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CHARACTERISATION OF GRAVITATIONAL-WAVE BURSTS TENSOR POLARISATIONS WITH THE *BAYESWAVE* PIPELINE

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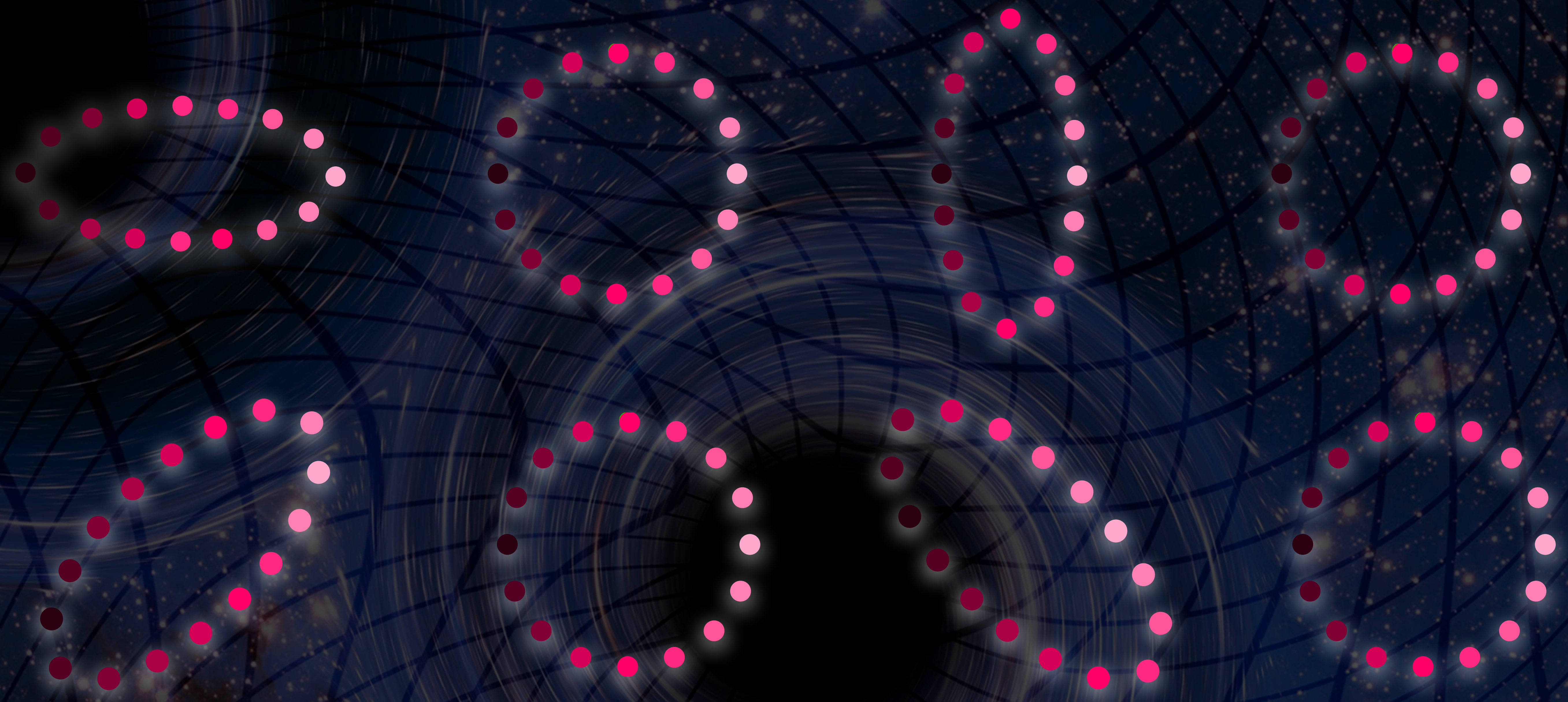
TALK OVERVIEW

- Gravitational-wave (GW) polarisations
- *BayesWave* signal models: Elliptical (E) and relaxed (R)
- Multi-detector network analyses:
  - **PART I:** Model selection - E vs. R
  - **PART II:** Measuring tensor polarisation content with R

GW POLARISATIONS

- According to General Relativity, GWs have two polarisations
  - Plus (+)
  - Cross (×)



Deformation of a ring of free-falling particles by each polarisation modes

DISENTANGLING GW POLARISATIONS

- Antenna pattern functions : $F_{\times}(\Omega, \psi)$ and $F_{+}(\Omega, \psi)$
  - Sensitivity of a detector to each polarisation state
  - Ω = the sky location of the source
  - ψ = the polarisation angle

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  - Sensitivity of a detector to each polarisation state
  - Ω = the sky location of the source
  - ψ = the polarisation angle
- Interferometric response of detector I (in the frequency domain)
  - $\tilde{h}_I = \left[F_I^{\times}(\Omega, \psi)\tilde{h}^{\times} + F_I^{+}(\Omega, \psi)\tilde{h}^{+}\right] e^{2\pi i f \Delta t_I(\Omega)}$
  - $\tilde{h}_+$ and $\tilde{h}_\times$ = amplitudes at a nominal reference location
  - Δt_I = light travel time from the reference location to detector I

DISENTANGLING GW POLARISATIONS

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DISENTANGLING GW POLARISATIONS

$$\tilde{h}_I = \left[F_I^{\times}(\Omega, \psi)\tilde{h}^{\times} + F_I^{+}(\Omega, \psi)\tilde{h}^{+}\right] e^{2\pi i f \Delta t_I(\Omega)}$$

- Contains up to four unknowns:
  - Two polarisation amplitudes: $\tilde{h}_+$ and $\tilde{h}_\times$
  - Sky location Ω
  - Source orientation ψ
- Need responses from multiple detectors to extract the polarisation components

EXPANDING GW DETECTOR NETWORK

- **Existing 2nd-generation ground-based detectors:**

  (1) LIGO - Hanford (H) and Livingston (L), USA

  (2) Virgo (V), Italy

  (3) KAGRA (K), Japan

[Image credits: LIGO Lab Caltech]

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  - (2) Virgo (V), Italy
  - (3) KAGRA (K), Japan
- **Compare multi-detector performances** in characterising polarisations:
  - HL (two-detector)
  - HLV (three-detector)
  - HLKV (four-detector)

[Image credits: LIGO Lab Caltech]

BAYESWAVE: ALGORITHM OVERVIEW

- An unmodelled transient gravitational wave (burst) analysis algorithm

BayesWave publications:
Cornish + Littenberg, Class. Quant. Grav 32, 130512 (2015)
Cornish et al., Phys. Rev. D 103, 044006 (2021)

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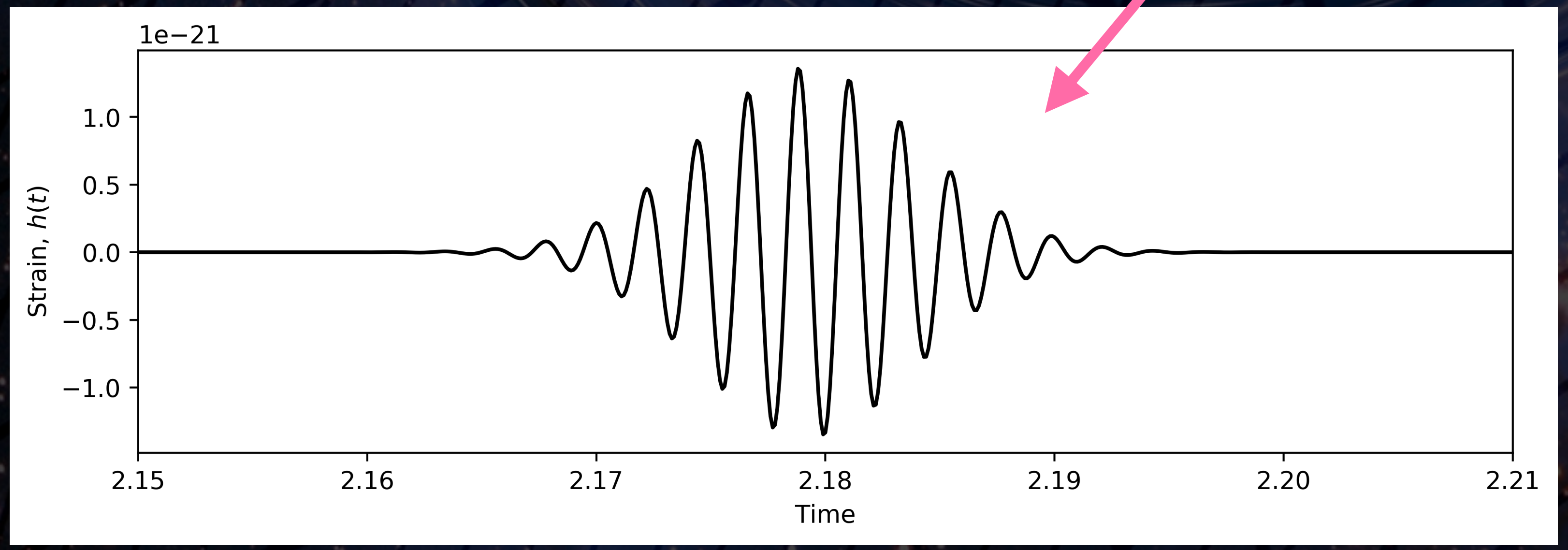

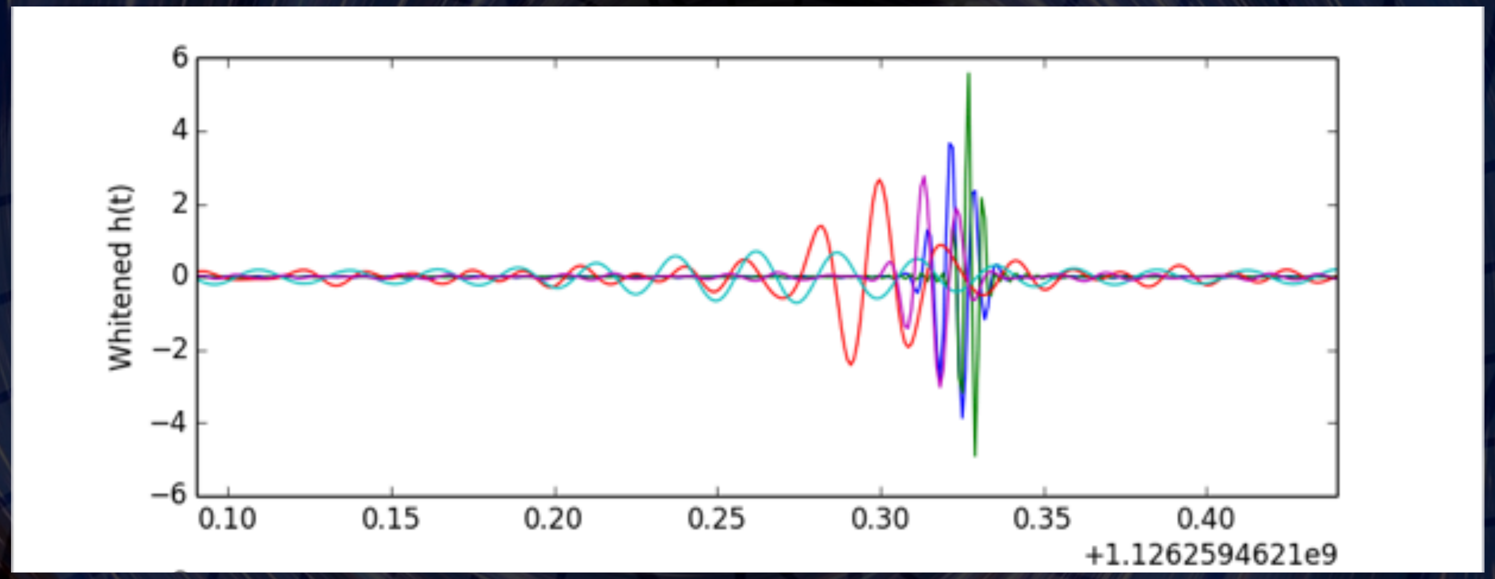

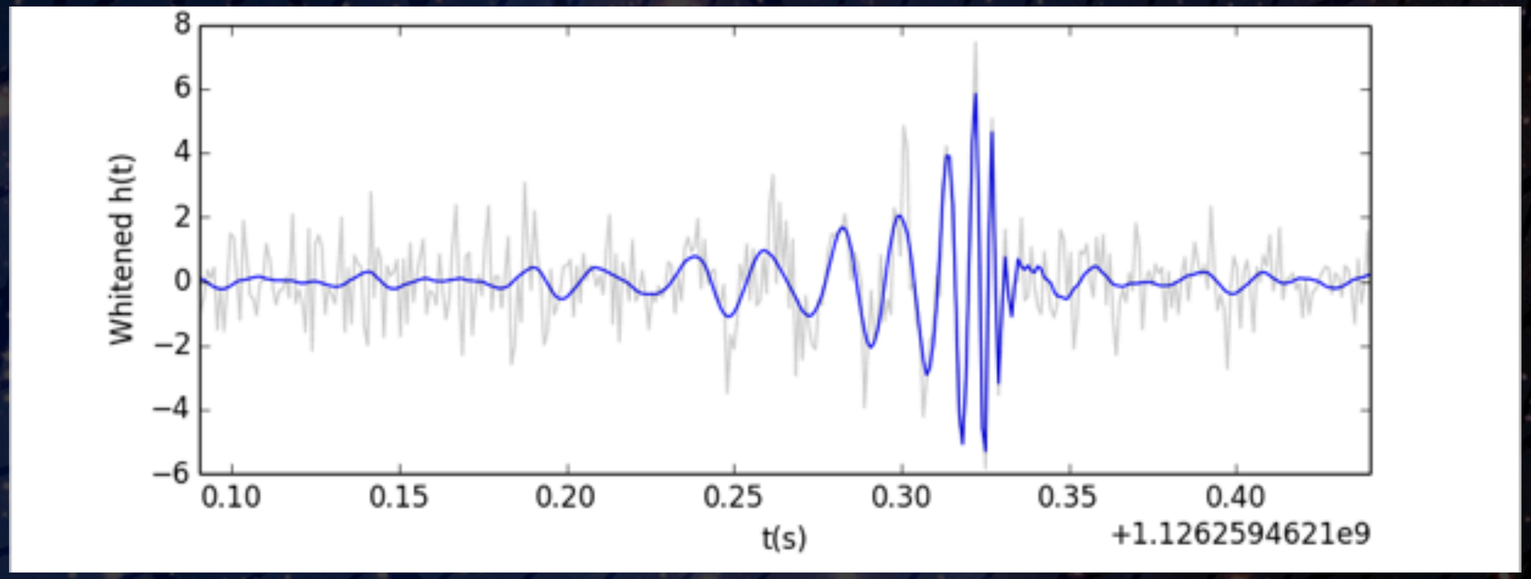

Images courtesy of Meg Millhouse

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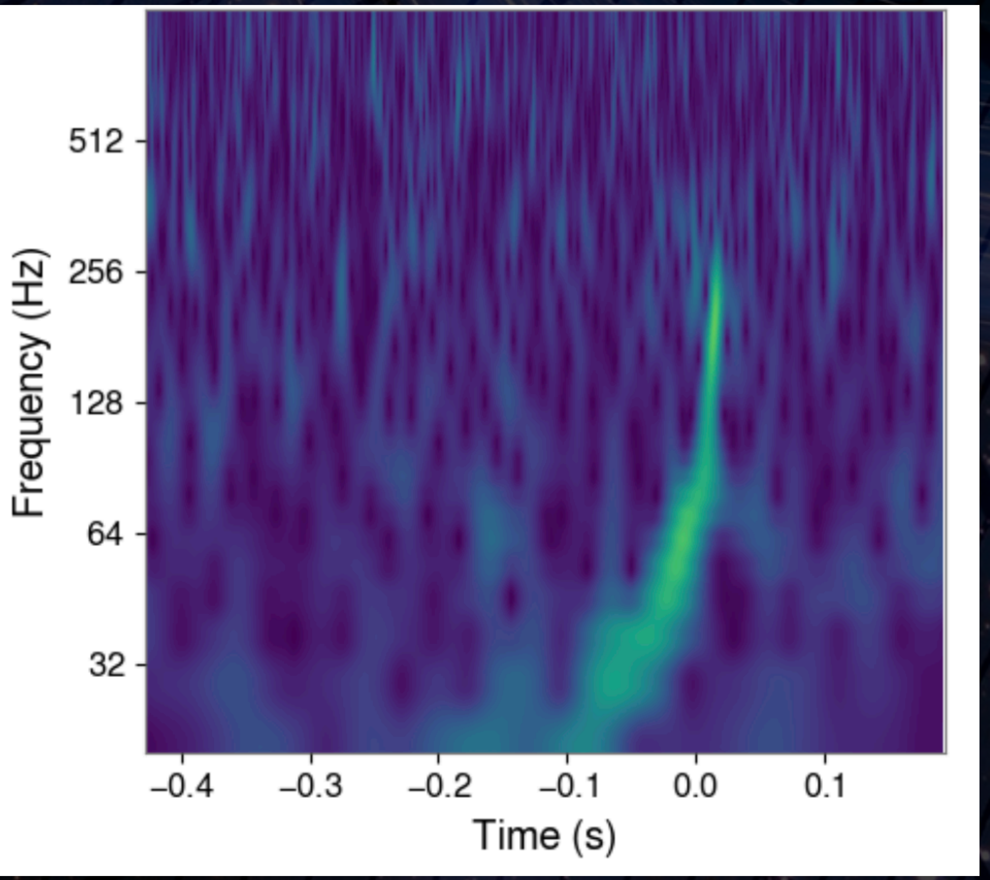
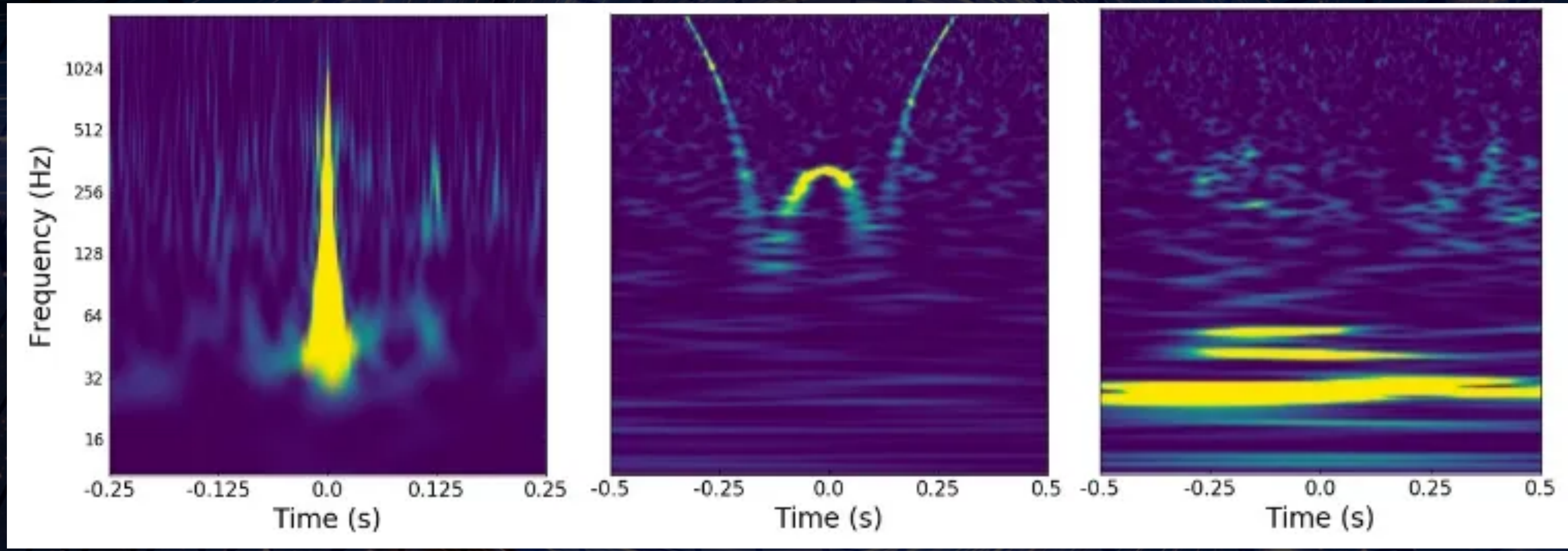
BAYESWAVE MODELS

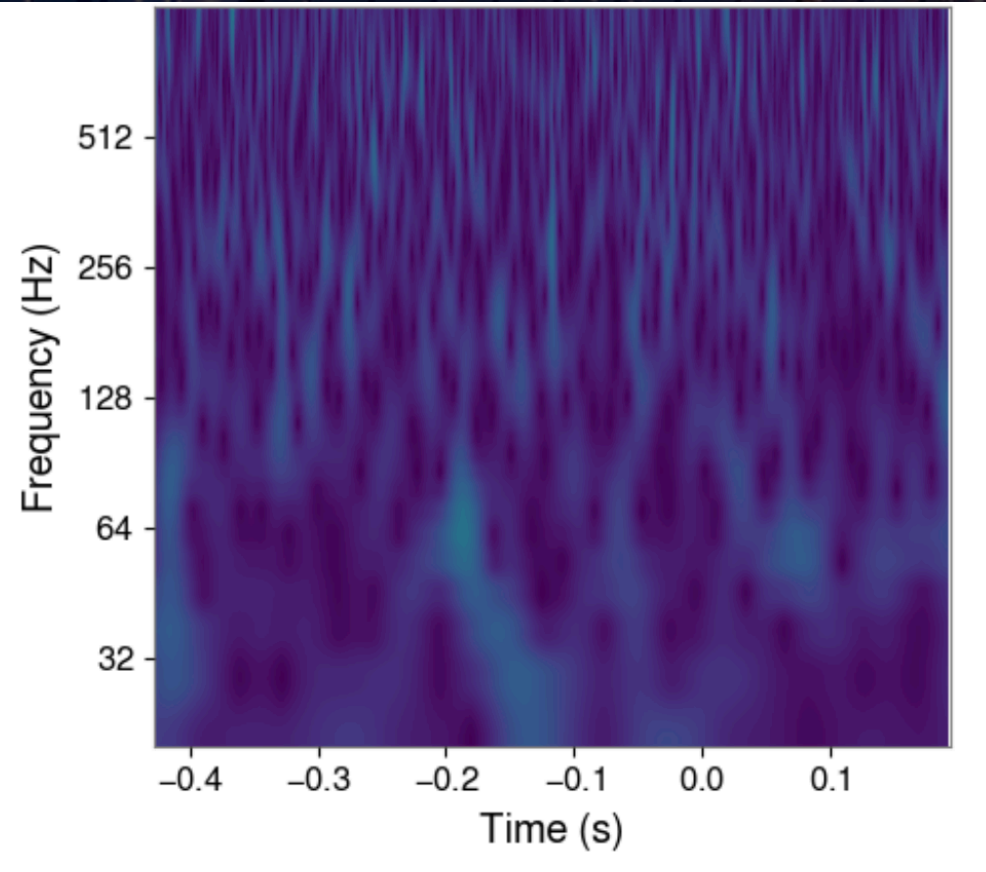
- Reconstructing transient features with three independent models:
  - Signal plus Gaussian-noise model, $\mathscr{S}$
  - Glitch plus Gaussian-noise model, $\mathscr{G}$
  - Gaussian-noise only model, $\mathscr{N}$



Signal



Glitches



Gaussian noise

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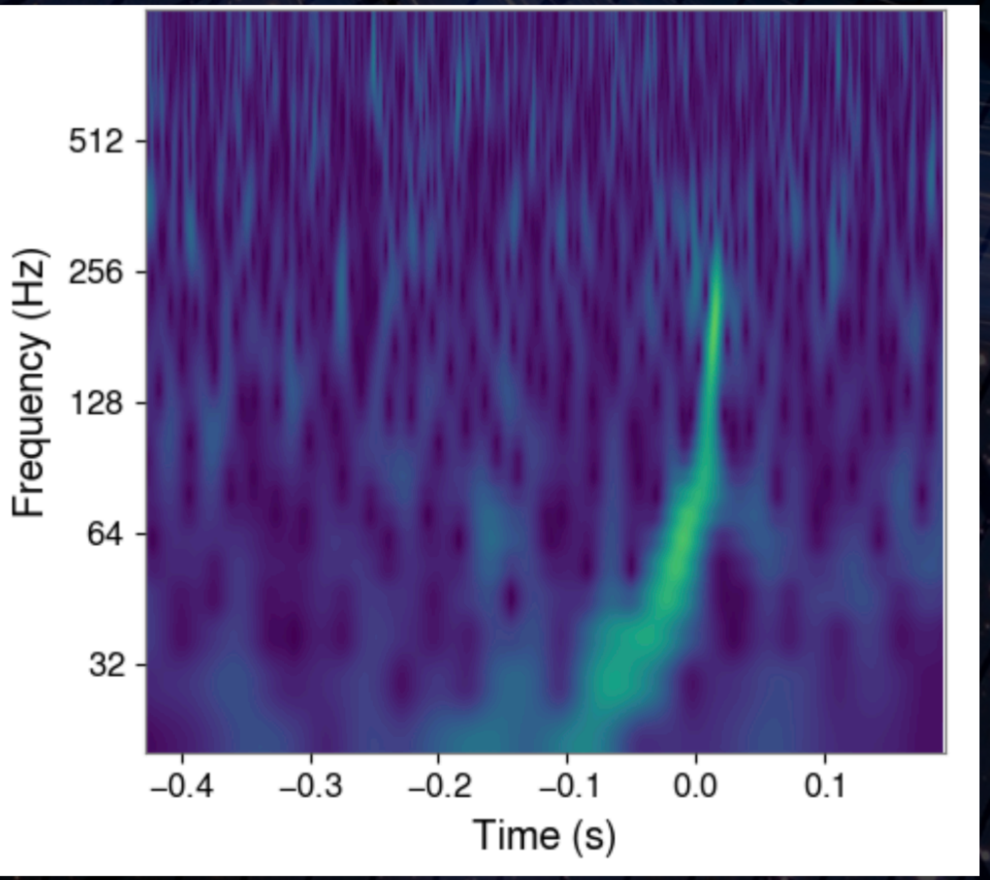

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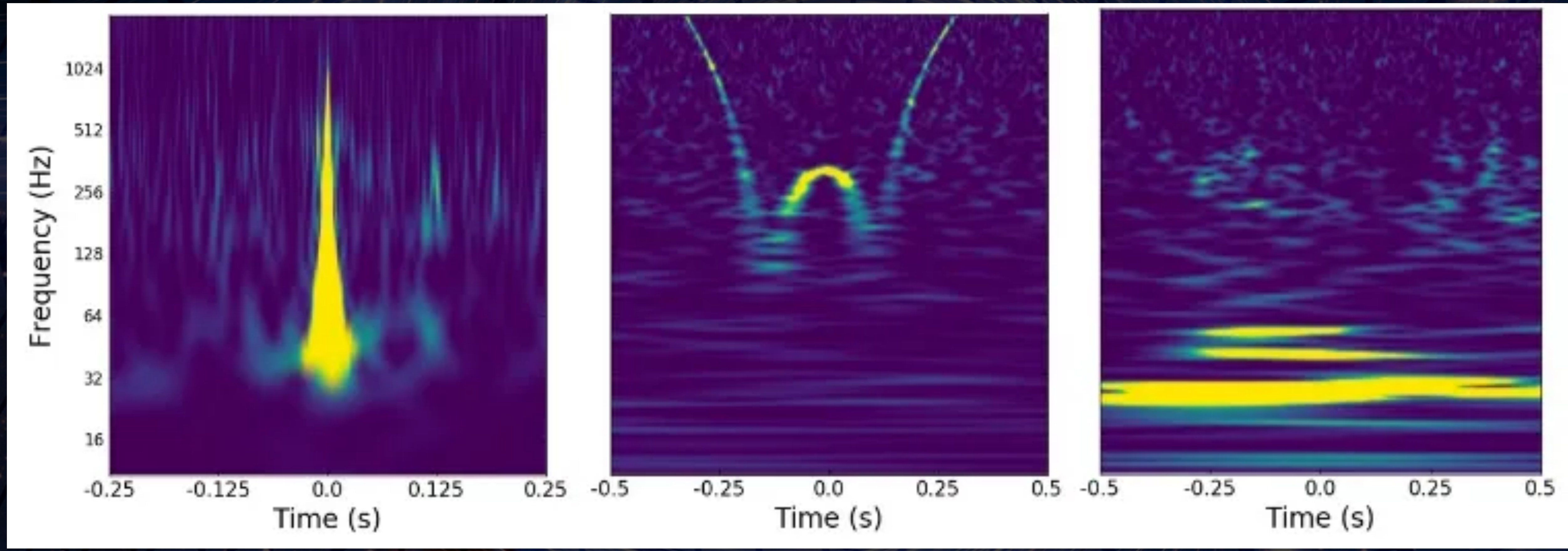

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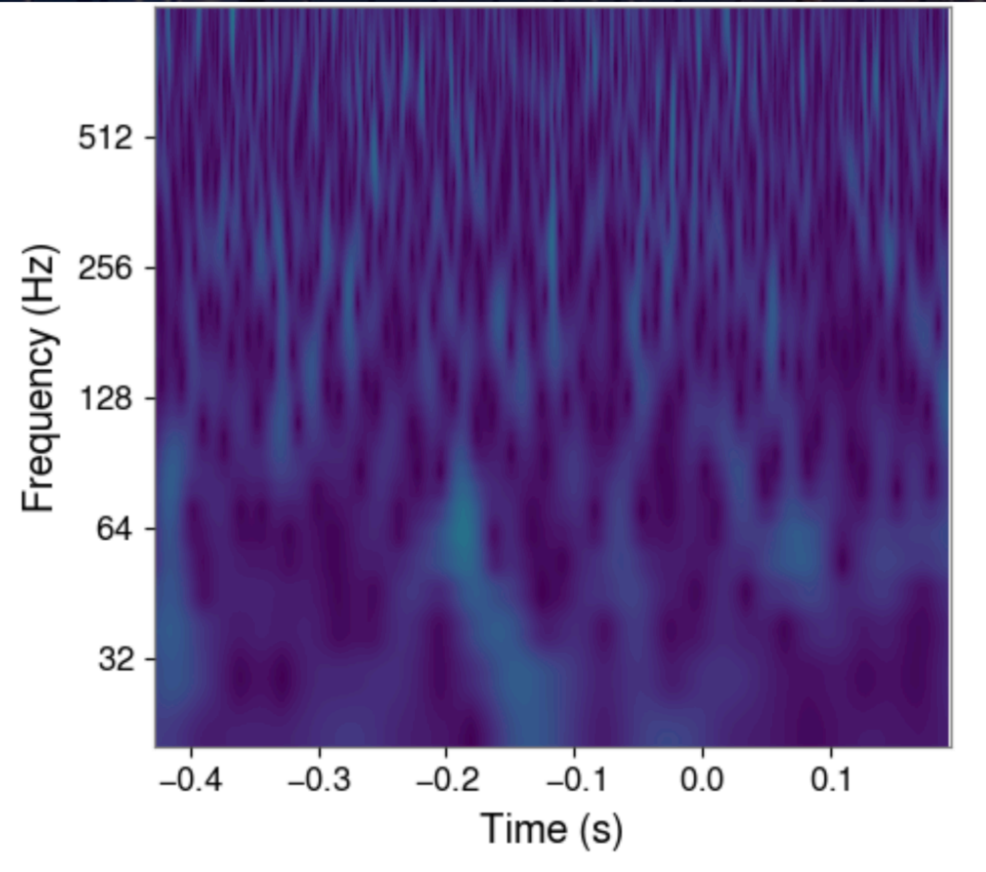

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BAYESWAVE **SIGNAL** MODELS

Λ : sine-Gaussian wavelet

ϵ : ellipticity

N : number of wavelets

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BAYESWAVE **SIGNAL** MODELS

Elliptical polarisation, E

$$\tilde{h}_+ = \sum_{n=1}^{N} \Lambda\left(f;\, t_0^n, f_0^n, Q^n, A^n, \phi^n\right)$$

$$\tilde{h}_\times = i\epsilon \tilde{h}_+$$

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To avoid degeneracies with the glitch model

- Λ : sine-Gaussian wavelet
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WHY RELAXED POLARISATION (R) MODEL?

Cornish et al., Phys. Rev. D 103, 044006 (2021)

WHY **RELAXED POLARISATION** ($\mathcal{R}$) MODEL?

- E does not hold for CBCs with time-varying polarisations, e.g.
  - Distinctive higher-order modes
  - Spin-precessing
- Other transient signals like supernovae are also generally unpolarised

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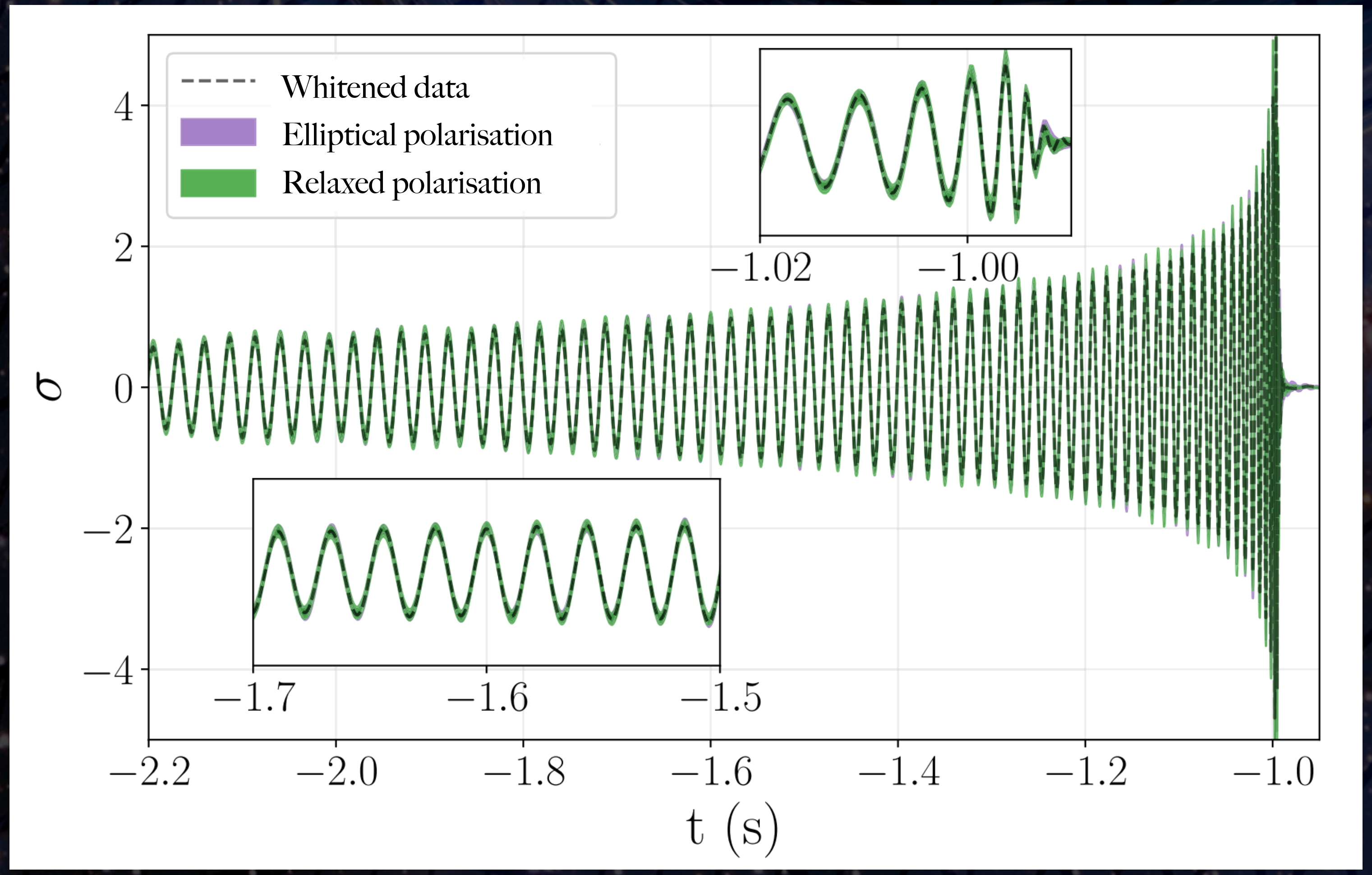

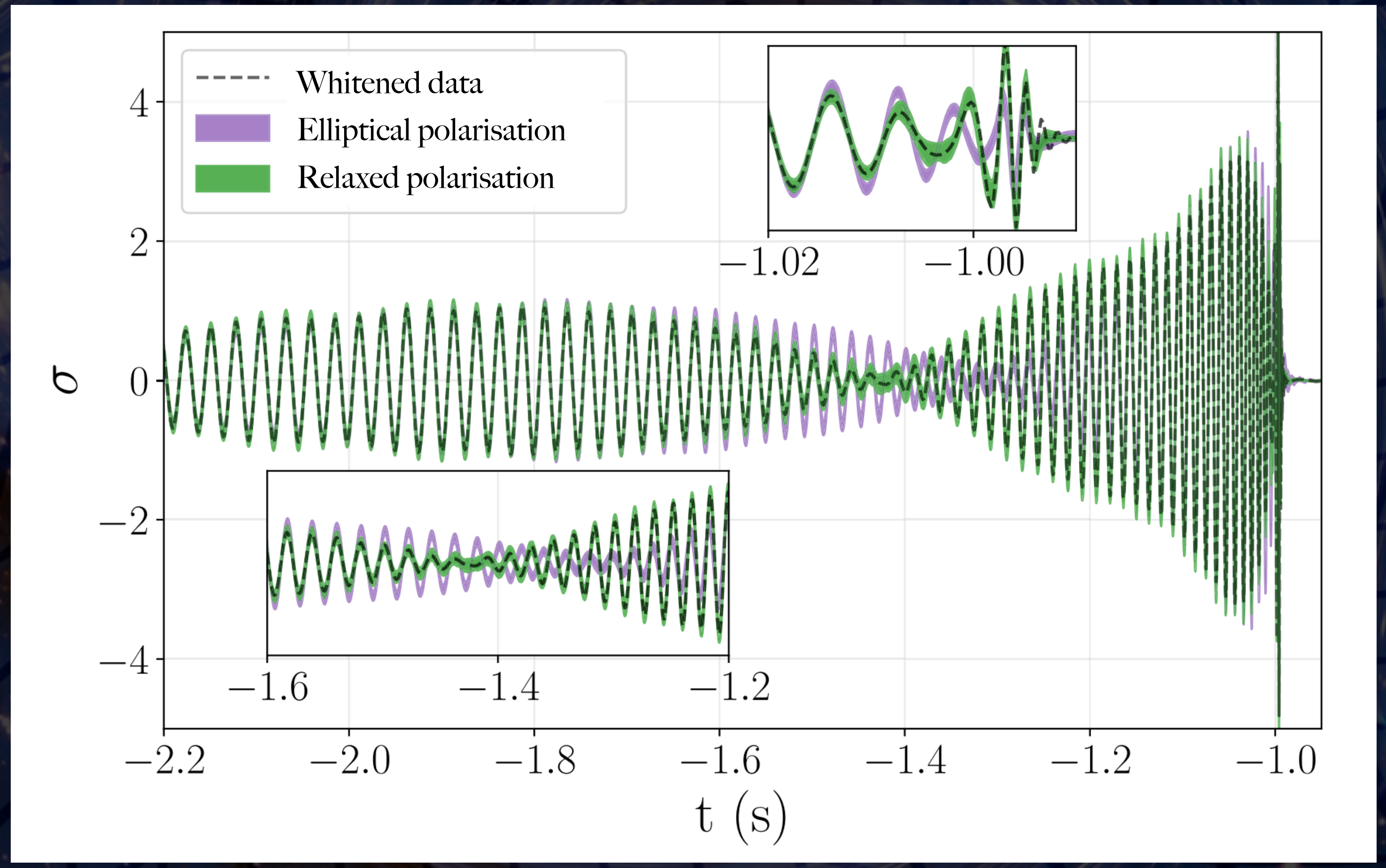

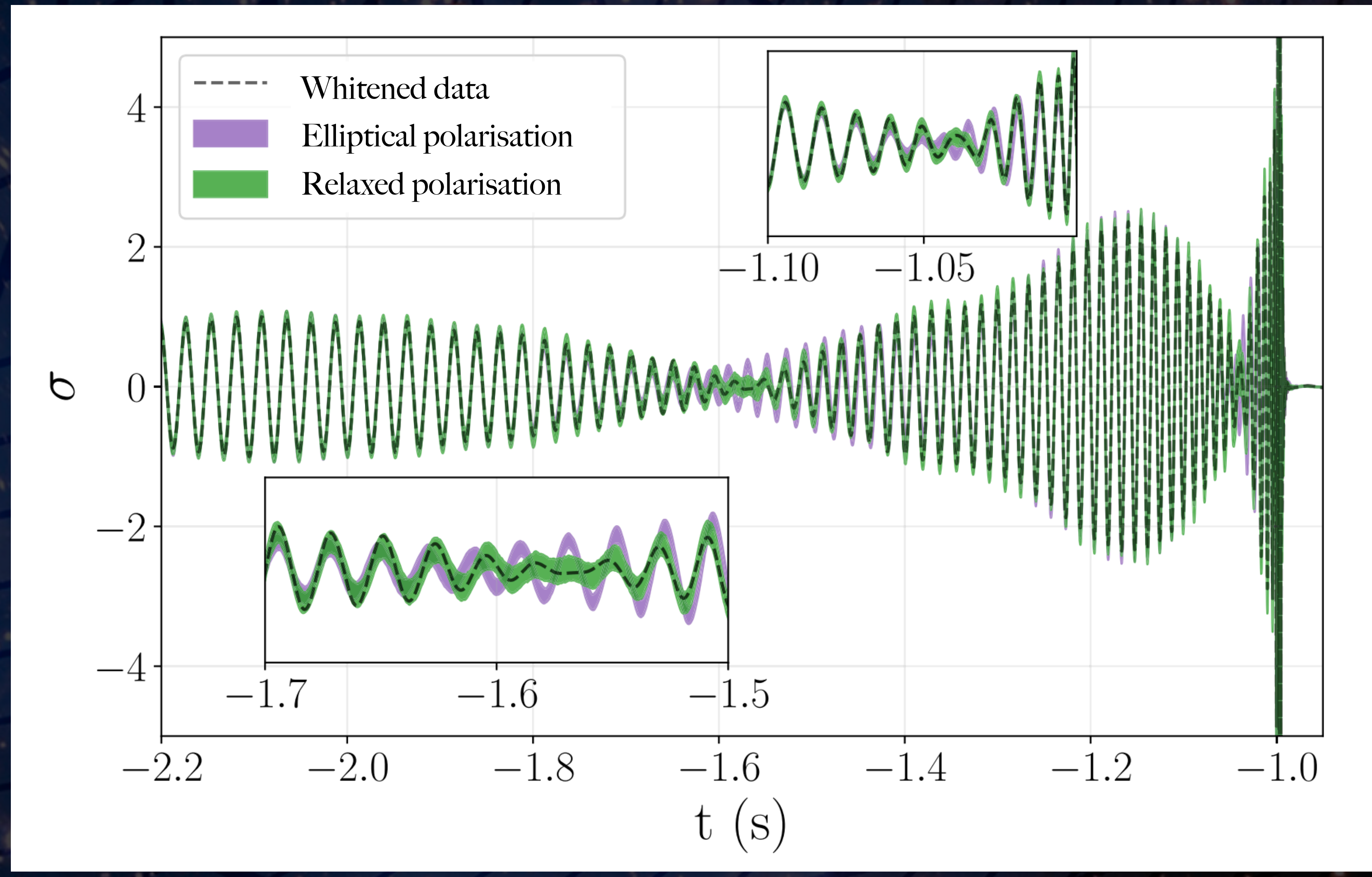

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PART 1

ELLIPTICAL (E) VS. RELAXED (R)

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Is there a preferred model?

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ELLIPTICAL (E) VS. RELAXED (R)

How well do the E and R polarisation models represent elliptical and nonelliptical GW signals?

Is there a preferred model?

Is the preference affected by the size of detector network?

TWO INJECTION SETS

- 200 injections
- High mass ratio $40M_{\odot} - 8M_{\odot}$
- High network signal-to-noise ratio: SNR ~ 50 (HLV)
- Uniform sky location and polarisation angle
- **Injected into** simulated detector noise

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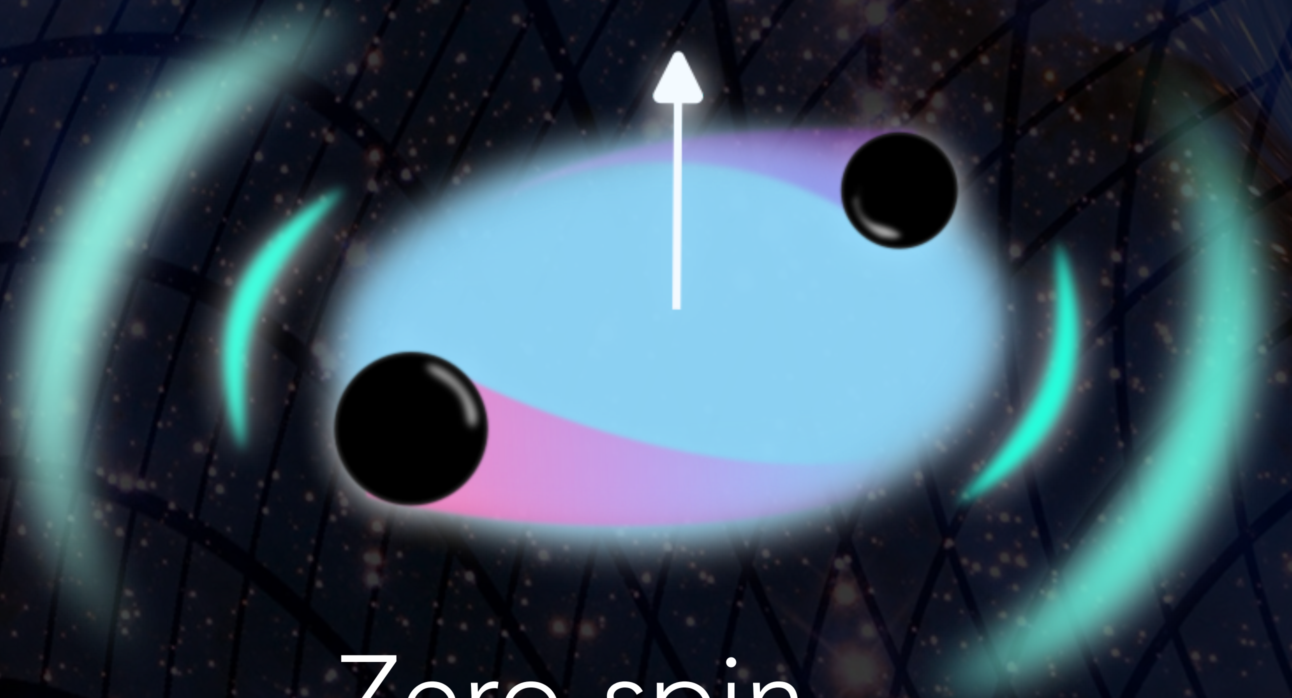
Nonprecessing (elliptical)



Zero-spin

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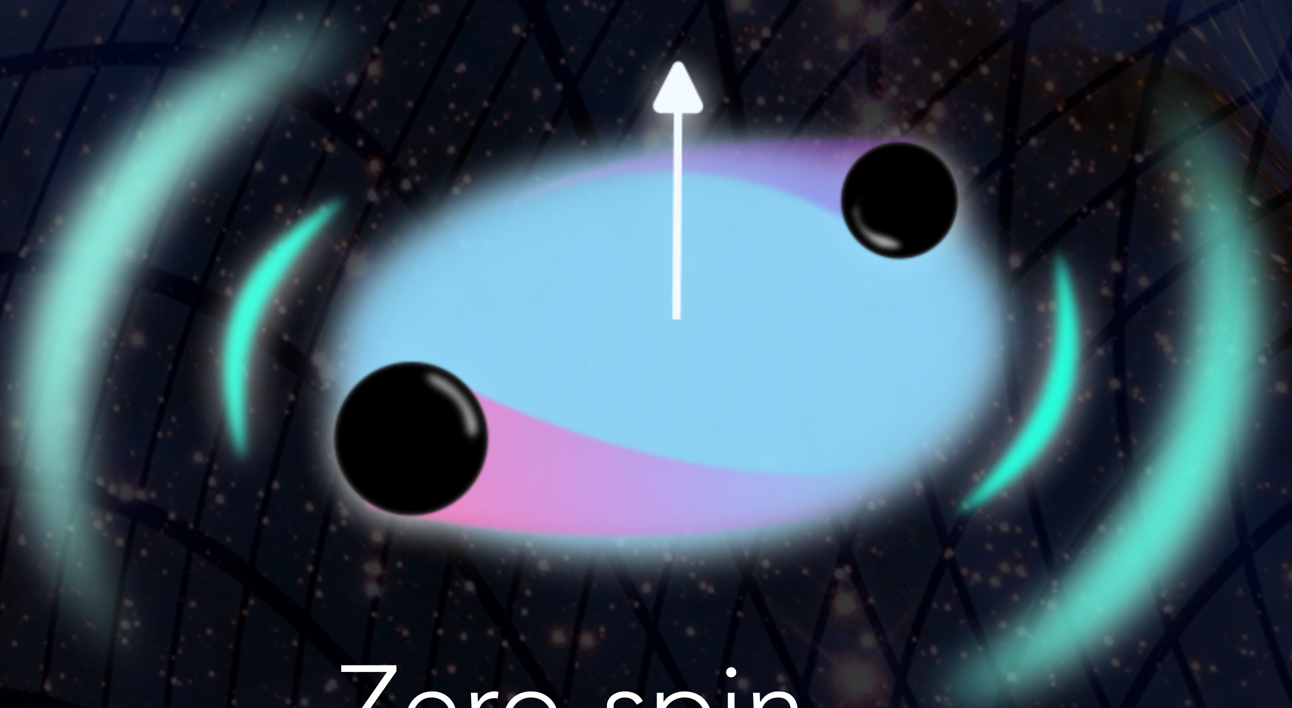
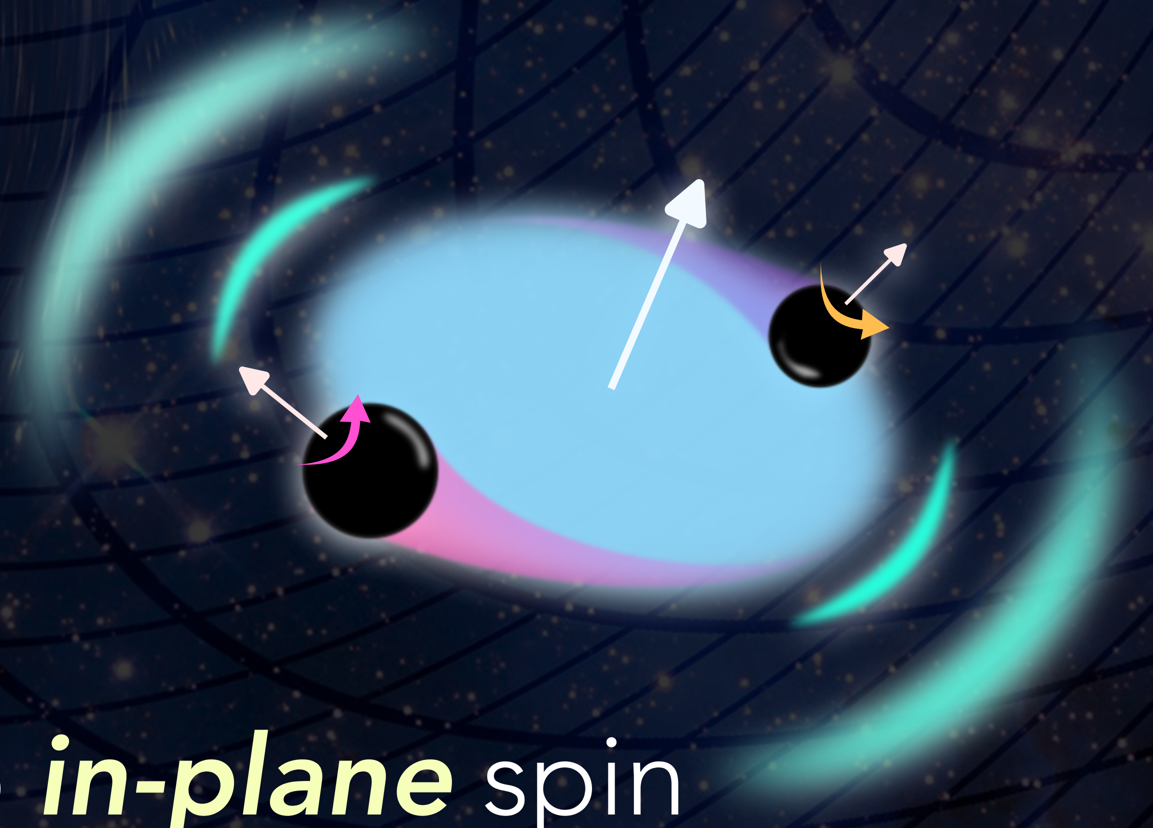


Zero-spin

Precessing (nonelliptical)



Non-zero ***in-plane*** spin

ELLIPTICAL (E) VS. RELAXED (R): FIGURE OF MERITS

Bayes Factor

$$\ln \mathscr{B}_{R,E} = \ln p(\vec{s}\,|\,R) - \ln p(\vec{s}\,|\,E)$$

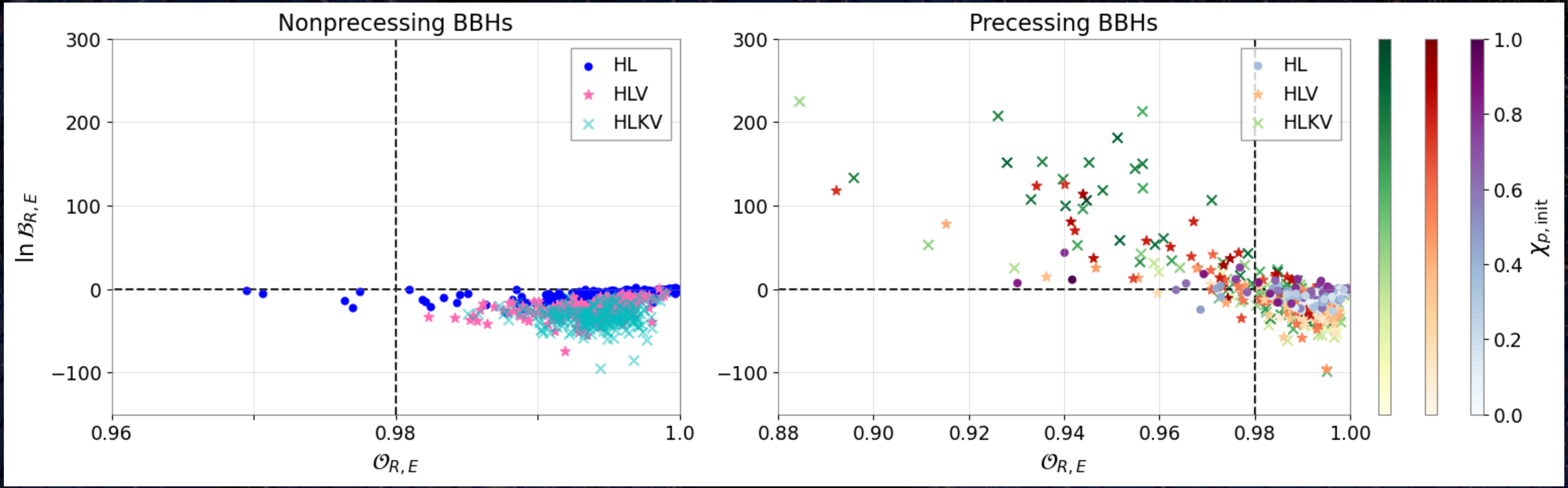
Network overlap (i.e. match)

$$\mathcal{O}_{R,E} = \frac{\sum_i \left(h_R^i \mid h_E^i\right)}{\sqrt{\sum_i \left(h_R^i \mid h_R^i\right) \sum_i \left(h_E^i \mid h_E^i\right)}}$$

where h^i is the *BayesWave*-recovered waveform for the i-th detector

ELLIPTICAL (E) VS. RELAXED (R)

Non-precessing BBHs



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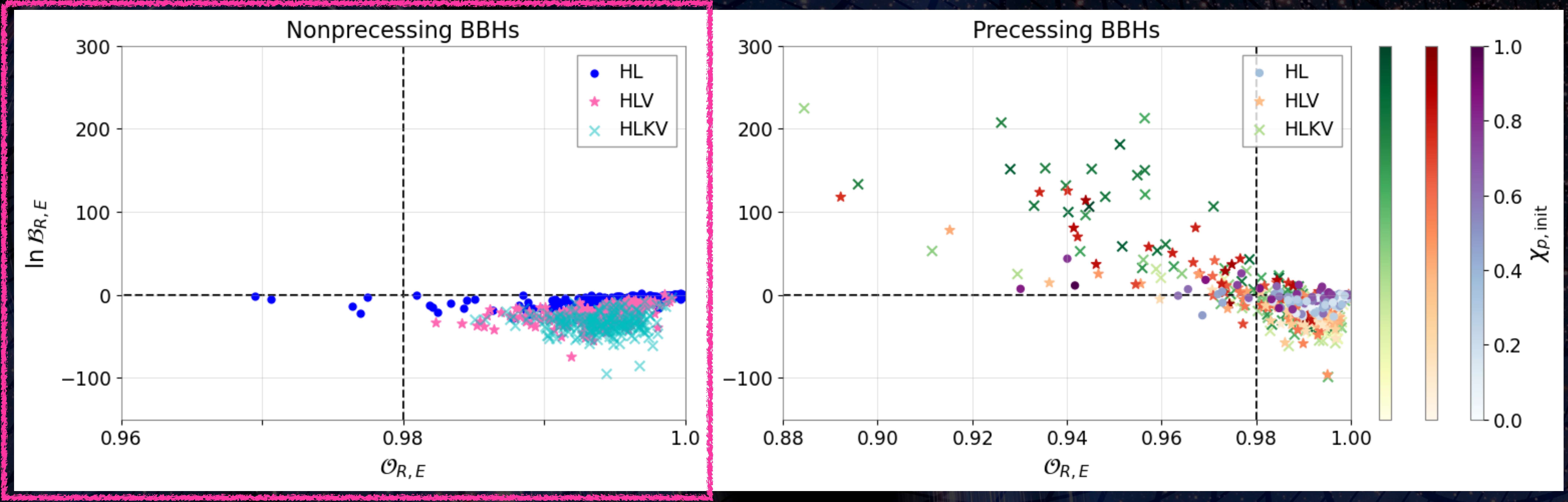

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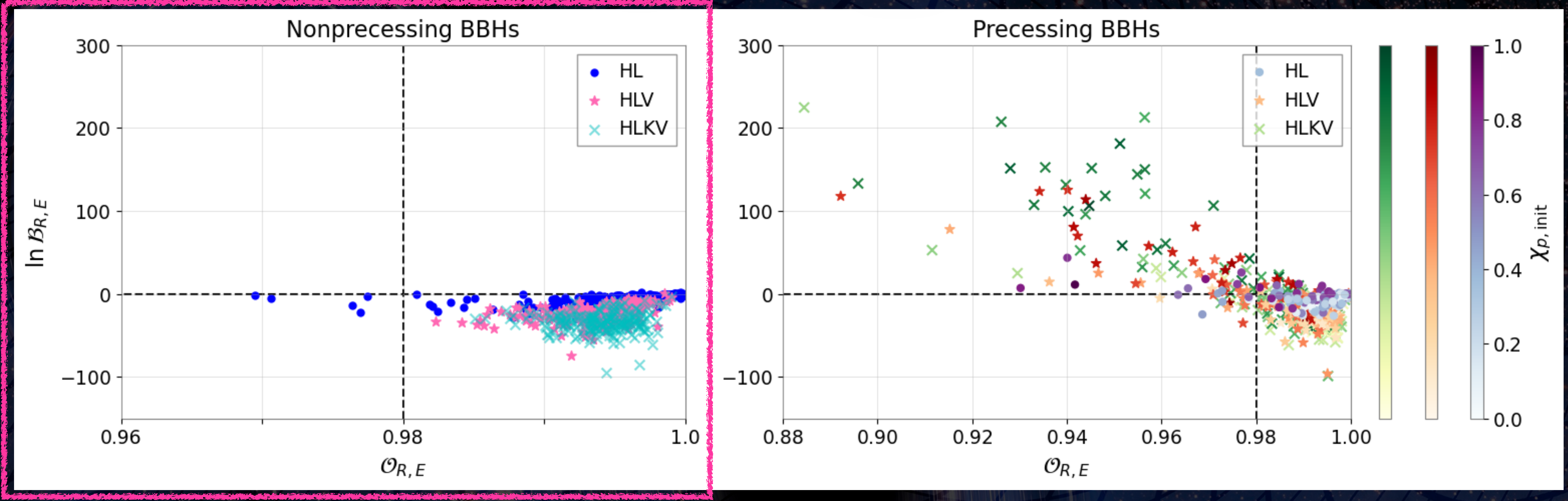
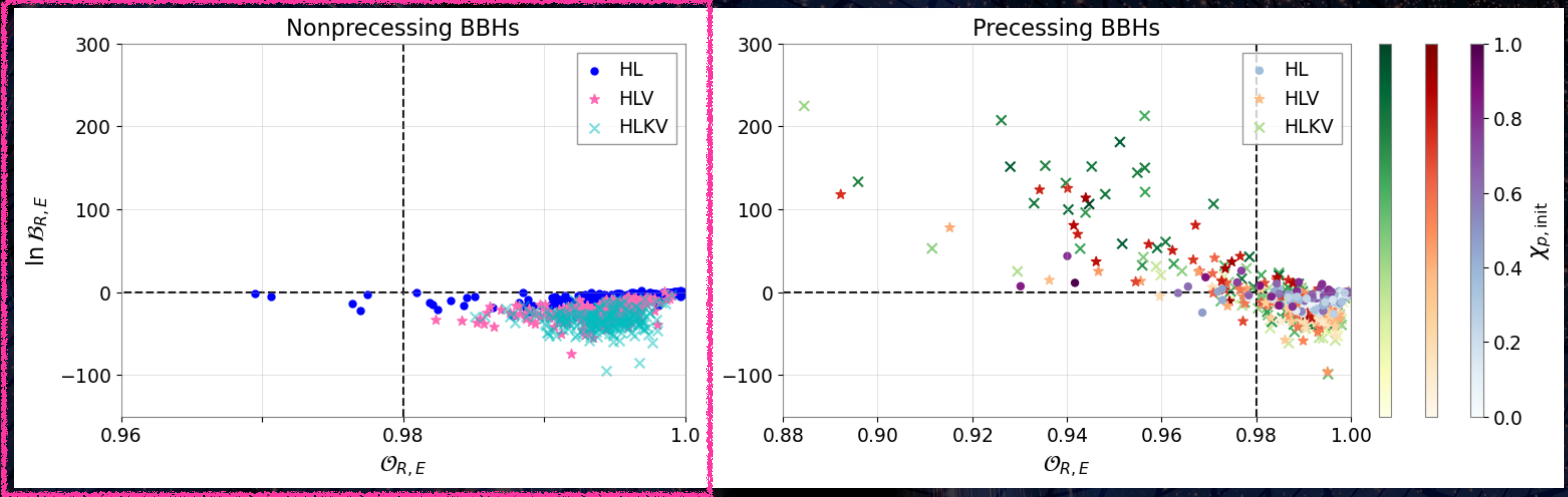


$$\text{Evidence of model } \mathscr{M} \simeq \text{Likelihood} \times \frac{\Delta V_{\mathscr{M}}}{V_{\mathscr{M}}}$$

ELLIPTICAL (E) VS. RELAXED (R)

Non-precessing BBHs

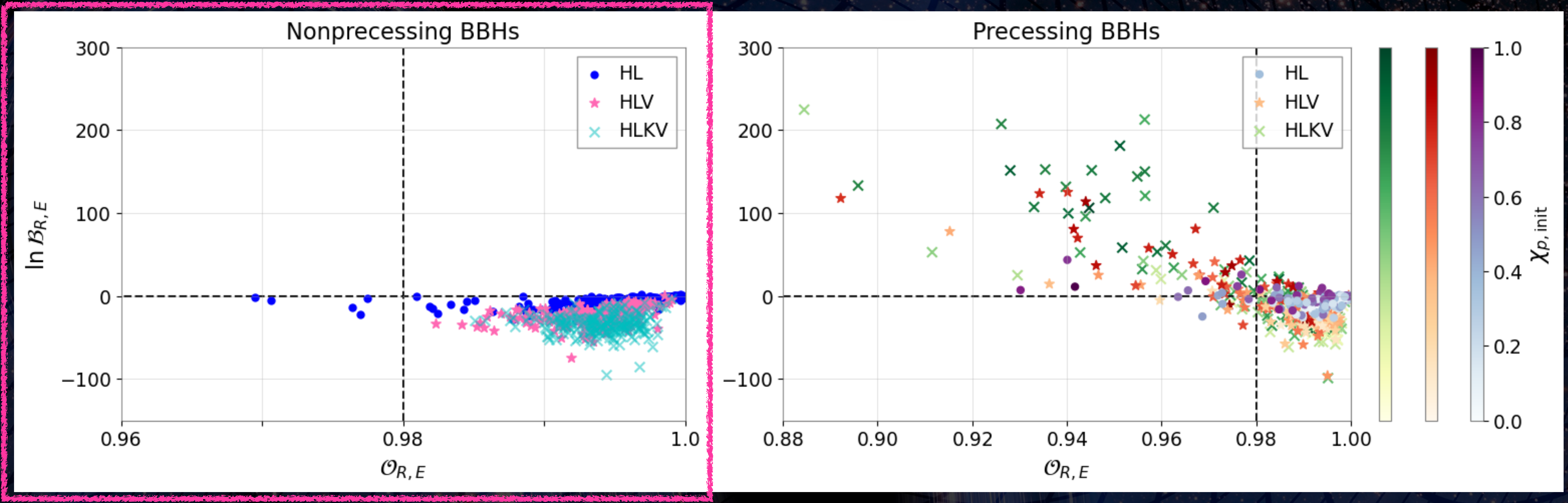


Evidence of model $\mathscr{M} \simeq$ Likelihood $\times \dfrac{\Delta V_{\mathscr{M}}}{V_{\mathscr{M}}}$

Bayes factor, $\mathscr{B}_{R,E} \simeq$ Likelihood ratio $\times \dfrac{\Delta V_R}{\Delta V_E}\dfrac{V_E}{V_R}$

ELLIPTICAL (E) VS. RELAXED (R)

Non-precessing BBHs



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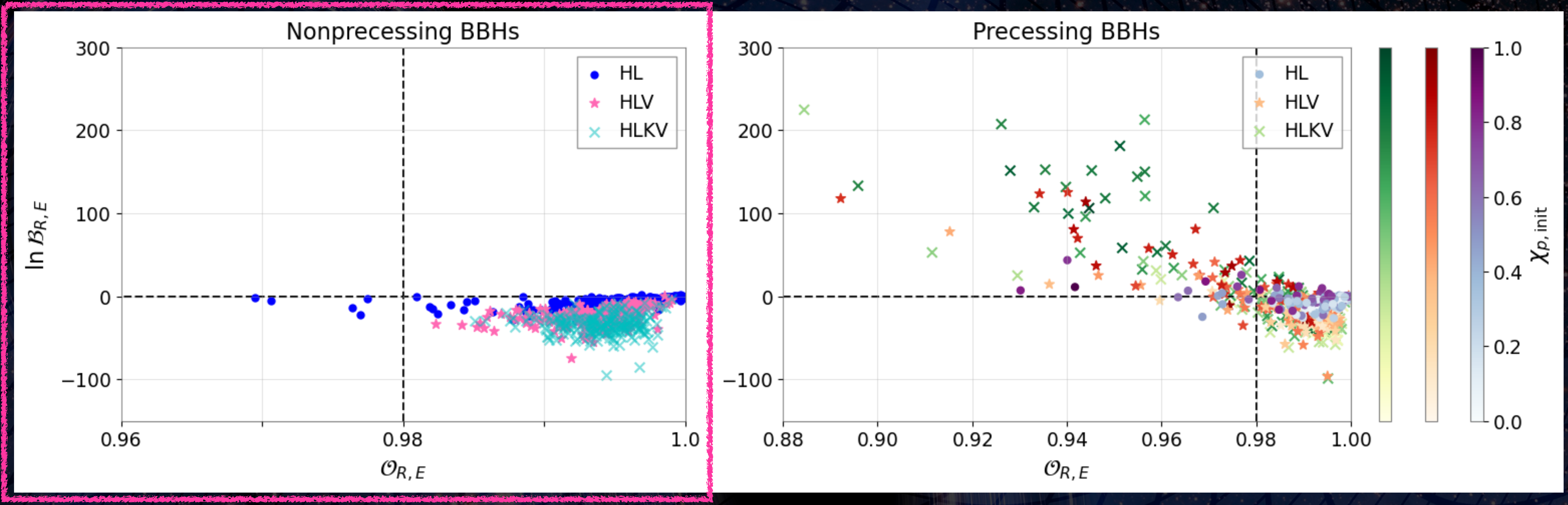
Bayes factor, $\mathscr{B}_{R,E} \simeq$ Likelihood ratio $\times \dfrac{\Delta V_R}{\Delta V_E} \dfrac{V_E}{V_R}$

$\Delta V_{\mathscr{M}}$: Posterior volume

$V_{\mathscr{M}}$: Total parameter space volume of model $\mathscr{M}$

ELLIPTICAL (E) VS. RELAXED (R)

Non-precessing BBHs (cont.)



$\mathcal{O}_{R,E} \sim 1 \Rightarrow$ Appx. equal likelihood and posterior volumes (i.e. $\Delta V_E \approx \Delta V_R$)

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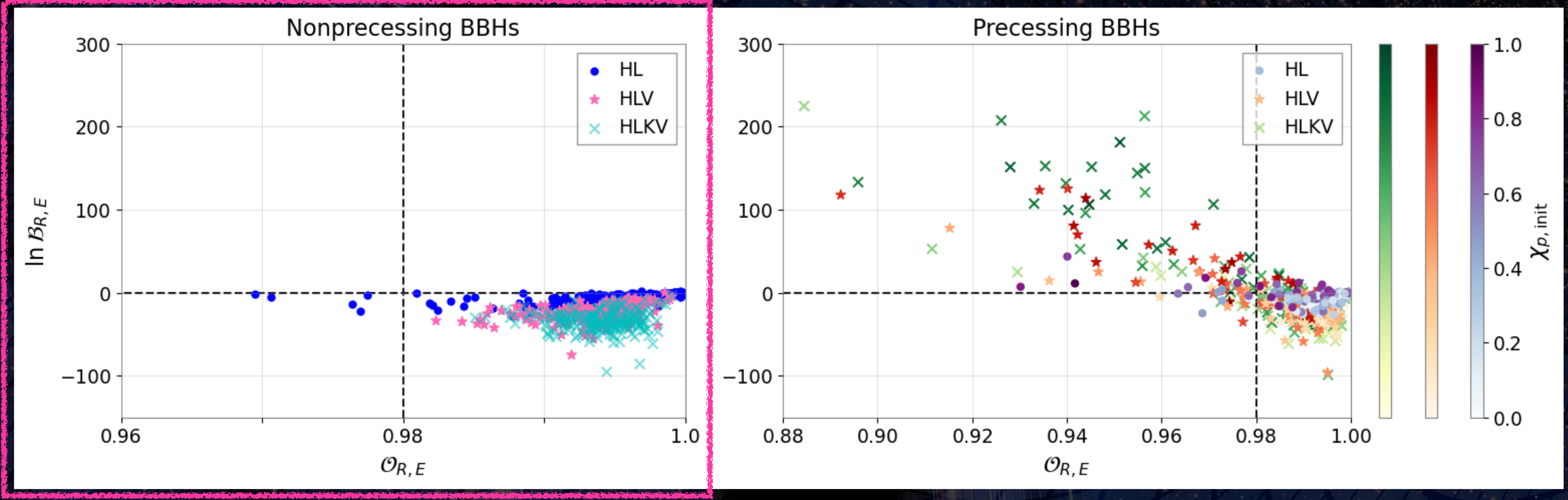
ELLIPTICAL (E) VS. RELAXED (R)

Non-precessing BBHs (cont.)



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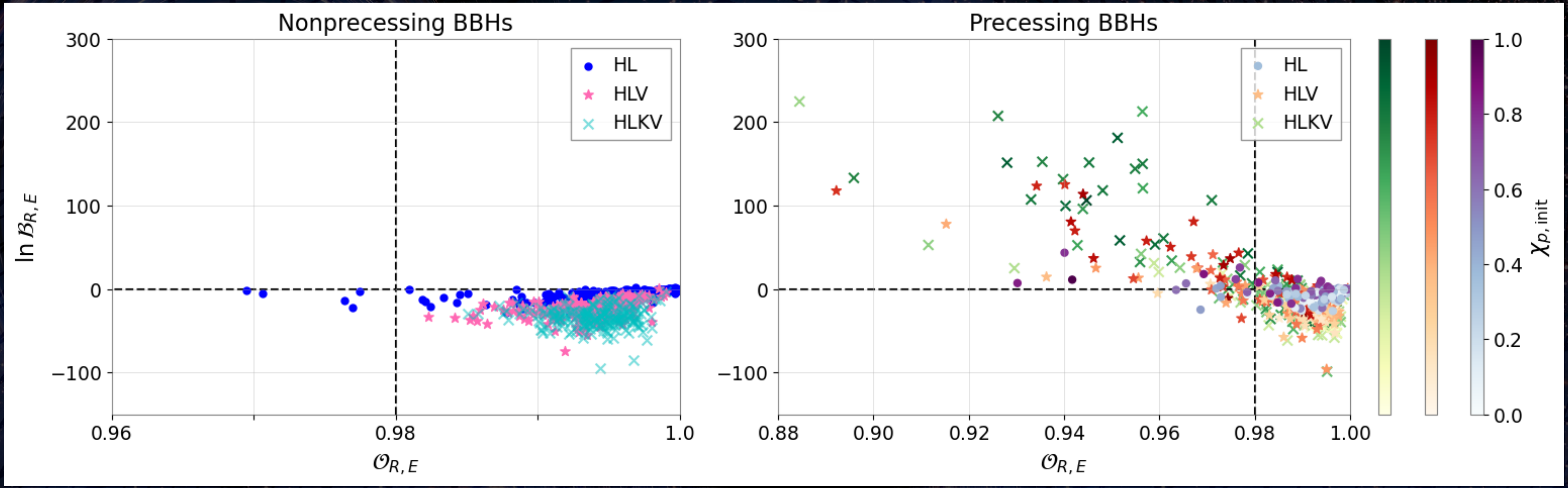
Bayes factor, $\mathcal{B}_{\mathbf{R,E}} \simeq 1 \times 1 \times \frac{V_\mathrm{E}}{V_\mathrm{R}} \Rightarrow \ln \mathcal{B}_{R,E} \lesssim 0$ for $V_E < V_R$

$\Delta V_{\mathcal{M}}$: Posterior volume
$V_{\mathcal{M}}$: Total parameter space volume of model $\mathcal{M}$

ELLIPTICAL (E) VS. RELAXED (R)

Precessing BBHs



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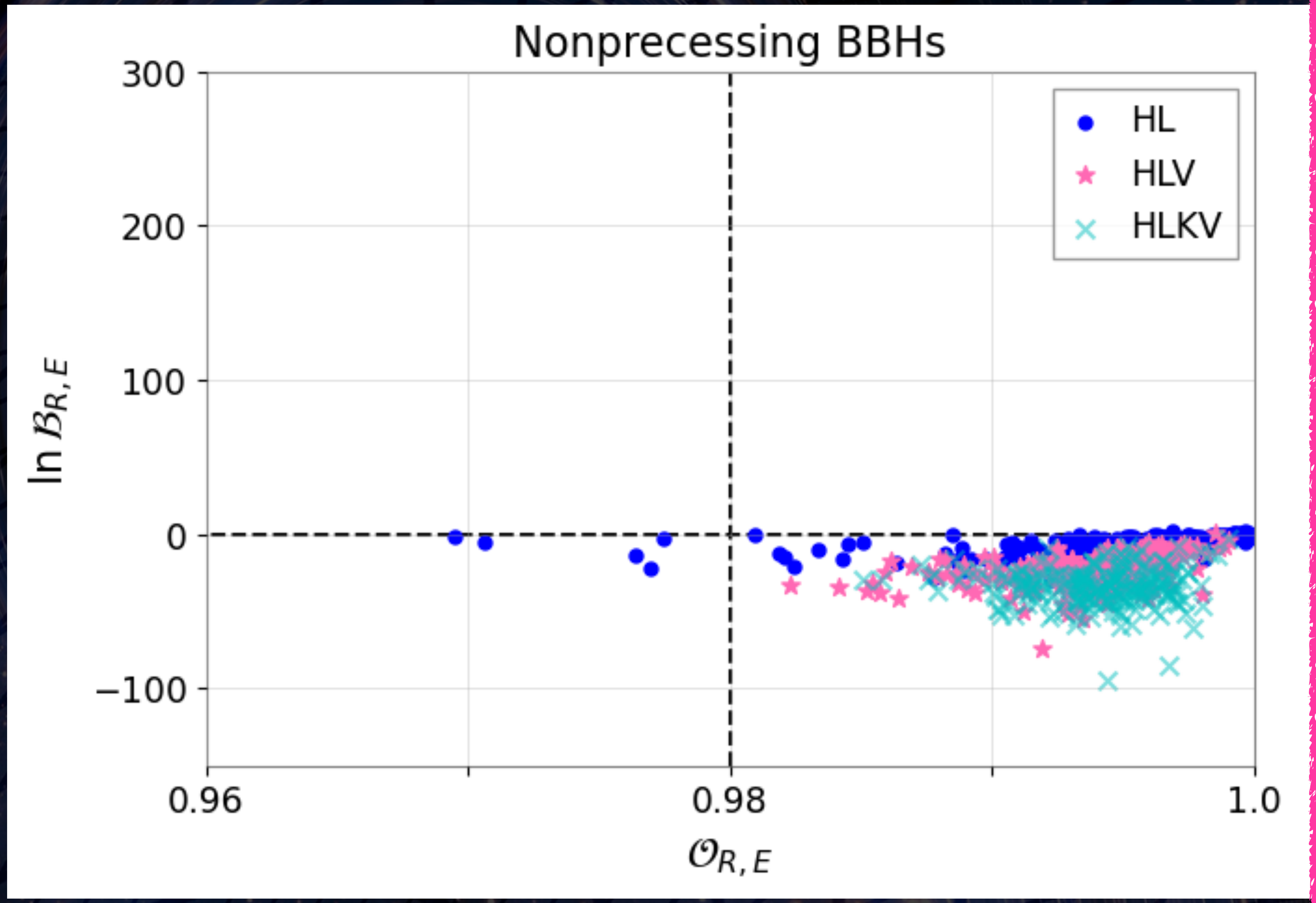

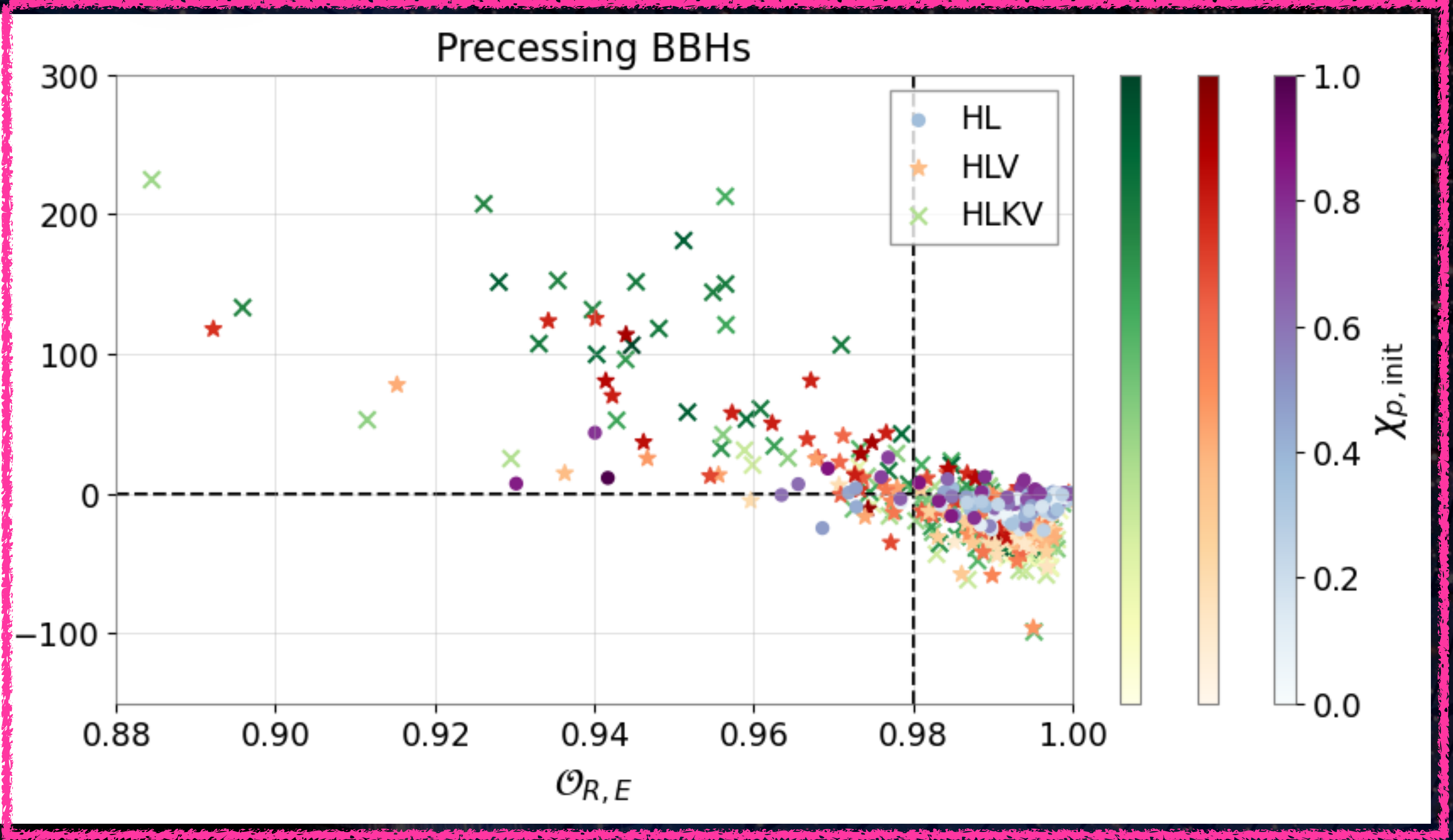

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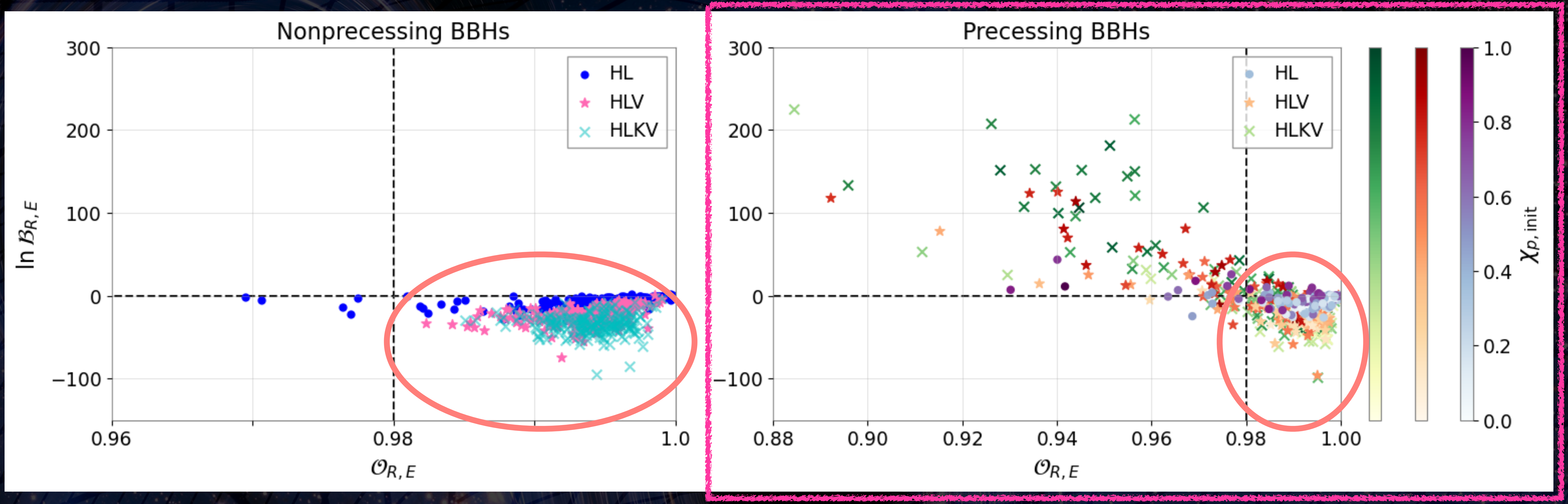
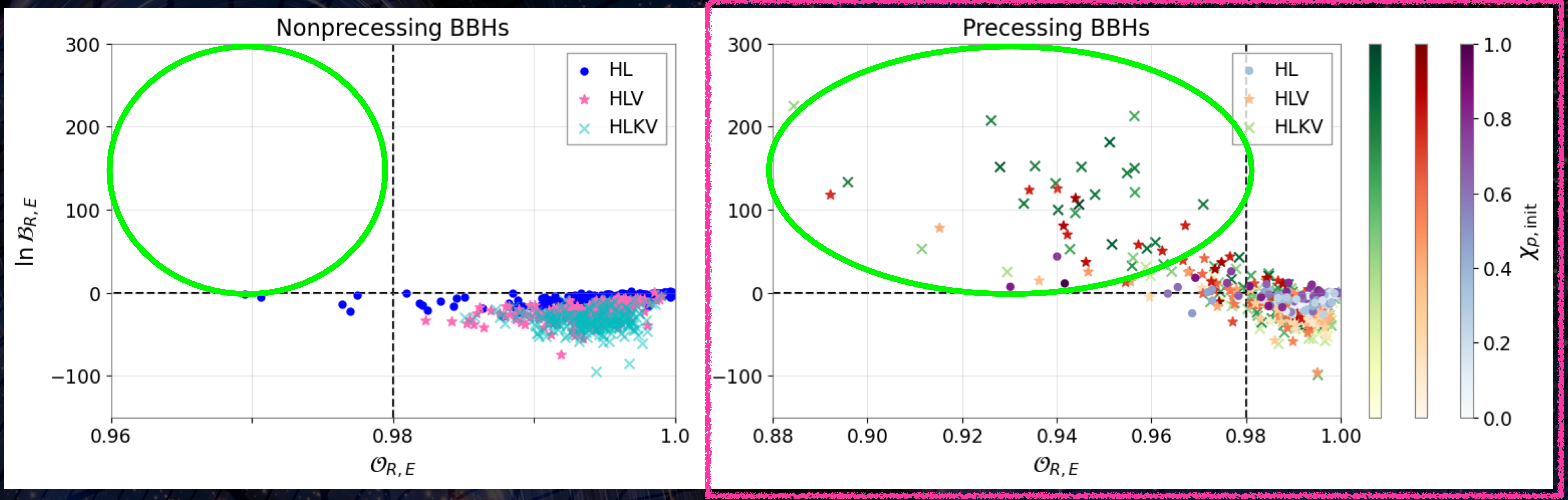


$\mathcal{O}_{R,E} \geq 0.98 \sim 1 \Rightarrow$ Similar behaviour (i.e. $\ln \mathcal{B}_{R,E} \lesssim 0$) for both non-precessing and precessing BBHs

ELLIPTICAL (E) VS. RELAXED (R)

Precessing BBHs (cont.)



(1) $\mathcal{O}_{R,E} < 0.98 \Rightarrow \ln \mathcal{B}_{R,E} > 0$ for some precessing BBHs

Mostly high $\chi_{p,\mathrm{init}}$ events

(2) $\ln \mathcal{B}_{R,E}$ is more positive with larger detector networks

Better reconstruction of non-elliptical features with R

KEY TAKEAWAYS

ELLIPTICAL (E) VS. RELAXED (R)

- Non-precessing BBHs are equally well-represented by both E and R, so if we had to choose one…
  Occams Razor says to pick the simpler one (E)
- Same for *most* precessing BBHs, BUT…
  High in-plane spin $\Rightarrow$ likely to have more precession, so generally better represented by R

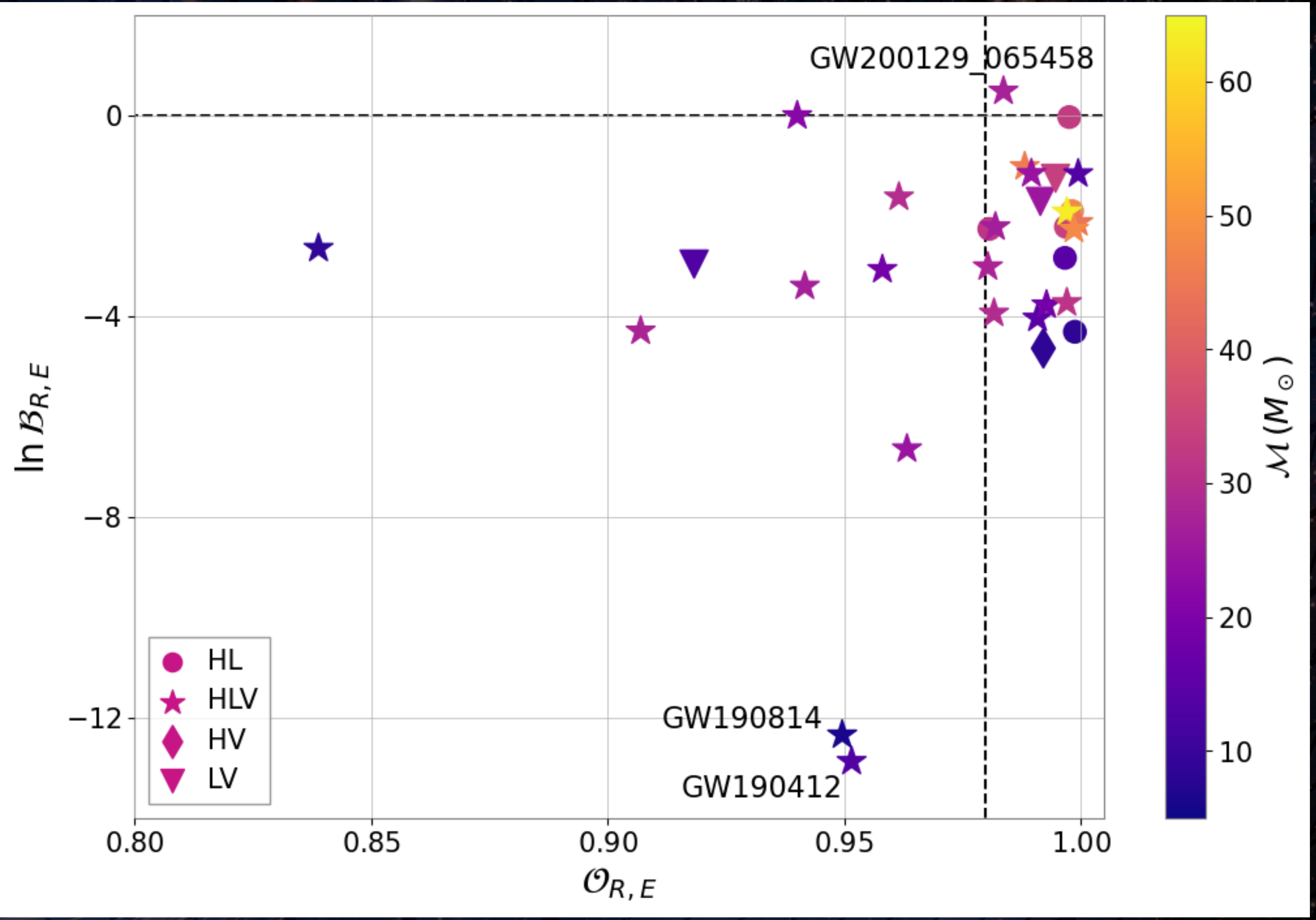
ELLIPTICAL (*E*) VS. RELAXED (*R*)

with real data - O3 events

$\mathcal{O}_{R,E} \gtrsim 0.90$

$\Downarrow$

E and *R* reconstructions are comparable



GWTC-2: Phys. Rev. X 11, 021053 (2021),
GWTC-3: Phys. Rev. X 13 041039 (2023).

ELLIPTICAL (E) VS. RELAXED (R)

with real data - O3 events

$\mathcal{O}_{R,E} \gtrsim 0.90$

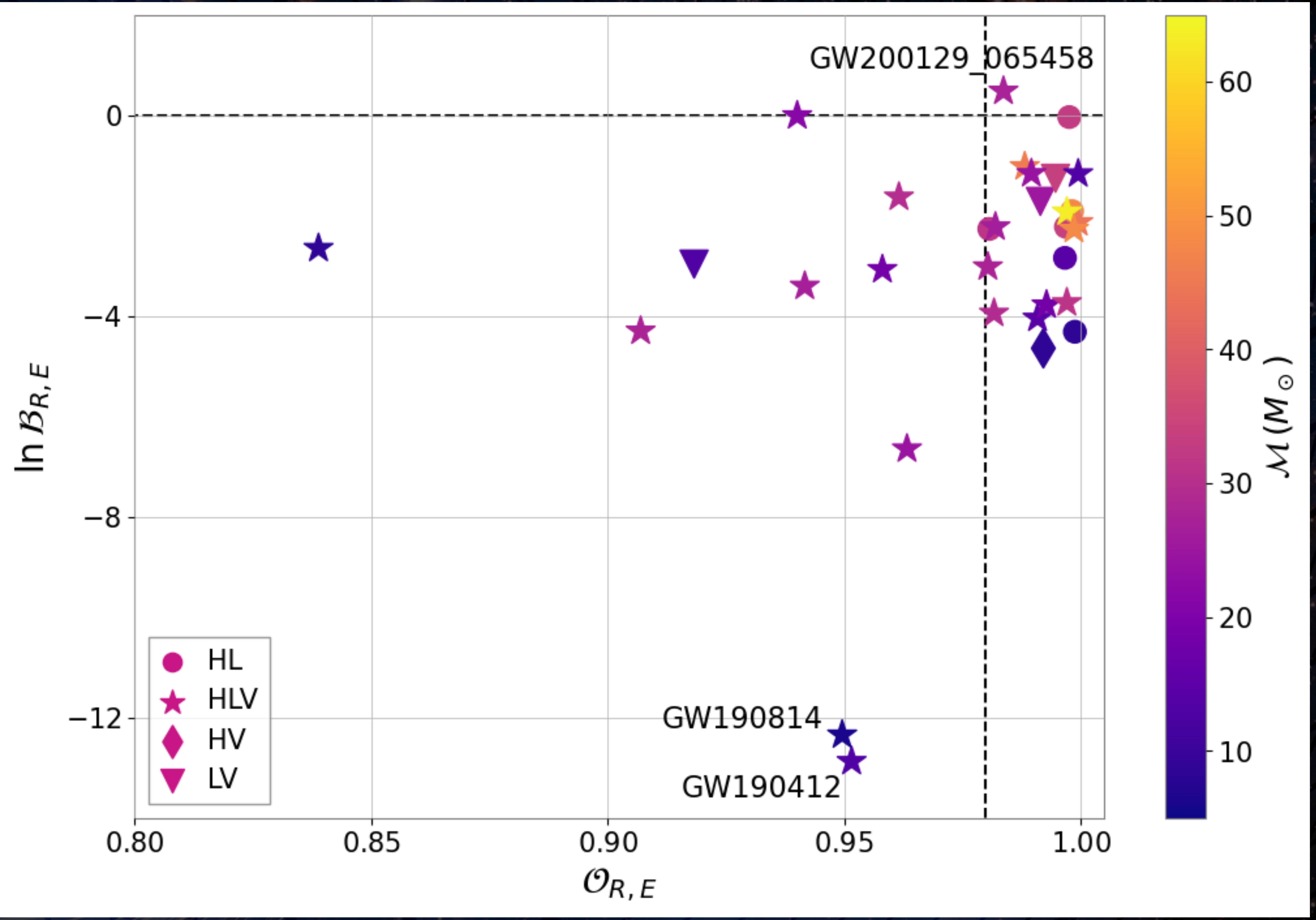
$\Downarrow$

E and R reconstructions are comparable

$\ln \mathcal{B}_{R,E} < 0$

$\Downarrow$

O3 events are generally prefers the elliptical polarisation model E



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GWTC-3: Phys. Rev. X 13 041039 (2023).

PART 2

WHAT ELSE CAN THE RELAXED POLARISATION (R) MODEL DO?

- E assumes that the GW signal is elliptical by constraining $\tilde{h}_\times = i\epsilon\tilde{h}_+$

- R models $\tilde{h}_+$ and $\tilde{h}_\times$ separately
  i.e. no prior assumption of the polarisation structure

- So R can be used to **measure generic polarisation content**

STOKES PARAMETERS (IN LINEAR BASIS)

$$I = |\tilde{h}_+|^2 + |\tilde{h}_\times|^2$$

$$Q = |\tilde{h}_+|^2 - |\tilde{h}_\times|^2$$

$$U = \tilde{h}_+\tilde{h}^*_\times + \tilde{h}_\times\tilde{h}^*_+$$

$$V = i(\tilde{h}_+\tilde{h}^*_\times - \tilde{h}_\times\tilde{h}^*_+)$$

- I → Total intensity
- Q, U → Linear polarisation
- V → Circular polarisation

GWs are polychromatic

$\therefore I, Q, U, V$ are functions of frequency

FRACTIONAL POLARISATION

Linear fraction

$$F_L = \frac{\sqrt{Q^2 + U^2}}{I}$$

Circular fraction

$$F_C = \frac{V}{I}$$

(Total) degree of polarisation

$$F_T = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

FRACTIONAL POLARISATION

Linear fraction

$$F_L = \frac{\sqrt{Q^2 + U^2}}{I}$$

Circular fraction

$$F_C = \frac{V}{I}$$

(Total) degree of polarisation

$$F_T = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

I, Q, U, V are real numbers

$$I^2 \leq Q^2 + U^2 + V^2$$

$0 \leq F_{\mathscr{P}} \leq 1$ for $\mathscr{P} \in \{L, C, T\}$

MEASUREMENT ACCURACY:

ROOT MEAN SQUARED RESIDUALS, $\mathscr{R}_{\mathrm{RMS}}$

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- RMS residuals between injected and recovered $F_{\mathscr{P}}$

$$\mathscr{R}_{\mathrm{RMS}}(F_{\mathscr{P}}) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[F_{\mathscr{P},\mathrm{rec}}(f_i) - F_{\mathscr{P},\mathrm{inj}}(f_i)\right]^2}$$

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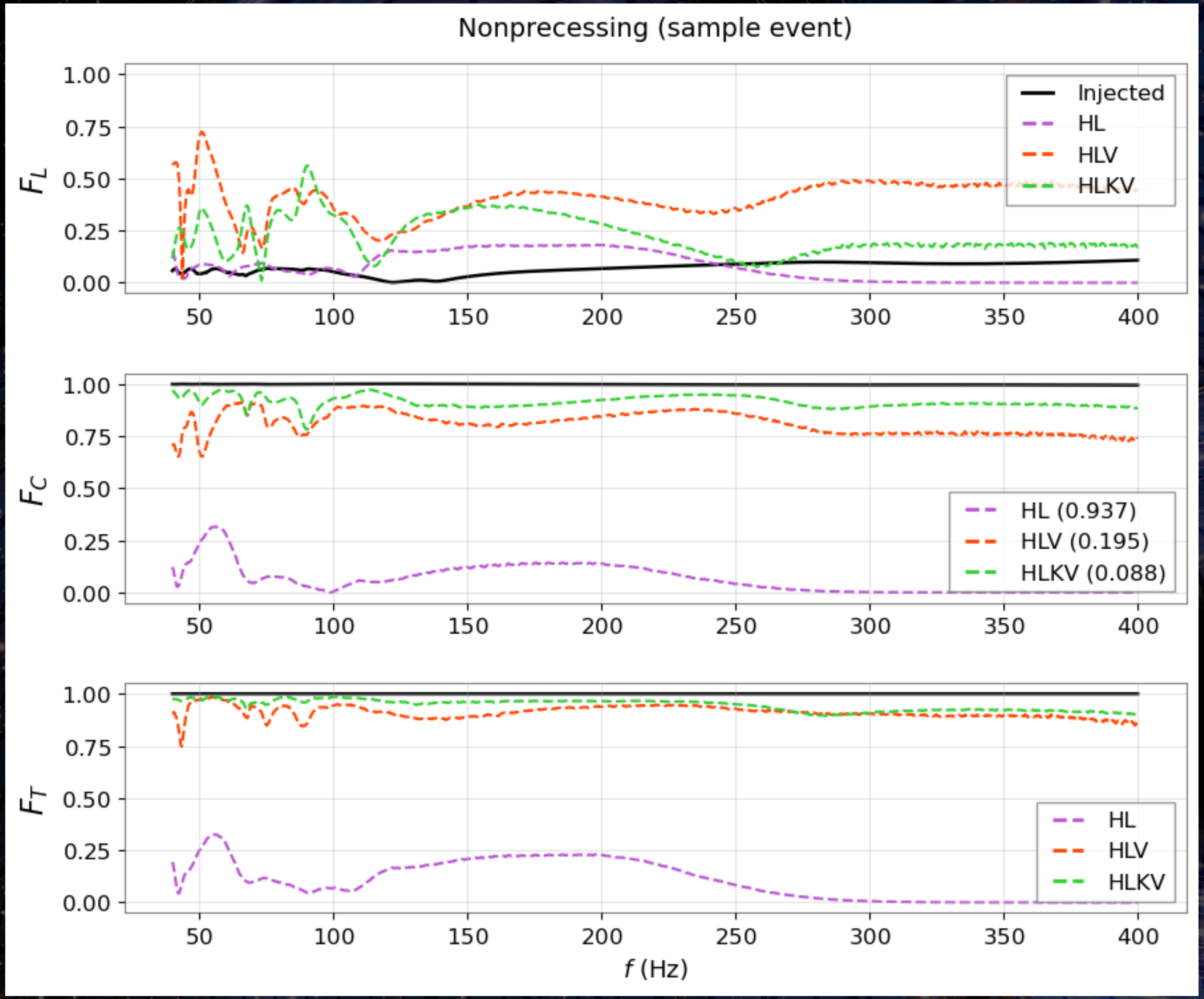
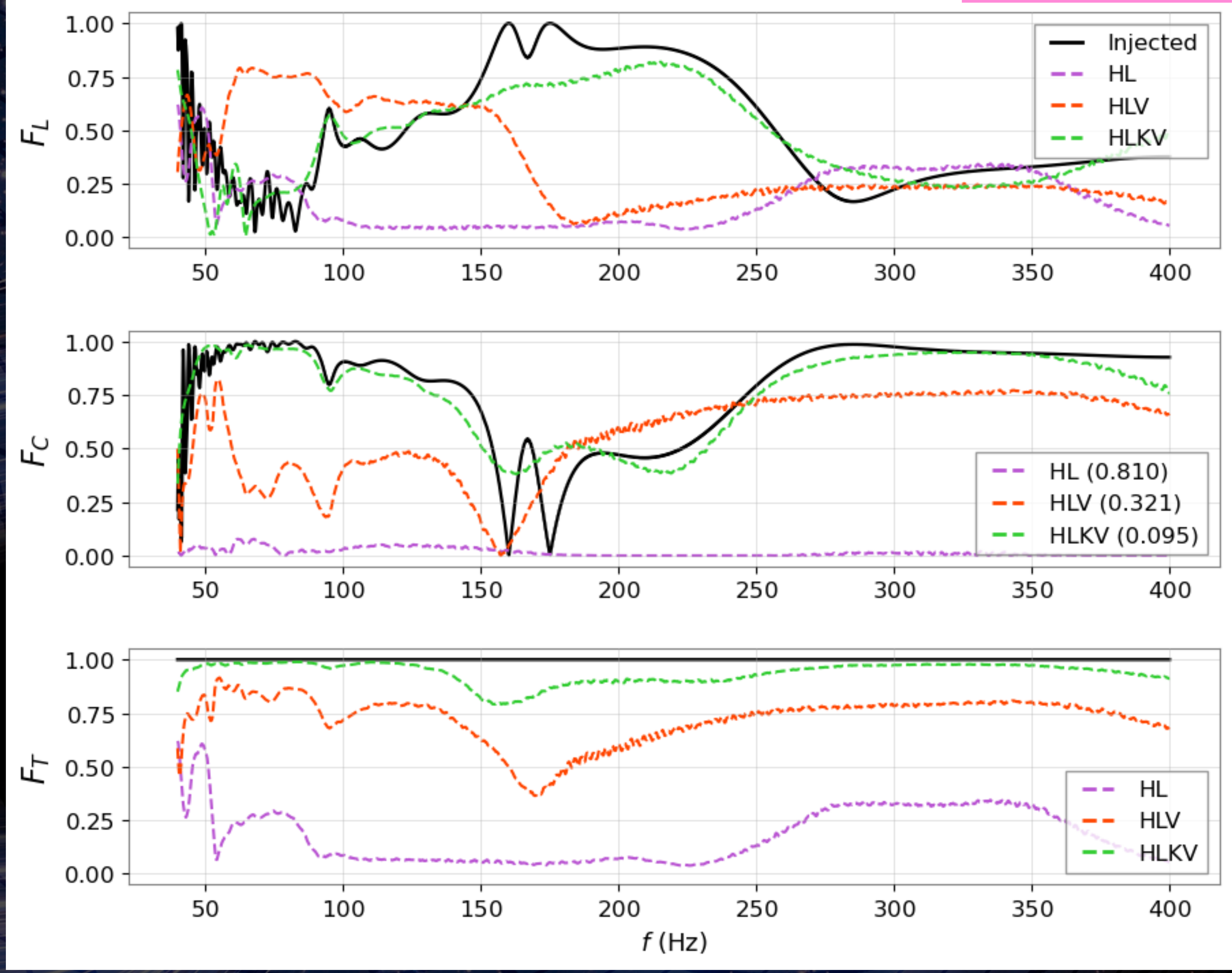
- n = Number of frequency intervals

ROOT MEAN SQUARED RESIDUALS, $\mathscr{R}_{\mathrm{RMS}}$

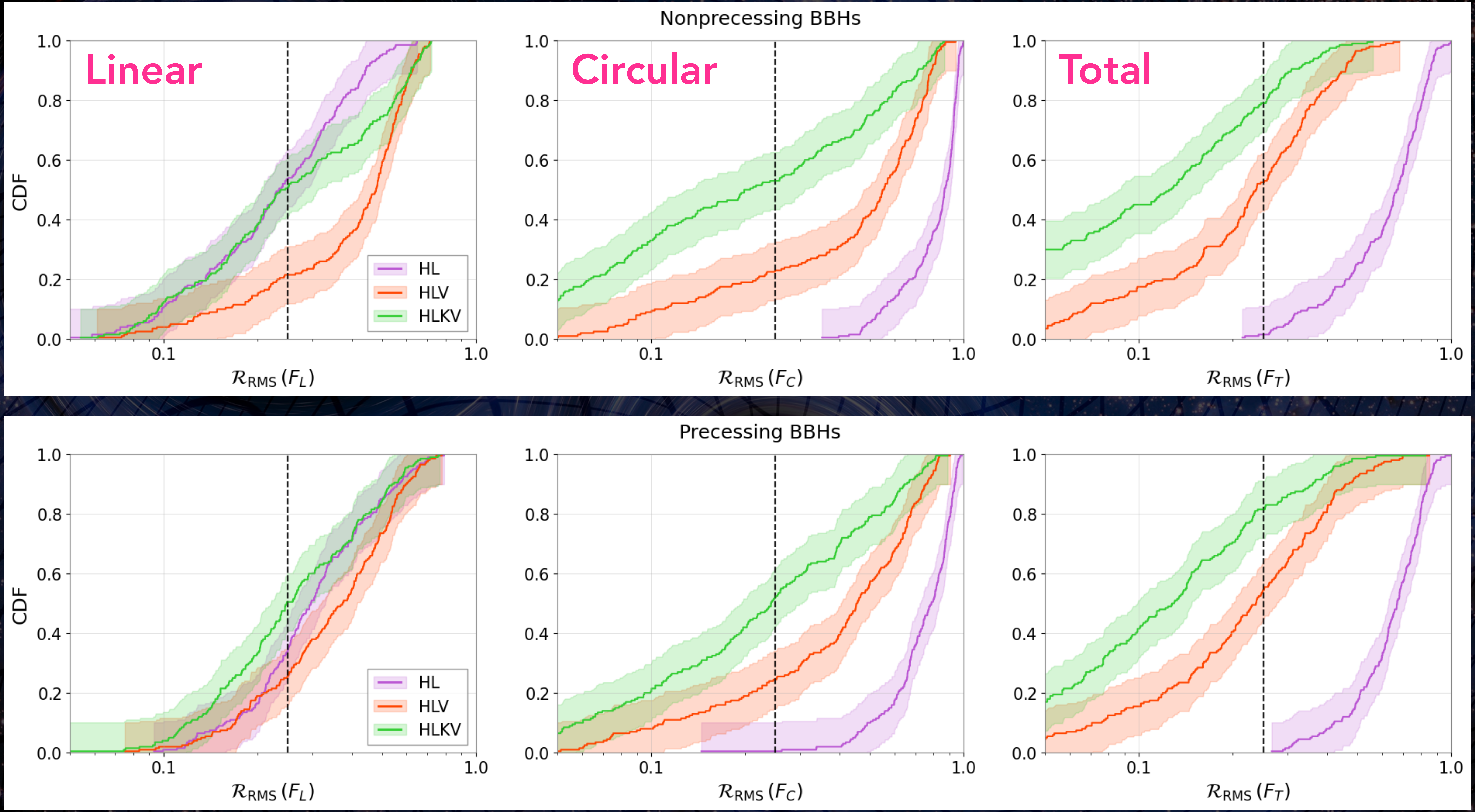
$\chi_{p,\mathrm{init}} = 0.77$





Lower $\mathscr{R}_{\mathrm{RMS}}(F_{\mathscr{P}})$ = Higher measurement accuracy

MEASURING FRACTIONAL POLARISATIONS WITH $\mathcal{R}$



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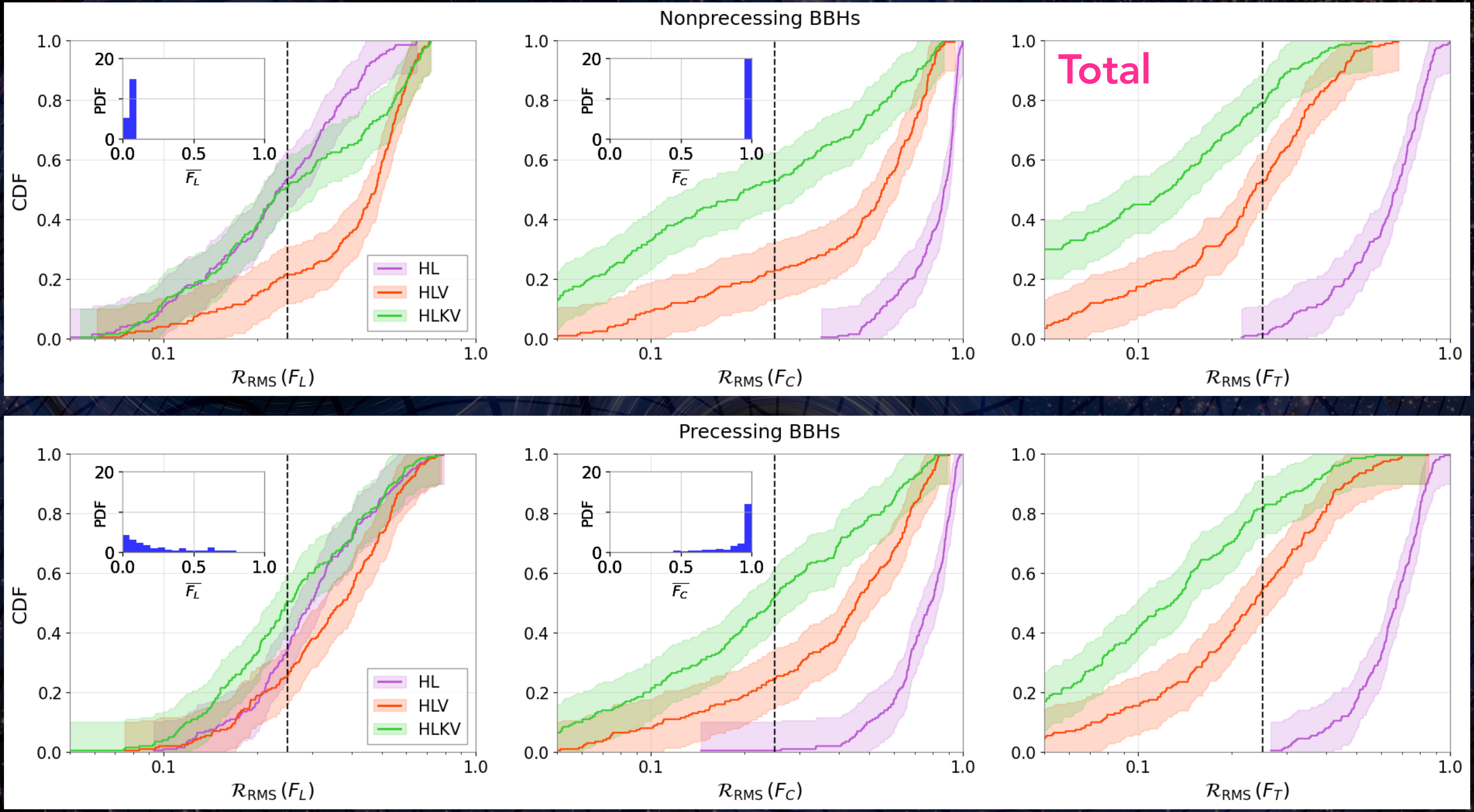

KEY TAKEAWAYS

MEASURING POLARISATION CONTENT WITH R

- R recovers fractional polarisations more accurately as the detector network expands
- When detector network is sufficiently large: Accuracy of polarisation measurements is not affected by signal morphology

⚠️ H and L are approximately coaligned

SUMMARY

- *BayesWave* can potentially distinguish between elliptical and nonelliptical GW signals through model selection via $\ln \mathscr{B}_{R,E}$
- The R model can be used to measure tensor polarisation content of GW burst signals
- Both of the above are enhanced by expanded detector networks
- **FUTURE WORK:**
  - Extend analyses to generic burst signals e.g. CCSN or WNB
  - Model selection between tensor (GR) and non-tensor (non-GR) polarisations