Asymptotic Safety and the Litim Sannino Model

Daniele Rizzo

In collaboration with

Daniel Litim

During my visit at

Sussex University
Brighton, UK

Graduate Seminar
NCBJ

NARODOWE CENTRUM NAUKI

NATIONAL CENTRE FOR NUCLEAR RESEARCH ŚWIERK

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11/04/2024

Legend

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Legend

Simple Slide:
understandable by everyone,
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this is your moment to check your phone
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It will never really be technical,
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e.g. $\epsilon^3 < \epsilon^2$ if $\epsilon < 1$

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PUT
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How to build a model of particles?

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Gauge Symmetry

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Fermionic Fields

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Interactions between Fields

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$$\mathcal{L}_{\text{int}} \supset \bar{\psi} H \psi$$

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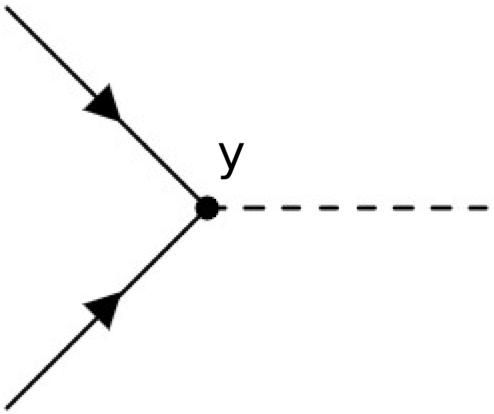

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Running of Coupling Constants

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Running of Coupling Constants

In quantum field theory
coupling constants are not constant,
they depend on the energy scale of the
process under consideration

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Beta Function

$$\beta(g) \equiv \frac{dg}{d\log\mu}$$

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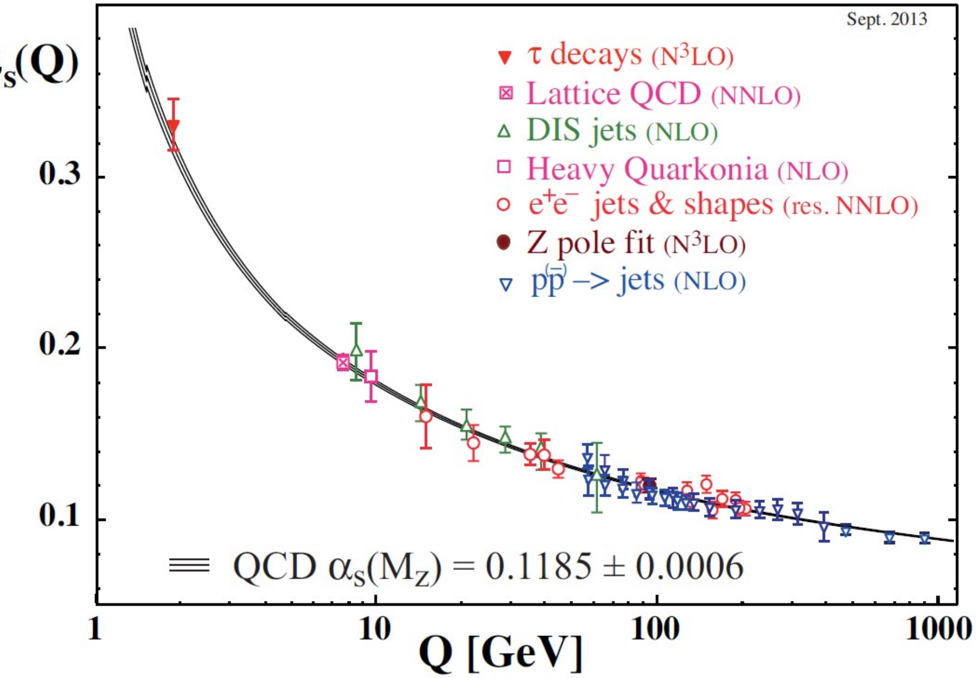
In quantum field theory coupling constants are not constant, they depend on the energy scale of the process under consideration

$$\beta(g) \equiv \frac{dg}{d\log\mu}$$

$$\frac{g_c^2}{4\pi} \equiv \alpha_s(Q)$$



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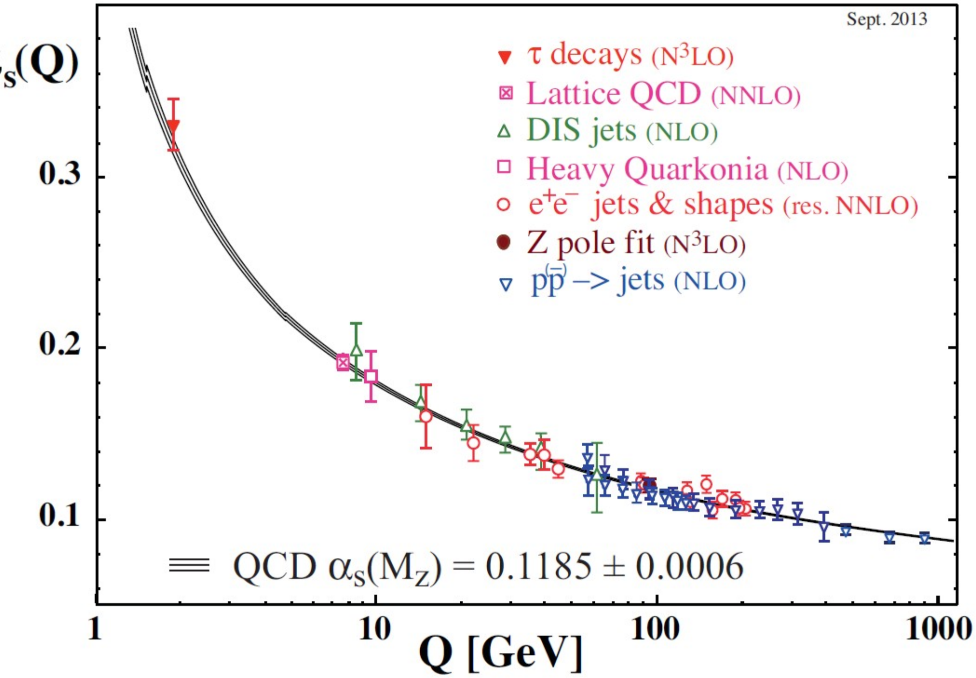
Fine structure
constant in QCD



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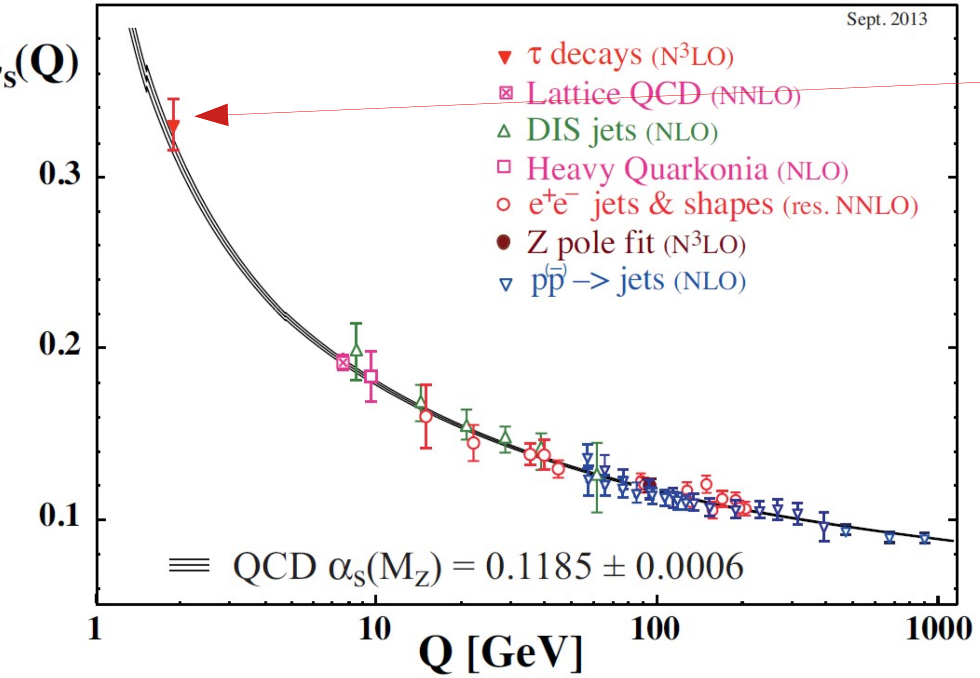

At low energies, the interaction between quarks and gluons is incredibly strong

Fine structure constant in QCD

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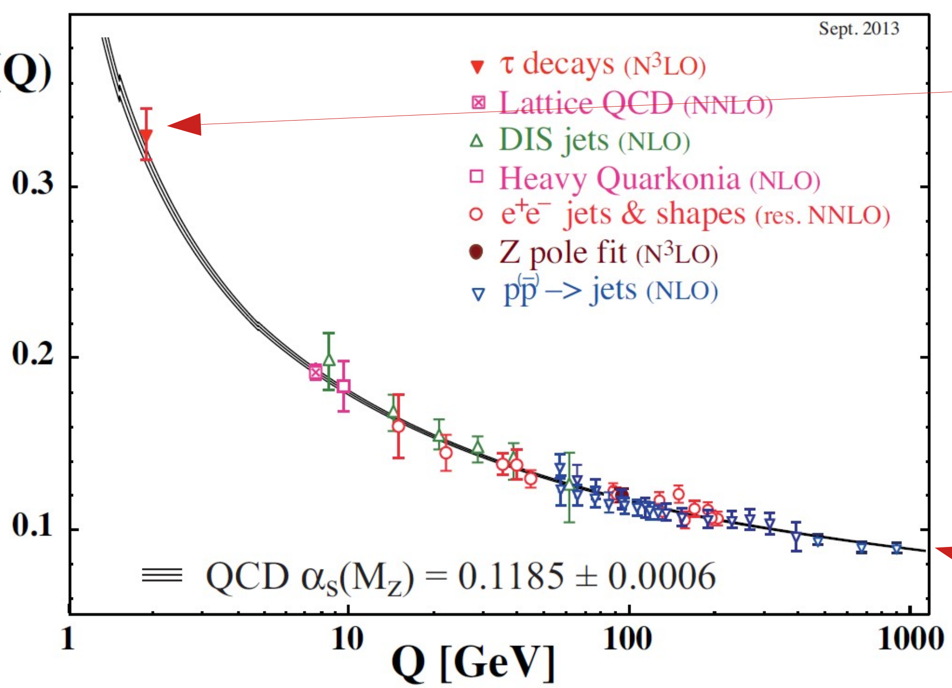

At low energies, the interaction between quarks and gluons is incredibly strong

At high energies, quarks and gluons do not interact

Perturbation Theory

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Perturbation Theory

$$H = H_0 + \lambda H_1$$

Perturbation Theory

$$H = H_0 + \lambda\, H_1$$

Full Hamiltonian
of the system

We do not know
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eigenvalue problem
for this Hamiltonian

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Part of the Hamiltonian for
which we know how to
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$$H_0 \left| n^{(0)} \right\rangle = E_n^{(0)} \left| n^{(0)} \right\rangle$$

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Part of the Hamiltonian that we do not understand and so we do not know how to solve the eigenvalue problem

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A small parameter
$\lambda < 1$

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Part of the Hamiltonian that we do not understand and so we do not know how to solve the eigenvalue problem

Perturbation

What is the solution of the eigenvalue problem for the full Hamiltonian?

$$H \left| n \right\rangle = E_n \left| n \right\rangle$$

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Perturbation Theory

$$H = H_0 + \lambda H_1$$

$$H_0 |n^{(0)}\rangle = E_n^{(0)} |n^{(0)}\rangle$$

$$H |n\rangle = E_n |n\rangle$$

Perturbation theory is the statement that the solution of the eigenvalue problem for the full Hamiltonian is given by a power series in the small parameter:

The eigenvectors are: $|n\rangle = |n^{(0)}\rangle + \lambda |n^{(1)}\rangle + \lambda^2 |n^{(2)}\rangle + \ \dots$

The eigenvalues are: $E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \ \dots$

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We know these from the unperturbed problem

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We know these from the unperturbed problem

There are “simple” formulas to systematically compute all these

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Running of Coupling Constants

Quantities in quantum field theory
can be computed in perturbation theory,
with the role of small parameter
being played by the Planck constant

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Running of Coupling Constants

Quantities in quantum field theory
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being played by the Planck constant → **LOOP EXPANSION**

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→ **LOOP EXPANSION**

Also Beta functions can be computed in a loop expansion

$$\beta(g) \equiv \frac{dg}{d\log\mu} = A\,g^2 + B\,g^3 + C\,g^4 + D\,g^5 + \ \ldots$$

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1-loop coefficient (→ A)

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2-loop coefficient (B)

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- A: 1-loop coefficient
- B: 2-loop coefficient
- C: 3-loop coefficient
- D: 4-loop coefficient

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Asymptotic Behaviors

$$\beta(g) \equiv \frac{dg}{d \log \mu} = A\, g^2$$

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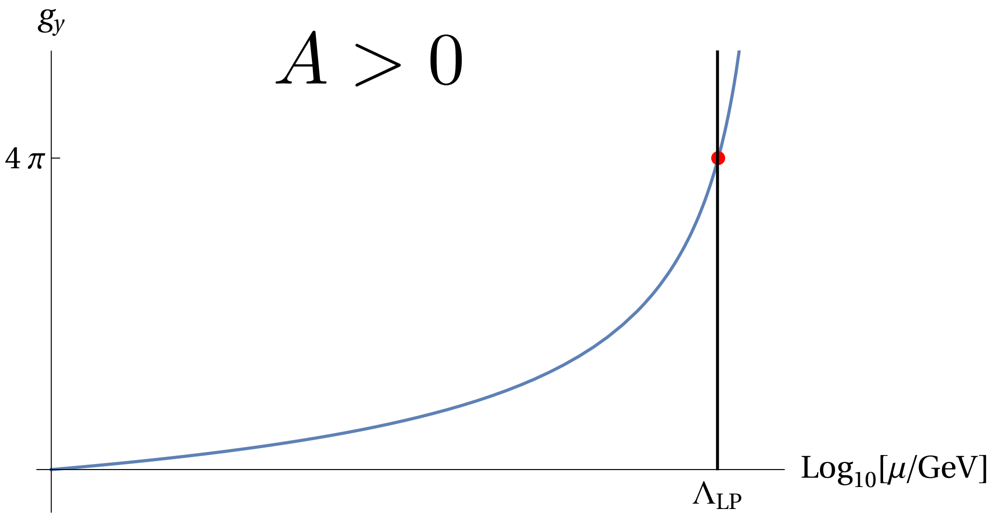
$$A > 0$$



Landau pole

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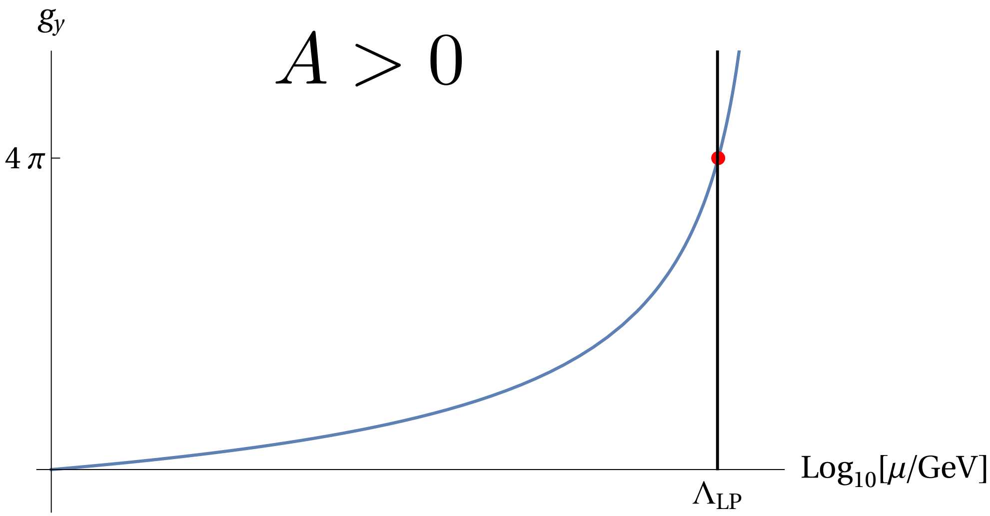

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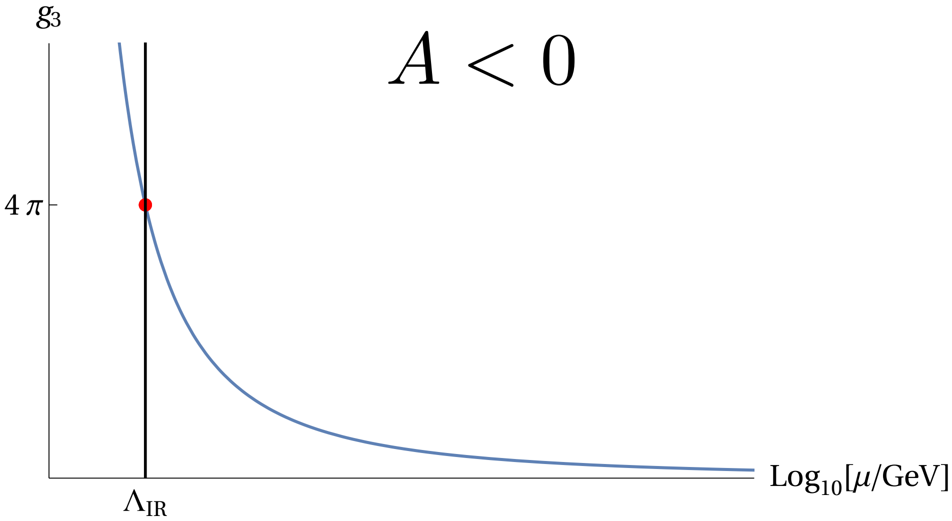

Asymptotic freedom

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Asymptotic Safety

$$\beta(g) \equiv \frac{dg}{d\log\mu} = A\,g^2 + B\,g^3$$

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Asymptotic Safety

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There is a specific value $g^* = -B/A$

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Fixed Point!

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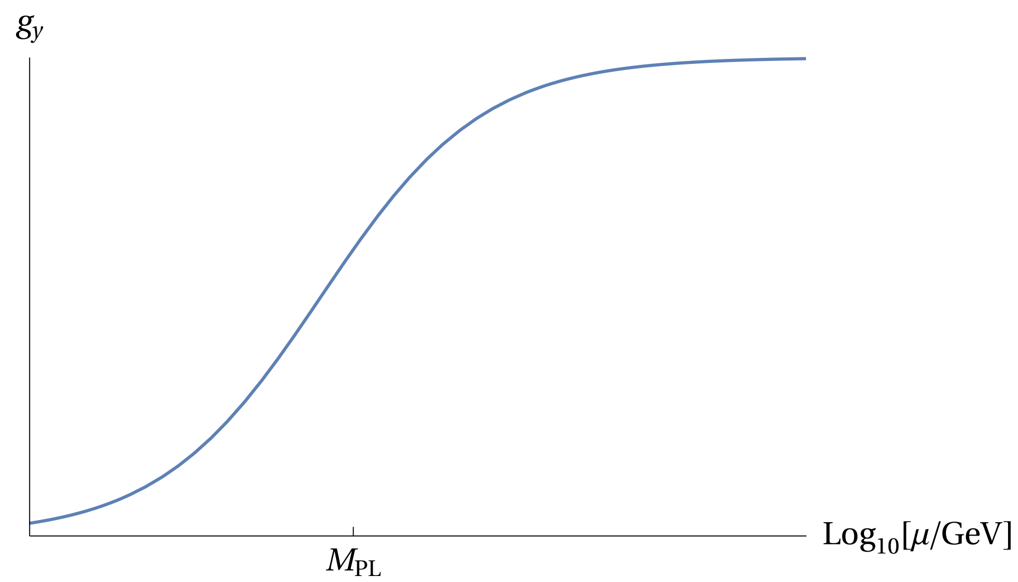

Asymptotic safety

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The Litim-Sannino (LiSa) Model

Litim, Sannino (2014)

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The Litim-Sannino (LiSa) Model

Gauge $F^a_{\mu\nu}\,(a = 1, ..., N_C^2 - 1)$

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Fermions $Q_i\,(i = 1, ..., N_F)$

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Scalars $H \in N_F \times N_F$

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The Lagrangian of the model: (Draw diagrams)

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} + \mathrm{Tr}\left(\partial_\mu H^\dagger \partial^\mu H\right) + \mathrm{Tr}\left(\bar{Q}\,i\not{D}\,Q\right)$$
$$-y\,\mathrm{Tr}\left(\bar{Q}_L H Q_R + \bar{Q}_R H^\dagger Q_L\right) - u\,\mathrm{Tr}\left(H^\dagger H\right)^2 - v\left(\mathrm{Tr}\,H^\dagger H\right)^2$$

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Scalars $H \in N_F \times N_F$

Veneziano parameter → $\epsilon \equiv \frac{N_F}{N_C} - \frac{11}{2}$

$$-\frac{11}{2} < \epsilon < +\infty$$

The Lagrangian of the model:

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$$(g,\ y,\ u,\ v)$$

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$0 < \epsilon \ll 1$

AT THE FP THE WHOLE MODEL DEPENDS ONLY ON THE VENEZIANO PARAMETER

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$$- y \, \mathrm{Tr} \left(\bar{Q}_L H Q_R + \bar{Q}_R H^\dagger Q_L \right) - u \, \mathrm{Tr} \left(H^\dagger H \right)^2 - v \left(\mathrm{Tr} \, H^\dagger H \right)^2$$

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Perturbation Theory in LiSa

The β functions are obtained in perturbation theory

$$\beta_g = ... \qquad \beta_y = ... \qquad \beta_u = ... \qquad \beta_v = ...$$

Perturbation Theory in LiSa

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$$\beta_g = \ldots \qquad \beta_y = \ldots \qquad \beta_u = \ldots \qquad \beta_v = \ldots$$

We will always consider loop expansions with the β function of the gauge at one extra loop level then the other couplings:

1-0-0   2-1-1   3-2-2   4-3-3

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Perturbation Theory in LiSa: 1-0-0

$$\beta_g = \frac{4}{3}\epsilon\, g^2$$

$$\beta_y = 0$$

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BORING

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Perturbation Theory in LiSa: 2-1-0

$$\beta_g = g^2 \left[\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon \right) g - 2 \left(\frac{11}{2} + \epsilon \right)^2 y \right]$$

$$\beta_y = y \left[(13 + 2\epsilon) y - 6g \right]$$

$$\beta_u = 0$$

$$\beta_v = 0$$

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Perturbation Theory in LiSa: 2-1-0

$$\beta_g = g^2 \left[\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right) g - 2\left(\frac{11}{2} + \epsilon\right)^2 y\right]$$

$$\beta_y = y \left[(13 + 2\epsilon) y - 6g\right]$$

$$\beta_u = 0$$

$$\beta_v = 0$$

$$y^* = \frac{6g^*}{13 + 2\epsilon} \approx \frac{6}{13} g^*$$

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$$g^* = \frac{26}{57}\epsilon + \mathcal{O}(\epsilon^2)$$

$$y^* = \frac{6g^*}{13+2\epsilon} \approx \frac{6}{13}g^*$$

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$$\beta_u = 0$$

$$\beta_v = 0$$

$$g^* = \frac{26}{57}\epsilon + \mathcal{O}(\epsilon^2)$$

$$y^* = \frac{6g^*}{13 + 2\epsilon} \approx \frac{6}{13} g^* = \frac{4}{19}\epsilon + \mathcal{O}(\epsilon^2)$$

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Perturbation Theory in LiSa: 2-1-0

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$$\beta_u = 0$$

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$$g^* = 0.456\epsilon + \mathcal{O}(\epsilon^2) \qquad y^* = 0.211\epsilon + \mathcal{O}(\epsilon^2)$$

Perturbation Theory in LiSa: 2-1-1

$$\beta_g = g^2 \left[\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right) g - 2\left(\frac{11}{2} + \epsilon\right)^2 y \right]$$

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$$\beta_u = -2\left(\frac{11}{2} + \epsilon\right) y^2 + 4u(y + 2u)$$

$$\beta_v = 12u^2 + 4v(v + 4u + y)$$

$$g^* = 0.456\epsilon + \mathcal{O}(\epsilon^2) \qquad\qquad y^* = 0.211\epsilon + \mathcal{O}(\epsilon^2)$$

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Perturbation Theory in LiSa: 2-1-1

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$$v^* = -0.137\epsilon + \mathcal{O}(\epsilon^2)$$

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Perturbation Theory in LiSa: 3-2-2

$$\beta_g = g^2\left[\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)g - 2\left(\frac{11}{2} + \epsilon\right)^2 y + \frac{701}{6}g^2 - \frac{3267}{8}g\,y + 605\,y^2 + \mathcal{O}(\epsilon^3)\right]$$

$$g^* = 0.456\epsilon + \mathcal{O}(\epsilon^2)$$

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$$u^* = 0.200\epsilon + \mathcal{O}(\epsilon^2)$$

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Perturbation Theory in LiSa: 3-2-2

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Perturbation Theory in LiSa: 3-2-2

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$\mathcal{O}(\epsilon^2)$

$$g^* = 0.456\epsilon + \mathcal{O}(\epsilon^2)$$

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Perturbation Theory in LiSa: 3-2-2

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$\mathcal{O}(\epsilon^2)$

UNIVERSALITY OF THE LEADING ORDER

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Perturbation Theory in LiSa: 2-1-1

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2-1-1

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Perturbation Theory in LiSa: 3-2-2

$$g^* = +0.456\epsilon + 0.781\epsilon^2 + \mathcal{O}(\epsilon^3)$$

$$y^* = +0.211\epsilon + 0.508\epsilon^2 + \mathcal{O}(\epsilon^3)$$

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$$v^* = -0.137\epsilon - 0.632\epsilon^2 + \mathcal{O}(\epsilon^3)$$

3-2-2

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Perturbation Theory in LiSa: 4-3-3

$$g^* = +0.456\epsilon + 0.781\epsilon^2 + 6.610\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$y^* = +0.211\epsilon + 0.508\epsilon^2 + 3.322\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$u^* = +0.200\epsilon + 0.440\epsilon^2 + 2.693\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$v^* = -0.137\epsilon - 0.632\epsilon^2 - 4.313\epsilon^3 + \mathcal{O}(\epsilon^4)$$

4-3-3

Conformal Window

$$g^* = +0.456\epsilon + 0.781\epsilon^2 + 6.610\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$y^* = +0.211\epsilon + 0.508\epsilon^2 + 3.322\epsilon^3 + \mathcal{O}(\epsilon^4)$$

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Let us be a little bit more quantitative and ask the questions:

- For what values of the Veneziano parameter do we actually have a fixed point?
- What can cause a fixed point to disappear?

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Conformal Window

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Let us be a little bit more quantitative and ask the questions:

- For what values of the Veneziano parameter do we actually have a fixed point?
- What can cause a fixed point to disappear?

The values of the Veneziano parameter for which the fixed point exist is called

CONFORMAL WINDOW

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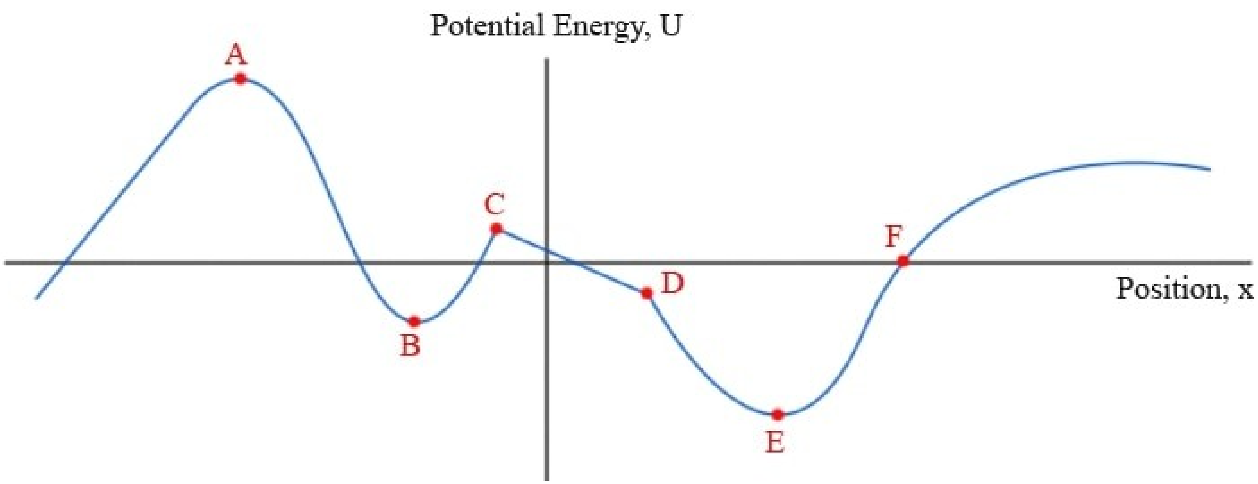
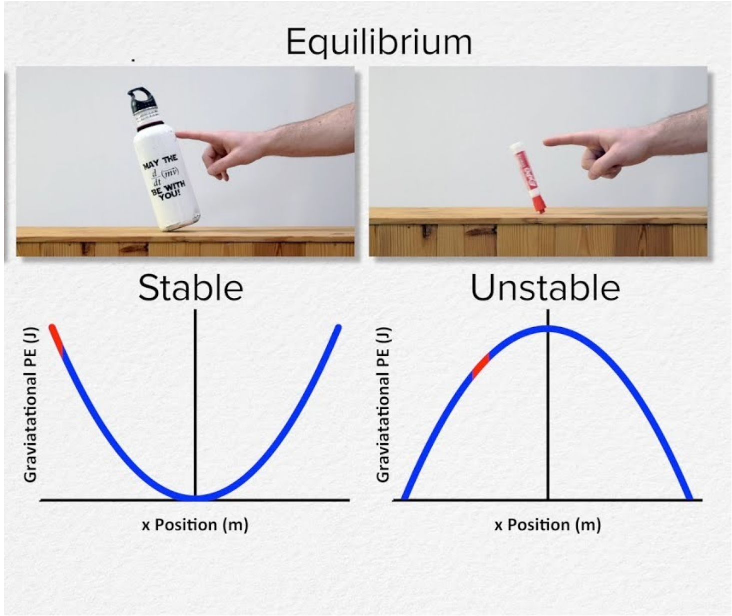
Vacuum Stability

The conformal window can “close” because of vacuum stability.





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Vacuum Stability at the Fixed Point

At the fixed point, the potential is given by:

$$V^*(H) = u^* \, \mathrm{Tr} \left(H^\dagger H \right)^2 + v^* \left(\mathrm{Tr} \, H^\dagger H \right)^2$$

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It was shown that the vacuum stability is obtained provided

$$\begin{cases} u^* > 0 \\ u^* + v^* \geq 0 \end{cases}$$

OR

$$\begin{cases} u^* < 0 \\ u^* + v^*/N_F \geq 0 \end{cases}$$

Litim, Mojaza, Sannino (2015)

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$$u^* = +0.200\epsilon + \mathcal{O}(\epsilon^2)$$

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2-1-1 $$u^* + v^* = 0.063\epsilon \geq 0$$

Vacuum Stability at the Fixed Point

$$u^* = +0.200\epsilon + \mathcal{O}(\epsilon^2)$$

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3-2-2 $\quad u^* + v^* = 0.063\epsilon - 0.192\epsilon^2 \geq 0$

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4-3-3 $\quad u^* + v^* = 0.063\epsilon - 0.192\epsilon^2 - 1.62\epsilon^3 \geq 0 \qquad \epsilon \leq 0.147$

Using Padè resummation this number is tighten further more to be $\epsilon \leq 0.087$

Fixed Point Merger

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Running of Coupling Constants

Quantities in quantum field theory can be computed in perturbation theory, with the role of small parameter being played by the Planck constant → **LOOP EXPANSION**

Also Beta functions can be computed in a loop expansion

$$\beta(g) \equiv \frac{dg}{d\log\mu} = A\,g^2 + B\,g^3 + C\,g^4 + D\,g^5 + \;\ldots$$

1-loop coefficient (A)

2-loop coefficient (B)

3-loop coefficient (C)

4-loop coefficient (D)

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Fixed Point Merger

$$\beta(g) \equiv \frac{dg}{d\log\mu} = g^2(A + B\,g + C\,g^2 + \cancel{D\,g^3} + \ \ldots) = 0$$

$$C = 1$$

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Fixed Point Merger

$$A + B\,g + g^2 = 0$$

Fixed Point Merger

$$A + B\,g + g^2 = 0 \qquad\qquad g^*_\pm = \frac{-B \pm \sqrt{B^2 - 4A}}{2}$$

- If the expression inside the squared root is negative, we have a pair of complex conjugate poles.
- On the other hand, if the expression inside squared root is positive, we have two real solutions, with a split given by the squared root term.

Fixed Point Merger

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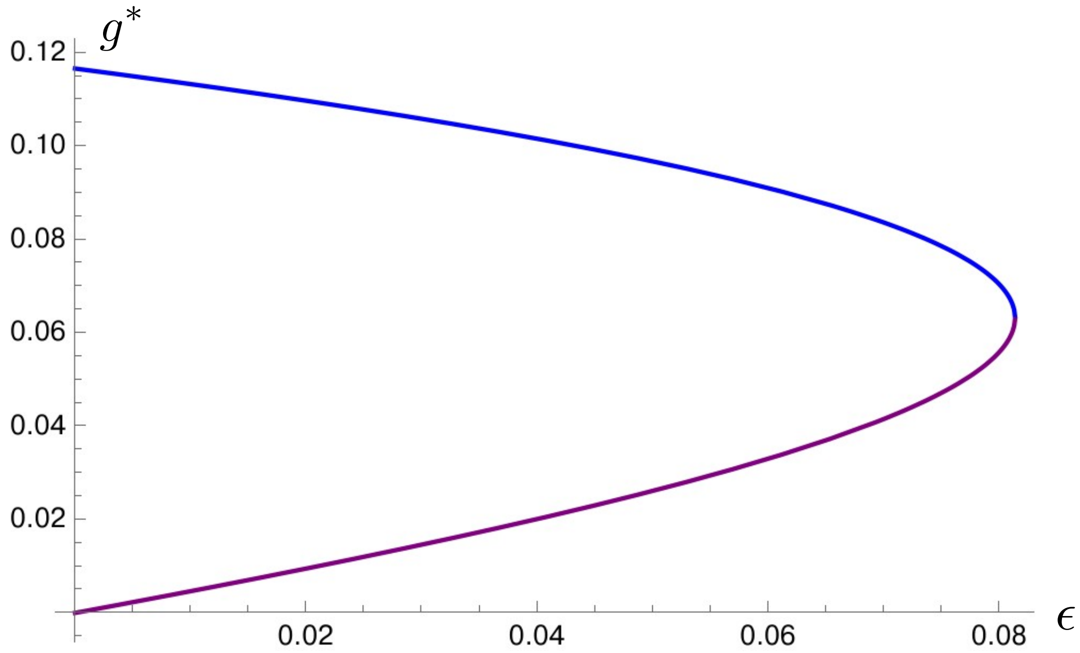

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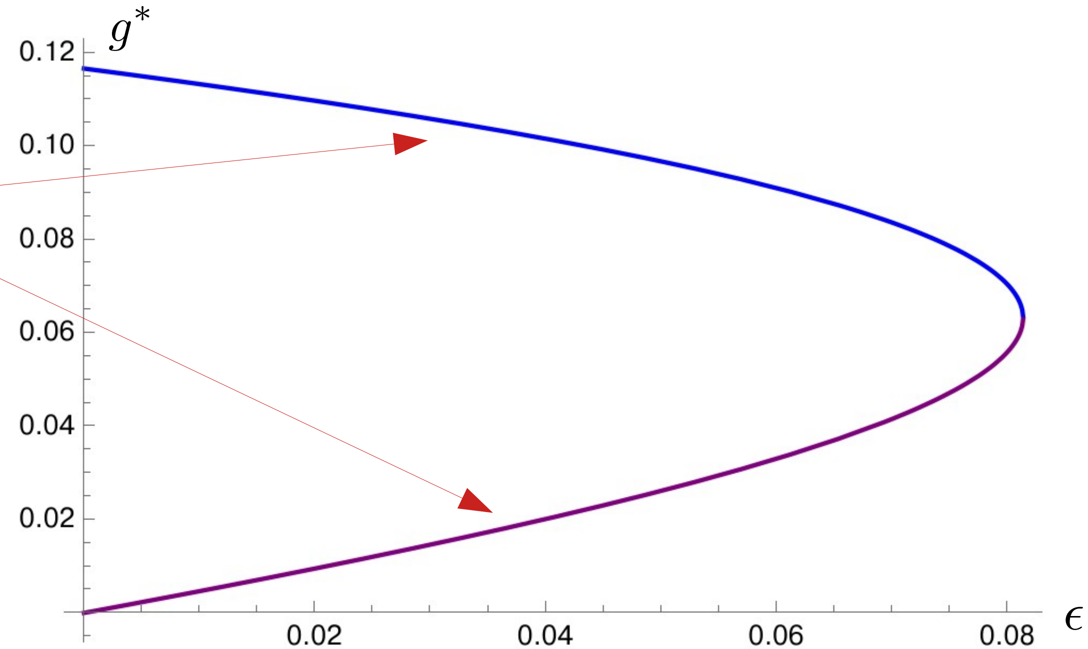
For small values of the Veneziano parameter the FPs are real



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Fixed Point Merger

$$A + B\,g + g^2 = 0 \qquad\qquad g^*_\pm = \frac{-B \pm \sqrt{B^2 - 4A}}{2}$$

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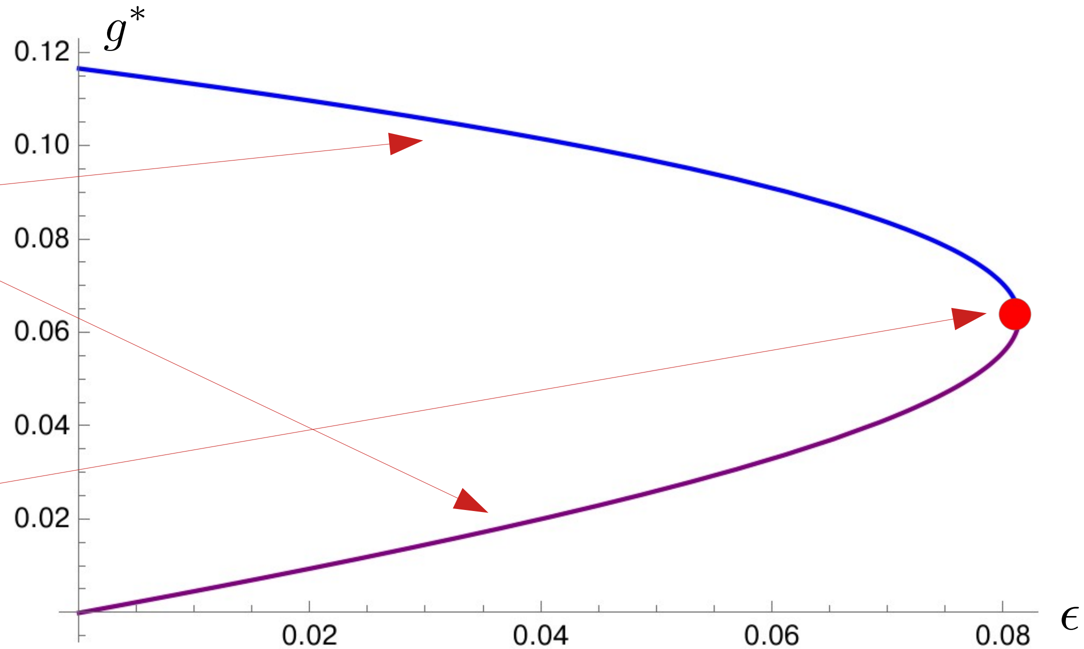
For small values of the Veneziano parameter the FPs are real

At a certain value, the solutions become complex and the FPs disappear



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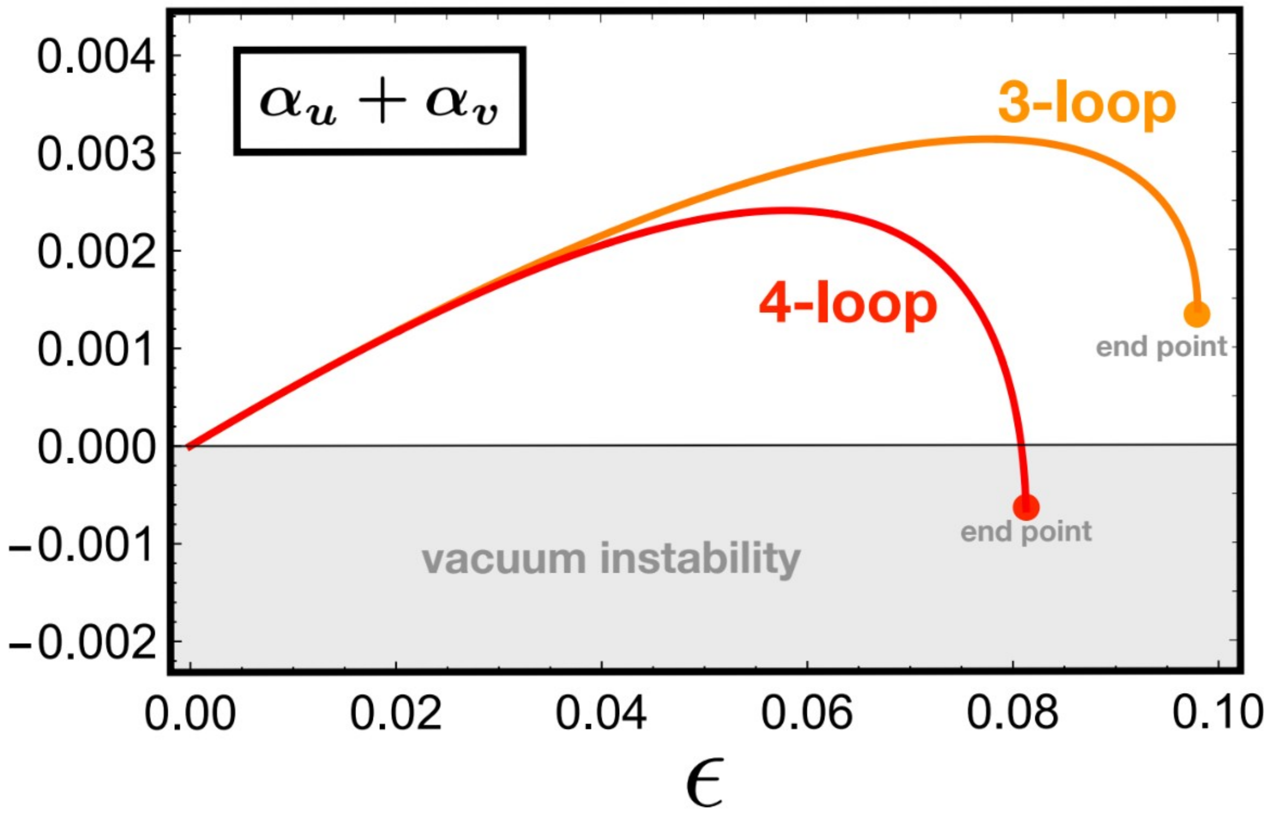
Conformal Window

Is the conformal window closing because of Vacuum Stability or a Fixed Point merger?



Plot kindly shared by Nahzaan Riyaz.

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Beyond marginal operators

$$v\left(\mathrm{Tr}\,H^{\dagger}H\right)^{2}$$

$$u\,\mathrm{Tr}\left(H^{\dagger}H\right)^{2}$$

$$y\,\mathrm{Tr}(\bar{Q}HQ)$$

Beyond marginal operators

$$v\,\left(\mathrm{Tr}\,H^\dagger H\right)^2 \quad\longrightarrow\quad \sum_n \gamma_n (\mathrm{Tr} H^\dagger H)^{n-2} (\mathrm{Tr} H^\dagger H)^2$$

$$u\,\mathrm{Tr}\,\left(H^\dagger H\right)^2$$

$$y\,\mathrm{Tr}(\bar{Q} H Q)$$

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Beyond marginal operators

$$v\left(\mathrm{Tr}\,H^{\dagger}H\right)^{2} \longrightarrow \sum_{n}\gamma_{n}(\mathrm{Tr}H^{\dagger}H)^{n-2}(\mathrm{Tr}H^{\dagger}H)^{2}$$

$$u\,\mathrm{Tr}\left(H^{\dagger}H\right)^{2} \longrightarrow \sum_{m}\alpha_{m}(\mathrm{Tr}H^{\dagger}H)^{m-2}\,\mathrm{Tr}(H^{\dagger}H)^{2}$$

$$y\,\mathrm{Tr}(\bar{Q}HQ)$$

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Beyond marginal operators

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$$y\,\mathrm{Tr}(\bar{Q} H Q) \longrightarrow \sum_l Y_l\,(\mathrm{Tr} H^\dagger H)^l\ \mathrm{Tr}(\bar{Q} H Q)$$

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Beyond marginal operators

$$v\,\left(\mathrm{Tr}\,H^\dagger H\right)^2 \longrightarrow \sum_n \gamma_n (\mathrm{Tr} H^\dagger H)^{n-2} (\mathrm{Tr} H^\dagger H)^2$$

$$U(\mathrm{Tr} H^\dagger H)$$

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Beyond marginal operators

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Beyond marginal operators

$$v\left(\mathrm{Tr}\,H^\dagger H\right)^2 \longrightarrow \underbrace{\sum_n \gamma_n (\mathrm{Tr} H^\dagger H)^{n-2}}_{U(\mathrm{Tr} H^\dagger H)} (\mathrm{Tr} H^\dagger H)^2$$

$$u\,\mathrm{Tr}\left(H^\dagger H\right)^2 \longrightarrow \underbrace{\sum_m \alpha_m (\mathrm{Tr} H^\dagger H)^{m-2}}_{C(\mathrm{Tr} H^\dagger H)}\,\mathrm{Tr}(H^\dagger H)^2$$

$$y\,\mathrm{Tr}(\bar{Q} H Q) \longrightarrow \underbrace{\sum_l Y_l\,(\mathrm{Tr} H^\dagger H)^l}_{Y(\mathrm{Tr} H^\dagger H)}\,\mathrm{Tr}(\bar{Q} H Q)$$

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Beyond marginal operators

$$v\left(\mathrm{Tr}\,H^\dagger H\right)^2 \xrightarrow{\partial_t U(\mathrm{Tr}H^\dagger H)} \underbrace{\sum_n \gamma_n (\mathrm{Tr}H^\dagger H)^{n-2}(\mathrm{Tr}H^\dagger H)^2}_{U(\mathrm{Tr}H^\dagger H)}$$

$$u\,\mathrm{Tr}\left(H^\dagger H\right)^2 \xrightarrow{\partial_t C(\mathrm{Tr}H^\dagger H)} \underbrace{\sum_m \alpha_m (\mathrm{Tr}H^\dagger H)^{m-2}}_{C(\mathrm{Tr}H^\dagger H)}\,\mathrm{Tr}(H^\dagger H)^2$$

$$y\,\mathrm{Tr}(\bar{Q}HQ) \xrightarrow{\partial_t Y(\mathrm{Tr}H^\dagger H)} \underbrace{\sum_l Y_l\,(\mathrm{Tr}H^\dagger H)^l}_{Y(\mathrm{Tr}H^\dagger H)}\,\mathrm{Tr}(\bar{Q}HQ)$$

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Beta Functions
Beyond Marginal Operators

$$\partial_t u = -4u + (2+\eta_H)\rho u' + \frac{1}{2}\left(\frac{1}{1+u'+4\rho c} + \frac{1}{1+u'}\right) - \frac{2N_C}{N_F}\frac{1}{1+\rho y^2}$$

$$\partial_t c = 2\eta_H c + (2+\eta_H)\rho c' - \frac{2N_C}{N_F}\frac{y^4}{(1+\rho y^2)^3} + \frac{1}{2}\Bigg(-\frac{128\rho^3c^5}{(1+u')^3(1+4\rho c+u')^3} + \frac{64\rho^2c^3(c-\rho c')}{(1+u')^2(1+4\rho c+u')^3} - \frac{8\rho c c'}{(1+4\rho c+u')^3} - \frac{48\rho^2c^2c'}{(1+u')(1+4\rho c+u')^3} + \frac{16c^2}{(1+4\rho c+u')^3} - \frac{2c'}{(1+4\rho c+u')^2}\Bigg)$$

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$$\partial_t y = -3\alpha_g y(0) + \frac{1}{2}(2\eta_\psi+\eta_H)y + (2+\eta_\phi)\rho y' - \frac{1}{2}\left(\frac{y'}{(1+4\rho c+u')^2}+\frac{y'}{(1+u')^2}\right) + \frac{y^3}{2(1+\rho y^2)(1+4\rho c+u')}\left(\frac{1}{1+4\rho c+u'}+\frac{1}{1+\rho y^2}\right) - \frac{y^3}{2(1+u')(1+\rho y^2)}\left(\frac{1}{1+\rho y^2}+\frac{1}{1+u'}\right)$$

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Fixed Point

$$\partial_t u = 0 \qquad \partial_t c = 0 \qquad \partial_t y = 0$$

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$$u(\rho) = \sum_{n=0}^{\infty} \alpha_n \rho^{n+1} \qquad c(\rho) = \sum_{n=1}^{\infty} \gamma_n \rho^{n-1} \qquad y(\rho) = \sum_{n=0}^{\infty} y_n \rho^{n}$$

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Fixed Point & Power Counting in ε

Coupling	FP	Coupling	FP	Coupling	FP
γ_1	$+0.199781\epsilon$	α_1	$+0.0625304\epsilon$	y_0	$+0.458831\sqrt{\epsilon}$
γ_2	$-0.404135\epsilon^3$	α_2	$-0.0844283\epsilon^3$	y_1	$+0.318417\sqrt{\epsilon^5}$
γ_3	$+0.558651\epsilon^4$	α_3	$+0.0721923\epsilon^4$	y_2	$-0.468528\sqrt{\epsilon^7}$
γ_4	$-0.812282\epsilon^5$	α_4	$-0.0699564\epsilon^5$	y_3	$+0.626392\sqrt{\epsilon^9}$
γ_5	$+1.16104\epsilon^6$	α_5	$+0.0706016\epsilon^6$	y_4	$-0.798058\sqrt{\epsilon^{11}}$
	$\vdots$		$\vdots$		$\vdots$

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Fixed Point & Power Counting in ε

$$u^*(\rho) = \sum_{n=0}^{\infty} \alpha_n^* \rho^{n+1}$$

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	⋮		⋮		⋮

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Vacuum stability at the fixed point

$$u^*(\rho) = \sum_{n=0}^{\infty} \alpha_n^* \rho^{n+1}$$

Vacuum stability at the fixed point

$$u^*(\rho) = \sum_{n=0}^{\infty} \alpha_n^* \rho^{n+1}$$

At leading order in ε a re-summation of the couplings can be performed:

$$u^*(\rho) = \alpha_1^* \rho^2 + \frac{A^2\,\rho^2}{4}\log(1 + A\,\rho) + \frac{B^2\,\rho^2}{4}\log(1 + B\,\rho) - \frac{N_C}{N_F} D^2\,\rho^2 \log(1 + D\,\rho)$$

$$A \equiv 2\alpha_1^*$$

$$B \equiv 2\alpha_1^* + 4\gamma_1^*$$

$$D \equiv \frac{N_F}{N_C}\alpha_y^*$$

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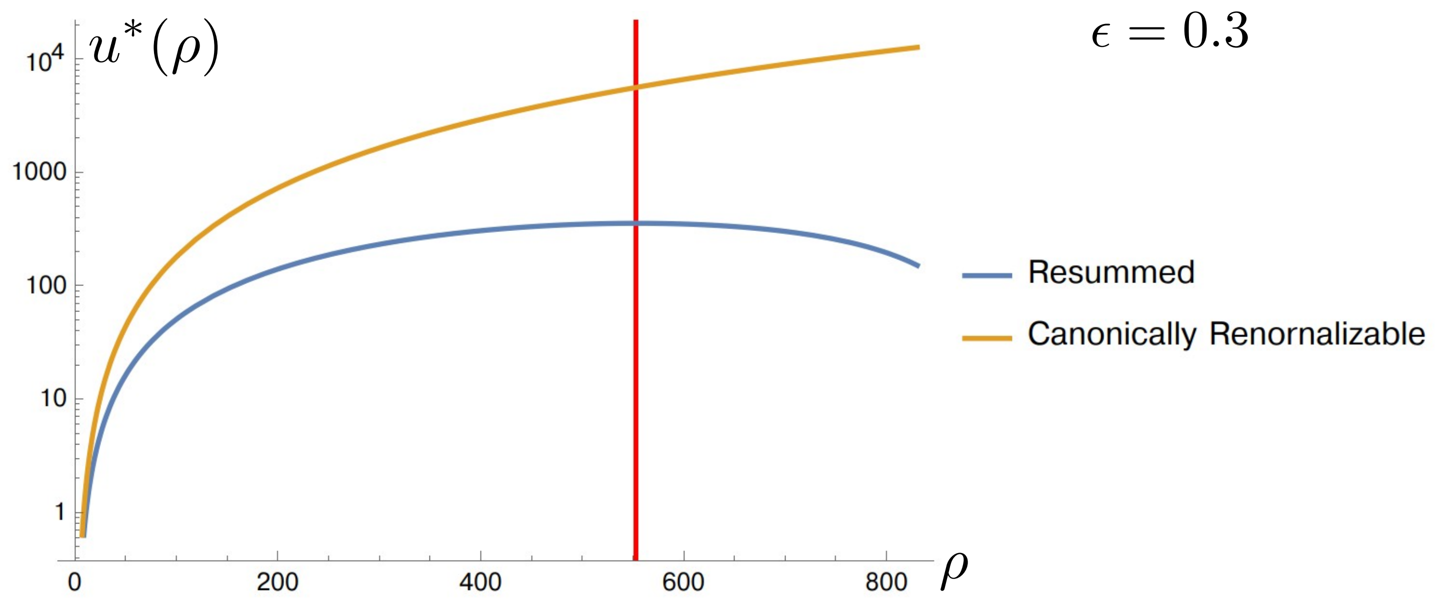

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Conclusion

- When studying the asymptotic behaviour of the running of a coupling constant, the presence of a Fixed Point can preserve the theory from running into infinity.

- A toy model (LiSa) was presented in very details as an example of a theory where such interacting Fixed Point is controlled by a small parameter.

- A perturbative analysis of the Conformal Window of LiSa was described and the main result is that it is still not clear whether the lost of conformality is due to vacuum instability or a Fixed Point Merger.

- Within Functional Renormalization Group techniques, the running and the Fixed Point of Beyond Marginal Operators have been studied. The main result is that at the Fixed Point it is possible to perform a re-summation of a power series. In this way, the scalar potential at the Fixed Point can be studied for arbitrarily large values of the field. **The potential remains stable!**

- An immediate future directions is to integrate the flow numerically, within having to perform any power series, but this is still WORK IN PROGRESS

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Scalar potential close to the FP

Stability Matrix

$$M^i_k \equiv \left[\frac{\partial \beta^i(g)}{\partial g^k}\right]_{g=g^*}$$

Couplings

$$\alpha_{n-1}\,(\mathrm{Tr}H^\dagger H)^n$$

$$\gamma_{n-1}\,(\mathrm{Tr}H^\dagger H)^{n-2}\mathrm{Tr}(H^\dagger H)^2$$

$$y_n\,(\mathrm{Tr}H^\dagger H)^n\,\mathrm{Tr}(\bar{Q}HQ)$$

Critical Exponents

$$\theta_{\alpha_{n-1}} = (2n-4) + n\gamma_M$$

$$\theta_{\gamma_{n-1}} = (2n-4) + (n-2)\gamma_M + \gamma_m$$

$$\theta_{y_n} = 2n + n\gamma_M + \left(\frac{\eta_H}{2} + \eta_Q\right)$$

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Flow

We define dimension-less couplings:

$$U = k^4 \, u \qquad \mathrm{Tr}\, H^\dagger H = \rho \, k^2 \qquad \mathrm{Tr}(H^\dagger H)^2 = \tau \, k^4$$

And compute the flow:

$$\partial_t u = -4u + (2 + \eta_H)\rho u' + \frac{1}{2}\left(\frac{1}{1 + u' + 4\rho \, c} + \frac{1}{1 + u'}\right) - \frac{2N_C}{N_F}\frac{1}{1 + \rho \, y^2}$$

Canonical dimension

Anomalous dimension

Quantum corrections from the scalar potential

Quantum corrections from the yukawa

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Flow

$$\partial_t U(\mathrm{Tr} H^\dagger H) \qquad \partial_t C(\mathrm{Tr} H^\dagger H) \qquad \partial_t Y(\mathrm{Tr} H^\dagger H)$$

Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left[\partial_t R_k \cdot \left(\Gamma_k^{(2)} + R_k\right)^{-1}\right]$$

Regulator

$$R_k = Z_k\,(k^2 - q^2)\,\Theta(k^2 - q^2)$$