Jakub Gizbert-Studnicki

in collaboration with

J. Ambjørn, A. Görlich and D. Németh

From IR to UV
Making contact of CDT with FRG

10th Conference of POTOR

Kazimierz Dolny, 18th September 2024

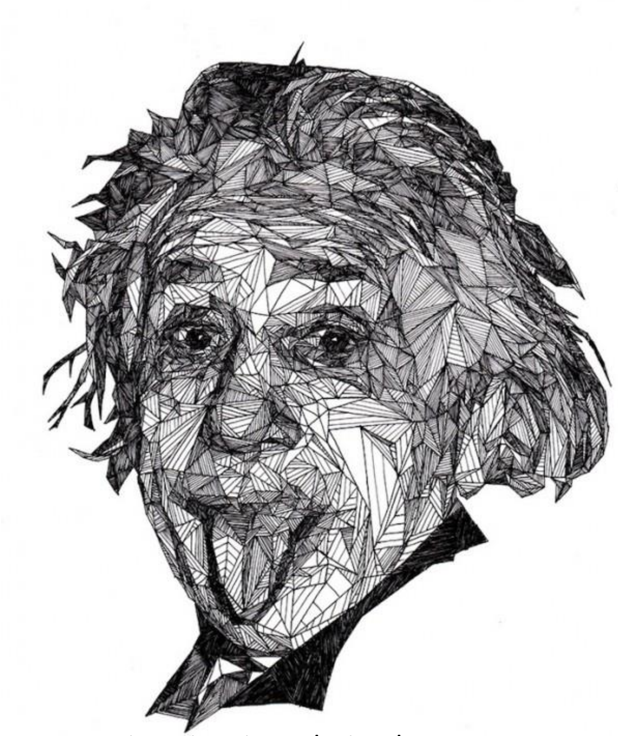
Is Lattice Quantum Gravity Asymptotically Safe?
Making contact between Causal Dynamical Triangulations and the Functional Renormalization Group.
[arXiv:2408.07808]

UNIWERSYTET JAGIELLOŃSKI W KRAKOWIE

UCZELNIA BADAWCZA
INICJATYWA DOSKONAŁOŚCI

Challenges of Quantum Gravity (QG)

- *As early as in 1916 Einstein* pointed out that „quantum theory would have to modify not only Maxwellian electrodynamics, but also the new theory of gravitation"*
- *After more than 100 years a complete, consistent quantum theory of gravity is still missing*
- *We have a number of interesting but incomplete research programs*
  - *string theory*
  - *loop quantum gravity*
  - *group field theory*
  - *causal set theory*
  - *noncommutative geometry*
  - *asymptotic safety (functional RG flow)*
  - *lattice QFT approaches (CDT, quantum Regge calc., …)*
  - …


A. Einstein triangulation by J. Bryan

* Sitzungsber. Preuss. Akad. Wiss. Berlin (1916) 688

Challenges of Quantum Gravity (QG)

- ✧ *Lack of experimental guidance*
- ✧ *Conceptual issues*
  - ✧ *QFT based on Einstein's GR is perturbatively non-renormalizable in D > 2 dimensions*
  - ✧ Renormalizable extensions (eg. with R^2 terms) have problems with unitarity
- ✧ *One is forced to use non-perturbative & background-independent approaches (difficult !)*
- ✧ *Non-perturbative renormalizability of QG ? Asymptotic safety (AS) conjecture:*
  - ✧ *renormalization group (RG) flow leads to a non-Gaussian UV fixed point*
  - ✧ *where QG becomes scale invariant (UV complete)*
  - ✧ *in the vicinity of the UV fixed point the RG flow trajectories lie in a finite dim. hypersurface (in the theory space of coupling constants): only a finite numer of relevant couplings should be fixed (measured) to make QG predictive at all scales*

M. Goroff, A. Sagnotti, NPB 266 (1986) 709

K. S. Stelle, Phys. Rev. D 16 (1977) 953

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  Asymptotic safety (AS) conjecture: S. Weinberg, 1980
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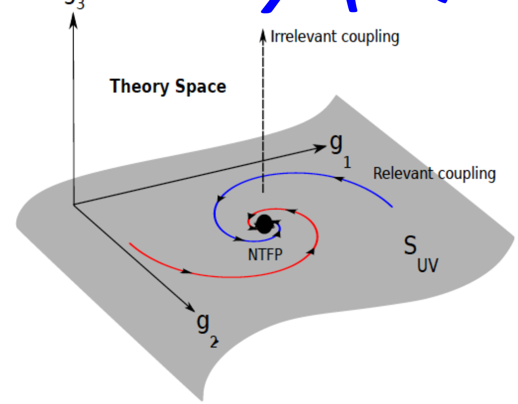
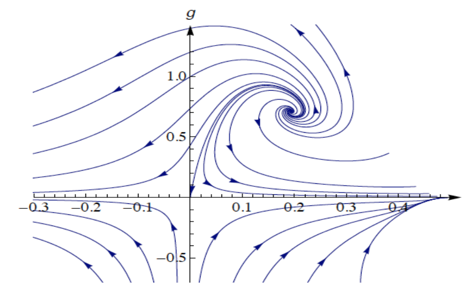
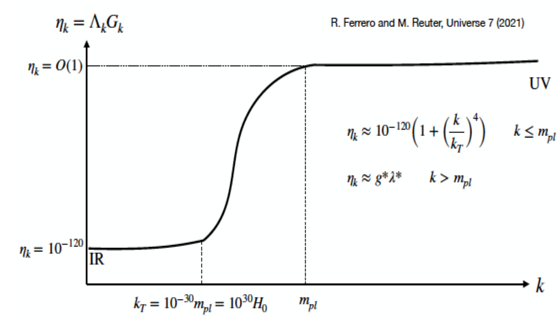
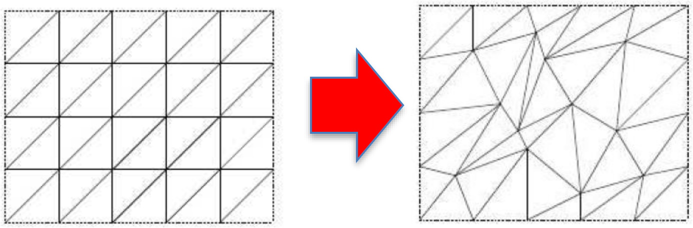


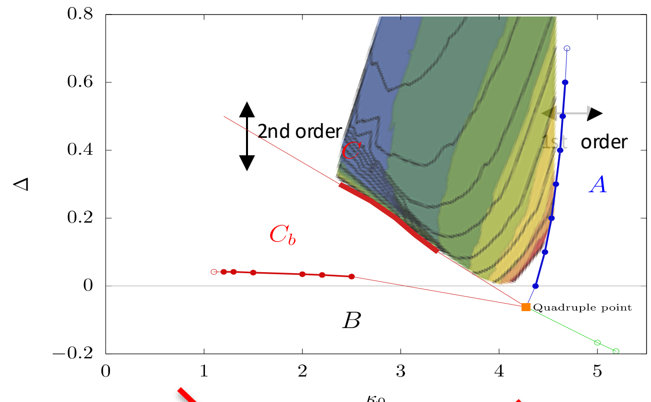


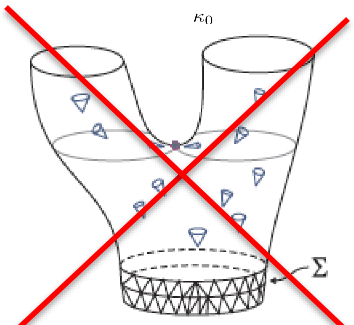


Challenges of Quantum Gravity (QG)

- ✧ *Lattice QFT formulation can allow to check AS in a unitary, non-perturbative, background-independent & diffeomorphism-invariant QG*
  - ✧ *we need dynamical lattices (DT) to encode geometric d.o.f.*
  - ✧ *continuum limit (UV fixed point ?) should be associated with a phase transition*
  - ✧ *one must be able to reproduce semi-classical gravity (IR limit)*
  - ✧ *causal structure is an important ingredient: CDT (J. Amjørn, J. Jurkiewicz, R. Loll)*
- ✧ *Main goals of QG as a Lattice QFT:*
  - ✧ *Formulate a UV complete (non-perturbatively renormalizable) and unitary theory of QG*
  - ✧ *with a correct IR limit (consistent with GR)*
  - ✧ *Study the emerging background geometry*
  - ✧ *and fluctuations around this geometry*
  - ✧ *Study properties of a quantum spacetime at the Planck scale; find some non-trivial predictions / observational effects (construct phenomenology ?)*
  - ✧ *(Unify with QFT(s) of the matter content)*
  - ✧ *…*







2D: J. Ambjorn, R. Loll, Nucl.Phys. B 536 (1998) 407

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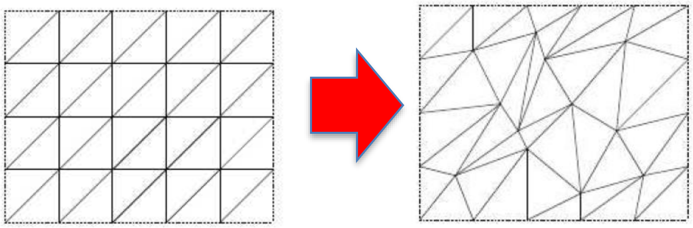

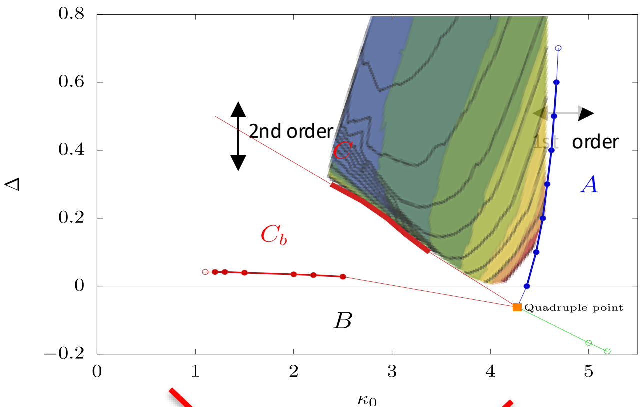

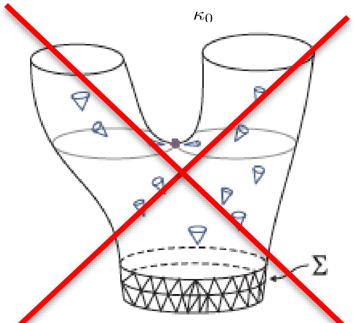

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Outline

- ✧ *Causal Dynamical Triangulations (CDT)*
- ✧ *Phase structure and phase transitions*
- ✧ *Semi-classical phase*
- ✧ *Functional Renormalization Group*
- ✧ *RG flow on the lattice*
- ✧ *IR limit*
- ✧ *UV limit*
- ✧ *Conclusions*

Causal Dynamical Triangulations

- CDT approach to QG is via a lattice QFT, using the path integral (PI) quantization
- One has to give a precise meaing to:
  - what class of geometries should be included in the PI
  - what (classical) action should be used
  - which symmetries (GR diffeomorhisms ?) should be preserved and how to do that
  - what is the integration measure
  - how to compute the PI in practice
  - how to deal with unitarity and (generic) PI divergencies
  - (how to include matter fields)

$$Z_{QG} = \int_{g \in \frac{Lor(M)}{Diff(M)}} D[g] \exp(i\, S_{\text{grav}}[g])$$

(Lorenzian) geometries

Causal Dynamical Triangulations

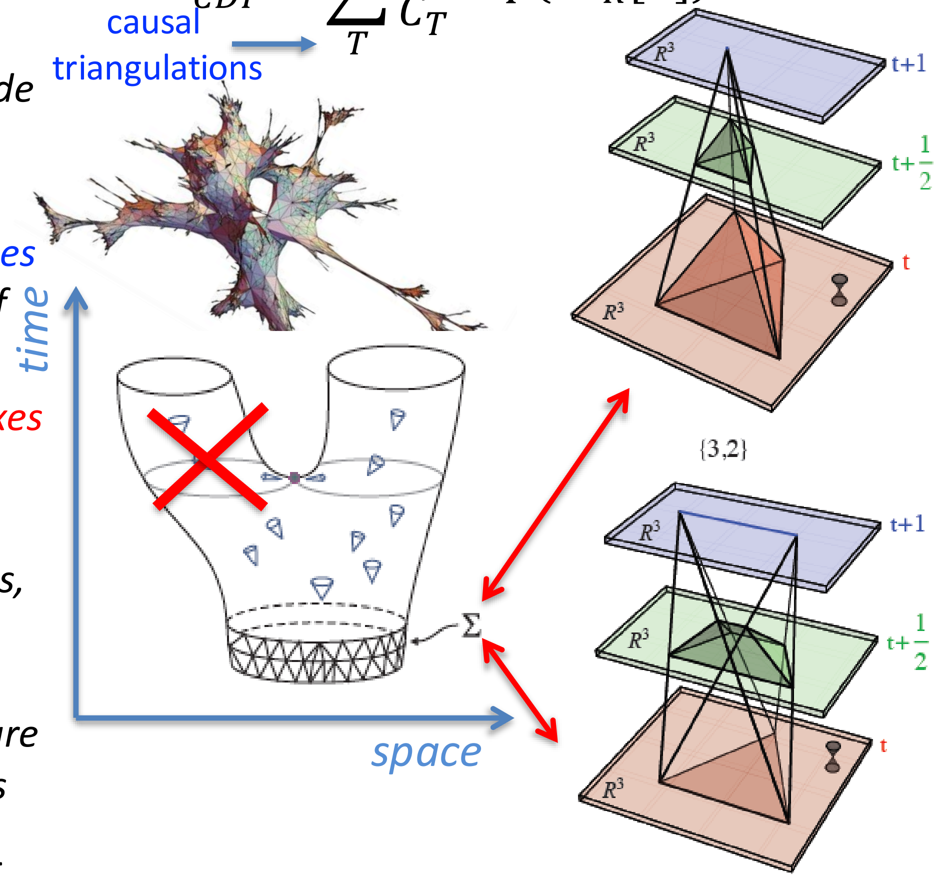
- *CDT approach to QG is via a lattice QFT, using the path integral (PI) quantization*
- *CDT (quantum) geometries:*
  - *In classical GR one deals with smooth (pseudo-)Riemannian manifolds*
  - *In the PI one should most likely also include non-smooth, but continuous (Lorenzian) geometries*
  - *CDT assumes globally hypebolic spacetimes which can be foliated into spacial slices of equal cosmological proper time**
  - *CDT takes a minimalistic approach and fixes topology of the manifold, with compact space & time periodic b.c. (S^1xS^3 or S^1xT^3)*
  - *We use a subset of continuous geometries, the so-called piecewise linear manifolds, that can be constructed from identical simplicial building blocks. We hope they are dense in the set of continuous geometries*

$$Z_{QG} = \int_{g\in\frac{Lor(M)}{Diff(M)}} D[g]\exp(i\, S_{\text{grav}}[g])$$

(Lorenzian) geometries

$$Z^{(L)}_{CDT} = \sum_T \frac{1}{C_T}\exp(i\, S_R[T])$$

causal triangulations



** Nature of the imposed foliation is not fully understood: 4-d diffeo. symetry breaking vs convenient gauge choice*

Causal Dynamical Triangulations

- *CDT approach to QG is via a lattice QFT, using the path integral (PI) quantization*
- *CDT action & diffeom. symmetry:*
  - *We use the Einstein-Hilbert action*
  - *For a piesewise linear manifold (triangulation) it takes the form of Regge's action*

    T. Regge, Nuovo Cim. A19 (1961) 558
  - *Curvature is defined by deficit angles around D-2 dim. „hinges" (triangles in 4D)*
  - *Regge's formulation uses only geometric invariants (geodesic edge lengths and deficit angles) making it coordiante free and therefore manifestly (at least spatial*) diffeomorphism invariant*
  - *As CDT uses only 2 types of bulding blocks with fixed edge lenghts the Regge action is very simple*

$$Z_{QG} = \int_{g \in \frac{Lor(M)}{Diff(M)}} D[g] \exp(i\, S_{\text{grav}}[g])$$

(Lorenzian) geometries

$$Z_{CDT}^{(L)} = \sum_T \frac{1}{C_T} \exp(i\, S_R[T])$$

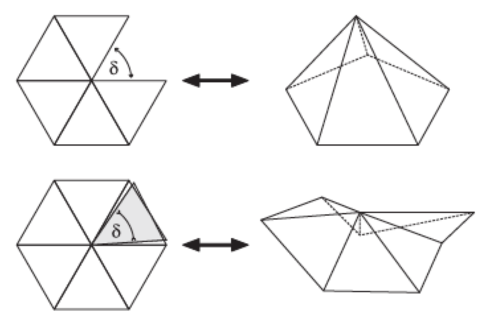
causal triangulations

$$S_{\text{grav}} = \frac{1}{16\pi G} \int_M d^4x \sqrt{-\det g}\,(R - 2\Lambda)$$

\# 4-simplices   # {4,1} 4-simpl.   # vertices

$$S_R = -k_0 N_0 + K_4 N_4 + \Delta\left(N_4^{(4,1)} - 6N_0\right)$$

$1/G$ Λ α $(l_t^2 = -\alpha l_s^2)$



* *Nature of the imposed foliation is not fully understood: 4-d diffeo. symetry breaking vs convenient gauge choice*

Causal Dynamical Triangulations

- *CDT approach to QG is via a lattice QFT, using the path integral (PI) quantization*
- *CDT PI measure and MC computations:*
  - *CDT uses a trivial measure $1/C_T$ (plans to investigate other measures)*
  - *In order to investigate the 4D PI one has to use Monte Carlo (MC) simulations*
  - *MC requires Euclidean formulation (Wick's rotation)*
  - *Due to the CDT imposed time foliation each Lorentzian geometry can be Wick-rotated to an Euclidean geom.*
  - *A the level of the Regge action Wick's rotation is achieved by an analytical continuation ($\alpha \to -\alpha$): coupling constants are appropriately changed, but general form of the action S_R remains the same in (L) and (E)*

$$Z_{QG} = \int_{g \in \frac{Lor(M)}{Diff(M)}} D[g] \exp(i\, S_{\text{grav}}[g])$$

(Lorenzian) geometries

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causal triangulations

symmetries of T

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$\alpha \to -\alpha$

$$Z_{CDT}^{(E)} = \sum_T \frac{1}{C_T} \exp(-S_R[T])$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \sum_T \frac{1}{C_T} \mathcal{O}[T] e^{-S_R[T])} \approx \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} \mathcal{O}[T^{(i)}]$$

Causal Dynamical Triangulations

- *CDT approach to QG is via a lattice QFT, using the path integral (PI) quantization*
- *CDT unitarity and PI divergencies:*
  - *CDT has a (time) reflection positive*, symmetric and bounded transfer matrix. In standard lattice QFT it guarantees unitarity if continuum limit exists*
  - *For finite N_4 there is no problem with the PI convergence as the number of configurations is exponentially bounded*
  - *Lattice spacing (l_s) plays a role of the UV cutoff l_s^{-1} as in ordinary lattice QFT*
  - *We want to investigate continuum limit ($l_s \to 0$, $N_4 \to \infty$), hopefully consistent with the UV fixed point of QG*
  - *CDT does not assume that spacetime is discrete in any sense !*

$$Z_{QG} = \int_{g \in \frac{Lor(M)}{Diff(M)}} D[g] \exp(i\, S_{\text{grav}}[g])$$

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** TM² is positive-definite*

Phase structure & phase transitions

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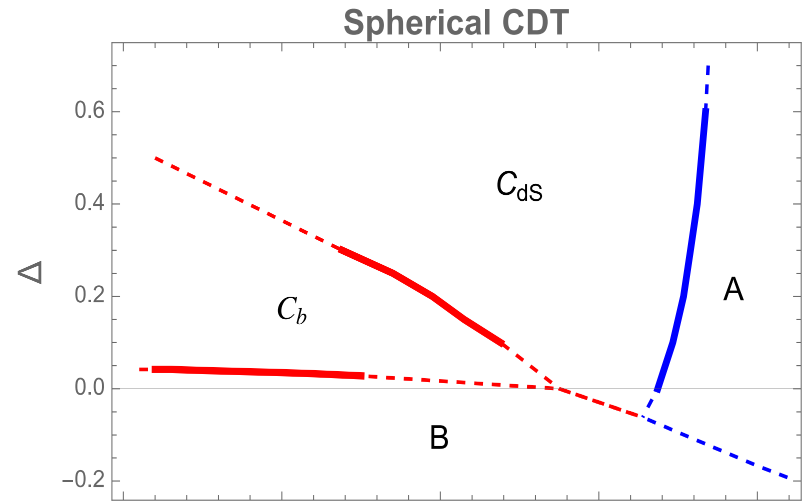
- *Phase structure:*
  - *We perform MC simulations with fixed N_4. The cosmological constant K_4 is tuned to N_4 and we effectively have two coupling constants: k_0 and Δ*
  - *Four phases (A, B, C_{dS}, C_b) of different generic geometries were discovered*
  - *The observable*: physical 3-volume of spatial layers: $V_3(t_i) \propto N_3(i) \cdot l_s^3$*
  - *In toroidal CDT one can even visualize generic geometries using „harmonic" coordinates defined by scalar fields*
  - *The difference between phases C_{dS} and C_b is captured by effective dimensions*
  - *One can also use „automatic" Machine Learning classification methods !*



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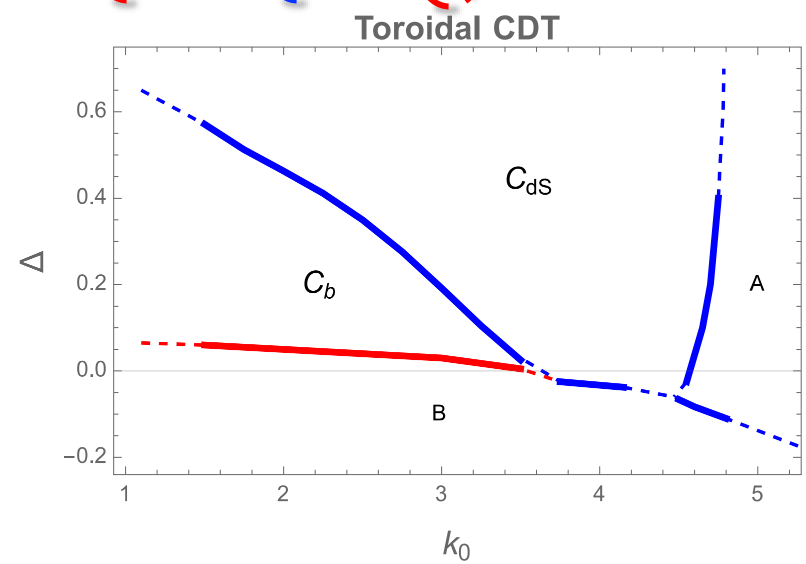

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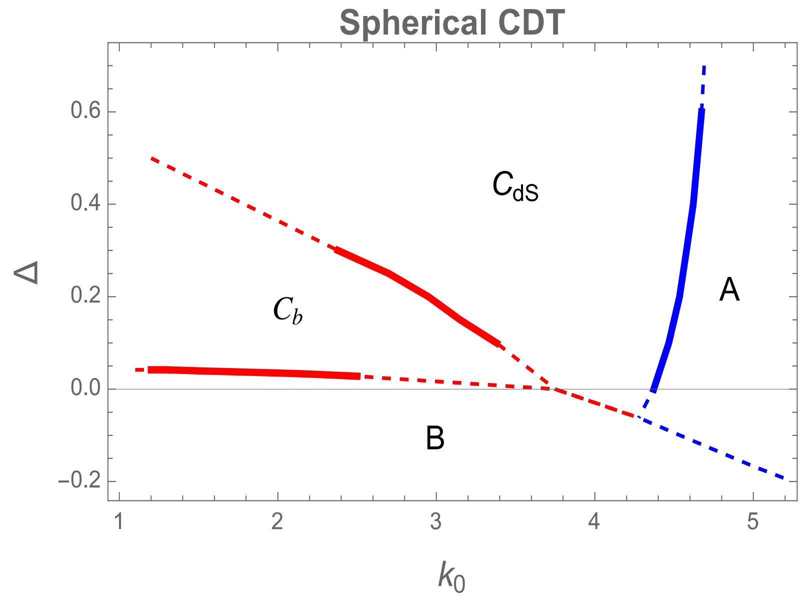

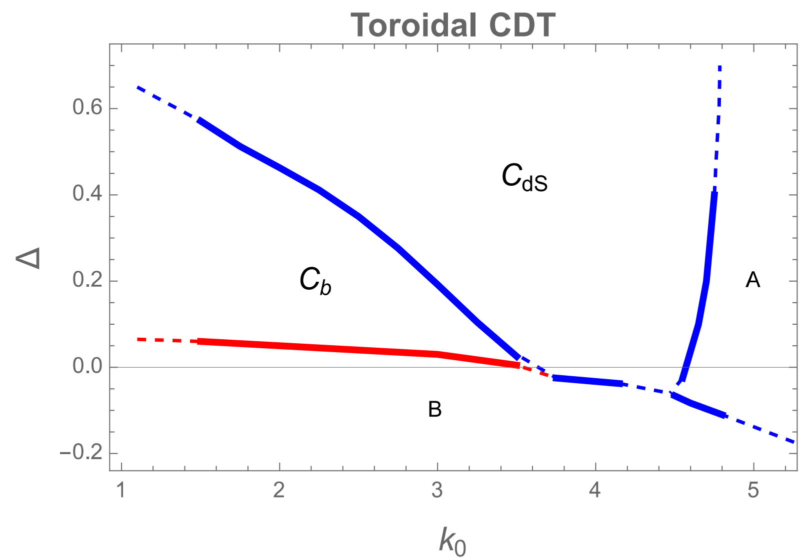

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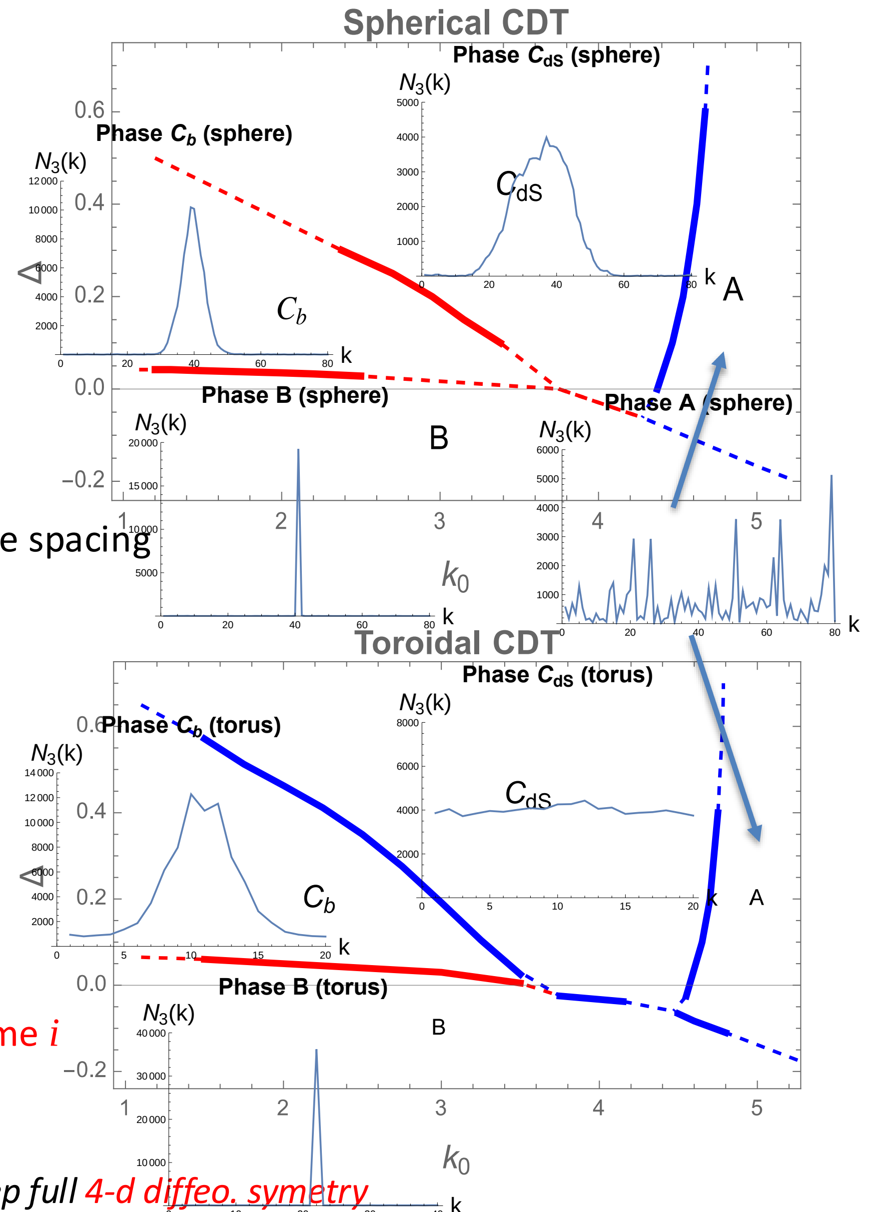

physical proper time $t_i = i \cdot l_t$

lattice time

lattice spacing

\# of tetrahedra at lattice time i

* *This is formally not a gauge invariant observable if we insist to keep full 4-d diffeo. symetry*

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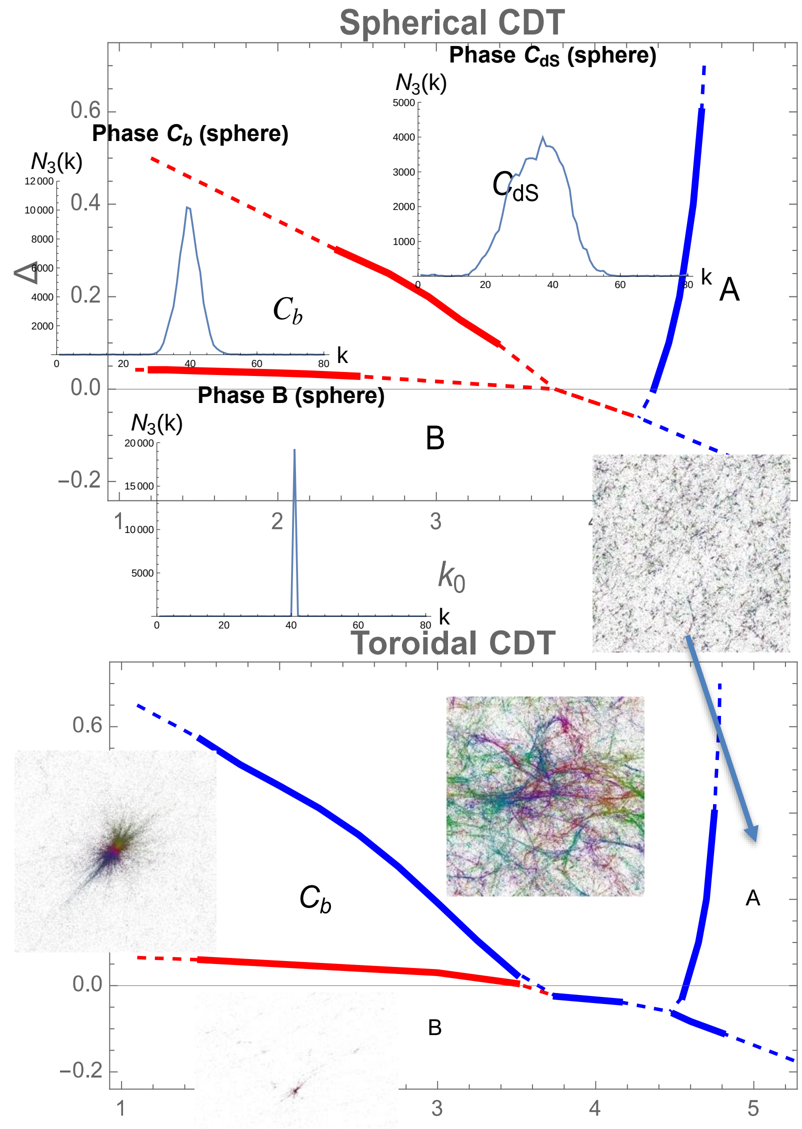

Eur.Phys.J.C 81 (2021) 708
Class.Quant.Grav. 38 (2021) 195030

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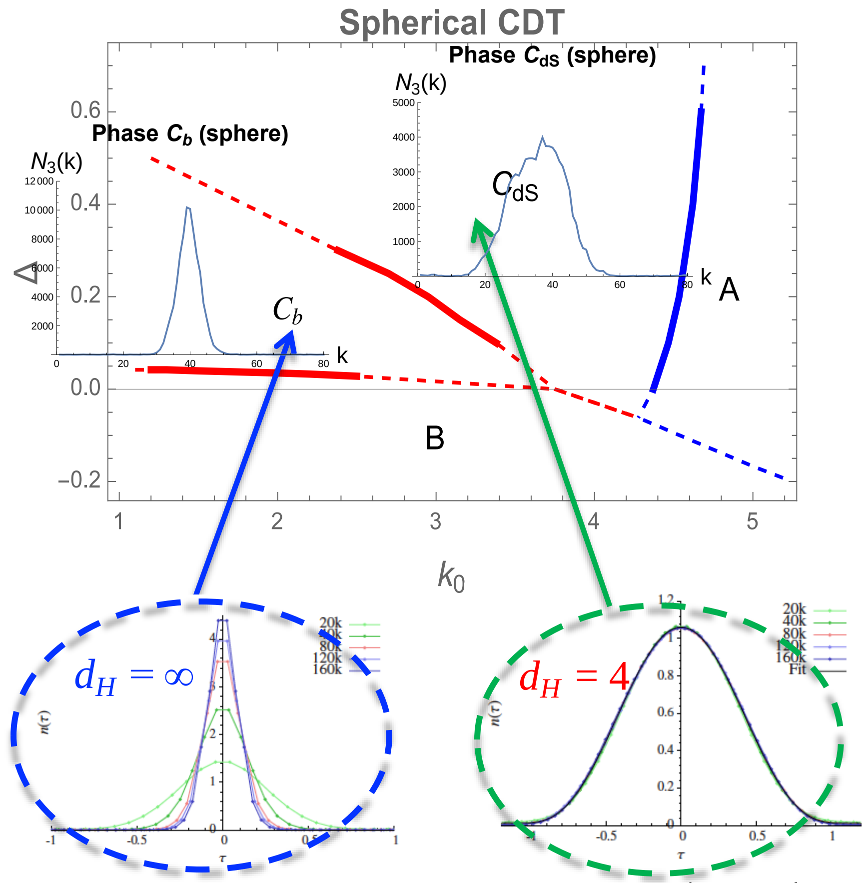
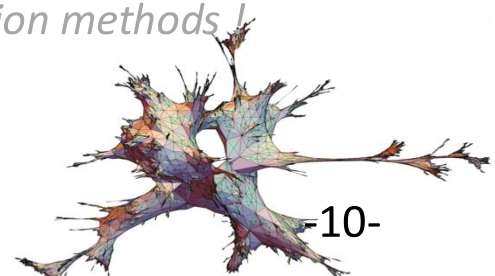


Hausdorff dimension: $\langle N_3(i) \rangle \rightarrow \dfrac{\langle N_3(i) \rangle}{N_4^{1-1/d_H}}$

rescaled average volume profiles (scaling for $d_H = 4$)

$i \rightarrow \dfrac{i}{N_4^{1/d_H}}$



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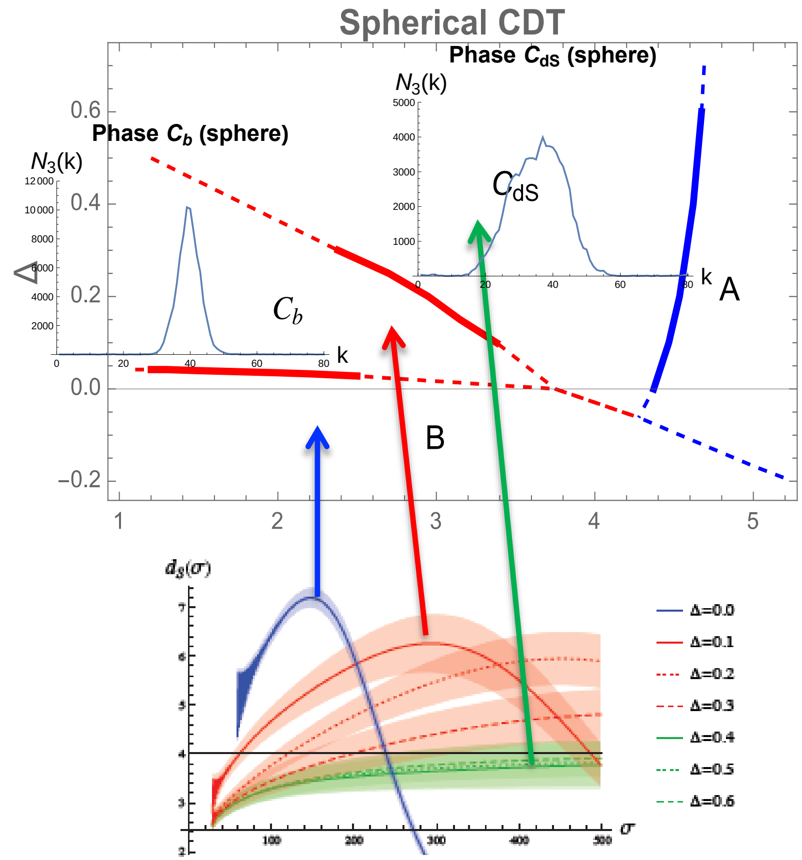


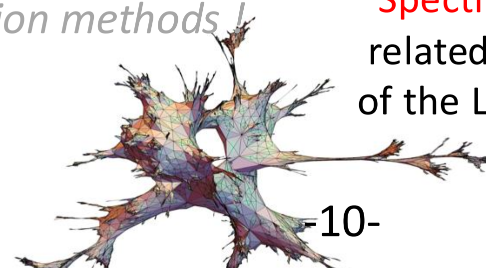
Spectral dimension*: related to heat kernel of the Laplace operator

$$\frac{\partial}{\partial\sigma} K(x, x_0;\ \sigma) = \Delta_g K(x, x_0;\ \sigma)$$

$$P_R(\sigma) = \frac{1}{V}\int dx\sqrt{g}K(x, x;\ \sigma)$$

$$d_S(\sigma) = -2\frac{d \log P_r(\sigma)}{d \log\ \sigma}$$

* Dimensional reduction appears in many approaches to QG (S. Carlip, CQG 34 (2017) 193001)



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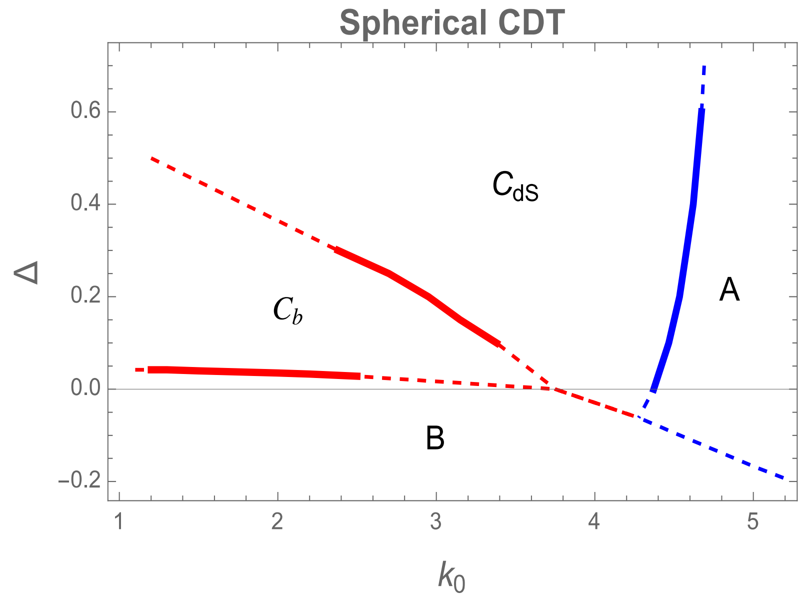
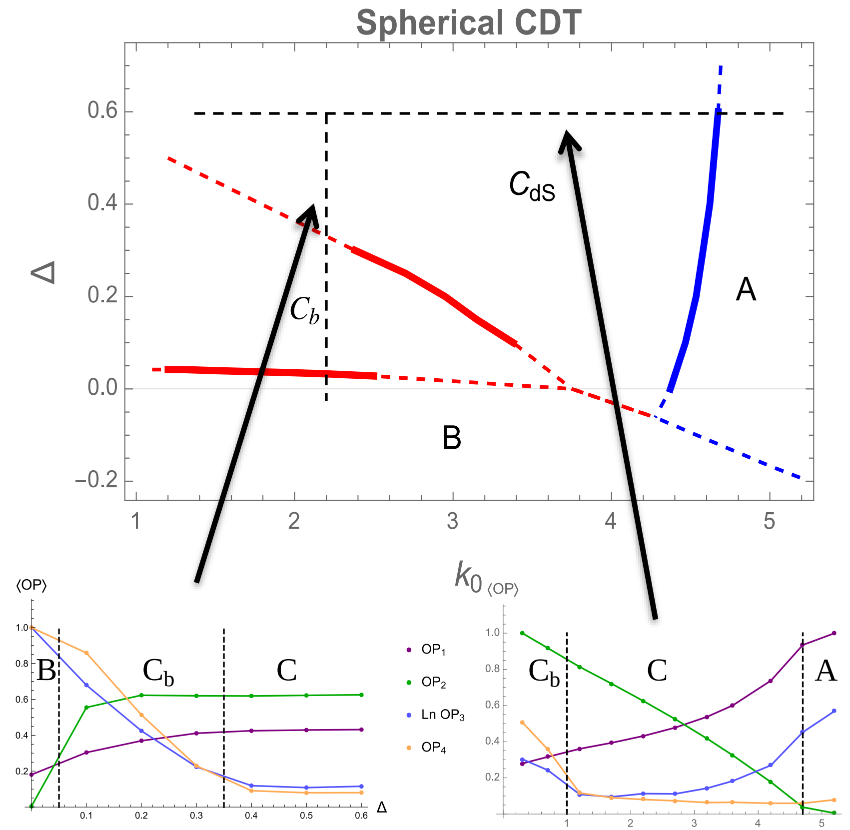


		A-C ("weak" I order)	A-B (I order)	B-Cb (II order)
SUPERVISED learning	Decision Tree			
	Gradient Boosted Trees			
	Logistic Regression			
	Nearest Neighbors			
	Neural Network			
	Random Forest			
	Support Vector Machines			
UNSUPERVISED learning	Agglomerate (2)			
	DBSCAN			
	Gaussian Mixture			
	K-Means (2)			
	MeanShift			
	Neighborhood Contraction			
	Spectral (2)			

Phase structure & phase transitions

- *Phase transitions:*
  - *Even though we don't study changes of the field configurations on the fixed spacetime but the changes of the spacetime geometry itself, we assume that it makes sense to use standard (lattice) statistical physics techniques*
  - *They require to define order parameters which capture symmetry differences between generic configurations (geometries) in different phases*
  - *Finite size scaling analysis is used to distinguish between 1st and 2nd order phase transitions*
  - *We are especially intersted in phase transitions sourrounding (the semi-classical, see next slides) phase C_{ds}*



$$OP_1 = N_0/N_4$$

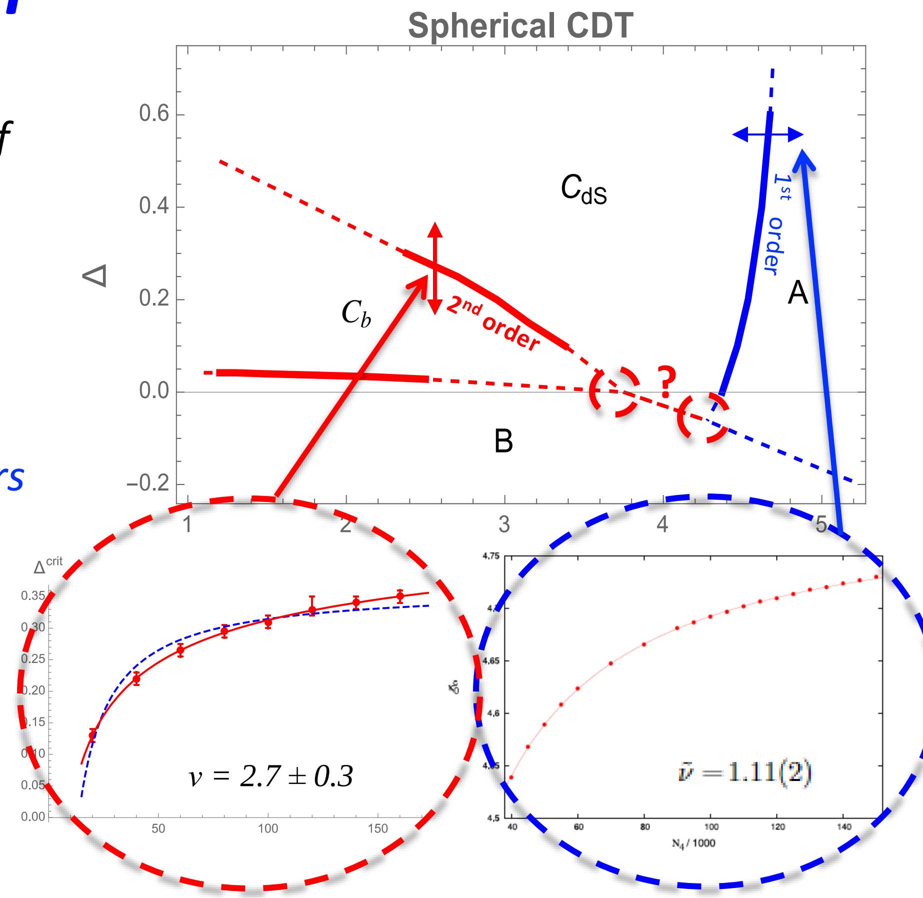
$$OP_3 = \sum_i (N_3(i+1) - N_3(i))^2$$

$$OP_2 = N_4^{(3,2)}/N_4^{(4,1)}$$

$$OP_4 = \max_v O(v)$$

OP	Phase A	Phase B	Phase C_{ds}	Phase C_b
OP_1	large	small	medium	medium
OP_2	small	small	large	large
OP_3	medium	large	small	medium
OP_4	small	large	small	large

Phase structure & phase transitions



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JHEP 02 (2016) 144
Phys. Rev. D 95 (2017) 124029

Phys. Rev. D 85 (2012) 124044
Class.Quant.Grav. 36 (2019) 224001

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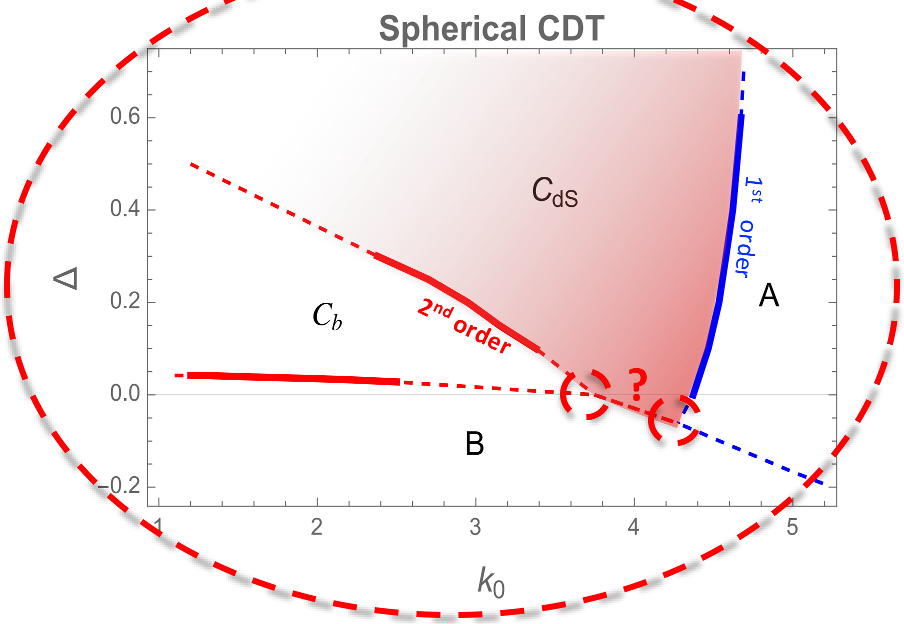

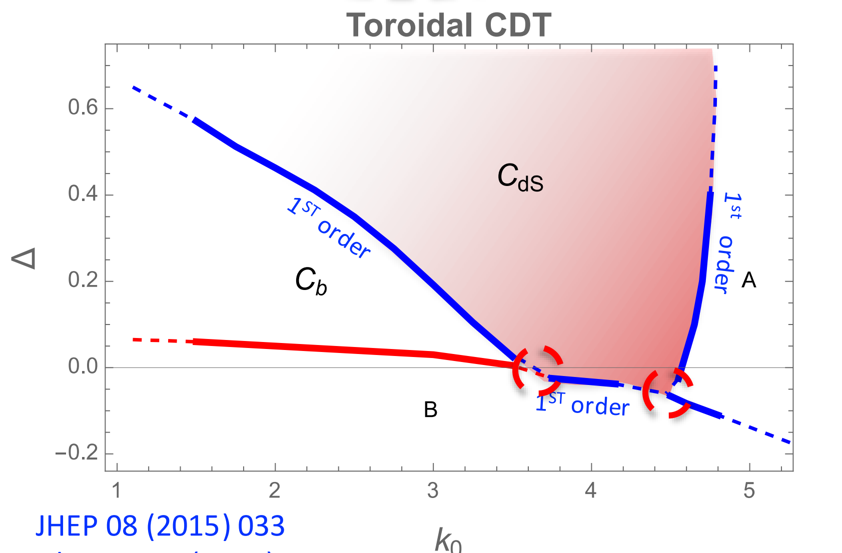

JHEP 08 (2015) 033
Eur.Phys.J.C 77 (2017) 152

Semi-classical phase

Semi-classical phase

- *Phase* *C_{ds}* *(de Sitter phase) has good semi-classical properties !*
- *Effective dimensions consistent with d = 4*
- *Dynamicaly emerging background geom.* ⊲
  - $\langle N_3(i)\rangle$ *profile of elongated ($\widetilde{\omega} \neq \omega_0$) 4-sphere*
  - *renormalizing* $l_t \to l_t = l_s \left(\frac{\omega_0}{\widetilde{\omega}}\right)^{4/3}$ *one obtains symmetric S^4, i.e., classicaly:* Euclidean de Sitter universe (max. sym. space with *Λ>0*)
  - local (average) curvature* consistent with S^4
  - ~homogenous and isotropic** on large scales
- *Minisuperspace behaviour of the scale factor*
  - *From quantum fluctuations of $N_3(i)$ one can recover the effective action of the scale factor*
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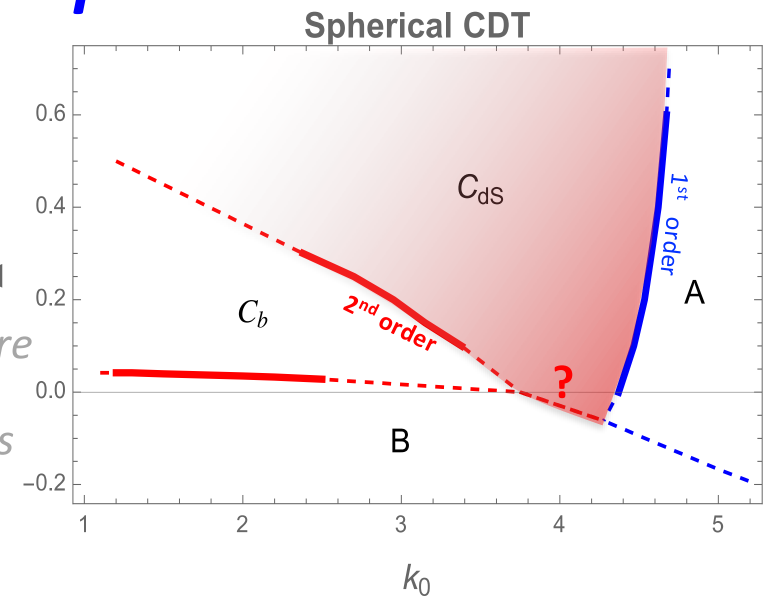
- *This was „derived" from first principles !*



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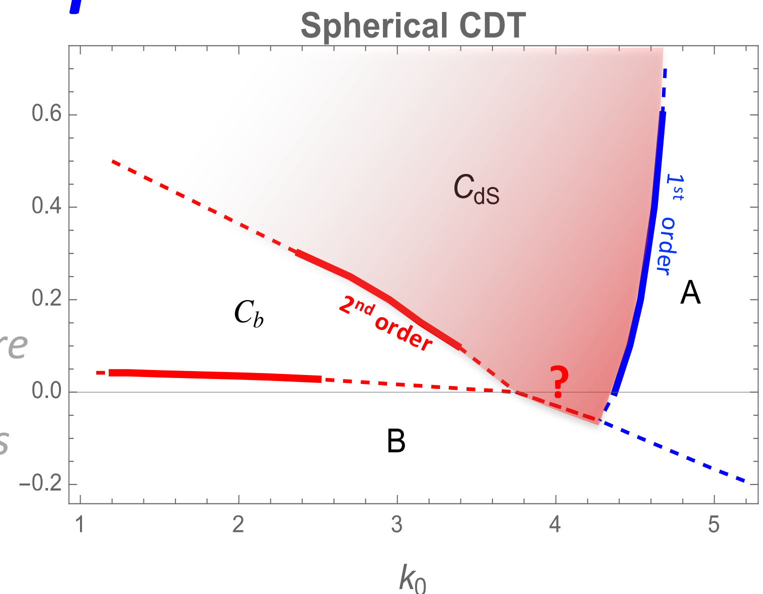

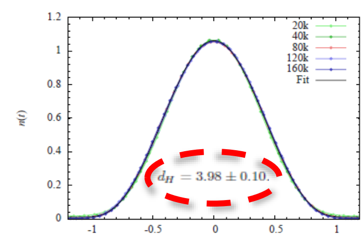

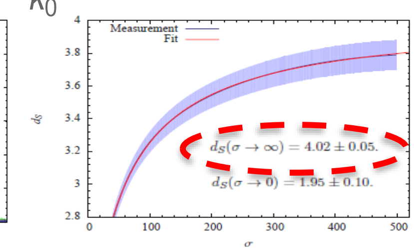

$\bar{n}_t \circledR \frac{\bar{n}_t}{N_4^{1-1/d_H}}$ $\quad t \circledR \frac{t}{N_4^{1/d_H}}$ $\quad d_S(s) = -2\frac{d\log P_R(s)}{d\log s}$

J. Ambjorn, J. Jurkiewicz R. Loll, PRL 95 (2005) 171301

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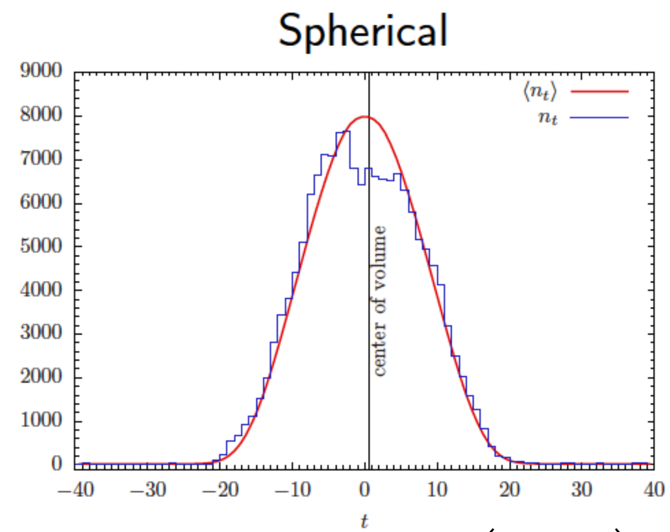

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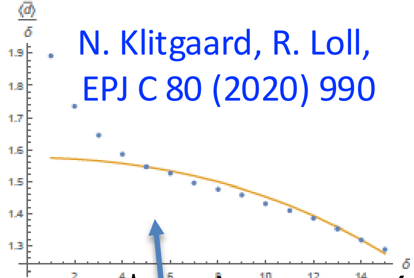
N. Klitgaard, R. Loll, EPJ C 80 (2020) 990



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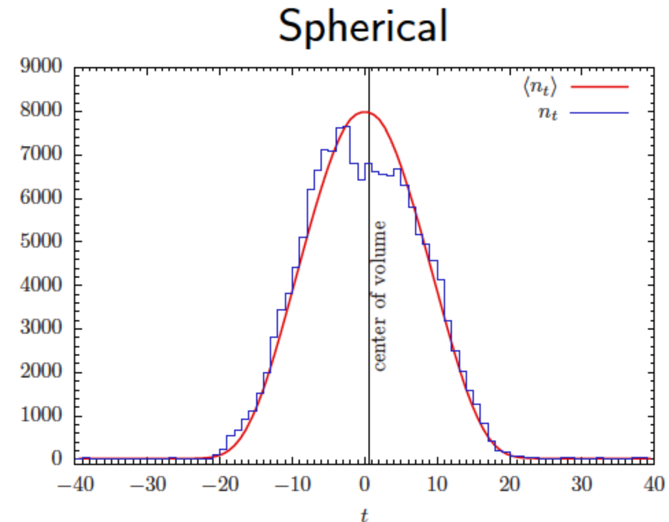

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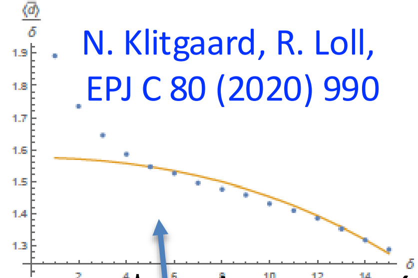
** Homogeneity measures in CDT: R. Loll , A. Silva, PRD 107 (2023) 086013



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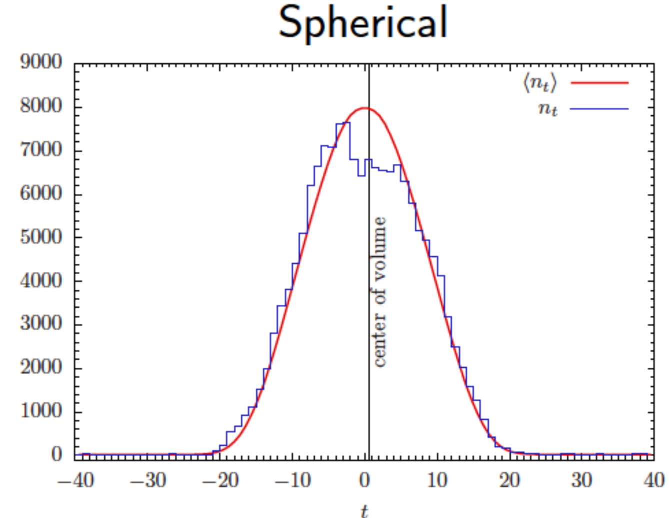

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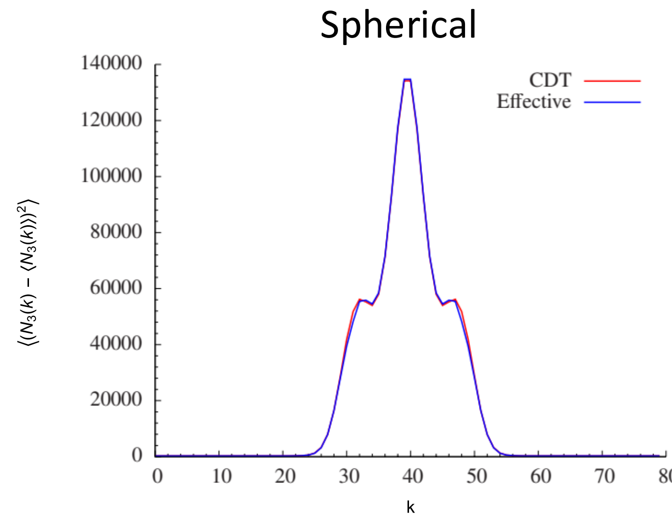

$$S = \frac{1}{\tilde{\Gamma}} \sum_i \left(\frac{(N_3(i+1) - N_3(i))^2}{N_3(i)} + \tilde{\mu} N_3(i)^{1/3} \right)$$

$$V_3(t_i) = N_3(i) l_s^3 \qquad \mu_0 = 9(2\pi^2)^{2/3}$$

$$S = \frac{l_t}{l_s} \frac{1}{\tilde{\Gamma} l_s^2} \sum_i l_t \left(\frac{(V_3(t_i + l_t) - (V_3(t_i))^2}{l_t^2 V_3(t_i)} + \frac{l_s^2}{l_t^2} \tilde{\mu} V_3(t_i)^{1/3} \right)$$

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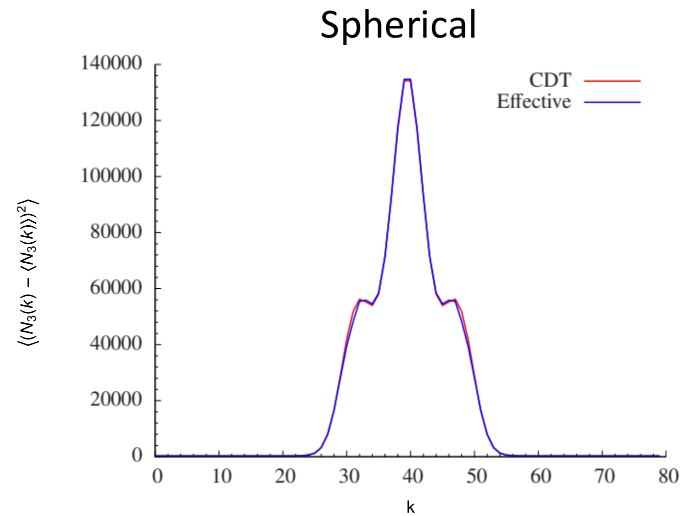

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Nucl. Phys. B 849 (2011) 144

agrees with Hartle–Hawking „noboundary" proposal

$\mu_0 = 9(2\pi^2)^{2/3}$

$$S_{MS} = \frac{1}{24\pi G} \int dt \left(\frac{\dot{V}_3(t)^2}{V_3(t)} + \mu_0 V_3(t)^{1/3} \right)$$

$$\boxed{24\pi G = \left(\frac{\tilde{\omega}}{\omega_0}\right)^{4/3} \tilde{\Gamma}\, l_s^2}$$

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Functional Renormalization Group

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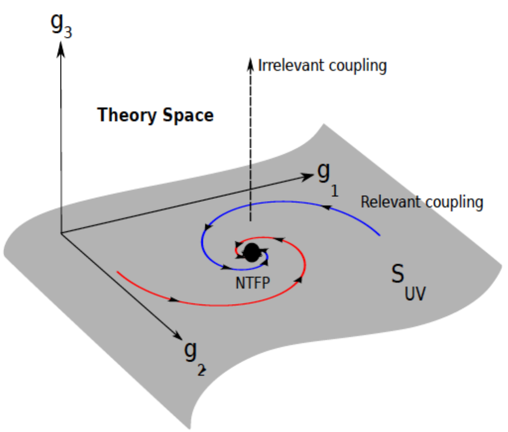
- *As CDT the FRG is also based on „traditional" non-petrurbative QFT framework to quantize gravity*
  - *Consider a (potentially ∞ dim.) space of all effective actions* of QG (or in practice their truncations)*
  - Alternatively one has a space of scale-dependent dimensionless couplings related to operators appering in the effective actions
  - *Compute the RG flow (based on solving* β-functions) *of the couplings with the cutoff scale* k
  - *Find RG trajectories linking IR* ($k \to 0$) *and UV* ($k \to \infty$) *fixed points* ($\beta = 0$) *of the RG flow*



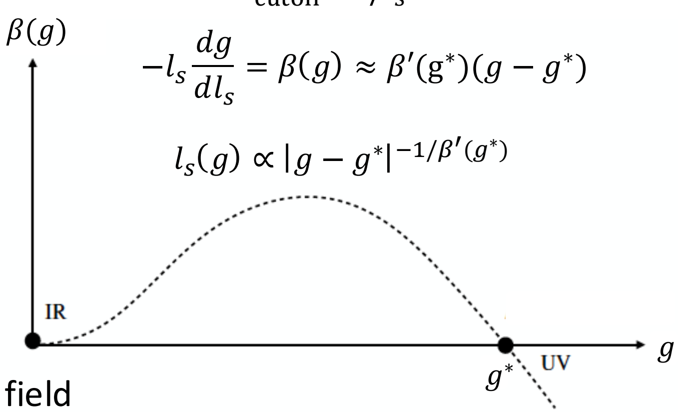
- *Asymptotic Safety conjecture (S. Weinberg)*
  - *Scale invariance of the UVFP imposes strong constraints on most operators (couplings)*
  - *On RG flow trajectories leading from IR to UV fixed points there is only a finite numer of relevant operators (finite dim. subspace of relevant couplings)*
  - *Even though the values of the couplings in the UV limit are not small one one can get a predictive theory of QG at all scales (nonperturbative renormalizability)*
  - *There is growing evidence from FRG in favour of AS*

Close to UVFP:

$$k_{\text{cutoff}} \sim 1/l_s$$

$$-l_s \frac{dg}{dl_s} = \beta(g) \approx \beta'(g^*)(g - g^*)$$

$$l_s(g) \propto |g - g^*|^{-1/\beta'(g^*)}$$



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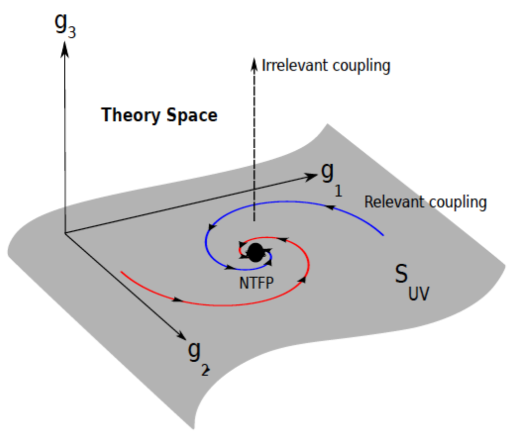

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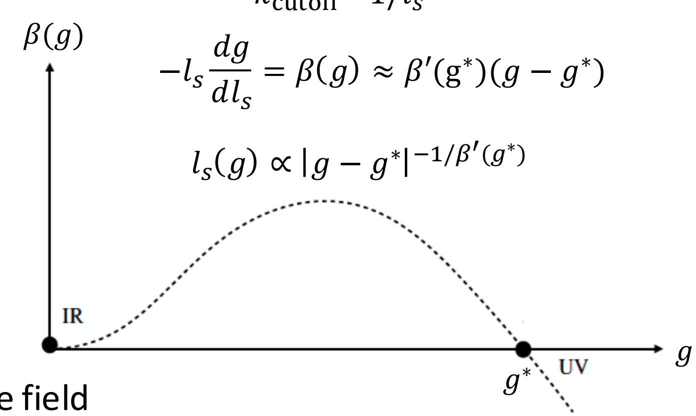

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$$S_k = \cancel{-}\frac{1}{16\pi G_k}\int d^4x\sqrt{\det g}(R - 2\Lambda_k) + gauge + ghost$$

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  - *In CDT one measures the (minisuperspace) Einstein-Hilbert effective action*
  - *Therefore in FRG we take the simplest Einstein-Hilbert truncation of the (Euclidean) effective actions with two scale-dependent couplings: G_k , Λ_k*
  - *An extremum of the E-H effective action is a de Sitter universe (the four-sphere S^4) with a 4-volume given by the cosmological constant $V_4 \propto \Lambda_k^{-2}$*
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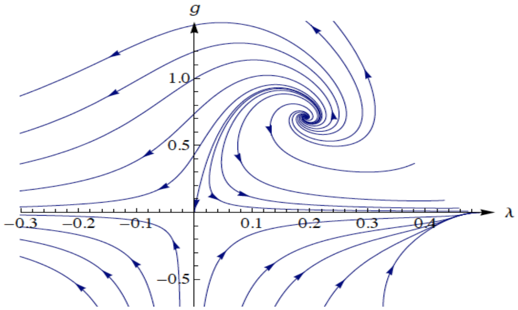

- *There are both the IR and the UV fixed points*
  - *In the IR ($k \to 0$): $G_k\Lambda_k \to 0$ as $G_k \to G_0 \approx G_N$, $\Lambda_k \to 0$ so one recovers semiclassical universe with $V_4 \to \infty$*
  - *In the UV ($k \to \infty$): $G_k\Lambda_k \to g^*\lambda^* \sim 1$ as $G_k \to g^*k^{-2} \to 0$, $\Lambda_k \to \lambda^*k^2 \to \infty$ so $V_4 \to 0$*

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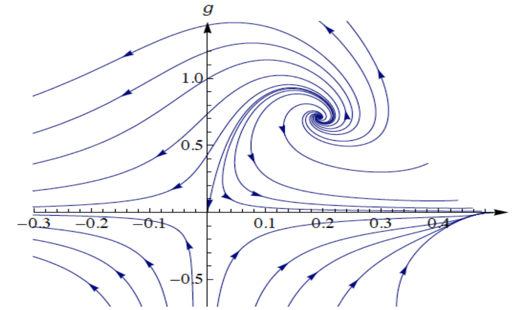

$$V_3(t) = \frac{3}{4}V_4\frac{1}{\omega_0 V_4^{1/4}}\cos^3\left(\frac{t}{\omega_0 V_4^{1/4}}\right)$$

$$S_{MS} = \frac{1}{24\pi G_k}\int dt\left(\frac{\dot V_3(t)^2}{V_3(t)} + \mu_0 V_3(t)^{1/3}\right)$$

$$\int dt\, V_3(t) = V_4 \propto \Lambda_k^{-2}$$

$$v_3 \equiv V_3/V_4^{3/4} \qquad s \equiv t/V_4^{1/4}$$

$$v_3(t) = \frac{3}{4}\frac{1}{\omega_0}\cos^3\left(\frac{t}{\omega_0}\right)$$

$$S_{MS} = \boxed{\frac{\sqrt{6}}{4\, G_k\Lambda_k}}\int ds\left(\frac{\dot v_3(t)^2}{v_3(t)} + \mu_0 v_3(t)^{1/3}\right)$$

$$\int ds\, v(s) = 1$$

Functional Renormalization Group

- *Making contact of FRG with CDT:* $S_k = \frac{1}{16\pi G_k}\int d^4x\sqrt{\det g}(R - 2\Lambda_k) + gauge + ghost$
  - *In CDT one measures the (minisuperspace) Einstein-Hilbert effective action*
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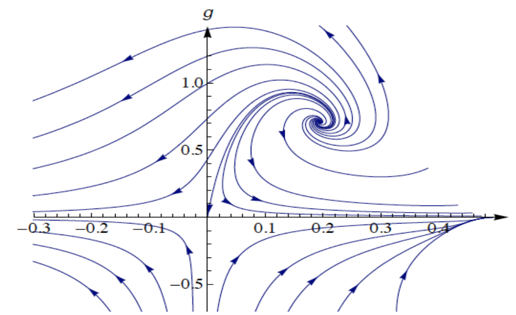
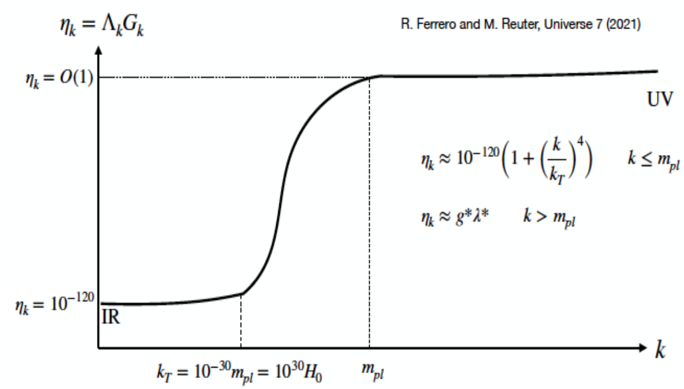
  - *The (relative) fluctuations are governed by a dimensionless effective coupling $g_{ef}^2 \propto G_k\Lambda_k$*





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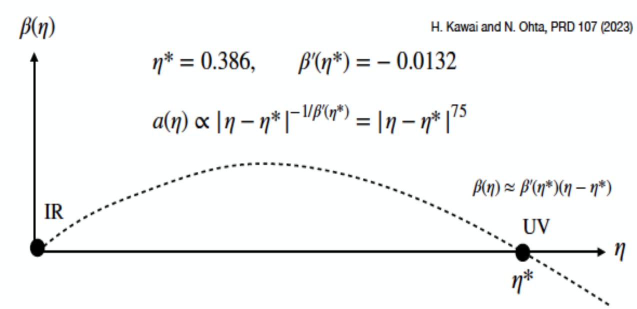

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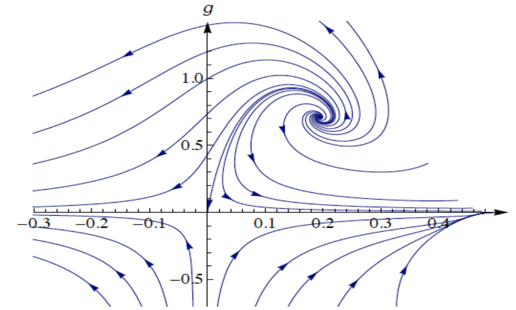

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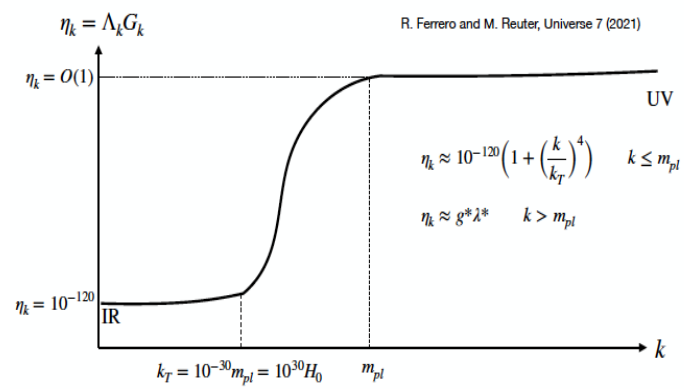

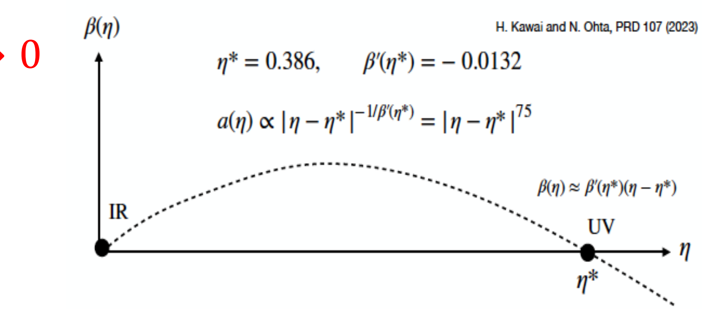

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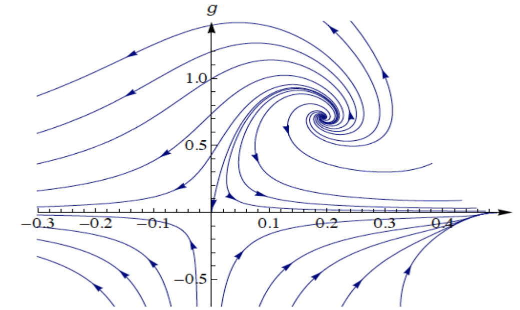
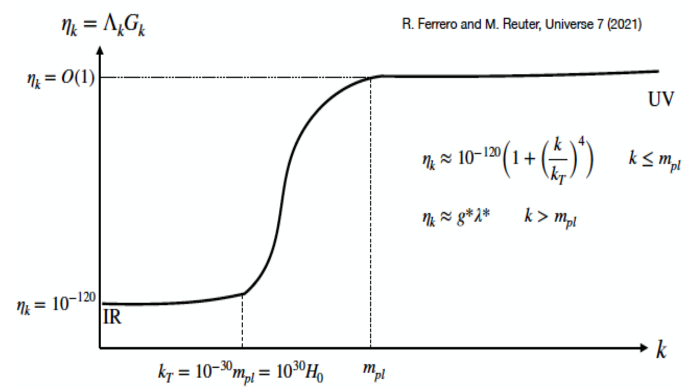
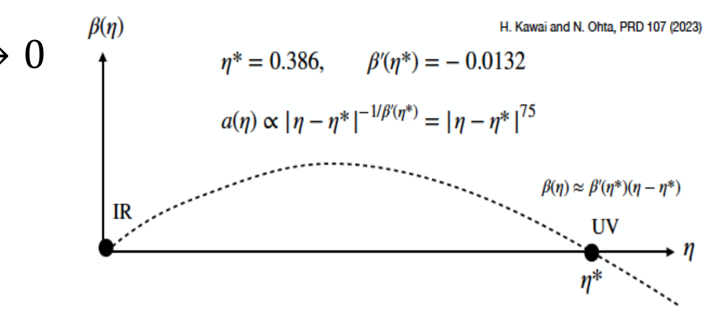


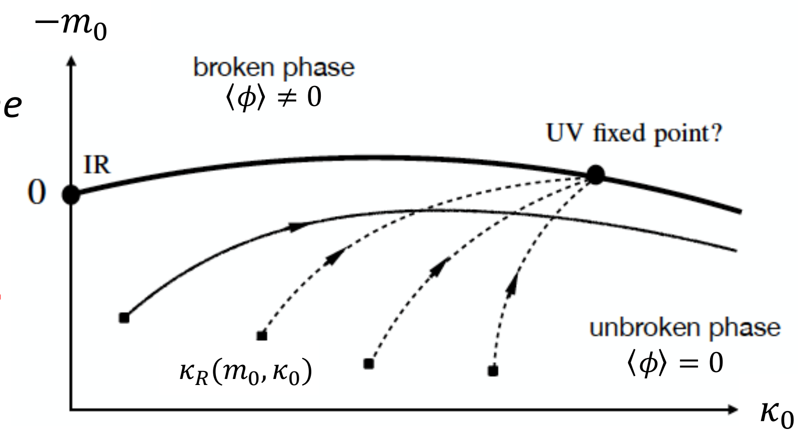




RG flow on the lattice

RG flow on the lattice (ϕ^4 example)

- *4D ϕ^4 (lattice) field theory example**
  - *2 dimensionless bare couplings: m_0, κ_0*
  - *for each choice of m_0, κ_0 one can compute the renormalized m_R, κ_R and the correl. length ξ*
  - *physical correl. length $\xi_{ph} = m_R^{-1} = \xi\, l_s$*
  - one can find RG flow where $\kappa_R, m_R = const.$
  - *there is a phase transition (where $\xi \to \infty$ so following the RG flow trajectory $l_s \to 0$)*
- *The IR limit*
  - *we approach the phase transition ($\xi \to \infty$)* keeping the bare coupling *κ_0 fixed*
  - *we cross the $\kappa_R = const$ RG trajectories in the direction of $\kappa_R \to \kappa_R^{ir}$*
- *The UV limit*
  - *we approach the phase transition ($\xi \to \infty$)* keeping the renormalized coupling *κ_R fixed*
  - *in order to do that we have to tune the bare coupling κ_0*



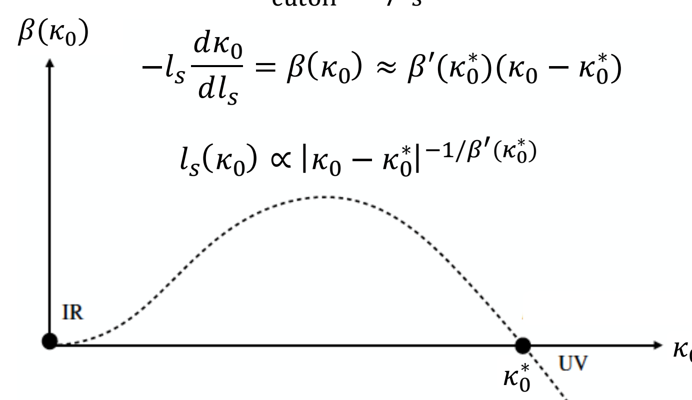
$$L = \left(\partial_\mu \phi\right)^2 + m_o \phi^2 + \kappa_0 \phi^4$$

$$\kappa_R \propto \Gamma_4(p_i = 0; m_0, \kappa_0)$$

$$k_{\text{cutoff}} \sim 1/l_s$$

$$-l_s \frac{d\kappa_0}{dl_s} = \beta(\kappa_0) \approx \beta'(\kappa_0^*)(\kappa_0 - \kappa_0^*)$$

$$l_s(\kappa_0) \propto |\kappa_0 - \kappa_0^*|^{-1/\beta'(\kappa_0^*)}$$



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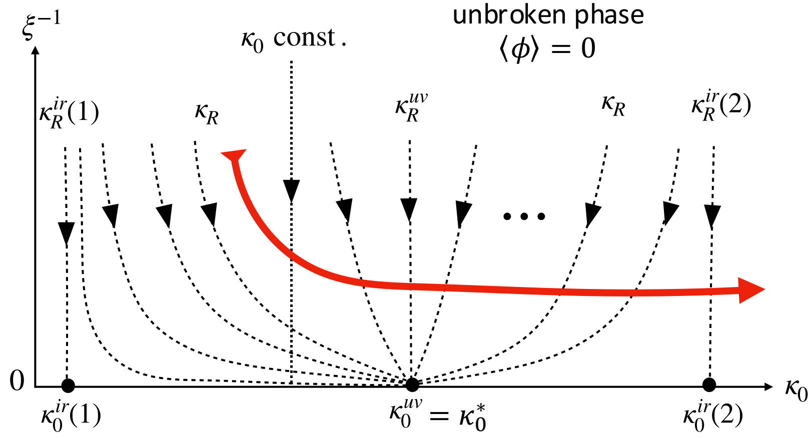

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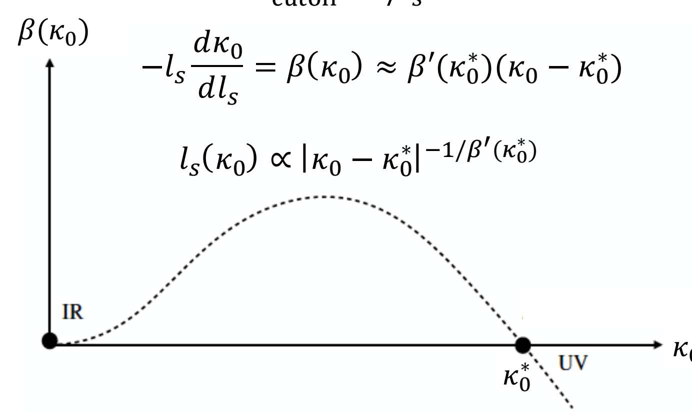

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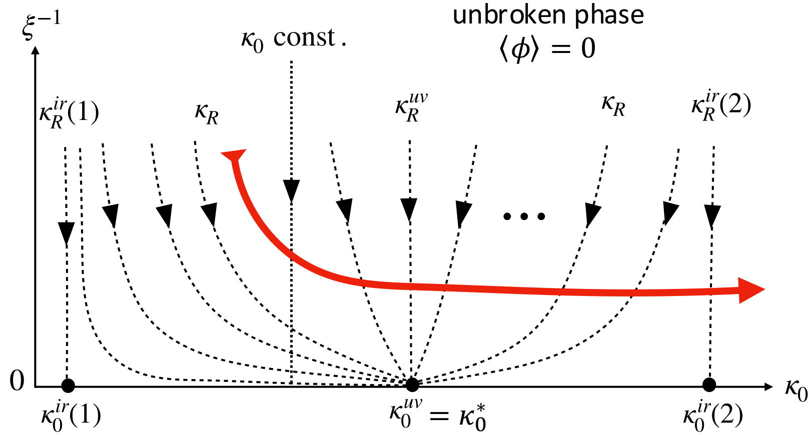

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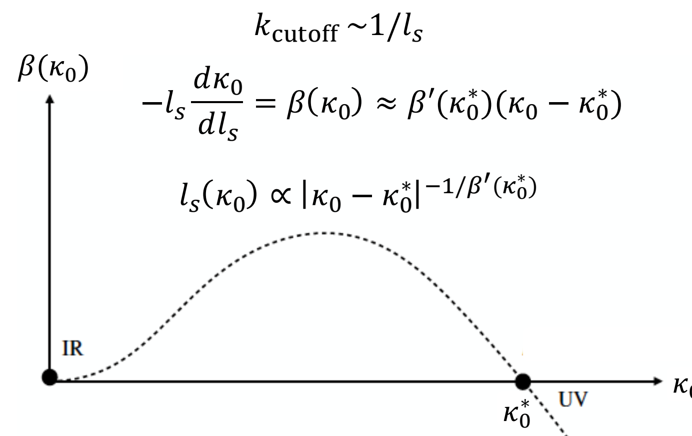

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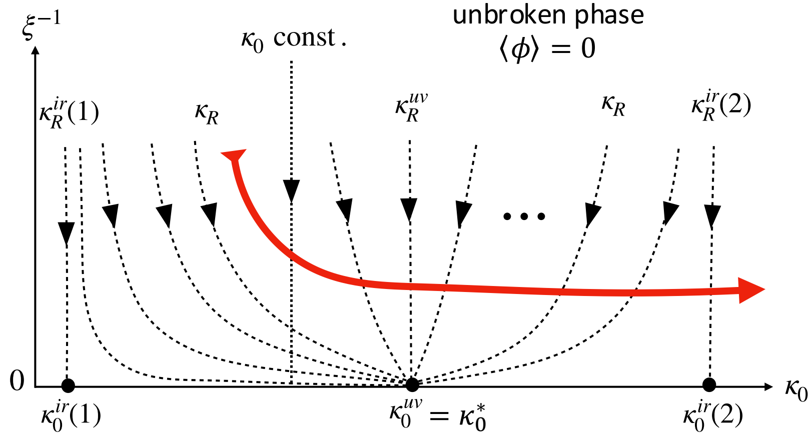

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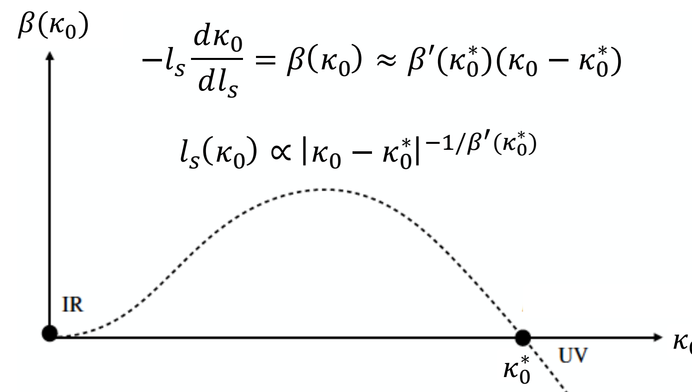

**Unfortunately there is no UV fixed point in ϕ^4*

RG flow on the lattice (CDT)

- *CDT*
  - *3 dimensionless bare couplings: k_0, Δ, K_4*
  - *The bare cosmol. const. K_4 is related to lattice volume N_4 : $K_4 \to K_4^{crit}(k_0, \Delta)$ when $N_4 \to \infty$*
  - *One can argue (based on 2-dim CDT results) that incide phase C_{dS} the correl. length: $\xi \propto N_4^{1/4}$*
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  - *we will approach the* $K_4^*(k_0, \Delta)$ critical surface ($\xi \to \infty$) keeping the bare couplings k_0 *, Δ fixed*
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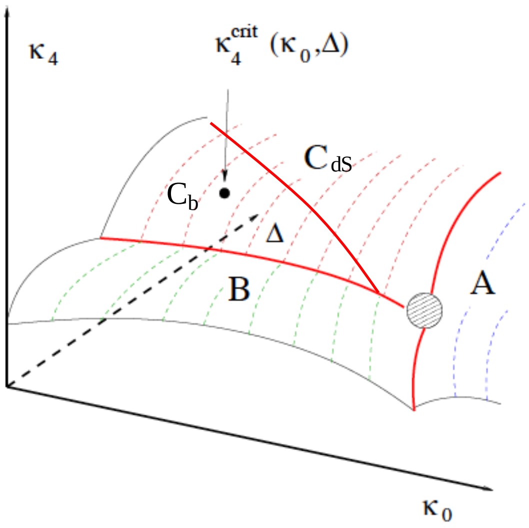

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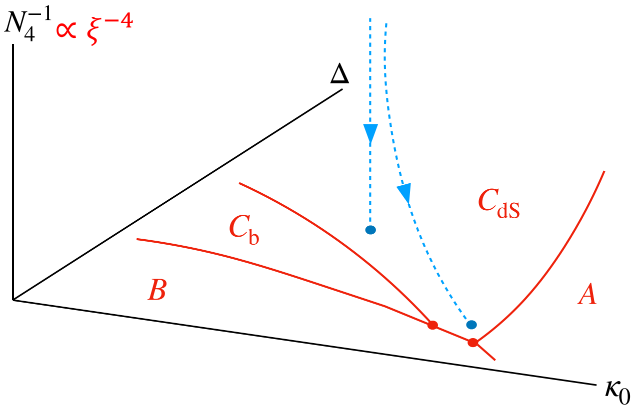

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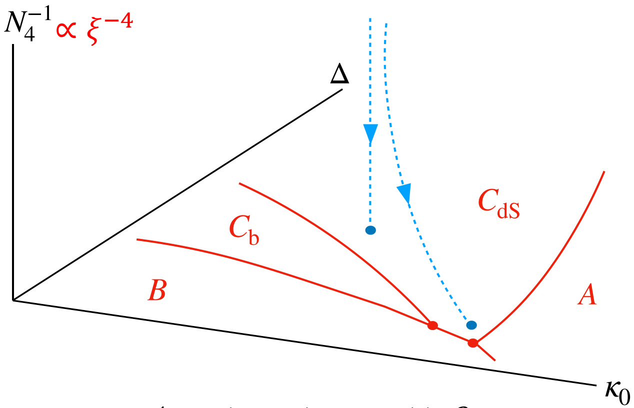

$$S = \frac{1}{\tilde{\Gamma}} \sum_i \left(\frac{(N_3(i+1) - N_3(i))^2}{N_3(i)} + \tilde{\mu} N_3(i)^{1/3} \right)$$

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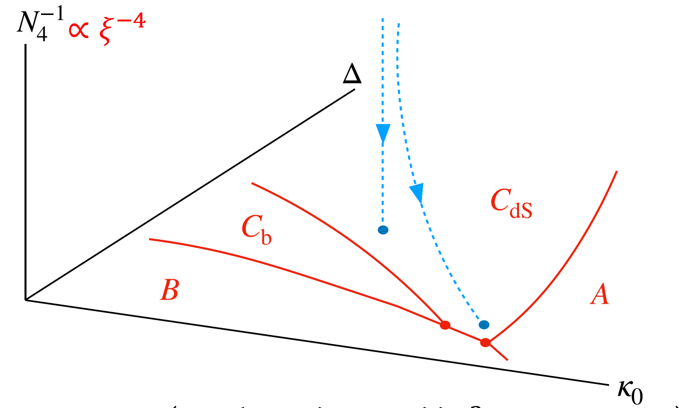

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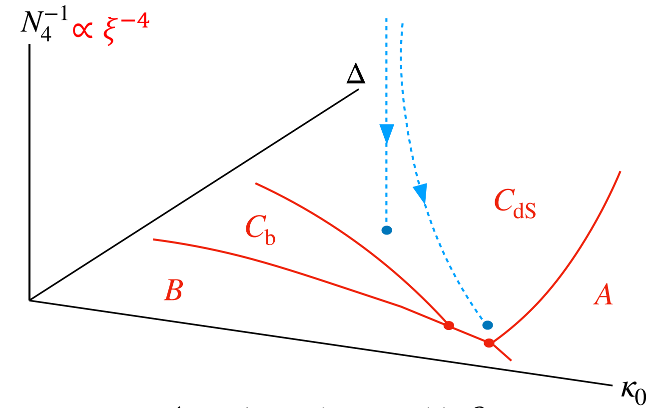

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$$\boxed{\begin{array}{c} k \to 0 : \; G_k \Lambda_k \to 0 \\ G_k \to G_0 \approx G_N, \; \Lambda_k \to 0 \end{array}}$$

- ✧ *we will approach the $K_4^*(k_0, \Delta)$ critical surface* ($\xi \to \infty$) keeping the bare couplings k_0 , Δ *fixed*
- ✧ *we associate it with the IR limit of FRG*

✧ *The UV limit*

$$\boxed{\begin{array}{c} k \to \infty : \; G_k \Lambda_k \to g^* \lambda^* \\ G_k \to g^* k^{-2}, \; \Lambda_k \to \lambda^* k^2 \end{array}}$$

- ✧ *we will approach the $K_4^*(k_0, \Delta)$* critical surface ($\xi \to \infty$) tuning the bare couplings k_0 , Δ *such that the effective coupling $G\Lambda$ stays fixed*
- ✧ *we associate it with the UV limit of FRG*

$$\boxed{\frac{\tilde{\Gamma}}{\sqrt{N_4}} \left(\frac{\tilde{\omega}}{\omega_0} \right)^2 \propto G_k \Lambda_k}$$

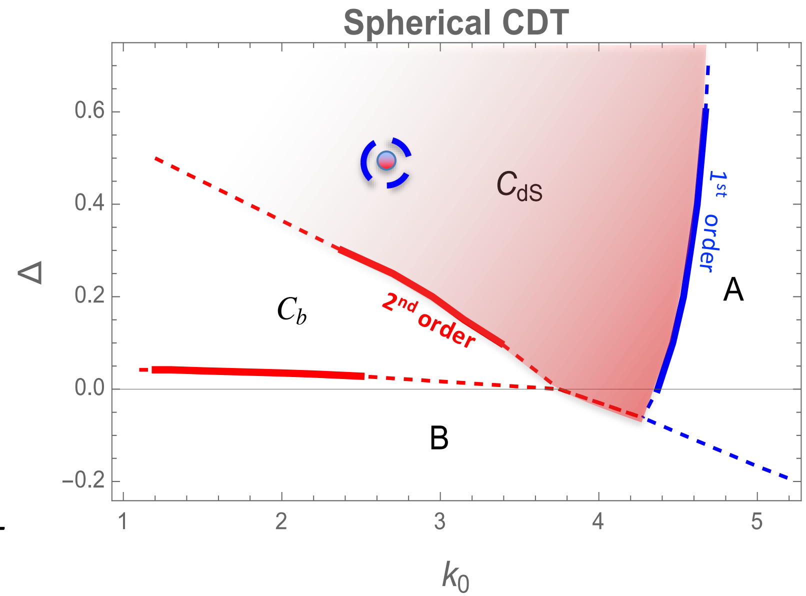
$$\boxed{24\pi G = \left(\frac{\tilde{\omega}}{\omega_0} \right)^{4/3} \tilde{\Gamma} \, l_s^2}$$

$$\boxed{V_4 = l_s^4 \left(\frac{\omega_0}{\tilde{\omega}} \right)^{4/3} N_4 \propto \frac{1}{\Lambda^2}}$$

IR limit

IR limit

- ✧ *IR limit*
  - ✧ *we approach the* $K_4^*(k_0, \Delta)$ critical surface ($\xi \to \infty$, i.e. $N_4 \to \infty$) keeping the bare couplings k_0 , *Δ fixed*
  - ✧ *for fixed* k_0 , *Δ* we have $\tilde{\Gamma}$, $\tilde{\omega} = const. > 0$
  - ✧ *from FRG for* $k \to 0$: $G_k \to G_0 \approx G_N = \ell_{Pl}^2$
  - ✧ *therefore in CDT lattice spacing remains constant :* $l_s \sim \ell_{Pl}$
  - ✧ *as* $N_4 \to \infty$ *and* $l_s > 0$ *the volume of the CDT universe* $V_4 \to \infty$
  - ✧ *this is consistent with FRG as for* $k \to 0$: $\Lambda_k \to 0$ so $V_4 \propto \Lambda_k^{-2} \to \infty$
  - ✧ *CDT (relative) fluctuations vanish and one reproduces (semi) classical spacetime*
  - ✧ *this is also consistent with FRG where* $G_k \Lambda_k \to 0$



$$\langle N_3(i)\rangle = N_4^{3/4}\frac{3}{4\tilde{\omega}}\cos^3\left(\frac{i}{\tilde{\omega}N_4^{1/4}}\right)$$

$$\langle|\delta N_3(i)|\rangle = \tilde{\Gamma}^{1/2}N_4^{1/2}\ \mathrm{F}\left(\frac{i}{\tilde{\omega}N_4^{1/4}}\right)$$

$$\frac{\langle|\delta V_3(t_i)|\rangle}{\langle V_3(t_i)\rangle} = \frac{\langle|\delta N_3(i)|\rangle}{\langle N_3(i)\rangle} \propto \frac{\sqrt{\tilde{\Gamma}\tilde{\omega}^2}}{N_4^{1/4}} \to 0$$

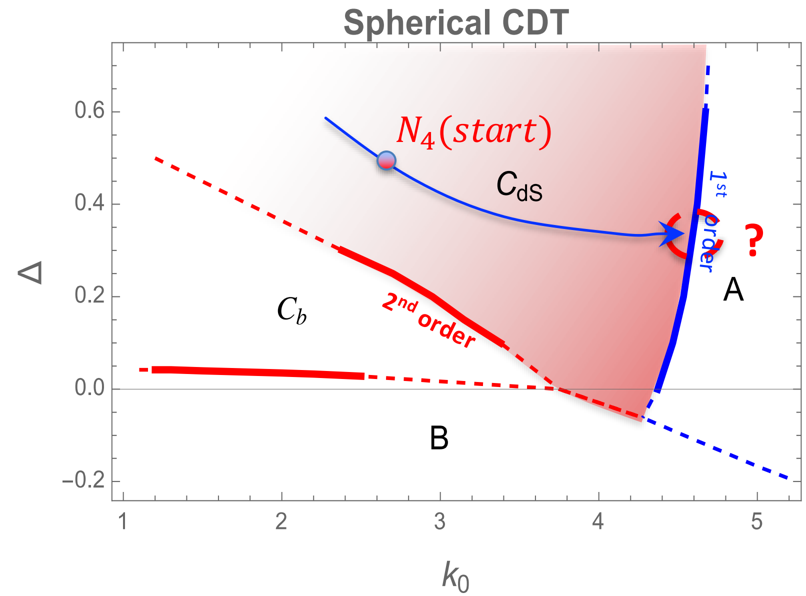
$$\boxed{24\pi G = \left(\frac{\tilde{\omega}}{\omega_0}\right)^{4/3}\tilde{\Gamma}\, l_s^2}$$

$$\boxed{V_4 = l_s^4\left(\frac{\omega_0}{\tilde{\omega}}\right)^{4/3} N_4 \propto \frac{1}{\Lambda^2}}$$

UV limit

UV limit

- *UV limit*
  - *we approach the* $K_4^*(k_0, \Delta)$ critical surface ($\xi \to \infty$, i.e. $N_4 \to \infty$) tuning the bare couplings k_0 *, Δ such that the effective* $G_k \Lambda_k = g^* \lambda^* = const$
  - *from FRG:* $G_k \to g^* k^{-2} \to 0$
  - *therefore in CDT :* $l_s \sim k^{-1} \to 0$
  - *(relative) fluctuations stay constant*
- *This requires finding RG flow trajectories* $\left(k_0(N_4), \Delta(N_4)\right)$ *parametrized by* N_4
- *Is only possible by approaching the* $C_{dS} - A$ *phase transition line*
  - *we fix* Δ $(\Delta = 0)$** and change only* k_0
  - *one can compute critical exponents related to scaling of* $\tilde{\Gamma}$ *and* $\tilde{\omega}$ *at the transition*
  - *the results show that it may be possible to approach the UV limit*
  - *however it is done at the 1st order transition*



$$\tilde{\Gamma}\left(k_0(N_4), \Delta(N_4)\right) \tilde{\omega}^2\left(k_0(N_4), \Delta(N_4)\right) \propto \sqrt{N_4}$$

$$\frac{\tilde{\Gamma}}{\sqrt{N_4}} \left(\frac{\tilde{\omega}}{\omega_0}\right)^2 \propto G_k \Lambda_k = g^* \lambda^* = const$$

$$\frac{\langle |\delta V_3(t_i)| \rangle}{\langle V_3(t_i) \rangle} = \frac{\langle |\delta N_3(i)| \rangle}{\langle N_3(i) \rangle} \propto \frac{\sqrt{\tilde{\Gamma} \tilde{\omega}^2}}{N_4^{1/4}} \to const \sim 1$$

$$24\pi G = \left(\frac{\tilde{\omega}}{\omega_0}\right)^{4/3} \tilde{\Gamma}\, l_s^2$$

$$V_4 = l_s^4 \left(\frac{\omega_0}{\tilde{\omega}}\right)^{4/3} N_4 \propto \frac{1}{\Lambda^2}$$

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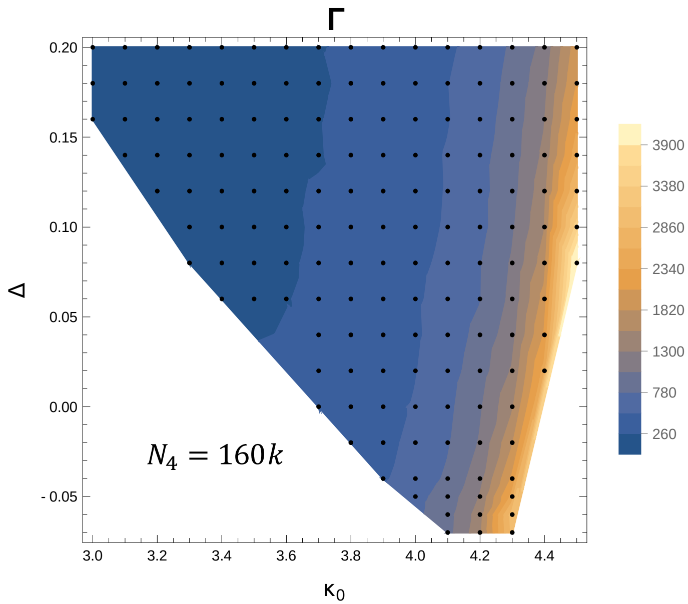

$$\tilde{\Gamma}\left(k_0(N_4), \Delta(N_4)\right) \widetilde{\omega}^2\left(k_0(N_4), \Delta(N_4)\right) \propto \sqrt{N_4}$$

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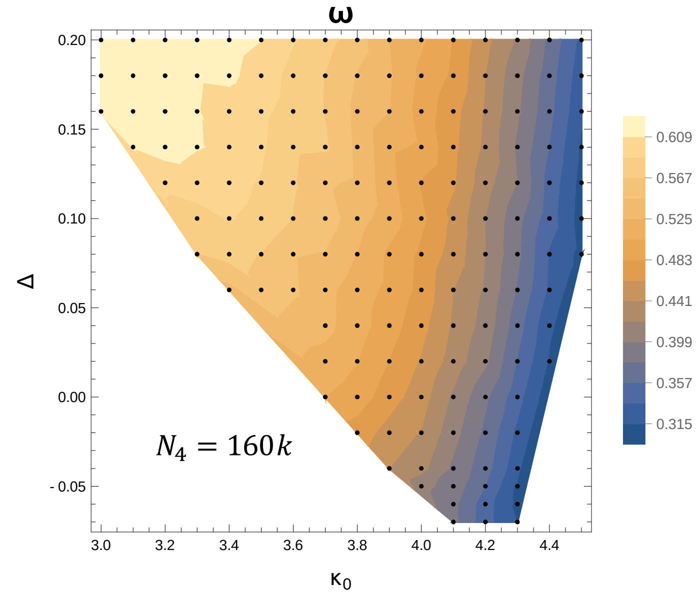

$$\boxed{\tilde{\Gamma}(k_0(N_4), \Delta(N_4))\,\tilde{\omega}^2(k_0(N_4), \Delta(N_4)) \propto \sqrt{N_4}}$$

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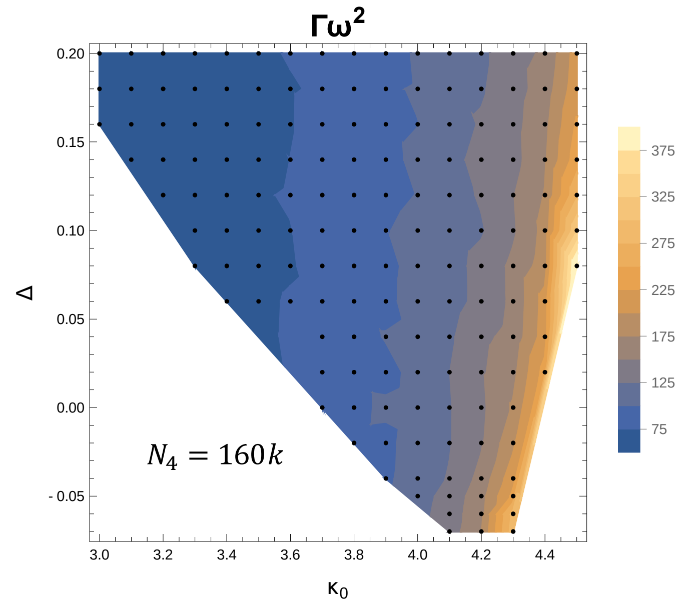

$$\boxed{\tilde{\Gamma}\big(k_0(N_4), \Delta(N_4)\big)\tilde{\omega}^2\big(k_0(N_4), \Delta(N_4)\big) \propto \sqrt{N_4}}$$

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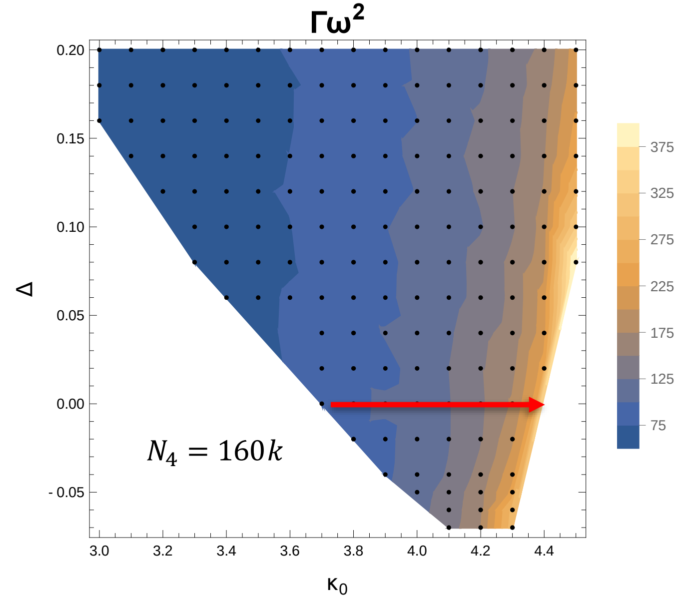
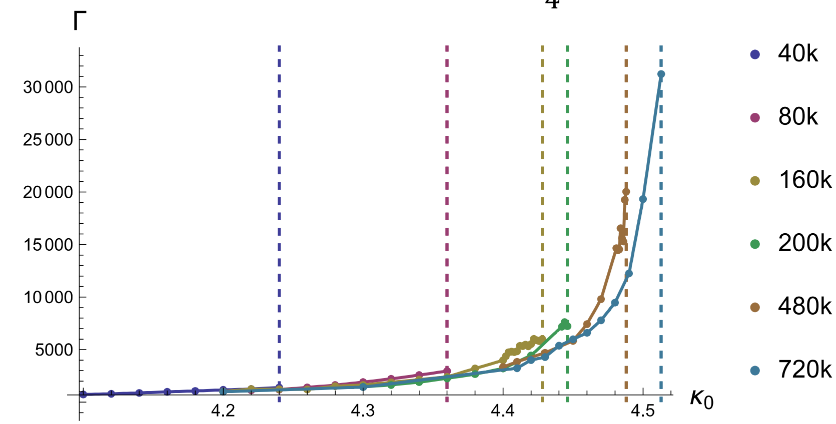


$$\boxed{\tilde{\Gamma}(k_0(N_4), \Delta(N_4))\,\tilde{\omega}^2(k_0(N_4), \Delta(N_4)) \propto \sqrt{N_4}}$$

UV limit

$$k_0^{uv}(N_4) = k_0^{uv} - \frac{c}{N_4^{1/4\nu_{uv}}}$$



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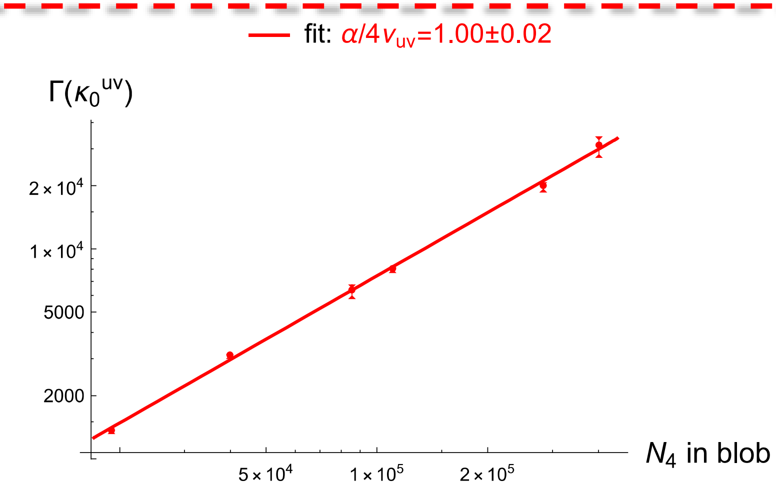
$$\tilde{\Gamma}\left(k_0^{uv}(N_4)\right) \propto N_4^{\alpha/4\nu_{uv}}$$

$$\boxed{\tilde{\Gamma}\left(k_0(N_4), \Delta(N_4)\right) \tilde{\omega}^2\left(k_0(N_4), \Delta(N_4)\right) \propto \sqrt{N_4}}$$



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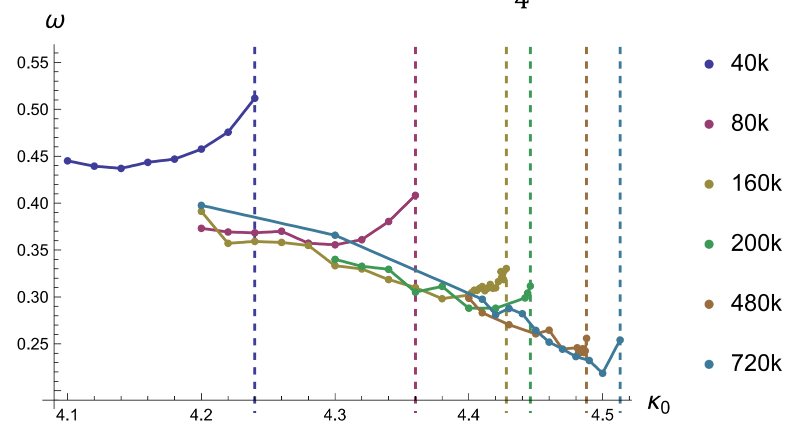

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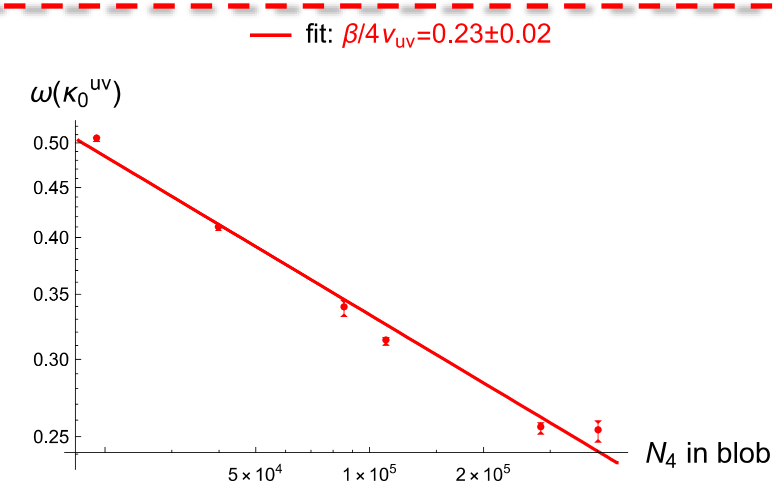
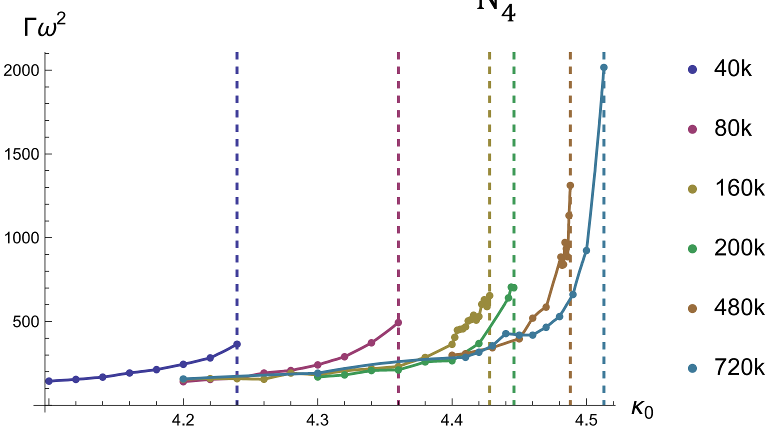
$$\tilde{\omega}\left(k_0^{uv}(N_4)\right) \propto N_4^{-\beta/4\nu_{uv}}$$

$$\boxed{\tilde{\Gamma}\left(k_0(N_4),\Delta(N_4)\right)\tilde{\omega}^2\left(k_0(N_4),\Delta(N_4)\right) \propto \sqrt{N_4}}$$



UV limit

$$k_0^{uv}(N_4) = k_0^{uv} - \frac{c}{N_4^{1/4\nu_{uv}}}$$



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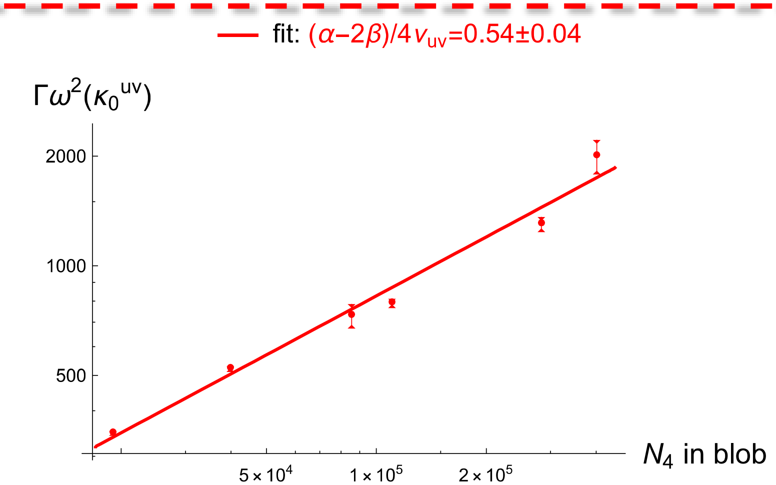
$$\boxed{(\alpha - 2\beta)/4\nu_{uv} \geq 1/2}$$

$$\tilde{\Gamma}\tilde{\omega}^2\big(k_0^{uv}(N_4)\big) \propto N_4^{(\alpha-2\beta)/4\nu_{uv}}$$

$$\boxed{\tilde{\Gamma}\big(k_0(N_4),\Delta(N_4)\big)\tilde{\omega}^2\big(k_0(N_4),\Delta(N_4)\big) \propto \sqrt{N_4}}$$



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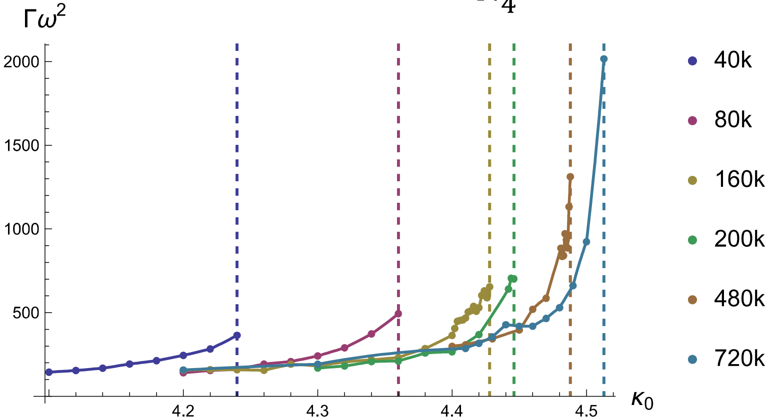

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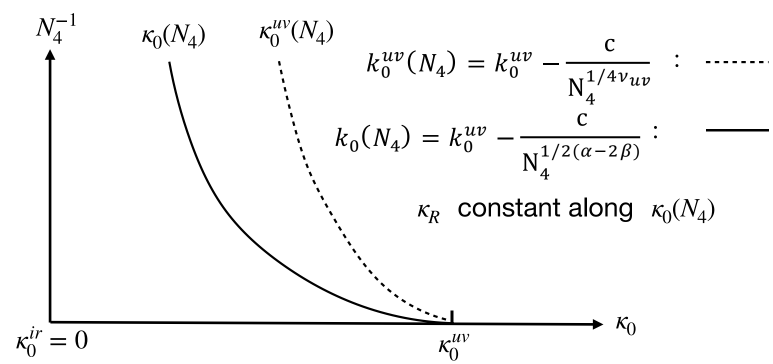

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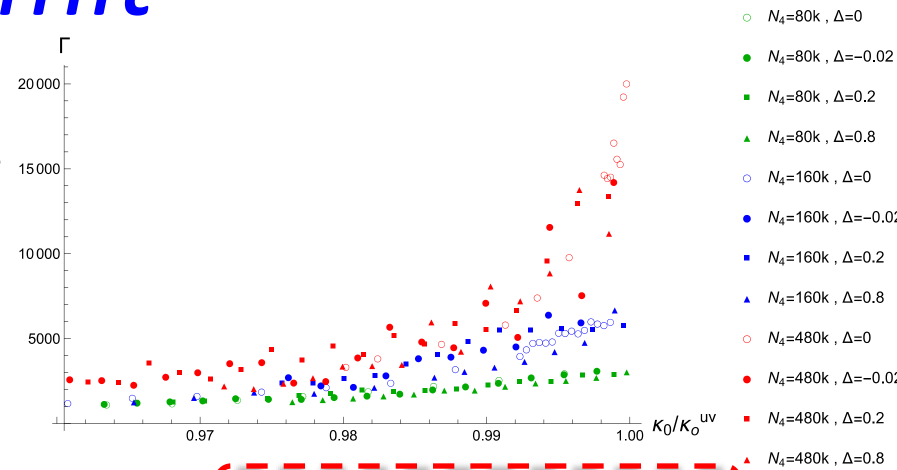
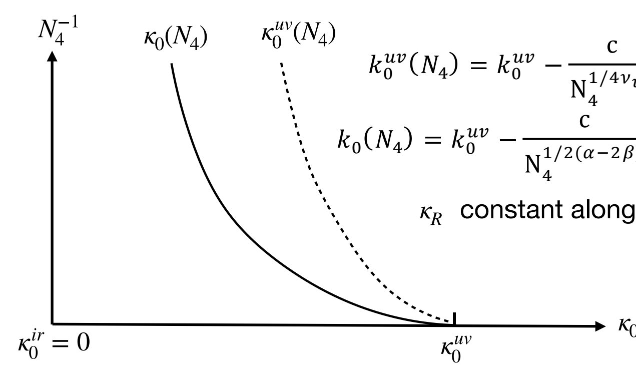


$$\boxed{(\alpha - 2\beta)/4\nu_{uv} \geq 1/2}$$

$$\tilde{\Gamma}\tilde{\omega}^2\big(k_0^{uv}(N_4)\big) \propto N_4^{(\alpha-2\beta)/4\nu_{uv}}$$

$$\boxed{\tilde{\Gamma}\big(k_0(N_4),\Delta(N_4)\big)\tilde{\omega}^2\big(k_0(N_4),\Delta(N_4)\big) \propto \sqrt{N_4}}$$



$$k_0^{uv}(N_4) = k_0^{uv} - \frac{c}{N_4^{1/4\nu_{uv}}} \quad : \quad \text{------}$$

$$k_0(N_4) = k_0^{uv} - \frac{c}{N_4^{1/2(\alpha-2\beta)}} \quad : \quad \text{———}$$

κ_R constant along $\kappa_0(N_4)$

*Results seem independent of Δ choice !

UV limit

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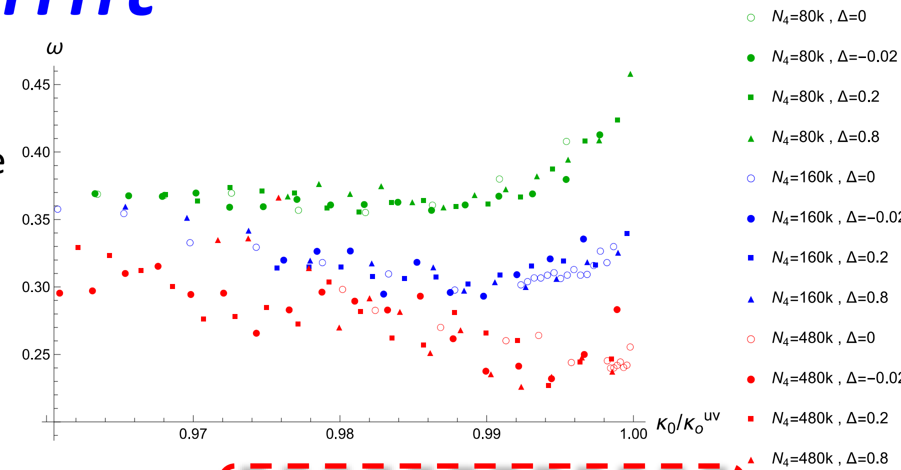

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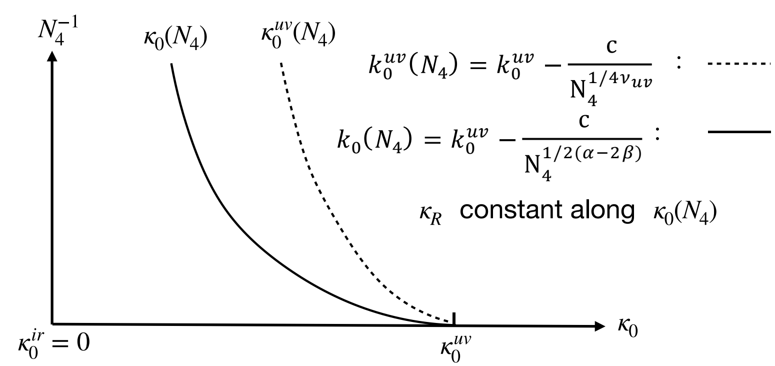
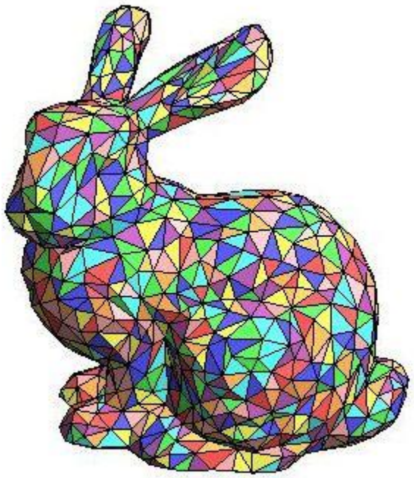
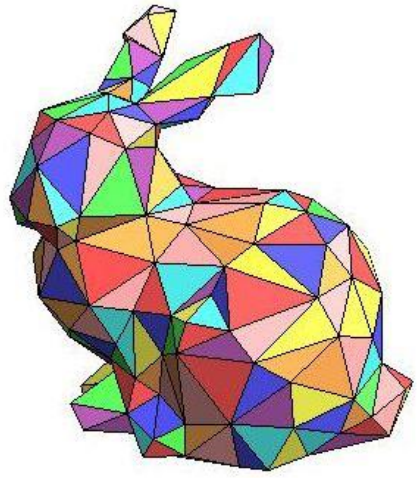


*Results seem independent of Δ choice !

Conclusions

- CDT is a lattice QFT and a promising candidate for a unitary and (if asymptotic safety is valid) UV complete (?) theory of QG formulated in a fully non-perturbative and background independent way
- One can study dynamically emerging background geometry and quantum fluctuations
- CDT has a rich phase structure including the semi-classical phase C_{dS}
  - correct IR limit of the scale factor (spatial volume) consistent with (Eucl.) de Sitter space
  - quantum fluctuations very well described by the minisuperspace action
- Phase C_{dS} is surrounded by a 2nd order phase transition (in S^3 CDT) & 1st order phase transitions with potentially higher order endpoints
- CDT can provide independent tests of the asymptotic safety conjecture in a fully non-perturbative setting, not dependent on FRG truncations
- One can make contact with FRG approach to QG by defining RG flow in CDT and search for the UV fixed point (?). The results seem promising.
- Open problems and questions:
  - the UV limit of CDT is obtained at the 1st order phase transition (non-standard)
  - this is possible because we assumed (following 2-dim CDT) that $\xi \propto N_4^{1/4}$
  - flow in FRG cutoff (k) seems to be independent from the flow in the CDT bare coupling space (k_0, Δ) and thus renormalized $\tilde{\Gamma}$, $\tilde{\omega}$. Probably more "observables" needed.

Thank You !

UNIWERSYTET JAGIELLOŃSKI W KRAKOWIE

UCZELNIA BADAWCZA
INICJATYWA DOSKONAŁOŚCI