Improvement of time-of-flight resolution of PET scanner using additional prompt photon

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J-PET

Agenda:

- Introduction
- Position resolution improvement algorithm
- Simulation results
- Summary

Introduction

- Positron Emission Tomography (PET) is a functional imaging method.

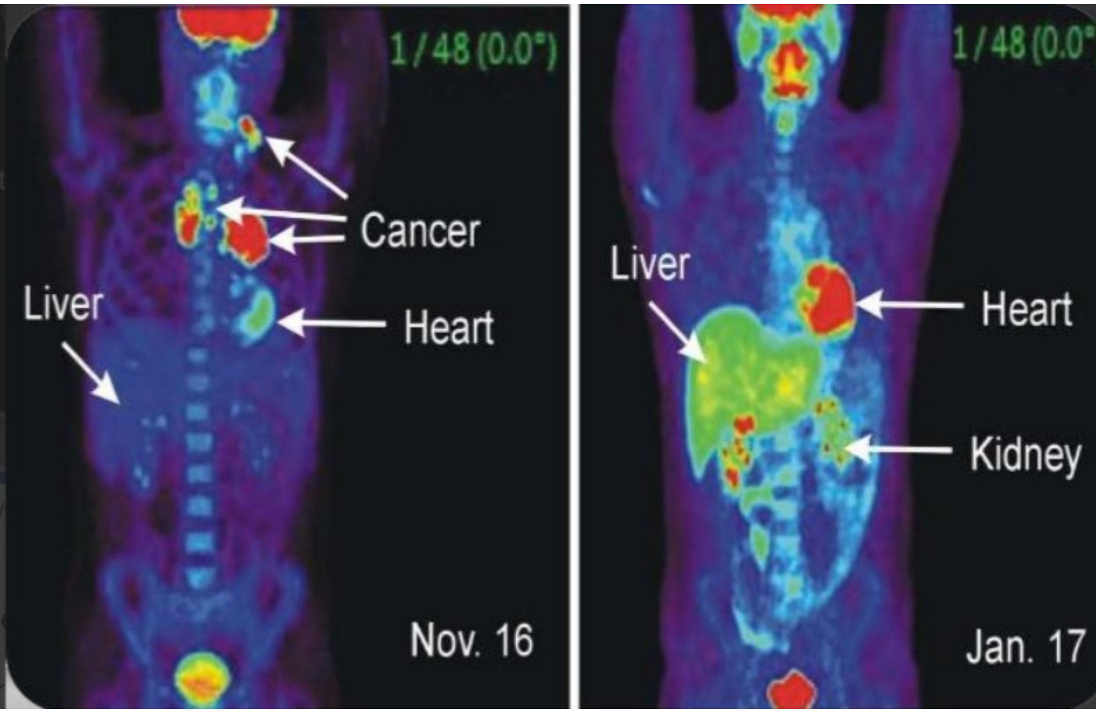


*Figure from:
https://www.openpr.com/news/738094/global-positron-emission-tomography-pet-scanners-market-2017-toshiba-hitachi-neusoft-topgrade-ealthcare.html

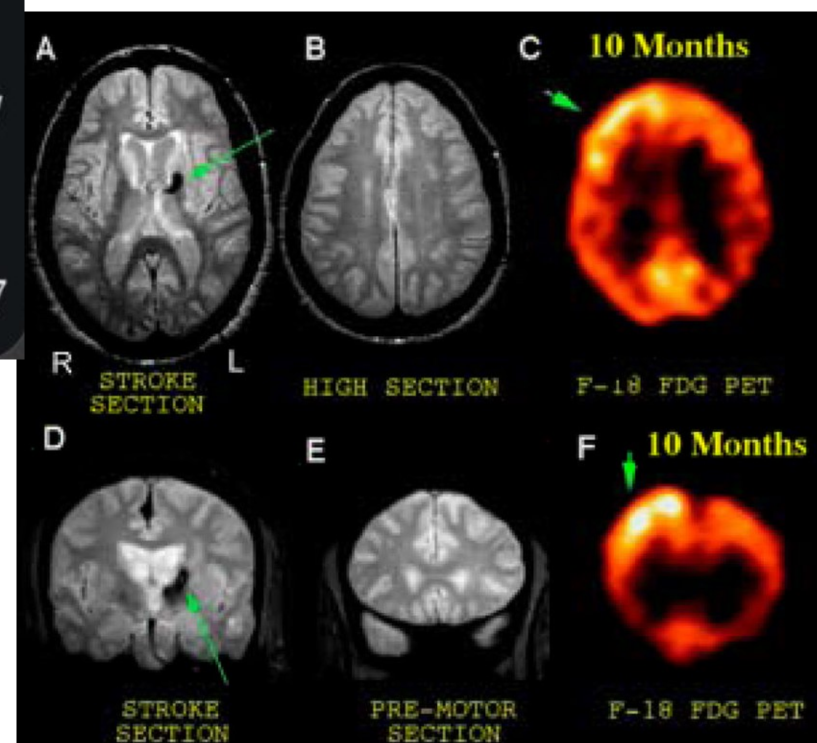


*Figure from:
James C. Eliassen et al. *Brain-Mapping Techniques for Evaluating Poststroke Recovery and Rehabilitation: A Review.* Top Stroke Rehabil. 2008 ; 15(5): 427–450

Introduction

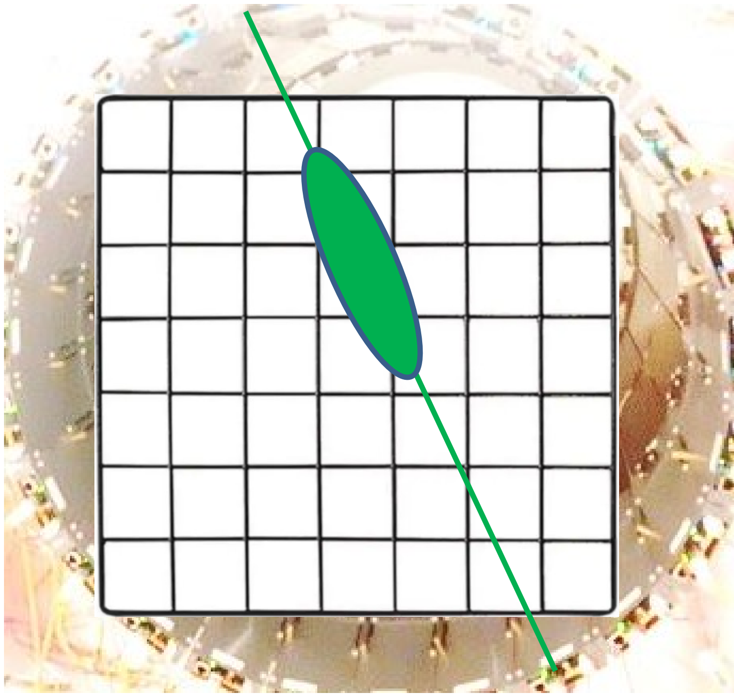
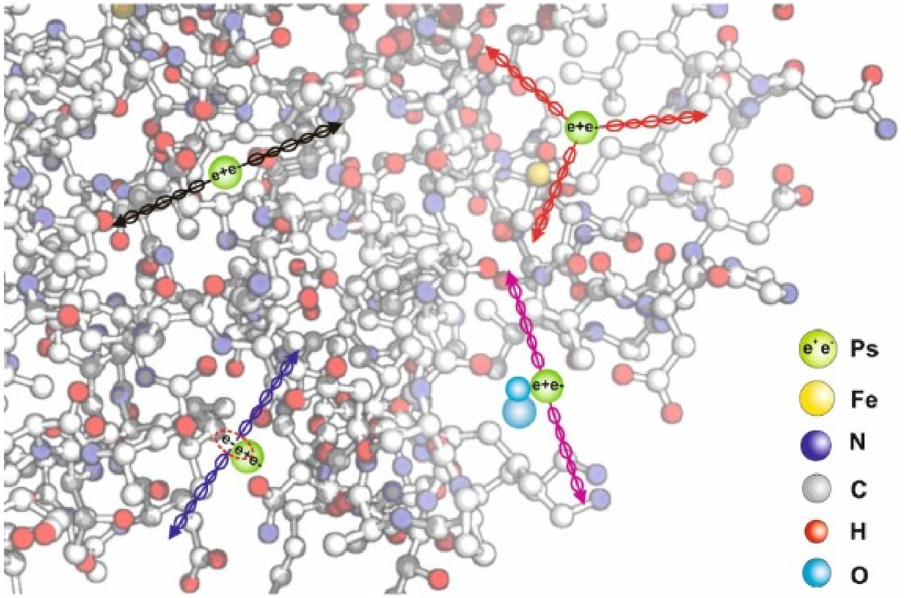


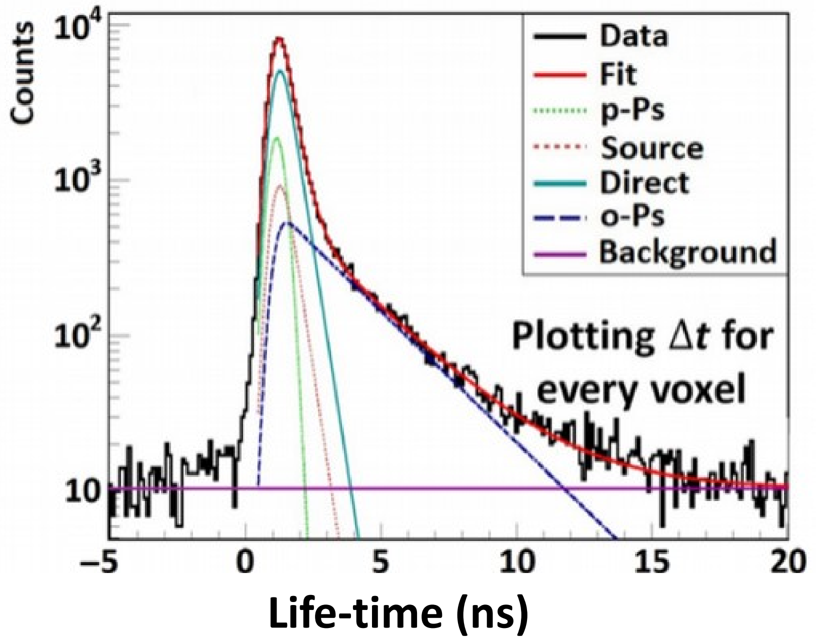
*Figure is adapted from: P. Moskal et al., EJNMMI Phys. 7 (2020) 44.

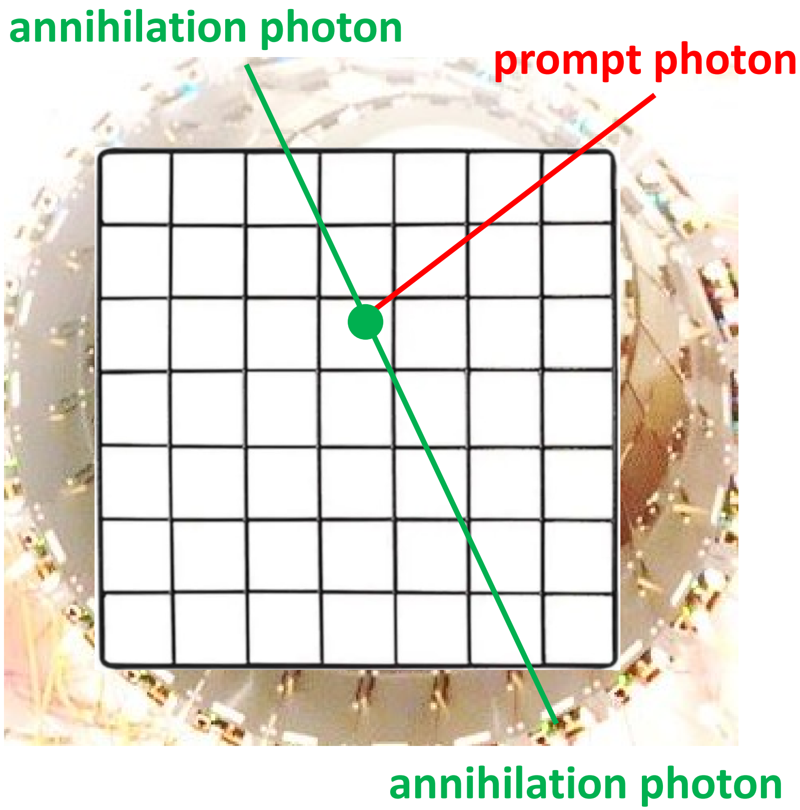


- Radioactive tracer, emitting positrons, is injected into the patient's body.
- Nowadays, in PET imaging two photons along LOR are considered.
- Reconstruction of annihilation position (green ellipse) is based on time measurement of arrival of the two photons at the detectors (left).

Introduction

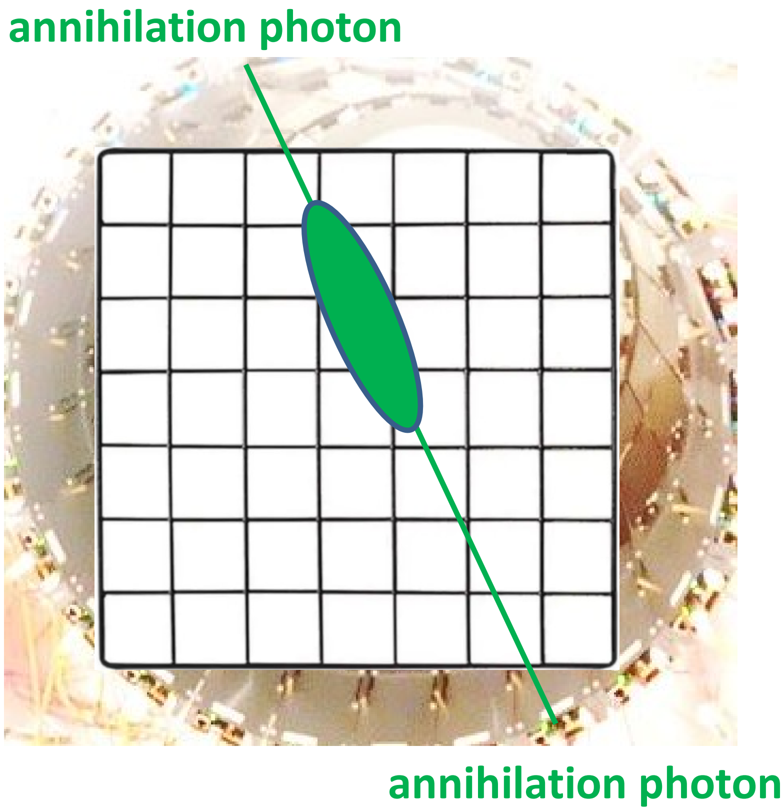
*Figure is adapted from: P. Moskal et al., Science Advance, 2021 ; 7, eabh4394.

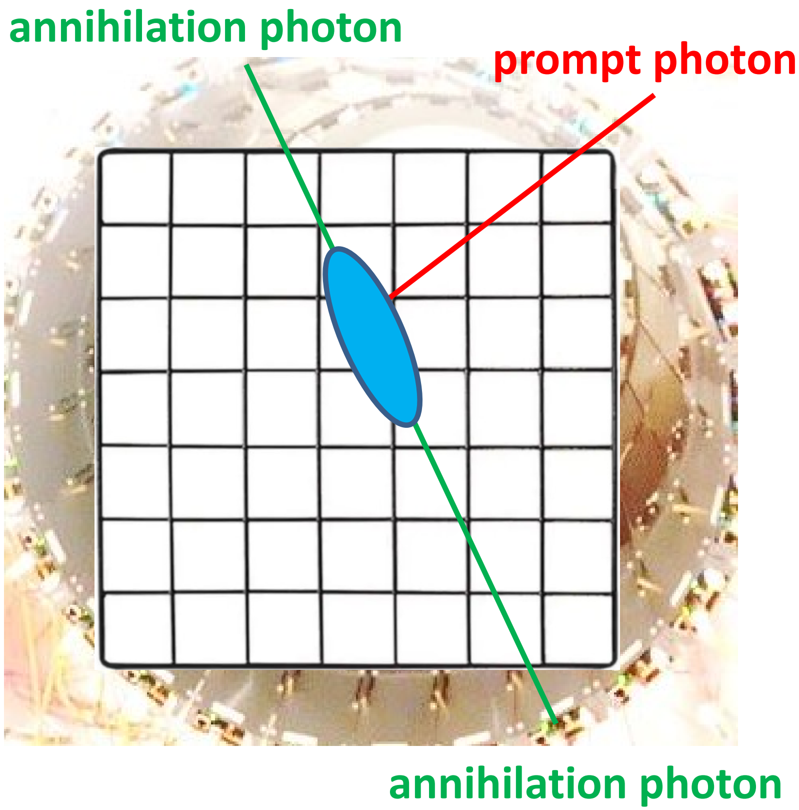




- In the Positronium Imaging, a special radiotracer that emits an additional prompt photon is required.
- The lifetime of Positronium may be estimated using two back-to-back annihilation photons and prompt photon.

Introduction





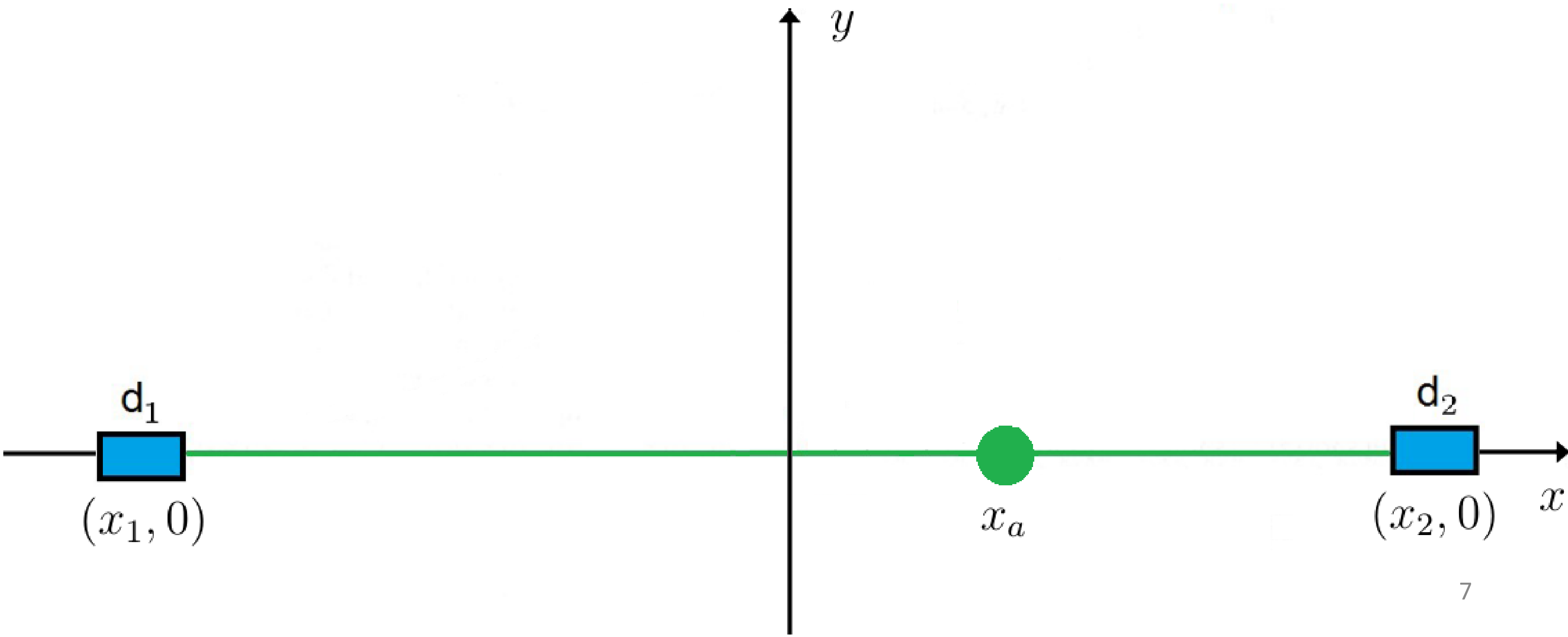
- Reconstruction of radiotracer activity using two annihilation photons only (left) and three photons (right).
- Is it possible to improve the position resolution (reduce the ellipse area) ?

Algorithm 1 Position reconstruction of emission point

Require: $x_1, x_2, t_1, t_2, x_3, y_3, t_3, \mu_\tau, \sigma_\tau$

1: $x_a \Leftarrow \frac{c\,(t_1 - t_2)}{2}$

2: $t_a \Leftarrow \frac{c\,(t_1 + t_2) - |x_1 - x_2|}{2\,c}$



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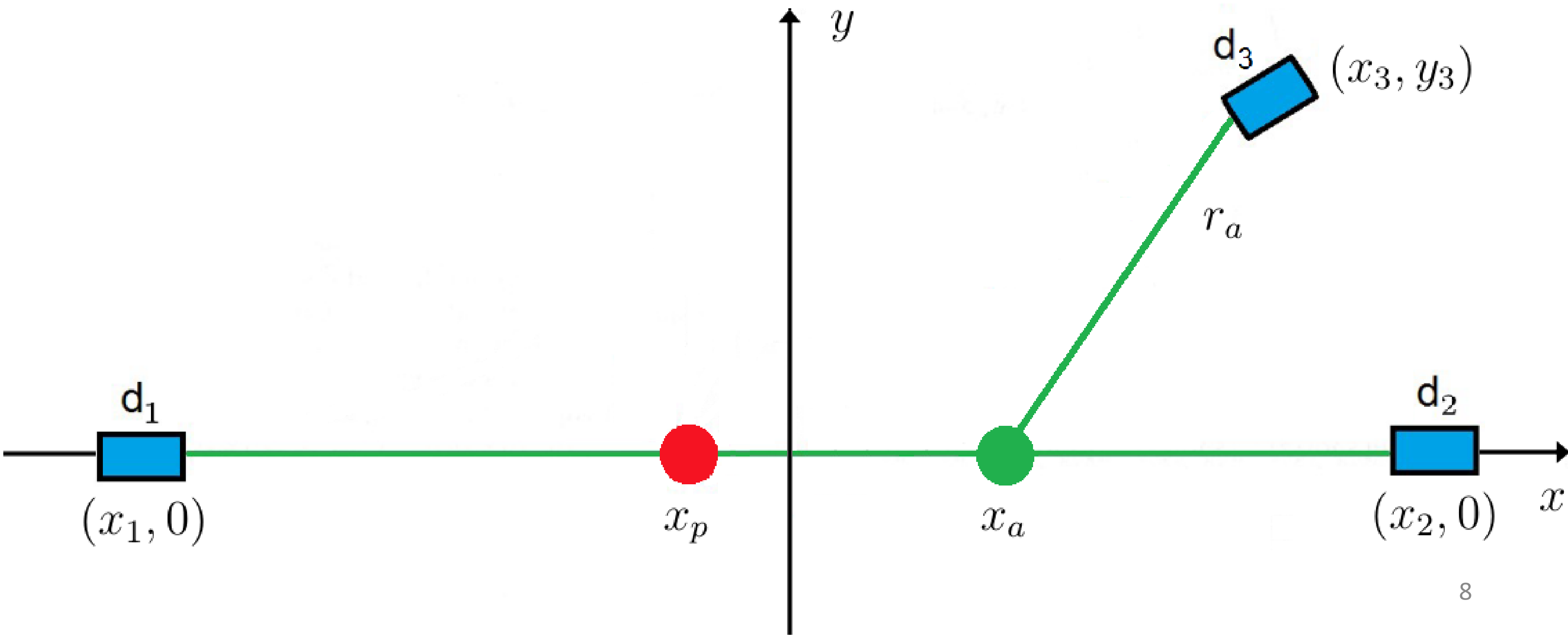
3: $r_a \Leftarrow \sqrt{(x_a - x_3)^2 + y_3^2}$ ▷ calculate distance r_a using annihilation position

4: $\tau \Leftarrow t_a - t_3 + \frac{r_a}{c}$ ▷ calculate lifetime



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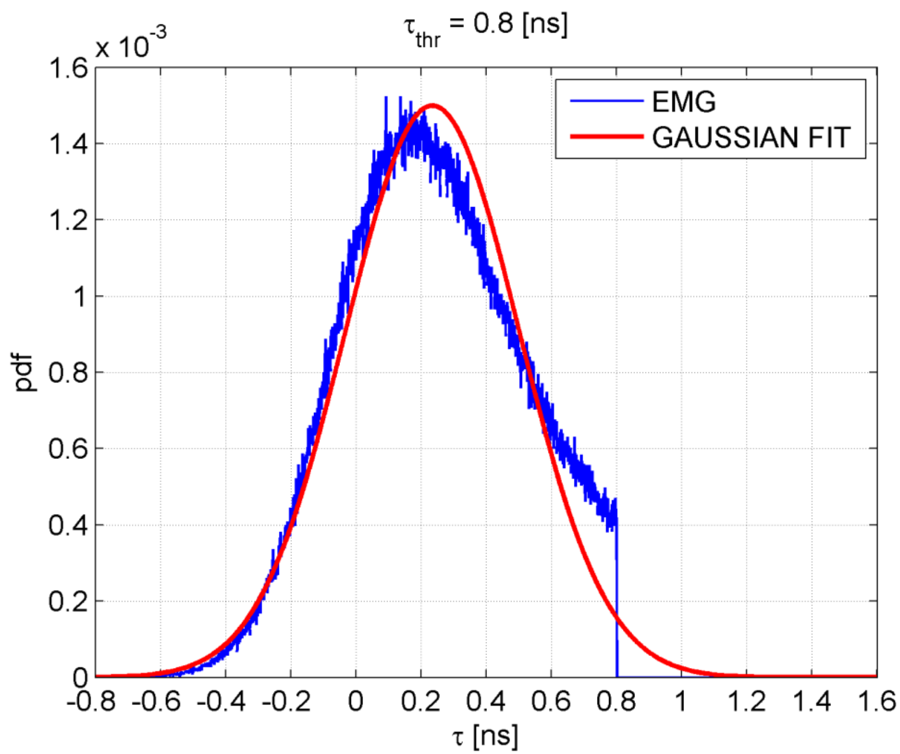
$$\tau \sim \sum_{k=1}^{3} I_k \cdot \mathrm{EMG}\left(\lambda_k, \frac{3}{2}\sigma_t^2\right)$$

$$\tau \sim \mathcal{N}\left(\mu_\tau, \sigma_\tau^2\right)$$

	Intensity (I_k)	Mean lifetime ($1/\lambda_k$)
Direct	0.65	0.388 ns
p-Ps	0.15	0.125 ns
o-Ps	0.20	2.000 ns



*Data is adapted from: P. Moskal et al., Science Advance, 2021 ; 7, eabh4394.

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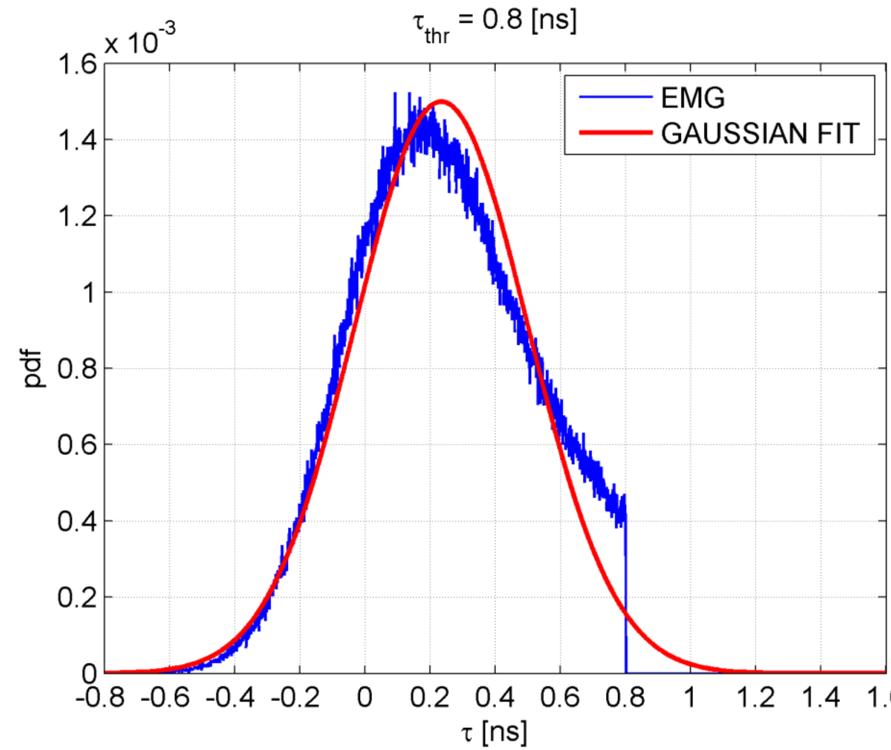
4: $\tau \Leftarrow t_a - t_3 + \frac{r_a}{c}$ ▷ calculate lifetime

5: **if** $\tau < \tau_{\text{thr}}$ **then**

6: $r_p \Leftarrow c\,(t_3 - t_a + \mu_\tau)$ ▷ calculate distance r_p using prior distribution

$$\tau \sim \mathcal{N}\left(\mu_\tau, \sigma_\tau^2\right)$$



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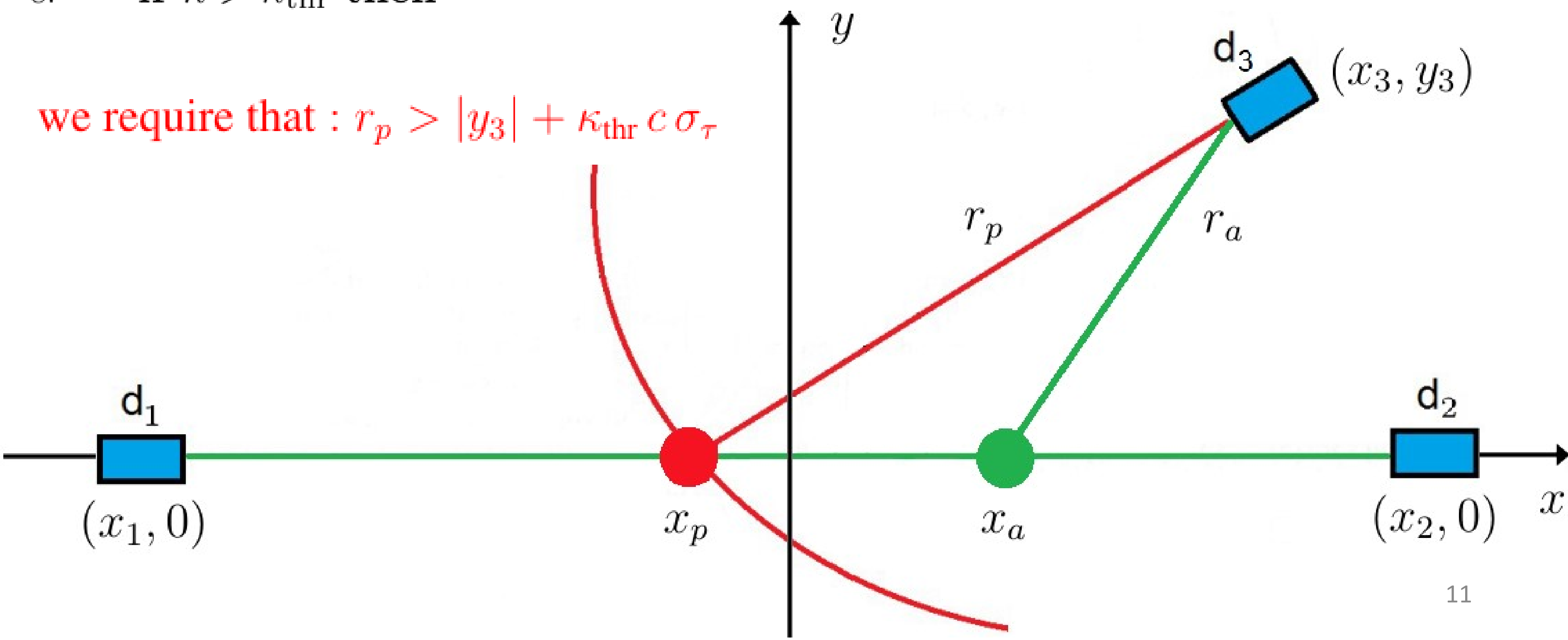
$\tau \Leftarrow t_a - t_3 + \frac{r_a}{c}$ ▷ calculate lifetime

if $\tau < \tau_{\text{thr}}$ **then**

  $r_p \Leftarrow c\,(t_3 - t_a + \mu_\tau)$ ▷ calculate distance r_p using prior distribution

  $\kappa \Leftarrow \frac{r_p - |y_3|}{c\,\sigma_\tau}$

  if $\kappa > \kappa_{\text{thr}}$ **then**



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$r_a \Leftarrow \sqrt{(x_a - x_3)^2 + y_3^2}$ ▷ calculate distance r_a using annihilation position

$\tau \Leftarrow t_a - t_3 + \frac{r_a}{c}$ ▷ calculate lifetime

if $\tau < \tau_{\mathrm{thr}}$ **then**

  $r_p \Leftarrow c\,(t_3 - t_a + \mu_\tau)$ ▷ calculate distance r_p using prior distribution

  $\kappa \Leftarrow \frac{r_p - |y_3|}{c\,\sigma_\tau}$

  if $\kappa > \kappa_{\mathrm{thr}}$ **then**

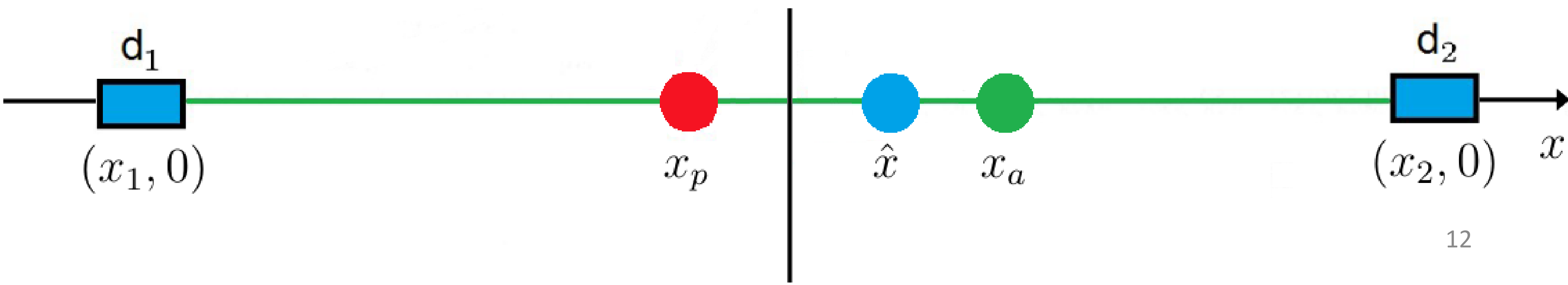
    $x_p \Leftarrow x_3 - \mathrm{sign}(x_3)\sqrt{r_p^2 - y_3^2}$ ▷ calculate position x_p using prompt photon

    $\sigma_p \Leftarrow c\,\sigma_\tau \frac{r_p}{\sqrt{r_p^2 - y_3^2}}$

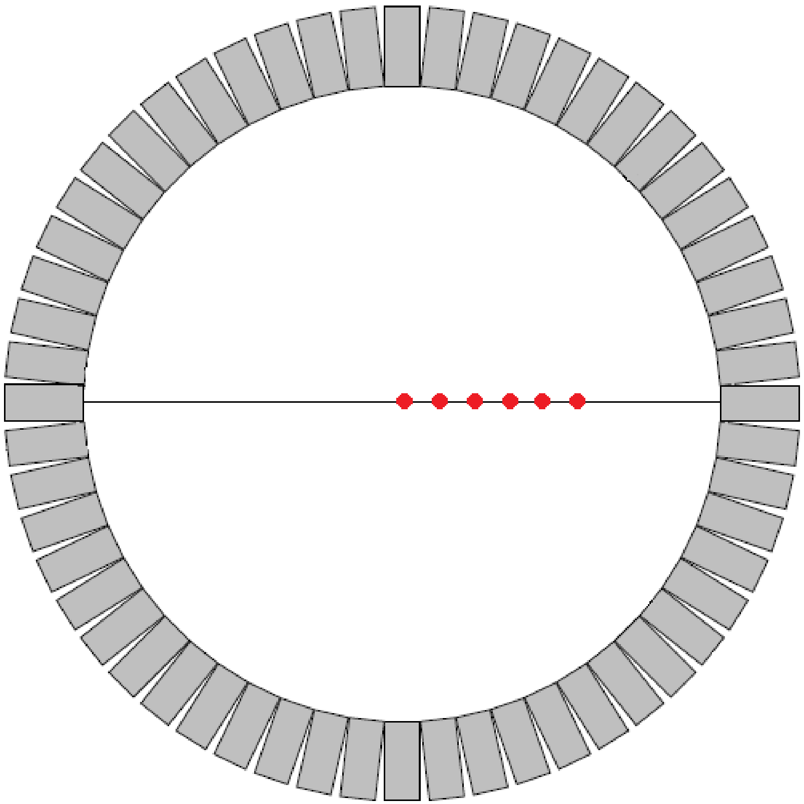
    $\hat{x} \Leftarrow x_a \frac{\sigma_p^2}{\sigma_a^2 + \sigma_p^2} + x_p \frac{\sigma_a^2}{\sigma_a^2 + \sigma_p^2}$ ▷ calculate final position $\hat{x}$

  end if

end if



Simulation results



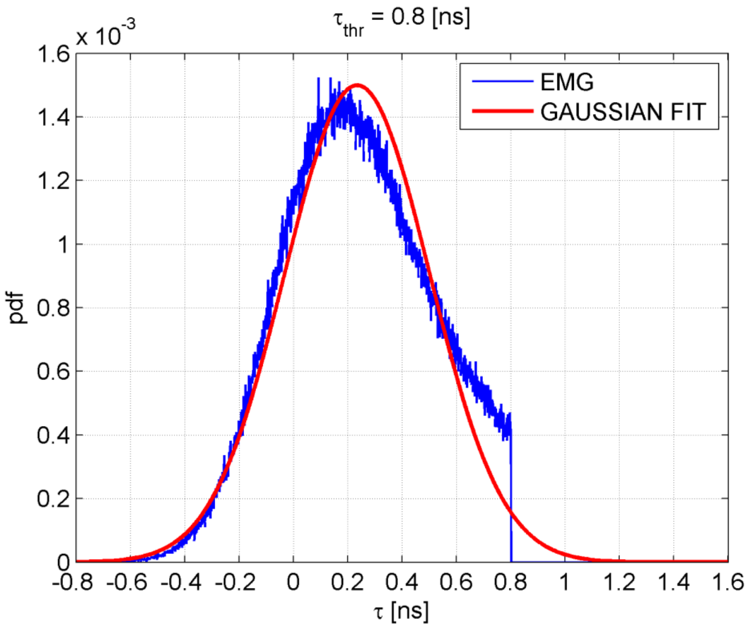
<u>Measurement setup:</u>

- 2-dimensional geometry
- Detector radius: 40 [cm]
- **Coincidence resolving time (CRT): 500 [ps]**
- Radial positions of source [cm]:
  0, 2, 4, 6, 8, 10, 12, 14, 15
- Number of events at each position: 100,000

We simulate only time uncertainty (positions are exact!)

Simulation results

- **Coincidence resolving time (CRT):   500 [ps]**



	Intensity (I_k)	Mean lifetime ($1/\lambda_k$)
Direct	0.65	0.388 ns
p-Ps	0.15	0.125 ns
o-Ps	0.20	2.000 ns

$$\tau \sim \sum_{k=1}^{3} I_k \cdot \mathrm{EMG}\left(\lambda_k, \frac{3}{2}\sigma_t^2\right) \approx \mathcal{N}\left(\mu_\tau, \sigma_\tau^2\right)$$

Prior gaussian distribution of lifetime:

$\mu_\tau = 250$ ps

$\sigma_\tau = 250$ ps

Simulation results

- **Coincidence resolving time (CRT): 500 [ps]**

Standard deviation along LOR based on prompt photon only:

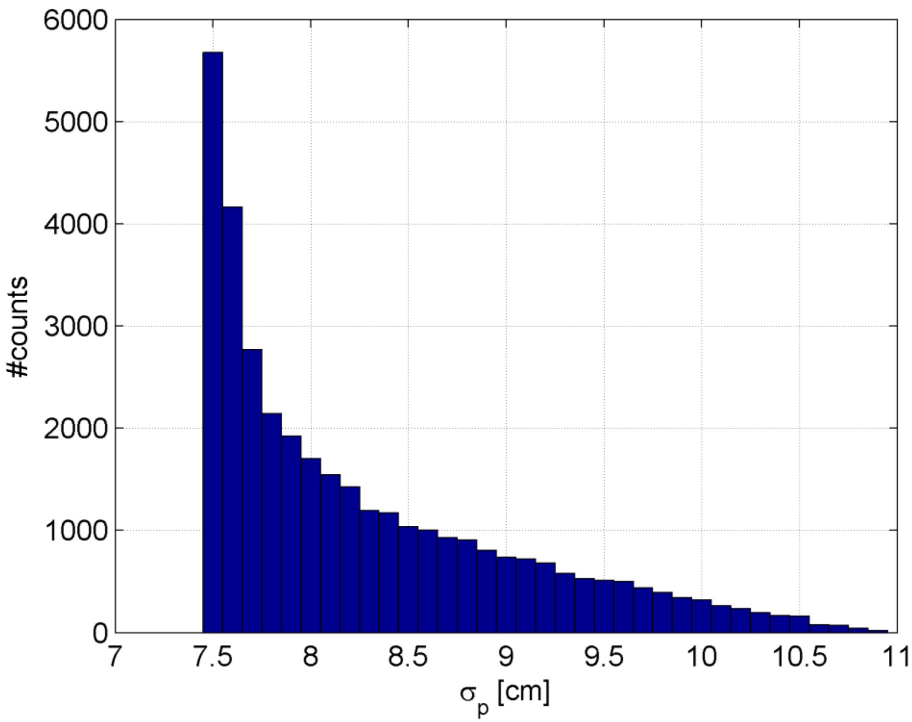
$$\sigma_p = \sigma_r \frac{r_p}{\sqrt{r_p^2 - y_3^2}} \geq 7.5 \text{ cm} \quad (\sigma_r = c\,\sigma_\tau = 7.5 \text{ cm})$$

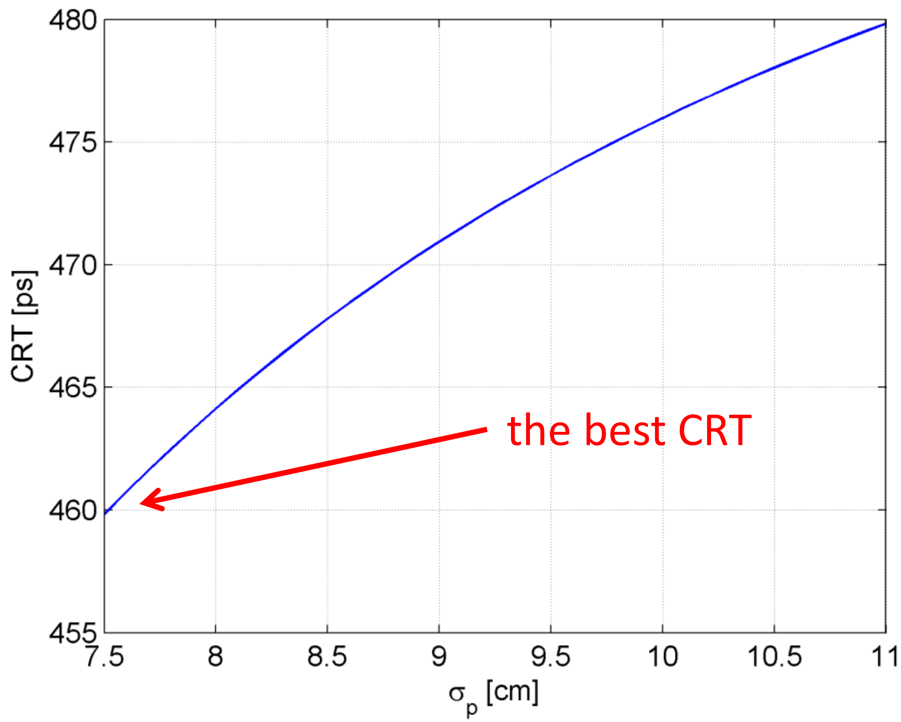
Standard deviation along LOR based on two annihilation photons only:

$$\sigma_a = 3.18 \text{ cm}$$

Simulation results

- Calculation of CRT for point source in (0,0):





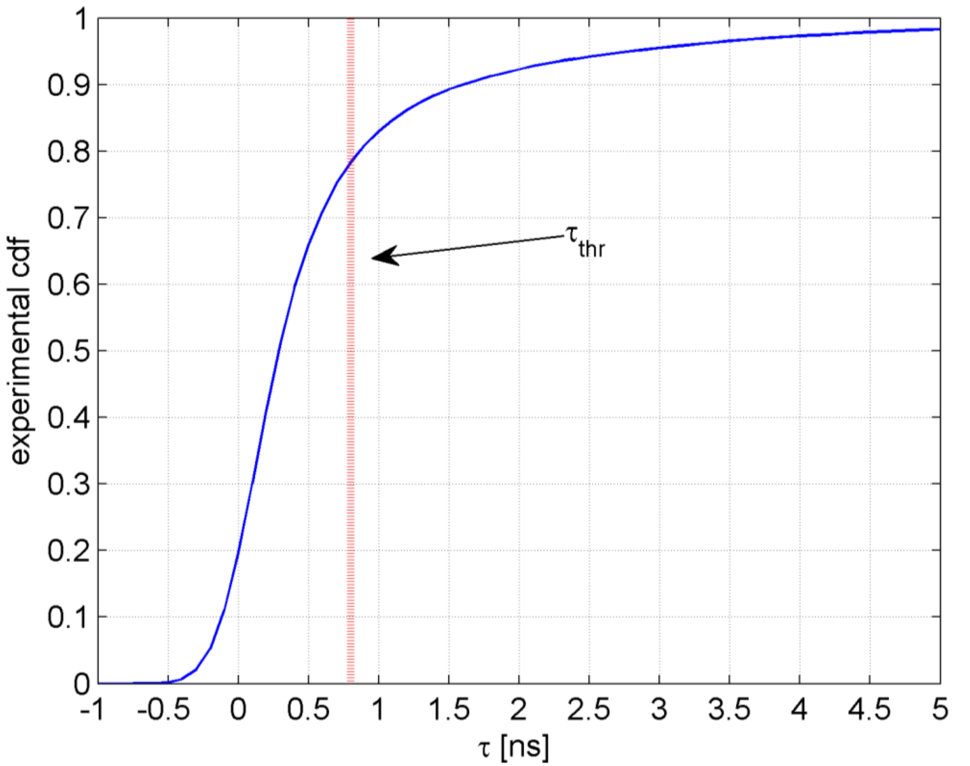
$$\sigma_p = \sigma_r \frac{r_p}{\sqrt{r_p^2 - y_3^2}} \geq 7.5 \text{ cm}$$

$$\hat{\sigma} = \frac{\sigma_a \sigma_p}{\sqrt{\sigma_a^2 + \sigma_p^2}}$$

- For the smallest σ_p (left figure) we get best possible CRT of about **460 ps** (right figure).

Simulation results

CONDITION I		
$\tau < \tau_{\mathrm{thr}}$	YES	77%
	NO	23%



Simulation results

CONDITION I		
$\tau < \tau_{\mathrm{thr}}$	YES	77%
	NO	23%

CONDITIONS I and II		$\kappa > \kappa_{\mathrm{thr}}$	
		YES	NO
$\tau < \tau_{\mathrm{thr}}$	YES	35%	42%
	NO	-	23%

Simulation results

- Calculation of CRT for point source in (0,0)
- Take into account that reconstruction with prompt photon may be provided in 35% of cases:

$$\sigma_a^2 = \frac{1}{N_a} \sum_{i}^{N_a} \left(x_a^i - \mu\right)^2$$

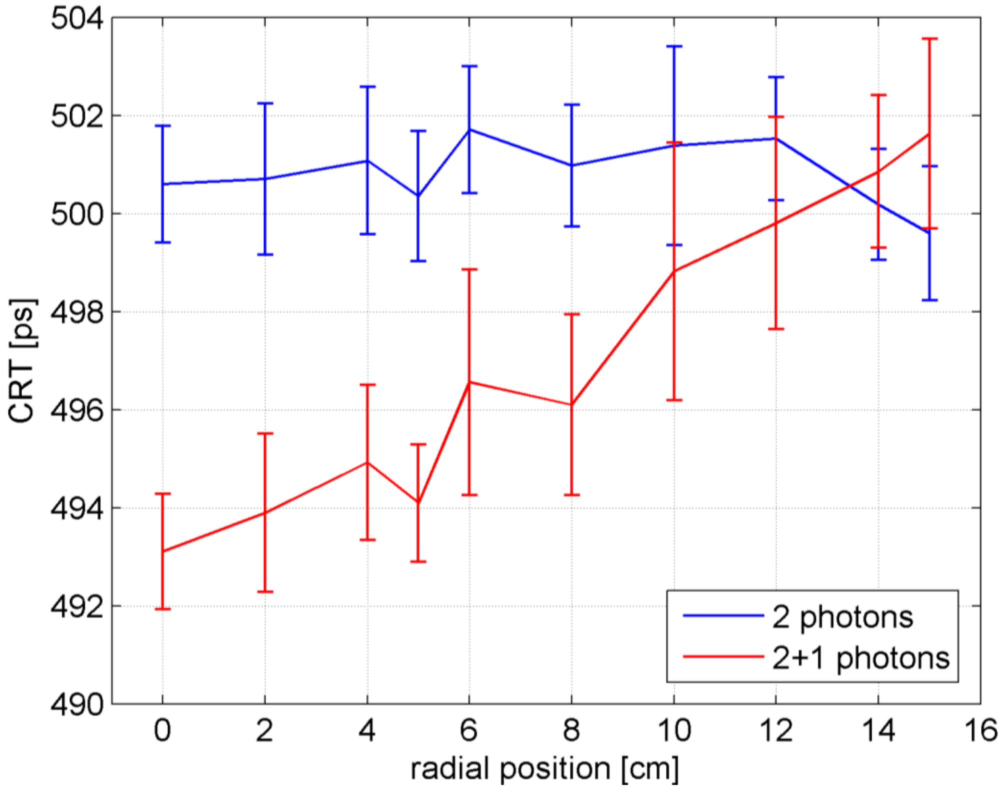
$$\hat{\sigma}^2 = \frac{1}{\hat{N}} \sum_{i}^{\hat{N}} \left(\hat{x}^i - \mu\right)^2$$

$$\sigma_{tot}^2 = \frac{1}{N_{tot}} \left(\sum_{i}^{N_a} \left(x_a^i - \mu\right)^2 + \sum_{j}^{\hat{N}} \left(\hat{x}^j - \mu\right)^2 \right)$$

where: $N_{tot} = N_a + \hat{N}$ and $\frac{N_a}{N_{tot}} = 0.65$, $\frac{\hat{N}}{N_{tot}} = 0.35$.

Therefore: $\mathrm{CRT}_{tot} \approx \sqrt{0.65\,(500)^2 + 0.35\,(466)^2} \approx$ 489 ps

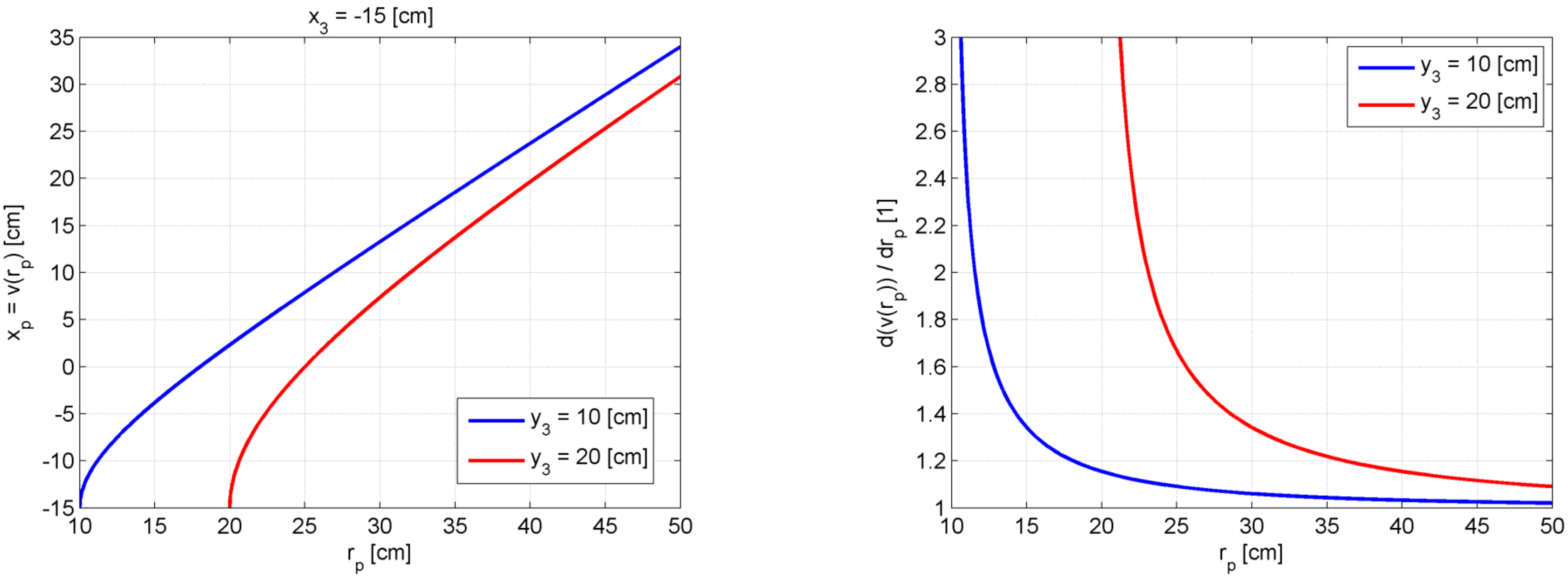
Simulation results



- Calculation of CRT for point sources in positions from 0 to 15 cm.
- The reference reconstruction with annihilation photons only does not depend on radial position (CRT about 500 ps – see blue curve).
- In the proposed algorithm the worsening of the CRT with increasing radial distance is observed.

Summary

- The algorithm for position reconstruction using two annihilation photons and prompt photon was proposed.
- There are two reasons why information from additional prompt photon does not improve significantly the CRT resolution:
    1. The proposed reconstruction with prompt photon may be provided in only 1/3 of cases.
    2. The estimate provided by prompt photon alone is smaller than expected (wide distribution of positronium lifetime).
- Future work:
    - ❑ Investigation of resolution improvement for CRTs smaller than 500 ps.
    - ❑ Consideration of both time and position uncertainties (in presented results positions were exact).



$$x_p = v(r_p) = \begin{cases} x_3 - \sqrt{r_p^2 - y_3^2} & x_3 \geq 0 \\ x_3 + \sqrt{r_p^2 - y_3^2} & x_3 < 0 \end{cases}$$

The linear approximation of a function $v()$:

$$x_p \approx x_3 \pm \left(\sqrt{\mu_r^2 - y_3^2} + (r_p - \mu_r) \frac{\partial}{\partial r_p}(v(\mu_r)) \right)$$