

The 10th Conference of the Polish Society on Relativity

16 – 20 September 2024, Kazimierz Dolny, Poland

Strong Lensing – New Opportunities in Multimessenger Era

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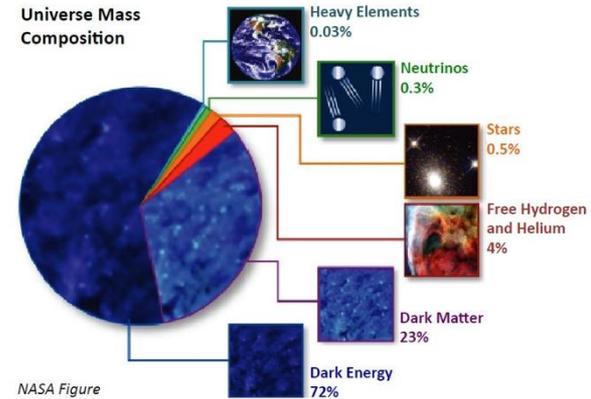
Broad perspective of the talk

- Gravitational lensing
one of the first predictions of GR
propagation of light in inhomogeneous universe
- Era of multimessenger astronomy
gravitational waves
whole EM spectrum
neutrinos
cosmic rays
- Transient events
GW from CBC
GRBs
FRBs
SNIa, SNII

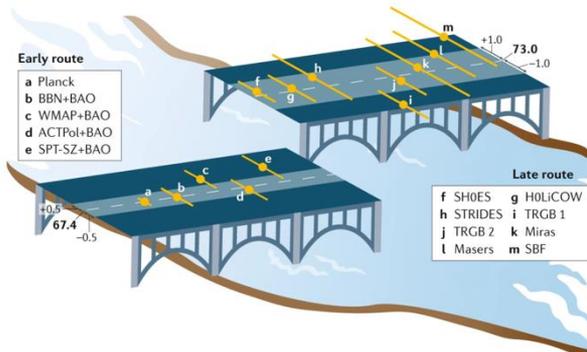
Broad perspective of the talk

- Cosmic mysteries and challenges
composition of the Universe

accelerated expansion



cracks in the concordance LCDM model



$$H_0$$

$$S_8$$

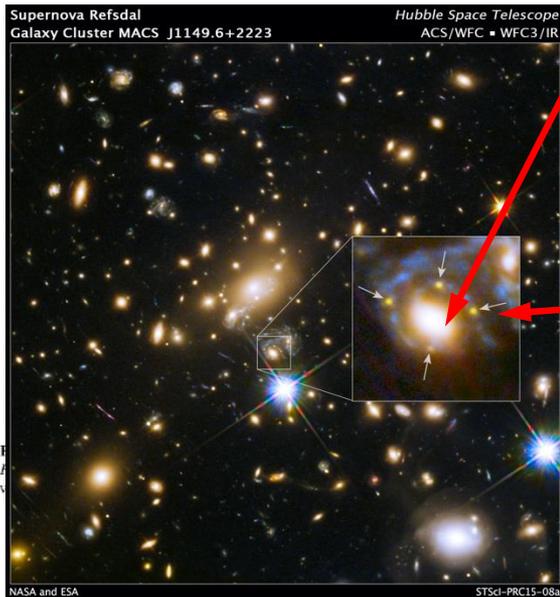
$$\Omega_k$$



There is a crack, a crack in everything
That's how the light gets in

„Refsdal“ supernova

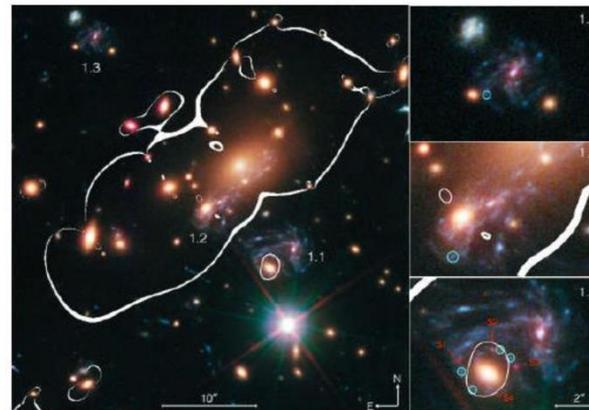
„Refsdal supernova“ discovered 11 Nov. 2014
Kelly et al. (2015) *Science* 347,1123



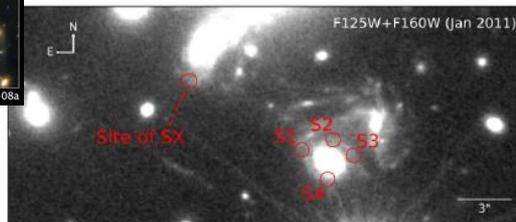
$z=0.54$ elliptical galaxy
belonging to
MACS J1149.6+2223
cluster

$z = 1.49$ source - spiral
galaxy

host of SNII

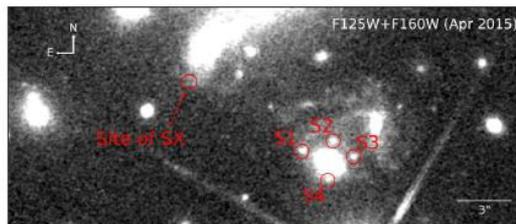


future reappearance
expected in ca. 1 yr

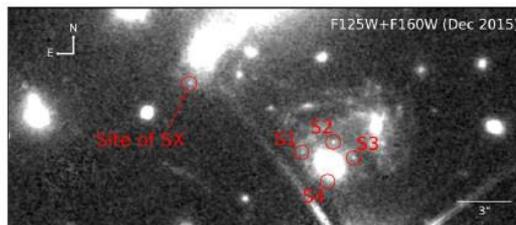


Kelly et al. (2016) *ApJL*

11 Dec. 2015
SNII found in SX image
as predicted !!!

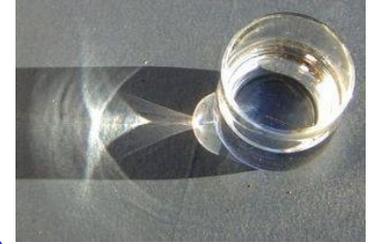


Great success of GR
(mass distribution modeling
from strong lensing)

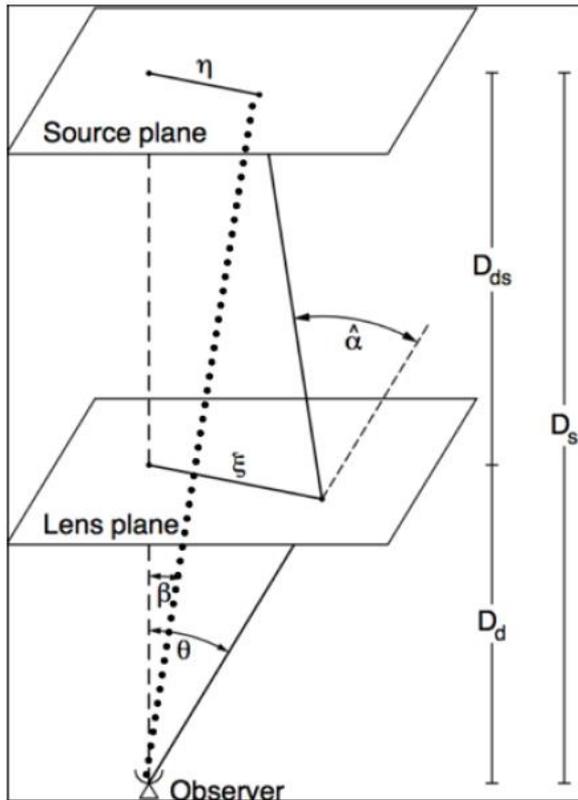


Success comparable
to the greatest triumphs of
celestial mechanics in
XIX century
(discovery of Neptune)

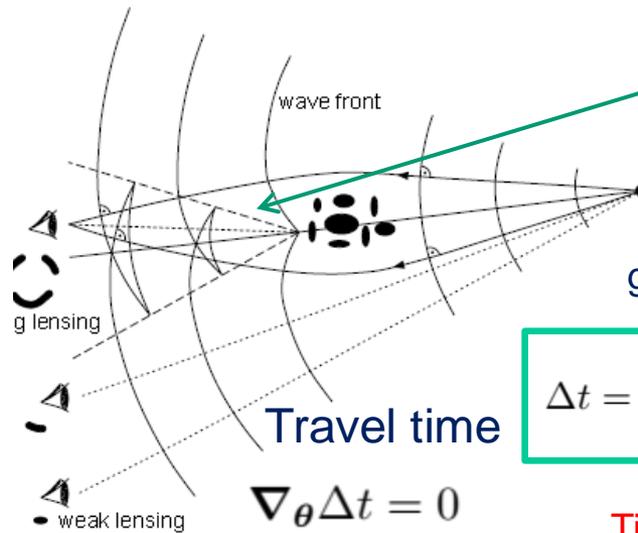
Gravitational lensing – geometric optics



Light rays formalism



Wavefront formalism (Fermat principle)



caustics

$$\phi(\theta) = \frac{D_{ls}}{D_l D_s} \frac{2}{c^2} \int \Phi(D_l \theta, z) dz$$

Newtonian potential at lens plane

geometrical term

$$\Delta t = \frac{1 + z_l}{c} \frac{D_{ol} D_{os}}{D_{ls}} \left[\frac{(\theta - \beta)^2}{2} - \phi(\theta) \right]$$

Time delay distance

Fermat potential

Lens equation

$$\hat{\alpha}(\theta) D_{ls} + \beta D_s = \theta D_s$$

$$\theta - \beta - \nabla_{\theta} \phi = 0$$

$$\theta_E = \sqrt{\frac{4GM(r < \theta_E)}{c^2} \frac{D_{ls}}{D_l D_s}}$$

Einstein radius

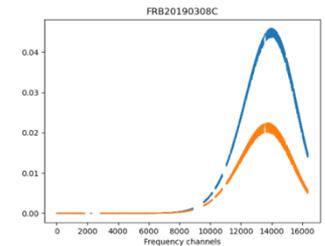
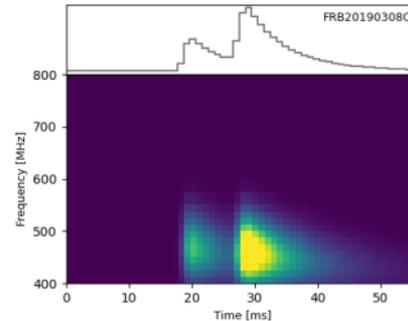
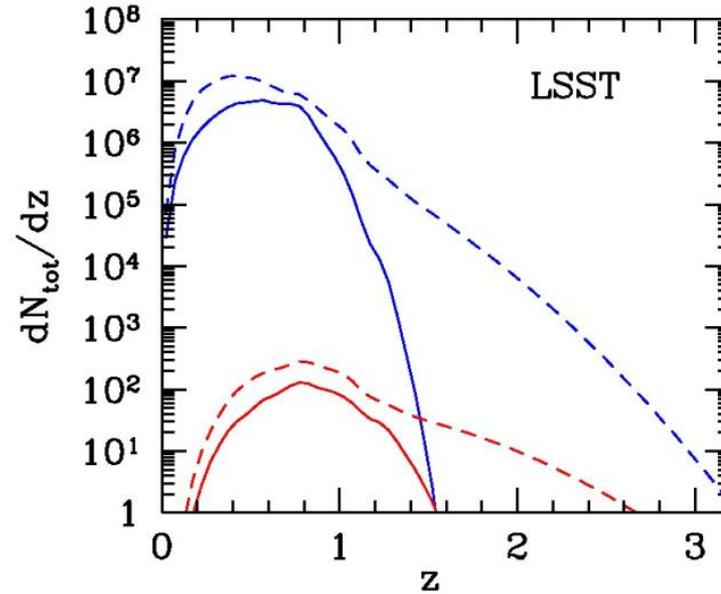
- Observables:
- * image positions and shape distortions
 - * time delay between images
 - * flux ratios magnification ratios

$$A(\theta) = \frac{\partial \beta}{\partial \theta} \quad \mu(\theta) = \frac{1}{\det A(\theta)}$$

$\alpha = \nabla_{\theta} \phi$ magnification

Strong lensing of transients

- LSST – Sne
900 strongly lensed SN
expected
ca. 650 SNIa
- GRBs
median $z \sim 2$
lensed GRB expected long
time ago (Paczynski 1996)
no fully confirmed case so far
- FRBs
bright ms long radio flashes
discovered 2007
lying at cosmological distances
expected to be lensed
first possible lensed FRB20190308C



Transients – advantage: high temporal resolution (time delays)
Radio band – high spatial resolution (image positions)

Strong lensing of gravitational waves



PRL 116, 061102 (2016)

Selected for a Viewpoint in *Physics*
 PHYSICAL REVIEW LETTERS



Observation of Gravitational Waves from a Binary Black Hole

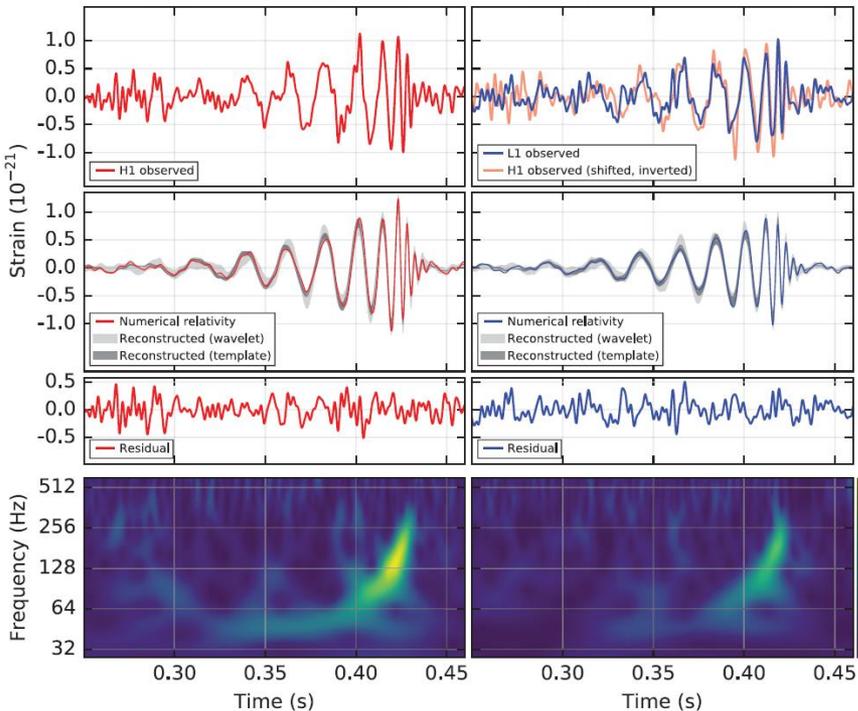
B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)

Hanford, Washington (H1)

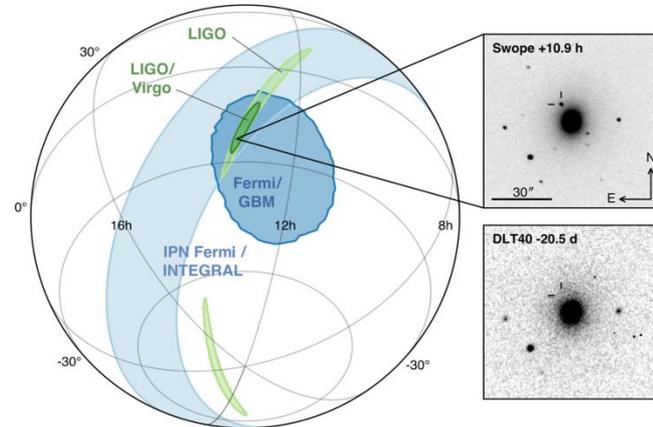
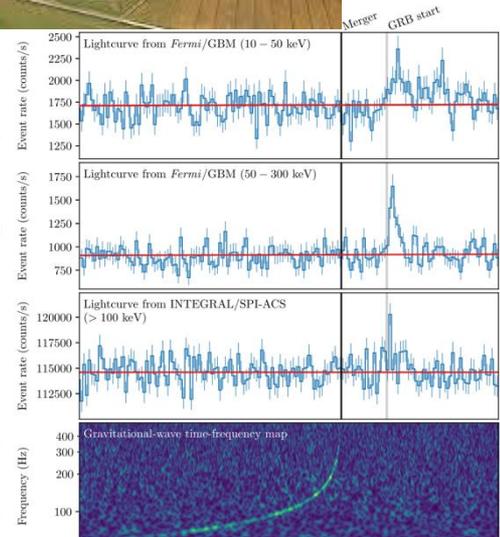
Livingston, Louisiana (L1)



tors of the Laser Interferometer Gravitational-wave signal. The signal sweeps across the sky with an average strain of 1.0×10^{-21} . It matches the waveform of a pair of black holes merging 1.3 billion years, equivalent to 100 Mpc corresponding

So far 90 registered gravitational signals in O1-O3a,b runs

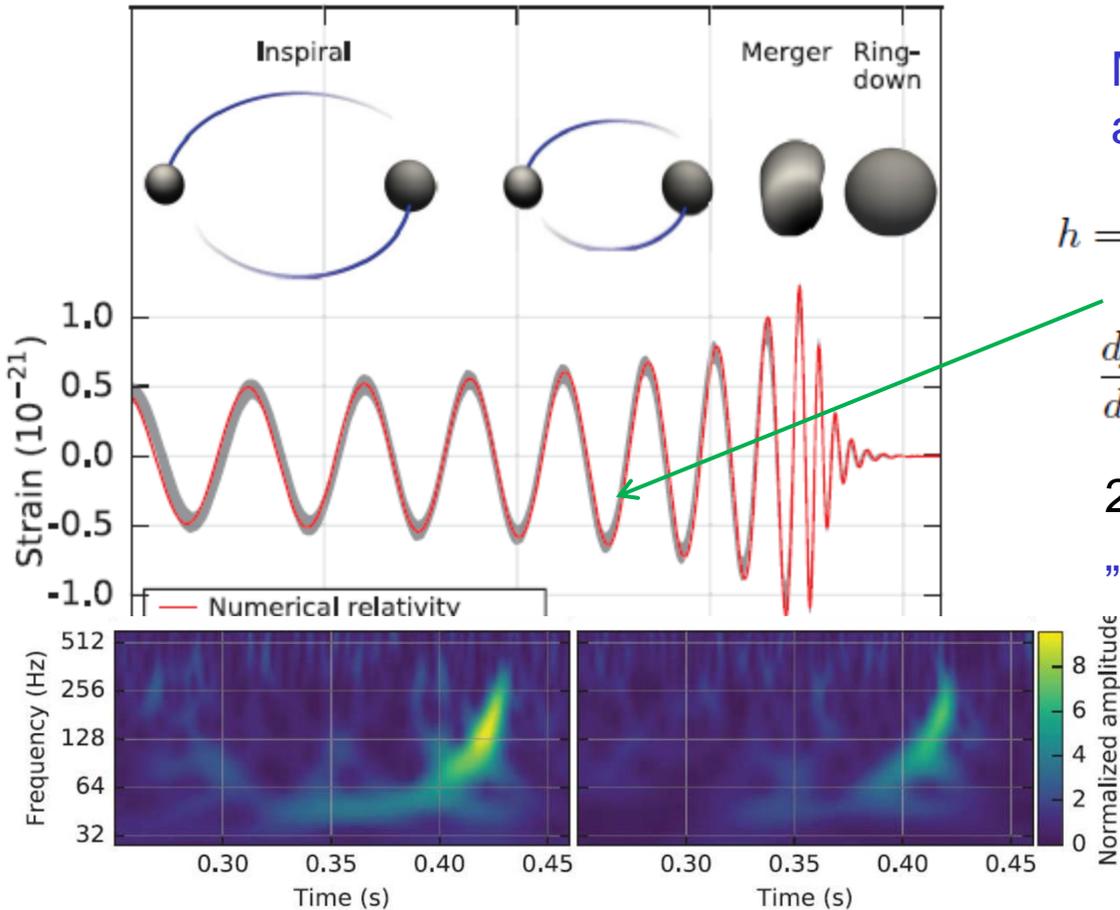
- GW150914 first ever**
- GW170817 first NS-NS**
- GW190426 first BH-NS**



The idea of „standard sirens”

B. Schutz 1986

B.Schutz, A. Królak 1987



Measure the strain $h(t)$
and frequency drift df/dt

$$h = \frac{4\pi^{2/3}(GM)^{5/3}}{c^4 D} f(t)^{2/3} \cos \left[\int_0^t f(t') dt' \right]$$

$$\frac{df}{dt} = \frac{96\pi^{8/3}}{5} \left(\frac{GM}{c^3} \right)^{5/3} f^{11/3}$$

2 equations for 2 unknowns:
„chirp mass” M & distance D

$$M = \frac{c^3}{G} \left(\frac{5}{96\pi^{8/3}} \frac{df}{dt} \right)^{3/5} f^{-11/5}$$

$$D = \frac{5c}{24\pi^2 h(t)} \frac{df(t)}{dt} f^{-3} \cos \left(\int_0^t f(t') dt' \right)$$

for cosmological sources

$$M_c \rightarrow (1+z) M_c$$

$$\begin{aligned} t &\rightarrow (1+z)t \\ f &\rightarrow \frac{1}{1+z}f \\ \frac{df}{dt} &\rightarrow \frac{1}{(1+z)^2} \frac{df}{dt} \end{aligned}$$

The distance inferred is the **luminosity distance**

$$D_L = (1+z) D \quad 9$$

Gravitational lensing in wave optics limit

Consider monochromatic wave

$$\psi(\mathbf{x}, t) = \tilde{\psi}(\mathbf{x})e^{-2\pi i f t}$$

Propagation equation in presence of weak gravitational potential U

(Helmholtz equation)

$$(\nabla^2 + \omega^2)\tilde{\psi} = 4\omega^2 U \tilde{\psi}$$

Diffraction integral

$$F(f, \beta) = \frac{1 + z_1}{c} \frac{D_{ol} D_{os}}{D_{ls}} \frac{f}{i} \int d^2 \theta \exp [2\pi i f \Delta t(\theta, \beta)]$$

$$w \lesssim 1$$

Wave effects become important and significantly modify geometric optics predictions

$$|F(f, \beta)|^2 = \frac{\pi w}{1 - e^{-\pi w}} \left| {}_1F_1 \left(\frac{i}{2} w, 1; \frac{i}{2} w \hat{\beta}^2 \right) \right|^2 \quad \text{Point mass}$$

$$|F(f, \beta)|^2 = \left| \sum_{n=0}^{\infty} \frac{\Gamma(1 + n/2)}{n!} g(w, \hat{\beta}) \right|^2 \quad \text{SIS lens}$$

$$g(w, \hat{\beta}) = \left(2w e^{(3\pi/2)i} \right)^{n/2} {}_1F_1 \left(-\frac{n}{2}, 1; \frac{i}{2} w \hat{\beta}^2 \right)$$

$$w = 2\pi f(1 + z_1) \frac{1}{c} \left(\frac{4\pi\sigma^2}{c^2} \right)^2 \frac{D_{ol} D_{ls}}{D_{os}}$$

Schneider P, Ehlers J and Falco E E 1992 *Gravitational Lenses* (Berlin: Springer);

Takahashi R and Nakamura T 2003 Wave effects in the gravitational lensing of gravitational waves from chirping binaries *Astrophys. J.* **595** 1039–51

Amplification factor

$$F = \frac{\tilde{\psi}^L}{\tilde{\psi}}$$

← wave amplitude with lensing
← without lensing

Strong gravitational lensing of explosive transients

Diffraction integral in geometric optics limit
i.e. only stationary points of time delay
contribute

$$|F(f, \beta)|^2 \approx \sum_j |\mu(\theta_j)| \text{ magnifications in g.o.} \\ + 2 \sum_{j < k} |\mu(\theta_j)\mu(\theta_k)|^{1/2} \cos [w\Phi(\theta_j, \theta_k) - \pi\Delta n_{jk}]$$

interference between images

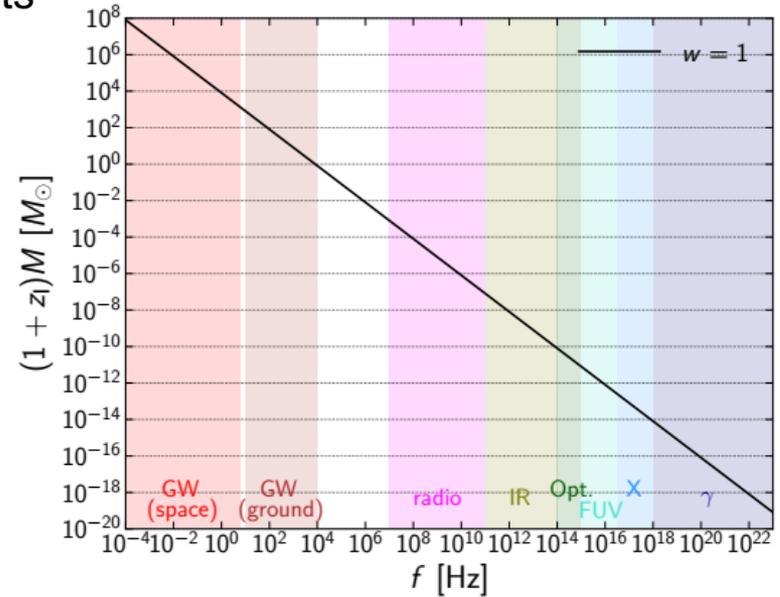


Figure 11. The relation between the frequency f and the (redshifted) mass $(1+z_1)M$ of a point mass lens for the dimensionless parameter $w = 1$, where w is defined in equation (44). The region below the solid line corresponds to the case that the gravitational lensing magnification is significantly suppressed due to the diffraction, which is one of wave optics effects.

Summary

wave optics effects:

- * diffraction
- * interference – beat patterns

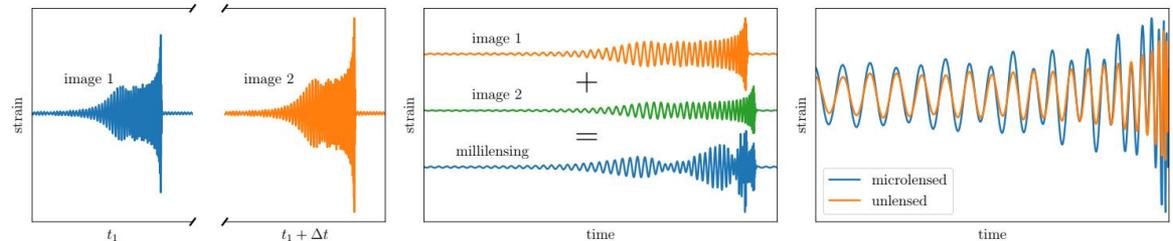


Figure 1. A representation of the effects of the different types of lensing considered in this work on gravitational wave strain. From left to right, we have strong lensing—where one has multiple distinct images—, millilensing—where one has multiple images with a time separation such that they overlap, giving a modulated signal in the detector—, and microlensing—where one has frequency-dependent beating patterns.



Einstein Telescope

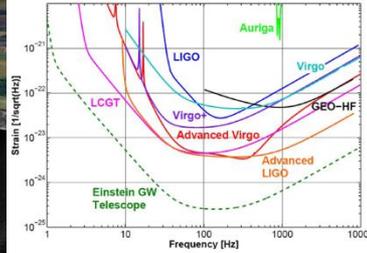


Figure 5: Sensitivities of gravitational wave detectors from the first to the third generation.

- Increased sensitivity great expectations
- Big catalogs of inspiral events up to cosmological distances
- Some of them would be gravitationally lensed

Table 2: Expected coalescence rates per Mpc^3 per Myr in the local universe ($z \simeq 0$). Also shown are predicted event rates in Advanced LIGO (aLIGO) and ET.

Source	BNS	NS-BH	BBH
Rate ($\text{Mpc}^{-3} \text{Myr}^{-1}$)	0.1–6	0.01–0.3	2×10^{-3} –0.04
Event Rate (yr^{-1}) in aLIGO	0.4–400	0.2–300	2–4000
Event Rate (yr^{-1}) in ET	$\mathcal{O}(10^3\text{--}10^7)$	$\mathcal{O}(10^3\text{--}10^7)$	$\mathcal{O}(10^4\text{--}10^8)$

A. Piórkowska *et al.* JCAP10(2013)022 (NS-NS only)

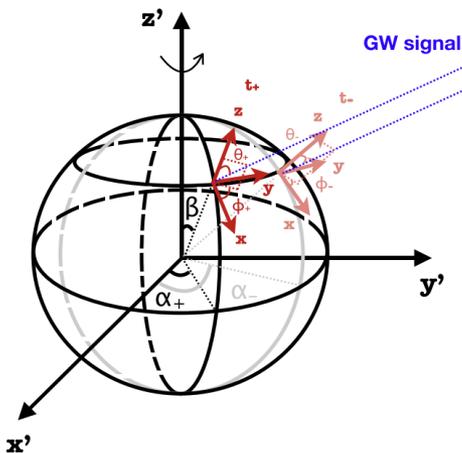
M. B. *et al.* JCAP10(2014)080 (full DCO: NS-NS, BH-NS, BH-BH)

X. Ding *et al.* JCAP12(2015)006

(relaxing intrinsic SNR=8 demand; magnification bias)

50 – 100 lensed events per year

BH-BH systems contribute 91 – 95%;
NS-NS systems 1 – 4%



THE ASTROPHYSICAL JOURNAL, 874:139 (6pp), 2019 April 1
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<https://doi.org/10.3847/1538-4357/ab095c>



How Does the Earth's Rotation Affect Predictions of Gravitational Wave Strong Lensing Rates?

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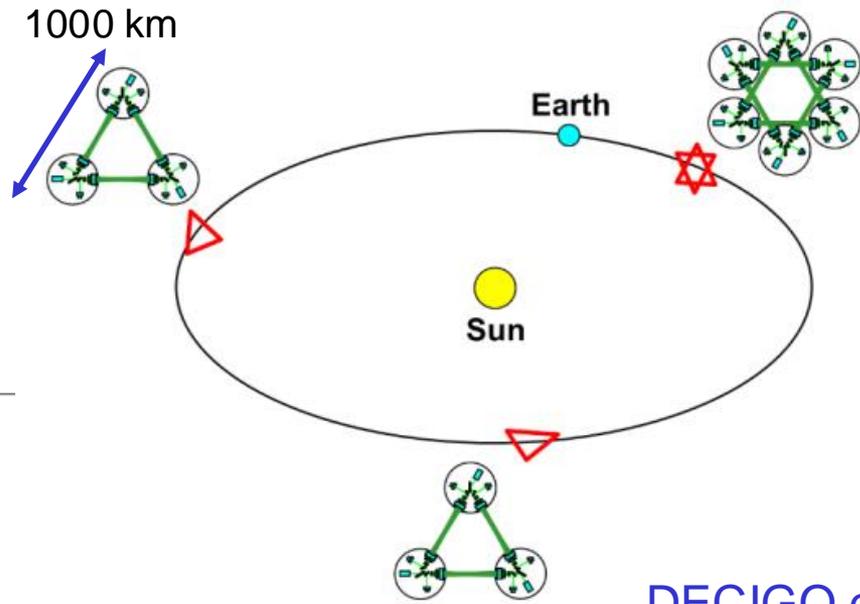
Received 2018 September 3; revised 2019 January 10; accepted 2019 February 20; published 2019 April 1

The future –

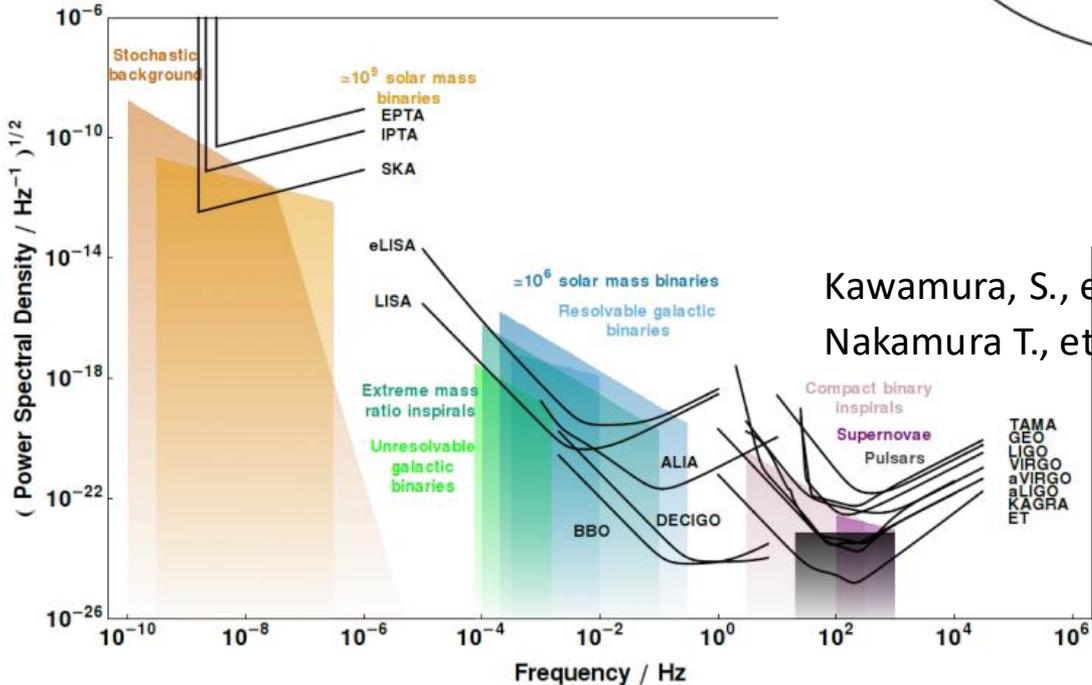
widen the frequency band of ground based detectors

LISA – 0.1 mHz – 100 mHz

DECIGO – 1 mHz – 100 Hz



DECIGO orbit



Kawamura, S., et al. 2019, IJMPD, 28, 1845001

Nakamura T., et al. 2016, Prog. Theor. Exp. Phys. 093E01

Detector noise power spectrum density of different ground and space detectors



B-DECIGO smaller scale detector

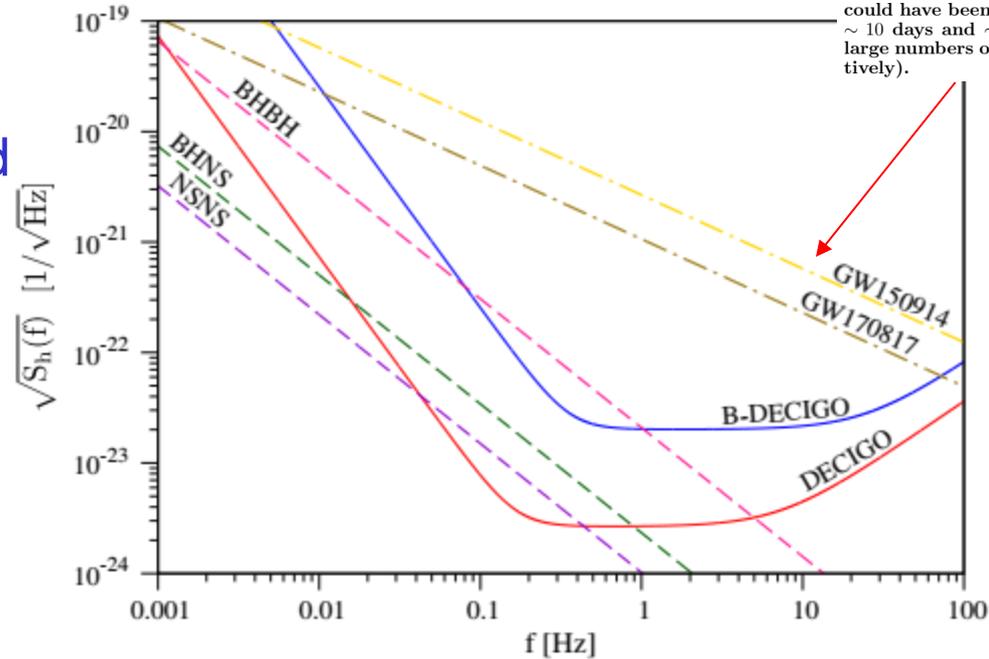


Inspiring Double Compact Object Detection and Lensing Rate: Forecast for DECIGO and B-DECIGO

Aleksandra Piórkowska-Kurpas^{1,2}, Shaoqi Hou³, Marek Biesiada^{1,4} , Xuheng Ding^{3,5} , Shuo Cao¹ , Xilong Fan³,
 Seiji Kawamura⁶, and Zong-Hong Zhu^{1,3}

DECIGO sensitivity will be significantly affected by unresolved BH-BH systems

B-DECIGO affected much less



tivity band. As discussed in details in [Isoyama et al. \(2018\)](#), time to coalescence can be estimated as: $t_c = 1.03 \times 10^6 \text{ s } (M_z/30.1 M_\odot)^{-5/3} (f/0.1 \text{ Hz})^{-8/3}$, which means that GW150914 and GW170817 could have been visible in (B-)DECIGO band for ~ 10 days and ~ 7 yrs prior to coalescence with large numbers of GW cycles (10^5 and 10^7 , respectively).

These GW events could have been detected by DECIGO or B-DECIGO

ET: $r_0 = 1917$ Mpc
 DECIGO $r_0 = 6709$ Mpc
 B-DECIGO $r_0 = 535$ Mpc

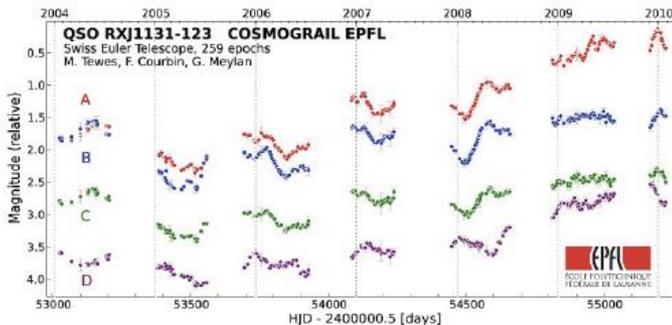
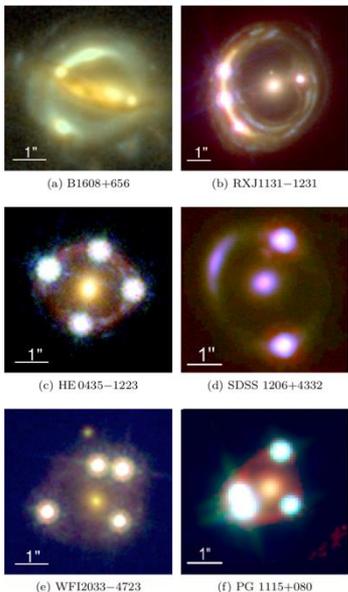
**50 lensed events per year
 few lensed events per year**

DECIGO and B-DECIGO only BH-BH systems

Strong lensing - applications

- Cosmology
measuring H_0
cosmic equation of state
cosmic curvature
- Testing modified gravity
PPN parameter
- Constraining Dark Matter models
- Testing fundamental physics
LIV
speed of GWs
mass of graviton

Ia. Strong lensing cosmography – H_0 from time delay



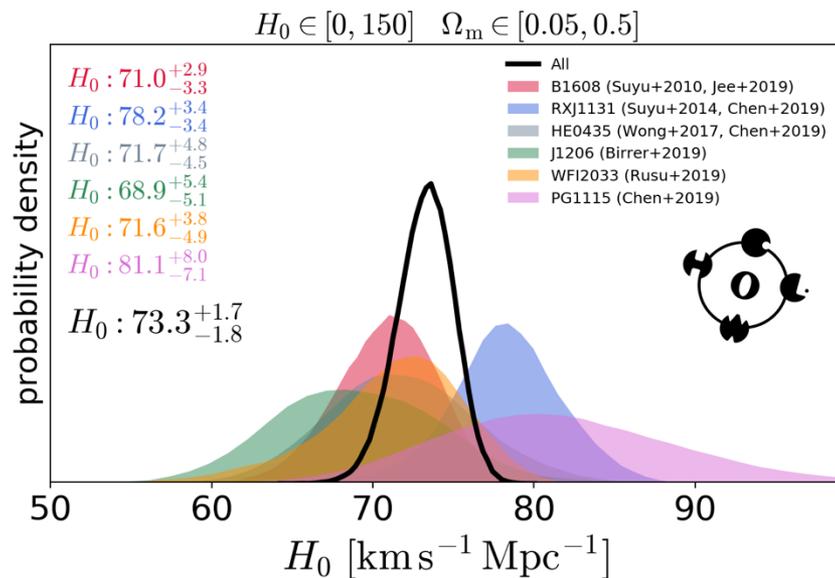
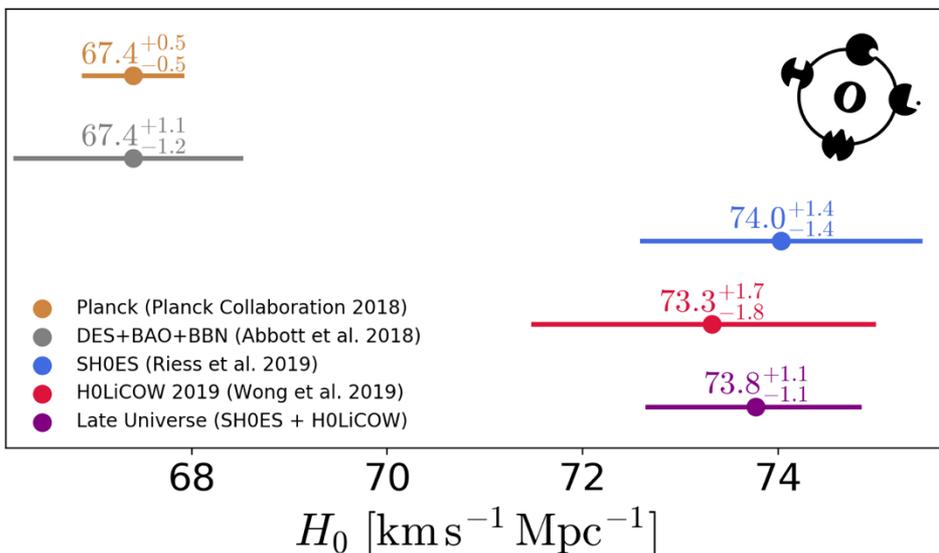
$$\frac{D_l}{D_s D_{ls}} = \frac{1}{H_0} f(z_l, z_s, \Omega_m, \Omega_\Lambda)$$

$$t(\vec{\theta}) = \frac{(1+z_d)}{c} \frac{D_d D_s}{D_{ds}} \left[\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \psi(\vec{\theta}) \right]$$

Annotations: Time delay distance (points to $D_d D_s / D_{ds}$), Shapiro delay (points to $\psi(\vec{\theta})$), Excess time delay (points to the entire equation), geometric time delay (points to $\frac{1}{2} (\vec{\theta} - \vec{\beta})^2$).

HOLiCOW currently 6 +1 lenses

flat Λ CDM



Precision cosmology from future lensed gravitational wave and electromagnetic signals

Kai Liao^{1,2}, Xi-Long Fan³, Xuheng Ding^{1,4,5}, Marek Biesiada^{4,6} & Zong-Hong Zhu^{1,4}

The standard siren approach of gravitational wave cosmology appeals to the direct luminosity distance estimation through the waveform signals from inspiralling double compact binaries, especially those with electromagnetic counterparts providing redshifts. It is limited by the calibration uncertainties in strain amplitude and relies on the fine details of the waveform. The Einstein telescope is expected to produce 10^4 - 10^5 gravitational wave detections per year, 50-100 of which will be lensed. Here, we report a waveform-independent strategy to achieve precise cosmography by combining the accurately measured time delays from strongly lensed gravitational wave signals with the images and redshifts observed in the electromagnetic domain. We demonstrate that just 10 such systems can provide a Hubble constant uncertainty of 0.68% for a flat lambda cold dark matter universe in the era of third-generation ground-based detectors.

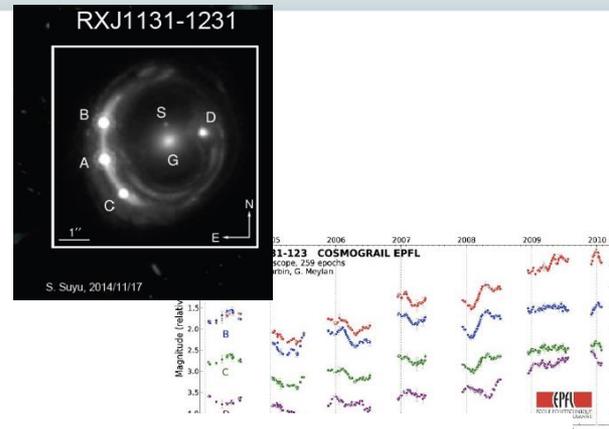


Table 1 Relative uncertainties of three factors contributing to the accuracy of time-delay distance measurement

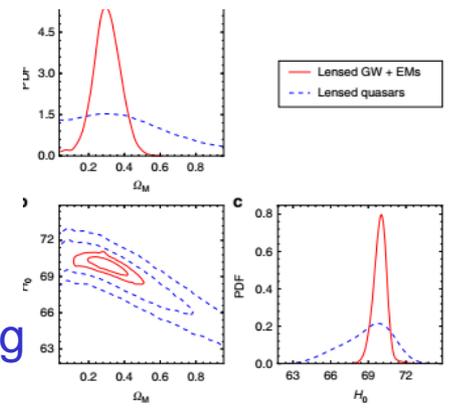
	$\delta\Delta t$	$\delta\Delta\psi$	δLOS
Lensed GW + EM	0%	0.6%	1%
Lensed quasar	3%	3%	1%

$\delta\Delta t$, $\delta\Delta\psi$, δLOS correspond to time delay, Fermat potential difference, and light-of-sight environment, respectively. We show the case for lensed gravitational wave (GW) + electromagnetic (EM) signals compared with standard technique in the EM domain using lensed quasars

Table 2 The average constraining power of 10 lensed gravitational wave + electromagnetic systems

	Flat Λ CDM (Ω_M fixed)	Flat Λ CDM		Flat ω CDM			Open Λ CDM		
	H_0	H_0	Ω_M	H_0	Ω_M	w	H_0	Ω_M	Ω_k
Uncertainty	0.37%	0.68%	27%	2.2%	36%	25%	1%	38%	± 0.18

We concern cosmological parameters in different scenarios: flat lambda cold dark matter (Flat Λ CDM) with or without dimensionless matter density Ω_M fixed, flat ω CDM where the dark energy equation of state ω is a free parameter, and open Λ CDM where cosmic curvature Ω_k is a free parameter. For the same number of lensed quasars, the power is weaker by a factor of ~4 according to the uncertainty propagation using Eq. (1) and Table 1

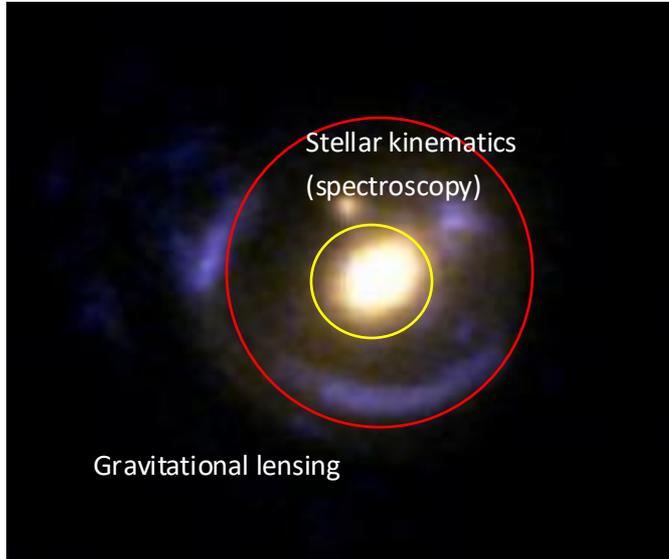


Sub percent – percent accuracy of H_0 possible with 10 lensed GW+EM

Similar perspectives for measuring curvature parameter

S.Cao, ..., M.B. et al., *Scientific Reports* 9, 11608, 2019

Ib. Strong lensing combined with stellar kinematics



1. The simplest lens model

Singular Isothermal Sphere (SIS)

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$$

$$\theta_E = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \frac{D_{ls}}{D_s}$$

2. **Spherically symmetric power-law mass density profile**

$$\rho(r) \sim r^{-\gamma}$$

stellar dynamics (spherically symmetric Jeans equation):
 mass projected inside the aperture radius
 scaled to the Einstein radius

Strong lensing: mass inside the Einstein radius

$$M_{lens} = \frac{c^2}{4G} \frac{D_l D_s}{D_{ls}} \theta_E^2 \quad \oplus \quad M_{dyn} = \frac{\pi}{G} \sigma_{ap}^2 D_l \theta_E \left(\frac{\theta_E}{\theta_{ap}}\right)^{2-\gamma} f(\gamma)$$

$$\theta_E = 4\pi \frac{\sigma_{ap}^2}{c^2} \frac{D_{ls}}{D_s} \left(\frac{\theta_E}{\theta_{ap}}\right)^{2-\gamma} f(\gamma)$$

3. **More general spherically symmetric power-law model**

$$\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\alpha} \quad \text{total mass}$$

$$\nu(r) = \nu_0 \left(\frac{r}{r_0}\right)^{-\delta} \quad \text{luminous matter}$$

$$\beta(r) = 1 - \sigma_t^2 / \sigma_r^2 \quad \text{stellar anisotropy}$$

$$\theta_E = 2\sqrt{\pi} \frac{\sigma_{ap}^2}{c^2} \frac{D_{ls}}{D_s} \left(\frac{\theta_E}{\theta_{ap}}\right)^{2-\alpha} (\xi - 2\beta)\lambda(\alpha)$$

Ic. Strong lensing cosmography – e.o.s. of the Universe

PHYSICAL REVIEW D 73, 023006 (2006)

Strong lensing systems as a probe of dark energy in the universe

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(Received 3 November 2005; published 23 January 2006)



JCAP03 (2012) 016 Constraints on cosmological models from strong gravitational lensing systems

Shuo Cao,^a Yu Pan,^{a,b} Marek Biesiada,^c Włodzimierz Godłowski^d and Zong-Hong Zhu^{a,1}

10 cluster lenses
70 galaxy lenses
from SLACS

Cosmological model	Best-fitting parameters ($n = 80$)	Best-fitting parameters ($n = 46$)
Λ CDM	$\Omega_m = 0.20^{+0.07}_{-0.07}$	$\Omega_m = 0.26^{+0.11}_{-0.10}$
w CDM	$w = -1.02^{+0.26}_{-0.26}$	$w = -1.15^{+0.34}_{-0.35}$
CPL	$w_0 = 0.60 \pm 1.76$ $w_a = -7.37 \pm 8.05$	$w_0 = -0.24 \pm 2.42$ $w_a = -6.35 \pm 9.75$

Cosmic equation of state from strong gravitational lensing systems

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Accepted 2010 March 22. Received 2010 March 18; in original form 2010 February 5

20 galaxy lenses
from SLACS + LSD

WMAP7+BAO+H0

$$\Omega_m = 0.272$$

$$w = -1.10 \pm 0.14$$

$$w_0 = -0.93 \pm 0.13$$

$$w_a = -0.41 \pm 0.71$$

Komatsu et al. 2011

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doi:10.1088/0004-637X/806/2/185

COSMOLOGY WITH STRONG-LENSING SYSTEMS

SHUO CAO¹, MAREK BIESIADA^{1,2}, RAPHAËL GAVAZZI³, ALEKSANDRA PIÓRKOWSKA², AND ZONG-HONG ZHU¹

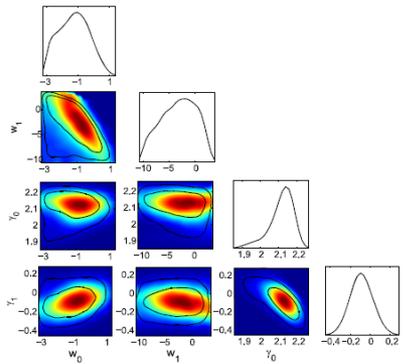
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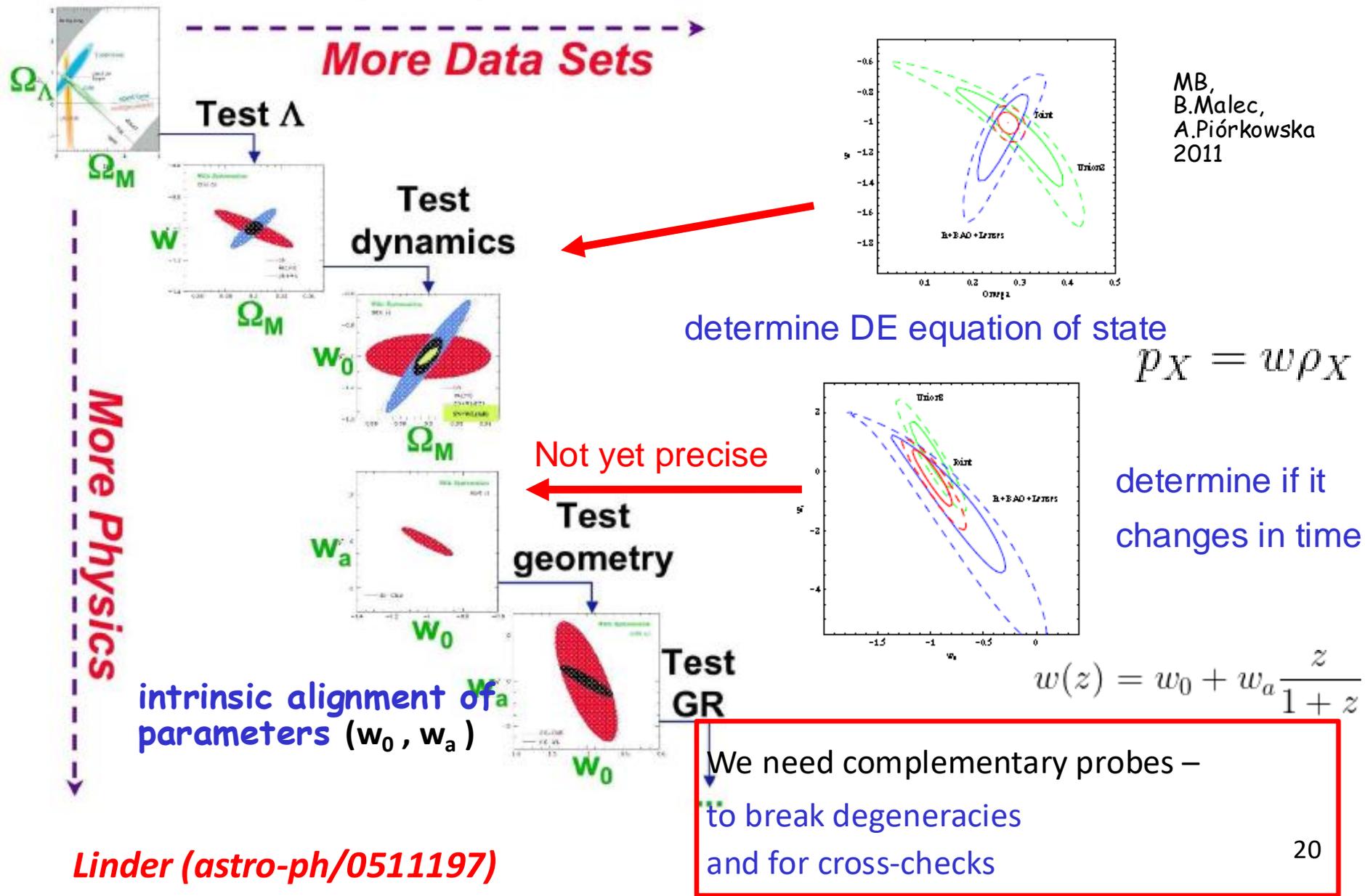
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Received 2015 January 23; accepted 2015 May 1; published 2015 June 17

118 lenses from
SLACS sample

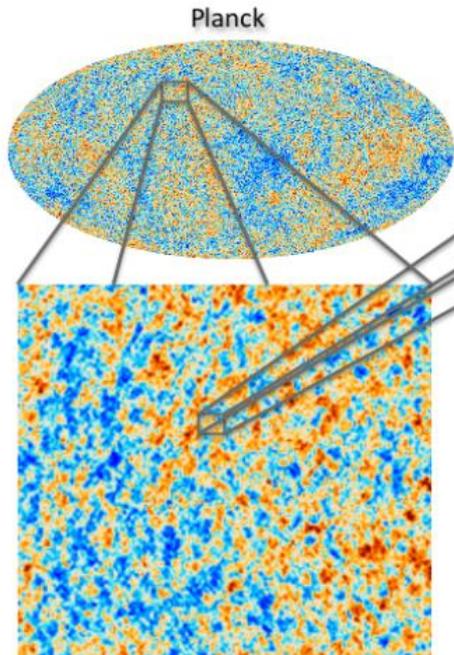


Modern cosmology: Incremental Exploration of the Unknown



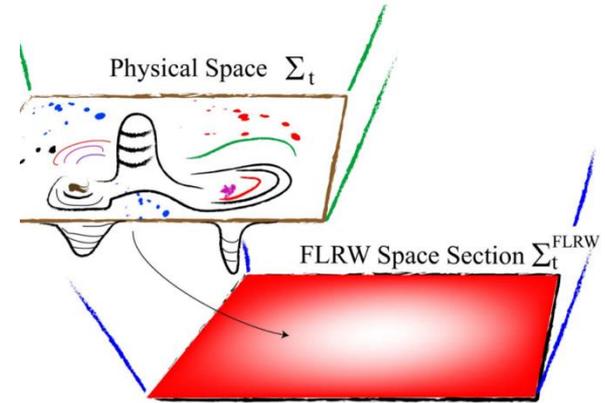
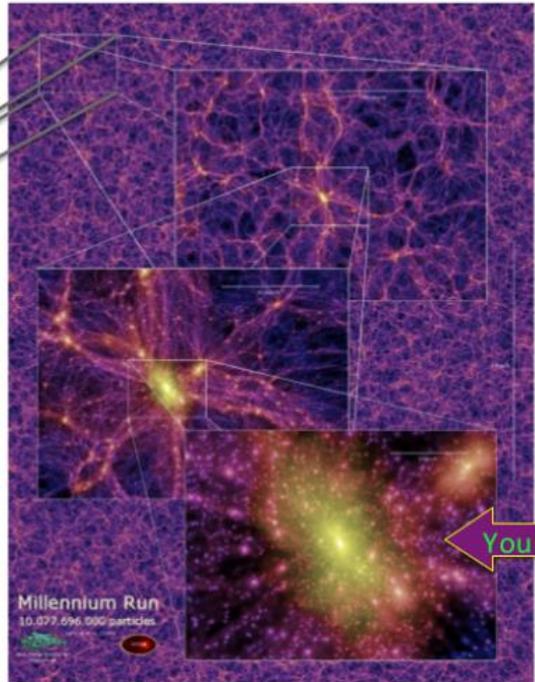
II. Strong lensing and curvature of the Universe

Emerging spatial curvature



Primordial quantum perturbations as seen in the Cosmic Microwave Background

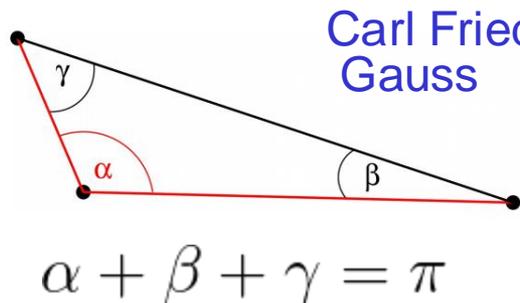
Dark matter distribution today (simulated)



Buchert, Carfora, *Class. Quant. Grav.* 25, 195001 (2008)

Formation of the large scale structure induces non-zero curvature at local scales

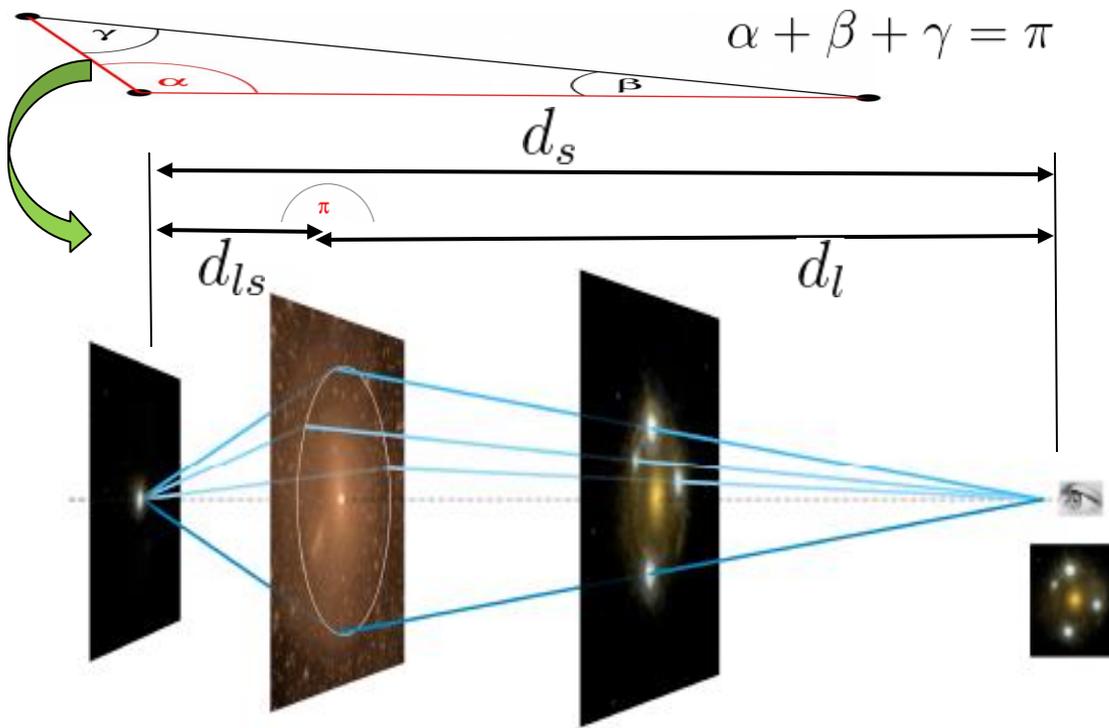
Credit: F. Leclercq, A. Pisani, B.D. Wandelt arXiv:1403.1260v1



Carl Friedrich Gauss



It is important to measure curvature with local objects



Strong lensing systems offer us „degenerated triangles”

One can obtain Ω_k if

d_l, d_s, d_{ls} are known

Observations:

z_l, z_s – known

Images -- $>$ d_{ls} / d_s

Time delays -- $>$ $d_l d_s / d_{ls}$

So: d_l is measurable

=====

d_s – match by redshift to some standard candle (or ruler)

Distance sum rule – valid in any FLRW metric

$$d_{ls} = \sqrt{1 + \Omega_k d_l^2} d_s - \sqrt{1 + \Omega_k d_s^2} d_l$$

$$\Omega_k(z_l, z_s) = \frac{d_l^4 + d_s^4 + d_{ls}^4 - 2d_l^2 d_s^2 - 2d_l^2 d_{ls}^2 - 2d_s^2 d_{ls}^2}{4d_l^2 d_s^2 d_{ls}^2}$$

This is a function of two redshifts, but within the FLRW metric it should be just a single number !

**Strongly gravitationally lensed type Ia supernovae:
Direct test of the Friedman-Lemaître-Robertson-Walker metric**

Jingzhao Qi,^{1,2} Shuo Cao,^{2,*} Marek Biesiada,^{2,3} Xiaogang Zheng,⁴ Xuheng Ding,⁴ and Zong-Hong Zhu^{2,4,†}

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Received 12 October 2018; published 19 July 2019

	$\delta\theta_E$	$\delta\sigma_{ap}$	$\delta\gamma$
Multiple images	1%	5%	1%
	$\delta\Delta t$	$\delta\Delta t$ (ML)	$\delta\Delta\psi$ (LOS)
Time delay	1%	1%	$\propto (\delta\theta_E, \delta\gamma)$
	$\Delta\mu_D$ (sta)	$\Delta\mu_D$ (ML)	$\delta\mu$
Lensed SNe Ia	σ_{stat}	0.70 mag	$\propto (\delta\theta_E, \delta\gamma)$
			σ_{sys}

Friedman-Lemaître-Robertson-Walker (FLRW) metric. Gravitational lensing systems with type Ia supernovae will provide a model-independent test of the FLRW metric independently of the lensing geometry. Our results are derived by the Large Synoptic Survey Telescope (LSST) with accuracy $\Delta\Omega_k = 0.04$.

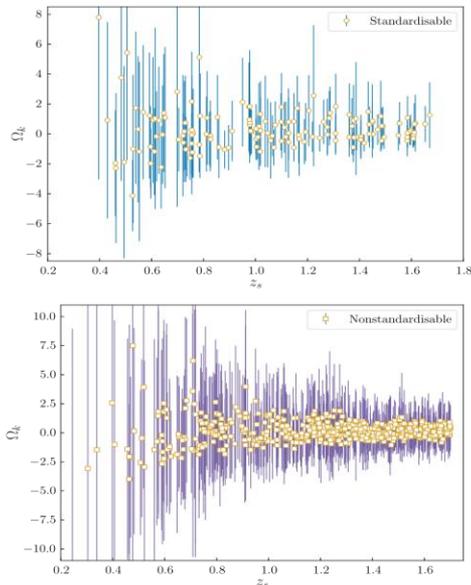


FIG. 1. An example of the simulated measurements of Ω_k from future observations of SGLSNe Ia: without and with the effect of microlensing. The blue lines denote the associated error bars (68.3% C.L.) of Ω_k when all the uncertainties are included.

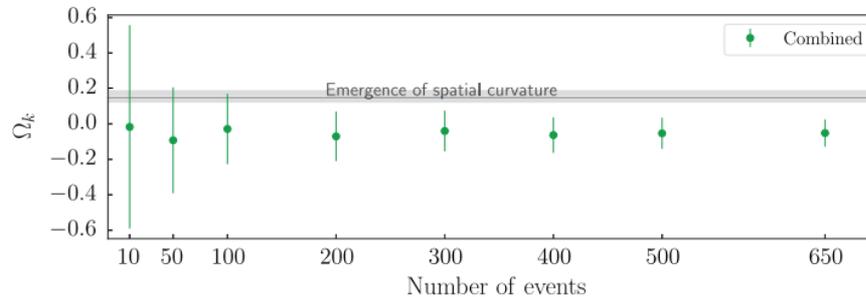
Curvature parameter from lensed SN Ia

$$\frac{d_{ls}}{d_s} = \sqrt{1 + \Omega_k d_l^2} - \frac{d_l}{d_s} \sqrt{1 + \Omega_k d_s^2}.$$

$$D_l = (1 + z_l) D_{\Delta t} \frac{D_{ls}^A}{D_s^A}.$$

$$D_s = \frac{10^{(\mu_D + 2.5 \log \mu)/5 - 5}}{1 + z_s} \text{ (Mpc)}.$$

$$\Omega_k(z_l, z_s) = \frac{d_l^4 + d_s^4 + d_{ls}^4 - 2d_l^2 d_s^2 - 2d_l^2 d_{ls}^2 - 2d_s^2 d_{ls}^2}{4d_l^2 d_s^2 d_{ls}^2}$$



650 lensed
SN Ia from LSST

D. A. Goldstein,
P. E. Nugent
2017

FIG. 3. Inferred Ω_k parameter as a function of the number of SGLSNe Ia, with the prediction of a silent universe added for comparison.

Testing the cosmic curvature at high redshifts: the combination of LSST strong lensing systems and quasars as new standard candles

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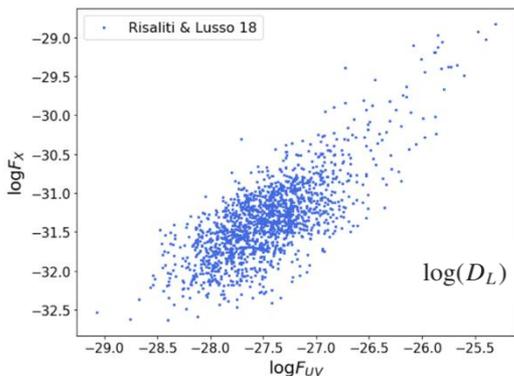
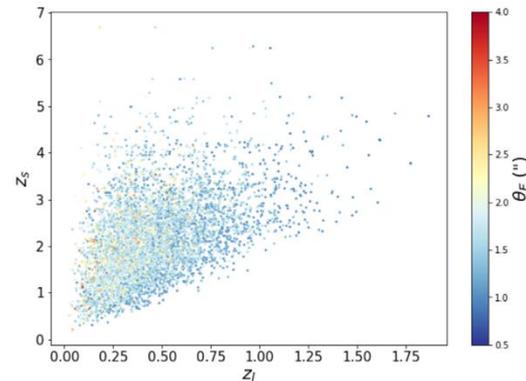


Simulated sample of SGL systems detectable in LSST (code of Collett 2015) → 5000 ellipticals
 $200 \text{ km s}^{-1} < \sigma_{ap} < 300 \text{ km s}^{-1}$

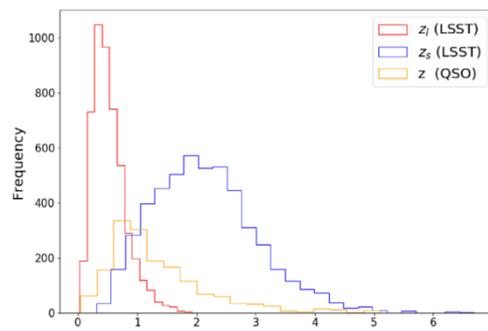
Distances matched to QSO[UV-X] standard candles (Risaliti & Lusso 2019)

$$\frac{d_{ls}}{d_s}$$

$$\frac{d_s}{d_l}$$



$$\log(D_L) = \frac{1}{2 - 2\hat{\gamma}} \times [\hat{\gamma} \log(F_{UV}) - \log(F_X) + \hat{\beta}]$$



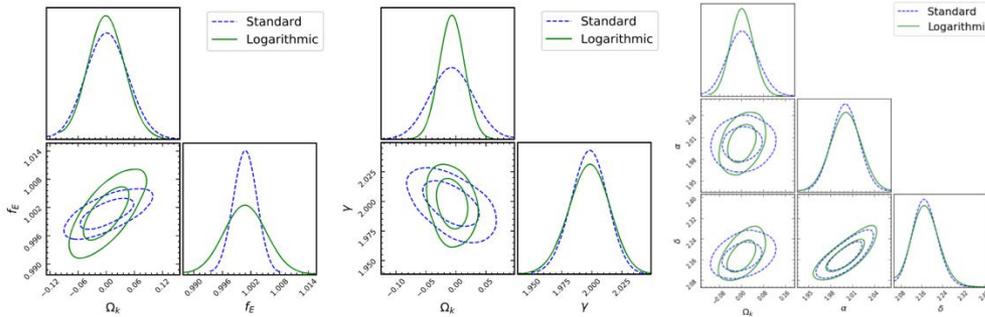
SGL systems & QSO[UV-X] overlap well in z

Figure 1. Scatter plot of the flux measurements of 1598 quasars (Risaliti & Lusso 2019).

Results

Table 1. Constraints on the cosmic curvature and lens profile parameters for three types of lens models, in the framework of standard polynomial and logarithmic polynomial cosmographic reconstructions.

Standard polynomial	Ω_k	f_E	γ	α	δ
SIS	0.002 ± 0.035	1.000 ± 0.002	\square	\square	\square
Power-law spherical	-0.007 ± 0.029	\square	2.000 ± 0.012	\square	\square
Extended power law	0.003 ± 0.045	\square	\square	2.000 ± 0.014	2.171 ± 0.035
Power-law spherical (with <i>HST</i> imaging)	-0.008 ± 0.028	\square	2.000 ± 0.012	\square	\square
Logarithmic polynomial	Ω_k	f_E	γ	α	δ
SIS	-0.001 ± 0.030	1.000 ± 0.003	\square	\square	\square
Power-law spherical	-0.007 ± 0.016	\square	2.000 ± 0.013	\square	\square
Extended power law	0.002 ± 0.031	\square	\square	2.002 ± 0.016	2.172 ± 0.035



redshift bins

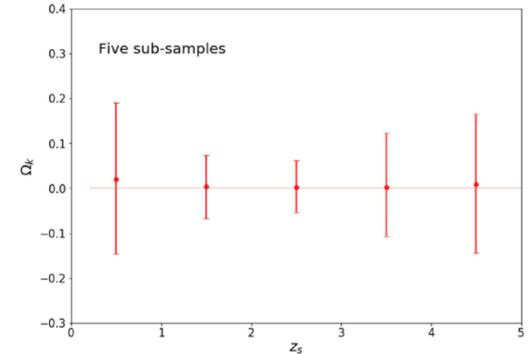


Figure 7. Determination of cosmic curvature with five subsamples $0 < z < 1.0$, $1.0 < z < 2.0$, $2.0 < z < 3.0$, $3.0 < z < 4.0$ and $4.0 < z < 5.0$ based on the source redshifts of SGL sample characterized by the SIS lens model.

Different lens models +
different cosmographic distance reconstructions

Conclusion: LSST data (+follow-up) would allow **sub-percent accuracy** of local Ω_k measurement

If we observe strongly lensed GW signals and their EM counterparts

Advantages:

very precise measurement of time delay

(however affected by microlensing and l.o.s. contamination)

Time delay distance – Fermat potential reconstructed from EM images of lensed host galaxy

EM images + spectroscopy of the lens

Sources are standard sirens

(up to magnification factor, which can be assessed from flux ratio and lens reconstruction)

$$\Omega_k(z_l, z_s) = \frac{1}{4} \frac{(1+z_l)^2 d_{\Delta t}^2}{d_s^4} + \frac{1}{4} \frac{(1+z_l)^2}{d_{\Delta t}^2 (d_{ls}/d_s)^4} + \frac{1}{4} \frac{1}{(1+z_l)^2 d_{\Delta t}^2} - \frac{1}{2} \frac{1}{(d_{ls}/d_s)^2 d_s^2} - \frac{1}{2} \frac{1}{d_s^2} - \frac{1}{2} \frac{1}{(1+z_l)^2 d_{\Delta t}^2 (d_{ls}/d_s)^2}$$

Cosmic curvature in terms of measureable quantities

Uncertainty budget: covariances by propagating uncorrelated measurement uncertainties of the observables

$$\gamma, \theta_E, \theta_{ab}, \Delta t, \Delta \psi \text{ (LOS)}, d_\epsilon^L, \text{ and } F$$

$$\delta \Omega_k(z_l, z_s) \sim (\delta \gamma, \delta \theta_E, \delta \theta_{ab}, \delta(\Delta t), \delta(\Delta \psi \text{ (LOS)}), \delta d_s^L, \delta F)$$

Direct test of the FLRW metric from strongly lensed gravitational wave observations

Shuo Cao¹, Jingzhao Qi², Zhoujian Cao³, Marek Biesiada^{1,3}, Jin Li⁴, Yu Pan⁵ & Zong-Hong Zhu¹

$$D_{\Delta t} \equiv \frac{D_l^A D_s^A}{D_{ls}^A} = \frac{c}{1+z_l} \frac{\Delta t_{i,j}}{\Delta \phi_{i,j}}$$

$$\frac{d_{ls}}{d_s} = \frac{D_{ls}^A}{D_s^A} = \frac{\theta_E}{4\pi \sigma_{ap}^2} \left(\frac{\theta_E}{\theta_{ap}} \right)^{\gamma-2} f(\gamma, M_E)^{-1}$$

$$D_l = (1+z_l) D_{\Delta t} \frac{D_{ls}^A}{D_s^A}$$

$$D_s = \frac{1}{1+z_s} D_s^L$$

	$\delta\theta_E$	$\delta\sigma_{ap}$	$\delta\gamma$
Image configuration	1%	5%	1%
	$\delta\Delta t$	$\delta\Delta\psi$	$\delta\Delta\psi(LOS)$
Time delay	0%	$\sim(\delta\theta_E, \delta\gamma)$	1%
	δd_s^L (SNR)	δd_s^L (WL)	δF (SL + ML)
Lensed GW	$2/p_{net}$	$0.05z$	10% (50%)

Mock catalog:
 100 lensed GW for ET
 1000 lensed GW for BBO

Merger rates –
 from StarTrack pop syn code
 masses and orientations sampled
 randomly

Lenses – elliptical galaxies, VDF
 from Schechter function fitted to
 SDSS DR3

Reconstruction and magnification
 uncertainties from the budget above

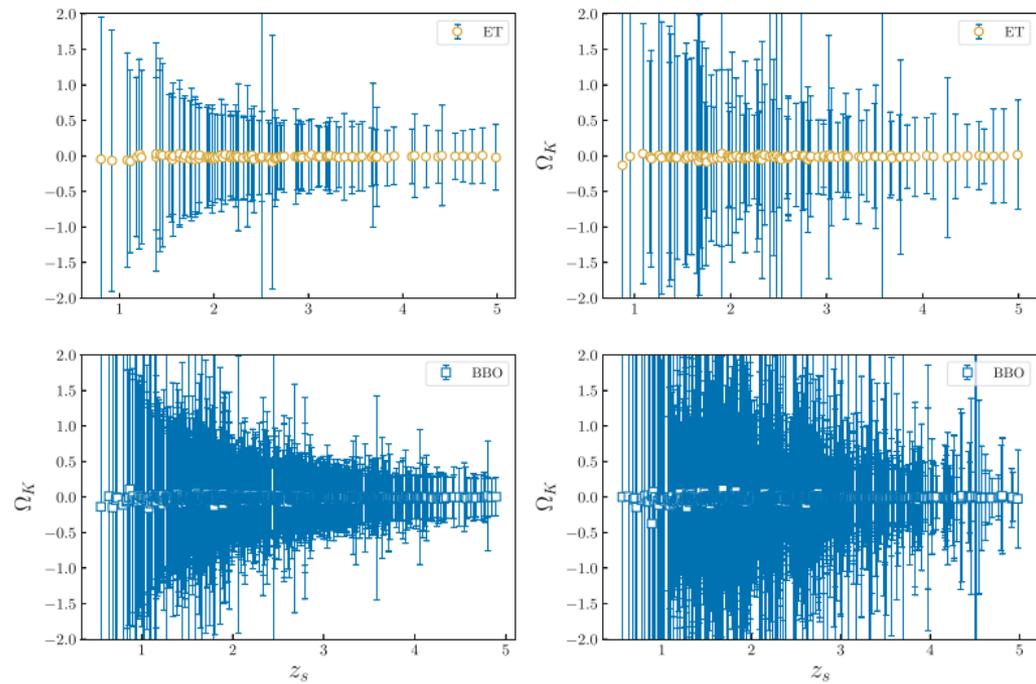


Figure 1. An example of the simulated measurements of the cosmic curvature from future observations of lensed GWs. We simulated 100 lensed GW signals detectable by the ET (upper panel) and 1000 signals detectable by the BBO (lower panel). Left and right panel respectively show the results with 10% and 50% uncertainty in the amplification factor measurements.

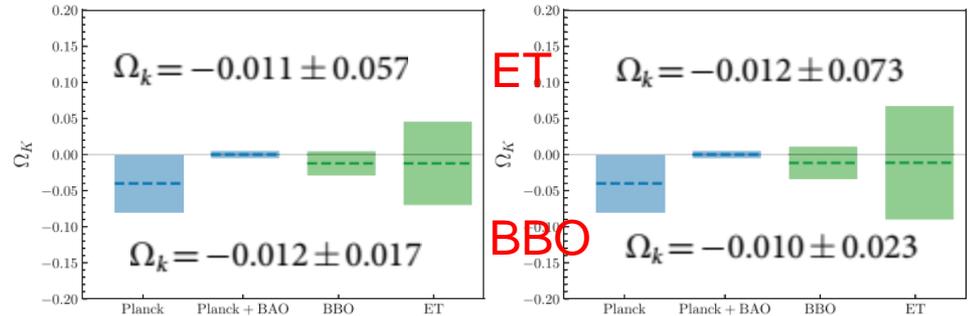


Figure 2. Statistical summary of simulated predictions of the Ω_k parameter measurements (inverse variance weighting) from future observations of lensed GWs. Left and right panel respectively show the results with 10% and 50% uncertainty in the amplification factor measurements. Predictions for the ET and the BBO are confronted with constraints achievable from the CMB and BAO measurements.

III. Strong lensing and as a new probe of parametrized post-Newtonian (PPN) gravity

Parametrized post-Newtonian (PPN) formalism is a very convenient way to study and compare gravity theories beyond GR

One useful PPN parameter γ measures the amount of spatial curvature generated by unit mass

In the weak field limit the metric is characterized by two potentials

$$ds^2 = a^2(\tau) \left[\left(1 + \frac{2\Phi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\Psi}{c^2}\right) g_{ij} dx^i dx^j \right] \quad \gamma = \frac{\Psi}{\Phi}$$

Motion of massive bodies (e.g. stellar dynamics) is sensitive to the Newtonian potential

Trajectory of light is sensitive to both potentials, as a result:

deflection angle is $\hat{\alpha}_{PPN} = \frac{1 + \gamma}{2} \hat{\alpha}_{GR}$

and the Einstein radius of spherically symmetric lens is $\theta_E = \sqrt{\frac{1 + \gamma}{2}} \left(\frac{4GM_E}{c^2} \frac{D_{ls}}{D_s D_l} \right)^{1/2}$



TEST OF PARAMETRIZED POST-NEWTONIAN GRAVITY WITH GALAXY-SCALE STRONG LENSING SYSTEMS

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Received 2016 September 11; revised 2016 November 18; accepted 2016 December 2; published 2017 January 20

We used a catalog of 80 intermediate mass lensing systems from SLACS, BELLS, LSD and SL2S $200 \text{ km s}^{-1} < \sigma_{ap} \leq 300 \text{ km s}^{-1}$
(Cao et al. 2015, ApJ 806:185)

lens model

total mass $\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\alpha}$

luminosity $\nu(r) = \nu_0 \left(\frac{r}{r_0}\right)^{-\delta}$

anisotropy $\beta(r) = 1 - \sigma_t^2 / \sigma_r^2$
 $\beta = 0.18 \pm 0.13$

lensing gives $\frac{GM_E}{R_E} = \frac{2}{(1 + \gamma)} \frac{c^2}{4} \frac{D_s}{D_{ls}} \theta_E$

stellar velocity dispersion gives

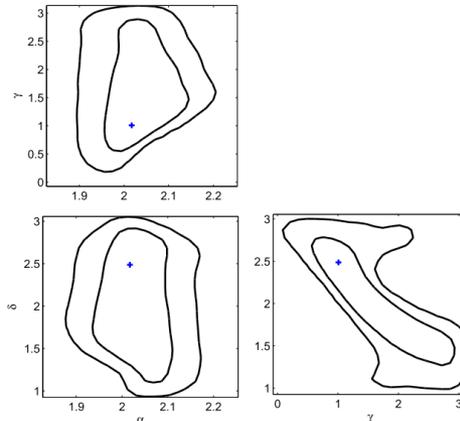
$$\sigma_r^2(r) = \left[\frac{GM_E}{R_E} \right] \frac{2}{\sqrt{\pi} (\xi - 2\beta) \lambda(\alpha)} \left(\frac{r}{R_E}\right)^{2-\alpha}$$

Best fits

$$\alpha = 2.017^{+0.093}_{-0.082},$$

$$\delta = 2.485^{+0.445}_{-1.393},$$

$$\gamma = 1.010^{+1.925}_{-0.452}.$$



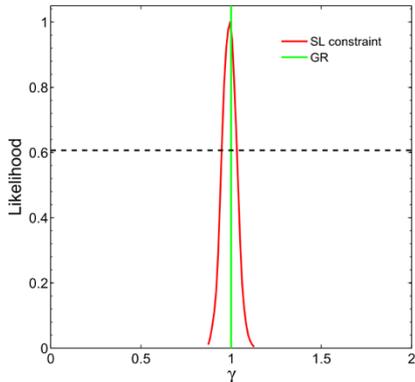


Figure 3. Normalized posterior likelihood of the PPN γ parameter obtained with rigid priors on the nuisance parameters (α , β , δ).

With priors

$$\langle \alpha \rangle = 2.00; \sigma_\alpha = 0.08$$

$$\langle \delta \rangle = 2.40; \sigma_\delta = 0.11.$$

$$\beta = 0.18 \pm 0.13$$

$$\gamma = 0.995^{+0.037}_{-0.047}$$



LSST simulated strong lensing systems with prior on Ω_k

$$\frac{\Delta\gamma}{\gamma} \sim 10^{-3} - 10^{-4}$$

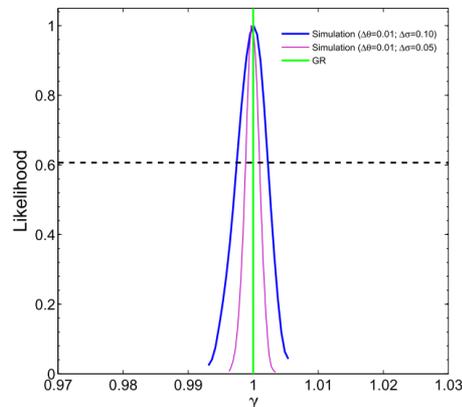


Figure 8. Constraints on the PPN parameter from simulated LSST strong lensing data, with a prior on the cosmic curvature $-0.007 < \Omega_k < 0.006$ from Planck.

GRAVITATION

A precise extragalactic test of General Relativity

Thomas E. Collett^{1*}, Lindsay J. Oldham², Russell J. Smith³, Matthew W. Auger², Kyle B. Westfall^{1,4}, David Bacon¹, Robert C. Nichol¹, Karen L. Masters^{1,5}, Kazuya Koyama¹, Remco van den Bosch⁶

Einstein's theory of gravity, General Relativity, has been precisely tested on Solar System scales, but the long-range nature of gravity is still poorly constrained. The nearby strong gravitational lens ESO 325-G004 provides a laboratory to probe the weak-field regime of gravity and measure the spatial curvature generated per unit mass, γ . By reconstructing the observed light profile of the lensed arcs and the observed spatially resolved stellar kinematics with a single self-consistent model, we conclude that $\gamma = 0.97 \pm 0.09$ at 68% confidence. Our result is consistent with the prediction of 1 from General Relativity and provides a strong extragalactic constraint on the weak-field metric of gravity.

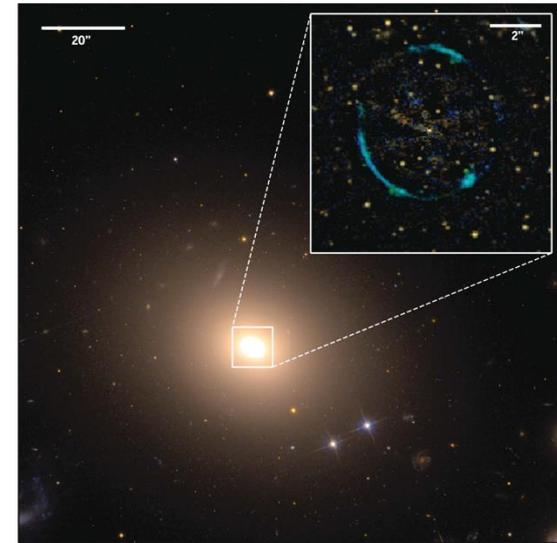


Fig. 1. Color composite image of ESO325-G004. Blue, green, and red channels are assigned to the F475W, F606W, and F814W HST imaging. The inset shows a F475W and F814W composite of the arcs of the lensed background source after subtraction of the foreground lens light. Scale bars are in arc seconds.

IV. QG phenomenology

Lorentz Invariance Violation

Long searched-for Quantum Gravity is often expected to provide „foamy space-time” at short distances

i.e. a possible breakdown of Lorentz invariance could be expected at high energies

The simplest manifestation of QG phenomenology – energy dependent relativistic dispersion relation, e.g.

$$E^2 - p^2 c^2 = \epsilon E^2 \left(\frac{E}{\xi_n E_{QG}} \right)^n \quad \text{for photons}$$

$$E_\nu^2 - p_\nu^2 c^2 - m_\nu^2 c^4 = \epsilon E_\nu^2 \left(\frac{E_\nu}{\xi_n E_{QG}} \right)^n \quad \text{for massive particles}$$

„sign parameter”
 $\epsilon = \pm 1$

„Quantum Gravity”
energy scale

COSMOLOGICAL DISTANCES

GALACTIC DISTANCES

$$t_{LIV} = \int_0^z \left[1 + \epsilon \frac{n+1}{2} \left(\frac{E}{\xi_n E_{QG}} \right)^n (1+z')^n \right] \frac{dz'}{H(z')}$$

GRBs

Time of flight

AGNs



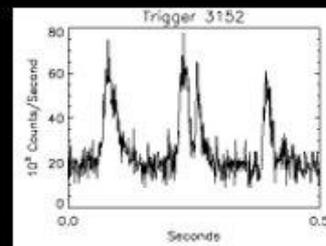
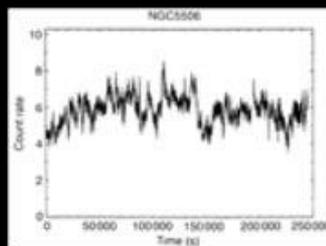
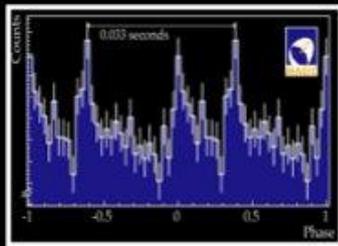
emission up to MeV detected so far (>100MeV - rare photons)

GeV up to TeV photons

PULSARS



GeV photons



OBSERVED TIME STRUCTURE

$$\Delta t = \frac{1}{H_0} \int_0^z \left(\frac{1+n}{2} \left(\frac{E}{\xi E_{pl}} \right)^n (1+z')^n \right) \frac{dz'}{\sqrt{\Omega_m (1+z')^3 + \Omega_\Lambda}}$$

Tests of quantum gravity from observations of γ -ray bursts

G. Amelino-Camelia, John Ellis, N. E. Mavromatos, D. V. Nanopoulos & Subir Sarkar

Nature 393, 763–765 (1998)



H.E.S.S.



MAGIC

HAWC



LHAASO

CTA

Time delay low vs. H.E.

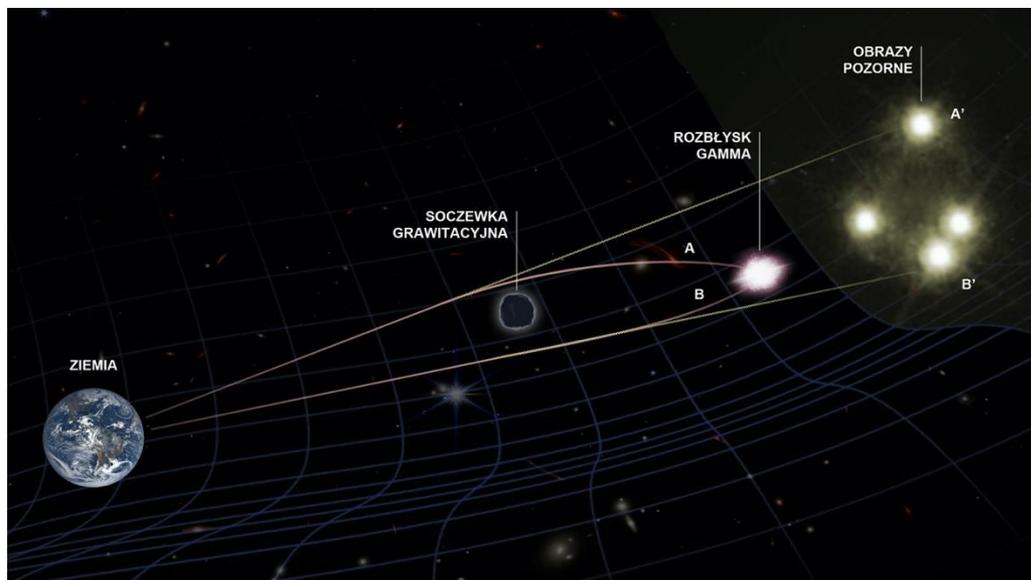
Gravitational lensing time delays as a tool for testing Lorentz-invariance violation

Marek Biesiada[★] and Aleksandra Piórkowska[★]

Department of Astrophysics and Cosmology, Institute of Physics, University of Silesia, Uniwersytecka 4, 40-007 Katowice, Poland

Classically light propagates with the speed of light c , at all wavelengths but in QG phenomenology velocity of light might be energy dependent. In a strongly lensed event time delays between images in low and high energy channels would be different.

This method is **free** from any assumptions regarding intrinsic source time lag between low and high energy signal emission.



lensing galaxy
velocity dispersion

magnification ratio

$$\Delta t_{\text{SIS,LIV}} - \Delta t_{\text{SIS}} = \frac{32\pi^2}{H_0} \left(\frac{\sigma_v}{c}\right)^4 \left(\frac{\alpha + 1}{\alpha - 1}\right) \left(\frac{\Delta E_0}{E_{\text{QG}}}\right) J_1(z_l, z_s)$$

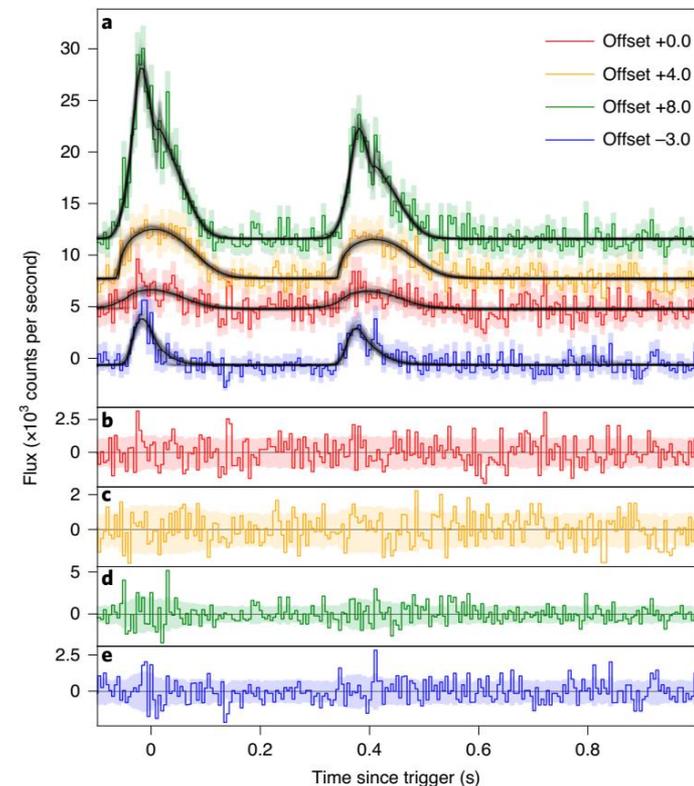
$$I_n(z_1, z_2) = \int_{z_1}^{z_2} \frac{(1+z')^n dz'}{h(z')} \quad J_n(z_l, z_s) \equiv \left[\frac{I_n(0, z_l)}{I_0(0, z_l)} + \frac{I_n(z_l, z_s)}{I_0(z_l, z_s)} - \frac{I_n(0, z_s)}{I_0(0, z_s)} \right]$$



Evidence for an intermediate-mass black hole from a gravitationally lensed gamma-ray burst

James Paynter¹✉, Rachel Webster¹✉ and Eric Thrane^{2,3}

If gamma-ray bursts are at cosmological distances, they must be gravitationally lensed occasionally^{1,2}. The detection of lensed images with millisecond-to-second time delays provides evidence for intermediate-mass black holes, a population that has been difficult to observe. Several studies have searched for these delays in gamma-ray burst light curves, which would indicate an intervening gravitational lens^{3–6}. Among the $\sim 10^4$ gamma-ray bursts observed, there have been a handful of claimed lensing detections⁷, but none have been statistically robust. Here we present a Bayesian analysis identifying gravitational lensing in the light curve of GRB 950830. The inferred lens mass M_l depends on the unknown lens redshift z_l , and is given by $(1+z_l)M_l = 5.5^{+1.7}_{-0.9} \times 10^4 M_\odot$ (90% credibility), which we interpret as evidence for an intermediate-mass black hole. The most probable configuration, with a lens redshift $z_l \approx 1$ and a gamma-ray burst redshift $z_s \approx 2$, yields a present-day number density of about $2.3^{+4.9}_{-1.6} \times 10^3 \text{ Mpc}^{-3}$ (90% credibility) with a dimensionless energy density $\Omega_{\text{IMBH}} \approx 4.6^{+9.8}_{-3.3} \times 10^{-4}$. The false alarm probability for this detection is $\sim 0.6\%$ with trial factors. While it is possible that GRB 950830 was lensed by a globular cluster, it is unlikely as we infer a cosmic density inconsistent with predictions for globular clusters $\Omega_{\text{GC}} \approx 8 \times 10^{-6}$ at 99.8% credibility. If a significant intermediate-mass black hole population exists, it could provide the seeds for the growth of supermassive black holes in the early Universe.





Lorentz Invariance Violation Test from Time Delays Measured with Gravitationally Lensed GRB Candidates 950830 and 200716C

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Time of flight effect

$$\Delta t_{\text{LIV}} = t_l - t_h$$

$$= -\frac{n+1}{2} \left(\frac{E}{E_{\text{QG}}} \right)^n \frac{I_n(0, z)}{H_0},$$

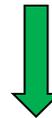
$$I_n(z_1, z_2) = \int_{z_1}^{z_2} \frac{(1+z')^n dz'}{h(z')}$$



Time delay for point mass lens (IMBH)

$$\Delta t_{\text{PM}} = \frac{2GM_l}{c^3} (1+z_l) \left[\frac{r-1}{\sqrt{r}} + \ln r \right]$$

magnification ratio



LIV effect on time delay for point mass lens (IMBH)

$$\Delta_{\text{LIV}}(\Delta t_{\text{PM}}) := \Delta t_{\text{LIV,PM}} - \Delta t_{\text{PM}}$$

$$= -\frac{4GM_l}{c^3} (1+z_l) \frac{r-1}{\sqrt{r}} \frac{n+1}{4} \left(\frac{E}{E_{\text{QG}}} \right)^n J_n(z_l, z_s)$$

$$\left[\frac{I_n(0, z_l)}{I_0(0, z_l)} + \frac{I_n(0, z_s)}{I_0(0, z_s)} - \frac{I_n(z_l, z_s)}{I_0(z_l, z_s)} \right]$$

Table 1

The Gravitational Lensing Time Delays and Flux Ratios in Different Energy Bands for Millilensing Events

GRB Name	Energy Channels (keV)	Δt_{obs} (s)	r
950830	(20–60)	0.405 ± 0.001	1.079 ± 0.172
	(60–110)	0.400 ± 0.002	1.253 ± 0.071
	(110–320)	0.395 ± 0.002	1.543 ± 0.037
	(320–2000)	0.385 ± 0.003	0.995 ± 0.165
200716C	(8–26)	1.972 ± 0.002	1.334 ± 0.618
	(26–85)	1.969 ± 0.003	1.519 ± 0.455
	(85–276)	1.964 ± 0.008	1.595 ± 0.439
	(276–900)	1.954 ± 0.018	1.367 ± 1.014
	(900–40000)	1.947 ± 0.025	1.501 ± 0.433

$$E_{\text{QG},1} \geq 3.2 \times 10^9 \text{ GeV}$$

Many unknowns (e.g. lens redshift),
but the method works !

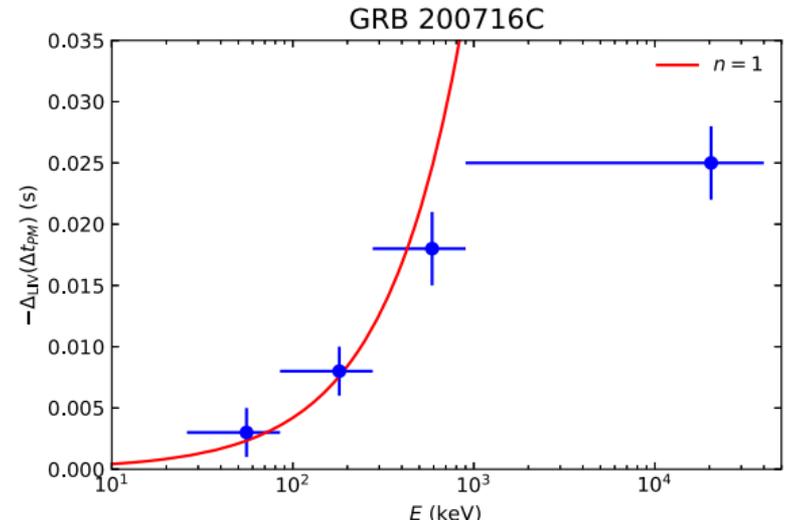
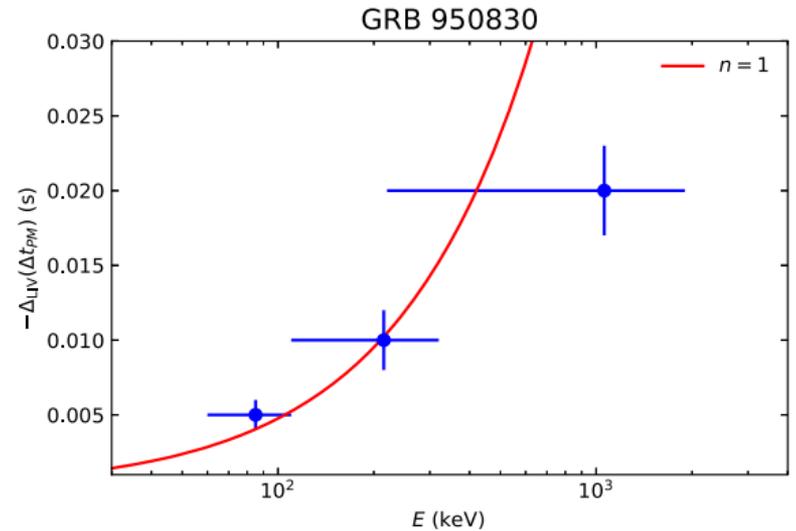


Figure 2. Energy dependence of the observed gravitational lensing time delays $\Delta_{\text{LIV}}(\Delta t_{\text{PM}})$ (relative to the softest energy band), and the best-fit theoretical curves (red solid line)—the linear ($n = 1$) LIV model.

Challenge for the future:

Find lensed GRBs or better yet TeV sources ...

Prompt GRB emission works for small time delays (mililensing, IMBHs)

For galaxy lensing – time delay ~ days, weeks, months ...

one should change the strategy

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Searching for Gravitationally Lensed Gamma-Ray Bursts with Their Afterglows

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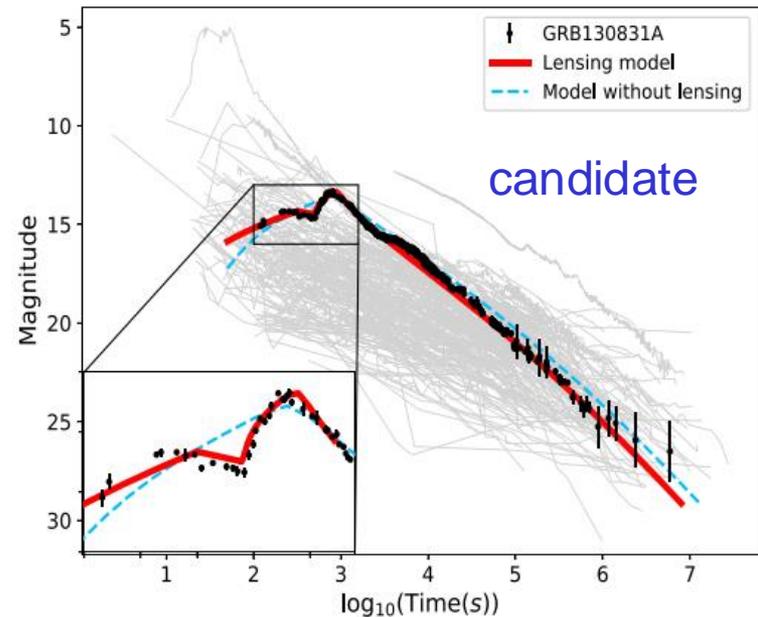
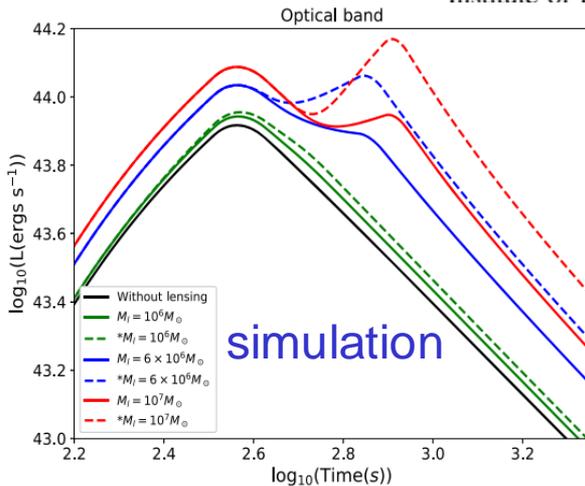
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revised 2021 October 8; accepted



In GR GWs propagate along null geodesics like photons

-no dispersion

In modified gravity theories (also QG phenomenology): graviton satisfies modified dispersion relation

• Can be massive

$$E^2 = p^2 c^2 + m_g^2 c^4 \quad \frac{v_g^2}{c^2} = 1 - \frac{m_g^2 c^4}{E^2}$$

• Massive GW + LIV term leads to dephasing of GW relative to the phase evolution in GR

$$E^2 = p^2 c^2 + m_g^2 c^4 + \Lambda p^\alpha c^\alpha \quad \frac{v_g^2}{c^2} = 1 - \frac{m_g^2 c^4}{E^2} - \Lambda E^{\alpha-2} \left(\frac{v}{c}\right)^\alpha$$

$$m_g \leq 7.7 \times 10^{-23} \frac{\text{eV}}{c^2}$$

but

$$m_g < (4.99 - 6.79) \times 10^{-29} \text{ eV}$$

Gravitational lensing can be helpful

A single lensed gravitational-wave signal enables us to measure the graviton mass with an accuracy comparable with the combined measurement across $O(10^3)$ unlensed signals

PRL 118, 221101 (2017) PHYSICAL REVIEW LETTERS week ending
2 JUNE 2017



GW170104: Observation of a 50-Solar-Mass Binary Black Hole Coalescence at Redshift 0.2

B. P. Abbott *et al.**

Journal of High Energy Astrophysics 33 (2022) 37–43



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Graviton mass from X-COP galaxy clusters

Aleksandra Piórkowska-Kurpas^{a,*}, Shuo Cao^b, Marek Biesiada^c



Lensing of Gravitational Waves as a Novel Probe of Graviton Mass

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Speed of Gravitational Waves from Strongly Lensed Gravitational Waves and Electromagnetic Signals

Xi-Long Fan,^{1,2,*} Kai Liao,³ Marek Biesiada,^{4,5} Aleksandra Piórkowska-Kurpas,⁴ and Zong-Hong Zhu^{1,5,†}

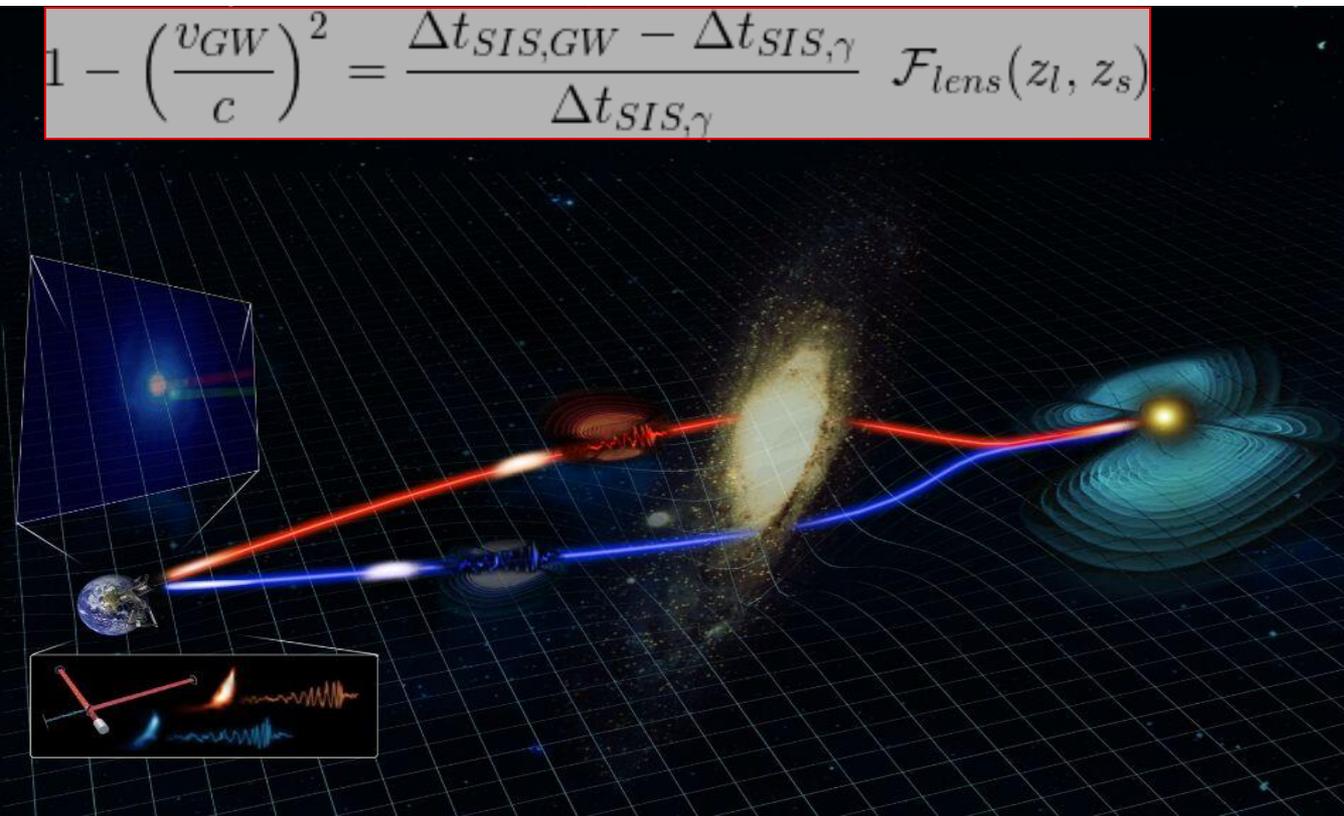
Classically GW propagates with the speed of light c , but in some theories of modified gravity (QG phenomenology) it can propagate with v_{GW} different from c

In a strongly lensed event time delays between images in GW and in EM would be different. If

This method is **free** from any assumptions regarding intrinsic time lag between EM and GW signal emission.

The same idea allows to test LIV (MB, A.Piórkowska 2009; Lan et al. 2022)

$$1 - \left(\frac{v_{GW}}{c}\right)^2 = \frac{\Delta t_{SIS,GW} - \Delta t_{SIS,\gamma}}{\Delta t_{SIS,\gamma}} \mathcal{F}_{lens}(z_l, z_s)$$



See also

Multimessenger time delays from lensed gravitational waves

Tessa Baker and Mark Trodden
Phys. Rev. D 95, 063512 (2017)

The idea of seeking for the difference in time delays of lensed EM and GW signals as a way of testing the speed of GWs is independent of many pre-assumptions inherent to other alternative methods.

- It is free from any assumptions concerning moments of emission of GW and EM signals (intrinsic time lag)
- Does not rely on detailed analysis of waveforms – only on detection trigger
- It does not really depend even on lens model (SIS served for illustration) – if GW and EM signals propagate at different speeds (detectable with this technique) it would be revealed as a difference of time delays anyway (EM and GW traverse the same lensing potential whatever it is).
- Only for qualitative interpretation of such a difference, lensing potential should be known precisely – this can be achieved with dedicated follow up study of the lensing system in the optical.

Challenge for the future: cosmic strings !

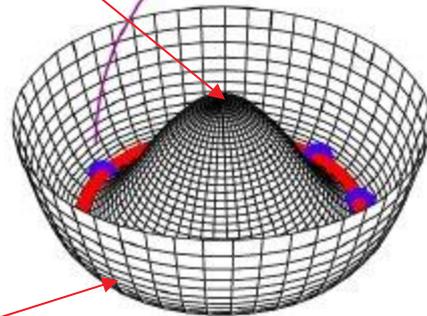
As the universe cooled down it went through at least three phase-transitions:

1. The GUT transition occurs between 10^{-37} s and 10^{-35} s after the Big Bang. The Grand Unification Theory (GUT) predicts that at very high-energy scales the electroweak-nuclear and strong-nuclear forces are unified into one force. The GUT symmetries are broken by the rapid expansion that caused a cooling down of the universe.
2. Around 10^{-11} s after the Big Bang the electroweak symmetry was broken. The electroweak symmetry unified electromagnetism and the weak interaction.
3. The quark-hadron transition at 10^{-5} s after the Big Bang caused the plasma of free quarks and gluons to convert into hadrons (baryons and mesons, the more well-known baryons are protons and neutrons).

Idea of topological defects

False vacuum

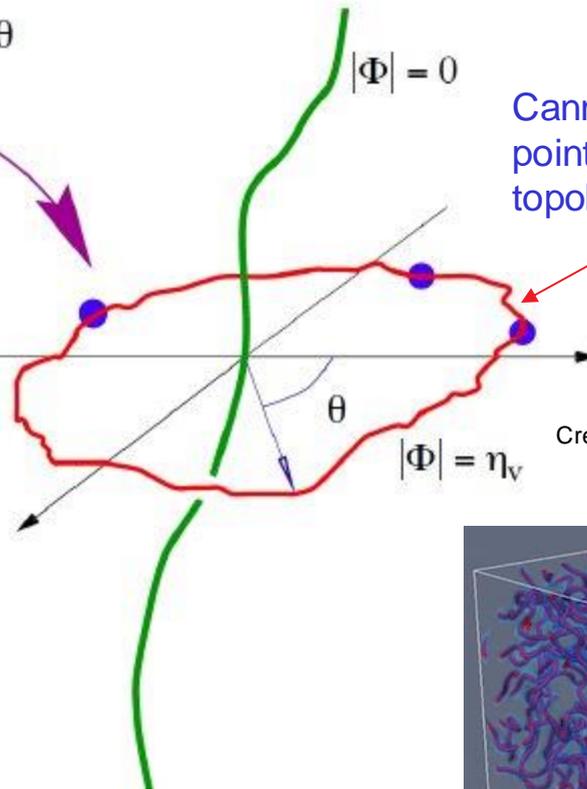
$$\langle \Phi \rangle = \eta_v e^{i\theta}$$



$$\mathcal{M} = G/H$$

True vacuum:
in general forms a
manifold \mathcal{M} of non-
trivial topology

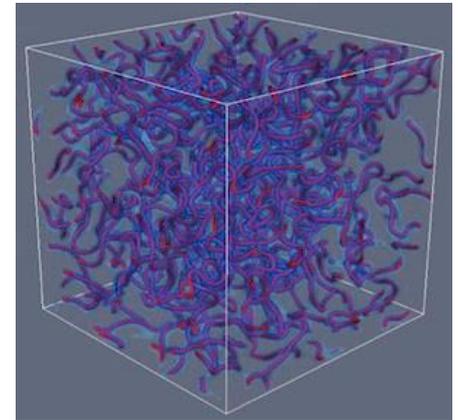
$$\pi_1(\mathcal{M})$$



Cannot be shrunk to a
point – string is a
topological defect here

Credit: Vachaspati, Pogosian, Steer

Universe populated with
strings, they interact,
form loops, oscillate -
emit GWs !



Cosmic string – cuts the wedge from the space-time

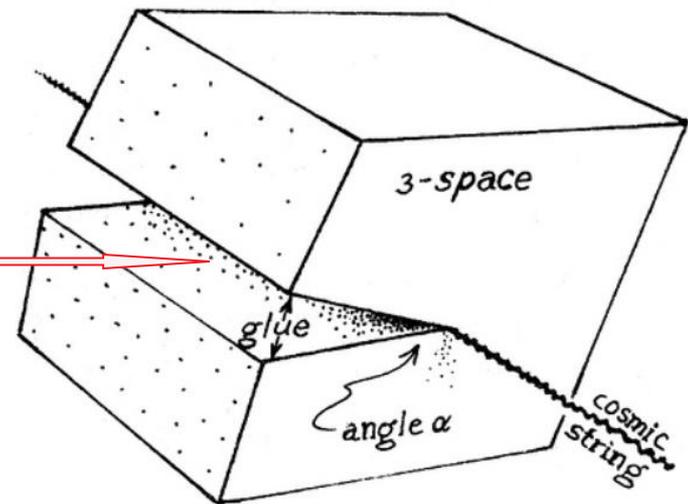
String is locally fully characterized by its tension
= mass per unit length μ

Deficit angle cut by the String

$$\delta = \frac{8\pi G\mu}{c^2}$$

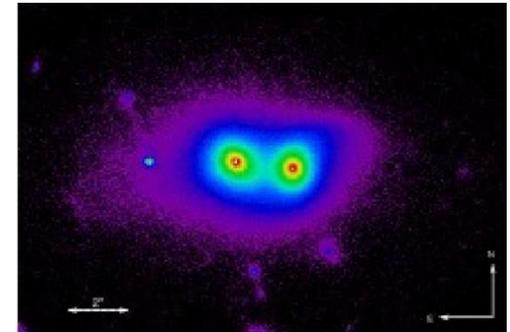
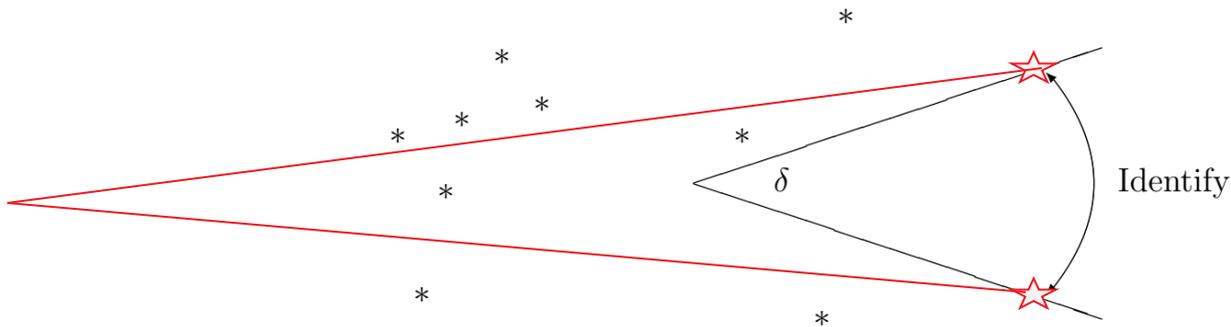
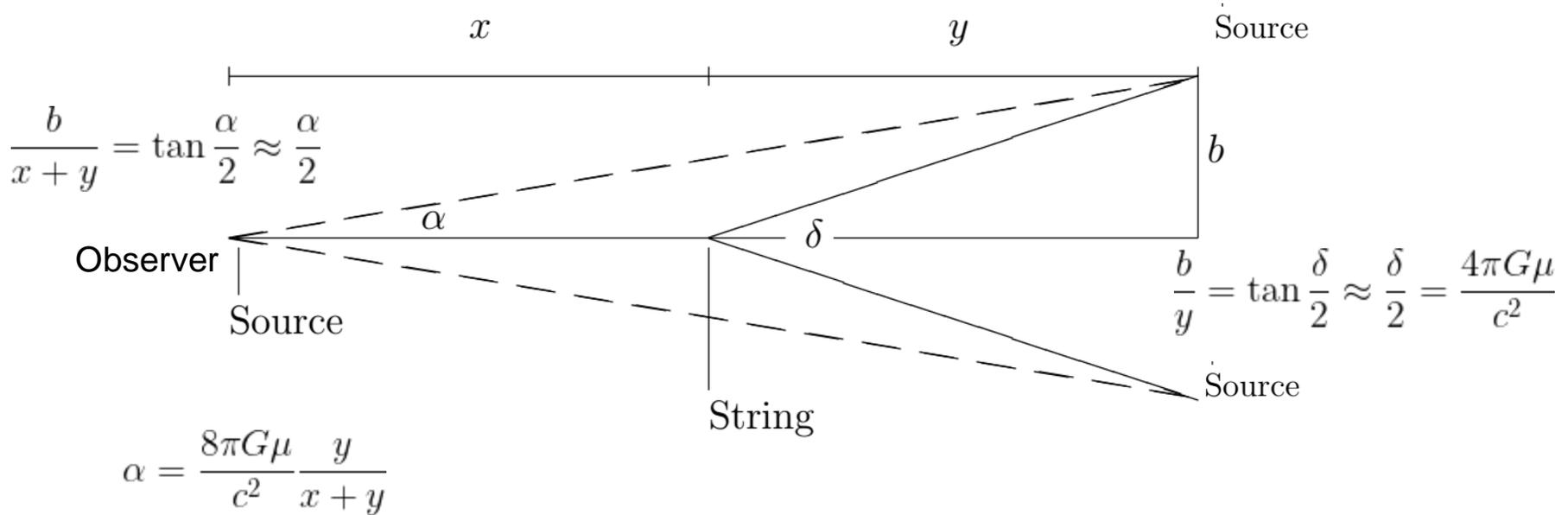
The only direct way to observe the String
is by gravitational lensing !

$$\Delta\vartheta = \frac{4\pi G\mu}{c^2}$$



Credit: Roger Penrose

Lensing by the cosmic string



Credit: Vachaspati, Pogosian, Steer

Source behind the cosmic string – split in two images (non-magnified !)

CSL – 1 once the most promising candidate

Lensing by the cosmic string

The relativistic calculation of lensing by a cosmic string is due to

J. R. Gott, *Astrophys. J.* 288, 422 (1985).

The anisotropy in the cosmic microwave background due to cosmic strings was calculated by

N. Kaiser and A. Stebbins, *Nature* 310, 391 (1984).

Exciting candidate of elliptical galaxy lensed by cosmic string

M. V. Sazhin et al., *Mon. Not. Roy. Astron. Soc.* 343, 353 (2003) [astro-ph/0302547].

M. V. Sazhin et al., astro-ph/0406516.

M. V. Sazhin et al., *Mon. Not. Roy. Astron. Soc.* 376, 1731 (2007) [astro-ph/0611744].

CSL -1 case

CSL-1: chance projection effect or serendipitous discovery of a gravitational lens induced by a cosmic string?

M. Sazhin^{1,2}, G. Longo^{3,4}, M. Capaccioli^{1,3}, J. M. Alcalá¹, R. Silvotti¹, G. Covone⁴, O. Khovanskaya², M. Pavlov¹, M. Pannella¹, M. Radovich¹, V. Testa⁵

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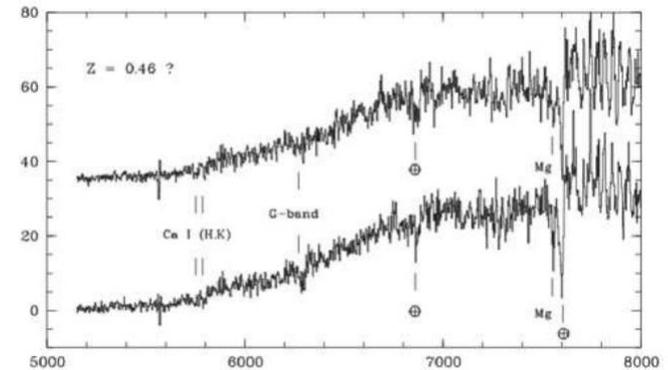
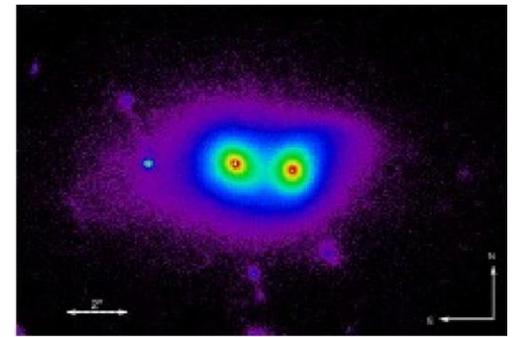
³ Dipartimento di Scienze Fisiche, Univ. Federico II, Polo delle Scienze e della Tecnologia, via Cinthia, 80126 Napoli, Italy

⁴ INAF - Telescopio Nazionale Galileo, Roque de Los Muchachos, Santa Cruz de La Palma, 38700-TF, Spain P.O. Box 565

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ABSTRACT

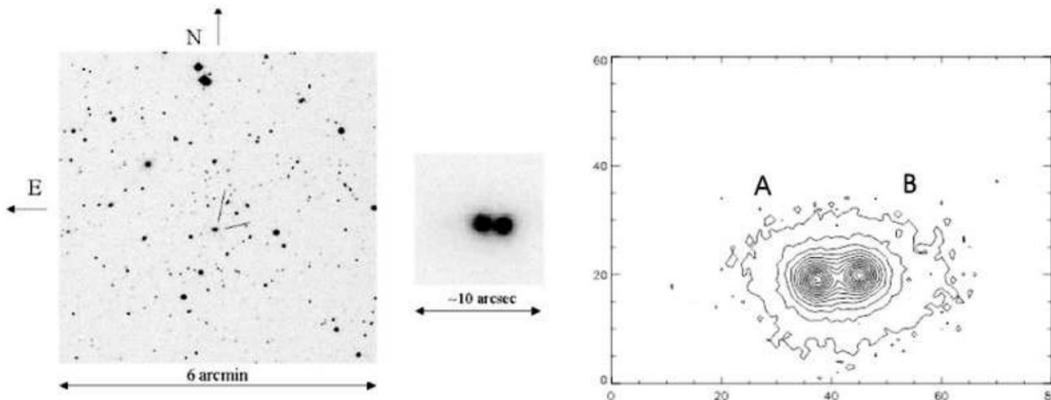
CSL-1 (Capodimonte–Sternberg–Lens Candidate, No.1) is an extragalactic double source detected in the OACDF (*Osservatorio Astronomico di Capodimonte - Deep Field*). It can be interpreted either as the chance alignment of two identical galaxies at $z = 0.46$ or as the first case of gravitational lensing by a cosmic string. Extensive modeling shows in fact that cosmic strings are the only type of lens which (at least at low angular resolution) can produce undistorted double images of a background source. We propose an *experimentum crucis* to disentangle between these two possible explanations. If the lensing by a cosmic string should be confirmed, it would provide the first measurements of energy scale of symmetry breaking and of the energy scale of Grand Unified Theory (GUT).



vertical shift in spectra – only for visualization

Eventually HST revealed a matter bridge (merger) ...

But in LSST era it is worth seeking for lensing by string



Conclusions

- Vera Rubin Observatory (LSST survey) will become a game changer in precision cosmology

10 000 strong lensing systems including 1000 quasar lenses.

- Great opportunity to advance alternative cosmological probes

strong lensing systems

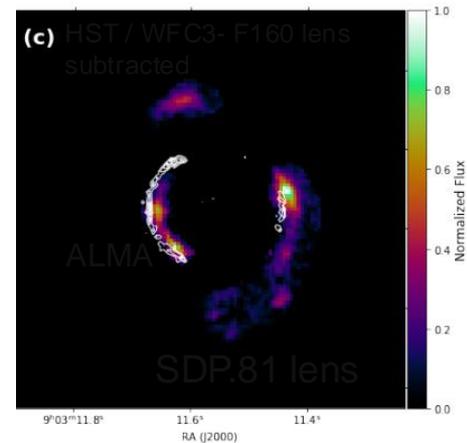
strongly lensed SN Ia (and other transient events)

anomalies in time delays and DM substructure, fuzzy DM ?

[K.Liao, ... ,M.B., et al. ApJ 867:69, 2018](#)

multiwavelength (optical, IR, radio) study of SL systems

lensed GRBs



- New generation of ground-based and space-borne GW detectors (ET, CE, DECIGO, LISA) will considerably enhance the statistics of GW events – lensed signals will be detected
- Opportunity to explore the fundamental questions in Physics, gravity theory beyond GR, QG, nature of DM.