(1+1)-dimensional Quantum Gravity from the Corner Proposal

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Presentation Plan

- Introduction
- Symmetries and gauge theories
- The corner proposal
- (1+1)-d gravity from symmetries

Introduction: Symmetries

Symmetries play a crucial role in all of physics

- Great tool to find solutions of complicated problems (Cosmology, Newton gravity, classical mechanics,...)
- They describe fundamental interactions through gauge theories

$$\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$$

- They are the defining property of fundamental physics
  Poincaré $\rightarrow$ Quantum Field Theory and fundamental particles
  Diffeomorphisms $\rightarrow$ General covariance and gravity

Introduction: Corner proposal

- Very interesting physics at the boundary of gauge theories (Quantum Hall effect, Holography, Black holes, ...)
- Gravity is a gauge theory of diffeomorphisms, at the boundary there exists a universal *corner symmetry group*
- The corner proposal states that this symmetry group is the fundamental ingredient of gravity

Symmetries and Gauge theories

Symmetries: Why?

From the Poincaré symmetry algebra $(\vec{J}, \vec{K}, \vec{P}, E)$

$$[J_m, P_n] = \epsilon_{mnk} P_k, \quad [K_i, P_k] = \eta_{ik} E, \quad [K_i, E] = -P_i,$$
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- Unitary irreducible representations $\rightarrow$ Quantum field theory

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From only the symmetry algebra, one can get both the classical and quantum theory.

Symplectic geometry

A symplectic manifold (phase space) $(\mathcal{F}, \Omega)$ is equipped with a symplectic 2-form

$$\Omega_{\mu\nu} = -\Omega_{\nu\mu}, \qquad \partial_\rho \Omega_{\mu\nu} = 0.$$

Non-degenerate (invertible)

$$\left(\Omega^{-1}\right)^{\mu\nu} \Omega_{\nu\rho} = \delta^\mu_\rho .$$

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The (inverse) symplectic form generates a Poisson bracket

$$\{f^\mu, f^\nu\} = \left(\Omega^{-1}\right)^{\mu\nu},$$

where f^μ are coordinates on $\mathcal{F}$.

Example: Classical mechanics

Defined by the action

$$S = \int \mathrm{d}t \left(\frac{p^2}{2m} - V(x) \right) ,$$

the phase space

$$\mathcal{F} = \{(x, p)\} ,$$

the symplectic form

$$\Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} ,$$

and the Poisson bracket

$$\{x, p\} = 1.$$

Noether Theorems

To each symmetry of the theory V^μ, there exists an associated charge H_V

$$V^\mu \Omega_{\mu\nu} = \partial_\nu H_V,$$

where the charge is the integral of a current

$$H_V = \int_\Sigma \mathrm{d}^3 x \, J_V(x).$$

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For gauge symmetries V_ϵ, Noether 2nd theorem says

$$H_{V_\epsilon} = \int_\Sigma \mathrm{d}^3 x \, \nabla Q_{V_\epsilon} = \int_{\partial\Sigma} Q_{V_\epsilon} .$$

It has support only on the boundary.

Charge algebra

The symplectic form also gives the charge algebra

$$V^\mu W^\nu \Omega_{\mu\nu} = \{H_V, H_W\}\,,$$

for two symmetries V, W.
This algebra represents the symmetry algebra of the theory.

Symmetries $\rightarrow$ Vectors $\rightarrow$ Charge $\rightarrow$ Symmetries

Gauge theories and edge modes

The fact that in gauge theories (without boundaries)

$$V_\epsilon^\mu \Omega_{\mu\nu} = 0,$$

is important. It allows to identify the gauge directions and quotient them out

$$\mathcal{P}_{\text{phys}} = \mathcal{P}/G,$$

where G is the gauge group. This is symplectic reduction.

Gauge theories and edge modes

When there is a boundary

$$V_\epsilon^\mu \Omega_{\mu\nu} = \partial_\nu H_{V_\epsilon}.$$

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We can introduce edge modes to the phase space

$$\phi : \partial\Sigma \longrightarrow G$$

and an associated symplectic structure Ω^ϕ such that

$$V_\epsilon^\mu \left(\Omega_{\mu\nu} + \Omega^\phi_{\mu\nu}\right) = 0.$$

The edge modes "eat-up" the charge and restore gauge symmetry.

Example Electromagnetetism in a ball

The phase space of E&M is

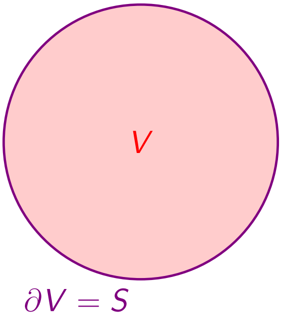
$$\mathcal{F} = \left\{ \vec{A}(x), \vec{E}(x) \right\}.$$

There a $U(1)$ gauge symmetry

$$\vec{A}(x) \mapsto \vec{A}(x) + \vec{\nabla}\alpha(x),$$

The associated charge is

$$H_\alpha = \int_V \vec{\nabla} \cdot (\alpha \vec{E})\, \mathrm{d}V = \int_S \alpha \vec{E} \cdot \mathrm{d}\vec{S}.$$



Example Electromagnetism in a ball

The phase space of E&M is

$$\mathcal{F} = \left\{ \vec{A}(x), \vec{E}(x), \phi(\theta, \varphi) \right\}.$$

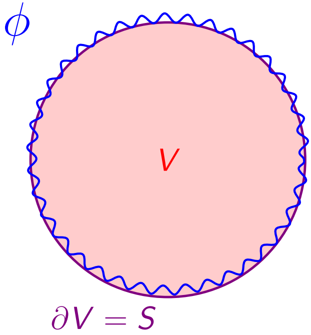
There a $U(1)$ gauge symmetry

$$\vec{A}(x) \mapsto \vec{A}(x) + \vec{\nabla}\alpha(x),$$

$$\phi(\theta, \varphi) \mapsto \phi(\theta, \varphi) e^{i\alpha(\theta,\varphi)},$$

The associated charge is

$$H_\alpha = 0.$$



For more details

A (great) review paper on edge modes in Yang-Mills theories and gravity with an accessible mathematical language:

"*On the covariant formulation of gauge theories with boundaries*" (2312.01918)
M. Assanioussi, J. Kowalski-Glikman, I. Mäkinen, L. Varrin (2023)

Recap

- To each symmetry of the theory there exists an associated charge (Noether)
- For gauge symmetries, these charges vanish except in the presence of boundaries. In that case, some gauge symmetries become physical
- For gauge symmetries with boundaries, one can choose between having physical gauge symmetries on the boundary or edge modes on the boundary

The Corner Proposal

The Universal Corner Group

The gauge symmetry in gravity is $G = \mathrm{Diff}(M)$. In the presence of a corner S, the universal corner group is given by[1]

[1] Luca Ciambelli and Robert G. Leigh. "Isolated surfaces and symmetries of gravity". In: *Physical Review D* (Aug. 2021).

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$$\boxed{\mathrm{UCS} = \left(\mathrm{Diff}(S) \ltimes \mathrm{GL}(2,\mathbb{R})^S\right) \ltimes (\mathbb{R}^2)^S}$$

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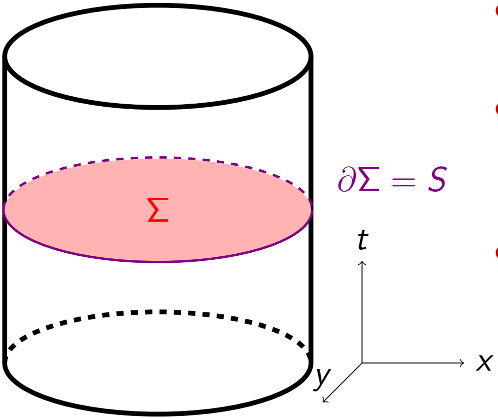
$$\boxed{\mathrm{UCS} = \left(\mathrm{Diff}(S) \ltimes \mathrm{GL}(2,\mathbb{R})^S\right) \ltimes (\mathbb{R}^2)^S}$$

- Universal $\rightarrow$ does not depend on the dynamics
- Infinite dimensional in several ways (but still smaller than $\mathrm{Diff}(M)$)
- The rest of $\mathrm{Diff}(M)$ is left uncharged

[1] Ciambelli and Leigh, "Isolated surfaces and symmetries of gravity".

Picture in 2+1 dimensions



- The Cauchy (spatial) slice Σ
- The circle S is the boundary of the corner
- The UCS is supported on the corner

The corner proposal

- The UCS symmetry group is the fundamental ingredient of gravity
- Every Quantum Gravity theory should hold a representation of that group (True for LQG[2])
- If we can find all of the unitary irreducible representations of the UCS we have all possible Quantum Gravity states

[2] Laurent Freidel, Marc Geiller, and Wolfgang Wieland. *Corner symmetry and quantum geometry*. 2023. arXiv: 2302.12799 [hep-th].

Link with Strominger triangle

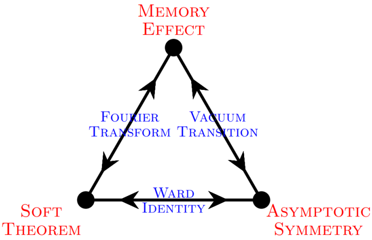


Figure: Strominger Infrared Triangle[a]

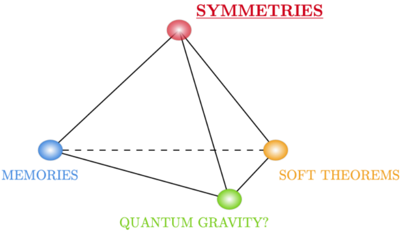


Figure: The Corner Proposal Pyramid[a]

[a] Andrew Strominger. *Lectures on the Infrared Structure of Gravity and Gauge Theory*. 2018.

[a] Luca Ciambelli and LV et al. "Cornering quantum gravity". In: *PoS* QG-MMSchools (2023), p. 010.

The extended corner symmetry

The ECS contains the asymptotic symmetries (BMSW group) and more. If we specialize to finite distance corners in the Einstein-Hilbert theory we get the extended corner symmetry group

$$\mathrm{ECS} = \left(\mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S\right) \ltimes (\mathbb{R}^2)^S$$

(1+1)-dimensional Gravity

Corner symmetry group in two dimensions

In $1+1$ dimensional spacetime, the extended corner symmetry group becomes

$$\mathrm{ECS}_2 = \mathrm{SL}(2,\mathbb{R}) \ltimes \mathbb{R}^2.$$

Corner symmetry group in two dimensions

In $1+1$ dimensional spacetime, the extended corner symmetry group becomes

$$\mathrm{ECS}_2 = \mathrm{SL}(2,\mathbb{R}) \ltimes \mathbb{R}^2.$$

The associated Lie algebra is spanned by the five generators $(L_0, L_\pm, P_\pm)$

$$[L_0, L_\pm] = \pm L_\pm, \quad [L_-, L_+] = 2L_0,$$
$$[L_0, P_\pm] = \frac{1}{2}P_\pm, \quad [L_\pm, P_\mp] = \mp P_\pm,$$

$$[L_\pm, P_\pm] = 0$$

Quick comment on the classical theory

You can define the classical phase space as the coadjoint orbits of the ECS_2

$$\mathfrak{ecs}_2 \xrightarrow{\text{Dual}} \mathfrak{ecs}_2^* \xrightarrow{\text{Coadjoint action}} \mathcal{O}_\omega = \mathcal{F}$$

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You can define the classical phase space as the coadjoint orbits of the ECS_2

$$\mathfrak{ecs}_2 \xrightarrow{\text{Dual}} \mathfrak{ecs}_2^* \xrightarrow{\text{Coadjoint action}} \mathcal{O}_\omega = \mathcal{F}$$

Since the algebra admits one Casimir

$$\mathcal{C}_3 = 2L_0 P_- P_+ - L_+ P_-^2 - L_- P_+^2 .$$

The phase space (orbits) is four dimensional. You can get a worldline action from the coadjoint orbit method. We expect the resulting theory to describe the edge modes.

Quantum Theory from the ECS: $\mathrm{SL}(2, \mathbb{R})$

There exists a oscillator representation of the $SL(2, \mathbb{R})$ algebra

$$L_+ = \frac{1}{2} a^\dagger a^\dagger,$$
$$L_- = \frac{1}{2} aa,$$
$$L_0 = \frac{1}{2} a^\dagger a + \frac{1}{4},$$

with

$$[a, a^\dagger] = 1.$$

But it can not be extanded to the full ECS.

Adding translations

In order to add translations → bipartite structure

$$L_0 = \frac{1}{2}(a_1^\dagger a_1 + a_2^\dagger a_2 + 1),$$
$$L_+ = a_1^\dagger a_2^\dagger,$$
$$L_- = a_1 a_2,$$

acting on states

$$|n\rangle_1 \otimes |m\rangle_2 \equiv |nm\rangle.$$

And now,

$$P_- = a_1, \qquad P_+ = a_2^\dagger,$$

realises the full $\mathrm{ECS} = \mathrm{SL}(2,\mathbb{R}) \ltimes \mathbb{R}^2$

(1+1)-d Quantum Gravity Hilbert space

The Hilbert space is

$$\mathcal{H} = \{|\psi\rangle = |mn\rangle \mid n, m \in \mathbb{N}\}$$

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The unique Casimir of the algebra

$$\mathcal{C}_3 = 2L_0 P_- P_+ - L_+ P_-^2 - L_- P_+^2,$$

vanishes in this representation. This is just one particular representation.

Entanglement entropy in gauge theories

Entanglement entropy between two space regions: For two Cauchy surfaces Σ and $\bar{\Sigma}$ we form the Hilbert space

$$\mathcal{H} = \mathcal{H}_\Sigma \otimes \mathcal{H}_{\bar{\Sigma}},$$

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Form the reduced density matrix

$$\rho_{\Sigma} = \mathrm{Tr}_{\bar{\Sigma}} \rho,$$

calculate the entanglement entropy of the Σ region

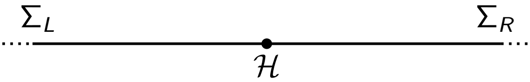
$$S(\Sigma) = -\mathrm{Tr}[\rho_{\Sigma} \ln(\rho_{\Sigma})].$$

Factorization failure

In gauge theories however

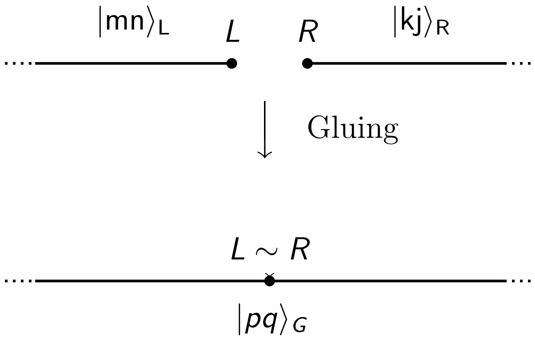
$$\mathcal{H} \textcolor{red}{\subset} \mathcal{H}_{\Sigma} \otimes \mathcal{H}_{\bar{\Sigma}},$$

because of constraints.



$\mathcal{H}$ is made of states that are invariant under boundary transformations.

Gluing procedure and Entanglement Entropy



- One bipartite Hilbert space for each segment
- After gluing, one bipartite Hilbert space for the unique segment
- The glued state is an entangled state

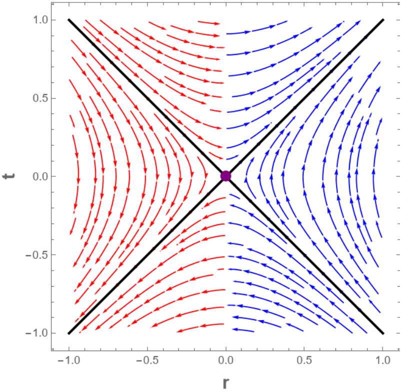
Speculations

Warning, Speculations!

Modular Hamiltonian

L_0 can be represented in two dimensions (t, r)

$$L_0 = t\partial_r + r\partial_t$$



The integral lines of the L_0 vector fields create left and right Rindler wedges. The lines are described by $r^2 - t^2 = cste = \lambda$.

$\Rightarrow \frac{L_0}{\lambda}$ can be taken as the modular Hamiltonian.

(1+1)-d Gravity entanglement entropy

The density matrix is given by

$$\rho^{(\lambda)} = \frac{\exp(-\beta \frac{L_0}{\lambda})}{\mathrm{Tr}[\exp(-\beta \frac{L_0}{\lambda})]}.$$

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Using the glued states $|pq\rangle_G$ we can calculate the entanglement entropy

$$S_R^{(\lambda)} = \frac{\beta}{4\lambda}\left(\coth\left(\frac{\beta}{4\lambda}\right) + 1\right) - \ln(e^{\frac{\beta}{2l}} - 1).$$

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For small length scale (high temperature) $\lambda/\beta << 1$

$$S_R^{(\lambda)} = 1 + \ln\left(\frac{2\lambda}{\beta}\right) + \mathcal{O}\left(\left(\frac{\beta}{\lambda}\right)^2\right).$$

Recap and conclusions

Conclusions

- Gauge theories and gravity admit an algebra of non-vanishing charges on the corners
- In the 2D case, we constructed *one* representation of the gravity algebra (ECS) and used it to glue two spatial region together
- We calculated the entanglement entropy of the subregion and recovered the Bekenstein-Hawking area law and quantum corrections in 2D

$$S_R^{(\lambda)} \sim \mathrm{Cste} + \ln(\frac{\lambda}{\beta})$$