Quantum computing of gauge fields

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Based mainly on:

- J. M. and T. Trześniewski, *Gauge fields and quantum entanglement*, Phys. Lett. B **810** (2020), 135808
- G. Czelusta and J. M, *Quantum simulations of a qubit of space*, Phys. Rev. D **103** (2021) no.4, 046001
- G. Czelusta and J. M, *Quantum circuits for the Ising spin networks*, Phys. Rev. D **108** (2023) no.8, 086027

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Gauge fields

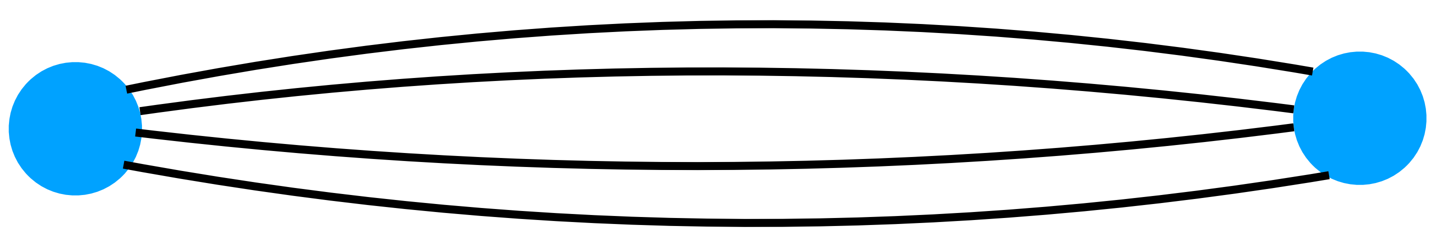
Provide description of the known „fundamental" interactions.

Electrodynamics - QED - U(1) - Yang-Mills theory

Electroweak interactions - SU(2)xU(1) - Yang-Mills theory

Chromodynamics - QCD - SU(3) - Yang-Mills theory

Gravitation - SU(2) - GR in the Ashtekar-Sen formalism



Difficult to study for non-Abelian cases due to the self-interaction of bosons mediating the interaction…

U(1) gauge field

One can notice that the U(1) Yang-Mills Lagrangian:

$$S = \int d^4x \mathcal{L} = -\frac{1}{4}\int d^4x F^{\mu\nu}F_{\mu\nu} \qquad \mu, \nu = 0, 1, 2, 3$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and A_μ is the four-potential (gauge field)

is invariant with respect to the gauge transformation:

$$A_\mu \to A'_\mu = U^\dagger A_\mu U - iU^\dagger \partial_\mu U$$

where

$U = e^{i\lambda(x^\mu)} \in U(1)$ and $\lambda \in \mathbb{R}$, which leads to

$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \lambda$$

where

The canonical momenta are: $\pi^a = \dfrac{\partial \mathcal{L}}{\partial \dot{A}_a} = -E^a \qquad a, b = 1, 2, 3$

...

… and

$$\pi^0 = \frac{\partial \mathcal{L}}{\partial \dot{A}_0} = 0$$ (primary constraint)

A_0 is a non-dynamical variable (Lagrange multiplier).

Employing the above, the ED Hamiltonian writes as:

$$H = \int d^3(\pi^\mu \dot{A}_\mu - \mathcal{L}) = \frac{1}{2}\int d^3x(\vec{E}^2 + \vec{B}^2) + \int d^3x A^0 \partial_a E^a$$

From here, we find

$$0 = \dot{\pi}_0 = \{\pi_0, H\} = \partial_a E^a := C(\vec{E})$$ (secondary constraint)

which is the Gauss law: $$\vec{\nabla} \cdot \vec{E} = 0$$ (or Gauss constraint)

no charges here!

In electrodynamics (U(1)) the Gauss law is a secondary constraint of the theory, which generates gauge transformations.

One can consider the smeared Gauss constraint:

$$C[\lambda] := \int d^3x \lambda(x) C(\vec{E})$$

which forms the first class algebra:

$$\{C[\lambda_1], C[\lambda_2]\} = 0$$

and, therefore, the Gauss constraint is a generator of the underlying symmetry. Indeed, because:

$$\delta A_a = \{A_a, C[\lambda]\} = \partial_a \lambda$$

we find that the Gauss constraint generates the *residual* U(1) gauge symmetry:

$$A_a \to A'_a = A_a + \delta A_a = A_a + \partial_a \lambda$$

SU(2) gauge field

Let us consider the SU(2) connection (1-form):

$$A = A_a^i \tau_i dx^a$$

The τ_i are generators of the $\mathfrak{su}(2)$ algebra $[\tau_i, \tau_j] = \epsilon_{ijk}\tau_k$

The SU(2) connection field is canonically conjugated to the SU(2) „electric" field $\vec{E}_a = E_a^i \tau_i$, where the arrow corresponds to the internal (SU(2)) space.

The canonical pair satisfies the following bracket:

$$\left\{ E_i^a(x), A_b^j(y) \right\} = \delta_b^a \delta_i^j \delta^{(3)}(x-y)$$

where $i, j = 1, 2, 3$ and $a, b = 1, 2, 3$

For theories invariant with respect to the local SU(2) transformations (e.g. Yang-Mills theory, GR in the Ashtekar formalism):

$$A_a \to A'_a = U^\dagger A_a U + U^\dagger \partial_a U$$

where $U \in SU(2)$, the Gauss constraint takes the following form:

$$C_i := D_a E^a_i = \partial_a E^a_i + \epsilon_{ij}{}^k A^j_a E^a_k = 0$$

The smeared SU(2) Gauss constraint is

$$C[\vec{\lambda}] := \int d^3x \lambda^i(x) C_i(E, A)$$

which satisfies the first class algebra:

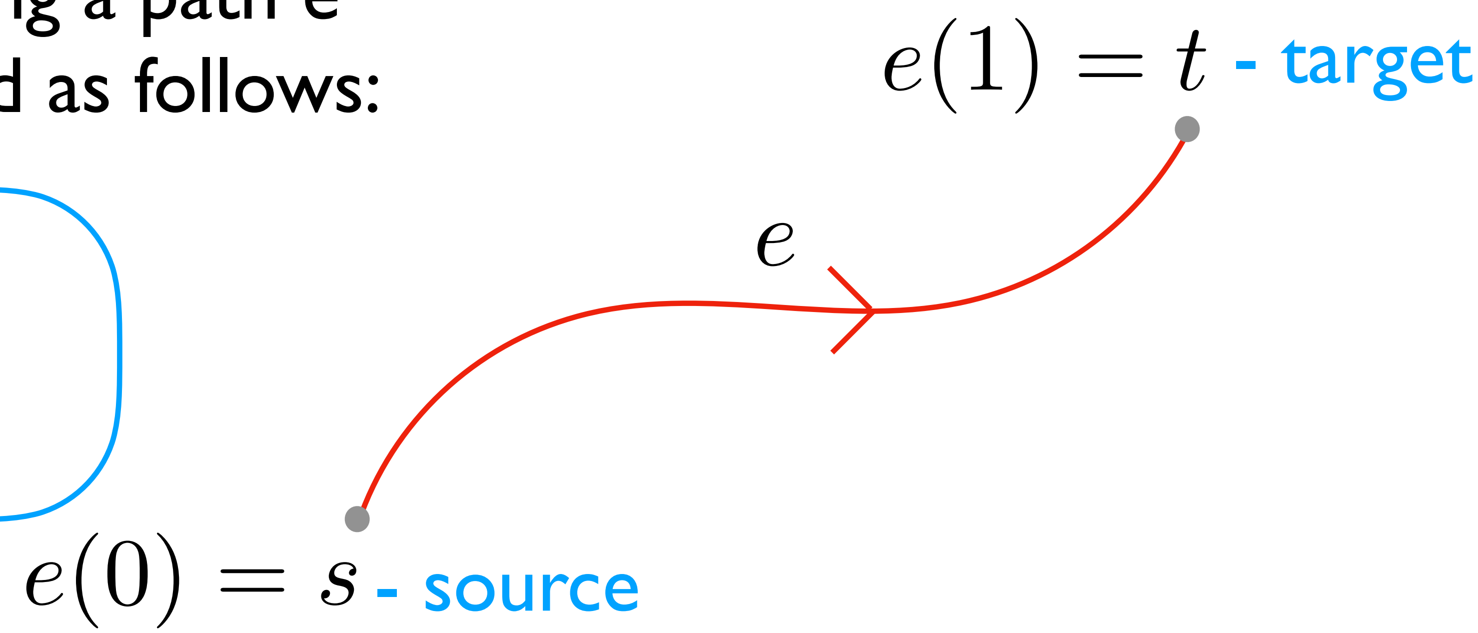
$$\left\{ C\left[\vec{\lambda}_1\right], C\left[\vec{\lambda}_2\right] \right\} = C\left[[\vec{\lambda}_1, \vec{\lambda}_2] \right]$$

Holonomies - field theoretical viewpoint

of the SU(2) connection along a path e
are non-local objects defined as follows:

$$h_e[A] := \mathcal{P} \exp \int_e A$$

path $e : [0, 1] \to \Sigma$



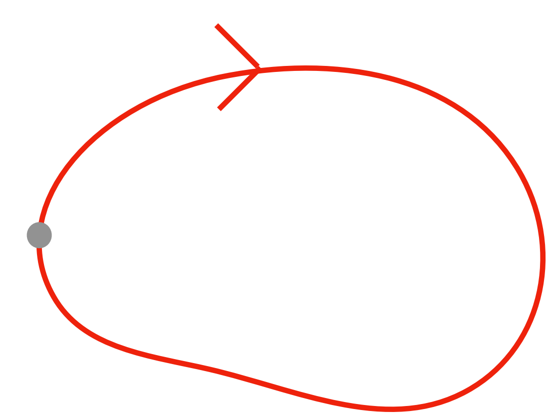
Under the gauge transformation the holonomy transforms as:

$$h_e[A] \to h'_e[A] = U^\dagger(e(0))\, h_e[A]\, U(e(1)) = U_s^\dagger h_e[A] U_t$$

where $U_s := U(e(0))$ and $U_t := U(e(1))$

Gauge invariant objects
- Wilson loops:

$$W_e[A] := \mathrm{tr}(h_e[A])$$



Holonomy-flux algebra

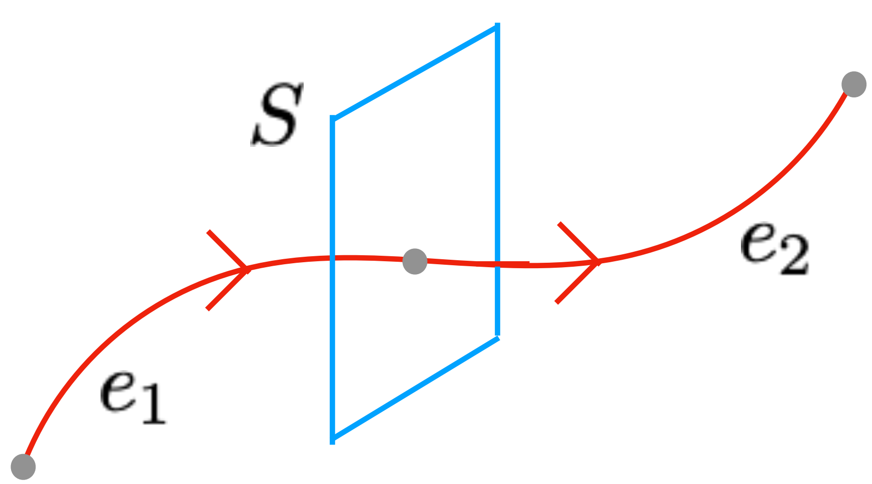
One can introduce flux of the SU(2) „electric" field thought a 2-surface S:

$$\vec{F}_S[E] := \int_S \epsilon_{abc} E_i^a \tau^i dx^b \wedge dx^c$$

which satisfies the holonomy-flux algebra:

$$\{F_S^i[E], h_e[A]\} = -\iota(e,S) h_{e_1}[A] \tau^i h_{e_2}[A]$$

where $\iota(e,S) = \pm 1, 0$ is the intersection number and $e = e_1 \cup e_2$.



Holonomies - quantum mechanical viewpoint

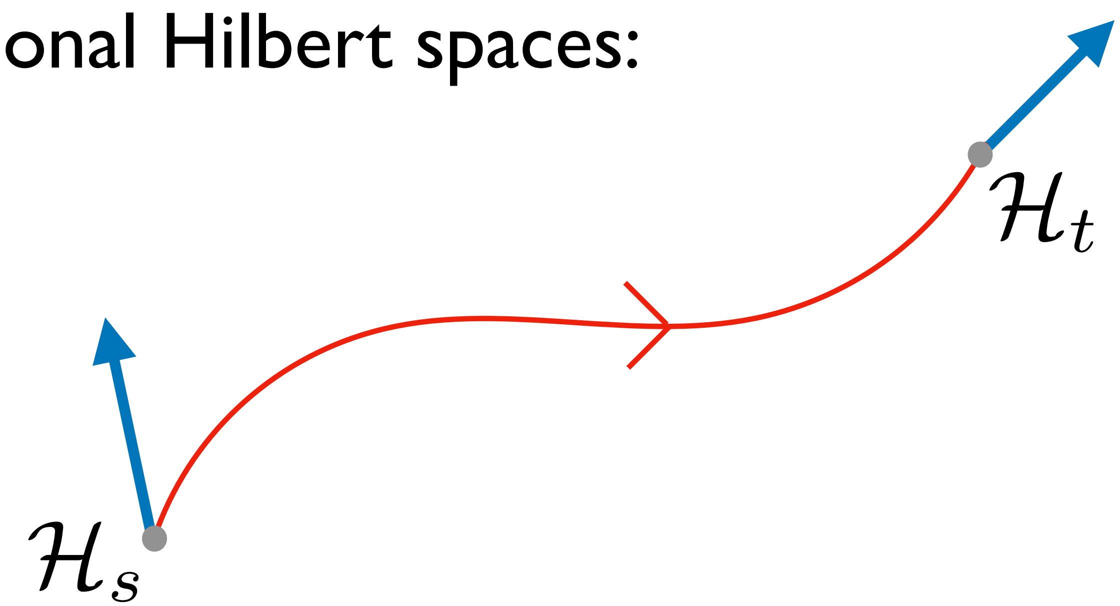
In the fundamental (j=1/2) representation of SU(2), the holonomies are 2x2 SU(2) matrices, which belong to the automorphism group of $\mathbb{C}^2$ (i.e. the space of non-relativistic spinors).

$\mathbb{C}^2$ equipped with the natural scalar product becomes the Hilbert space of a *qubit* system. Pure quantum states correspond to rays in $\mathbb{C}^2$.

The SU(2) holonomy becomes then an isomorphism (unitary map) between the two 2-dimensional Hilbert spaces:

$$\mathcal{H}_s = \text{span}\,\{|0\rangle_s, |1\rangle_s\}$$

$$\mathcal{H}_t = \text{span}\,\{|0\rangle_t, |1\rangle_t\}$$



For general *j*-representation of SU(2), the holonomies are *(2j+1)x(2j+1)* SU(2) matrices, such that $\dim \mathcal{H}_s = 2j+1 = \dim \mathcal{H}_t$

Holonomy as a unitary map

Employing the basis elements of the source and target Hilbert spaces, it is convenient to express an arbitrary holonomy map as:

$$h = h_{IJ} |I\rangle_{s\,t}\langle J| \in \mathcal{H}_s \otimes \mathcal{H}_t^*$$ where $I, J = 0, 1$

The action of this unitary map can be either left-handed or right-handed:

$$h_L : \mathcal{H}_s^* \to \mathcal{H}_t^* \qquad h_R : \mathcal{H}_t \to \mathcal{H}_s$$

The Hermitian conjugation of h: $h^\dagger = h_{IJ}^* |J\rangle_{t\,s}\langle I| \in \mathcal{H}_t \otimes \mathcal{H}_s^*$

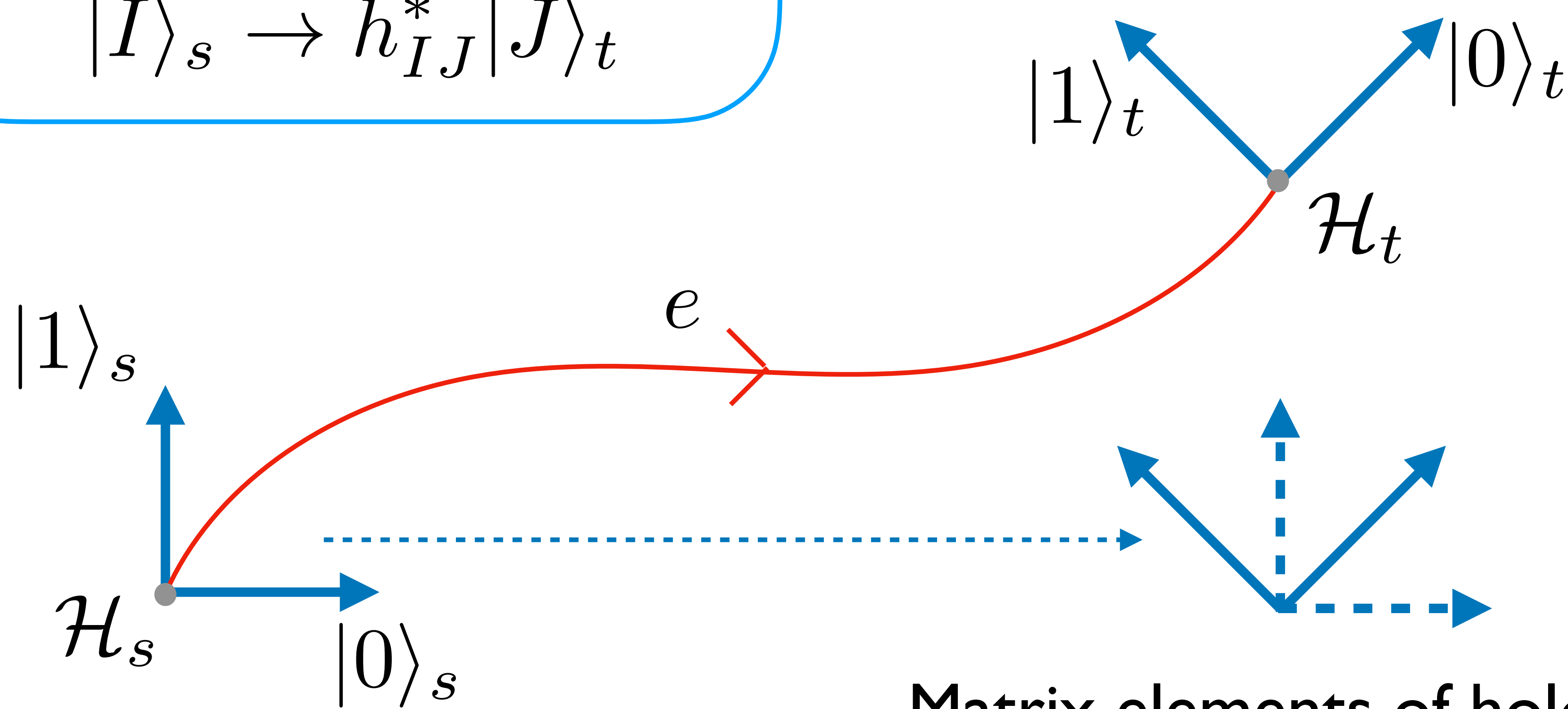
acts as $h_L^\dagger : \mathcal{H}_t^* \to \mathcal{H}_s^*$ and $h_R^\dagger : \mathcal{H}_s \to \mathcal{H}_t$

Example: A basis state ${}_s\langle K| \in \mathcal{H}_s^*$ at the point s is mapped into:

$${}_s\langle K|\, h = h_{IJ\,s}\langle K|I\rangle_{s\,t}\langle J| = h_{KJ\,t}\langle J| \in \mathcal{H}_t^*$$ at the point t.

Mapping of the basis states:

$$|I\rangle_s \to h^*_{IJ}|J\rangle_t$$

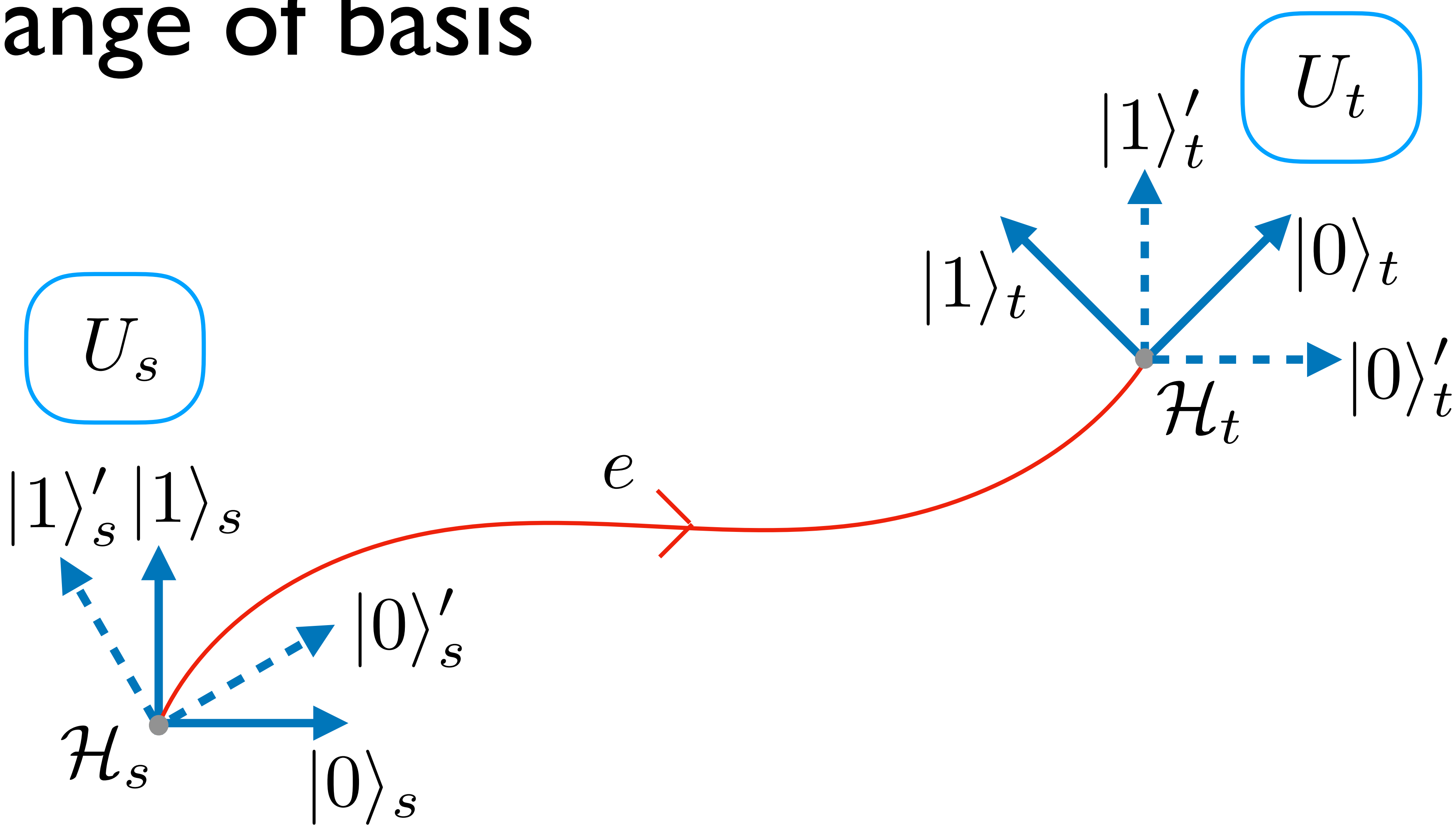


Matrix elements of holonomies are the coefficients of decomposition of one basis into the other

Physical interpretation: holonomies describe displacement of a quantum system from point s to point t in the gauge field:

$$|\psi\rangle \to \mathcal{P}e^{\int_e A}|\psi\rangle$$

Change of basis

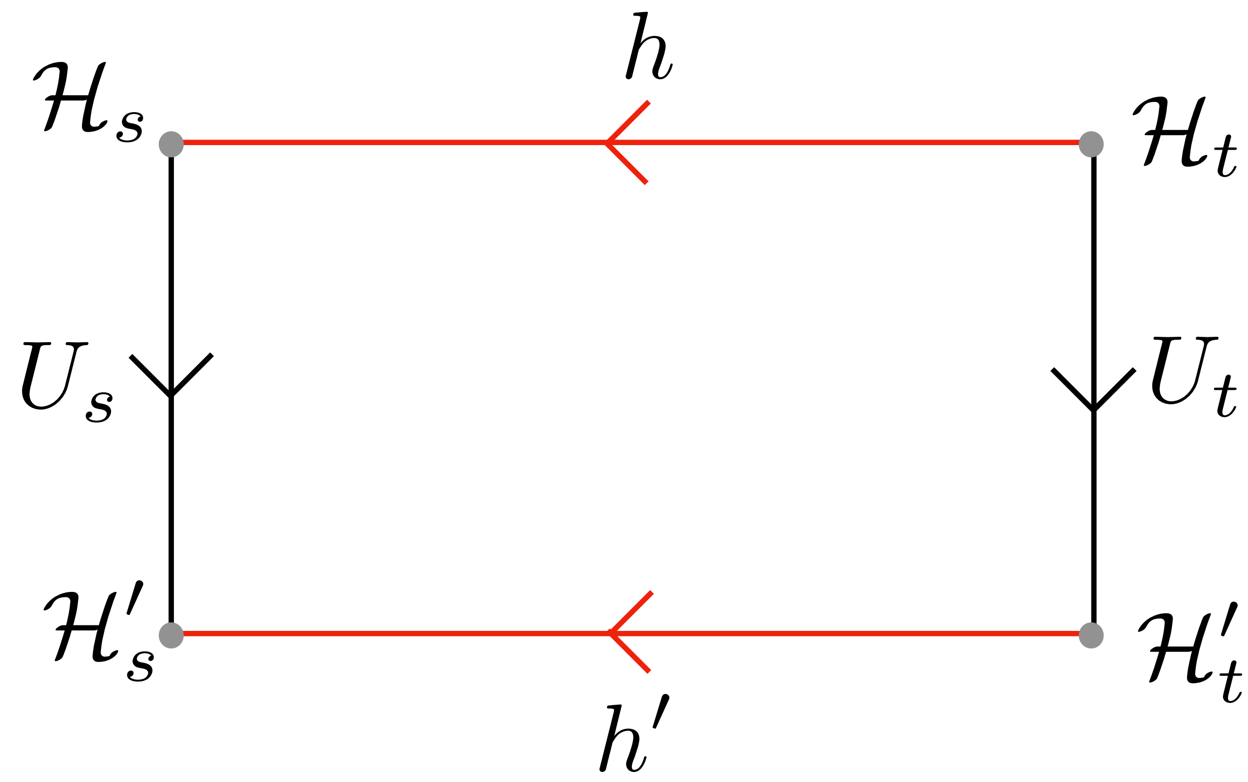


Physics does not depend on the choince of basis.

One can perform unitary transformation: $|I\rangle' = U|I\rangle$

or using components of the unitary matrix $|I\rangle' = U_{JI}|J\rangle$

How the holonomies transform under the change of bases?



The action of holonomy is preserved under the transformation of bases if

$$h'_{IJ}|I\rangle'_{s\,t}\langle J|' = U_{s,KI}h'_{IJ}U^{\dagger}_{t,JL}|K\rangle_{s\,t}\langle L| = h_{IJ}|I\rangle_{s\,t}\langle J|$$

which leads to the transformation rule: $h \rightarrow h' = U^{\dagger}_{s}hU_{t}$

It is clear that the change of h under unitary transformations in the source and target spaces is equivalent to the action of a SU(2) gauge transformation.

Holonomies as wave functions

Functions of holonomies, equipped with Haar measure on SU(2) Lie group form a Hilbert space:

$$\mathcal{H} = L^2(SU(2))$$

so that: $\varphi(h_e) \in L^2(SU(2))$. Following the Peter-Weyl therem:

$$L^2(SU(2)) = \oplus_j (\mathcal{H}_j \otimes \mathcal{H}_j^*)$$

where $\mathcal{H}_j$ is a spin-j Hilbert space.

The orthonormal basis states in the Hilbert space, for a given path e, are:

$$\varphi(h_e)^j = \frac{1}{\sqrt{2j+1}} (h_{IJ})^j |I\rangle_{st} \langle J| \in \mathcal{H}_{j,s} \otimes \mathcal{H}_{j,t}^* \qquad I, J = 0, \ldots, 2j$$

For spin-1/2: $\varphi(h_e)^{1/2} = \frac{1}{\sqrt{2}} h_{IJ} |I\rangle_{st} \langle J| \in \mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2}^* \quad I, J = 0, 1$

States in $\mathcal{H}_s \otimes \mathcal{H}_t$

What are the states in $\mathcal{H}_s \otimes \mathcal{H}_t$?

Because of the isomorphism between $\mathcal{H}_j$ and $\mathcal{H}_j^*$ one can map states from $\mathcal{H}_s \otimes \mathcal{H}_t^*$ to states in $\mathcal{H}_s \otimes \mathcal{H}_t$.

In particular, for spin-1/2:

$$\Psi(h_e) \in \mathcal{H}_{1/2} \otimes \mathcal{H}^*_{1/2} \rightarrow |\Psi\rangle = \frac{1}{\sqrt{2}} h^*_{IJ} |I\rangle_s |J\rangle_t \in \mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2}$$

where h_{IJ} are matrix components of the SU(2) matrix.

The isomorphism is the manifestation of the Choi-Jamiołkowski Isomorphism known in the theory of quantum channels.

The state can be used to introduce anti-linear map & relation to quantum teleportation (Czech, Lamprou & Susskind, 2018; Czech, De Boer, Ge & Lamprou, 2019)

Improved analysis, gravity, networks, etc. (JM & Trześniewski, 2020)

The state $|\Psi\rangle := \frac{1}{\sqrt{2}} h^*_{IJ} |I\rangle_s |J\rangle_t \in \mathcal{H}_s \otimes \mathcal{H}_t$

is a **maximally entangled** state. The density matrix:

$$\hat{\rho} = |\Psi\rangle\langle\Psi| = \frac{h^*_{IJ} h_{KL}}{2} (|I\rangle_{ss}\langle K|)(|J\rangle_{tt}\langle L|)$$

The reduced density matrix:

Unitarity of h: $h_{IK} h^\dagger_{KJ} = \delta_{IJ}$

$$\hat{\rho}_s := \mathrm{tr}_t(\hat{\rho}) = \frac{h^*_{IJ} h^T_{JK}}{2} (|I\rangle_{ss}\langle K|) = \frac{1}{2}\hat{I}$$

The same for $\hat{\rho}_t := \mathrm{tr}_s(\hat{\rho})$

The mutual information is maximal:

$$I(s:t) = S(\rho_s) + S(\rho_t) - S(\rho) = 2\ln 2$$

$$S(\rho_{s,t}) = -\mathrm{tr}(\rho_{s,t} \ln \rho_{s,t}) = \ln 2$$

$S(\rho) = 0$ (pure state)

Antilinear map

Equivalently to the case of holonomy, one can the following map:

$$\mathcal{H}_s^* \ni {}_s\langle I| \to \left[\sqrt{2}\,|\Psi\rangle \circ C\right]({}_s\langle I|) = h^*_{IJ}|J\rangle_t \in \mathcal{H}_t$$

where C is the complex conjugation operation.

Change of bases leads to

$$\begin{aligned}{}_s\langle I|' \to \left[\sqrt{2}\,|\Psi\rangle \circ C\right]({}_s\langle I|') &= \sqrt{2}\left(U^*_{s,JI}\right)^* {}_s\langle J|\Psi\rangle \\ &= U^T_{s,IJ} h^*_{JL}|L\rangle_t = U^T_{s,IJ} h^*_{JL} U^*_{t,LM}|M\rangle'_t\end{aligned}$$

which leads to the following transformation rule:

$$h_{JM} \to h'_{JM} = U^\dagger_{s,IJ} h_{JL} U_{t,LM}$$

The map is equivalent to the SU(2) gauge transformation.

Constructing discrete (lattice) SU(2) gauge theory

… employing the holonomies and fluxes.

Let us consider the L links e, which meet at N nodes.
The full quantum holonomy-flux algebra (between holonomies and conjugated fluxes) is:

$$[F_e^j, h_{e'}] = i\delta_{ee'} h_{e'} \tau^j$$

$$[h_e, h_{e'}] = 0$$

$$[F_e^i, F_{e'}^j] = i\delta_{ee'} \epsilon^{ij}{}_k F_e^k$$

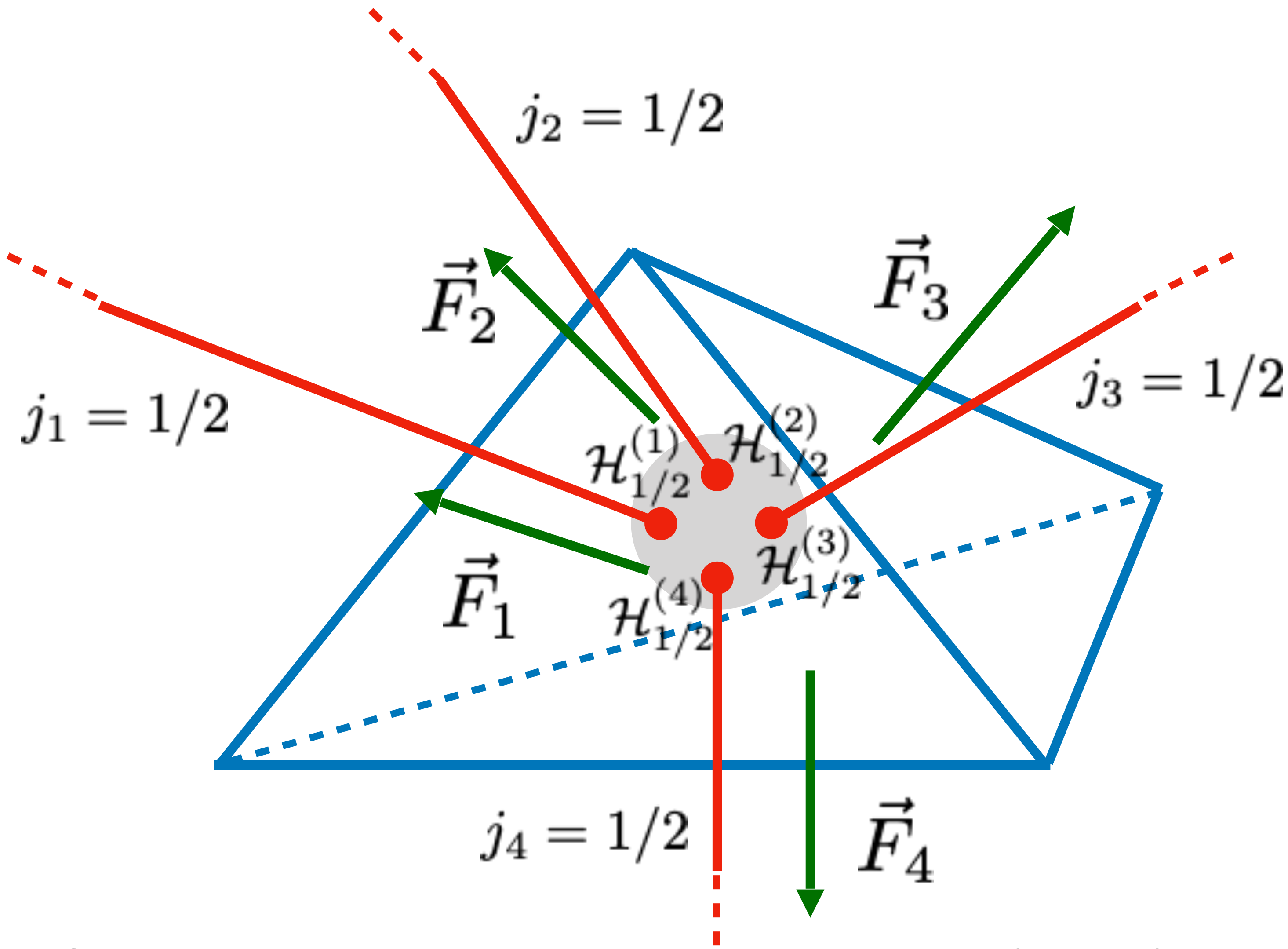
Therefore, the conjugated fluxes are just angular momenta: $\vec{F} = \vec{J}$

And the kinematical Hilbert space is:

$$L^2(SU(2)^L)$$

Imposing the Gauss constraint

For simplicity, let us consider 4-valent nodes and fundamental (spin-1/2) representations at the links.



$\hat{P}_{\rm G}$ - Projection onto spin-0 subspace by the virtue of the Gauss constraint.

The Gauss constraint tells us that the four fluxes (angular momenta) conjugated to the holonomies sum-up to zero:

$$\sum_{i=1}^{4} \vec{F}_i = 0$$

Equation of a tetrahedron with the areas of the faces:

$$A_i = ||\vec{F}_i||$$

Quantum tetrahedron

A state of the quantum tetrahedron is:

$$|\Psi\rangle \in \mathcal{H}_{j_1} \otimes \mathcal{H}_{j_1} \otimes \mathcal{H}_{j_3} \otimes \mathcal{H}_{j_4} \quad \text{such that} \quad (\sum_{a=1}^{4} \hat{\vec{J}}_a)|\Psi\rangle = 0$$

where $\mathcal{H}_{j_a} = \text{span}\{|j_a, -j_a\rangle, \ldots, |j_a, j_a\rangle\}$

and $\hat{\vec{J}}_a \cdot \hat{\vec{J}}_a |j_a, m_a\rangle = j_a(j_a+1)|j_a, m_a\rangle$

So, the $|\Psi\rangle$ state belongs to the SU(2)-invariant subspace of product of spins.

Please note that the states allows for quantum communication without a shared reference frame:

$$\hat{\rho} = \int_{SU(2)} dg \hat{U}(g)^4 \hat{\rho} \hat{U}^\dagger (g)^4$$

We call the subspace an intertwiner space:

$$|\mathcal{I}\rangle \in \text{Inv}(\mathcal{H}_{j_1} \otimes \mathcal{H}_{j_1} \otimes \mathcal{H}_{j_3} \otimes \mathcal{H}_{j_4})$$

dim Inv = number of linearly independent singlet states

The special case $j_1 = j_2 = j_3 = j_4 = j$

$$\dim \mathrm{Inv}(\mathcal{H}_j \otimes \mathcal{H}_j \otimes \mathcal{H}_j \otimes \mathcal{H}_j) = 2j + 1$$

In what follows we will focus on the fundamental representation of SU(2): j=1/2

$$\dim \mathrm{Inv}(\mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2}) = 2$$

This comes from the fact that:

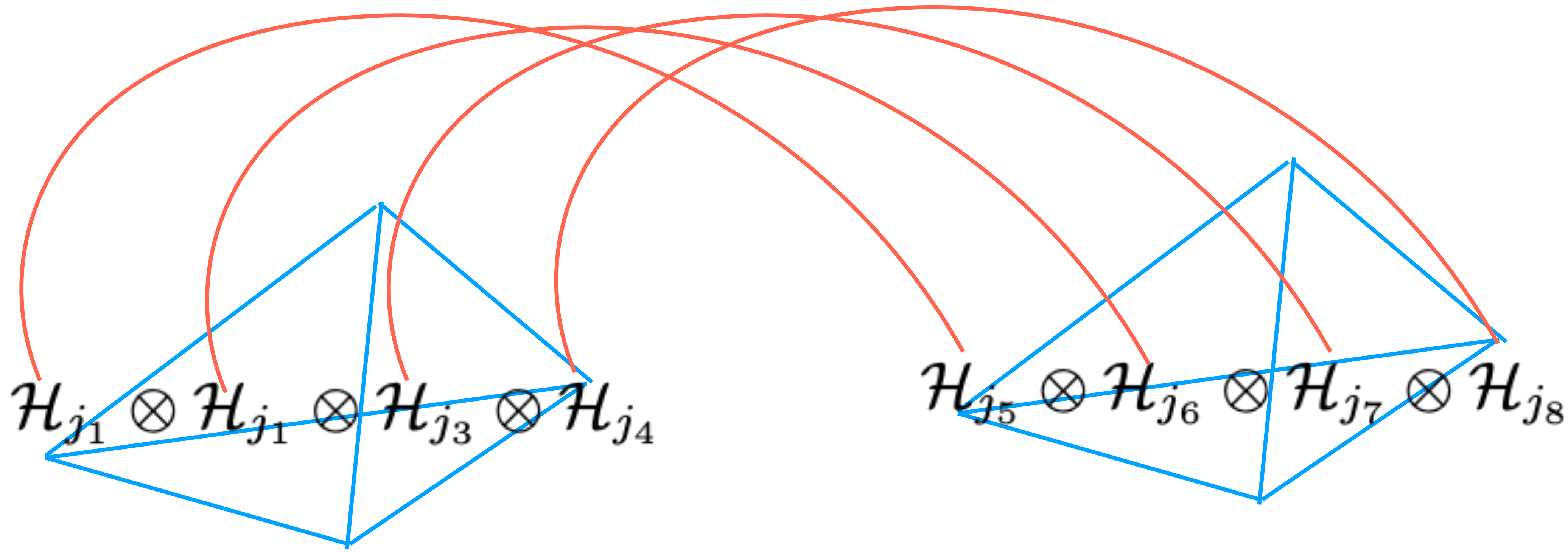
$$\mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2} \otimes \mathcal{H}_{1/2}$$
$$= \boxed{2\mathcal{H}_0} \oplus 3\mathcal{H}_1 \oplus \mathcal{H}_2$$

The invariant subspace is two-dimensional - intertwiner qubit

Consequently, the physical Hilbert space reduces to:

$$\boxed{L^2(SU(2)^L/SU(2)^N) = \otimes_{i=1}^N \mathcal{H}_{1/2}^{(i)}}$$

Building gauge invariant states from holonomies



One can begin with the product spaces in which holonomies live, i.e.:

$$\mathcal{H}_{j_1} \otimes \mathcal{H}_{j_5} \qquad \mathcal{H}_{j_2} \otimes \mathcal{H}_{j_6} \qquad \mathcal{H}_{j_3} \otimes \mathcal{H}_{j_7} \qquad \mathcal{H}_{j_4} \otimes \mathcal{H}_{j_8}$$

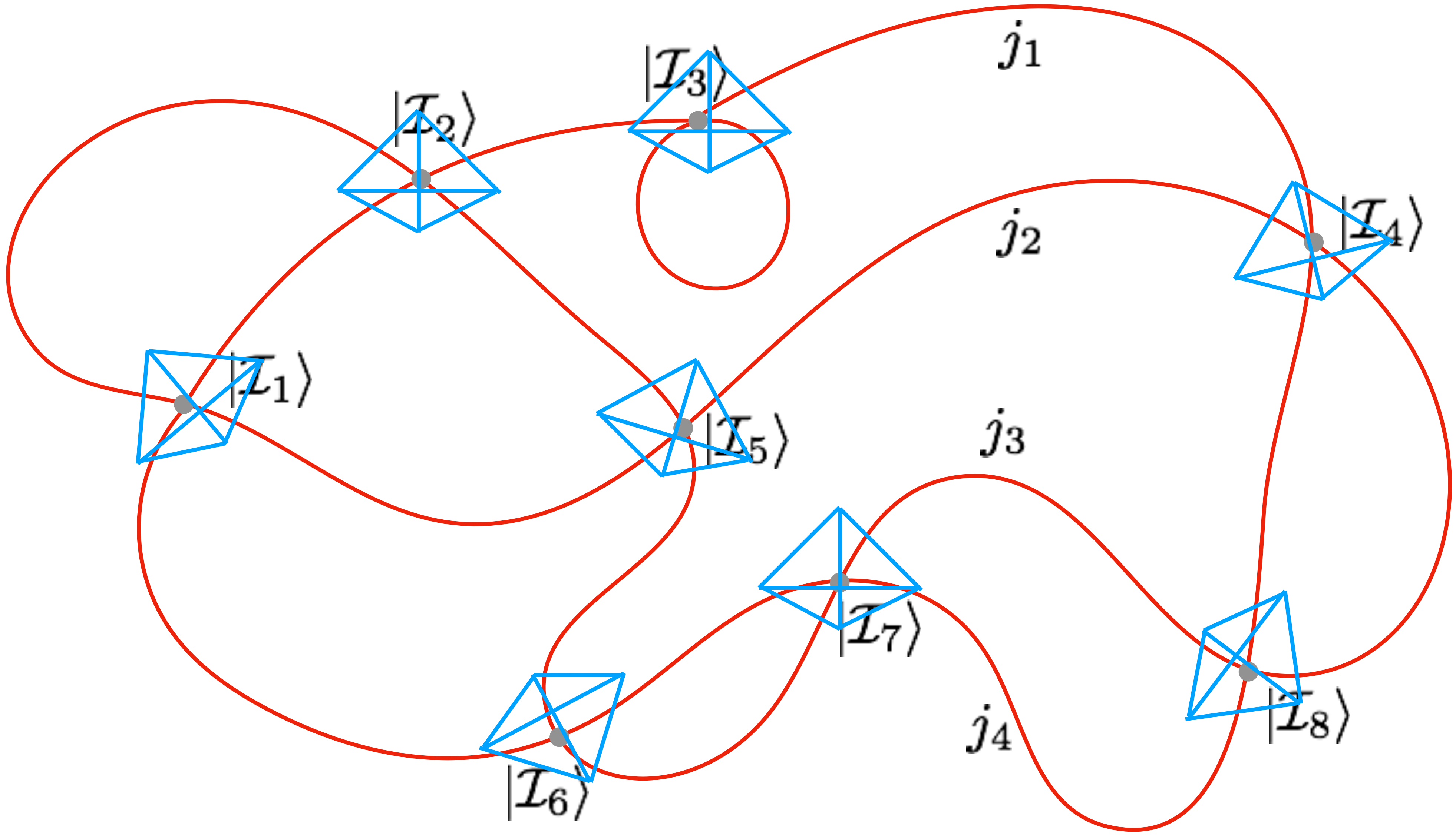
take their product $\otimes_{i=1}^{8} \mathcal{H}_{j_i}$

and impose the Gauss constraint, which leads to:

$$\mathrm{Inv}(\mathcal{H}_{j_1} \otimes \mathcal{H}_{j_2} \otimes \mathcal{H}_{j_3} \otimes \mathcal{H}_{j_4}) \otimes \mathrm{Inv}(\mathcal{H}_{j_5} \otimes \mathcal{H}_{j_6} \otimes \mathcal{H}_{j_7} \otimes \mathcal{H}_{j_8})$$

This construction generalizes to an arbitrary number of nodes and leads to a concept of spin networks. (Penrose, Rovelli, Smolin, …)

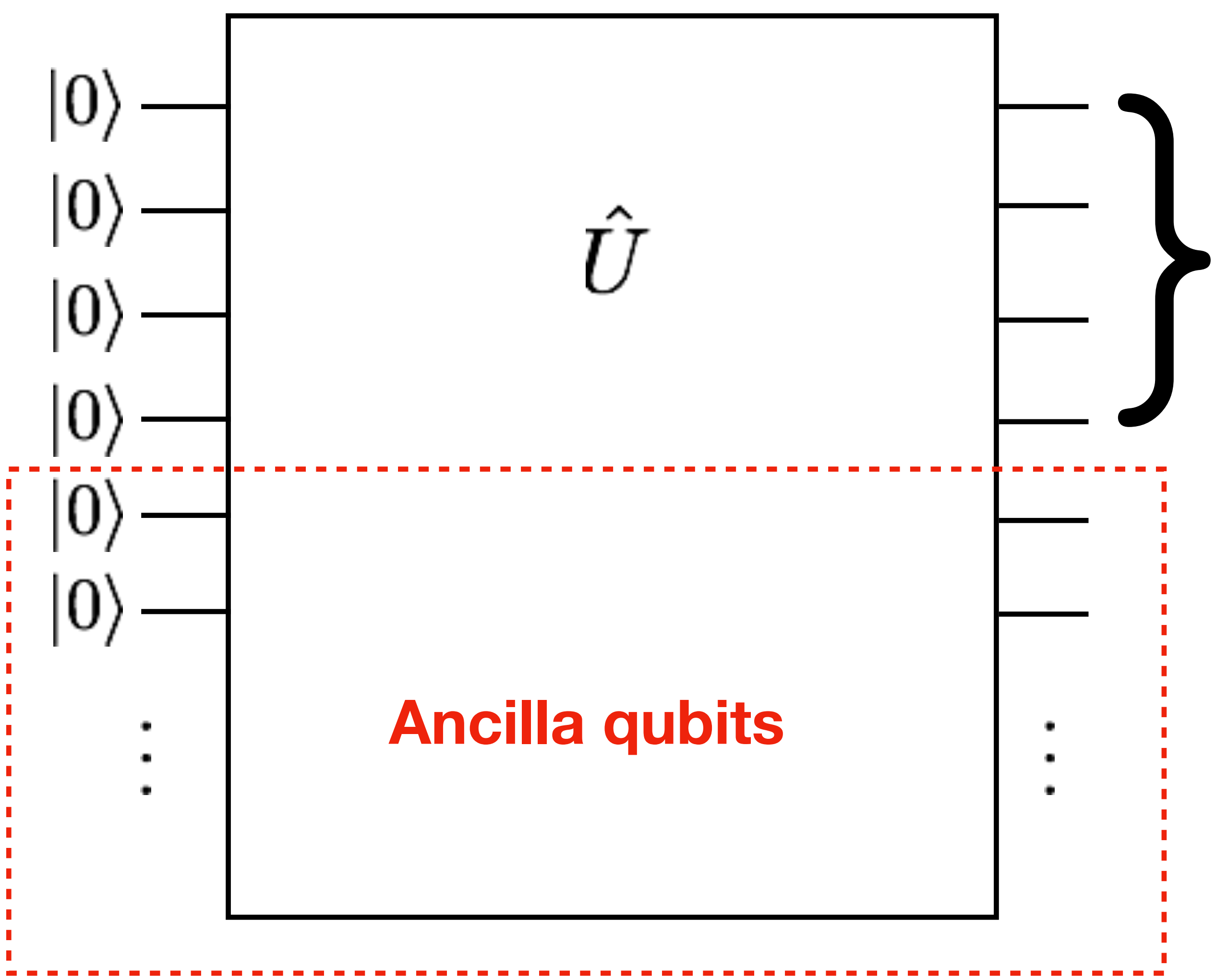
Spin networks - states of SU(2) gauge theory

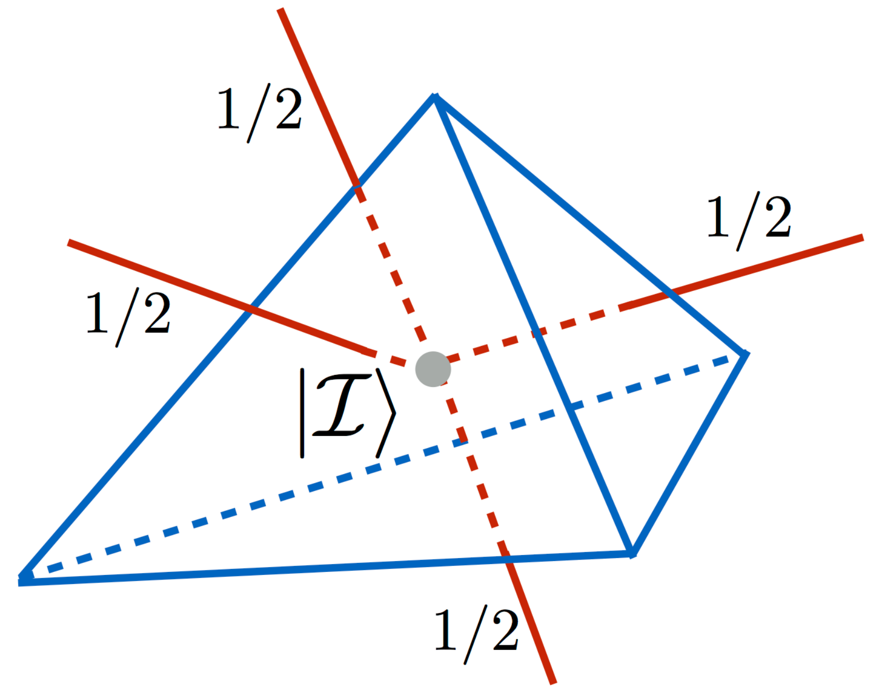


Spin labels - irreducible representations of the SU(2) group: $j_i = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$

Local SU(2) gauge invariance (Gauss constraint) implies that spins sum up to zero at the nodes - degeneracy leads to intertwiner spaces.

Quantum circuit for the quantum tetrahedron



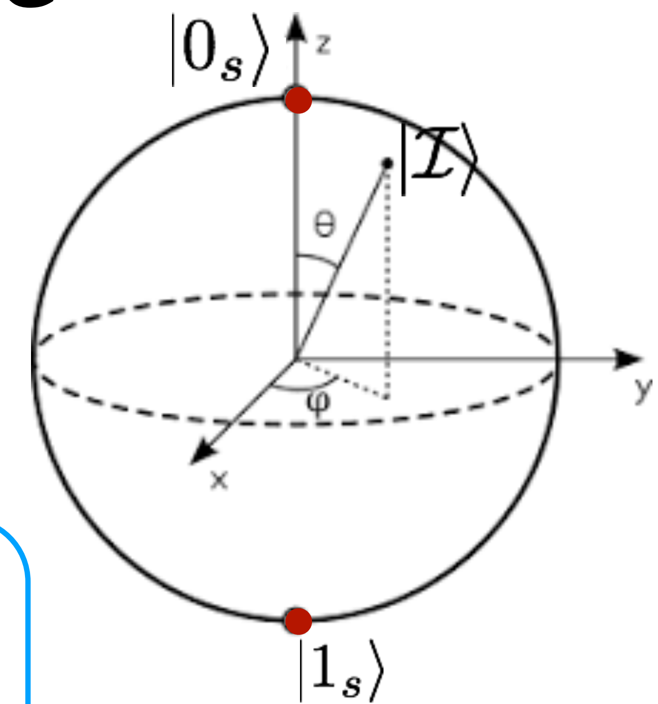


At least four qubits are needed to create a single intertwiner qubit state.

K. Li *et al.* (2019), JM (2019), G. Czelusta & JM (2021), L. Cohen *et al.* (2021)

A general intertwiner qubit state

$$|\mathcal{I}\rangle = \cos(\theta/2)|0_s\rangle + e^{i\phi}\sin(\theta/2)|1_s\rangle$$



Intertwiner qubit base states in the s-channel (four qubit singlets):

$$|0_s\rangle = |S\rangle \otimes |S\rangle,$$
$$|1_s\rangle = \frac{1}{\sqrt{3}}\left(|T_+\rangle \otimes |T_-\rangle + |T_-\rangle \otimes |T_+\rangle - |T_0\rangle \otimes |T_0\rangle\right).$$

Singlet and triplet states for two spin-1/2 particles:

$$|S\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right),$$
$$|T_+\rangle = |00\rangle,$$
$$|T_0\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right),$$
$$|T_-\rangle = |11\rangle.$$

Preparation of the state $|\mathcal{I}\rangle$

Task: Find a circuit U which generates a general intertwiner qubit state:

$$|\mathcal{I}\rangle = \hat{U}_{\mathcal{I}}|0000\rangle$$

The procedure is **not** unique!

$$|\mathcal{I}\rangle = \cos(\theta/2)|0_s\rangle + e^{i\phi}\sin(\theta/2)|1_s\rangle$$

can be expressed as:

$$|\mathcal{I}\rangle = \frac{c_1}{\sqrt{2}}(|0011\rangle + |1100\rangle) + \frac{c_2}{\sqrt{2}}(|0101\rangle + |1010\rangle) + \frac{c_3}{\sqrt{2}}(|0110\rangle + |1001\rangle)$$

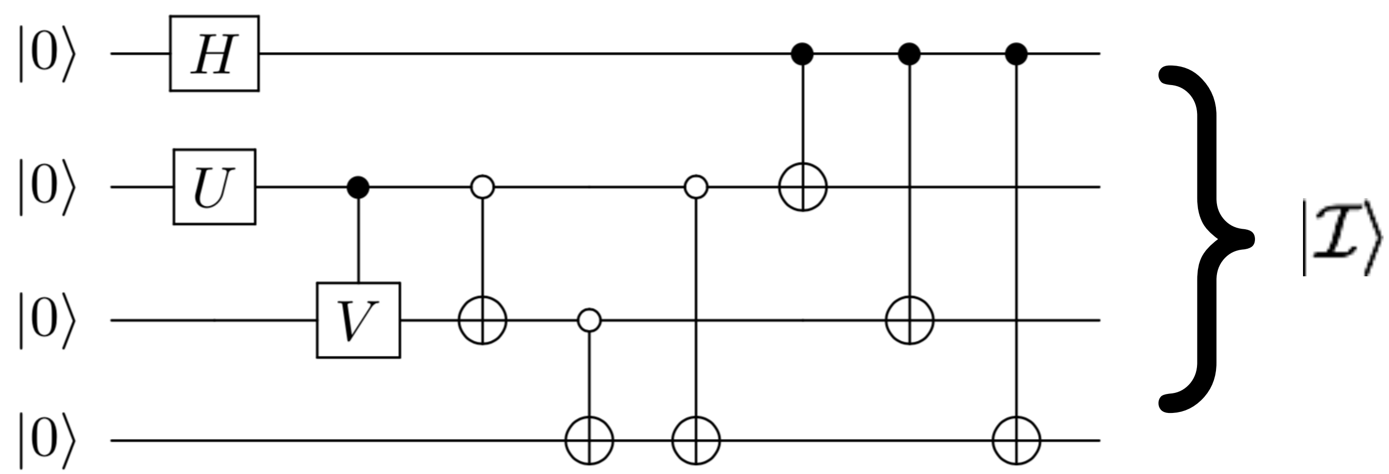
$$c_1 = \sqrt{\frac{2}{3}}e^{i\phi}\sin(\theta/2),$$

$$c_2 = \frac{1}{\sqrt{2}}\left(\cos(\theta/2) - \frac{1}{\sqrt{3}}e^{i\phi}\sin(\theta/2)\right) = \frac{e^{i\chi_+}}{\sqrt{2}}\sqrt{1 - \frac{2}{3}\sin^2(\theta/2) - \frac{\sin\theta\cos\phi}{\sqrt{3}}},$$

$$c_3 = \frac{1}{\sqrt{2}}\left(-\cos(\theta/2) - \frac{1}{\sqrt{3}}e^{i\phi}\sin(\theta/2)\right) = \frac{e^{i\chi_-}}{\sqrt{2}}\sqrt{1 - \frac{2}{3}\sin^2(\theta/2) + \frac{\sin\theta\cos\phi}{\sqrt{3}}},$$

$$\sum_{i=1}^{3}|c_i|^2 = 1 \qquad \sum_{i=1}^{3}c_i = 0$$

A quantum circuit for quantum tetrahedron:



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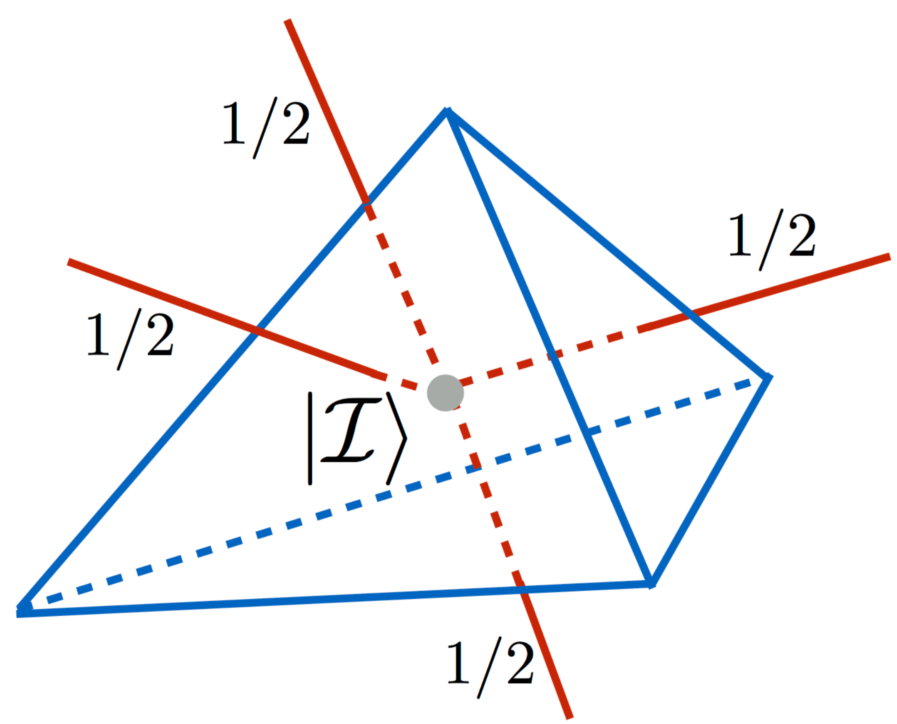


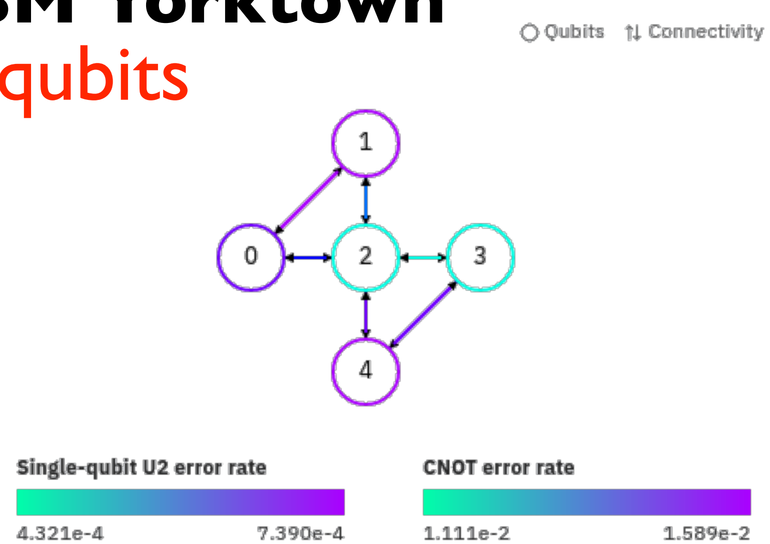
G. Czelusta & JM (2021)

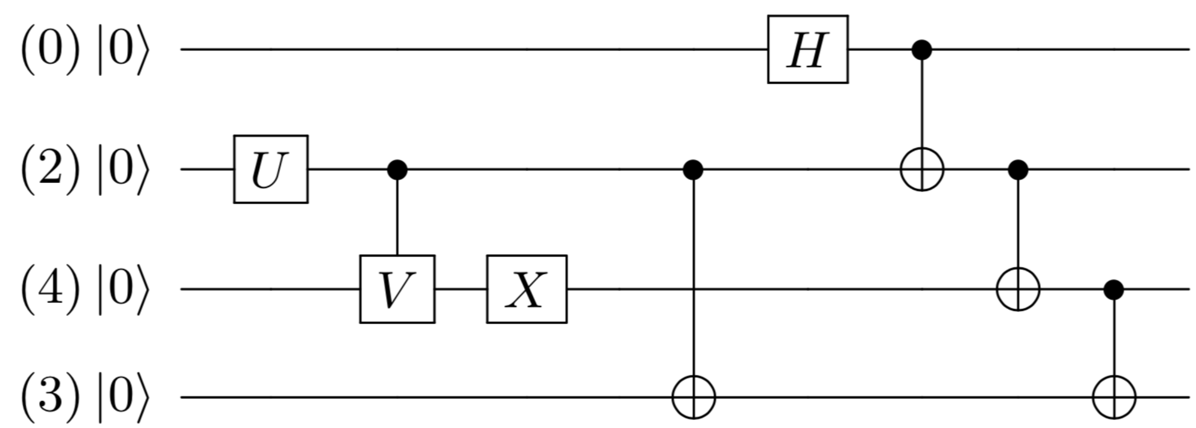
Transpilation

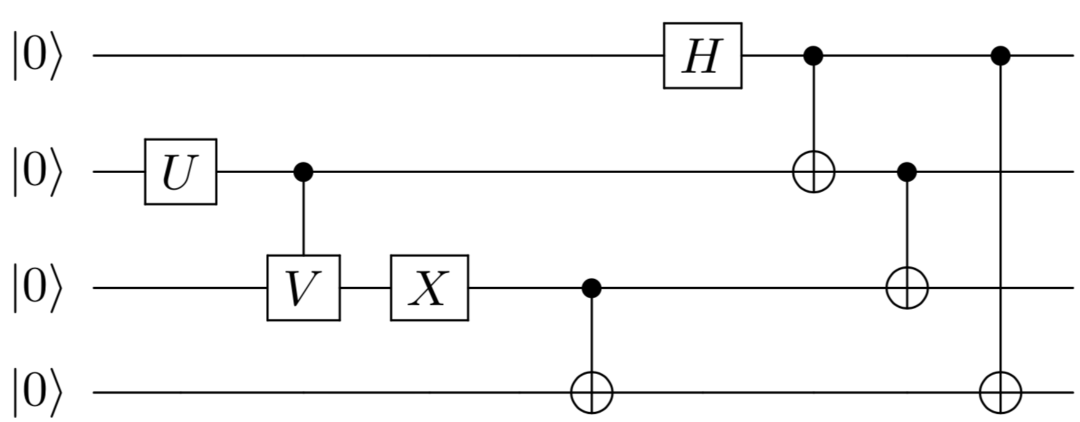
The circuit has to be fitted to the topology of a quantum processor:

IBM Yorktown

5 qubits

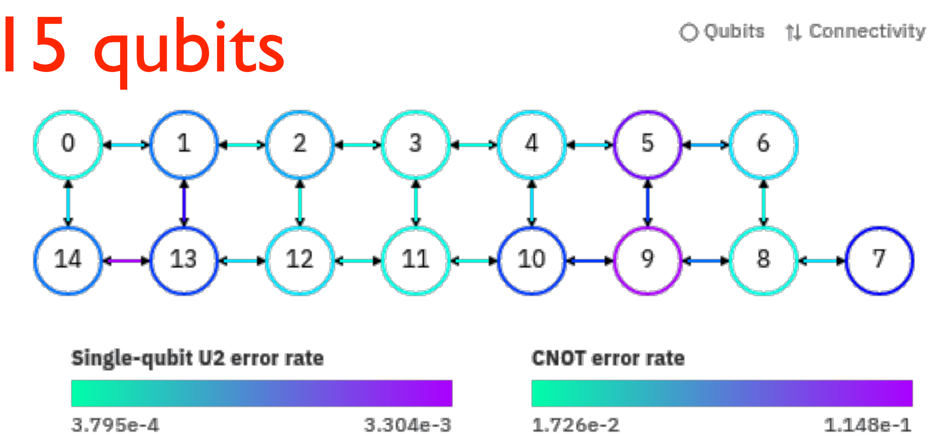




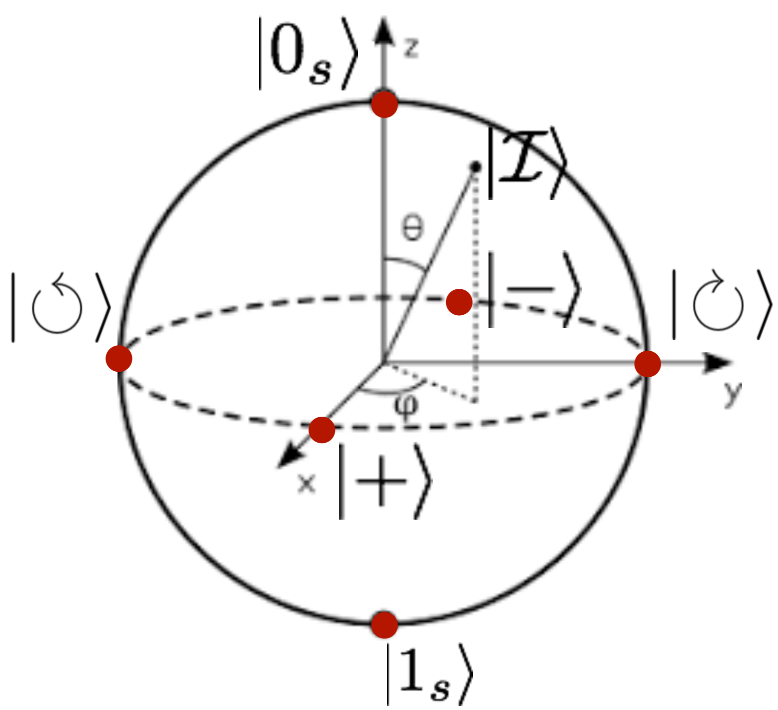


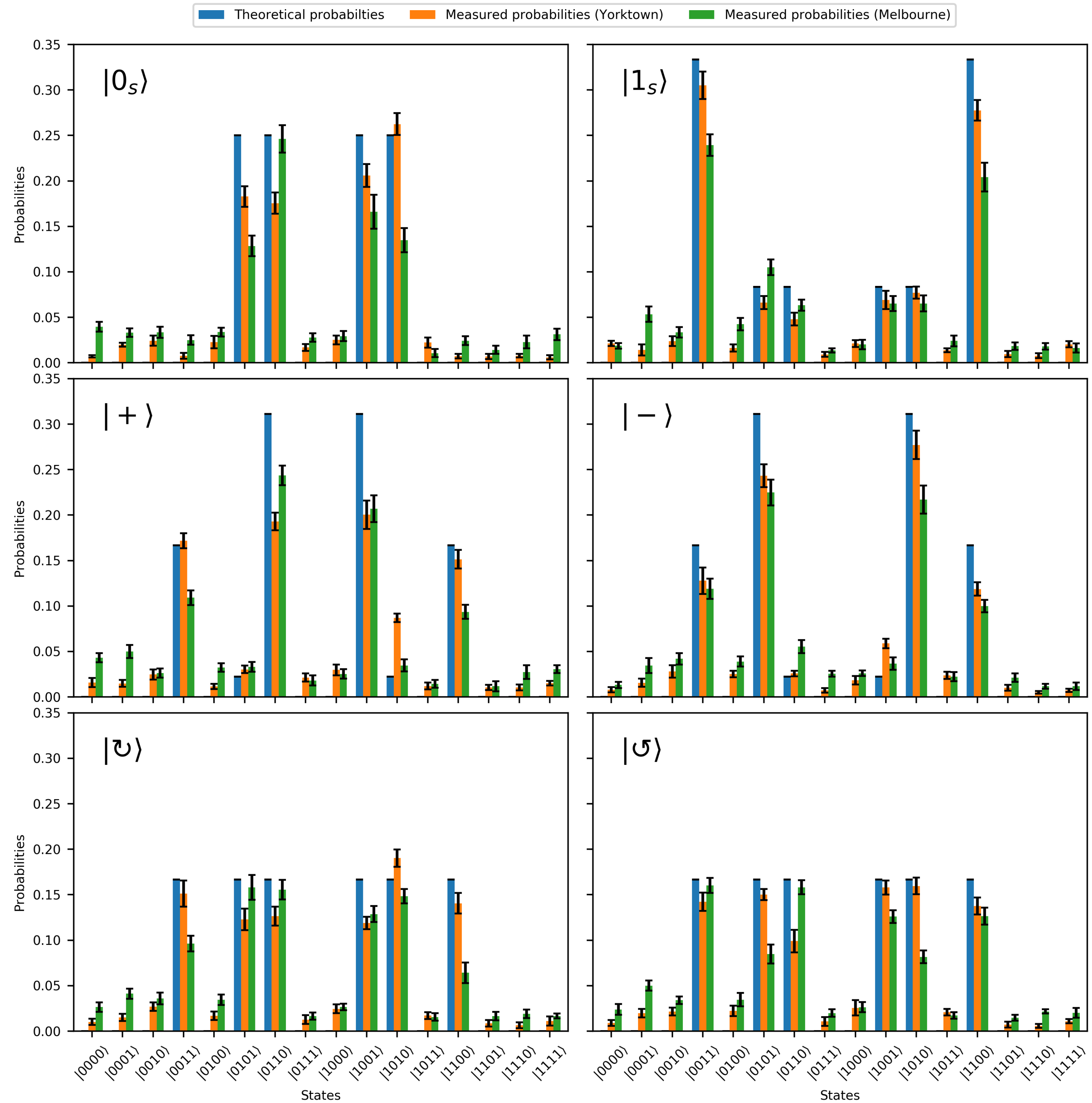
IBM Melbourne

15 qubits



Simulations





A sequence of 10 computational rounds each containing 1024 shots was performed for every of the considered states.

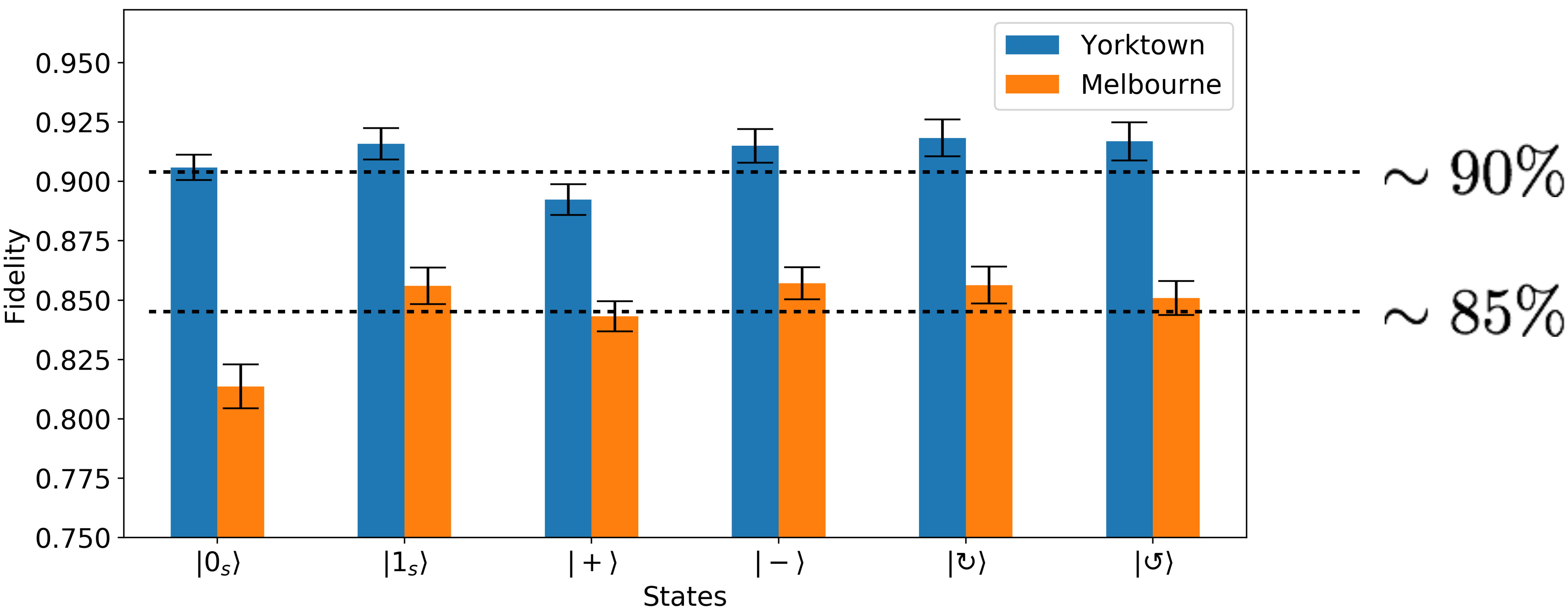
Fidelities

Classical fidelity:

$$F\left(p,q\right)=\sum_i \sqrt{p_i q_i}$$

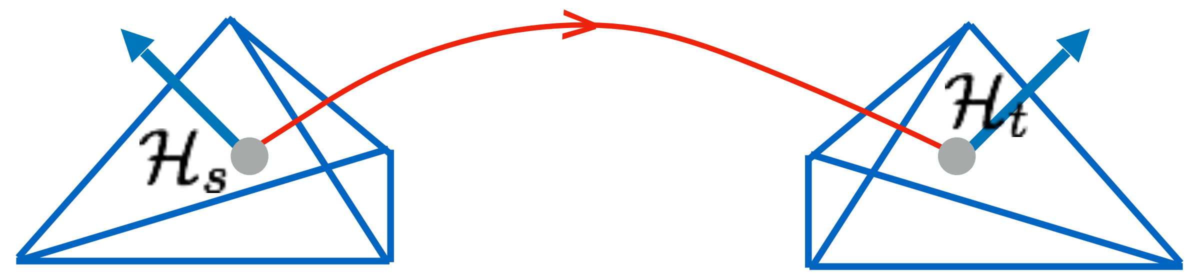
Experimental results:

State	Yorktown	Melbourne
$\lvert 0_s\rangle$	0.906 ± 0.005	0.814 ± 0.009
$\lvert 1_s\rangle$	0.916 ± 0.007	0.856 ± 0.008
$\lvert +\rangle$	0.892 ± 0.007	0.843 ± 0.006
$\lvert -\rangle$	0.915 ± 0.007	0.857 ± 0.007
$\lvert \circlearrowright\rangle$	0.918 ± 0.008	0.856 ± 0.008
$\lvert \circlearrowleft\rangle$	0.917 ± 0.008	0.851 ± 0.007



Beyond a single node…

SU(2) holonomies = maximal entanglement



Quantum entanglement is „gluing" together faces of tetrahedra.

The state associated with holonomy can be written as:

$$|\mathcal{E}\rangle = \frac{1}{\sqrt{2}} h^*_{IJ} |I\rangle_s |J\rangle_t \in \mathcal{H}_s \otimes \mathcal{H}_t$$

e.g. $|\mathcal{E}_l\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle)$

h_{IJ} are matrix components of the SU(2) holonomy.

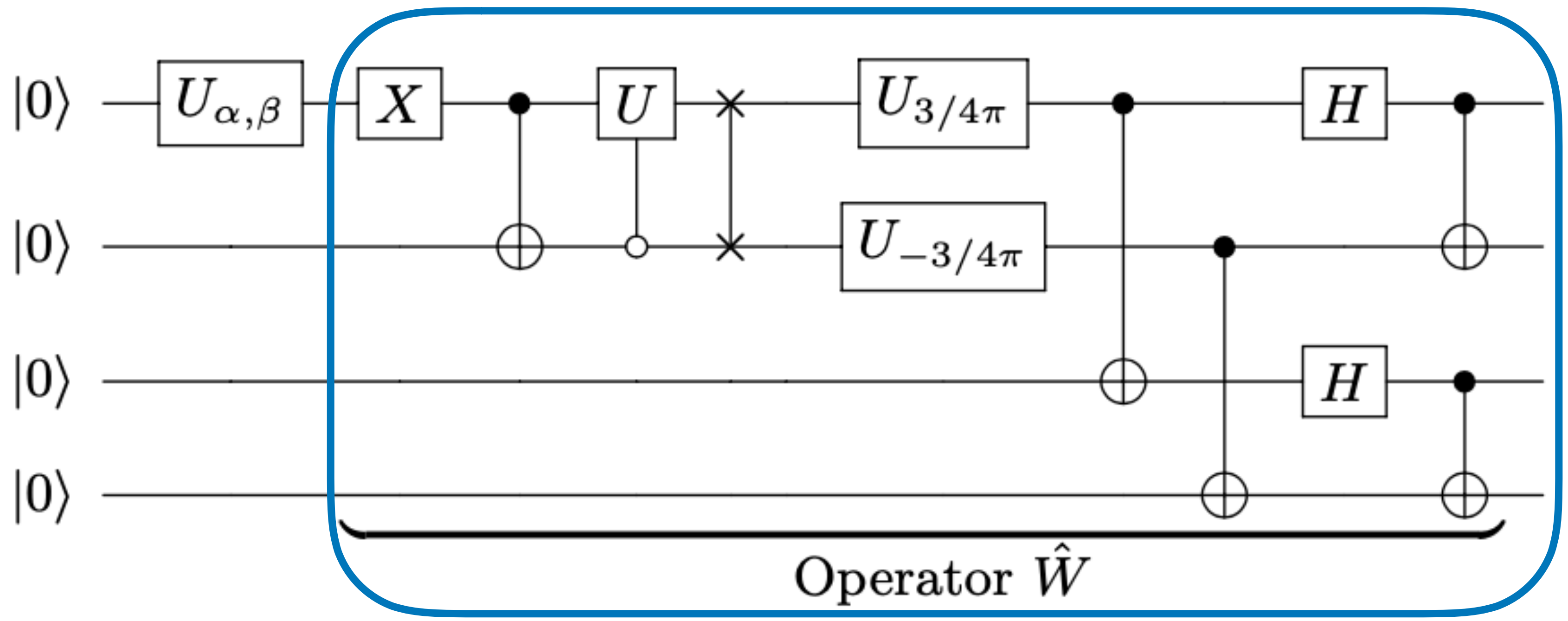
Based on this, Maximally Entangled Spin Network (MESN) states can be introduced:

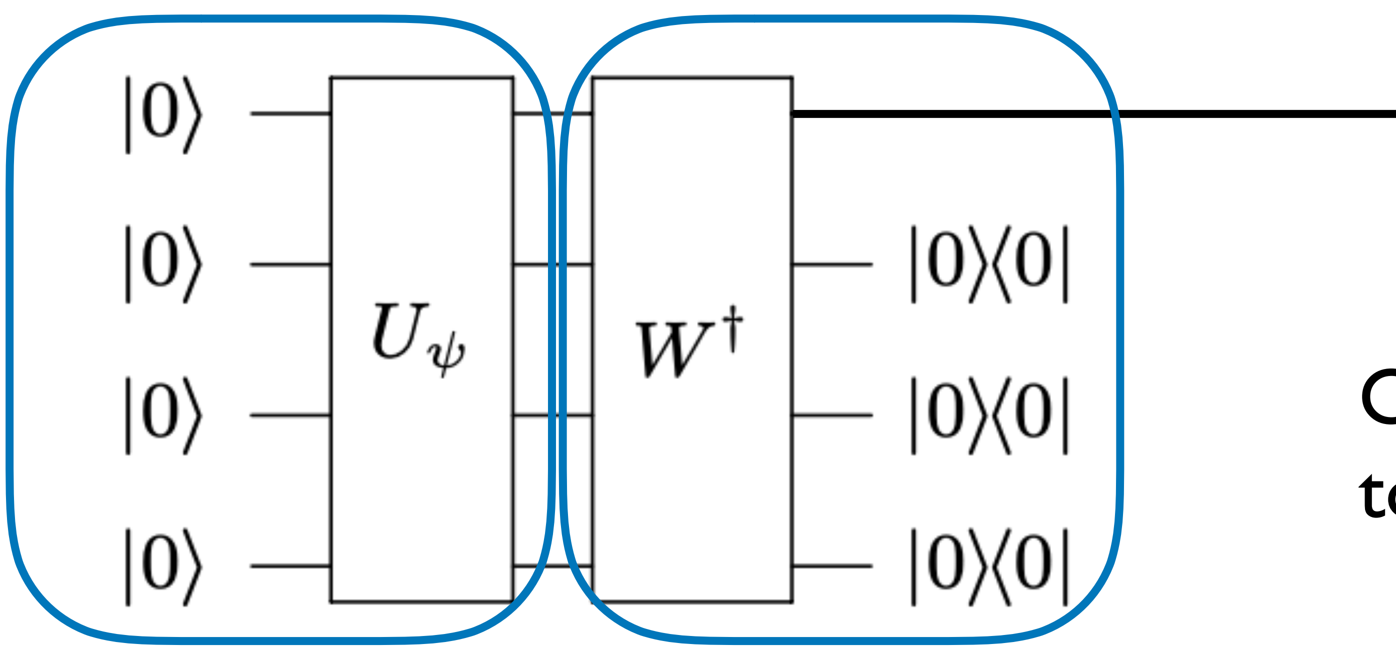
$$|\mathrm{MESN}\rangle := \hat{P}_G \bigotimes_l |\mathcal{E}_l\rangle$$

New circuit for an Ising node

G. Czelusta & JM (2023)

$$\hat{W}\left(\alpha|0\rangle + \beta|1\rangle\right)|000\rangle = |\mathcal{I}(\alpha,\beta)\rangle = \alpha|\iota_0\rangle + \beta|\iota_1\rangle$$

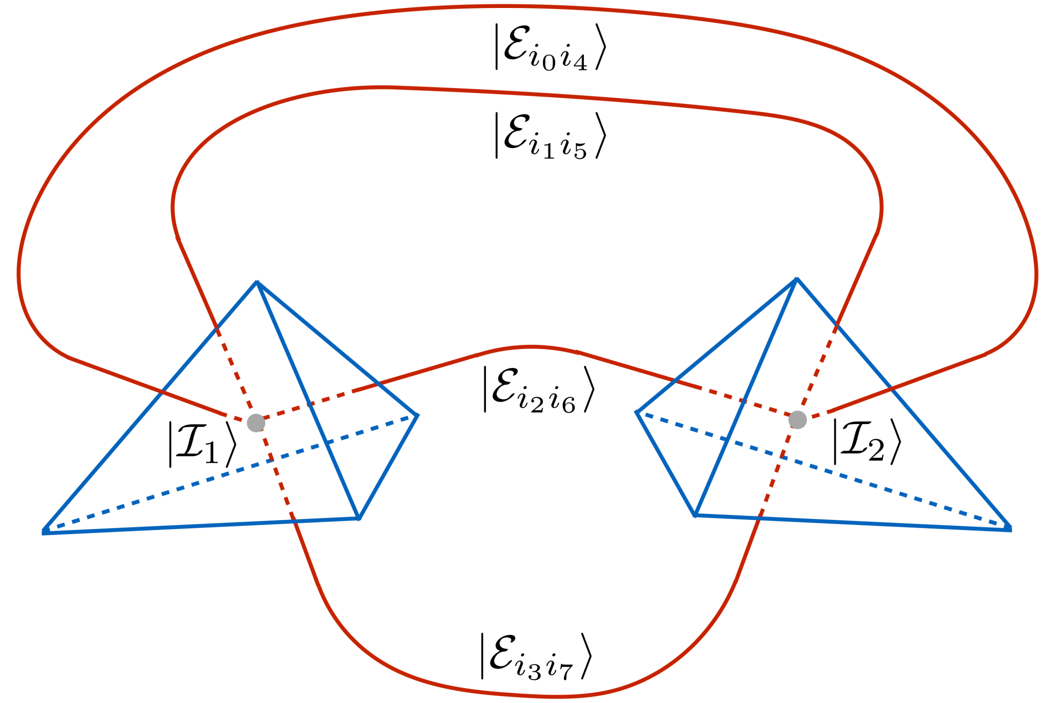


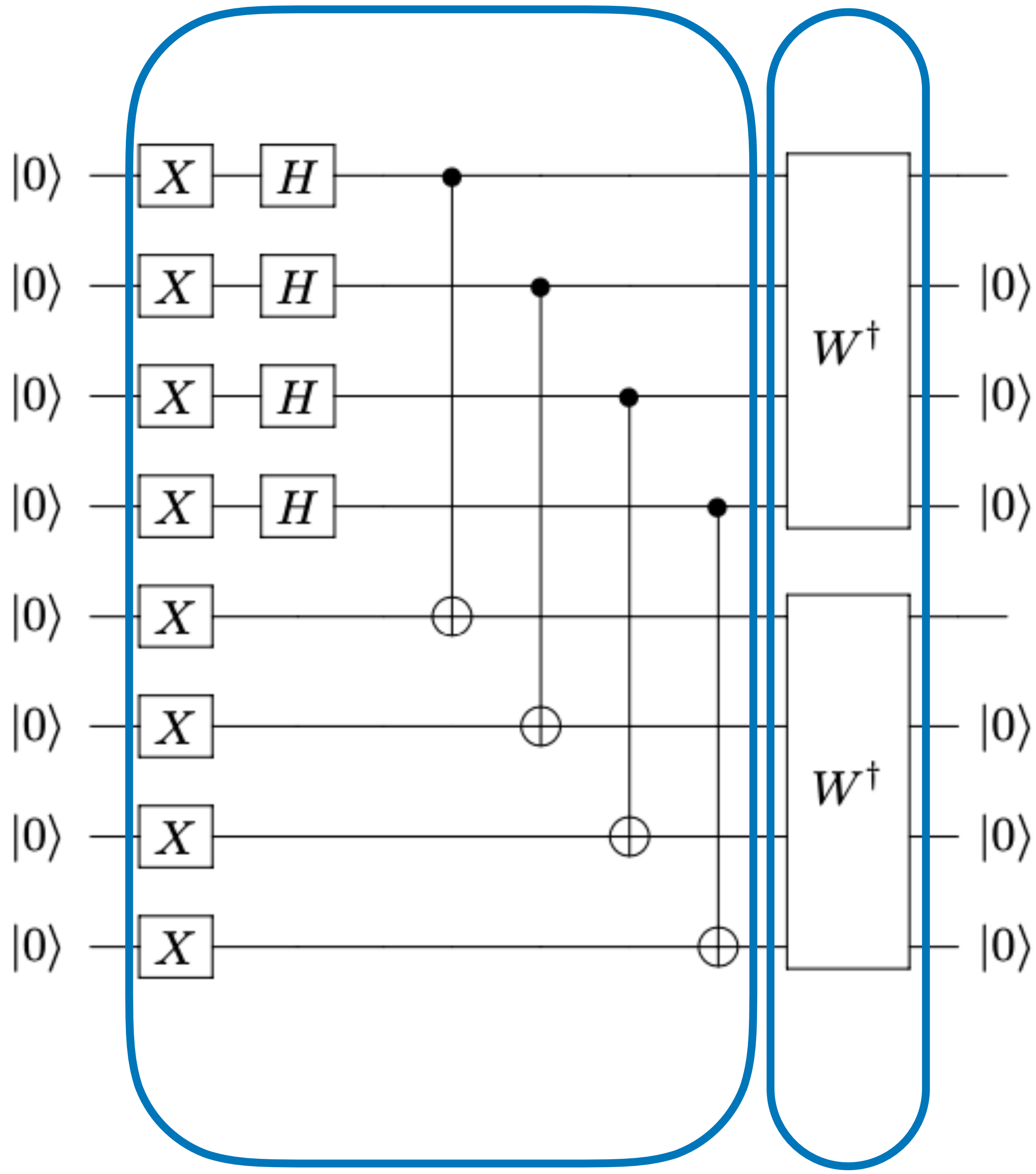


Operator $\hat{W}$ contributes to a „projection" operator

State preparation **Projection**

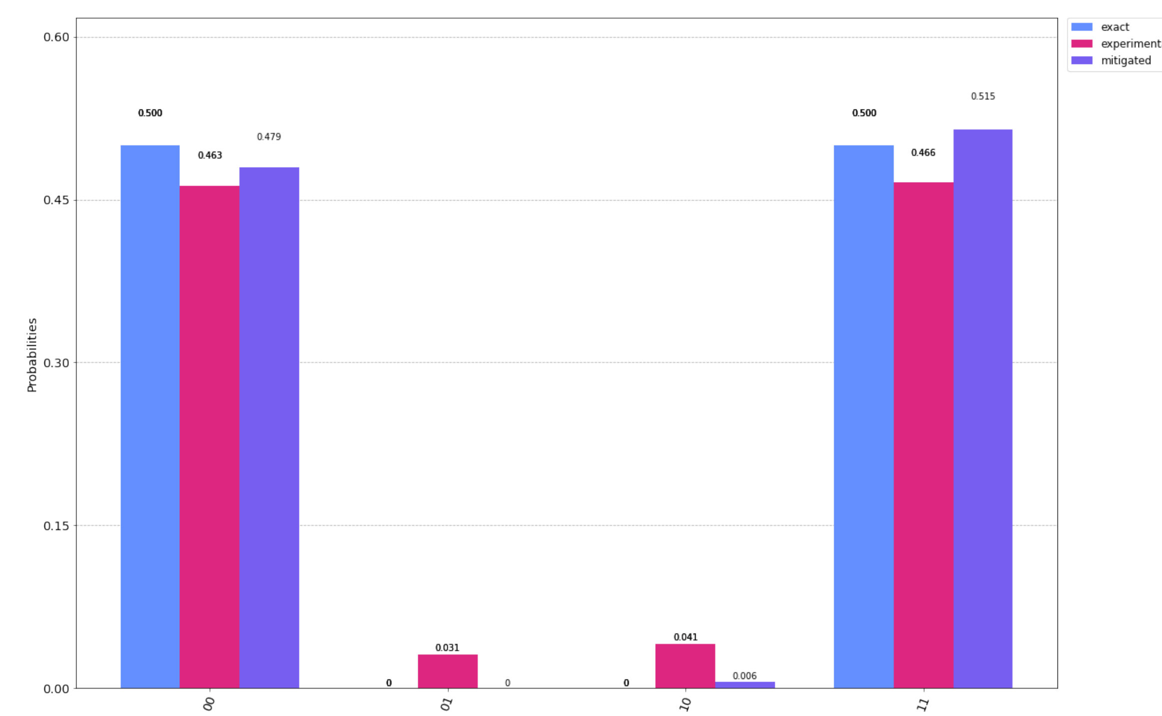
Dipole





Entangled pairs **Projections**

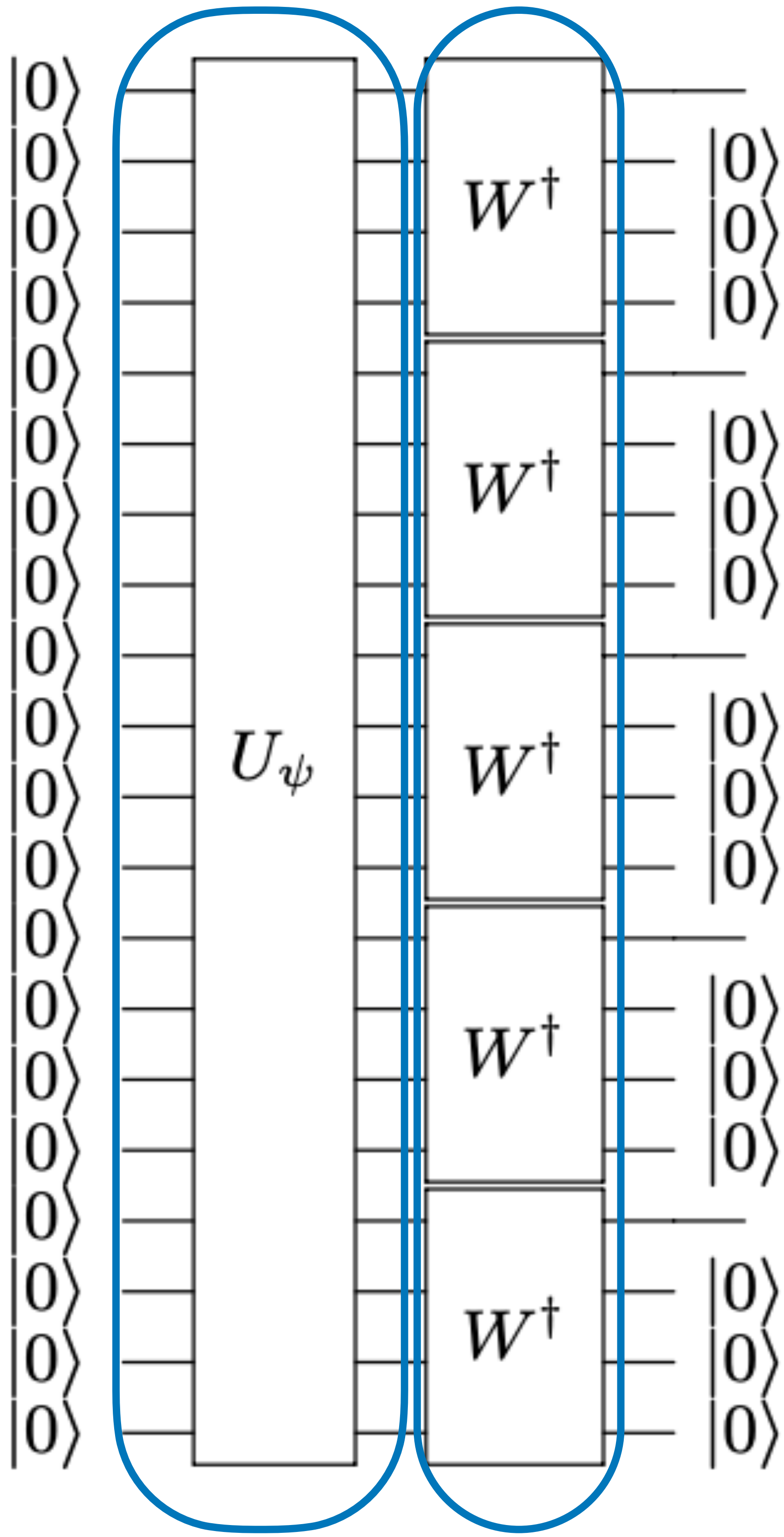
Measured and predicted probabilities:



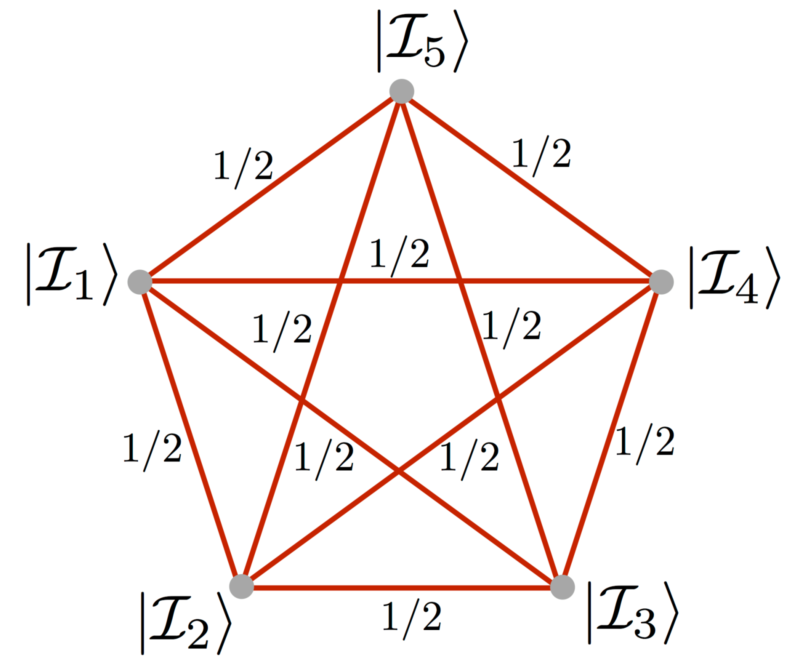
Manila IBM quantum computer

The quantum fidelity of the found state is ≈ 0.99

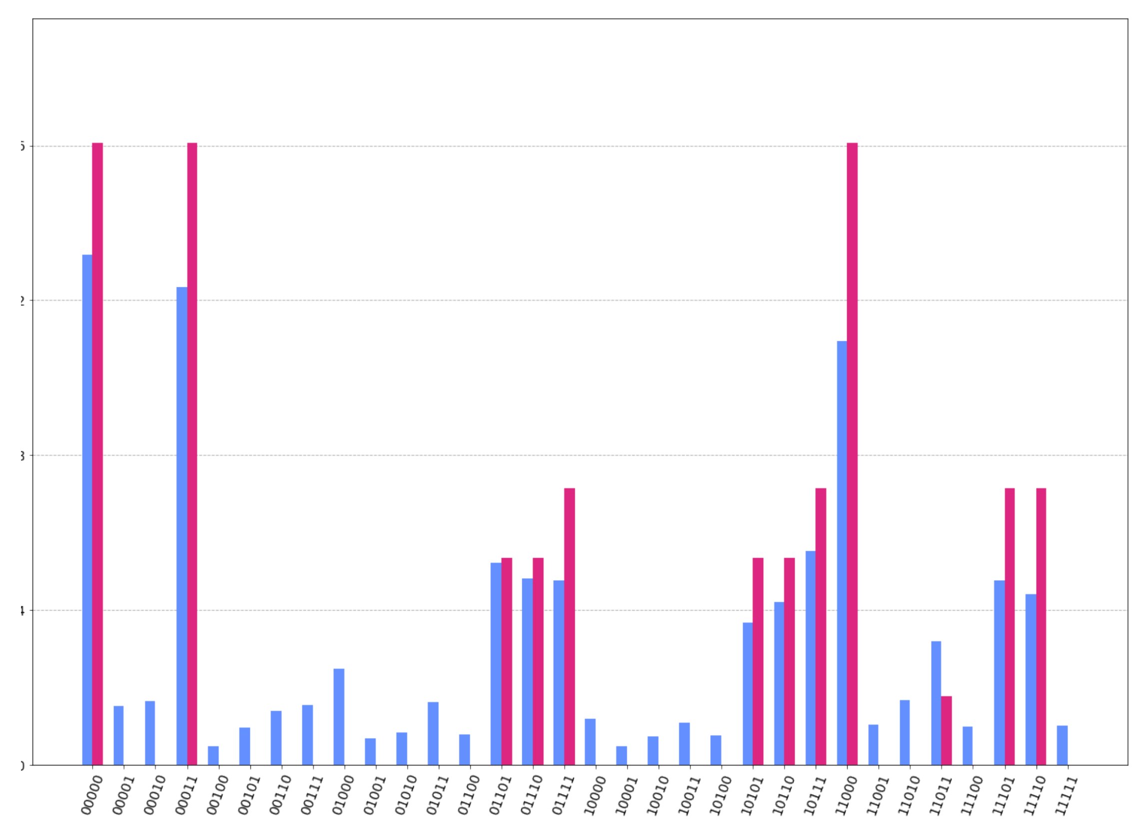
Pentagram



Entangled pairs **Projections**



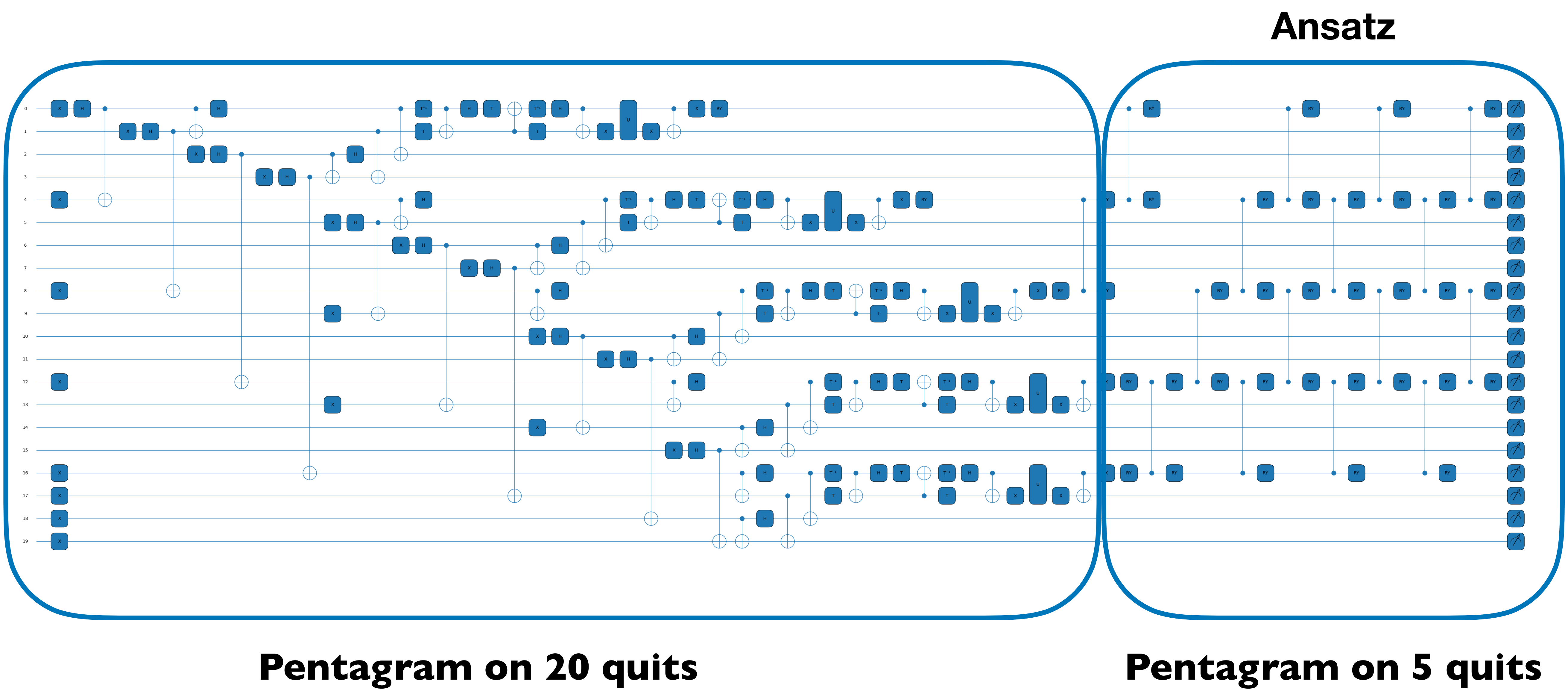
Measured and predicted (from the {15j} symbol) probabilities:



Manila IBM quantum computer

The quantum fidelity of the found state is: ≈ 0.77

Variational transfer of the 5-qubit state of the pentagram



The probability of the state $|0\rangle^{\otimes 20}$ is maximized.

Summary and future prospects

- Holonomies of SU(2) gauge field carry maximal entanglement.
- Gauge invariant states of the discrete SU(2) gauge theory can be introduced and represented as quantum circuits.
- First quantum simulations of SU(2) gauge invariant states have successfully been performed on quantum computers. Better quantum computing resources are needed!
- The quantum computing methods may bring advantage to simulations of the gauge theories - computational complexity to be explored (e.g. using geometric methods).
- Implementation of quantum dynamics is to be done.
- Extension of the construction to other gauge fields, e.g. SU(3) and beyond (large N limit) is an exciting research challenge.

Thank you!

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