Energy flow of λ in Hořava-Lifshitz cosmology

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Outline

- General Relativity, in spite of its wide theoretical success and experiment validations is not renormalizable.
- Adding higher spatial-derivative terms to the Lagrangian makes the theory renormalizable at high energies.
- However, that demands giving up on the idea of space-time invariance under four-dimensional diffeomorphisms.
- Hořava proposed gravity equipped with an anisotropic scaling at the Planck scale given in term of a critical Lifshitz exponent.
- The resulting theory was proven to be fully perturbatively renormalizable in all spatial dimensions.
- There have been several attempts to put observational bounds on Hořava gravity parameters.
- Our paper focuses on a parameter describing deviations from GR.
    - E. Czuchry, N.A. Nilsson, *On the energy flow of λ in Hořava-Lifshitz cosmology, Phys. Rev.* **D** *110 (2024), 043502.*

Non-renormalizabilty of GR

- In four-dimensional spacetimes G_N has the dimension of $(\text{mass})^{-2}$ (in units $\hbar = 1 = c$),
- It should be larger than or equal to zero in order for the theory to be renormalizable perturbatively
- The expansion of a given physical quantity F in terms of G_N must be in the form

$$F = \sum_{n=0}^{\infty} a_n \left(G_N E^2\right)^n,$$

  where E denotes the energy of the system, so $\left(G_N E^2\right)$ is dimensionless.
- When $E^2 \gtrsim G_N^{-1}$, such expansions diverge.
- Therefore, it is expected that perturbative effective QFT is broken down at such energies. It is in this sense that GR is often said to be not perturbatively renormalizable.

Higher order GR and non-unitarity

- Including high-order derivative corrections in the action improves ultraviolet (UV) behavior:

$$S_{EH} = \int d^4x\sqrt{-g}R,$$

  like for example adding a quadratic term $R_{\mu\nu}R^{\mu\nu}$.
- It causes the change of the gravitational propagator from $1/k^2$ to

$$\frac{1}{k^2} + \frac{1}{k^2}G_N k^4 \frac{1}{k^2} + \frac{1}{k^2}G_N k^4 \frac{1}{k^2}G_N k^4 \frac{1}{k^2} + ... = \frac{1}{k^2 - G_N k^4}.$$

- At UV the propagator is dominated by the term $1/k^4$, and the UV divergence can be cured.
- However, the modified theory is not unitary, there are two poles:

$$\frac{1}{k^2 - G_N k^4} = \frac{1}{k^2} - \frac{1}{k^2 - G_N^{-1}},$$

  The $1/k^2$ describes a massless spin-2 graviton, while $1/(k^2 - G_N^{-1})$ a massive one but with a wrong sign – actually a ghost.
- The existence of this ghost makes the theory not unitary.

Ostrogradsky's theorem and Lorentz Invariance

- The existence of the ghost is closely related with including time-derivatives higher than two.
- In the $R_{\mu\nu}R^{\mu\nu}$ case the field equations are fourth-orders.
- Ostrogradsky's theorem states that *a system is not (kinematically) stable if it is described by a non-degenerate higher time-derivative Lagrangian*.
- Therefore any higher derivative theory of gravity is not stable.
- A possible way to evade Ostrogradsky's theorem is to include only high-order spatial derivative terms in the Lagrangian, but keep the time derivative terms to the second order.
- This might be achieved by breaking Lorentz Invariance (LI) in the UV, while still maintaining in the IR.
- This is exactly what Hořava proposed.

Perturbatively renormalizable realistic quantum gravity model

- The theory obtained via controlled breaking of Lorentz Invariance turned out to be fully renormalizable in a strict sense in all space dimensions
- Exact calculation of its renormalization group (RG) flow was conducted in $2+1$ dimensions[1], revealing an asymptotically free UV fixed point.
- In the case of $3+1$ dimensions, partial results regarding the RG flow of projectable HG were obtained and potential candidates for asymptotically free UV fixed points were found and analyzed[2].
- Therefore, the theory serves as a realistic quantum gravity model.

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[1] A. O. Barvinsky, D. Blas, M. Herrero-Valea, S.M. Sibiryakov, and C. F. Steinwachs, *Horava gravity is asymptotically free in 2+1 dimensions*, Phys. Rev. Lett. 119, 211301 (2017).

[2] A. O. Barvinsky, A. V. Kurov, and S. M. Sibiryakov, *Beta functions of (3+1)-dimensional projectable Horava gravity*, Phys. Rev. D 105, 044009 (2022)

Breaking LI and Lifshitz's scaling

- The basic assumption of the Hořava theory was to consider anisotropic scaling between time and space:

$$t \to b^{-z} t, \quad x^i \to b^{-1} x'^i, \ (i = 1, 2, ..., d)$$

  where z denotes the dynamical critical exponent.
- LI requires $z = 1$, while power-counting renomalizibality requires $z \geq d$ (d is the spatial dimension of the spacetime).
- Usually spacetimes with $d = 3$ are considered and the minimal value $z = d$.
- Equation above is a reminiscent of Lifshitz's scalar fields in condensed matter physics hence Hořava gravity is called the Hořava-Lifshitz (HL) theory.

Breaking LI and Lifshitz's scaling

- In the described scaling the time and space have, respectively, the dimensions:
$$[t] = -z, \quad \left[x^i\right] = -1.$$
- Such a scaling breaks explicitly the LI and hence 4-dimensional diffeomorphism invariance.
- Hořava assumed that it is broken only down to the level
$$t \to \xi_0(t), \quad x^i \to \xi^i\left(t, x^k\right),$$
so the spatial diffeomorphism still remains.
- The above symmetry is often referred as to *the foliation-preserving diffeomorphism*: Diff(M, $\mathcal{F}$).

$(3+1)$-decomposition and ADM formalism

- Due to foliation-preserving diffeomorphism invariance the metric of Hořava-Lifshitz theory is written in the well-known ADM formalism:

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i - N^i dt)(dx^j - N^j dt),$$

where N, N_i and g_{ij} are dynamical variables.

- The most general form of the action:

$$S = \int d^3x dt N\sqrt{g}\left[K^{ij}K_{ij} - \lambda K^2 - \mathcal{V}(g_{ij})\right],$$

where λ is the running coupling and $\mathcal{V}$ is a (gravitational) potential. K_{ij} represents the extrinsic curvature.

- The square $K^{ij}K_{ij}$ and its trace-squared K^2 are individually invariant under Diff(M, $\mathcal{F}$), but for $\lambda = 1$ the full kinetic term $K^{ij}K_{ij} - K^2$ is invariant under four-diffeomorphisms.

$(3+1)$-decomposition and ADM formalism

- Under the Lifshitz scaling of t, x^i the variables N, N^i and g_{ij} scale as:
$$N \to N, \quad N^i \to b^2 N^i, \quad g_{ij} \to g_{ij},$$
so that their dimensions are
$$N = 0, \quad \left[N^i\right] = 2, \quad [g_{ij}] = 0.$$

- Under the Diff$(M, \mathcal{F})$, on the other hand, they transform as,
$$\begin{aligned}
\delta N &= \xi^k \nabla_k N + \dot{N}\xi_0 + N\dot{\xi_0}, \\
\delta N_i &= N_k \nabla_i \xi^k + \xi^k \nabla_k N_i + g_{ik}\dot{\xi^k} + \dot{N_i}\xi_0 + N_i\dot{\xi_0}, \\
\delta g_{ij} &= \nabla_i \xi_j + \nabla_j \xi_i + \xi_0 \dot{g}_{ij},
\end{aligned}$$

HL gravity: detailed balance and projectability

The detailed-balance condition reduces the number of terms in the action by assuming that it should be possible to derive $\mathcal{V}$ from a superpotential W:

$$\mathcal{V} = E^{ij}\mathcal{G}_{ijkl}E^{kl}, \quad E^{ij} = \frac{1}{\sqrt{g}}\frac{\delta W}{\delta g_{ij}}, \quad \mathcal{G}^{ijkl} = \frac{1}{2}\left(g^{ik}g^{jl} + g^{il}g^{jk}\right) - \lambda g^{ij}g^{kl}.$$

which for $\lambda = 1$ reduces to the standard Wheeler-DeWitt metric. Together with *projectability condition* $N = N(t)$ the most general action can be written as:

$$S_{db} = \int d^3x\, dt\sqrt{g}N\left[\frac{2}{\kappa^2}\left(K_{ij}K^{ij} - \lambda K^2\right) + \frac{\kappa^2}{2\omega^4}C_{ij}C^{ij} - \frac{\kappa^2\mu}{2\omega^2}\frac{\epsilon^{ijk}}{\sqrt{g}}R_{il}\nabla_j R^l_k \right.$$
$$\left. + \frac{\kappa^2\mu^2}{8}R_{ij}R^{ij} + \frac{\kappa^2\mu^2}{8(1-3\lambda)}\left(\frac{1-4\lambda}{4}R^2 + \Lambda R - 3\Lambda^2\right)\right],$$

where C^{ij} is the Cotton tensor, ϵ^{ijk} is the totally antisymmetric tensor, and the parameters κ, ω, and μ have mass dimension -1,0, and 1, respectively.

Hořava-Lifshitz gravity and IR limit

- The Cotton tensor, C_{ij}, is defined by

$$C^{ij} = \epsilon^{ikl}\nabla_k \left(R^j{}_l - \frac{1}{4}R\delta^j_l \right) = \epsilon^{ikl}\nabla_k R^j{}_l - \frac{1}{4}\epsilon^{ikj}\partial_k R.$$

- It is expected that the HL action reduces to the Einstein-Hilbert one in the IR limit of the theory.
- This is possible if the speed of light c and gravitational constant G correspond to HL parameters as follows:

$$G = \frac{\kappa^2}{32\pi c}, \quad c = \frac{\kappa^4\mu^2\Lambda}{8(3\lambda - 1)^2}.$$

Coupling constant λ

- The coupling constant λ is dimensionless.
- It runs logarithmically with energy, at $\lambda = 1$ the theory is supposed to reduce to the classical GR. One found UV fixed point is at $\lambda = \infty$, some possible for $\lambda < 1/3$.
- In the region $1/3 < \lambda < 1$ there are tachyonic ghosts and the corresponding quantum theory is not unitary. This region is excluded from all realistic considerations.
- The most physically interesting case is the regime $\lambda \geq 1$ allowing for a possible flow towards GR with $\lambda = 1$.
- Region $\lambda \leq 1/3$ is disconnected from $\lambda = 1$.
- The parameter λ is supposed to control the breaking of Lorentz invariance.

Coupling constant λ

- The theoretical works proved that parameter λ runs with energy, from asymptotic point(s) at UV ($\lambda = \infty$) to IR ($\lambda = 1$).
- Is is possible to detect small changes of its value calculated using different available cosmological data?
- Side note: Unfortunately data from binary and triple binary objects is not useful, as it was shown that theories with asymptotically flat spacetimes different from GR only when $\lambda \neq 1$ have been shown to be equivalent to GR.

Hořava-Lifshitz cosmology: detailed balance

To derive equations of HL cosmology one uses the spatial part of the metrics being the standard FLRW line element: $g_{ij} = a^2(t)\gamma_{ij}$, $N_i = 0$, where γ_{ij} denotes a maximally symmetric metric with constant curvature:

$$\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j = \frac{\mathrm{d}r^2}{1 - Kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2),$$

values $K = \{-1, 0, 1\}$ correspond respectively to closed, flat, and open Universe. This background metric implies that

$$C_{ij} = 0\,, \qquad R_{ij} = \frac{2K}{a^2}g_{ij}\,, \qquad K_{ij} = \frac{H}{N}g_{ij}\,,$$

where $H \equiv \dot{a}/a$ denotes the Hubble parameter.
On this background the gravitational action take the following form :

$$S_{\mathrm{FRW}} = \int dt\, d^3x\, Na^3 \left\{ \frac{3(1-3\lambda)}{2\kappa^2}\frac{H^2}{N^2} + \frac{3\kappa^2\mu^2\Lambda}{4(1-3\lambda)}\left(\frac{K}{a^2} - \frac{\Lambda}{3}\right) - \frac{\kappa^2\mu^2}{8(1-3\lambda)}\frac{K^2}{a^4} \right\}.$$

Hořava-Lifshitz cosmology: detailed balance

1. Varying the HL action w.r.t N and a and setting $N = 1$ at the end of calculations;
2. Populating our model with the canonical matter and radiation fields represented by the energy densities (and pressures) ρ_m (p_m) and ρ_r (p_r) and subject to the continuity equation $\dot{\rho} + 3H(\rho + p) = 0$;

leads to analogues of the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{6(3\lambda - 1)}\left[\rho_m + \rho_r\right] + \\ - \frac{\kappa^2}{6(3\lambda - 1)}\left[\frac{3\kappa^2\mu^2 K^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda - 1)}\right] + \frac{\kappa^4\mu^2\Lambda K}{8(3\lambda - 1)^2 a^2},$$

$$\frac{d}{dt}\frac{\dot{a}}{a} + \frac{3}{2}\left(\frac{\dot{a}}{a}\right)^2 = -\frac{\kappa^2}{4(3\lambda - 1)}\left[p_m + p_r\right] + \\ - \frac{\kappa^2}{4(3\lambda - 1)}\left[\frac{\kappa^2\mu^2 K^2}{8(3\lambda - 1)a^4} - \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda - 1)}\right] - \frac{\kappa^4\mu^2\Lambda K}{16(3\lambda - 1)^2 a^2}.$$

Hořava-Lifshitz cosmology

- Requiring that our cosmological equations coincide with the standard Friedmann equations results in $G_{\text{cosmo}} = \kappa^2/(3\lambda - 1)$ and $\kappa^4\mu^2\Lambda = 8(3\lambda - 1)^2$.
- $|G_{\text{cosmo}}/G_{\text{grav}}| \approx 3\lambda/2$ and when Lorentz invariance is restored ($\lambda = 1$): $G_{\text{grav}} = G_{\text{cosmo}}$.
- Under DB and in the units $8\pi G_{\text{grav}} = 1$: $\kappa^2 = 4$, $\mu^2\Lambda = 2$.
- Introducing the standard density parameters leads to the Friedmann equation suitable for numerical analysis:

$$H^2 = H_0^2\left[\frac{2}{3\lambda - 1}(\Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z^4)) + \Omega_{K0}(1+z)^2 + \omega + \frac{\Omega_{K0}^2}{4\omega}(1+z)^4\right]$$

  H denotes the Hubble parameter, the subscript 0 the value as measured today, $\omega = \Lambda/(2H_0^2)$.
- A characteristic feature of HL theory is a dark radiation term $\Omega_{k0}^2/4\omega$, that could be expressed in terms of the effective number of neutrino species ΔN_{eff} present during the BBN epoque: $\Omega_{k0}^2/4\omega = 0.13424\Delta N_{\text{eff}}\Omega_{r0}$.
- We can also obtain a constraint from the $z = 0$ limit, where $H|_{z=0} = H_0$, which reads: $(1 - \Omega_{k0} - \omega - \Omega_{k0}^2/(4\omega))(3\lambda - 1)/2 = \Omega_{m0} + \Omega_{r0}$.

Beyond detailed balance formulation

- The detailed balance formulation has many theoretical flaws including violation of parity.
- The attempts to fix these issues include breaking DB or adding terms to superpotential, as well as breaking projectability condition.
- The simplest model is the Sotiriou-Visser-Weinfurtner (SVW) generalization, where DB is relaxed and the potential $\mathcal{V}$ contains also cubic terms, while suppressing parity violating terms.

SVW cosmology

The cosmological equations are following:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{2\sigma_0}{3\lambda - 1}(\rho_m + \rho_r) + \\ + \frac{2}{3\lambda - 1}\left[\frac{\Lambda}{2} + \frac{\sigma_3 K^2}{6a^4} + \frac{\sigma_4 K}{6a^6}\right] + \frac{\sigma_2}{3(3\lambda - 1)}\frac{K}{a^2},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\dot{a}}{a} + \frac{3}{2}\left(\frac{\dot{a}}{a}\right)^2 = -\frac{3\sigma_0}{3\lambda - 1}\frac{\rho_r}{3} + \\ - \frac{3}{3\lambda - 1}\left[-\frac{\Lambda}{2} + \frac{\sigma_3 K^2}{18a^4} + \frac{\sigma_4 K}{6a^6}\right] + \frac{\sigma_2}{6(3\lambda - 1)}\frac{K}{a^2},$$

where σ_i are arbitrary constants.

SVW cosmology

- As in DB we find $G_{\text{cosmo}} = 6\sigma_0/(8\pi(3\lambda - 1))$, where $\sigma_0 = \kappa^2/12$.
- Using the same procedure we rewrite cosmological equations to read:

$$H^2 = H_0^2 = \left[\frac{2}{3\lambda - 1}\left(\Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \right.\right.$$
$$\left.\left. +\omega_1 + \omega_3(1+z)^4 + \omega_4(1+z)^6\right] + \Omega_{k0}(1+z)^2\right],$$

  where we have introduced the following dimensionless parameters:
  $\omega_1 = \sigma_1/(6H_0^2)$, $\omega_3 = \sigma_3 H_0^2 \Omega_{k0}^2/6$, $\omega_4 = -\sigma_4 \Omega_{k0}/6$.
- Additionally, we impose $\omega_4 > 0$ for the Hubble parameter to be real for all z.
- We can extract a constraint from the $z = 0$ limit:
  $(1 - \Omega_{k0})(3\lambda - 1/2 = \Omega_{m0} + \Omega_{r0} + \omega_1 + \omega_3 + \omega_4$.
- Considering that the ω_4 term corresponds to a quintessence-like kinetic field we get the following constraint at the time of BBN:
  $\omega_3 = 0.13424\Delta N_{\text{eff}}\Omega_{r0} - \omega_4(1 + z_{BBN})^2$ (at $z_{BBN} \approx 4 \cdot 10^4$).
- We abbreviate the Beyond Detailed Balance case as BDB.

Numerical calculations

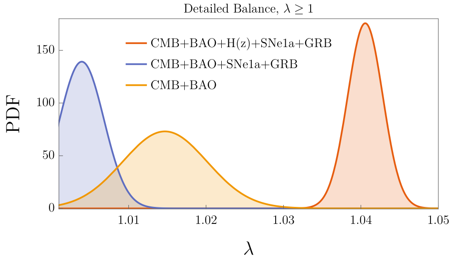
- We wanted to estimate the values of the several HL gravity parameters using the Markov-Chain Monte Carlo (MCMC) method and a large cosmological data set.
- The data includes expansion rates of elliptical and lenticular galaxies, Type Ia Supernovae, Baryon Acoustic Oscillations, Cosmic Microwave Background and priors on the Hubble parameter.
- We used the parallelised Markov-Chain Monte Carlo (MCMC) code developed in Mathematica.
- The code although slower than the Fortran or C ones makes it easier to add new data, and is also simple to modify.
- Therefore, things such as the cosmological model, statistical method and parameters used can easily be changed.
- During every step in the computation, the MCMC method calculates the χ^2, and in the end returns the parameter set which minimised the χ^2 function.

The astronomical data

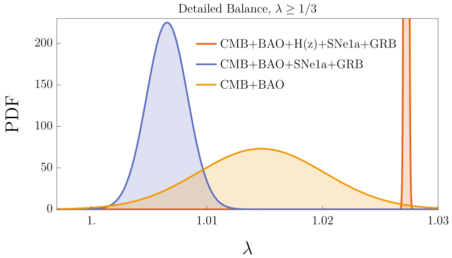
- At high redshift (early times), our probes are the Cosmic Microwave Background (CMB) and Baryon Acoustic Oscillations (BAO).
- The CMB originates at redshift $z_\star \sim 1100$, when the temperature of the Universe was around $T \geq 3000$K (0.26eV).
- This is complemented by BAO observations, that also affect the distribution of local galaxies and are more sensitive to different parameters compared to the CMB.
- Supernovae Type 1a are used as standard rulers and outputs more energy than the rest of its host galaxy (up to $\sim$ 40 MeV). Their energy is less important here as they are used as distance rulers.
- The Cosmic Chronometer (CC) data set is based on passively evolving galaxies.
- We also have gamma-ray bursts, which are the most violent explosions in the known Universe (up to 96 GeV).

Investigating the possible running of λ

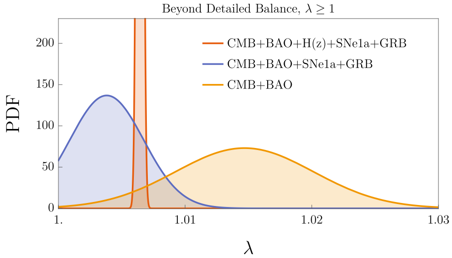
- We systematically remove data in two steps:
  1. The Hubble parameter measurements from Cosmic Chronometers (CC),
  2. Everything except the truly early-Universe probes, CMB and BAO.
- In DB removing CC data pushes λ to take values close to unity, but further removing data moves it back up toward higher values and with much larger error bars.
- In the BDB the situation here is somewhat reversed: removing CC data produces a higher value of λ than with the full dataset, in contrast to our other results.
- In almost all scenarios removing Hubble data from CC strongly pushes λ close to IR limit.



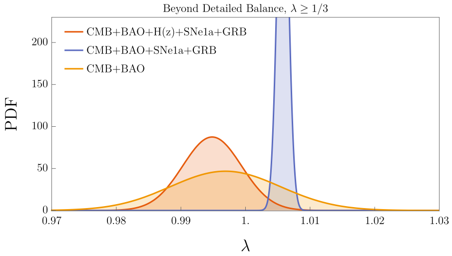
(a) DB, prior



(b) DB, no prior



(c) BDB, prior



(d) BDB, no prior

Parameter	Detailed balance, prior $\lambda \geq 1$: CMB+BAO+H(z) +SNe1a+Cepheids +GRB	Detailed balance, prior $\lambda \geq 1$: CMB+BAO +SNe1a+Cepheids +GRB	Detailed balance, prior $\lambda \geq 1$: CMB+BAO	Detailed balance, no prior on λ: CMB+BAO+H(z) +SNe1a+Cepheids +GRB	Detailed balance, no prior on λ: CMB+BAO +SNe1a+Cepheids +GRB	Detailed balance, no prior on λ: CMB+BAO
Ω_b	0.0049768 ± 0.0000016	$0.04898^{+0.00055}_{-0.000053}$	0.05104 ± 0.00080	0.049236 ± 0.000070	0.04907 ± 0.00058	$0.05139^{+0.00019}_{-0.00020}$
$\Omega_b h^2$	0.020952 ± 0.000074	0.02227 ± 0.00018	$0.02192^{+0.00019}_{-0.00021}$	0.02158 ± 0.00012	0.02217 ± 0.00017	0.02187 ± 0.00011
Ω_m	0.3336 ± 0.0018	$0.3153^{+0.0057}_{-0.0055}$	0.3398 ± 0.0083	0.3204 ± 0.0030	0.3170 ± 0.0056	$0.34437^{+0.00010}_{-0.00012}$
$\Omega_m h^2$	$0.14043^{+0.00024}_{-0.00025}$	0.14335 ± 0.00090	$0.1458^{+0.0011}_{-0.0010}$	$0.14047^{+0.00030}_{-0.00033}$	0.14323 ± 0.00092	0.14657 ± 0.00023
$\Omega_k 10^4$	-4.1364 ± 0.0040	$-6.116^{+1.55}_{-0.30}$	$-11.60^{+2.61}_{-1.76}$	-4.254 ± 0.019	-5.745 ± 0.029	$-13.5030^{+0.0088}_{-0.0086}$
$\Omega_r 10^5$	$9.937^{+0.036}_{-0.035}$	9.20 ± 0.12	$9.74^{+0.19}_{-0.18}$	$9.543^{+0.070}_{-0.065}$	$9.259^{+0.012}_{-0.011}$	$9.83^{+0.012}_{-0.018}$
h	0.6488 ± 0.0012	$0.6743^{+0.0043}_{-0.0044}$	$0.6553^{+0.0061}_{-0.0063}$	$0.6621^{+0.0023}_{-0.0024}$	0.6722 ± 0.0040	0.65239 ± 0.00040
M	-19.5051 ± 0.0013	$-19.437^{+0.012}_{-0.013}$	-	-19.4783 ± 0.0075	-19.442 ± 0.012	-
$\Delta N_{\rm eff}$	0.0046750 ± 0.0000076	$\mathbf{0.1104^{+0.0011}_{-0.0049}}$	$0.038^{+0.013}_{-0.015}$	0.005099 ± 0.000060	0.009670 ± 0.000062	0.05195 ± 0.00019
λ	1.0406 ± 0.0023	$\mathbf{< 1.0032}^\dagger$	$1.0146^{+0.055}_{-0.053}$	1.02726 ± 0.00012	1.0065 ± 0.0018	1.0159 ± 0.0014
$\left\lvert \frac{G_{\rm cosmo}}{G_{\rm grav}} - 1 \right\rvert$	0.0574 ± 0.0030	$\mathbf{< 0.0035}^\dagger$	$0.0214^{+0.0078}_{-0.0077}$	0.03928 ± 0.00017	$0.00997^{+0.0026}_{-0.0025}$	0.0232 ± 0.0020
$\chi^2_{\rm min}$	1778.27	1635.41	27.30	1705.04	1638.49	27.76

TABLE I: Parameter constraints at 1σ for the Detailed Balance case, with and without a hard prior on the parameter λ. † implies a one-sided upper bound resulting from a hard uniform prior, and **bold** indicates a particularly noisy parameter.

Parameter	Beyond detailed balance, prior $\lambda \geq 1$: CMB+BAO+H(z) +SNe1a+Cepheids +GRB	CMB+BAO +SNe1a+Cepheids +GRB	CMB+BAO	Beyond detailed balance, no prior on λ: CMB+BAO+H(z) +SNe1a+Cepheids +GRB	CMB+BAO +SNe1a+Cepheids +GRB	CMB+BAO
Ω_b	$0.049371^{+0.00049}_{-0.00048}$	0.05024 ± 0.00049	$0.05116^{+0.00062}_{-0.00063}$	0.04900 ± 0.00040	$0.05034^{+0.00017}_{-0.00016}$	0.05034 ± 0.00016
$\Omega_b h^2$	0.022922 ± 0.00023	0.02262 ± 0.00020	0.02226 ± 0.00018	0.022864 ± 0.00011	0.02250 ± 0.00011	0.02250 ± 0.00011
Ω_m	0.3198 ± 0.0053	$0.3194^{+0.0056}_{-0.0054}$	0.3349 ± 0.0073	$0.3109^{+0.0038}_{-0.0041}$	0.3232 ± 0.0031	$0.3284^{+0.0093}_{-0.0091}$
$\Omega_m h^2$	0.1484 ± 0.0017	$0.14386^{+0.00095}_{-0.00096}$	0.1458 ± 0.0011	$0.14509^{+0.00015}_{-0.00016}$	$0.14442^{+0.00046}_{-0.00049}$	$0.1458^{+0.0030}_{-0.0027}$
$\Omega_k 10^3$	$-9.71^{+1.73}_{-1.83}$	$-4.98^{+0.77}_{-0.46}$	-3.93 ± 0.15	$-5.399^{+0.022}_{-0.023}$	-4.338 ± 0.080	-6.32 ± 2.56
$\Omega_r 10^5$	9.01 ± 0.13	9.29 ± 0.11	$9.61^{+1.63}_{-1.61}$	$8.97^{+1.06}_{-1.11}$	9.36 ± 0.61	$9.431^{+0.028}_{-0.030}$
h	0.6813 ± 0.0048	$0.6711^{+0.0039}_{-0.0040}$	$0.6599^{+0.0056}_{-0.0055}$	$0.6831^{+0.0043}_{-0.0040}$	$0.6685^{+0.0022}_{-0.0021}$	$0.6660^{+0.0109}_{-0.0096}$
M	$-19.414^{+0.014}_{-0.015}$	-19.446 ± 0.0011	-	$-19.412^{+0.012}_{-0.011}$	-19.4525 ± 0.0040	-
ΔN_{eff}	0.61 ± 0.14	$0.258^{+0.028}_{-0.048}$	0.198 ± 0.015	$0.2578^{+0.0042}_{-0.0044}$	0.2166 ± 0.0070	$0.31^{+0.19}_{-0.15}$
λ	1.00644 ± 0.00020	$< 1.0068^{\dagger}$	1.0065 ± 0.0025	$0.9949^{+0.0045}_{-0.0046}$	1.00578 ± 0.00086	$0.9972^{+0.0081}_{-0.0088}$
$\left\lvert \frac{G_{\mathrm{cosmo}}}{G_{\mathrm{grav}}} - 1 \right\rvert$	0.00957 ± 0.00029	$< 0.010^{\dagger}$	0.0096 ± 0.0037	$0.0078^{+0.0070}_{-0.0053}$	0.0086 ± 0.0013	< 0.019
ω_1	0.69957 ± 0.00039	$0.6909^{+0.0063}_{-0.0062}$	$0.6789^{+0.0070}_{-0.0075}$	0.6866 ± 0.0036	$0.6898^{+0.0044}_{-0.0043}$	0.6898 ± 0.0043
$\omega_3 10^6$	$7.07^{+1.62}_{-1.71}$	$1.77^{+0.10}_{-0.15}$	$1.5201^{+0.068}_{-0.071}$	$2.9109^{+0.047}_{-0.0049}$	1.80 ± 0.15	1.79 ± 0.15
χ^2_{min}	1634.37	1632.36	23.82	1635.54	1633.85	21.85

TABLE II: Parameter constraints at 1σ for the Beyond Detailed Balance case, with and without a hard prior on the parameter λ. † implies a one-sided upper bound resulting from a hard uniform prior.

Other results

- Another interesting quantity is the difference between $G_{\rm grav}$ and $G_{\rm cosmo}$ expressed as $|G_{\rm cosmo}/G_{\rm grav} - 1|$.
    - We find that in DB it is smaller than $\sim 5.8\%$, which is linked to the values obtained on λ.
    - For BDB it is smaller $< 2\%$.
- We also find that Ω_{k0} is non-zero in the DB formulation of HL cosmology.
- Another interesting result is the value of $\Delta N_{\rm eff} > 0.2$ in almost all cases, and a 1σ upper limit of $\Delta N_{\rm eff} \leq 0.75$ for BDB (prior, all data) – this chain passed all convergence criteria.
    - Therefore, with all other parameters taking on reasonable values one may consider the possibility of a fourth neutrino species present in HL cosmology;
    - It was recently suggested (Carneiro, 2018) that a fourth neutrino might solve the H_0, however the authors arrive there at $N_{\rm eff} \approx 4$ (effective number of neutrino species), which is far higher than our results.

Other results

- The fact that $\Delta N_{\mathrm{eff}} \neq 0$ fits with the non-flatness results indicated by Ω_{k0}.
- A closed Universe Ω_{k0} has been found in several different analyses of Hořava-Lifshitz cosmology and the model does indeed prefer a closed Universe.
- Other studies have reported a strong preference for a closed Universe in the *Planck* data, a result which is sensitive to the amount of lensing in the sample.
- Other data sets, primarily BAO, strongly favour a closed Universe (under the assumption of ΛCDM) to the extent that the tension in Ω_{k0} has been estimated at $2.5 - 3\sigma$.

Summary

- The fitted value of λ varies with the kind of astronomical/cosmological sources and possibly their energies.
- Curvature parameter Ω_{k0} is non-zero in the DB formulation of HL cosmology.
- The value of $\Delta N_{\rm eff} > 0.2$ in almost all cases, and a 1σ upper limit of $\Delta N_{\rm eff} \leq 0.75$ for BDB (prior, all data).

Discussion

- Our main result is that in minimal versions of the Hořava the parameter controlling LI varies with energies of the astronomical messengers.
- However, we have to distinguish gravitational energy and energies of the probes, and take into account hidden assumptions on the model.
- We used the cosmological models of the simplest Hořava gravity models (DB and BDB) which are subject to several theoretical problems.
- These models have problems with strong coupling in the IR and unstable Minkowski limit, however they were proven to be **strictly renormalizable** in all dimensions, contain built-in dark matter arriving as integration constant and a mechanism of generating scale-invariant perturbations solving the horizon problem and the the flatness one.
- The whole theory is still not excluded by observational data, fitting more and more narrow parameter space and it might serve as realistic quantum gravity and cosmological models.