{E}_1\,\mathcal{E}_2}{f(r)} - \frac{1}{f(r)}\left[\mathcal{E}_1^2 - \frac{f(r)}{r^2}\left(l_1 - er^2\left(w^{(F)} + \frac{\alpha}{2}w^{(B)}\right)\right)^2 - f(r)\right]^{\frac{1}{2}} \times \left[\mathcal{E}_2^2 - f(r)\left(\frac{l_2^2}{r^2} + 1 - \sqrt{f(r)}\tilde{\beta}_2(F,B,\alpha)\right)\right]^{\frac{1}{2}} - \frac{1}{r^2}\left(l_1 - er^2\left(w^{(F)} + \frac{\alpha}{2}w^{(B)}\right)\right)l_2$$

where

$$w^{(F)} \equiv \frac{B_0^{(F)}}{2m} \quad \text{and} \quad w^{(B)} \equiv \frac{B_0^{(B)}}{2m}$$





Higher concentration of dark matter

6 Constant homogeneous magnetic field

We suppose that there exist magnetic fields stemming from those sectors in the vicinity of BH, being axisymmetric and homogenous at spatial infinity.

The conserved qualities are:

$$\mathcal{E} = -\frac{1}{m}\xi^{\mu}_{(t)}p_{\mu} = \dot{t}\, f(r)$$

$$l_z = \frac{1}{m}\xi^{\mu}_{(\phi)}p_{\mu} = r^2\sin^2\theta\left(\dot{\phi} + \tilde{e}_A\tilde{B}_0^{(\tilde{F})} + \tilde{e}_B\tilde{B}_0^{(\tilde{B})}\right)$$

where

$$\tilde{B}_0^{(\tilde{F})} = \frac{e}{2m}B_0^{(\tilde{F})}$$

$$\tilde{B}_0^{(\tilde{B})} = \frac{e}{2m}B_0^{(\tilde{B})}$$

$$B_0^{(\tilde{F})} = \frac{\sqrt{2-\alpha}}{2}\left(B_0^{(F)} - B_0^{(B)}\right)$$

$$B_0^{(\tilde{B})} = \frac{\sqrt{2+\alpha}}{2}\left(B_0^{(F)} + B_0^{(B)}\right)$$

The normalisation condition of the four-velocity $u^{\alpha}u_{\alpha} = -1$ leads to

$$\mathcal{E}^2 = \dot{r}^2 + r^2 f(r)\dot{\theta}^2 + U_{eff}$$

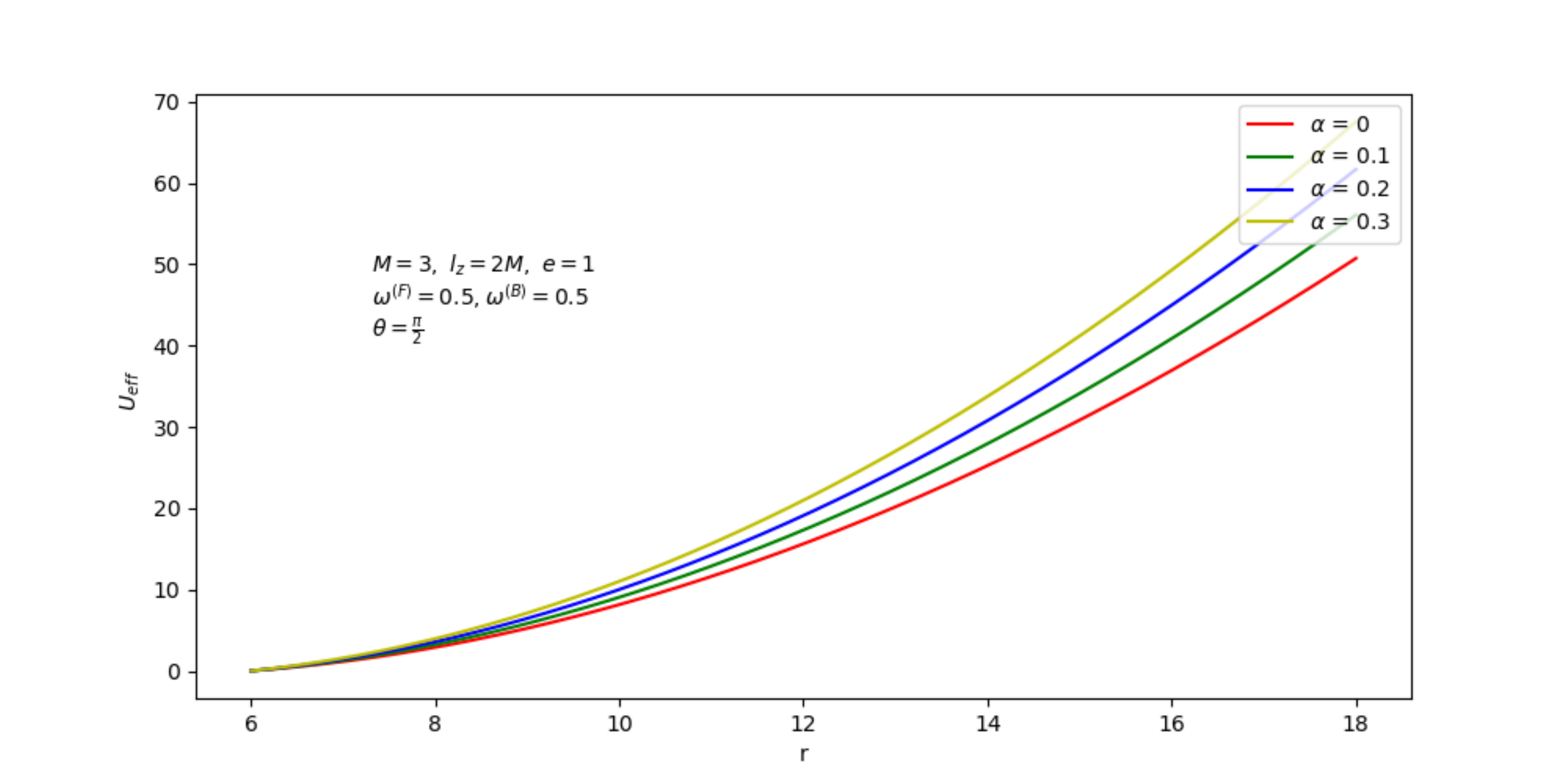
where

$$U_{eff} = f(r)\left[1 + r^2 \sin^2\theta\left(\frac{l_z}{r^2 \sin^2\theta} - e\left(w^{(F)} + \frac{\alpha}{2} w^{(B)}\right)\right)^2\right]$$

$$w^{(F)} \equiv \frac{B_0^{(F)}}{2m}$$

$$w^{(B)} \equiv \frac{B_0^{(B)}}{2m}$$

$$e_d = 0$$



The position of innermost circular orbit (ISCO) is determined by:

$$\partial_r U_{eff} = 0 \quad \text{and} \quad \partial_r^2 U_{eff} = 0$$

By solving above two conditions simultaneously, we get

$$e\left(w^{(F)} + \frac{\alpha}{2}w^{(B)}\right) = \frac{1}{\sqrt{2}\sin\theta} \times \frac{\sqrt{2M(6M - r_\pm)}}{r_\pm\left[4r_\pm^2 - 18Mr_\pm + 12M^2 \pm 2M\sqrt{\left(3r_\pm - 2M\right)\left(6M - r_\pm\right)}\right]^{1/2}}$$

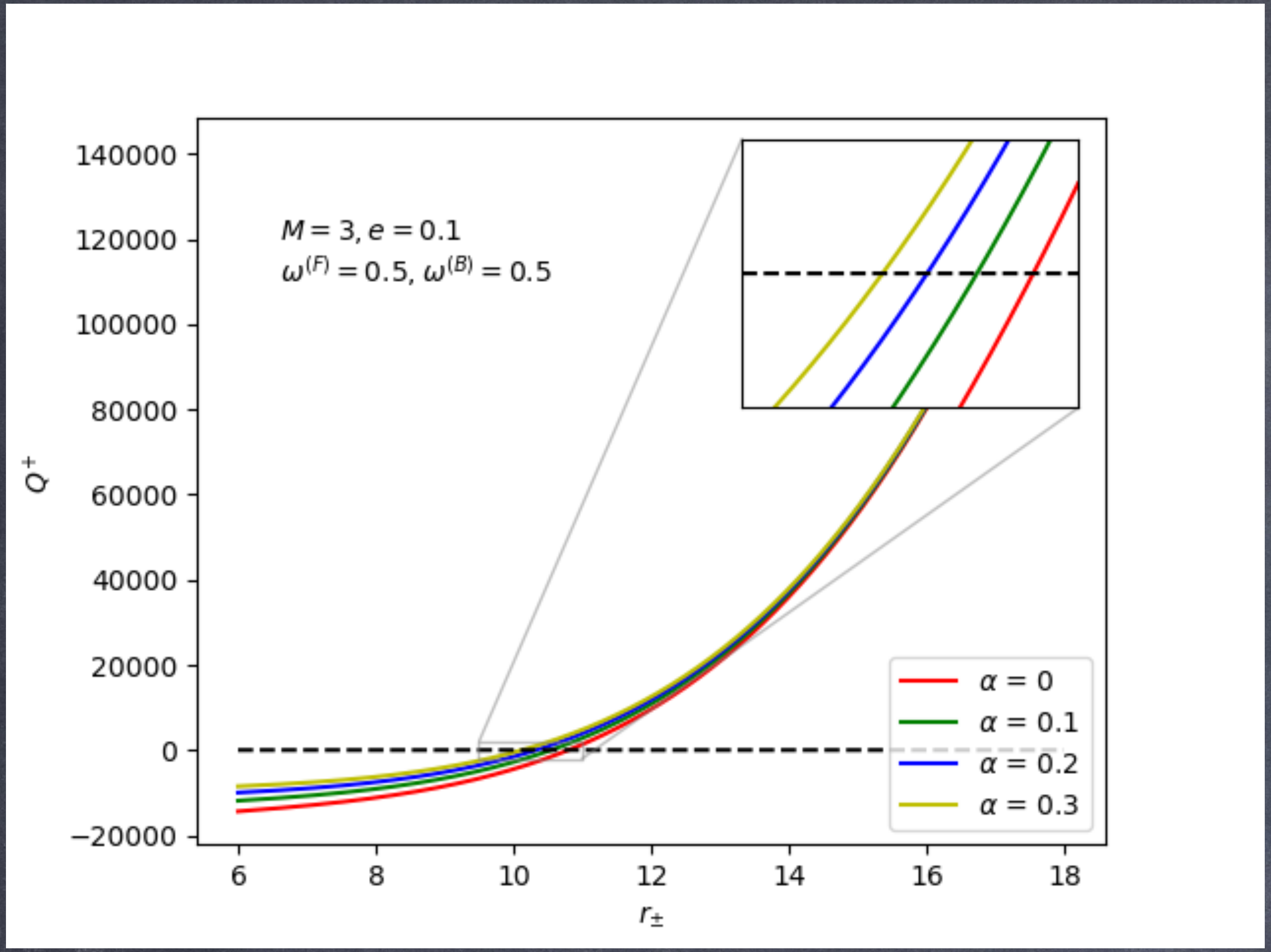
$$l_\pm = \pm\frac{1}{\sqrt{2}}\frac{\sqrt{2M}(\sin\theta)r_\pm}{r_\pm\left[4r_\pm^2 - 18Mr_\pm + 12M^2 \pm 2M\sqrt{\left(3r_\pm - 2M\right)\left(6M - r_\pm\right)}\right]^{1/2}}$$

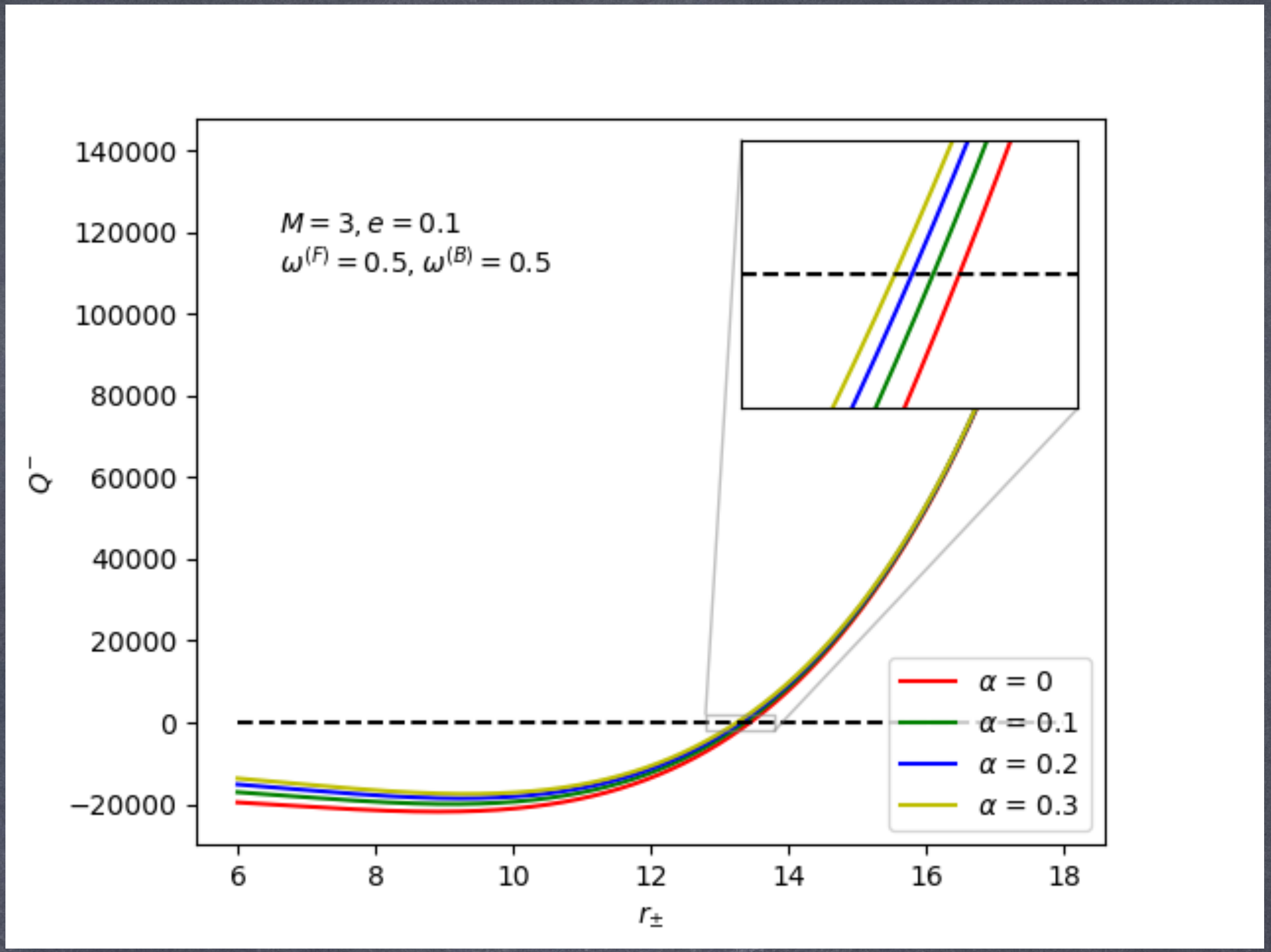
$$r_\pm \equiv \sqrt{r^2}$$

In order to visualise the dependency of ISCO on α, let us define two functions:

$$Q^+(r_\pm) \equiv 4r_\pm^4 - 18Mr_\pm^3 + 12Mr_\pm^2 + 2Mr_\pm^2\sqrt{(3r_\pm - 2M)(6M - r_\pm)} - \frac{M(6M - r_\pm)}{e^2\left(w^{(F)} + \frac{\alpha}{2}w^{(B)}\right)^2\sin^2\theta}$$

$$Q^-(r_\pm) \equiv 4r_\pm^4 - 18Mr_\pm^3 + 12Mr_\pm^2 - 2Mr_\pm^2\sqrt{(3r_\pm - 2M)(6M - r_\pm)} - \frac{M(6M - r_\pm)}{e^2\left(w^{(F)} + \frac{\alpha}{2}w^{(B)}\right)^2\sin^2\theta}$$





The radius of ISCO decreases as the coupling between dark photon and Maxwell fields increases.

Summary

- We have considered motion of magnetized and charged particles in the spacetime of spherically symmetric weakly magnetized BH.
- We take into account Einstein-Maxwell gravity with dark sector, using the so-called dark photon theory.
- U(1)-gauge field coupled to ordinary Maxwell one is responsible for the invisible sector.
- The weakly magnetized solution has been found using Wald's procedure.
- It was shown that magnetic coupling parameter, responsible for the strength of external magnetic fields, is influenced by dark matter sector magnetic field and α-coupling constant.

Thank you for your attention!!