

Studies of CPT with D^0 mesons

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Motivation

1. We want to test CPT in the charm sector.
2. Good place to study CPT - neutral D meson oscillations.
3. CPT invariance studied by probing space-time symmetry violations.
4. The framework to study these deviations - Standard Model Extension (SME).

Key symmetries in particle physics

$$D^0 \xrightarrow{\hat{C}} \bar{D}^0$$

$(c\bar{u}) \xrightarrow{\hat{C}} (u\bar{c})$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \xrightarrow{\hat{P}} \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}$$



C: $\hat{C} |c, \vec{x}, t\rangle = |\bar{c}, \vec{x}, t\rangle$, charge conjugation transformation

P: $\hat{P} |c, \vec{x}, t\rangle = |c, -\vec{x}, t\rangle$, parity transformation

T: $\hat{T} |c, \vec{x}, t\rangle = \langle c, \vec{x}, -t|$, time reversal transformation

CPT: $\hat{C}\hat{P}\hat{T}$

CPT assumed to be strictly conserved in SM !!!

$[H, X] = 0$ then X is conserved.

CPT theorem and Lorentz Violation (LV)

CPT theorem: Local QFTs with Lorentz symmetry \Rightarrow CPT invariant.

Greenberg: In interacting theories CPTV necessitates LV.

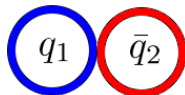
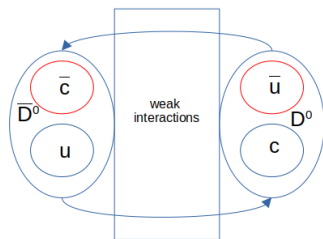
Take home message: Look for CPTV by searching for LV!

Flavoured Neutral Mesons

meson: quark (q_1) + antiquark (\bar{q}_2)

neutral meson: electric charge = 0

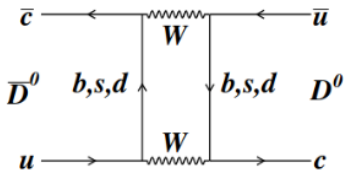
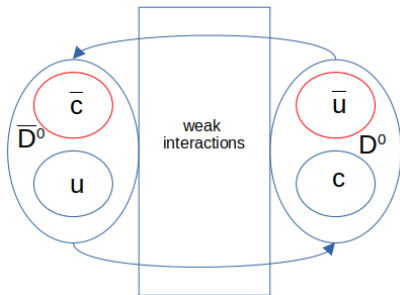
flavoured neutral meson: non-zero S, C, B content



Neutral meson $Q = 0$	Flavor F
\mathbf{K}^0 $d\bar{s}$	Strangeness $S = 1$
\mathbf{D}^0 $c\bar{u}$	Charm $C = 1$
\mathbf{B}^0 $d\bar{b}$ $s\bar{b}$	Beauty $B = 1$ $B = 1, S = -1$

credits: Agnes Roberts, Presentation IUCSS CPT 21.

D^0 oscillations



credits: Fernando Martinez Vidal arXiv:0910.5061

Neutral Meson Oscillations - math. description

1. $H = H_0 + H_{wk}$
2. H_{wk} perturbation to H_0 ,
3. $|D^0\rangle, |\bar{D}^0\rangle$ eigenstates of H^0 .
4. $|D_L\rangle, |D_H\rangle$ eigenstates of H .

Weiskopf Wigner Approximation yields 2x2 H^{eff} :

$$H^{eff} = \hat{\mathbf{M}} + i\hat{\mathbf{\Gamma}} = \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{bmatrix} + \frac{i}{2} \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{bmatrix} \quad (1)$$

H^{eff} is a sum of two hermitian matrices: $\hat{\mathbf{M}}$ and $\hat{\mathbf{\Gamma}}$, mass and decay matrix respectively.

H^{eff} eigenstates - mass states

$|D^0\rangle, |\bar{D}^0\rangle \rightarrow$ not eigenvectors of H^{eff} .

$|D_L\rangle, |D_H\rangle \rightarrow$ eigenvectors of H^{eff} .

System in one of two mass states $|D_L\rangle$ or $|D_H\rangle$.

Mass states - linear combinations of flavour states.

$$\begin{cases} |D_L\rangle & = a_1 |D^0\rangle + a_2 |\bar{D}^0\rangle, \\ |D_H\rangle & = b_1 |D^0\rangle + b_2 |\bar{D}^0\rangle, \\ |D_{L/H}(t)\rangle & = e^{-itH^{\text{eff}}} |D_{L/H}(0)\rangle. \end{cases} \quad (2)$$

Find $|D^0(t)\rangle, |\bar{D}^0(t)\rangle$ by solving the above:

$$\begin{cases} |D^0(t)\rangle & = c_1(t) |D^0\rangle + c_2(t) |\bar{D}^0\rangle, \\ |\bar{D}^0(t)\rangle & = d_1(t) |D^0\rangle + d_2(t) |\bar{D}^0\rangle. \end{cases} \quad (3)$$

The coefficients will change in time \rightarrow oscillations !!!

Neutral meson system - Wz parametrisation of CPT

$$H^{\text{eff}} = \left[\begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix} \right] \quad (4)$$

$$H^{\text{eff}} = \frac{1}{2} \Delta\lambda \begin{bmatrix} U + z & VW^{-1} \\ VW & U - z \end{bmatrix} \quad (5)$$

$$U = \frac{\lambda}{\Delta\lambda}, \quad V = \sqrt{1 - z^2}$$

$\lambda_{H,L}$ eigenvalues of H^{eff} , $\lambda_{H,L} = m_{H,L} - i\Gamma_{H,L}/2$

$$\Delta\lambda = \lambda_H - \lambda_L$$

$$W = w \exp(i\omega)$$

CPTV parameter $z = \frac{H_{11}^{\text{eff}} - H_{22}^{\text{eff}}}{\Delta\lambda}$. CPV parameter W .

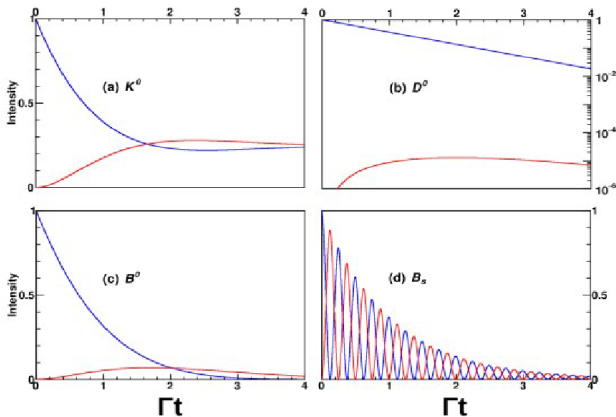
Oscillations - different species

$\lambda_{H/L}$ has a real $m_{H/L}$ and imaginary part $\Gamma_{H/L}$.

$x = \frac{m_H - m_L}{\Gamma}$, $y = \frac{\Gamma_H - \Gamma_L}{2\Gamma}$, where $\Gamma = \frac{\Gamma_H + \Gamma_L}{2}$.

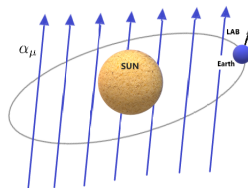
PDF($D^0 \rightarrow D^0$) $e^{-\Gamma t} (\cosh y\Gamma t + \cos x\Gamma t)$

PDF($D^0 \rightarrow \bar{D}^0$) $e^{-\Gamma t} (\cosh y\Gamma t - \cos x\Gamma t)$



Standard Model Extension (SME)

[Phys.Rev.D64:076001,2001]



LV/CPTV treated as a small deviation to Standard Model.

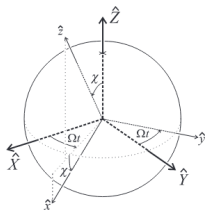
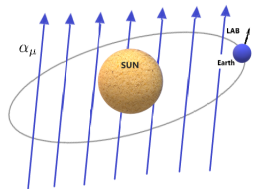
SME lagrangian perturbation $\delta L_{SME} = a_\mu^q \bar{q} \gamma^\mu q$

a_μ^q , quark couplings to LV field to quarks q .

$h_{cpt} = H_{11}^{eff} - H_{22}^{eff} \propto 4$ CPTV parameters $\Delta a_\mu \simeq a_\mu^{q1} - a_\mu^{q2}$.

Take home message: 4 Δa_μ CPTV parameters representing coupling of quarks to LV!

Sidereal modulations of parameter z



Phys.Rev.D61:016002,2000

$$z = \frac{\beta^\mu \Delta a_\mu}{\Delta m - \frac{i}{2} \Delta \Gamma}; \quad (6)$$

z is now a function of sidereal time!

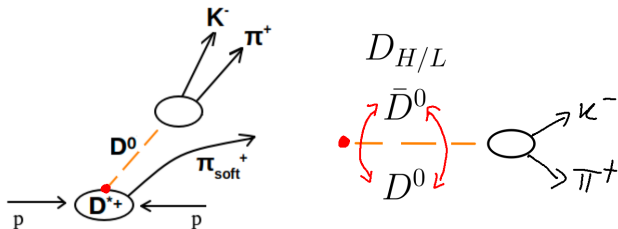
$$\beta^\mu \Delta a_\mu = \gamma [\Delta a_0 + \beta \Delta a_z \cos \chi + \beta \sin \chi (\Delta a_y \sin \Omega T + \Delta a_x \cos \Omega T)]$$

red – lab. reference frame

blue – fixed stars reference frame

green – geographical location dependence

General experimental idea



1. We want to measure z and Δa_μ .
2. Initial state D^0 and f final state. $D^0 \rightarrow K^- \pi^+$.
3. $PDF(\bar{D}^0 \rightarrow \bar{f})$, $PDF(D^0 \rightarrow f)$ expressible in $(z, \Delta a_\mu)$.
4. $N(\bar{D}^0 \rightarrow \bar{f})$, $N(D^0 \rightarrow f)$ can be used to construct A_{CPT} observable.
5. $PDFs$ can be used to construct a A_{CPT} model sensitive to $(z, \Delta a_\mu)$.
6. $(z, \Delta a_\mu)$ extracted from A_{CPT} model to data.

$$A_{CPT}(t, T) = \frac{N(\bar{D}^0 \rightarrow \bar{f}) - N(D^0 \rightarrow f)}{N(\bar{D}^0 \rightarrow \bar{f}) + N(D^0 \rightarrow f)}; \quad A_{CPT}(t, T) = \frac{PDF(\bar{D}^0 \rightarrow \bar{f}) - PDF(D^0 \rightarrow f)}{PDF(\bar{D}^0 \rightarrow \bar{f}) + PDF(D^0 \rightarrow f)}(7)$$

How to study CPT with D^0 mesons

1. Collect $D^0 \rightarrow K^- \pi^+$ and $\bar{D}^0 \rightarrow K^+ \pi^-$ decays.
2. Record both (t - lifetimes, T - sidereal times)
3. Collected data used to construct an observable $A_{CPT}(t, T)$.
4. CPTV parameters can be extracted from a fit of $A_{CPT}(t, T)$ model to data.
5. Since CP does not depend on T it is constructive to redefine the observable:

$$Diff(t, T_i, T_j) = A_{CPT}(t, T_i) - A_{CPT}(t, T_j). \quad (8)$$

FOCUS measurement [Phys.Lett.B556:7-13,2003]

1. E831/FOCUS @ FERMILAB 1996-1997.
3. $D^{*+} \rightarrow \pi_{\text{soft}}^+ D^0 (K^- \pi^+)$, $D^{*-} \rightarrow \pi_{\text{soft}}^- \bar{D}^0 (K^+ \pi^-)$
2. D^* produced: 180GeV γ beam with BeO target.
4. 17000 D^0/\bar{D}^0 decays.
5. Most stringent bounds on z and Δa_μ .
6. Assuming $z = \text{const}$ (pure Wz parametrisation)
 $-0.68 < \text{Re}(z) - \text{Im}(z) < 2.34$.
7. Under SME assumption $\Delta a_\mu < 10^{-13} \text{GeV}$,
(sidereal modulation amplitude $|z(\Delta a_\mu)| < 300$).
8. LHCb Run2 (2015-2018) 90 million, LHCb Run3 1 billion (ongoing).
7. My goal \rightarrow use LHCb Run 2 data and improve on FOCUS.

Back to FOCUS measurement

$$H^{\text{eff}} = \frac{1}{2} \Delta \lambda \begin{bmatrix} U+z & VW^{-1} \\ VW & U-z \end{bmatrix} \quad (9)$$

$$W = \frac{q}{p}; V = \sqrt{1-z^2}$$

Fitted model:

$$A_{CPT}(t) = \frac{PDF(\bar{D}^0 \rightarrow \bar{f})(t) - PDF(D^0 \rightarrow f)(t)}{PDF(\bar{D}^0 \rightarrow \bar{f})(t) + PDF(D^0 \rightarrow f)(t)}. \quad (10)$$

$$\begin{aligned} |D^0(t)\rangle &= (g_+(t) + zg_-(t)) |D^0\rangle - \sqrt{1-z^2} \frac{q}{p} g_-(t) |\bar{D}^0\rangle, \\ |\bar{D}^0(t)\rangle &= (g_+(t) - zg_-(t)) |\bar{D}^0\rangle - \sqrt{1-z^2} \frac{p}{q} g_-(t) |D^0\rangle, \end{aligned} \quad (11)$$

$$\begin{aligned} PDF(D^0 \rightarrow f)(t) &= |\langle f | D^0(t) \rangle|^2; \\ PDF(\bar{D}^0 \rightarrow \bar{f})(t) &= |\langle \bar{f} | \bar{D}^0(t) \rangle|^2; f = K^{-\pi^+}. \end{aligned}$$

FOCUS measurement

- A_{CPT} Taylor to the 3rd degree in x , y , and $|z|^2 \ll 1$:

$$\begin{aligned} A_{CPT}(t) = & (y\operatorname{Re}(z) - x\operatorname{Im}(z)) t - \sqrt{R} \sin \phi (x \cos \delta - y \sin \delta) t \\ & - \operatorname{Re}(z) \cos \phi \left(\frac{\sqrt{R}(x^2 + y^2)(x \cos \delta - y \sin \delta)}{2x} \right) t^2 \\ & + \frac{\operatorname{Re}(z)}{6} t^3 x^2 y + \frac{\operatorname{Re}(z)}{6} t^3 y^3. \end{aligned} \tag{12}$$

- $\sqrt{1 - z^2} \approx 1$ instead of $\sqrt{1 + |z|^4 - 2|z|^2 \cos 2\theta}$.

$$\begin{aligned} |D^0(t)\rangle &= (g_+(t) + zg_-(t)) |D^0\rangle - \sqrt{1 - z^2} \frac{q}{p} g_-(t) |\bar{D}^0\rangle, \\ |\bar{D}^0(t)\rangle &= (g_+(t) - zg_-(t)) |\bar{D}^0\rangle - \sqrt{1 - z^2} \frac{p}{q} g_-(t) |D^0\rangle. \end{aligned} \tag{13}$$

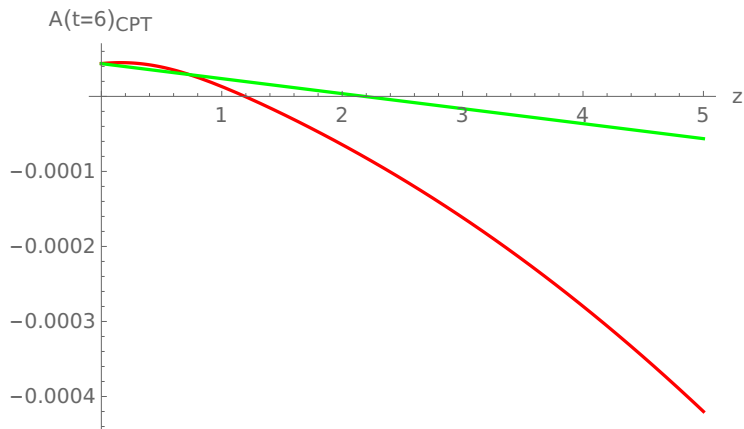
FOCUS measurement

For the modified formula and small x, y, t , A_{CPT} takes the form:

$$\begin{aligned} A_{CPT}(t) = & t(y|z|\cos(\theta) - x|z|\sin(\theta)) \\ & + \sqrt{Rt} \left[\frac{1}{2} y (1 + |z|^4 - 2|z|^2 \cos(2\theta))^{1/4} \cos\left(\delta - \phi + 0.5 \arctan 2 \left(\frac{|z|^2 \sin(2\theta)}{1 - |z|^2 \cos(2\theta)} \right)\right) \right. \\ & - \frac{1}{2} y (1 + |z|^4 - 2|z|^2 \cos(2\theta))^{1/4} \cos\left(\delta + \phi + 0.5 \arctan 2 \left(\frac{|z|^2 \sin(2\theta)}{1 - |z|^2 \cos(2\theta)} \right)\right) \\ & + \frac{1}{2} x (1 + |z|^4 - 2z^2 \cos(2\theta))^{1/4} \sin\left(\delta - \phi + 0.5 \arctan 2 \left(\frac{|z|^2 \sin(2\theta)}{1 - |z|^2 \cos(2\theta)} \right)\right) \\ & \left. - \frac{1}{2} x (1 + |z|^4 - 2|z|^2 \cos(2\theta))^{1/4} \sin\left(\delta + \phi + 0.5 \arctan 2 \left(\frac{|z|^2 \sin(2\theta)}{1 - |z|^2 \cos(2\theta)} \right)\right) \right] + \dots \end{aligned}$$

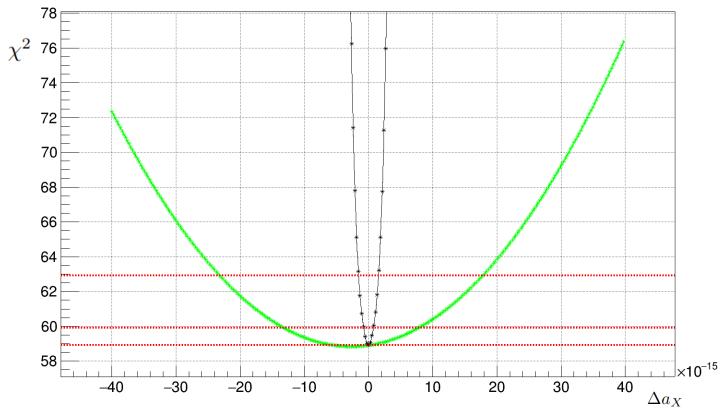
FOCUS measurement

modified A_{CPT} formula *red* and FOCUS formula *green*.



How does this change influence sensitivity?

For the corrected formula χ^2 approaches $\Delta a_X = 0$ more steeply (MC LHCb Run2 statistics):



Common misconception about z , Δa_μ in charm

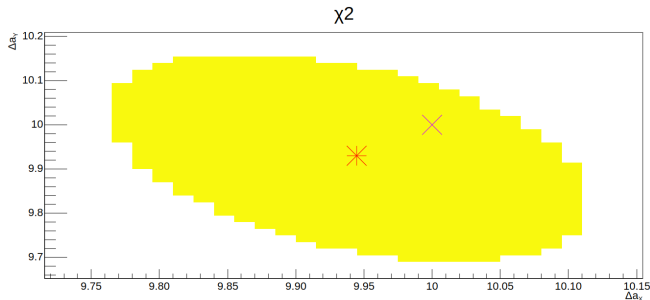
$$\begin{aligned} |D_L\rangle &= p\sqrt{1-z} |D^0\rangle + q\sqrt{1+z} |\bar{D}^0\rangle \\ |D_H\rangle &= p\sqrt{1+z} |D^0\rangle - q\sqrt{1-z} |\bar{D}^0\rangle \end{aligned} \quad (14)$$

$$\begin{aligned} |D^0(t)\rangle &= (g_+(t) + zg_-(t)) |D^0\rangle - \sqrt{1-z^2} \frac{q}{p} g_-(t) |\bar{D}^0\rangle, \\ |\bar{D}^0(t)\rangle &= (g_+(t) - zg_-(t)) |\bar{D}^0\rangle - \sqrt{1-z^2} \frac{p}{q} g_-(t) |D^0\rangle. \end{aligned} \quad (15)$$

1. No a priori constrained on $|z| < 1$.
2. Δa_μ derived in the context of general renormalisable EFT.
3. Δa_μ perturbative for $\Delta a_\mu < 10^{-2} \text{ GeV}$.
4. Amplitude $|z(\Delta a_\mu)| < 10^{13}$.

Exemplary fit recipe to extract $(\Delta a_X, \Delta a_Y)$

1. Place D^0/\bar{D}^0 events into (t, T) bins (6×6).
2. Construct observable $A_{CPT}(t, T_i) - A_{CPT}(t, T_j)$
3. Make a 2D fit of $A_{CPT}(t, T_i) - A_{CPT}(t, T_j)$ to data.
4. Δa_X and Δa_Y will be fit parameters.



Check the scale of $\Delta a_X, \Delta a_Y$ we are sensitive to.

1. Prepare MC decays for different pairs of $(\Delta a_X, \Delta a_Y)$.
2. We calculate χ^2 for each dataset in two modes, where $\Delta a_X, \Delta a_Y$: float freely, set to zero.
3. For each MC calculate: $\chi^2(0, 0) - \chi^2(\Delta a_X, \Delta a_Y)$.
4. The greater the overlap, the lower the discriminative power!

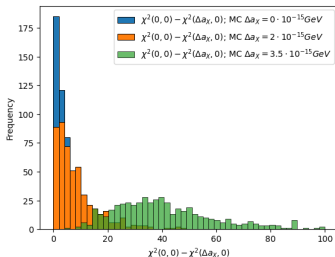


Figure: Profile likelihood ratios for different hypothesis of $\Delta a_X, \Delta a_Y = 0$

Summary

1. We test the SM by searching for CPTV.
2. We do it by looking at D^0 oscillations.
3. We interpret CPTV as small deviations to the SM.
4. We can study these deviations in pure Wz ($z = \text{const}$) or in the SME (z modulated).
5. Improvement of FOCUS constraints on CPTV parameters.
 $\Delta a_\mu < 10^{-13} \text{ GeV} \rightarrow \Delta a_\mu < 10^{-14} \text{ GeV}$.
7. New bounds on z and Δa_μ using LHCb Run2 data.
8. Establish sensitivity on z and Δa_μ in LHCb Run3.

BACKUP

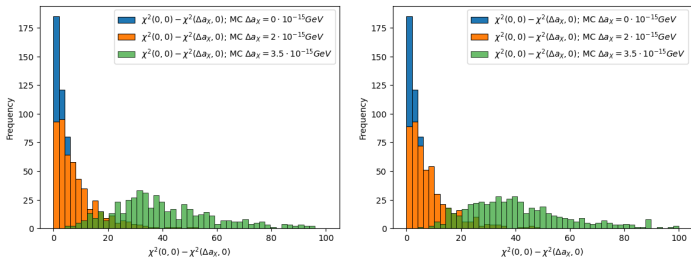


Figure: Profile likelihood ratios for different hypothesis of Δa_X , $\Delta a_Y = 0$. non-CPTV parameters modelled with gaussian prior.

LHCb data

Data from LHCb.

6 fb⁻¹ @13 TeV p-p, Run2

10⁸ events: $D^0 \rightarrow K^- \pi^+$ Run2

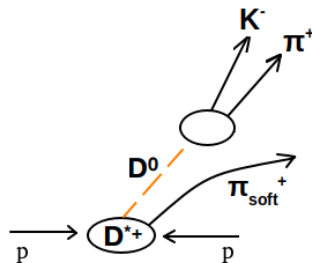
10⁹ events: $D^0 \rightarrow K^- \pi^+$ Run3

What do we do?

We create an observable based on events.

Observable described by a model including p, q, z .

$z \sim \beta \Delta a_\mu$ bounds from fit



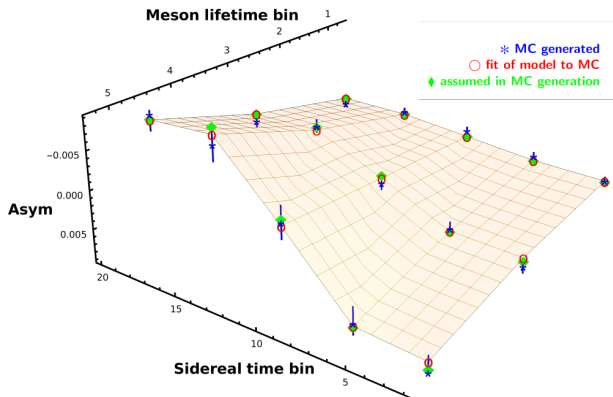
Removal of CPV from data

1. $A_{CPT}(t, T) = f_{CP}(t) + g_{CP}(t) \circ h(t, T)$
2. $f_{CP}(t)$ contribution can be removed by taking $A_{CPT}(t, T)$ differences:

$$\text{Diff}(t, T_i, T_j) = A_{CPT}(t, T_i) - A_{CPT}(t, T_j). \quad (16)$$

SME parameter extraction

	Δa_X [GeV]	Δa_Y [GeV]
fit	$(4.7 \pm 0.3) \cdot 10^{-13}$	$(3.5 \pm 2.9) \cdot 10^{-14}$
MC	$5 \cdot 10^{-13}$	0



Current Limits on CPTV

KLOE K^0 : $\Delta a_0, \Delta a_{X,Y,Z} \leq 10^{-18}$ GeV
FOCUS D^0 : $\Delta a_0, \Delta a_{X,Y,Z} \leq 10^{-13}$ GeV
LHCb B^0 : $\Delta a_0, \Delta a_{X,Y,Z} \leq 10^{-15}$ GeV
LHCb B_s^0 : $\Delta a_{0,Z} \leq 10^{-12}$, $\Delta a_{X,Y} \leq 10^{-14}$ GeV

Measurements in different sectors are complementary [Data tables: arXiv:0801.0287].

LHCb – good place to study CPTV in charm, 10^4 greater statistics compared to FOCUS and excellent decay time resolution 45fs.

Differences in accuracy stem from different masses!

CPT parametrisation in oscillations

Phenomenological complex parametrisation (p,q,z)

$$\begin{aligned} |D_L\rangle &= p\sqrt{1-z} |D^0\rangle + q\sqrt{1+z} |\bar{D}^0\rangle \\ |D_H\rangle &= p\sqrt{1+z} |D^0\rangle - q\sqrt{1-z} |\bar{D}^0\rangle \end{aligned} \quad (17)$$

T conserved ! $\left| \frac{p}{q} \right| = 1$

CPT conserved ! $z = 0$

CP conserved ! $z = 0, \left| \frac{p}{q} \right| = 1$

For $z = 0$, $jp^2 + jq^2 = 1$

CPT assumed to be strictly conserved in SM

The relation for z is given by: $z = \frac{H_{11}^{eff} - H_{22}^{eff}}{\Gamma(x - i2y)}$,
 $x, y \sim M, \Gamma$ eigenvalues.

Take home message: CPT conserved $\Rightarrow z = 0$!!!