Deeply virtual scattering in QCD

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Outline

- Standard model: EW + QCD
- QCD $\rightarrow$ deep inelastic scattering **(DIS)**
- Deeply virtual Compton scattering **(DVCS)** and collinear factorization:
  - The $x = \xi$ restriction
- Timelike Compton scattering **(TCS)**
- Double deeply virtual Compton scattering **(DDVCS):**
  - $x \neq \xi$ solution
  - Kleiss & Stirling (KS) techniques
  - **Latest results on theory and phenomenology**
- Experiments around the globe
- Summary

Standard model

Modern particle physics = Standard model:

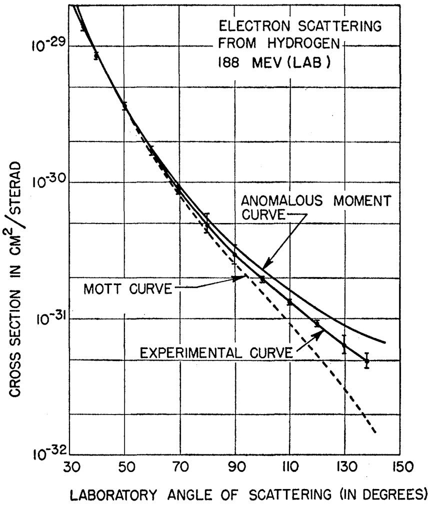
$$\underbrace{SU(3)_c}_{\substack{\text{QCD}\\\text{sector}}} \otimes \underbrace{SU(2)_L \otimes U(1)_Y}_{\substack{\text{Electroweak}\\\text{sector}}}$$

- $SU(3)_c$: strong interactions = quarks and gluons (*partons*) coupled to form hadrons
- Quarks and gluons: elementary, free *asymptotically* only
- Hadrons: composite objects, the states you find in nature

How to access partons?

- "Hitting" hadrons with high-energy particles: (virtual) photon
- Mid 1950s: SLAC proves the proton to be an extended object (**elastic scattering**)
- Late 1960s: SLAC & MIT prove the composite nature of the proton via **inelastic** scattering:
  1. Inclusive: **no control over all** final state particles
  2. Exclusive: **full control** over all final state particles

Elastic scattering of electron and proton at SLAC



Plot from: R. Hofstader & R. W. McAllister, Phys. Rev. **98**, 217 (1955)

- **Mott xsec: spinless and pointlike target**

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Mott}} = \underbrace{\frac{Z^2\alpha_{\text{em}}^2}{4E^2\sin^2(\theta/2)}}_{\text{Rutherford}}\cos^2(\theta/2)$$

- **Rosenbluth xsec: spin-1/2 and pointlike target**

$$\left.\frac{d^2\sigma}{d\Omega dE'}\right|_{\substack{\text{Ros,}\\ \text{point}}} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{Mott}}\left[1+2\frac{Q^2}{4M^2}\tan^2(\theta/2)\right]$$

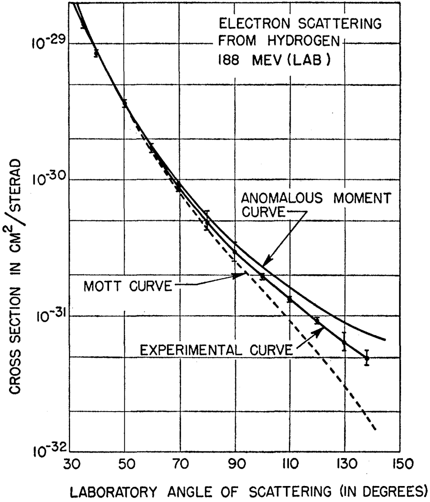
$$\times\delta\left(\frac{Q^2}{2M}-\nu\right),\ \nu=E-E'$$

- **Rosenbluth xsec: spin-1/2 and extended target**

$$\left.\frac{d^2\sigma}{d\Omega dE'}\right|_{\substack{\text{Ros,}\\ \text{extended}}} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{Mott}}\left[F_E^2(|\vec{q}|^2)\right.$$

$$\left.+2\frac{Q^2}{4M^2}\tan^2(\theta/2)F_M^2(|\vec{q}|^2)\right]\delta\left(\frac{Q^2}{2M}-\nu\right)$$

Elastic scattering of electron and proton at SLAC



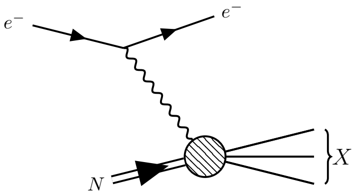
Plot from: R. Hofstader & R. W. McAllister, Phys. Rev. **98**, 217 (1955)

- **Rosenbluth xsec: spin-1/2 and extended target**

$$\left.\frac{d^2\sigma}{d\Omega dE'}\right|_{\substack{\text{Ros,}\\ \text{extended}}} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{Mott}} \left[F_E^2(|\vec{q}|^2) + 2\frac{Q^2}{4M^2}\tan^2(\theta/2) F_M^2(|\vec{q}|^2) \right] \delta\left(\frac{Q^2}{2M} - \nu\right)$$

- Taking the **proton to be an extended object** of radius $\sim 10^{-13}$ cm with spin-1/2 fits the experimental data

Deep ineslastic scattering (DIS)



Feynman graph for DIS

- DIS: inclusive experiment breaking the proton

$$d\sigma_{\text{DIS}} \sim L^{\mu\nu} W_{\mu\nu}$$

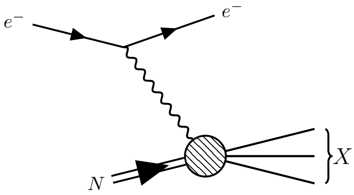
- $L^{\mu\nu} \rightarrow$ pQED tensor, calculable
- $W_{\mu\nu} \rightarrow$ non-trivial QCD part: perturbative coefficient function convoluted with a non-perturbative term (PDF)

Deep ineslastic scattering (DIS)

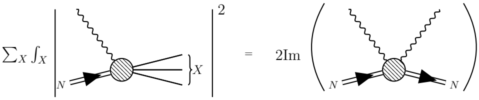
- DIS: inclusive experiment breaking the proton

$$d\sigma_{\text{DIS}} \sim L^{\mu\nu} W_{\mu\nu}$$

- $W_{\mu\nu} \rightarrow$ optical theorem:



Feynman graph for DIS



$\sum_X \int_X |X\rangle\langle X| = 1 \leftrightarrow$ **inclusiveness** in practice

Form factors & geometric shape

- DIS cross-section in terms of form factors (F_1, F_2):

$$\left.\frac{d^2\sigma}{d\Omega dE'}\right|_{\text{DIS}} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{Mott}} \cdot \left[\frac{1}{\nu} F_2(x,Q^2) + \frac{2}{M}\tan^2(\theta/2) F_1(x,Q^2)\right],$$

$$x = \frac{Q^2}{2pq} \underbrace{=}_{\substack{\text{proton}\\ \text{rest}\\ \text{frame}}} \frac{Q^2}{2M\nu} \leftrightarrow \text{Björken variable},$$

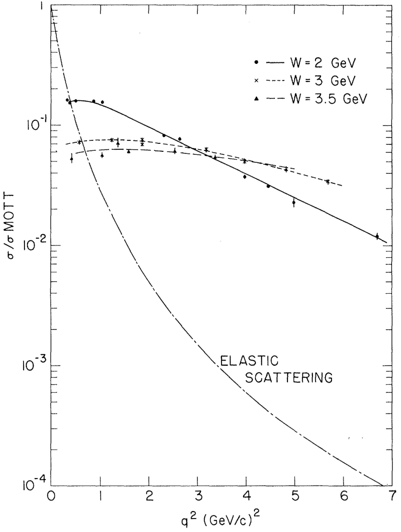
$$\nu = E - E'$$

- If F_1, F_2 are independent of Q^2, then DIS results on scattering on pointlike particles and the proton is not elementary:

$$\text{spatial distribution: } \rho_j(\vec{r}) \sim \int d^3\vec{q}\ e^{i\vec{q}\vec{r}} F_j(x,|\vec{q}|^2), \quad |\vec{q}|^2 = Q^2$$

- If $F_j(x, |\vec{q}|^2) = f_j(x)$, then $\rho_j(\vec{r}) \sim \delta(\vec{r}) f_j(x) \Rightarrow$ pointlike target

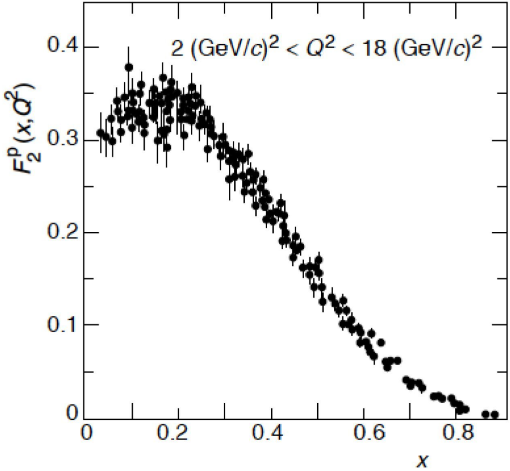
DIS at SLAC-MIT



Plot from: M. Breidenbach et al., Phys. Rev. Lett. **23**, 935 (1969)

- W is the invariant mass of the recoiling target system (X in the graph for DIS)
- **$W = 3$ and 3.5 GeV:** the (almost) independence with q^2 suggests scattering off elementary particles (pointlike or very small) particles

More from SLAC



Plot taken from Carolina Riedl's talk at the 61 Cracow summer school of theoretical physics: Electron-Ion Collider physics (2021). F_2^p means F_2 for the proton

- Almost independent of Q^2: very small particles inside the proton that are interacting with each other, i.e.,

$$F_2(x, Q^2) \sim F_2(x)$$

- Dependence on Q^2 $\rightarrow$ DGLAP and gluons

Parton Distribution Function (PDF)

- DIS cross-section can be factorized via PDFs as

$$\sigma_{\gamma N \to X} = \sum_f \int_0^1 dx \ \sigma_{\gamma \mathfrak{q}_f \to X}(x) \mathrm{PDF}_f(x), \quad f = \text{flavor},$$

$$x = \text{longitudinal momentum fraction},$$

in the Björken limit: $Q^2 \to \infty$, x fixed

- For PDFs, x coincides with the Björken variable: $x_B = \frac{Q^2}{2pq}$
- PDF is a 1D distribution:

$$\mathrm{PDF}_f(x) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ix\bar{p}^+ z^-} \langle N(p) | \bar{\mathfrak{q}}_f(-z/2) \gamma^+ \mathcal{W}[-z/2, z/2] \mathfrak{q}_f(z/2) | N(p) \rangle \Big|_{z_\perp = z^+ = 0}$$

- $z^+, z^-, z_\perp$: light-cone coordinates
- $\mathcal{W}$: Wilson line

Light-cone dominance

- In PDF: $z^2 = 2z^+z^- + z_\perp^2 \xrightarrow{z^+=z_\perp=0} 0$, **but why?**
- PDF comes from $W_{\mu\nu}$:

$$4\pi W_{\mu\nu} = \int d^4z \; e^{iqz} \langle p|j_\mu(z) j_\nu(0)|p\rangle$$

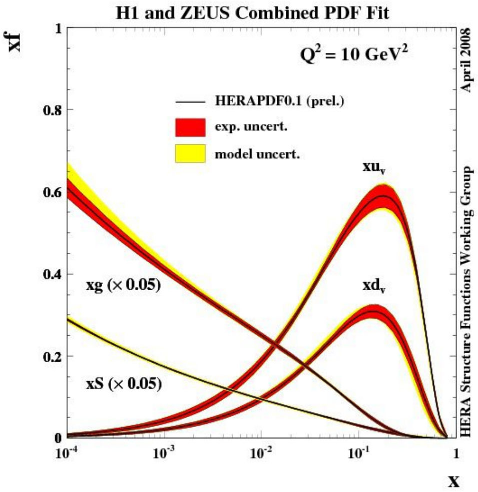
Light-cone dominance

$$4\pi W_{\mu\nu} = \int d^4z \; e^{iqz} \langle p | j_\mu(z) j_\nu(0) | p \rangle$$

What's the support of this integral?

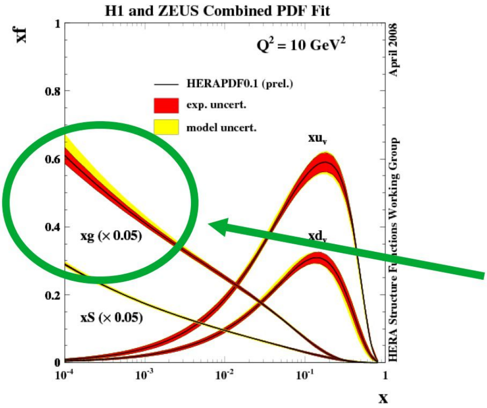
1. Causality: $z^2 \geq 0$
2. $qz = \frac{z^+ \nu}{p^+} - z^- x_B p^+ \,, \quad \nu = pq \to \infty$ in Bj. lim.
3. Riemann-Lebesgue: $\int d^n z \; f(z) e^{-i\eta z} \xrightarrow{|\eta| \to \infty} 0$
4. qz must be small $\Rightarrow z^+ \sim p^+/\nu \sim 0 \,, \quad z^- \sim \mathrm{const}/(x_B p^+)$
5. $qz \sim \mathrm{const} < \infty$
6. $z^2 \sim z_\perp^2 = -((z^1)^2 + (z^2)^2) \leq 0$
7. $(1) + (6) \to z^2 \sim 0 \Rightarrow$ **DIS is light-cone dominated**

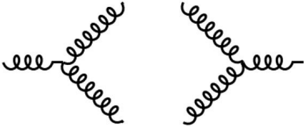
HERA's PDFs



- HERA = Hadron-Electron Ring Accelerator, at DESY in Germany
- Measurements of PDFs are key for experiments and calculations where hadrons play a role: pp collisions at the LHC are a primary example

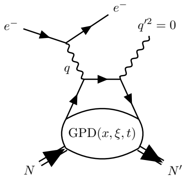
- At small-x, there must be a saturation scale Q_s^2 for which gluon splitting and fusion rates become equal and the gluon distribution for momentum xg stops growing:





Deeply virtual Compton scattering (DVCS)

- In the late 1990s, Ji, Müller and Radyushkin introduced the Generalized Parton Distributions (GPDs) through the DVCS process:



Feynman diagram for DVCS

- It is an exclusive scattering

- $\xi = \frac{-\bar{q}\Delta}{2\bar{p}\bar{q}}\,, \quad \bar{q} = \frac{q+q'}{2}\,, \quad \bar{p} = \frac{p+p'}{2}\,, \quad \Delta = p' - p\,, \quad t = \Delta^2$

Relevance of the GPDs

- Connected to QCD energy-momentum tensor, and so to spin. GPDs are a way to address the hadron's **spin puzzle.** Ji's sum rule:

$$\int_{-1}^{1} dx \; x[H_f(x,\xi,0)+E_f(x,\xi,0)]=2J_f$$

- **Tomography:** distribution of quarks in terms of the longitudinal momentum and in the transverse plane,

$$q_f(x,\mathbf{b}_\perp)=\int \frac{d^2\mathbf{\Delta}}{4\pi^2} \; e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}} H_f(x,0,t=-\mathbf{\Delta}^2)$$

But how do GPDs appear?

DVCS amplitude

- To 1st order in $\alpha_{\rm em}$ and under collinear factorization ($\mathcal{O}\left(\frac{\ell_\perp^2}{Q^2}, \frac{\Delta_\perp^2}{Q^2}, \frac{\ell_\perp \Delta_\perp}{Q^2}, \frac{\ell^- \bar{p}^+}{Q^2}\right)$ neglected):

$$\langle \gamma(q')N(p')|\mathcal{S}^{(2)}|\gamma^*(q)N(p)\rangle = -3i(2\pi)^4\delta(p+q-p'-q')\sum_f e_f^2(\varepsilon_{q\lambda})_\alpha(\varepsilon^*_{q'\lambda'})_\beta$$

$$\times g_\perp^{\alpha\beta}\int_{-1}^{1}dx\ \left(\frac{1}{x-\xi+i0}+\frac{1}{x+\xi-i0}\right)\underbrace{\frac{1}{2}\int\frac{dz^-}{2\pi}\ e^{ix\bar{p}^+z^-}\langle N(p')|\bar{\mathfrak{q}}_f(-z/2)\gamma^+\mathfrak{q}_f(z/2)|N(p)\rangle\Big|_{\substack{z^+=0\\ z_\perp=0}}}_{\text{GPD}}$$

$$+\ \text{(axial term)}$$

- GPD factorization happens in the amplitude as opposed to the factorization in cross-section via PDFs
- GPD $\simeq$ 3D version of a PDF

GPD & Compton Form Factor (CFF)

- CFF definition from our previous result ($t = \Delta^2$),

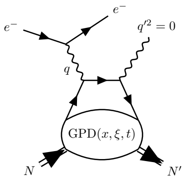
$$\mathrm{CFF}(\xi,t) = \mathcal{H}_f(\xi,t) = \int_{-1}^{1} dx \underbrace{\left(\frac{1}{x-\xi+i0} + \frac{1}{x+\xi-i0}\right)}_{\text{Coefficient function, } C_f(x,\xi), \text{ perturbative component}}$$

$$\times \underbrace{H_f(x,\xi,t)}_{\text{a particular GPD}(x,\xi,t), \text{ non-perturbative function}}$$

- In observables, GPDs will appear hidden inside CFFs

The need to go beyond DVCS

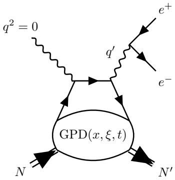
- LO amplitude for DVCS restricts the access to GPDs to the line $x = \xi$:



$$\mathrm{CFF}_{\mathrm{DVCS}} \sim \mathrm{PV}\left(\int_{-1}^{1} dx \frac{1}{x-\xi} \mathrm{GPD}(x,\xi,t)\right) - \int_{-1}^{1} dx \;\; i\pi\delta(x-\xi)\mathrm{GPD}(x,\xi,t) + \cdots$$

- Let us take a look at other processes that may solve this problem

- "Mirror" image of DVCS:



Feynman diagram for TCS

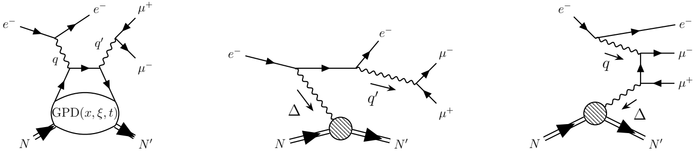
- GPDs enter the LO amplitude of TCS via the CFFs:

$$\mathrm{CFF}_{\mathrm{TCS}} \sim \mathrm{PV}\left(\int_{-1}^{1} dx \frac{1}{x+\xi}\mathrm{GPD}(x,\xi,t)\right) - \int_{-1}^{1} dx \;\; i\pi\delta(x+\xi)\mathrm{GPD}(x,\xi,t) + \cdots$$

- **1st measurement of TCS** by the CLAS collaboration at JLab: P. Chatagnon et al., PRL **127**, 262501 (2021)

Double deeply virtual Compton scattering (DDVCS)

- An extra virtuality with respect to DVCS and TCS to escape the $x = \pm\xi$ lines:



DDVCS (left), BH diagrams (middle and right). Crossed diagrams are not shown

- GPDs enter the DDVCS amplitude at LO via CFFs:

$$\mathrm{CFF}_{\mathrm{DDVCS}} \sim \mathrm{PV}\left(\int_{-1}^{1} dx \frac{1}{x-\rho}\mathrm{GPD}(x,\xi,t)\right) - \int_{-1}^{1} dx \;\; i\pi\delta(x-\rho)\mathrm{GPD}(x,\xi,t) + \cdots$$

$$\rho = -\frac{\bar{q}^2}{2\bar{p}\bar{q}}\,, \quad \xi = \frac{-\bar{q}\Delta}{2\bar{p}\bar{q}}$$

Original papers in DDVCS: Belitsky & Muller, PRL 90, 022001 (2003); Guidal & Vanderhaeghen, PRL 90, 012001 (2003); Belitsky & Muller, PRD 68, 116005 (2003)

DDVCS subprocess

- DDVCS subprocess amplitude:

$$i\mathcal{M}_{\rm DDVCS}=\frac{ie^4\bar{u}(\ell_-,s_\ell)\gamma_\mu v(\ell_+,s_\ell)\bar{u}(k',s)\gamma_\nu u(k,s)}{(q^2+i0)(q'^2+i0)}\,T^{\mu\nu}_{s_2 s_1}$$

- Compton tensor decomposition at LT:

$$T^{\mu\nu}_{s_2 s_1}=T^{(V)\mu\nu}\bar{u}(p',s_2)\Big[(\mathcal{H}+\mathcal{E})\not{n}-\frac{\mathcal{E}}{M}\bar{p}^+\Big]u(p,s_1)+T^{(A)\mu\nu}\bar{u}(p',s_2)\left[\widetilde{\mathcal{H}}\not{n}+\frac{\widetilde{\mathcal{E}}}{2M}\Delta^+\right]\gamma^5 u(p,s_1)$$

$$T^{(V)\mu\nu}=-\frac{1}{2}(g^{\mu\nu}-n^\mu n^{\star\nu}-n^\nu n^{\star\mu})\equiv-\frac{1}{2}g_\perp^{\mu\nu}$$

$$T^{(A)\mu\nu}=-\frac{i}{2}\epsilon^{\mu\nu}{}_{\rho\sigma}n^\rho n^{\star\sigma}\equiv-\frac{i}{2}\epsilon_\perp^{\mu\nu}$$

- Longitudinal plane is built with $\{\bar{q},\bar{p}\}$
- $q_\perp^\nu \sim \Delta_\perp^\nu \Rightarrow g_\perp^{\mu\nu}q_\nu \neq 0 \Rightarrow$ EM gauge-violation

DDVCS subprocess

- Longitudinal plane is built with $\{\bar{q}, \bar{p}\}$
- $q_\perp^\nu \sim \Delta_\perp^\nu \Rightarrow g_\perp^{\mu\nu} q_\nu \neq 0 \Rightarrow$ EM gauge-violation
- Gauge-violation can be cured by evaluating the **hard part of the process** at $t = t_0$ ($|t_0| \leq |t|$):

$$q_\perp^\nu|_{t=t_0} \sim \Delta_\perp^\nu|_{t=t_0} = 0$$

- This procedure is consistent with the collinear factorization which is at the core of the GPD description

Kleiss-Stirling (KS) techniques

- Avoid computation of traces of gamma-matrices
- Address amplitude first instead of its modulus squared
- **Reduce amplitudes to complex-numbers**
- 2 scalars as building blocks, a and b as light-like vectors:

$$s(a,b) = \bar{u}(a,+)u(b,-) = -s(b,a)$$

$$t(a,b) = \bar{u}(a,-)u(b,+) = [s(b,a)]^*$$

$$s(a,b) = (a^2 + ia^3)\sqrt{\frac{b^0 - b^1}{a^0 - a^1}} - (a \leftrightarrow b)$$

Kleiss & Stirling, Nucl. Phys. B 262 (1985) 235-262

DDVCS subprocess *à la* KS

- DDVCS subprocess amplitude:

$$i\mathcal{M}_{\text{DDVCS}} = \frac{-ie^4}{(Q^2 - i0)(Q'^2 + i0)} \left(i\mathcal{M}^{(V)}_{\text{DDVCS}} + i\mathcal{M}^{(A)}_{\text{DDVCS}} \right)$$

- Vector contribution:

$$i\mathcal{M}^{(V)}_{\text{DDVCS}} = -\frac{1}{2} \left[f(s_\ell, \ell_-, \ell_+; s, k', k) - g(s_\ell, \ell_-, n^\star, \ell_+) g(s, k', n, k) - g(s_\ell, \ell_-, n, \ell_+) g(s, k', n^\star, k) \right]$$
$$\times \left[(\mathcal{H} + \mathcal{E}) [Y_{s_2 s_1} g(+, r'_{s_2}, n, r_{s_1}) + Z_{s_2 s_1} g(-, r'_{-s_2}, n, r_{-s_1})] - \frac{\mathcal{E}}{M} \mathcal{J}^{(2)}_{s_2 s_1} \right]$$

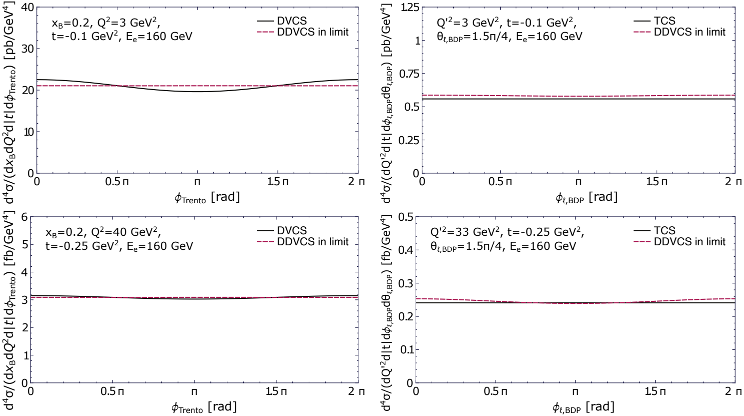
- Axial contribution:

$$i\mathcal{M}^{(A)}_{\text{DDVCS}} = \frac{-i}{2} \epsilon^{\mu\nu}_{\perp} j_\mu(s_\ell, \ell_-, \ell_+) j_\nu(s, k', k) \left[\widetilde{\mathcal{H}} \mathcal{J}^{(1,5)+}_{s_2 s_1} + \widetilde{\mathcal{E}} \frac{\Delta^+}{2M} \mathcal{J}^{(2,5)+}_{s_2 s_1} \right]$$

More details in:

K. Deja, V. Martínez-Fernández, B. Pire, P. Sznajder, J. Wagner, PRD **107** (2023), no. 9, 094035

DVCS & TCS limits of DDVCS



Comparison of DDVCS and (left) DVCS and (right) TCS cross-sections for pure VCS subprocess. **GK model for GPDs.**

Trento: PRD 70, 117504 (2004); **BDP:** EPJC23, 675 (2002)

Observables in DDVCS: beam-spin asymmetry

- Single beam-spin asymmetry for longitudinally polarized electrons:

$$A_{LU}(\phi_{\ell,\mathrm{BDP}}) = \frac{\Delta\sigma_{LU}(\phi_{\ell,\mathrm{BDP}})}{\sigma_{UU}(\phi_{\ell,\mathrm{BDP}})}$$

$$\Delta\sigma_{LU}(\phi_{\ell,\mathrm{BDP}}) = \int_0^{2\pi} d\phi \int_{\pi/4}^{3\pi/4} d\theta_{\ell,\mathrm{BDP}} \ \sin\theta_{\ell,\mathrm{BDP}}$$

$$\times\left(\frac{d^7\sigma^{\rightarrow}}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_{\ell,\mathrm{BDP}}} - \frac{d^7\sigma^{\leftarrow}}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_{\ell,\mathrm{BDP}}}\right)$$

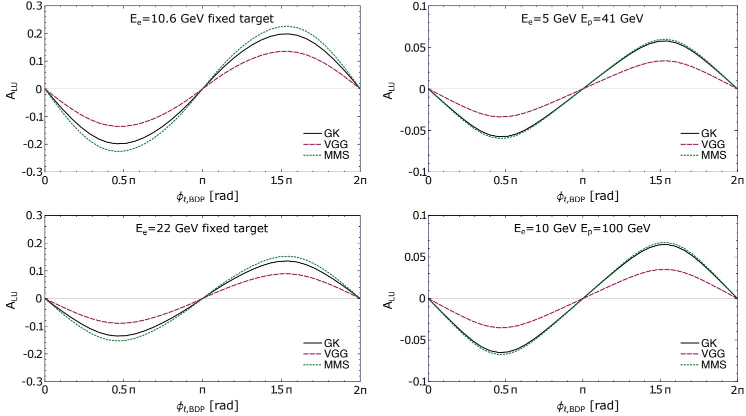
- We consider $Q'^2 > Q^2$: our DDVCS is “more” timelike than spacelike
- Variable for later use:

$$y = \frac{pq}{pk} \underbrace{=}_{\substack{\text{proton}\\ \text{rest}\\ \text{frame}}} \frac{E-E'}{E},$$

E=energy of incoming electron beam, E'=energy of recoiled electron

Observables in DDVCS: beam-spin asymmetry

K. Deja, V. Martínez-Fernández, B. Pire, P. Sznajder, J. Wagner,
PRD **107** (2023), no. 9, 094035

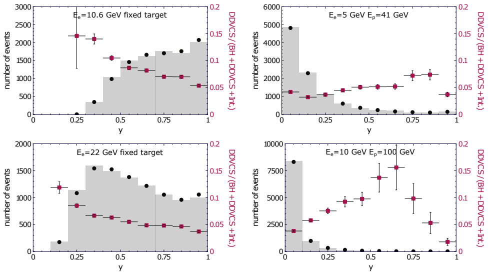


JLab12, JLab20+: **15-20%** EIC 5x41, EIC 10x100: **3-7%**

Experiment	Beam energies [GeV]	y	$\lvert t\rvert$ [GeV^2]	Q^2 [GeV^2]	Q'^2 [GeV^2]
JLab12	$E_e = 10.6,\ E_p = M$	0.5	0.2	0.6	2.5
JLab20+	$E_e = 22,\ E_p = M$	0.3	0.2	0.6	2.5
EIC	$E_e = 5,\ E_p = 41$	0.15	0.1	0.6	2.5
EIC	$E_e = 10,\ E_p = 100$	0.15	0.1	0.6	2.5

Monte Carlo study: distribution in y (DDVCS)

K. Deja, V. Martínez-Fernández, B. Pire, P. Sznajder, J. Wagner, PRD **107** (2023), no. 9, 094035



JLab12, JLab20+ EIC 5x41, EIC 10x100

EpIC MC
integrated cross-section
pure DDVCS fraction

Kinematic cuts:
$Q^2 \in (0.15, 5)$ GeV^2
$Q'^2 \in (2.25, 9)$ GeV^2
JLab: $-t \in (0.1, 0.8)$ GeV^2
EIC: $-t \in (0.01, 1)$ GeV^2
$\phi, \phi_\ell \in (0.1, 2\pi - 0.1)$ rad
$\theta_\ell \in (\pi/4, 3\pi/4)$ rad
JLab: $y \in (0.1, 1)$
EIC: $y \in (0.05, 1)$

10000 events/distribution. Neither acceptance nor detectors response are taken into account in this study

Experiment	Beam energies [GeV]	Range of $\|t\|$ [GeV^2]	$\sigma\|_{0<y<1}$ [pb]	$\mathcal{L}^{10k}\|_{0<y<1}$ [fb^{-1}]	$y_{\min}$	$\sigma\|_{y_{\min}<y<1}/\sigma\|_{0<y<1}$
JLab12	$E_e = 10.6,\ E_p = M$	$(0.1, 0.8)$	0.14	70	0.1	1
JLab20+	$E_e = 22,\ E_p = M$	$(0.1, 0.8)$	0.46	22	0.1	1
EIC	$E_e = 5,\ E_p = 41$	$(0.05, 1)$	3.9	2.6	0.05	0.73
EIC	$E_e = 10,\ E_p = 100$	$(0.05, 1)$	4.7	2.1	0.05	0.32

Worldwide picture

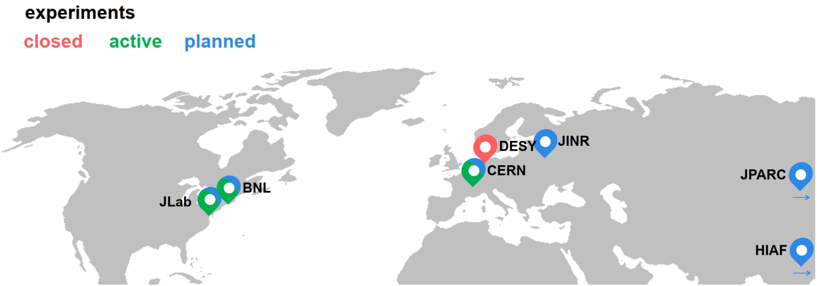
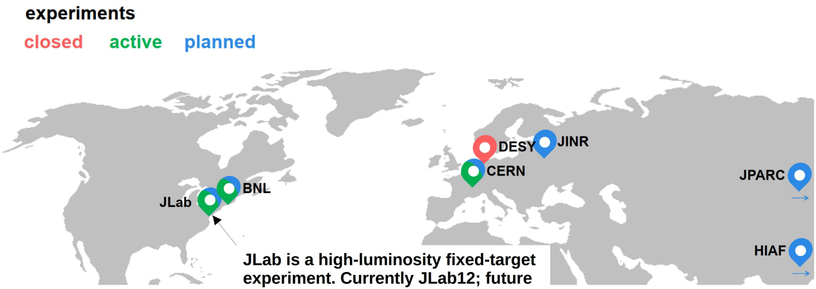


Image courtesy of Paweł Sznajder

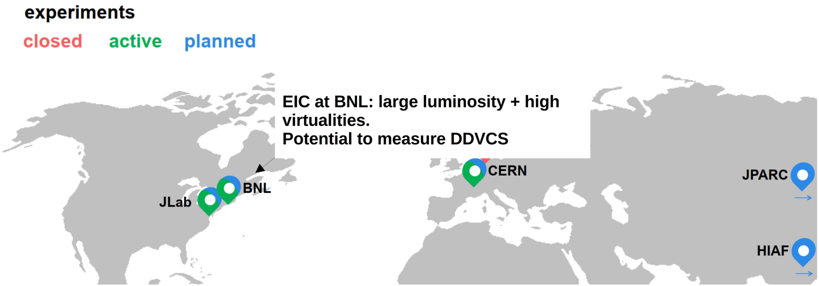
Worldwide picture



JLab is a high-luminosity fixed-target experiment. Currently JLab12; future JLab20+? DVCS and TCS can be done, potential for DDVCS

More info about JLab20+: A. Accardi et al., arXiV: nucl-ex/2306.09360

Worldwide picture



More info about EIC: R. Abdul Khalek et al., Nucl. Phys. A 1026 (2022) 122447 (Yellow Report)

Summary

- DIS and PDF have been explained and related
- GPDs as a 3D version of the PDFs have been introduced via exclusive inelastic scattering
- I have shown how DDVCS allows to map GPDs on their whole domain, i.e., $x \neq \xi$. Full knowledge of GPDs is fundamental in order to:
    1. Understand the different sources for the spin of the hadron
    2. Perform transverse imagining (tomography)
- The latest results in both the theoretical and phenomenological aspects of DDVCS have been discussed
- New experiments are ahead: EIC (and maybe JLab20+?)

Thank you!

Complementary slides

Light-cone coordinates

- A way to parameterize 4-vectors:

$$z^\mu = z^- n^\mu + z^+ n'^\mu + z_\perp^\mu \,,$$

$$n^2 = n'^2 = 0 \,, \quad nn' \neq 0 \,,$$

$$z_\perp n = z_\perp n' = 0$$

- n, n' define the "minus" and "plus" longitudinal directions, respectively

Wilson line

- It restores the gauge invariance of the PDF,

$$\mathcal{W}[z_1^-, z_2^-] = \mathbb{P} \exp\left[ig \int_{z_2^-}^{z_1^-} da^- A^+(a^-) \right]$$

- It is the result of resummming contributions to leading twist ($\mathcal{O}(\text{constant}/Q^2)$ neglected) from gluon exchanges between the hard part (parton-photon interaction) and the soft part (non-active partons):

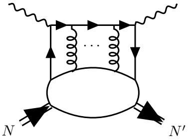


Diagram for a Wilson line

- Under the gauge $A^+ = 0$, $\mathcal{W} \equiv 1$ and can be disregarded

DVCS amplitude

- Photon-proton interaction, to order† $\mathcal{O}(\alpha_{\mathrm{em}})$:

$$\begin{aligned}\langle\gamma(q')N(p')|\mathcal{S}^{(2)}|\gamma^*(q)N(p)\rangle \;=\; & \tfrac{3}{4}(2\pi)^4\delta(p+q-p'-q')(\varepsilon_{q\lambda})_\alpha(\varepsilon^*_{q'\lambda'})_\beta \\ & \times\sum_f e_f^2\int d^4\ell\,\frac{d^4z}{(2\pi)^4}\;e^{i\ell z}\,\mathrm{tr}\left\{\left[\gamma^\beta\frac{i(\not\ell+\not q)}{(\ell+q)^2+i0}\gamma^\alpha+\gamma^\alpha\frac{i(\not\ell-\not q')}{(\ell-q')^2+i0}\gamma^\beta\right]\gamma^\mu\right\} \\ & \times\langle N(p')|{:}\bar{\mathrm{q}}_f(0)\gamma_\mu \mathrm{q}_f(z){:}|N(p)\rangle \\ & + \text{(axial term)}\end{aligned}$$

- ℓ is the 4-momentum of the active parton
- $\varepsilon_{q\lambda}$ is the polarization vector of the photon with momentum q and polarization λ

† $e_f^2 = e^2 Q_f^2$, Q_f = quark's fraction of proton electric charge

DVCS amplitude

- Momentum can be parameterized with the following variables:

  1. x, the fraction of the hadron longitudinal momentum that is carried away by the parton
  2. $\xi = -\frac{\Delta\bar{q}}{2\bar{p}\bar{q}}$, called skewness
     $(\bar{p} = (p + p')/2,\ \Delta = p' - p,\ \bar{q} = (q + q')/2)$

- This way, for instance,

$$(\ell - q')^\mu = (x - \xi)\bar{p}^+ n'^\mu + \left(\ell^- - \frac{Q^2}{4\xi\bar{p}^+}\right) n^\mu + \ell_\perp + \Delta_\perp$$

$$(\ell - q')^2 = -(x - \xi)\frac{Q^2}{2\xi} + \underbrace{\mathcal{O}\left(\frac{\ell_\perp^2}{Q^2}, \frac{\Delta_\perp^2}{Q^2}, \frac{\ell_\perp\Delta_\perp}{Q^2}, \frac{\ell^-\bar{p}^+}{Q^2}\right)}_{\text{Neglecting transverse dynamics renders collinear factorization}}$$

- Collinear factorization is justified on Q^2 being the scale of the process and, therefore, larger than any other momentum

GPDs of a spin-1/2 hadron

- Further decomposition of the correlator that has been called the GPD so far ($t = \Delta^2$):

$$F_f = \frac{1}{2}\int \frac{dz^-}{2\pi}\; e^{ix\bar{p}^+z^-}\langle N(p')|\bar{\mathfrak{q}}_f(-z/2)\gamma^+\mathfrak{q}_f(z/2)|N(p)\rangle\Big|_{\substack{z^+=0\\ z_\perp=0}}$$

$$= \frac{1}{2\bar{p}^+}\left[H_f(x,\xi,t)\bar{u}(p')\gamma^+u(p) \;+\; E_f(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\right],$$

$$\sigma^{\beta\alpha}=\frac{i}{2}\left[\gamma^\beta,\gamma^\alpha\right]$$

- Technically, the GPDs are H_f and E_f, although F_f is usually referred to as the GPD too. As usual, it is a matter of context...
- The axial term give rise similarly to another two GPDs: $\widetilde{H}_f$ and $\widetilde{E}_f$

Kleiss-Stirling (KS) techniques

- Define a massless spinor basis $\{u(\kappa_0, \pm)\}$ for momentum $\kappa_0^2 = 0$ and helicity $\pm$. The negative-helicity state is defined by:

$$\bar{u}(\kappa_0, -)u(\kappa_0, -) = \omega_- \not{\kappa}_0, \quad \omega_\lambda = \frac{1}{2}(1 + \lambda\gamma^5)$$

- Define a spacelike vector $\kappa_1^2 = -1$ such that $\kappa_0 \kappa_1 = 0$, then the positive-helicity state is:

$$u(\kappa_0, +) = \not{\kappa}_1 u(\kappa_0, -)$$

Kleiss & Stirling, Nucl. Phys. B 262 (1985) 235-262

Kleiss-Stirling (KS) techniques

- For any spinor associated to a massless fermion of momentum p, imposing Dirac equation, projection

$$u(p,\lambda)\bar{u}(p,\lambda) = \omega_\lambda \not{p}, \quad \lambda = \pm\,,$$

and using spinor basis above, one finds:

$$u(p,\lambda) = \frac{\not{p}\, u(\kappa_0, -\lambda)}{\sqrt{2p\kappa_0}}$$

- The only restriction to this formula is $p\kappa_0 \neq 0$ and, for computer purposes, not extremely small
- Massive spinors can be expanded by means of the massless ones described here

Kleiss & Stirling, Nucl. Phys. B 262 (1985) 235-262

DDVCS subprocess *à la* KS

- **f = contraction of 2 currents**

$$f(\lambda,k_0,k_1;\lambda',k_2,k_3)=\bar{u}(k_0,\lambda)\gamma^\mu u(k_1,\lambda)\bar{u}(k_2,\lambda')\gamma_\mu u(k_3,\lambda')=2[s(k_2,k_1)t(k_0,k_3)\delta_{\lambda-}\delta_{\lambda'+}$$

$$+t(k_2,k_1)s(k_0,k_3)\delta_{\lambda+}\delta_{\lambda'-}+s(k_2,k_0)t(k_1,k_3)\delta_{\lambda+}\delta_{\lambda'+}+t(k_2,k_0)s(k_1,k_3)\delta_{\lambda-}\delta_{\lambda'-}]$$

- **g = contraction of a current with a lightlike vector a**

$$g(s,\ell,a,k)=\bar{u}(\ell,s)\not{a}u(k,s)=\delta_{s+}s(\ell,a)t(a,k)+\delta_{s-}t(\ell,a)s(a,k)$$

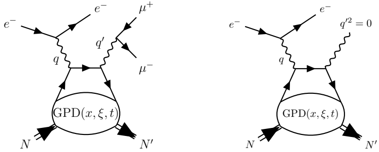
- **BH diagrams can be treated in a similar manner**

More details in:

K. Deja, V. Martínez-Fernández, B. Pire, P. Sznajder, J. Wagner, PRD **107** (2023), no. 9, 094035

DVCS limit of DDVCS

- **DDVCS to DVCS:**

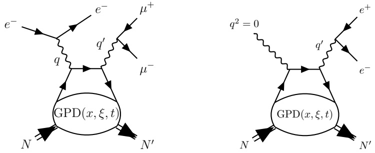


DDVCS (left), DVCS (right)

$$\int d\Omega_\ell \underbrace{\frac{d^7\sigma}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_\ell}}_{\text{DDVCS}} \xrightarrow{Q'^2\to 0} \underbrace{\left(\frac{d^4\sigma}{dx_B dQ^2 d|t| d\phi}\right)}_{\text{DVCS}} \frac{\mathcal{N}}{Q'^2}$$

$$\mathcal{N}=\alpha_{\rm em}/(3\pi)$$

- **DDVCS to TCS:**

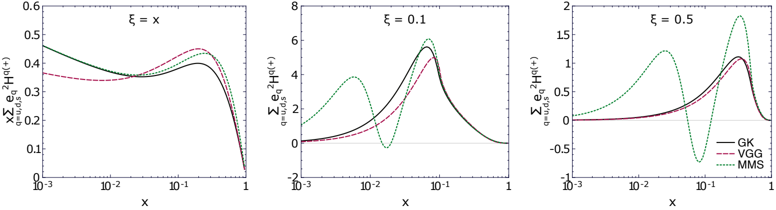


DDVCS (left), TCS (right)

$$\int d\phi \underbrace{\frac{d^7\sigma}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_\ell}}_{\text{DDVCS}} \xrightarrow{Q^2\to 0} \underbrace{\left(\frac{d^4\sigma}{dQ'^2 d|t| d\Omega_\ell}\right)}_{\text{TCS}} \underbrace{\frac{d^2\Gamma}{dx_B dQ^2}}_{\text{EPA photon flux}}$$

$$\frac{d^2\Gamma}{dx_B dQ^2} = \frac{\alpha_{\text{em}}}{2\pi Q^2}\left(1+\frac{(1-y)^2}{y}-\frac{2(1-y)Q^2_{\text{min}}}{yQ^2}\right)\frac{\nu}{Ex_B} \leftarrow \text{EPA photon}-\text{flux}$$

Models for the C-even part of GPD H^q



Distributions of $\sum_q e_q^2 H^{q(+)}(x, \xi, t)$ at $t = -0.1\ \mathrm{GeV}^2$, where $q = u, d, s$ flavors for (left) $\xi = x$, (middle) $\xi = 0.1$ and (right) $\xi = 0.5$. The solid black, dashed red and dotted green curves describe the GK, VGG and MMS GPD models, respectively. The C-even part of a given vector GPD is defined as:
$H^{q(+)}(x, \xi, t) = H^q(x, \xi, t) - H^q(-x, \xi, t)$. The scale is chosen as $\mu_F^2 = 4\ \mathrm{GeV}^2$.