Graduate Physics Seminar

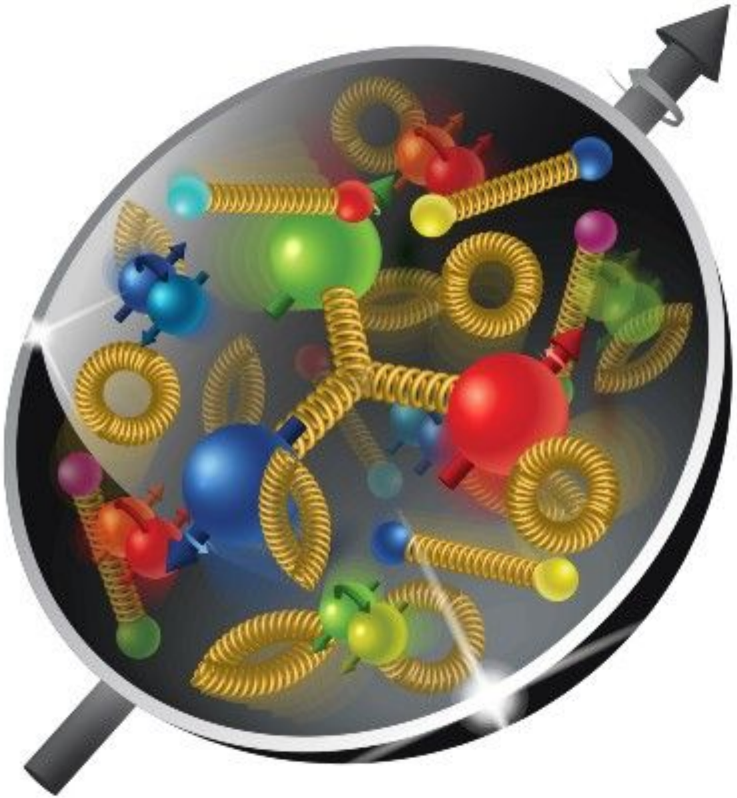
Single Inclusive Particle Production in CGC: Beyond Eikonal Order

-Swaleha Mulani

Supervisors: Dr. hab Tolga Altinoluk, Dr. Guillaume Beuf

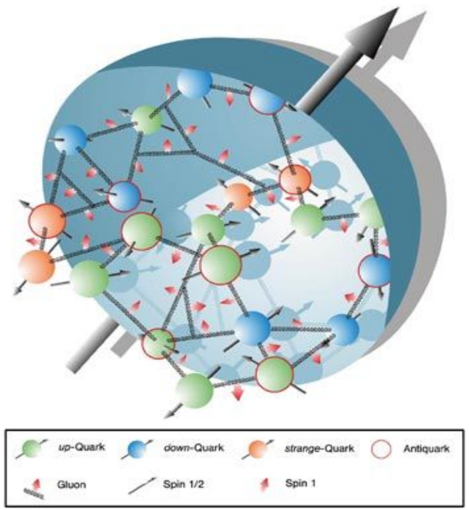
Outline

- Introduction
  - QCD and QED
  - Deep Inelastic Scattering
  - Saturation Physics
  - Color Glass condensate
  - Eikonal Approximation
- Beyond Eikonal Order
- Single Inclusive Particle Production
  - Cross section
    - Quarks
    - Gluons
- Summary

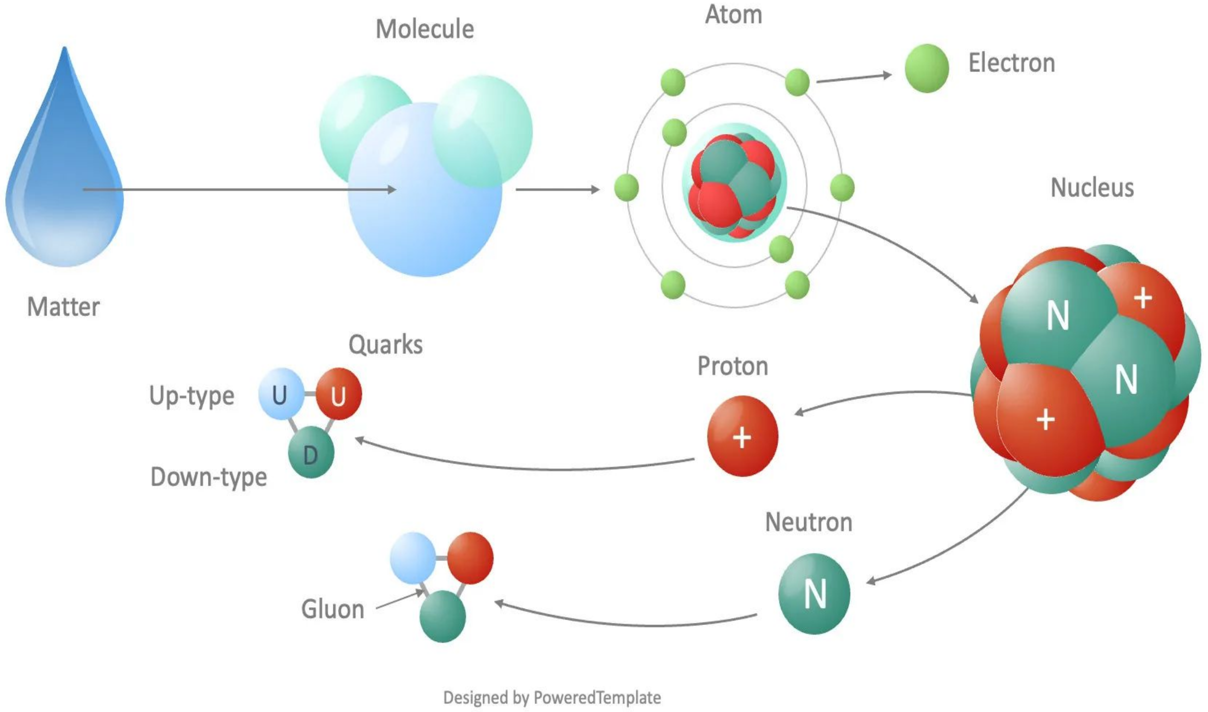


Matter from Molecule to Quark

"How many quarks in a gallon again?"



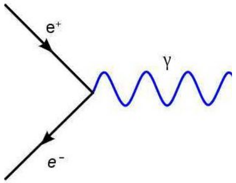
Credit: CERN



QED and QCD

QED

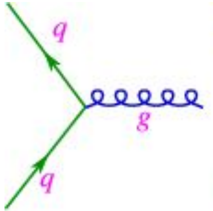
- Describes the interactions of charged particles with **electromagnetic fields**
- Particles exchange **photons** between them



- Photos are massless bosons, with spin 1, and are chargeless, do not self interact
- Coupling constant, strength of interaction, α~1/137

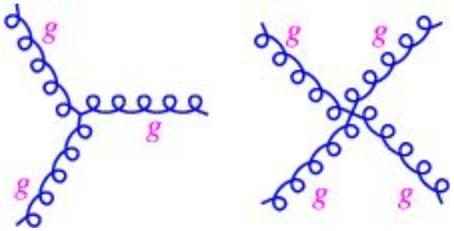
QCD

- Natural fundamental interaction that occurs between subatomic particles, **strong force**
- Particles exchange **gluons** between them



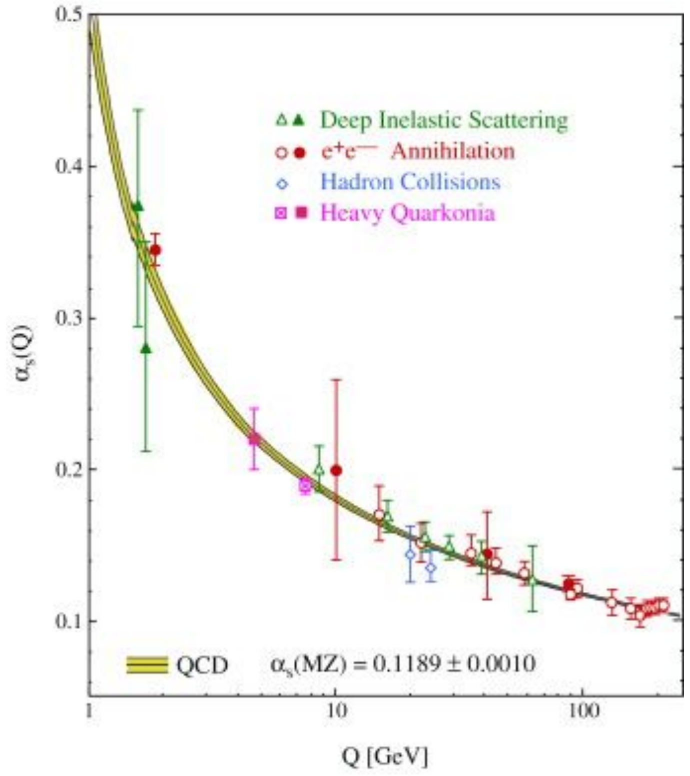
- Gluons are massless vector bosons, spin1, have color charge, self-interact
- Running coupling constant, α~α(Q)

QCD Only →



Coupling Constant (QCD)

Confinement



strong force confining quarks inside a proton (and keeping protons inside a nucleus)

Asymptotic Freedom

Credit: arXiv:hep-ex/0606035

Cross section

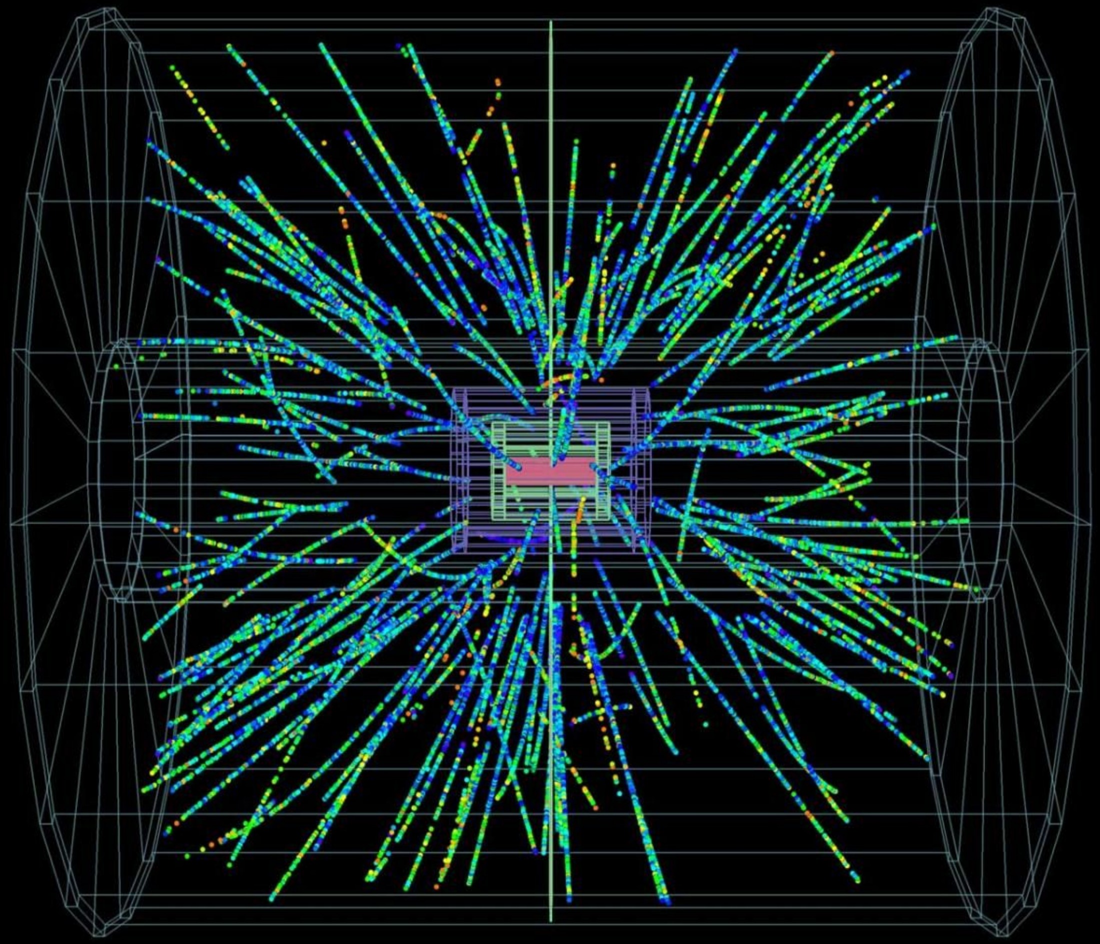
To study inside of hadrons

↓

We have to probe it

↓

Perform collision and measure the produced particles

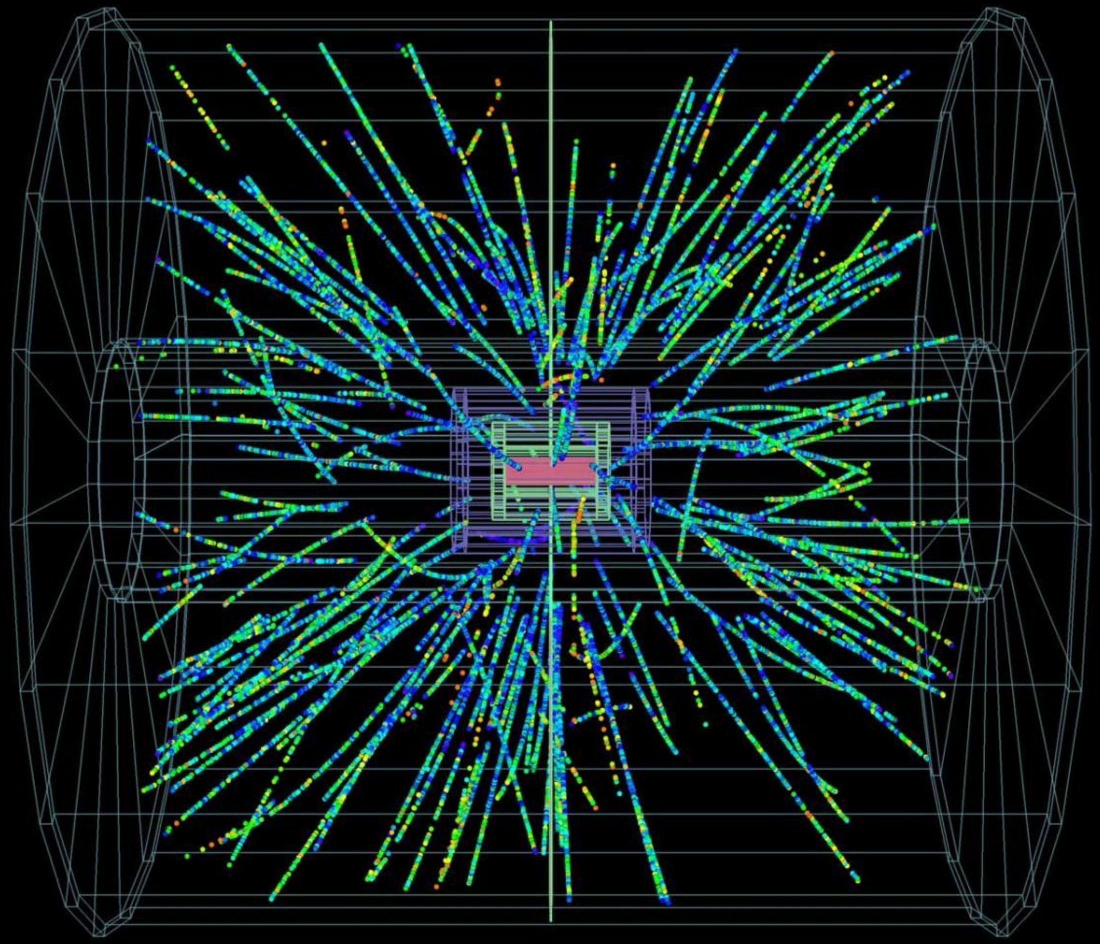


The first proton-lead collisions of 2013 send showers of particles through the ALICE detector (Image: CERN)

Cross section

probability of particles interacting or scattering off each other as they collide

Helps understanding properties, dynamics and interior of hadrons

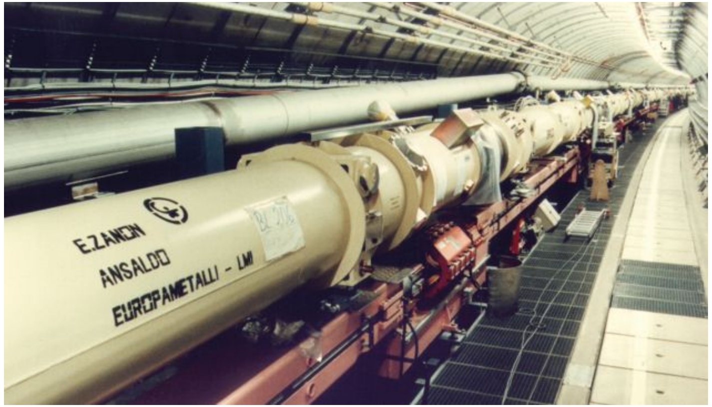


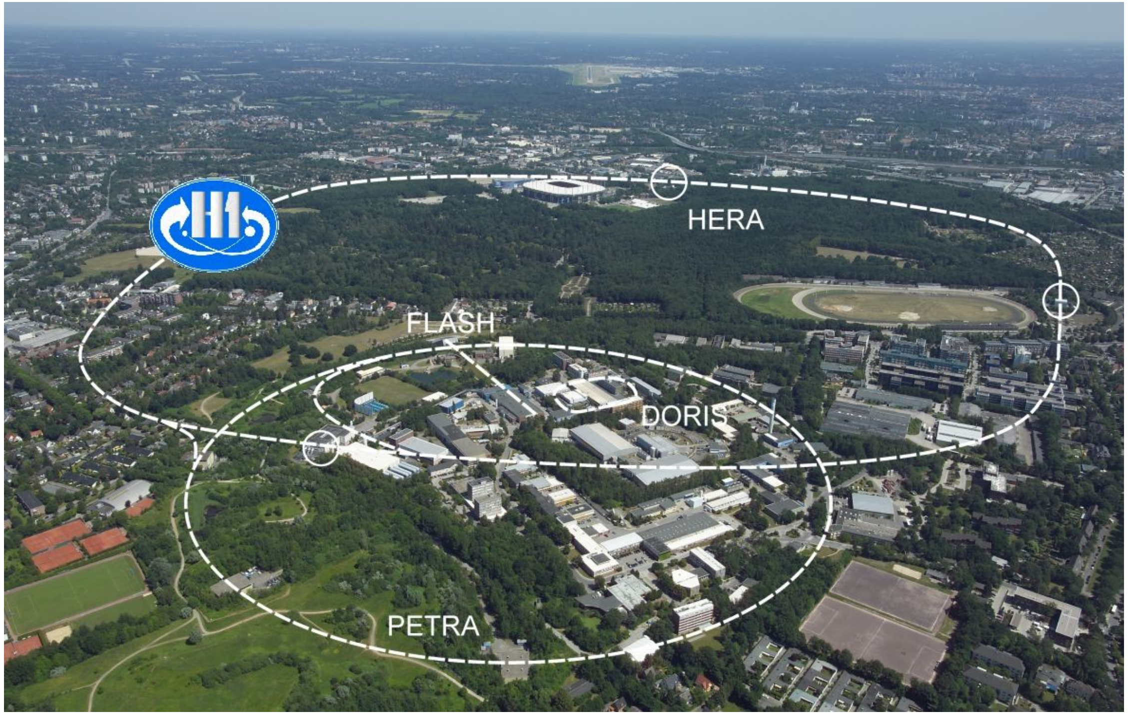
The first proton-lead collisions of 2013 send showers of particles through the ALICE detector (Image: CERN)

Collision Experiment: e-p collision at HERA (1992-2007)

Centre of mass energy= 320 GeV

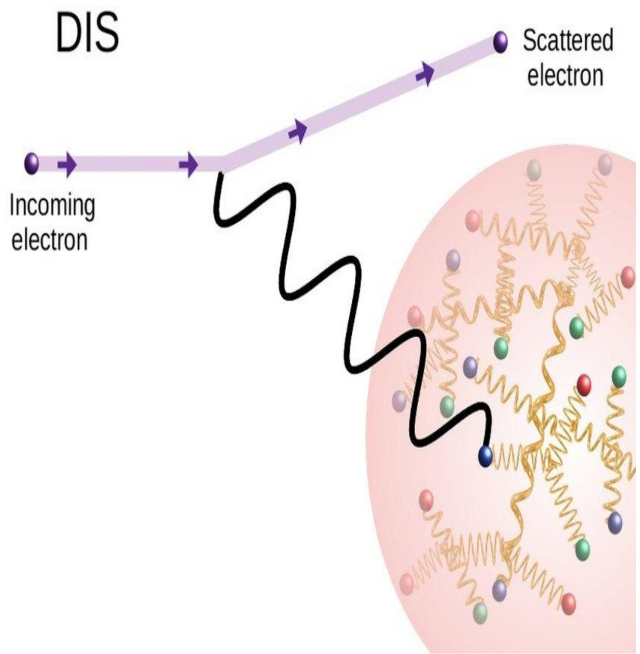
E_e = 27.5 GeV,
E_p = 920 GeV

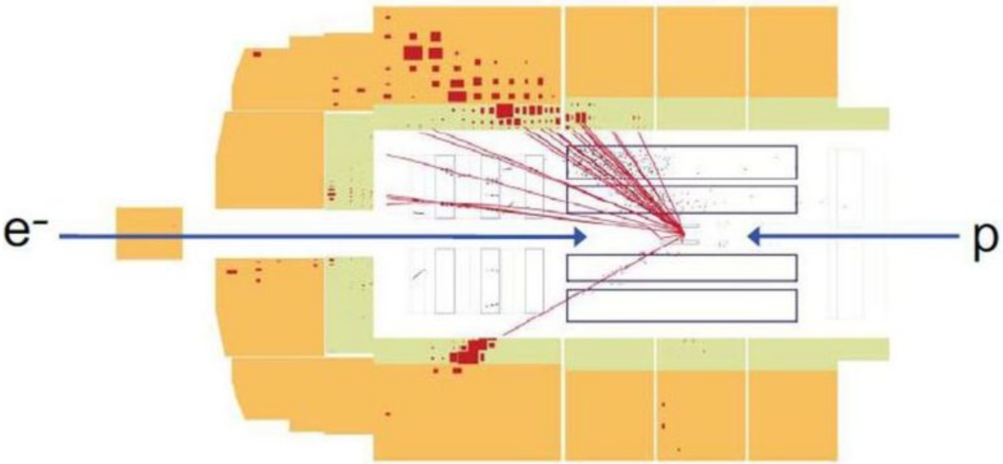




Credit: HERA

Deep Inelastic Scattering at HERA





Credit: HERA

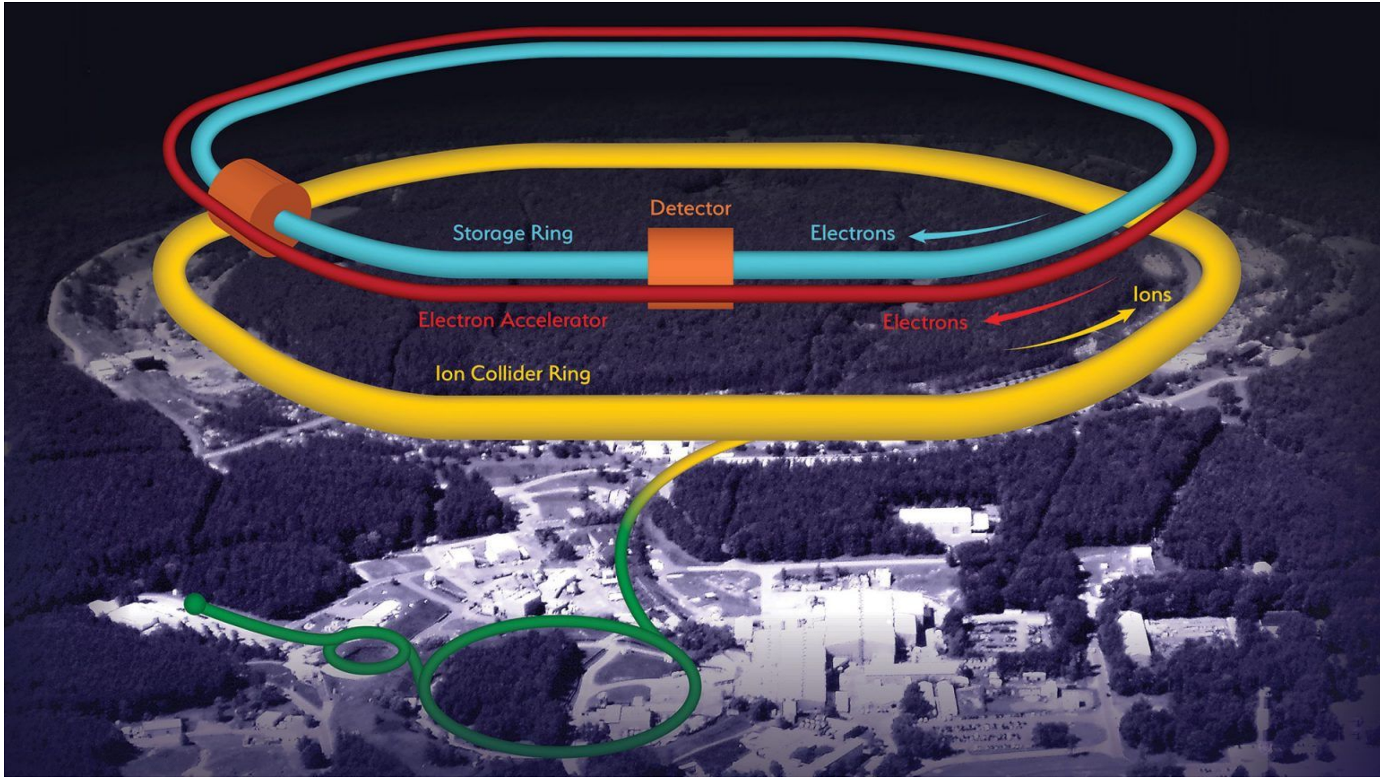
Deep: very high energy of leptons= very short wavelength= ability to probe the distances that are small compared to size of target hadron

Electron Ion Collider (Upcoming)

Lower centre of mass energy than HERA but Higher Luminosity (100-1000 times HERA)

↓

Cover wide Kinematic Range



Can probe proton as well as Ions(no. nucleus)

Credit: EIC

Deep Inelastic Scattering : Kinematics

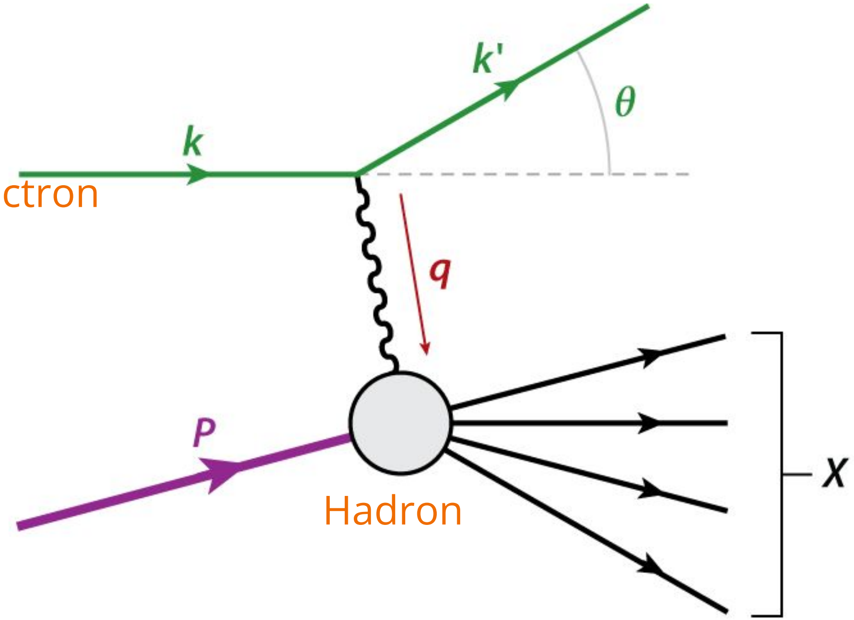
Inclusion cross-section in DIS in terms of Lorentz invariants:

- $x \equiv$ longitudinal momentum fraction carried by parton in the hadron;

  $x = p_i/P$

  p_i = initial momentum of struck parton

  P = momentum transfer of target
- Photon Virtuality: $q^2 = -Q^2 < 0$
- s= the square of center of mass energy; $s = (p+k)^2$
- $W^2 = (p+q)^2 = Q^2/x$



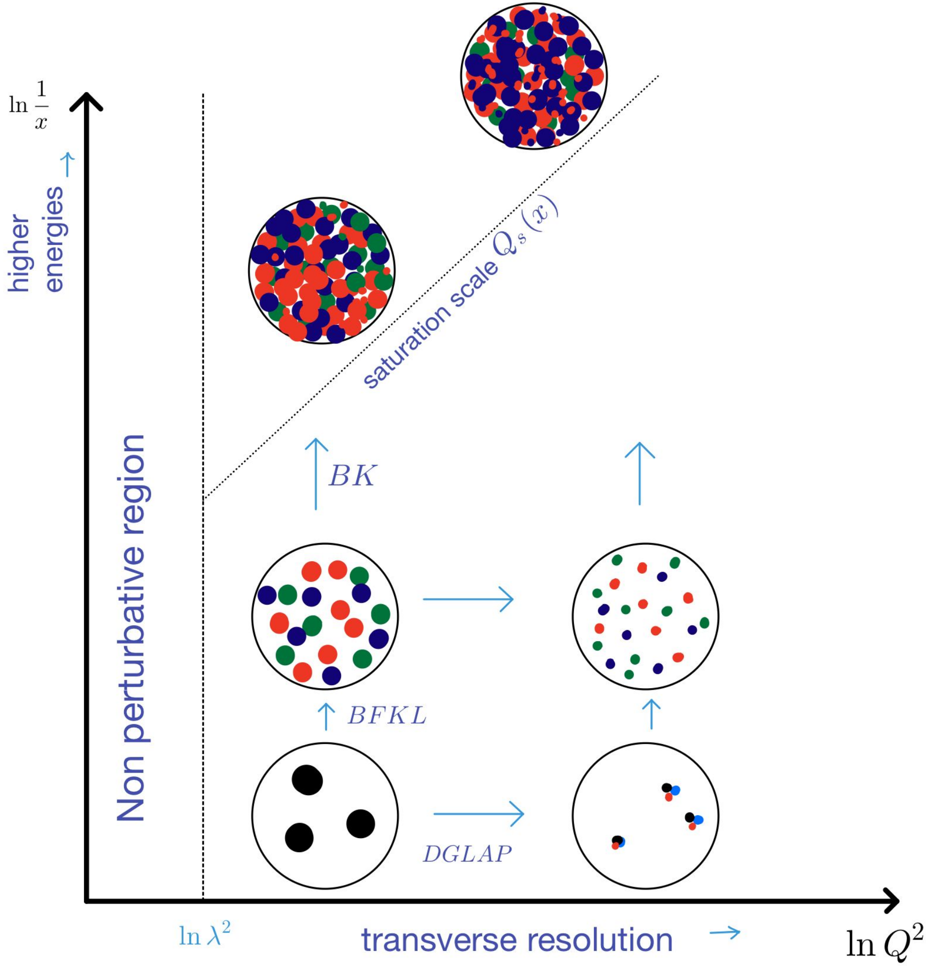
Gelis F, et al. 2010.
Annu. Rev. Nucl. Part. Sci. 60:463–89

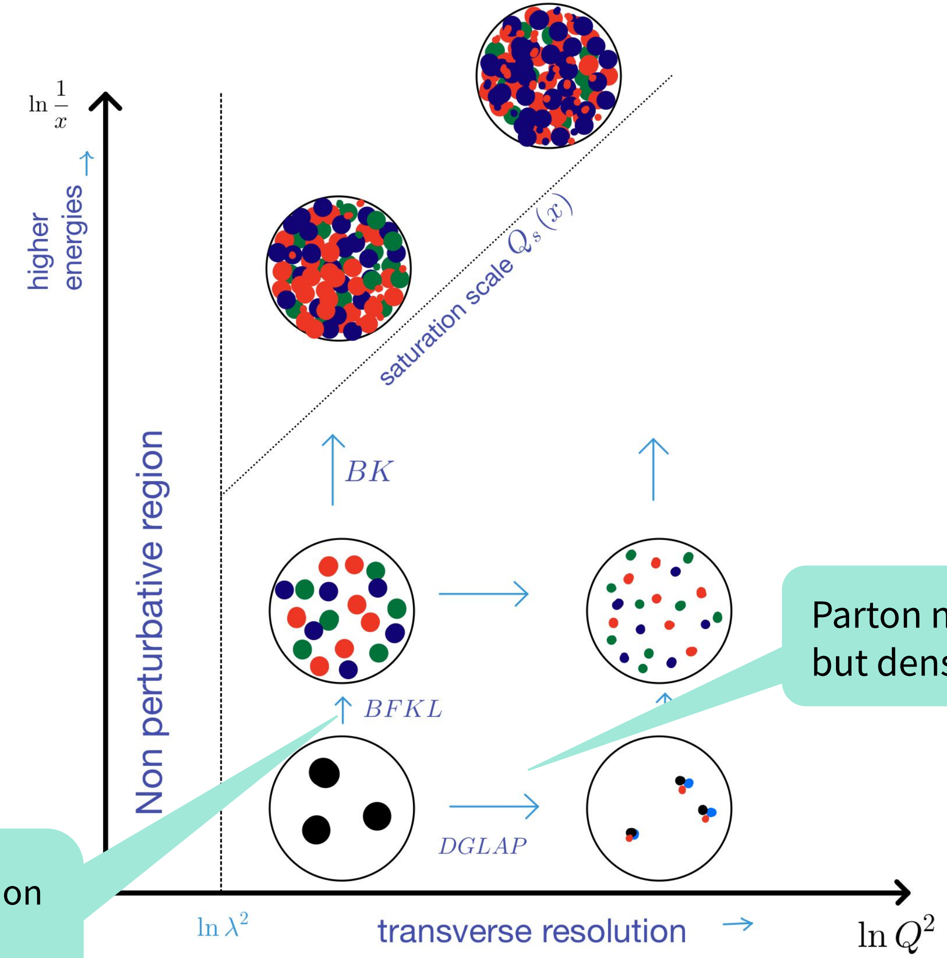
Saturation Physics:

Phase Diagram of Gluon Saturation

Two ways to increase the energy $s = Q^2/x$:

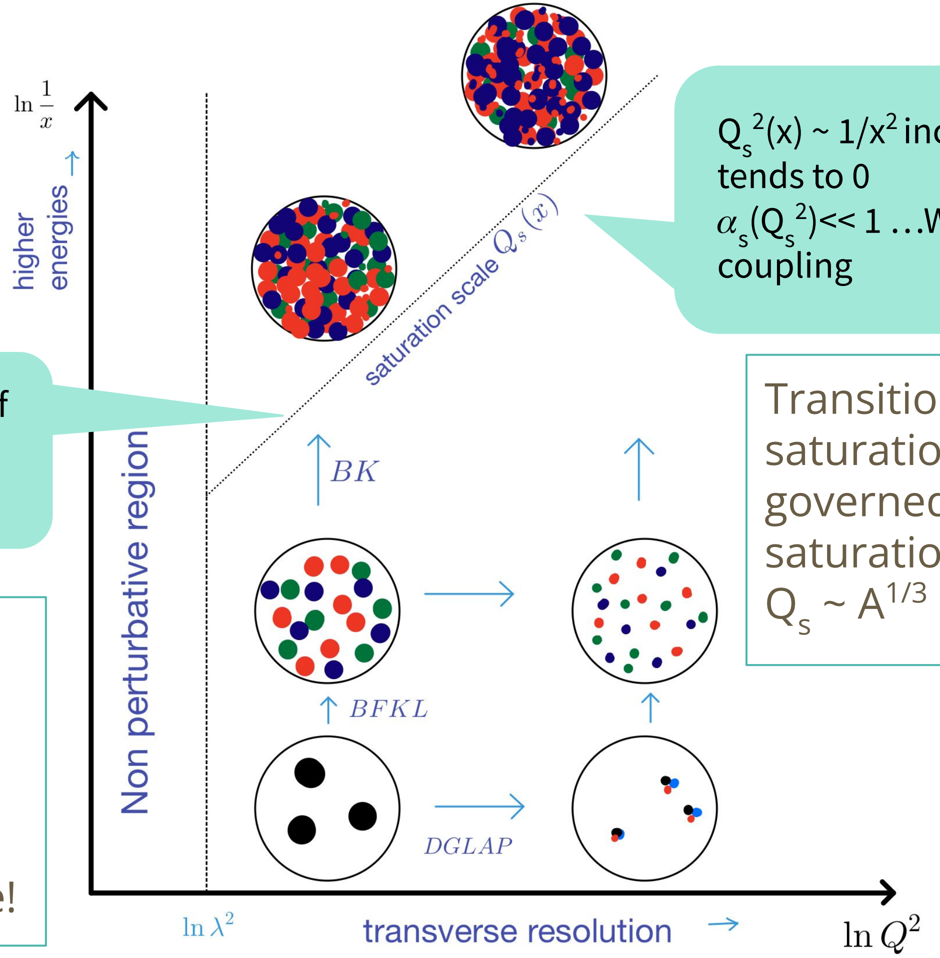
1. $Q^2 \longrightarrow$ infinity, fixed x
2. $x \longrightarrow 0$, fixed Q^2





Parton no. increases, but density decreases

Multiple Gluon emissions
$N_g \sim e^{□\ln(1/x)}$



$Q_s^{\,2}(x) \sim 1/x^2$ increases x tends to 0
$\alpha_s(Q_s^{\,2}) << 1$ …Weak coupling

Non linear interaction of gluons
unitarity

Transition to saturation region is governed by saturation scale Q_s.
$Q_s \sim A^{1/3} (1/x)^{\lambda}$

Saturation physics allows us to study region of high parton density with weak coupling methods, calculations are doable!

$Q_s^2(x) \sim 1/x^2$ increases x tends to 0

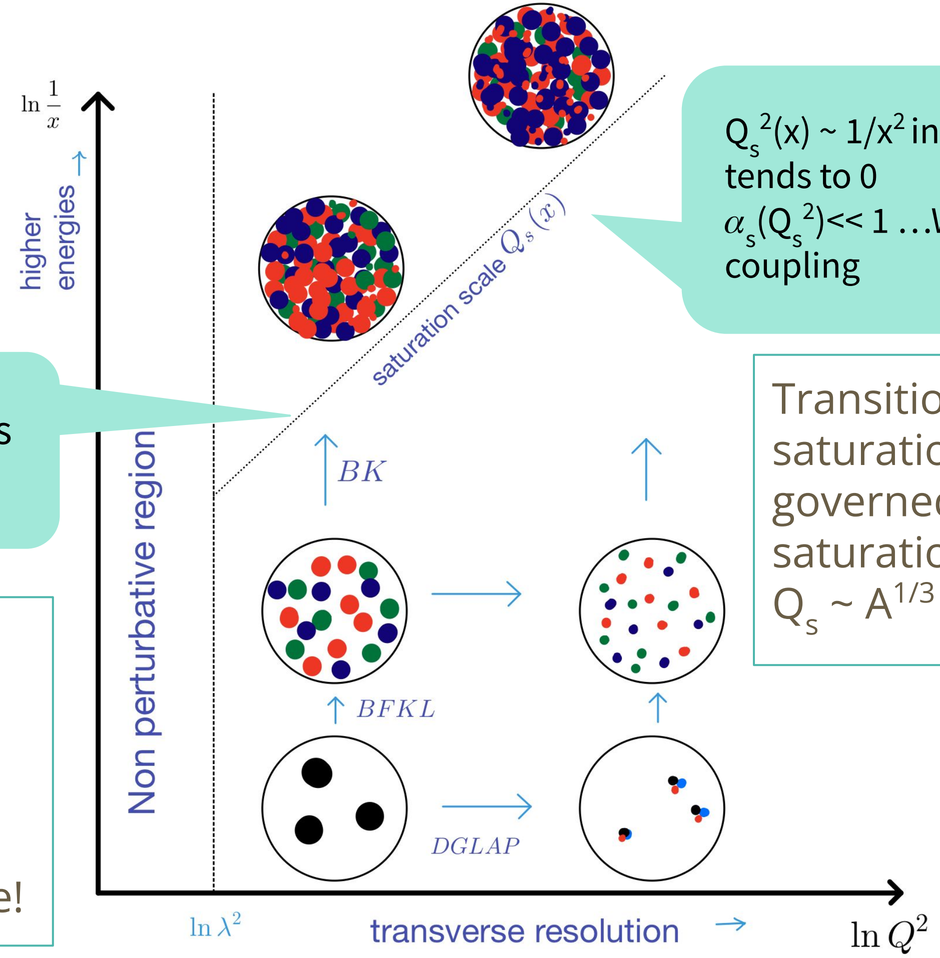
$\alpha_s(Q_s^2) \ll 1$ …Weak coupling

Color Glass Condensate

Non-linear interactions of gluons

$N_g \leq 1$

unitarity



Transition to saturation region is governed by saturation scale Q_s.

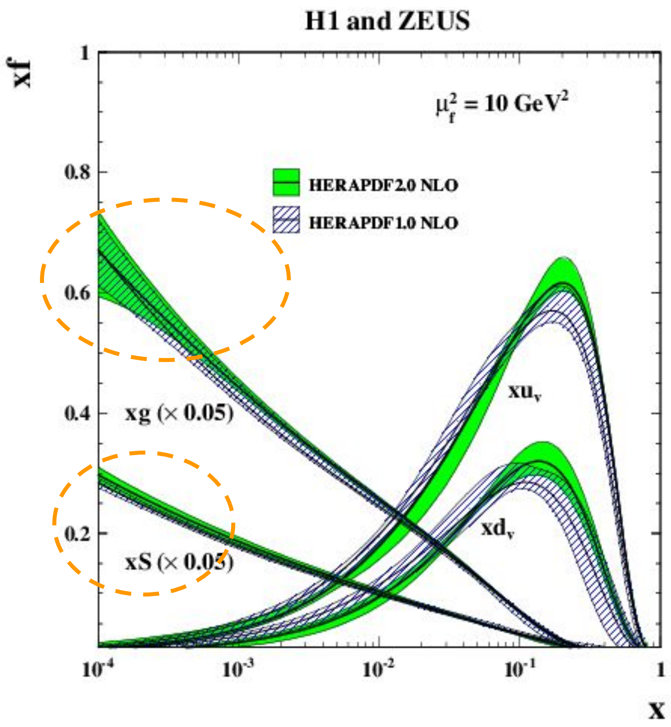
$Q_s \sim A^{1/3} (1/x)^{\lambda}$

Parton Density

Parton gas

Saturation physics allows us to study region of high parton density with weak coupling methods, calculations are doable!

- At very high energy: small-x, gluons and sea quarks dominate
- HERA Data compatible with saturation at small-x



Credit: DOI:10.5506/APHYSPOLBSUPP.8.957

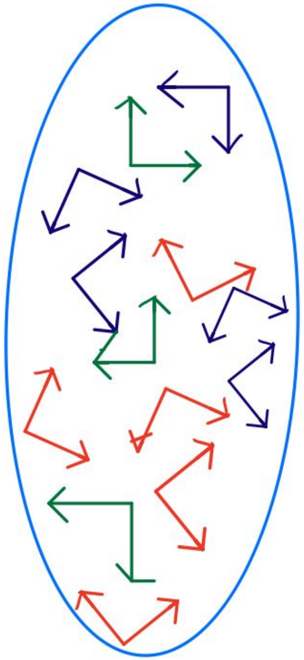
Color Glass Condensate (CGC)

Color : Gluons have "Color"

Glass: the small-x gluons are created by slowly moving patrons(with large x) which are randomly distributed over transverse plane ⟶ it looks like almost frozen over natural time scale of scattering(This is very similar to **spin glass**, where spins are distributed randomly and move very slowly)

Condensate: It is dense matter of gluons. Can be better described as fields rather than point particles!

Can be studied by weak coupling methods!



Generally in saturation physics in Color Glass Condensate (CGC) framework 2 approximations:

- **Semi-classical approximation:**
  Dense target given by Strong semi-classical gluon field $A_\mu(x)$
- **Eikonal approximation :**
  Limit of infinite boost of $A_\mu(x)$

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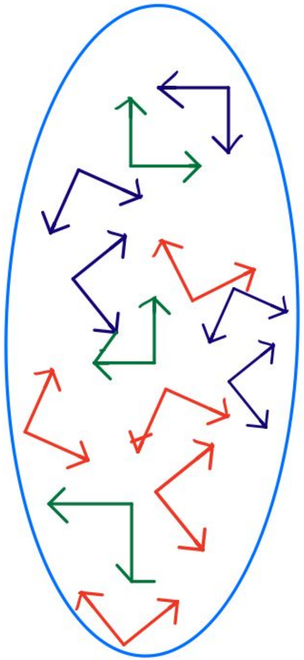
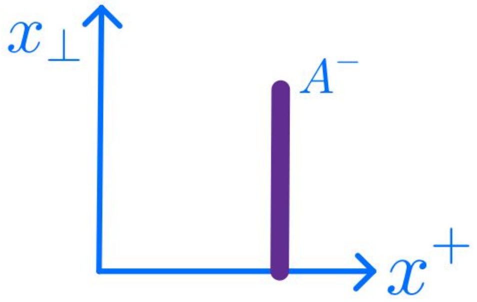


Generally in saturation physics in Color Glass Condensate (CGC) framework 2 approximations:

- **Eikonal approximation :**

  Limit of infinite boost of $A_\mu(x)$

- Taking into account **only leading power in** terms of high energy : (here, leading order component w.r.t. γ_t)
- Good enough approximation to describe physics at very high energy accelerators.



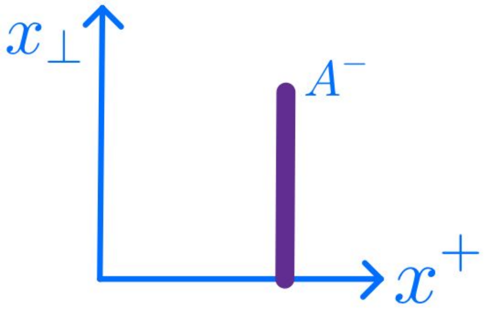
Eikonal Order: For $A_\mu(x)$

Eikonal Order

1. Shockwave approx.: target is localised in the longitudinal direction $x^+ = 0$ (**zero width**).
2. **Only leading - component of target considered**, subleading components are neglected (suppressed by γ_t)
3. Time dilation and static approximation: **x^- dependence of target neglected**

In light-cone coordinate, w.r.t. Lorentz boost factor of target (γ_t)

$$A^- = \mathcal{O}(\gamma_t) >> A^j = \mathcal{O}(1) >> A^+ = \mathcal{O}(1/\gamma_t)$$



Possible Applications:

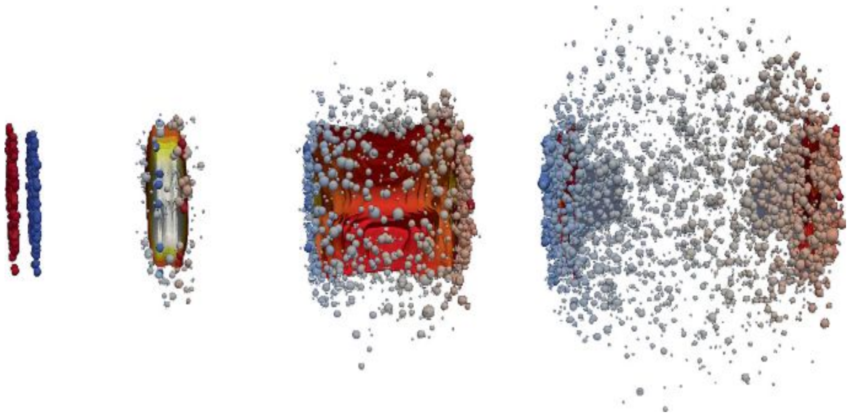
- Dilute-Dilute Scattering:
- Dense-Dense Scattering:
- Dilute-Dense Scattering:

Possible Applications:

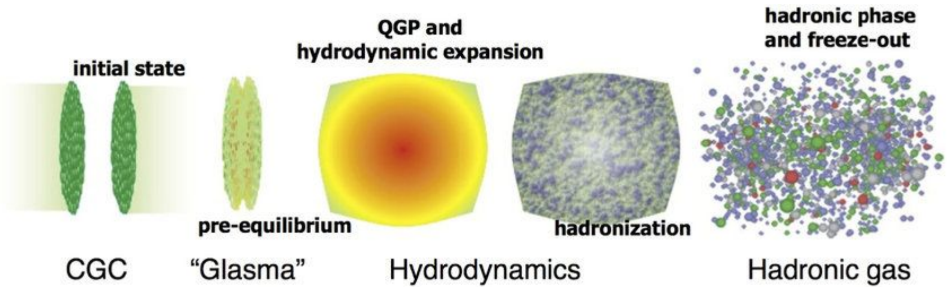
- **Dilute-Dilute Scattering:**
  - No saturation effect, BFKL formalism
- Dense-Dense Scattering:
- Dilute-Dense Scattering:

Possible Applications:

- **Dense-Dense Scattering:**
  - Target and projectile **both are saturated** (Non-linear dynamics of Yang-Mills fields)
  - Applied to heavy ion collision, pp at very high energies



Credit: Strongly Interacting Matter under Rotation, 2021, Volume 987,Karpenko



Credit: arXiv:1309.7616

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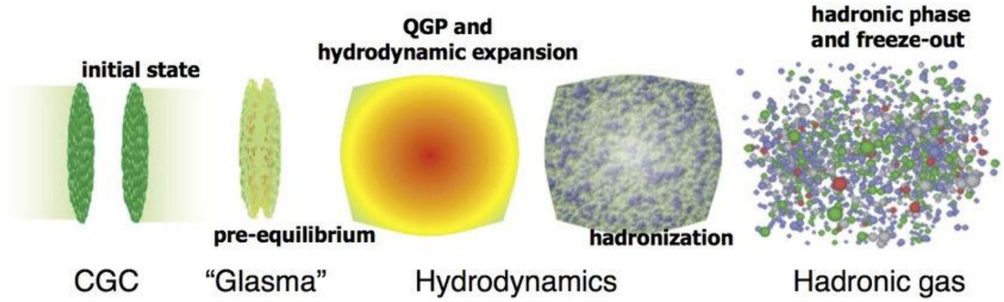

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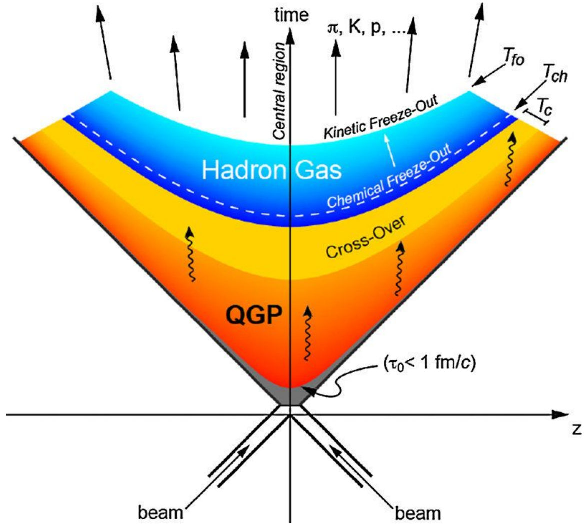

Credit: DOI:10.1016/j.nuclphysa.2019.02.006

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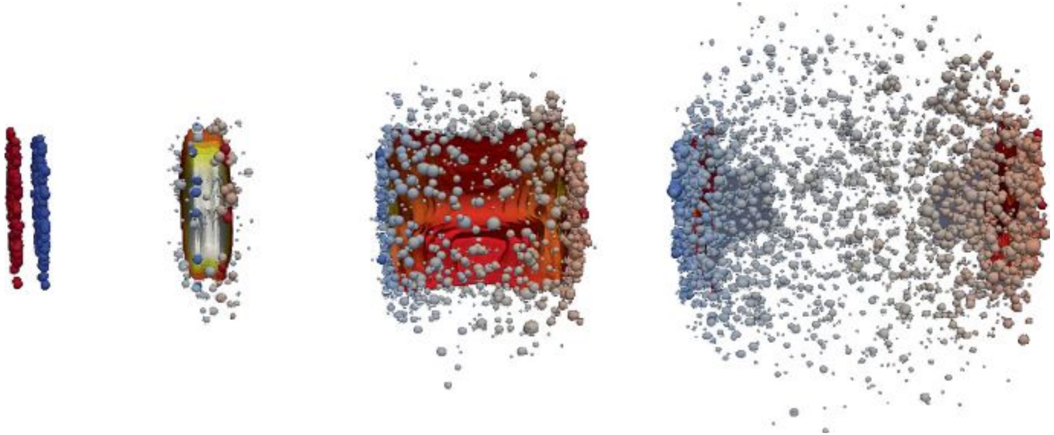

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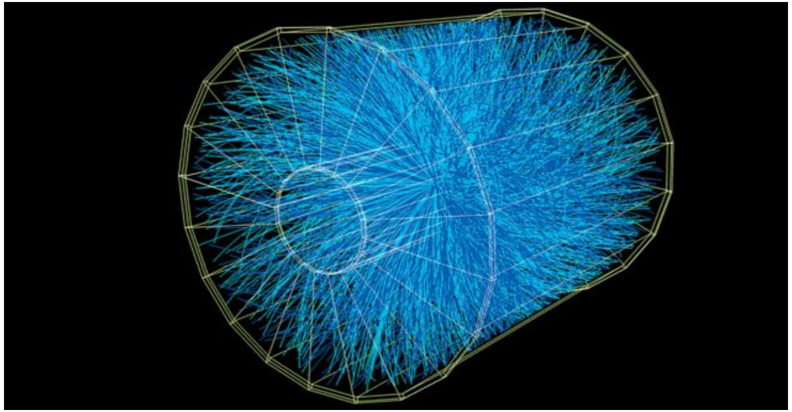
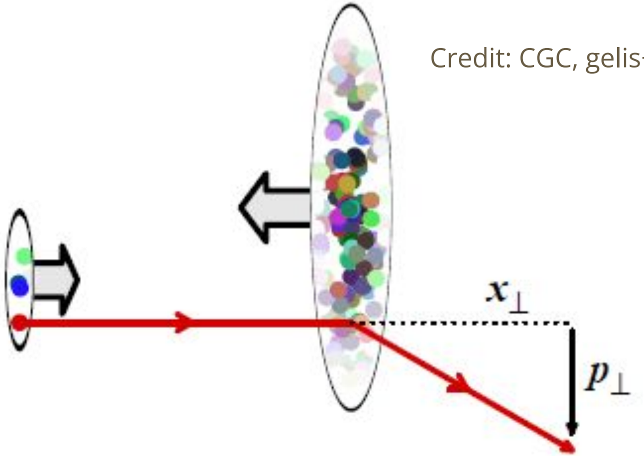


Credit: ALICE, pb-pb collision

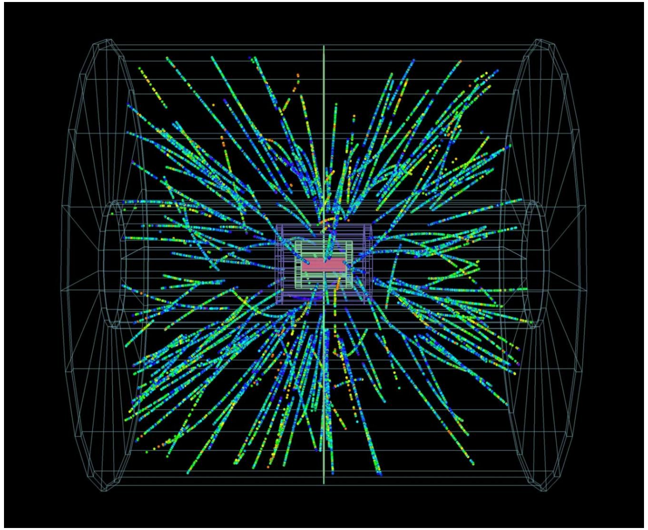
Possible Applications:

- **Dilute-Dense Scattering:**
  - Target is saturated (CGC formalism)
  - Can be applied to: DIS on A, pA collisions



Credit: CGC, gelis+

pA collisions: saturation sensitive observables are forward particle production/jet production........



The first proton-lead collisions of 2013 send showers of particles through the ALICE detector (Image: CERN)

Single Inclusive Particle Production in pA collision

Proton-nucleus (pA) collisions gives access to small-x region of nuclear wave function

Density in heavy nuclei are enhanced by $A^{1/3}$

Forward particle production in pA collision promising channel:

1. Look for signatures of gluon saturation
2. To study dynamics of non-linear QCD

Eikonal Order and Going beyond Eikonal Order

Eikonal Approximation:

- Taking into account only leading power in terms of high energy
- Good enough approximation to describe physics at very high energy accelerators.

Going Beyond Eikonal order:

- Taking into account terms suppressed in energy.
- In comparatively moderate energy accelerators (EIC and RHIC) sub-eikonal corrections might be sizable.

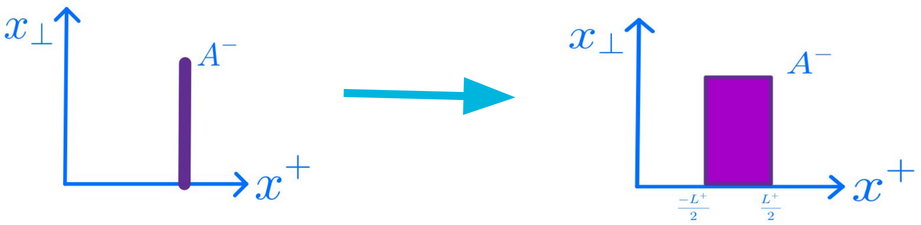
Main Objective:

Providing sub-eikonal corrections to the various observable in CGC Framework

Going Beyond Eikonal Order..................

w.r.t. Lorentz boost factor of target (γ_t)

$$A^- = \mathcal{O}(\gamma_t) >> A^j = \mathcal{O}(1) >> A^+ = \mathcal{O}(1/\gamma_t)$$



Going Beyond Eikonal Order: For $A_\mu(x)$

Eikonal Order

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3. Time dilation and static approximation: **x^- dependence of target neglected**

Next-to-eikonal Order

1. Instead of infinite thin shockwave as a target, we consider **finite width** of a target.
2. Include **transverse component** of background field(target).
3. Consider background field is x^- dependent: **dynamics of the target are considered**.

Going Beyond Eikonal Order: Quark Background Field

- Due to large boost of the target along x^-: its **localized in longitudinal x^+** direction around small support.
- If we consider projections on quark background field then,

$$\Psi(z) = \frac{\gamma^+\gamma^-}{2}\Psi(z) + \frac{\gamma^-\gamma^+}{2}\Psi(z) = \Psi^-(z) + \Psi^+(z)$$

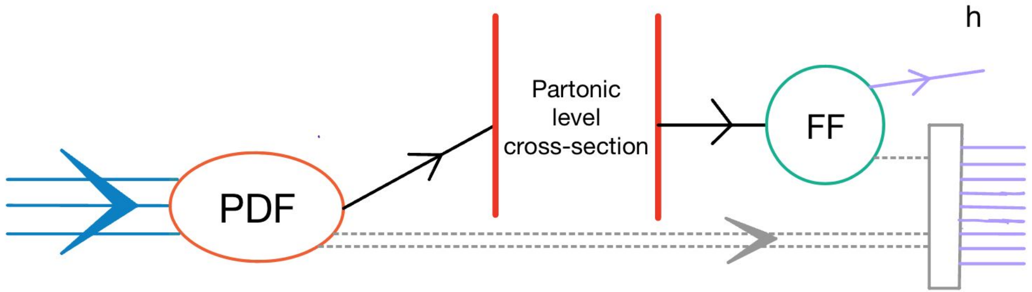
$\Psi^-(z)$: $\mathcal{O}(\sqrt{\gamma_t})$ — $\Psi^+(z)$: $\mathcal{O}(1/\sqrt{\gamma_t})$

- For **Next-to-eikonal (NEik) corrections, only - component** considered and + component is neglected (contribute at NNEik only).

Single Inclusive Particle Production: Cross-section

- In CGC framework cross section is given as:

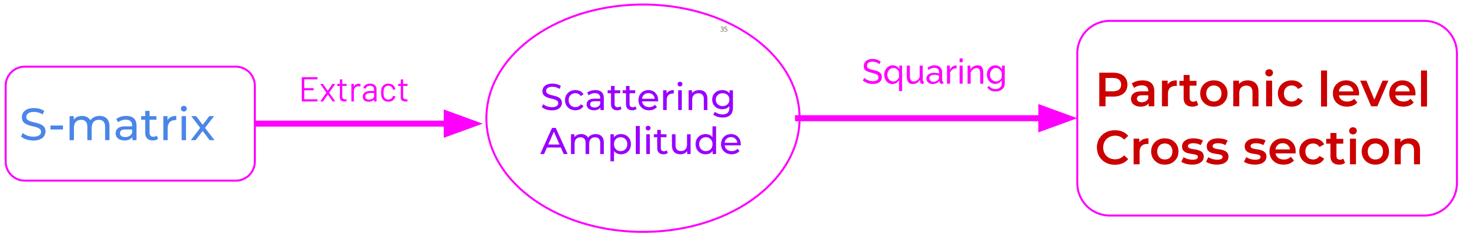
$$\sigma \propto \text{PDF} \otimes \text{Partonic level cross-section} \otimes \text{fragmentation function}$$



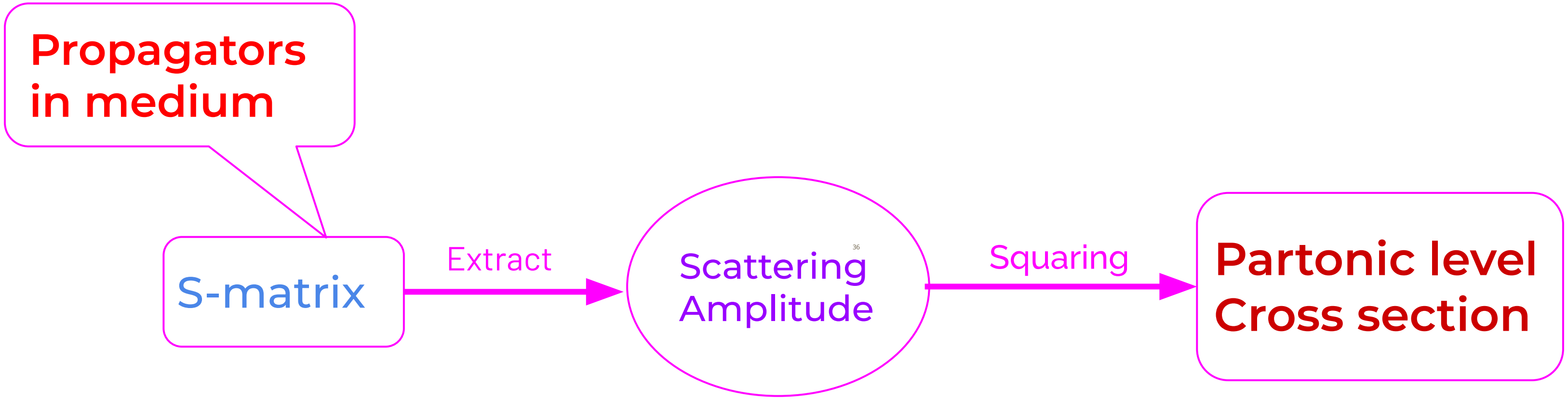
- Effect of Including NEik corrections **seen at Partonic level** Cross-section
- Our goal: ***To compute partonic level cross section at NEik order***

How do we compute the cross-section?

Single Inclusive Particle Production: Cross-section



Single Inclusive Particle Production: Cross-section



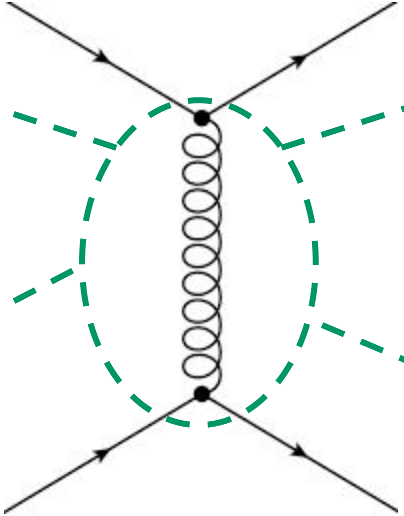
Propagators??

At NEik order: in finite width medium

Propagators??

associated with specific particles or fields

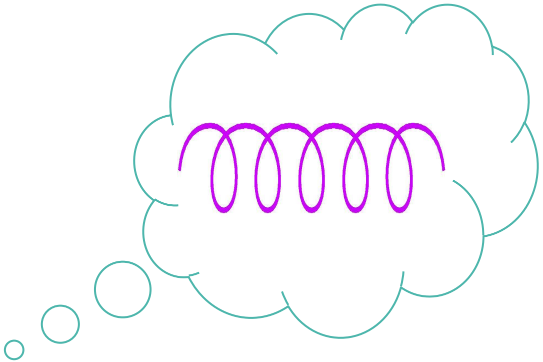
provides information about **propagation** of particle/field.



depends on 1.the momentum, mass, and energy of the particle or field involved 2. the interactions and forces present in the system.

fundamental in computing probabilities of particle interactions, scattering processes, and other quantum phenomena.

Gluon Propagator: in Vacuum



In vacuum Gluon propagator (without presence of medium) in momentum space is give as:

$$G^{\mu\nu}_{0,F}(p) = \frac{i}{p^2 + i\varepsilon}\left[-g^{\mu\nu} + \frac{p^\mu \eta^\nu + \eta^\mu p^\nu}{p \cdot \eta}\right]$$

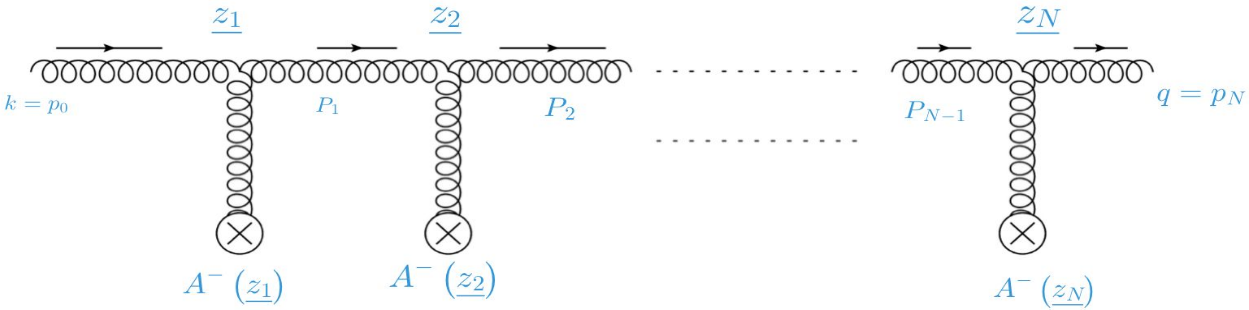
This is in Light-cone gauge, $A^+ = 0$ and $\eta^2=0$
Where, $\eta^\mu = g^{\mu +}$

How to obtain these propagators?

At NEik order: in finite width medium
INCLUDING INTERACTION WITH MEDIUM

Gluon Propagator at Eikonal order

- To calculate it, we **re-sum multiple interaction diagrams** of Gluon background field.



Similar for quarks in Altinoluk,Beuf, Czajka, Tymowska [2012.03886] , Altinoluk, Beuf [arXiv:2109.01620]

Gluon Propagator at NEik order

Recipe: For General gluon propagator at next-to-eikonal order travelling through entire medium

a. First compute Eikonal order Gluon Propagator in gluon background field.

    i. Only "-" leading component of classical gluon background field is considered.

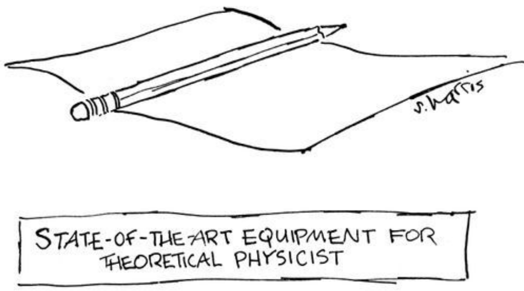
b. Use this computed gluon propagator to obtain next-to-eikonal (NEik) contributions

    i. Due to considering finite width of the target

    ii. Due to interaction with transverse components of medium

    iii. Due to dynamics of target (including x- dependence)

Total Gluon propagator at NEik order



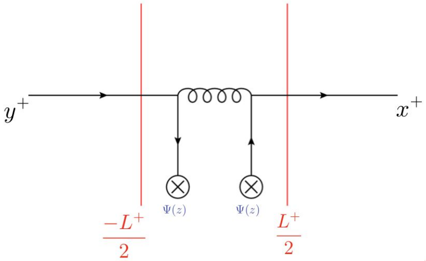
Total gluon propagator upto NEik order travelling through the entire medium (dynamic gluon background field) for the case $x^+ > L^+/2$ and

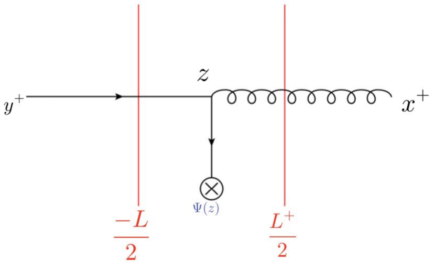
$y^+ < -L^+/2$ with $x^+ > y^+$ is:

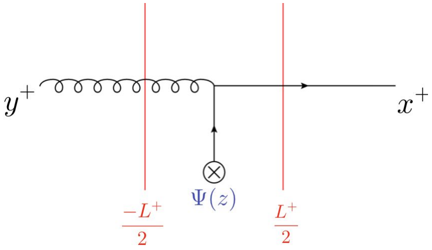
$$
\begin{aligned}
G_F^{\mu\nu}(x,y) = &\int \frac{d^3\underline{q}}{(2\pi)^3}\, e^{-ix\cdot\check{q}}\,\theta(q^+) \int \frac{d^3\underline{k}}{(2\pi)^3}\, e^{iy\cdot\check{k}}\,\theta(k^+)\, \frac{1}{q^+ + k^+} \\
&\times \left[-g^{\mu\nu} + \frac{\check{k}^\mu \eta^\nu}{k^+} + \frac{\eta^\mu \check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k})\right] \int d^2z_\perp e^{-i(q_\perp - k_\perp)z_\perp} \\
&\times \int dz^-\, e^{i(q^+-k^+)z^-}\, \mathcal{U}_A\Big(\frac{L^+}{2}, \frac{-L^+}{2}, z_\perp, z^-\Big) \\
&+ \int \frac{d^3\underline{q}}{(2\pi)^3} \frac{e^{-ix\cdot\check{q}}}{2q^+}\theta(q^+) \int \frac{d^3\underline{k}}{(2\pi)^3} \frac{e^{iy\cdot\check{k}}}{2k^+}\theta(k^+) \int dz^-\, e^{iz^-(q^+-k^+)} \\
&\times \int d^2z_\perp\, e^{-iz_\perp(q_\perp - k_\perp)} \Bigg\{\left(-g^{\mu\nu} + \frac{\check{k}^\mu\eta^\nu}{k^+} + \frac{\eta^\mu\check{q}^\nu}{q^+} - \frac{\eta^\mu\eta^\nu}{q^+k^+}(\check{q}\cdot\check{k})\right) \\
&\times \Bigg(-\frac{q^j+k^j}{2}\int_{\frac{-L^+}{2}}^{\frac{L^+}{2}} dz^+ \Big[\mathcal{U}_A\Big(\frac{L^+}{2}, z^+; z_\perp, z^-\Big)\Big(\overrightarrow{D}_{z^j} - \overleftarrow{D}_{z^j}\Big)\mathcal{U}_A\Big(z^+, -\frac{L^+}{2}; z_\perp, z^-\Big)\Big] \\
&- i\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \Big[\mathcal{U}_A\Big(\frac{L^+}{2}, z^+; z_\perp, z^-\Big)\Big(\overleftarrow{D}_{z^j}\overrightarrow{D}_{z^j}\Big)\, \mathcal{U}_A\Big(z^+, -\frac{L^+}{2}; z_\perp, z^-\Big)\Big]\Bigg) \\
&+ \left(g^{\mu j}g^{\nu i} - \frac{\eta^\mu g^{\nu i} q^j}{q^+} - \frac{g^{\mu j}k^i\eta^\nu}{k^+} + \frac{\eta^\mu\eta^\nu k^i q^j}{q^+k^+}\right) \\
&\times \left(\int dz^+\, \mathcal{U}_A\Big(\frac{L^+}{2}, z^+; z_\perp, z^-\Big)\, gT\cdot F_{ij}\, \mathcal{U}_A\Big(z^+, -\frac{L^+}{2}; z_\perp, z^-\Big)\right)\Bigg\}
\end{aligned}
$$

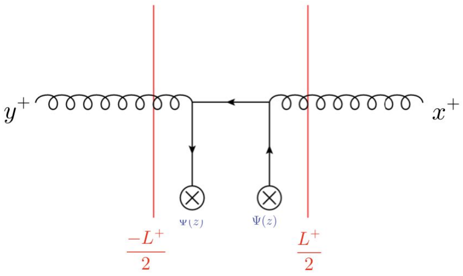
Single Inclusive Particle Production

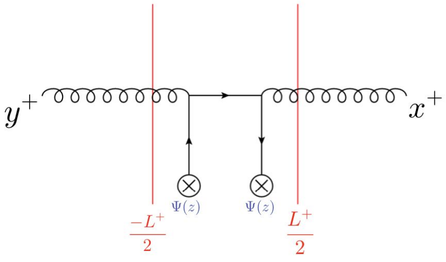
- Finally to compute Partonic level scattering cross-section of single inclusive particle production in forward pA collision at NEik order:
    - From quark to gluon Conversion
    - From gluon to quark Conversion
    - Gluon-target Scattering
    - Quark-target Scattering











Single Inclusive Particle Production

- Finally to compute Partonic level scattering cross-section of single inclusive particle production in forward pA collision at NEik order:
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  - **From gluon to quark Conversion**
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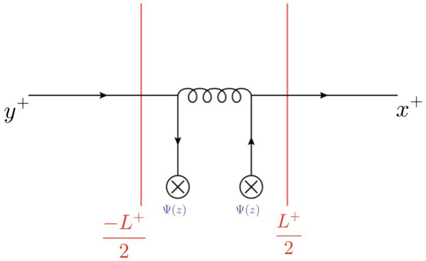
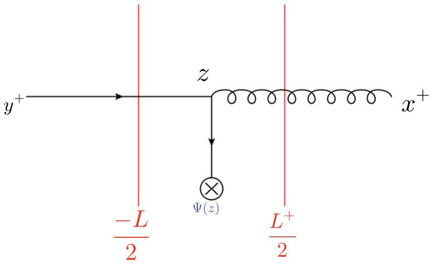
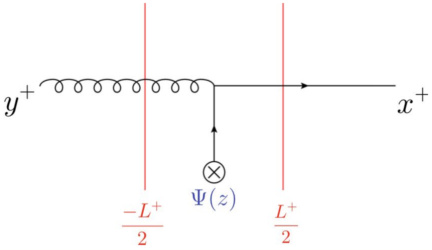
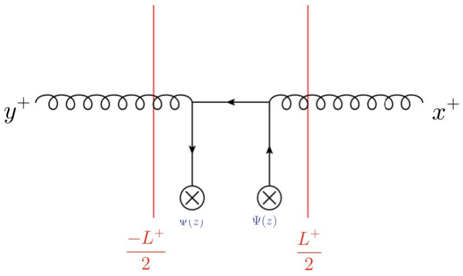
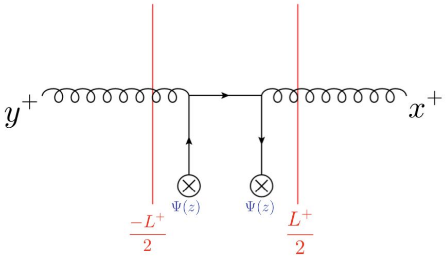
  - Quark-target Scattering







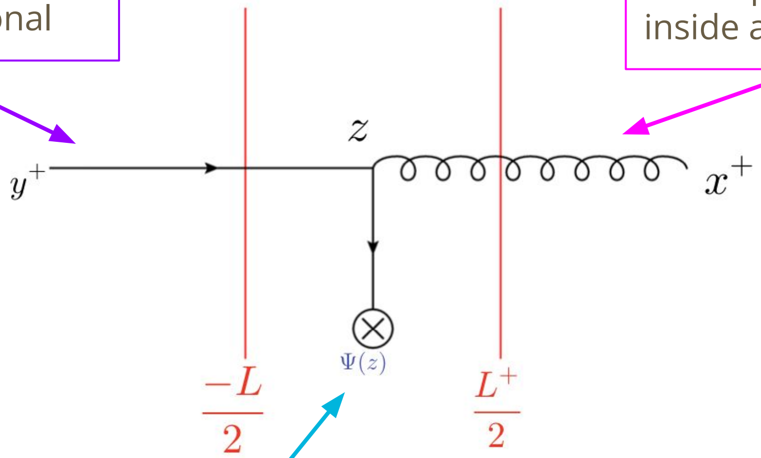




Single Inclusive Particle Production

Quark propagator
before inside, eikonal

Gluon propagator
inside after, eikonal

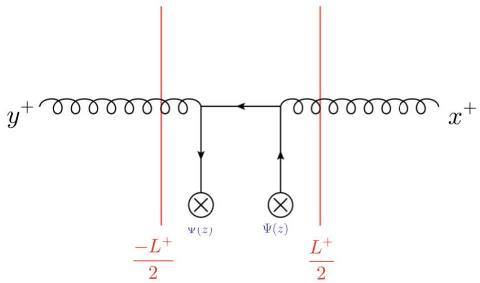


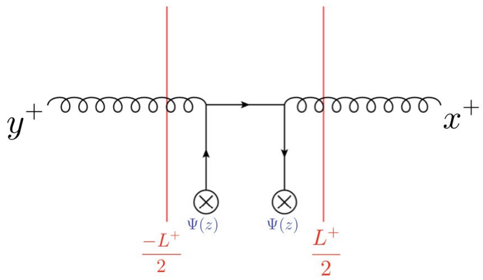
- Component, leading

Single Inclusive Particle Production: Gluons

- **Gluon-target Scattering:**
  - Gluon-target scattering in gluon background field
  - Gluon-target scattering considering effect of quark background field through t-channel quark exchange

Adding both of these contributions together to get total gluon-target scattering cross-section at NEik order

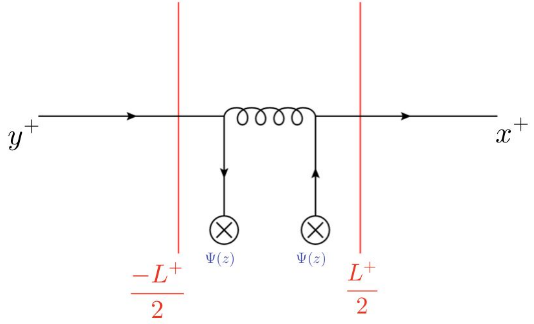




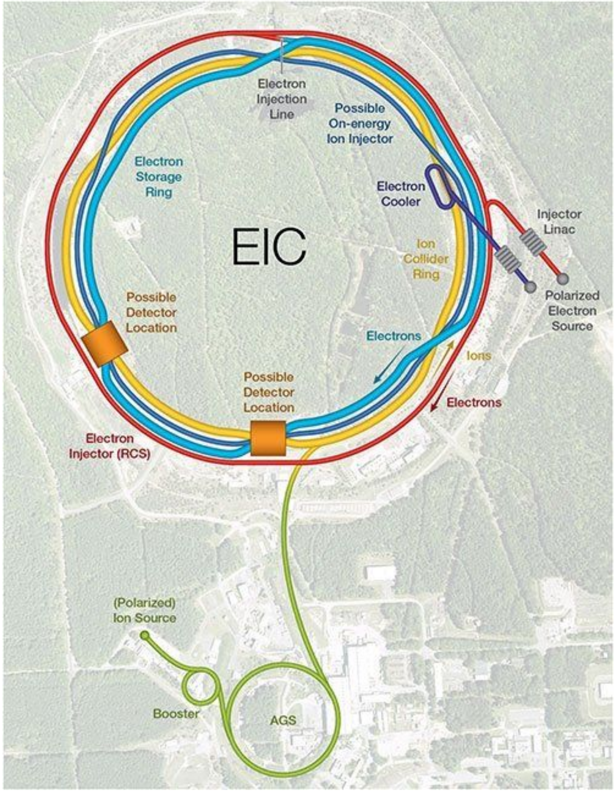
Single Inclusive Particle Production: Quarks

- **Quark-target Scattering:**
  - Quark-target scattering in gluon background field
  - Quark-target scattering including effect of quark background field

Summing up these contributions to get total scattering cross-section for quark-target scattering at NEik order for pA collisions

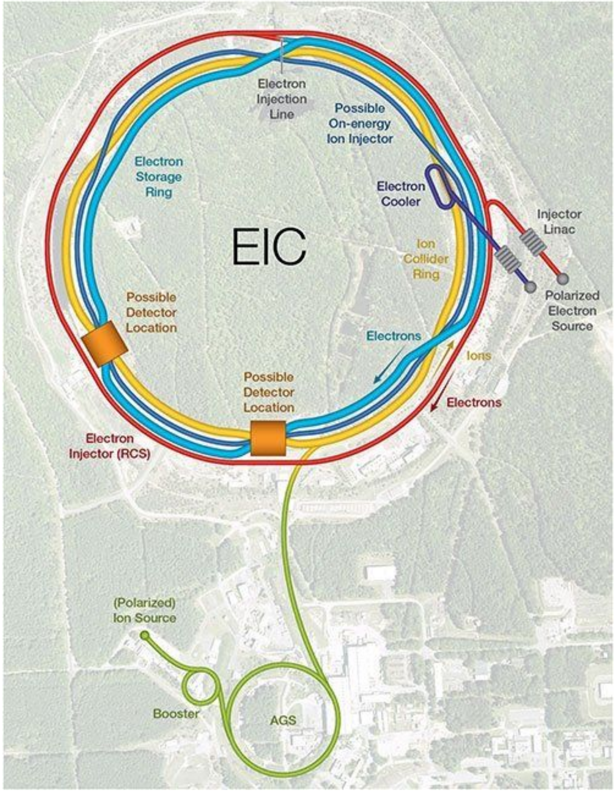


Summary



- Saturation Physics in relatively new physics
  - Lot of interesting things and physics yet to be explored
- Upcoming electron-ion collider brings prospect of precision era
  - So sub eikonal corrections might be sizable in upcoming future!

Summary



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Thank You!