# Studies of CPT with $D^0$ mesons

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# Motivation

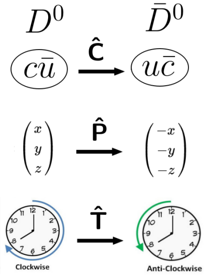
1. We want to test CPT in the charm sector.

2. Good place to study CPT - neutral D meson oscillations.

3. CPT invariance studied by probing space-time symmetry violations.

4. The framework to study these deviations - Standard Model Extension (SME).

# Key symmetries in particle physics




C: $\hat{\boldsymbol{C}}\,|c, \vec{x}, t\rangle = |\bar{c}, \vec{x}, t\rangle$, charge conjugation transformation
P: $\hat{\boldsymbol{P}}\,|c, \vec{x}, t\rangle = |c, -\vec{x}, t\rangle$, parity transformation
T: $\hat{\boldsymbol{T}}\,|c, \vec{x}, t\rangle = \langle c, \vec{x}, -t|$, time reversal transformation
CPT: $\hat{\mathbf{C}}\hat{\mathbf{P}}\hat{\mathbf{T}}$

**CPT** assumed to be strictly conserved in SM !!!
$[H, X] = 0$ then $X$ is conserved.

# CPT theorem and Lorentz Violation (LV)

CPT theorem: Local QFTs with Lorentz symmetry $\Rightarrow$ CPT invariant.

Greenberg: In interacting theories CPTV necessitates LV.

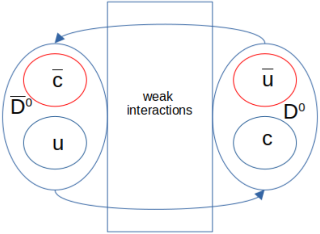
**Take home message: Look for CPTV by searching for LV!**

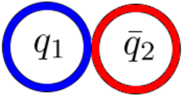
# Flavoured Neutral Mesons

meson: quark ($q_1$) + antiquark ($\bar{q}_2$)

neutral meson: electric charge = 0

flavoured neutral meson: non-zero S, C, B content

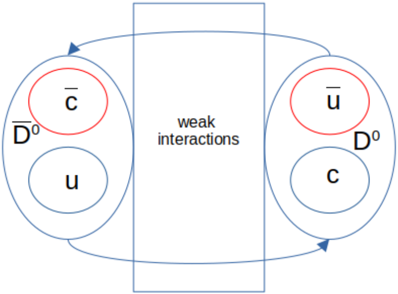


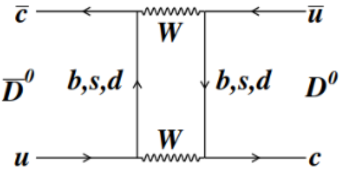



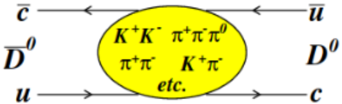

| Neutral meson Q = 0 | Flavor F |
|---|---|
| $\mathbf{K^0}$ <br> $d\bar{s}$ | Strangeness <br> $S = 1$ |
| $\mathbf{D^0}$ <br> $c\bar{u}$ | Charm <br> $C = 1$ |
| $\mathbf{B^0}$ <br> $d\bar{b}$ <br> $s\bar{b}$ | Beauty <br> $B = 1$ <br> $B = 1, S = -1$ |

credits: Agnes Roberts, Presentation IUCSS CPT 21.

# $D^0$ oscillations




credits: Fernando Martinez Vidal arXiv:0910.5061

# Neutral Meson Oscillations - math. description

1. $H = H_0 + H_{wk}$
2. $H_{wk}$ perturbation to $H_0$,
3. $\left|D^0\right\rangle$, $\left|\bar{D}^0\right\rangle$ eigenstates of $H^0$.
4. $|D_L\rangle$, $|D_H\rangle$ eigenstates of $H$.

Weiskopf Wigner Approximation yields 2x2 $H^{eff}$:

$$H^{eff} = \hat{\mathbf{M}} + i\hat{\mathbf{\Gamma}} = \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{bmatrix} + \frac{i}{2} \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{bmatrix} \tag{1}$$

$H^{eff}$ is a sum of two hermitian matrices: $\hat{\mathbf{M}}$ and $\hat{\mathbf{\Gamma}}$, mass and decay matrix respectively.

# $H^{eff}$ eigenstates - mass states

$\left|D^0\right\rangle$, $\left|\bar{D}^0\right\rangle \rightarrow$ not eigenvectors of $H^{eff}$.

$|D_L\rangle$, $|D_H\rangle \rightarrow$ eigenvectors of $H^{eff}$.

System in one of two mass states $|D_L\rangle$ or $|D_H\rangle$.

Mass states - linear combinations of flavour states.

$$\begin{cases} |D_L\rangle & = a_1 \left|D^0\right\rangle + a_2 \left|\bar{D}^0\right\rangle , \\ |D_H\rangle & = b_1 \left|D^0\right\rangle + b_2 \left|\bar{D}^0\right\rangle , \\ \left|D_{L/H}(t)\right\rangle & = e^{-itH^{eff}} \left|D_{L/H}(0)\right\rangle . \end{cases} \tag{2}$$

Find $\left|D^0(t)\right\rangle$, $\left|\bar{D}^0(t)\right\rangle$ by solving the above:

$$\begin{cases} \left|D^0(t)\right\rangle & = c_1(t) \left|D^0\right\rangle + c_2(t) \left|D^0\right\rangle , \\ \left|\bar{D}^0(t)\right\rangle & = d_1(t) \left|D^0\right\rangle + d_2(t) \left|D^0\right\rangle . \end{cases} \tag{3}$$

The coefficients will change in time $\rightarrow$ oscillations !!!

# Neutral meson system - $Wz$ parametrisation of CPT

$$H^{\text{eff}} = \left[\begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix}\right] \tag{4}$$

$$H^{\text{eff}} = \frac{1}{2}\Delta\lambda \begin{bmatrix} U + z & VW^{-1} \\ VW & U - z \end{bmatrix} \tag{5}$$

$U = \frac{\lambda}{\Delta\lambda}, V = \sqrt{1 - z^2}$

$\lambda_{H,L}$ eigenvalues of $H^{\text{eff}}$, $\lambda_{H,L} = m_{H,L} - i\Gamma_{H,L}/2$

$\Delta\lambda = \lambda_H - \lambda_L$

$W = w \exp(i\omega)$

CPTV parameter $z = \frac{H_{11}^{\text{eff}} - H_{22}^{\text{eff}}}{\Delta\lambda}$. CPV parameter $W$.

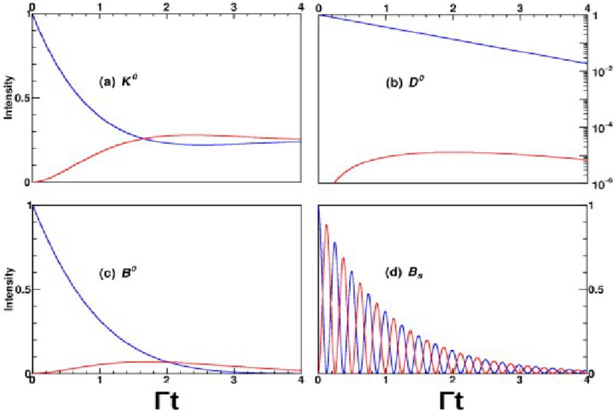
# Oscillations - different species

$\lambda_{H/L}$ has a real $m_{H/L}$ and imaginary part $\Gamma_{H/L}$.

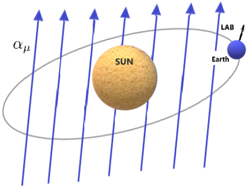
$x = \frac{m_H - m_L}{\Gamma}$, $y = \frac{\Gamma_H - \Gamma_L}{2\Gamma}$, where $\Gamma = \frac{\Gamma_H + \Gamma_L}{2}$.

$\mathrm{PDF}(D^0 \to D^0) \sim e^{-\Gamma t}\left(\cosh y\Gamma t + \cos x\Gamma t\right)$

$\mathrm{PDF}(D^0 \to \bar{D}^0) \sim e^{-\Gamma t}\left(\cosh y\Gamma t - \cos x\Gamma t\right)$



# Standard Model Extension (SME) [Phys.Rev.D64:076001,2001]




LV/CPTV treated as a small deviation to Standard Model.

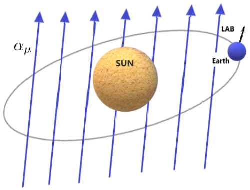
SME lagrangian perturbation $\delta L_{SME} = a^q_\mu \bar{q}\gamma^\mu q$

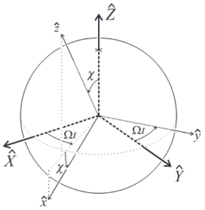
$a^q_\mu$, quark couplings to LV field to quarks $q$.

$h_{cpt} = H^{eff}_{11} - H^{eff}_{22} \propto$ 4 CPTV parameters $\Delta a_\mu \simeq a^{q_1}_\mu - a^{q_2}_\mu$.

Take home message: 4 $\Delta a_\mu$ CPTV parameters representing coupling of quarks to LV!

# Sidereal modulations of parameter z




Phys.Rev.D61:016002,2000

$$z = \frac{\beta^\mu \Delta a_\mu}{\Delta m - \frac{i}{2}\Delta\Gamma}; \tag{6}$$

**$z$ is now a function of sidereal time!**

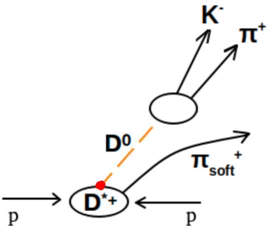
$$\beta^\mu \Delta a_\mu = \gamma \left[\Delta a_0 + \beta \Delta a_Z \cos\chi + \beta \sin\chi \left(\Delta a_Y \sin \Omega T + \Delta a_X \cos \Omega T\right)\right]$$

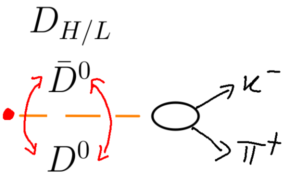
red – lab. reference frame

blue – fixed stars reference frame

green – geographical location dependence

# General experimental idea




1. We want to measure $z$ and $\Delta a_\mu$.
2. Initial state $D^0$ and $f$ final state. $D^0 \to K^-\pi^+$.
3. $PDF(\bar{D}^0 \to \bar{f}), PDF(D^0 \to f)$ expressible in $(z,\ \Delta a_\mu)$.
4. $N(\bar{D}^0 \to \bar{f}), N(D^0 \to f)$ can be used to construct $A_{CPT}$ observable.
5. $PDFs$ can be used to construct a $A_{CPT}$ model sensitive to $(z,\ \Delta a_\mu)$.
6. $(z,\ \Delta a_\mu)$ extracted from $A_{CPT}$ model to data.

$$A_{CPT}(t,T) = \frac{N(\bar{D}^0\to\bar{f})-N(D^0\to f)}{N(\bar{D}^0\to\bar{f}+N(D^0\to f))};\quad A_{CPT}(t,T) = \frac{PDF(\bar{D}^0\to\bar{f})-PDF(D^0\to f)}{PDF(\bar{D}^0\to\bar{f})+PDF(D^0\to f)} \quad (7)$$

# How to study CPT with $D^0$ mesons

1. Collect $D^0 \to K^-\pi^+$ and $\bar{D}^0 \to K^+\pi^-$ decays.
2. Record both (t - lifetimes, T - sidereal times)
3. Collected data used to construct an observable $A_{CPT}(t, T)$.
4. CPTV parametes can be extracted from a fit of $A_{CPT}(t, T)$ model to data.
5. Since CP does not depend on $T$ it is constructive to redefine the observable:

$$Diff(t, T_i, T_j) = A_{CPT}(t, T_i) - A_{CPT}(t, T_j). \qquad (8)$$

# FOCUS measurement [Phys.Lett.B556:7-13,2003]

1. E831/FOCUS @ FERMILAB 1996-1997.
3. $D^{*+} \to \pi^+_{\text{soft}} D^0 \, (K^- \pi^+) \, , D^{*-} \to \pi^-_{\text{soft}} \bar{D}^0 \, (K^+ \pi^-)$
2. $D^*$ produced: 180GeV $\gamma$ beam with BeO target.
4. 17000 $D^0/\bar{D}^0$ decays.
5. Most stringent bounds on $z$ and $\Delta a_\mu$.
6. Assuming $z = const$ (pure $Wz$ parametrisation)
$-0.68 < Re(z) - Im(z) < 2.34$.
7. Under SME assumption $\Delta a_\mu < 10^{-13} GeV$,
(sidereal modulation amplitude $|z(\Delta a_\mu)| < 300$).
8. LHCb Run2 (2015-2018) 90 million, LHCb Run3 1 billion (ongoing).
7. My goal $\to$ use LHCb Run 2 data and improve on FOCUS.

# Back to FOCUS measurement

$$H^{\text{eff}} = \frac{1}{2}\Delta\lambda \begin{bmatrix} U+z & VW^{-1} \\ VW & U-z \end{bmatrix} \tag{9}$$

$W = \frac{q}{p}$; $V = \sqrt{1-z^2}$

Fitted model:

$$A_{CPT}(t) = \frac{PDF(\bar{D}^0 \to \bar{f})(t) - PDF(D^0 \to f)(t)}{PDF(\bar{D}^0 \to \bar{f})(t) + PDF(D^0 \to f)(t)}. \tag{10}$$

$$\begin{aligned} \left|D^0(t)\right\rangle &= (g_+(t) + zg_-(t))\left|D^0\right\rangle - \sqrt{1-z^2}\frac{q}{p}g_-(t)\left|\bar{D}^0\right\rangle, \\ \left|\bar{D}^0(t)\right\rangle &= (g_+(t) - zg_-(t))\left|\bar{D}^0\right\rangle - \sqrt{1-z^2}\frac{p}{q}g_-(t)\left|D^0\right\rangle, \end{aligned} \tag{11}$$

$PDF(D^0 \to f)(t) = | < f|D^0(t) > |^2$;
$PDF(\bar{D}^0 \to \bar{f})(t) = | < \bar{f}|\bar{D}^0(t) > |^2$; $f = K^-\pi^+$.

# FOCUS measurement

- $A_{CPT}$ Taylor to the 3rd degree in $x$, $y$, and $|z|^2 << 1$:

$$A_{CPT}(t) = (y\mathrm{Re}(z) - x\mathrm{Im}(z))\, t - \sqrt{R}\sin\phi\,(x\cos\delta - y\sin\delta)\, t - \mathrm{Re}(z)\cos\phi \left( \frac{\sqrt{R}(x^2+y^2)(x\cos\delta - y\sin\delta)}{2x} \right) t^2 + \frac{\mathrm{Re}(z)}{6} t^3 x^2 y + \frac{\mathrm{Re}(z)}{6} t^3 y^3. \quad (12)$$

- $\sqrt{1-z^2} \approx 1$ instead of $\sqrt{1+|z|^4 - 2|z|^2\cos 2\theta}$.

$$\left|D^0(t)\right\rangle = (g_+(t) + zg_-(t))\left|D^0\right\rangle - \sqrt{1-z^2}\frac{q}{p}g_-(t)\left|\bar{D^0}\right\rangle,$$
$$\left|\bar{D^0}(t)\right\rangle = (g_+(t) - zg_-(t))\left|\bar{D^0}\right\rangle - \sqrt{1-z^2}\frac{p}{q}g_-(t)\left|D^0\right\rangle. \quad (13)$$

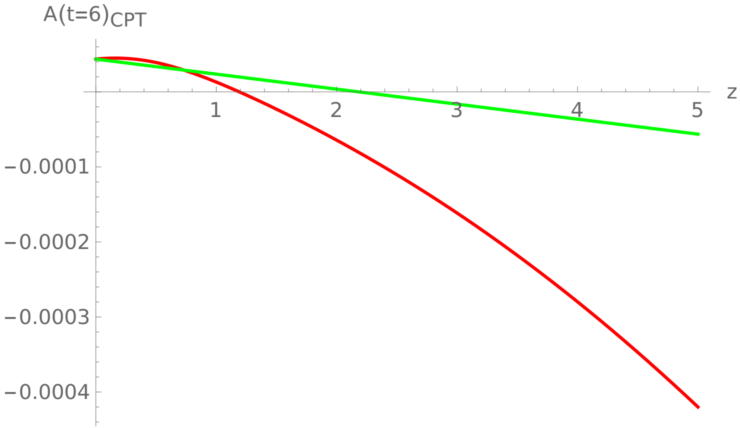
# FOCUS measurement

For the modified formula and small $x, y, t$, $A_{CPT}$ takes the form:

$$
\begin{aligned}
A_{CPT}(t) =& t\left(y|z|\cos(\theta) - x|z|\sin(\theta)\right) \\
&+ \sqrt{R}t[\frac{1}{2}y\left(1+|z|^4-2|z|^2\cos(2\theta)\right)^{1/4}\cos\left(\delta-\phi+0.5\arctan 2\left(\frac{|z|^2\sin(2\theta)}{1-|z|^2\cos(2\theta)}\right)\right. \\
&- \frac{1}{2}y\left(1+|z|^4-2|z|^2\cos(2\theta)\right)^{1/4}\cos\left(\delta+\phi+0.5\arctan 2\left(\frac{|z|^2\sin(2\theta)}{1-|z|^2\cos(2\theta)}\right)\right. \\
&+ \frac{1}{2}x\left(1+|z|^4-2z^2\cos(2\theta)\right)^{1/4}\sin\left(\delta-\phi+0.5\arctan 2\left(\frac{|z|^2\sin(2\theta)}{1-|z|^2\cos(2\theta)}\right)\right. \\
&- \frac{1}{2}x\left(1+|z|^4-2|z|^2\cos(2\theta)\right)^{1/4}\sin\left(\delta+\phi+0.5\arctan 2\left(\frac{|z|^2\sin(2\theta)}{1-|z|^2\cos(2\theta)}\right)\right.] + \ldots
\end{aligned}
$$

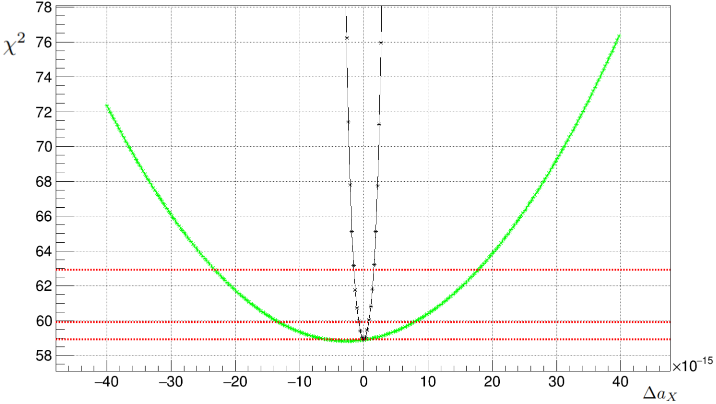
# FOCUS measurement

modified $A_{CPT}$ formula *red* and FOCUS formula *green*.



# How does this change influence sensitivity?

For the corrected formula $\chi^2$ approaches $\Delta a_X = 0$ more steeply (MC LHCb Run2 statistics):



# Common misconception about $z$, $\Delta a_\mu$ in charm

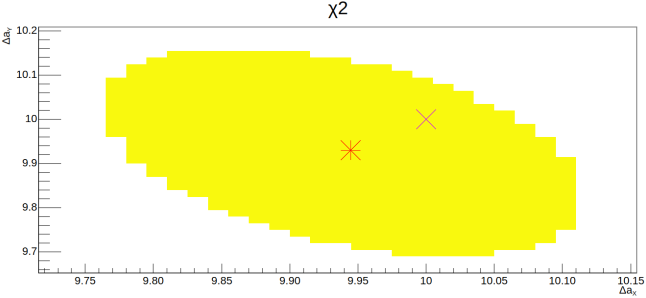
$$
\begin{aligned}
|D_L\rangle &= p\sqrt{1-z}\,\left|D^0\right\rangle + q\sqrt{1+z}\,\left|\bar{D}^0\right\rangle \\
|D_H\rangle &= p\sqrt{1+z}\,\left|D^0\right\rangle - q\sqrt{1-z}\,\left|\bar{D}^0\right\rangle
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
\left|D^0(t)\right\rangle &= \left(g_+(t) + z g_-(t)\right)\left|D^0\right\rangle - \sqrt{1-z^2}\,\frac{q}{p} g_-(t)\left|\bar{D}^0\right\rangle , \\
\left|\bar{D}^0(t)\right\rangle &= \left(g_+(t) - z g_-(t)\right)\left|\bar{D}^0\right\rangle - \sqrt{1-z^2}\,\frac{p}{q} g_-(t)\left|D^0\right\rangle .
\end{aligned} \tag{15}
$$

1. No a'priori constrained on $|z| < 1$.
2. $\Delta a_\mu$ derived in the context of general renormalisable EFT.
3. $\Delta a_\mu$ perturbative for $\Delta a_\mu < 10^{-2} GeV$.
4. Amplitude $|z(\Delta a_\mu)| < 10^{13}$.

# Exemplary fit recipe to extract ($\Delta a_X$, $\Delta a_Y$)

1. Place $D^0/\bar{D}^0$ events into (t,T) bins (6x6).
2. Construct observable $A_{CPT}(t, T_i) - A_{CPT}(t, T_j)$
3. Make a 2D fit of $A_{CPT}(t, T_i) - A_{CPT}(t, T_j)$ to data.
4. $\Delta a_X$ and $\Delta a_Y$ will be fit parameters.



# Check the scale of $\Delta a_X, \Delta a_Y$ we are sensitive to.

1. Prepare MC decays for different pairs of $(\Delta a_X, \Delta a_Y)$.
2. We calculate $\chi^2$ for each dataset in two modes, where $\Delta a_X, \Delta a_Y$: float freely, set to zero.
3. For each MC calculate: $\chi^2(0,0) - \chi^2(\Delta a_X, \Delta a_Y)$.
4. The greater the overlap, the lower the discriminative power!

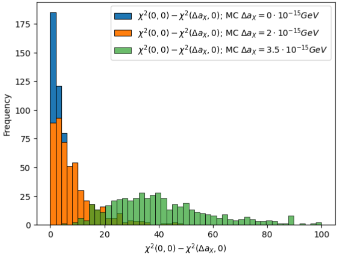



Figure: Profile likelihood ratios for different hypothesis of $\Delta a_X$, $\Delta a_Y = 0$

# Summary

1. We test the SM by searching for CPTV.
2. We do it by looking at $D^0$ oscillations.
3. We interpret CPTV as small deviations to the SM.
4. We can study these deviations in pure $Wz$ ($z = const$) or in the SME ($z$ modulated).
5. Improvement of FOCUS constraints on CPTV parameters.

$\Delta a_\mu < 10^{-13} GeV \rightarrow \Delta a_\mu < 10^{-14} GeV$.

7. New bounds on $z$ and $\Delta a_\mu$ using LHCb Run2 data.
8. Establish sensitivity on $z$ and $\Delta a_\mu$ in LHCb Run3.

# BACKUP

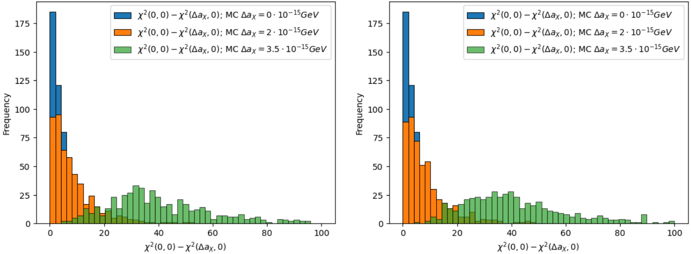



Figure: Profile likelihood ratios for different hypothesis of $\Delta a_X$, $\Delta a_Y = 0$. non-CPTV parameters modelled with gaussian prior.

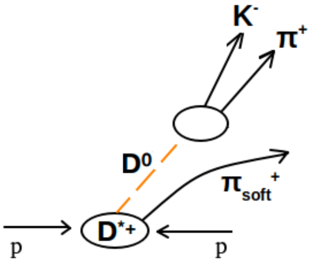
# LHCb data

Data from LHCb.

- 6 fb-1 @13 TeV p-p, Run2
- $10^8$ events: $D^0 \to K^- \pi^+$ Run2
- $10^9$ events: $D^0 \to K^- \pi^+$ Run3

What do we do?

- We create an observable based on events.
- Observable described by a model including $p, q, z$.
- $z \sim \beta \Delta a_\mu$ bounds from fit



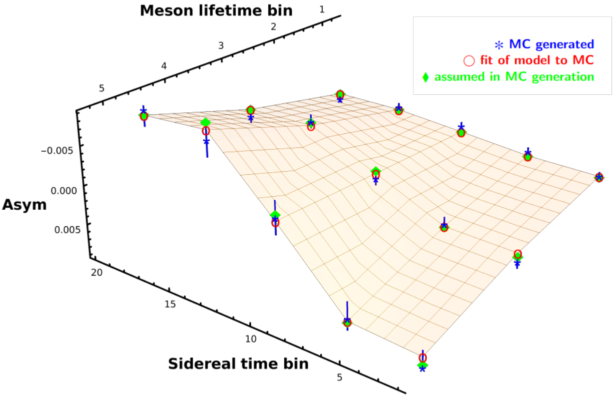
# Removal of CPV from data

1. $A_{CPT}(t, T) = f_{CP}(t) + g_{CP}(t) \circ h(t, T)$
2. $f_{CP}(t)$ contribution can be removed by taking $A_{CPT}(t, T)$ differences:

$$\mathrm{Diff}(t, T_i, T_j) = A_{CPT}(t, T_i) - A_{CPT}(t, T_j). \tag{16}$$

# SME parameter extraction

| | $\Delta a_X$ [GeV] | $\Delta a_Y$ [GeV] |
|---|---|---|
| fit | $(4.7 \pm 0.3) \cdot 10^{-13}$ | $(3.5 \pm 2.9) \cdot 10^{-14}$ |
| MC | $5 \cdot 10^{-13}$ | 0 |



# Current Limits on CPTV

| |
|---|
| **KLOE** $K^0$**:** $\Delta a_0$**,** $\Delta a_{X,Y,Z} \leq 10^{-18}$ **GeV** |
| **FOCUS** $D^0$**:** $\Delta a_0$**,** $\Delta a_{X,Y,Z} \leq 10^{-13}$ **GeV** |
| **LHCb** $B^0$**:** $\Delta a_0$**,** $\Delta a_{X,Y,Z} \leq 10^{-15}$ **GeV** |
| **LHCb** $B_s^0$**:** $\Delta a_{0,Z} \leq 10^{-12}$**,** $\Delta a_{X,Y} \leq 10^{-14}$ **GeV** |

Measurements in different sectors are complementary [Data tables: arXiv:0801.0287].

LHCb – good place to study CPTV in charm, $10^4$ greater statistics compared to FOCUS and excellent decay time resolution 45fs.

Differences in accuracy stem from different masses!

# CPT parametrisation in oscillations

## Phenomenological complex parametrisation (p,q,z)

$$
\begin{aligned}
|D_L\rangle &= p\sqrt{1-z}\,\left|D^0\right\rangle + q\sqrt{1+z}\,\left|\bar{D}^0\right\rangle \\
|D_H\rangle &= p\sqrt{1+z}\,\left|D^0\right\rangle - q\sqrt{1-z}\,\left|\bar{D}^0\right\rangle
\end{aligned} \tag{17}
$$

T conserved $\rightarrow \left|\frac{p}{q}\right| = 1$

CPT conserved $\rightarrow z = 0$

CP conserved $\rightarrow z = 0, \left|\frac{p}{q}\right| = 1$

For $z = 0$, $|p|^2 + |q|^2 = 1$

CPT assumed to be strictly conserved in SM

The relation for $z$ is given by: $z = \frac{H^{eff}_{11} - H^{eff}_{22}}{\Gamma(x - i2y)}$,
$x, y \sim M, \Gamma$ eigenvalues.

Take home message: CPT conserved $\Rightarrow z = 0$ !!!