Quantum Neural Networks: current status and next steps

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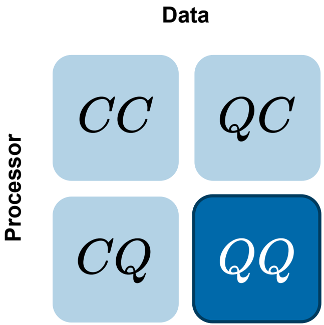
- Quantum Information Theory
- Bound Entanglement and Bell States
- Quantum Machine Learning



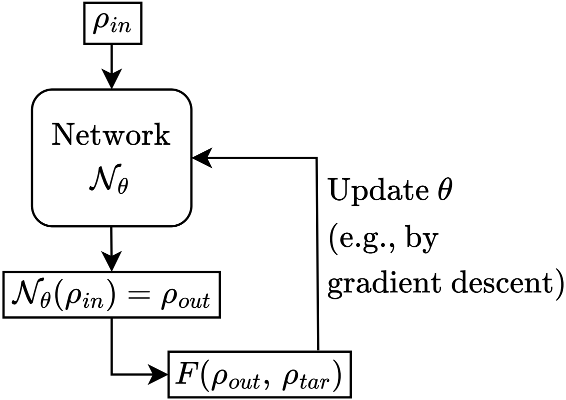
Overview

- Mathematical Framework of Supervised Quantum Machine Learning
- Dissipative Quantum Neural Networks (DQNNs)
- Numerical Results
- Outlook

Mathematical Framework of Supervised QML

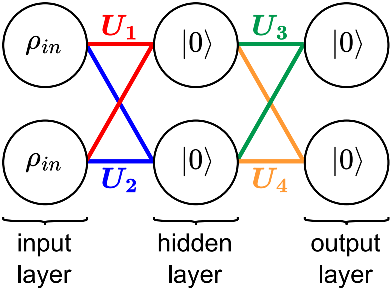
- **Input data (quantum state):** $\rho_{in} \in \mathcal{H}_{in}$
- **Output state:** $\rho_{out} \in \mathcal{H}_{out}$
- **Most general (linear) quantum map:** $\mathcal{N}_\theta : \mathcal{H}_{in} \to \mathcal{H}_{out}$
- **Target state (for training):** $\rho_{tar} \in \mathcal{H}_{out}$
- **Cost function (e.g., fidelity):** $F(\rho_{out}, \rho_{tar}) = \mathrm{Tr}\left(\sqrt{\sqrt{\rho_{tar}}\,\rho_{out}\,\sqrt{\rho_{tar}}}\right)^2$

Mathematical Framework of Supervised QML



Dissipative Quantum Neural Networks (DQNN)

- **Structure:** input layer - hidden layer(s) - output layer
- Neurons = qudits
  Weights/bias = unitary transformations



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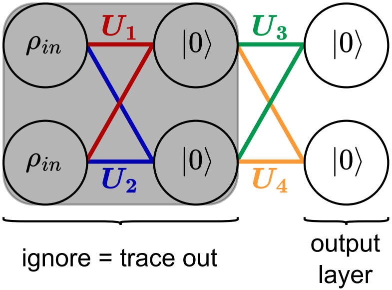
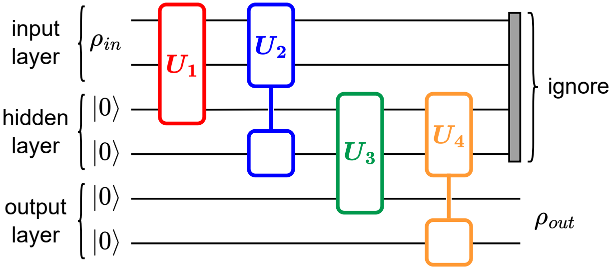


Dissipative Quantum Neural Networks (DQNN)

- **Quantum Circuit Diagram:**



- DQNN is variational quantum circuit with local unitaries and local output

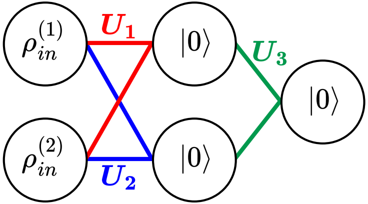
Learning Purity of Qudits

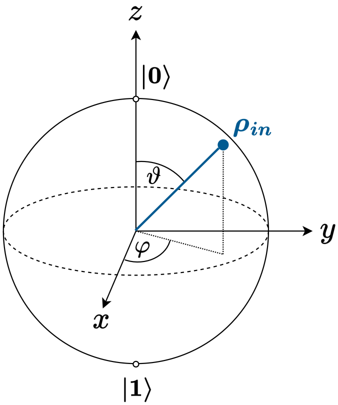
- **Purity of a quantum state ρ_{in}:**

$$\mathrm{Pur}(\rho_{in}) = \frac{d}{d-1}\left(\mathrm{Tr}(\rho_{in}^2) - \frac{1}{d}\right)$$

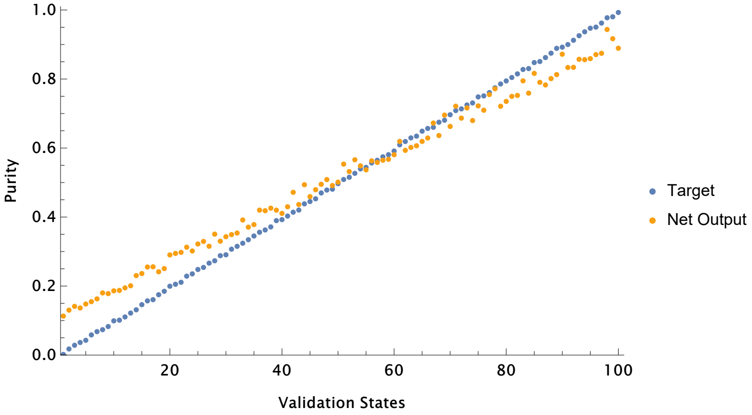
- **Target state:**

$$\rho_{tar} = \frac{1+\mathrm{Tr}(\rho_{in}^2)}{2}|0\rangle\langle 0| + \frac{1-\mathrm{Tr}(\rho_{in}^2)}{2}|1\rangle\langle 1|$$

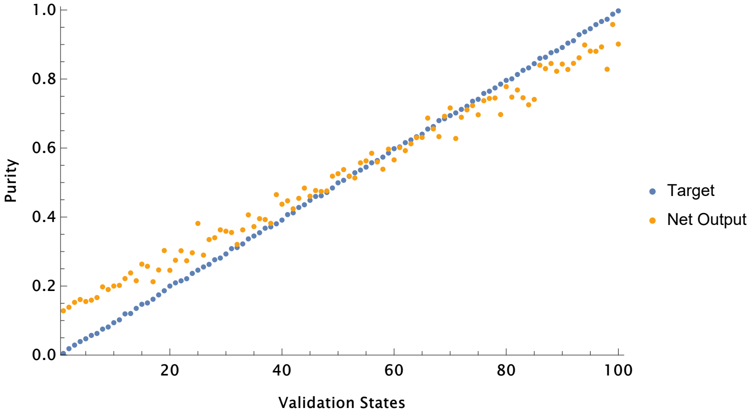




Learning Purity of Qubits



Learning Purity of Qutrits



Learning Entanglement of Pure Qubits

- **Pure bipartite qubit state:**

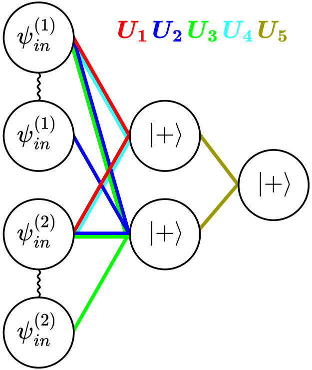
  $$|\psi\rangle \in \mathbb{C}^2 \otimes \mathbb{C}^2$$

- **Entanglement (concurrence) of state $|\psi_{in}\rangle$:**

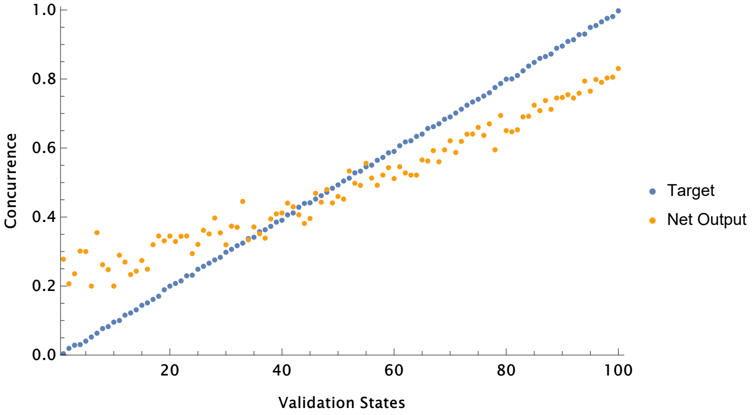
  $$\mathrm{Con}(\psi_{in}) = |\langle\psi_{in}|\,(\mathrm{Y}\otimes\mathrm{Y})\,|\psi_{in}^*\rangle|$$

- **Target state:**

  $$\rho_{tar} = \frac{4-\mathrm{Con}(\psi_{in})^2}{4}|0\rangle\langle 0| + \frac{\mathrm{Con}(\psi_{in})^2}{4}|1\rangle\langle 1|$$



Learning Entanglement of Pure Qubits



Outlook

- What is the best way to introduce non-linearities in quantum machine learning algorithms?
- Universality of DQNNs with local unitaries, small net size, and with/without feed-forward?
- Use qudit-networks for:
  - → Separability problem for qutrits
  - → Distillability of ququart Werner states