Strong Gravitational Lensing Applications on Cosmology and Galactic Evolution

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Supervisor: Marek Biesiada

NCBJ

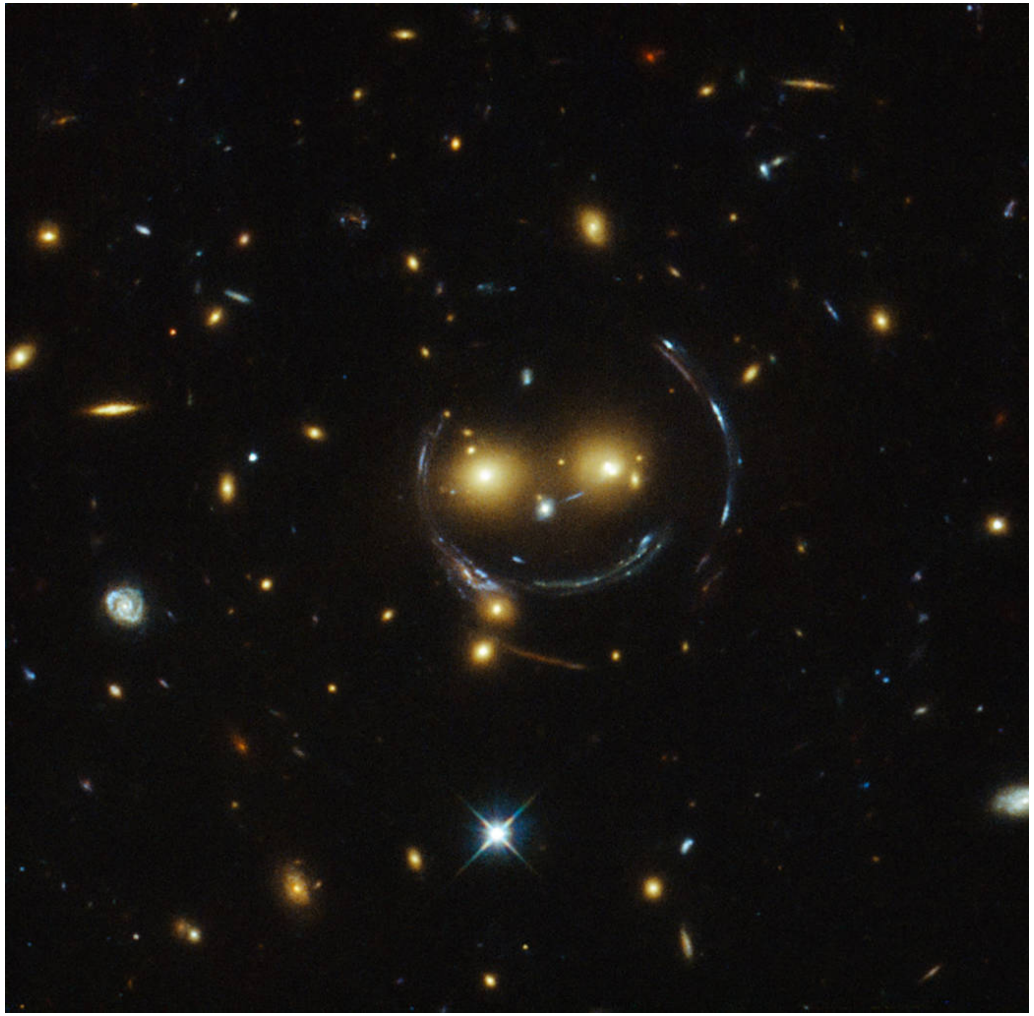
Outline

- Backgrounds
  - Strong gravitational lensing
  - Cosmology parameters
  - Galaxy evolution
- Galaxy global properties and their redshift evolution
- Cosmology constraints
- Future perspective

Strong Gravitational Lensing

Strong Gravitational lensing

galaxy cluster SDSS J1038+4849



NASA/ESA

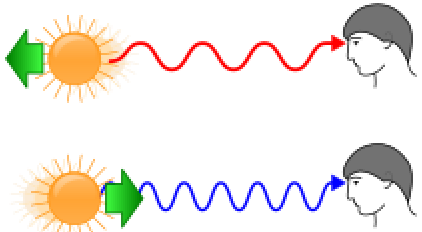
SPT-S J041839-4751.8
Redshift： 4.2248

NASA/ESA/CSA/STScI/j. Roger

Strong Gravitational lensing

Redshift z:

$$z = \frac{f_{emi} - f_{obs}}{f_{obs}} = \frac{\lambda_{obs} - \lambda_{emi}}{\lambda_{emi}}$$



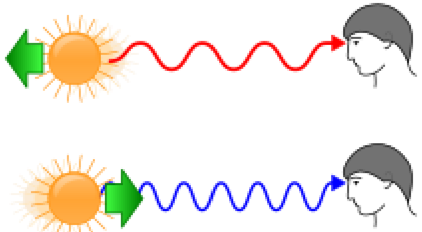
Credit: Wikipedia

Strong Gravitational lensing

Redshift z:

$$z = \frac{f_{emi} - f_{obs}}{f_{obs}} = \frac{\lambda_{obs} - \lambda_{emi}}{\lambda_{emi}}$$

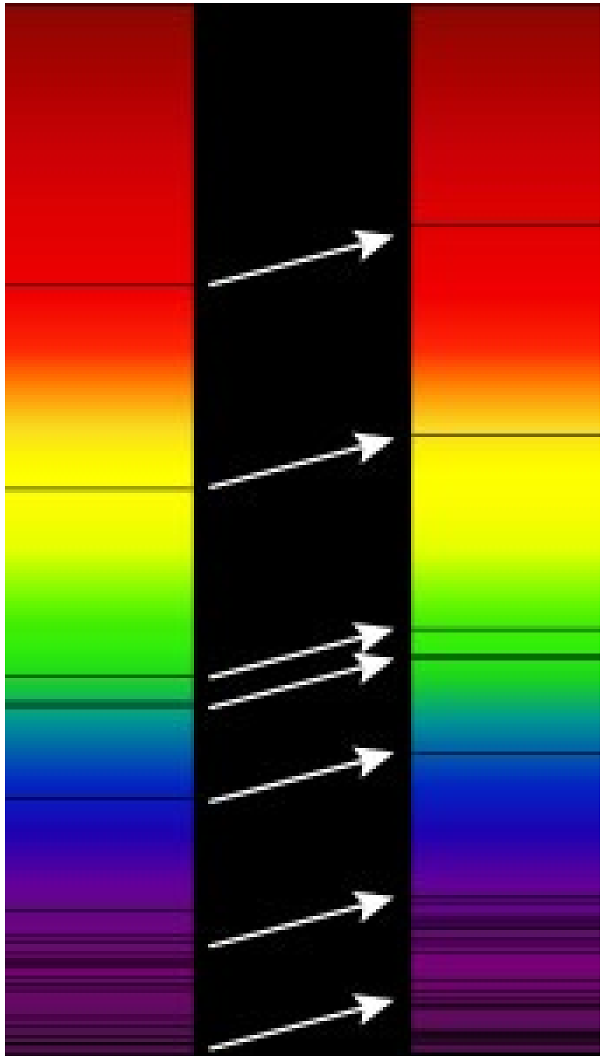
$$1 + z = \frac{\lambda_{obs}}{\lambda_{emi}} = \frac{\lambda_{now}}{a \cdot \lambda_{now}} = \frac{1}{a}$$



Credit: Wikipedia

Distance ← Redshift → Time

Credit: Wikipedia



Absorption line from the Sun

Absorption line from a distant galaxy

Strong Gravitational lensing – History

- Gravitational lensing **first proposed** by Soldner (1801).
  Deflection angle $\alpha = \frac{2GM}{c^2 r}$ (For sun gives 0.85”)
- Derived by Einstein using **Equivalence principle & Euclidean metric** (1911)
  Same deflection angle $\alpha = \frac{2GM}{c^2 r}$
- Deflection angle with **General Relativity** by Einstein (1915)
  $\alpha = \frac{4GM}{c^2 r}$ (For sun gives 1.7”)
- **Measured** by Eddington during a solar eclipse (1919)
  In Sobral: $1.98'' \pm 0.12''$; in Principe: $1.61'' \pm 0.30''$;
- **Lensed extragalactic nebulae (galaxies)** producing multi-images suggested by Zwicky (1937)
- ……

After this point, the curtain is gradually being unveiled on the study of cosmology using gravitational lensing ……

Strong Gravitational lensing – History

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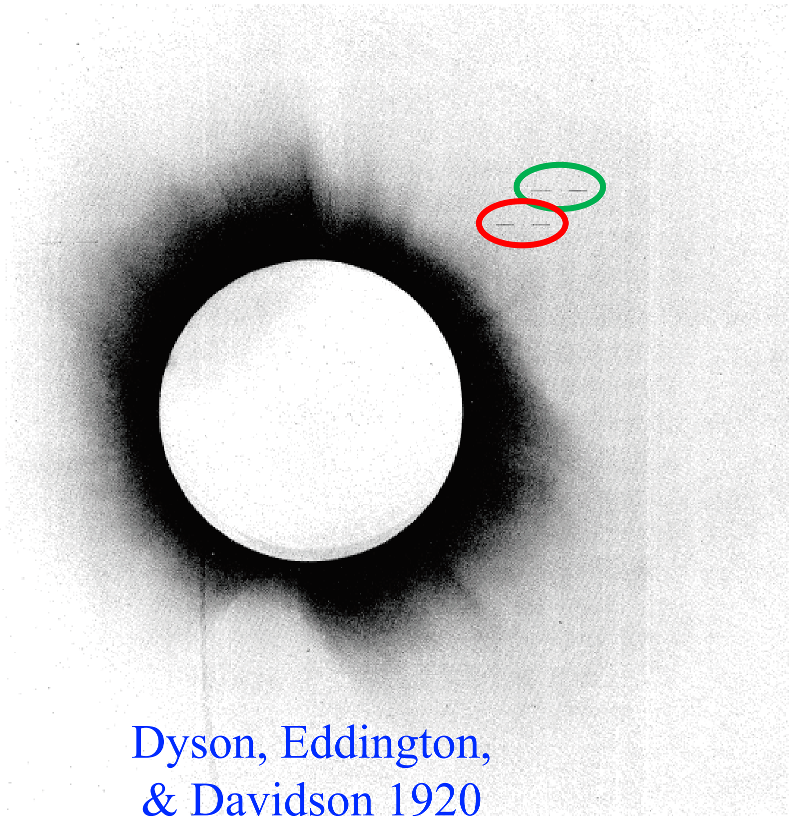


Dyson, Eddington, & Davidson 1920

Strong Gravitational lensing

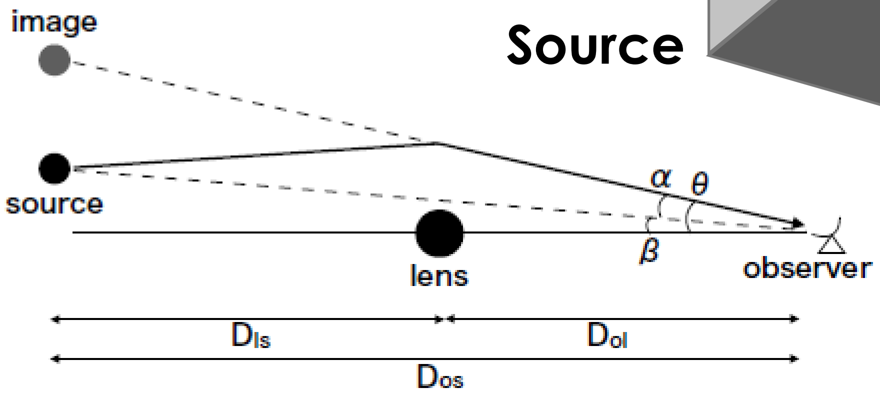
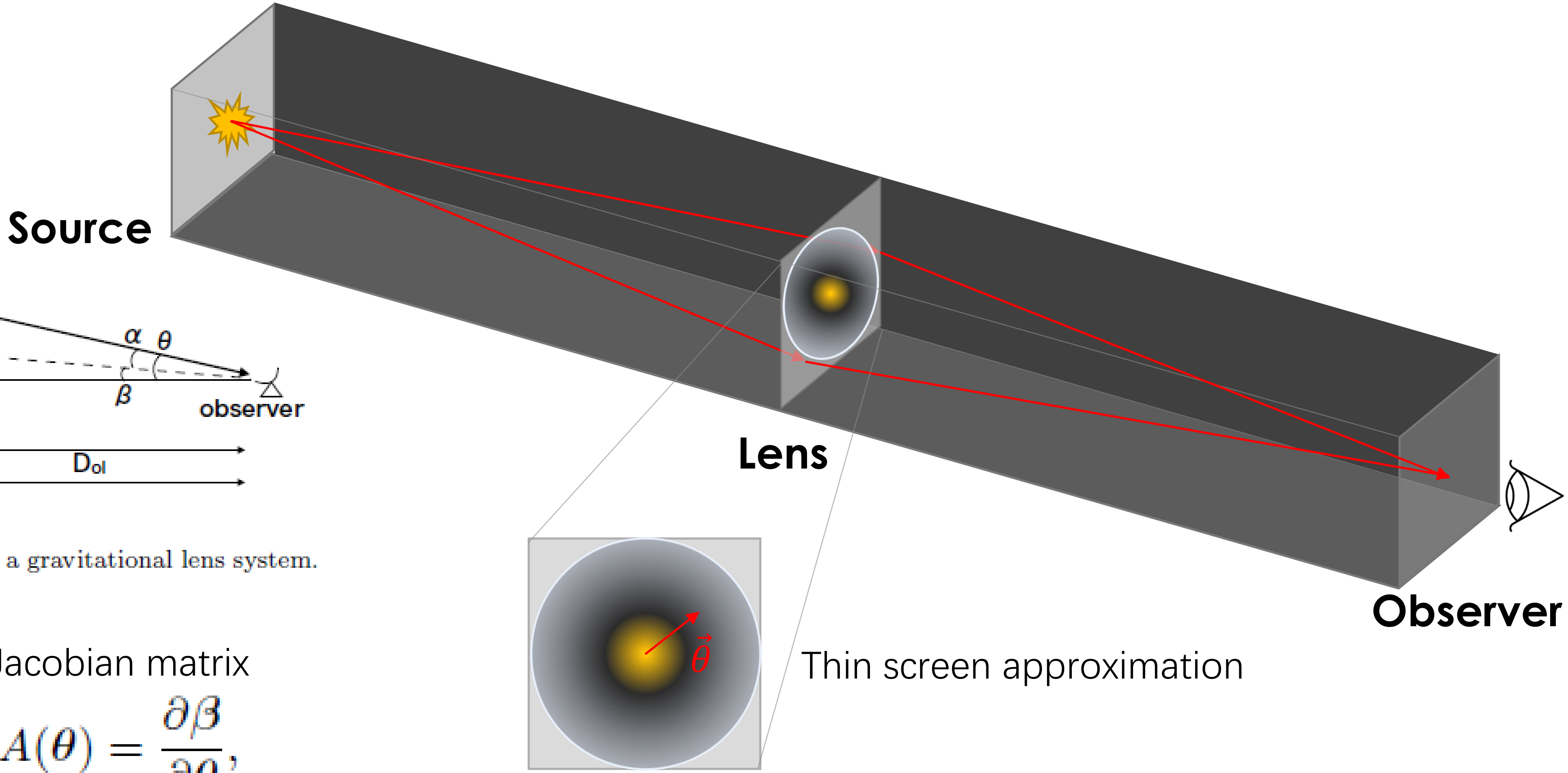


Figure 1. Schematic illustration of a gravitational lens system.

Oguri 2019



$$\alpha = \nabla_{\theta}\phi.$$

$$\beta = \theta - \alpha(\theta),$$

Jacobian matrix

$$A(\theta) = \frac{\partial\beta}{\partial\theta},$$

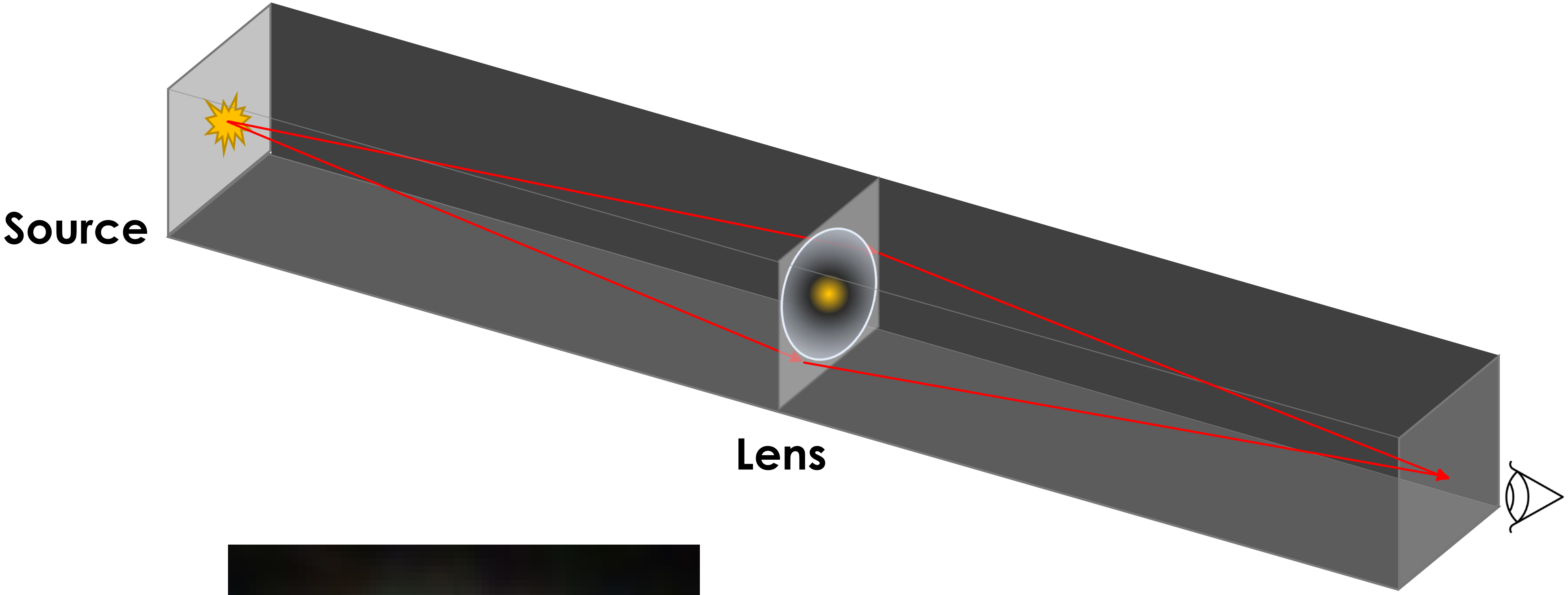
Magnification

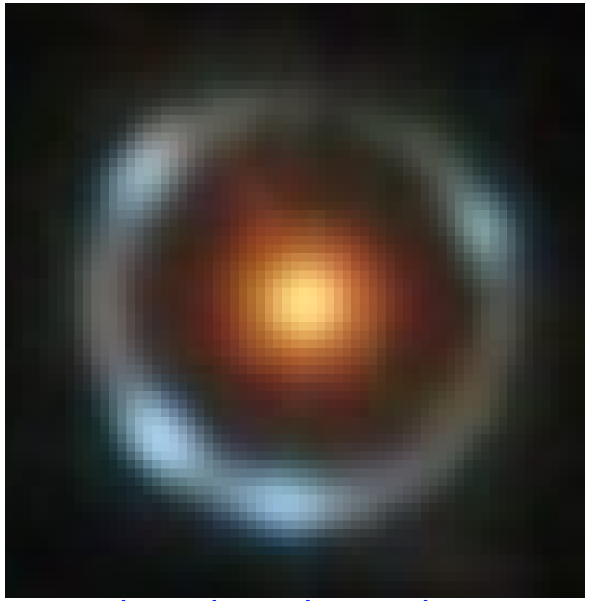
$$\mu(\theta) = \frac{1}{\det A(\theta)}.$$

Effective Lensing Potential

$$\phi(\vec{\theta}) = \frac{D_{ls}}{D_{ol}D_{os}}\frac{2}{c^2}\int \Phi(D_{ol}\vec{\theta}, z)dz$$

Observables





NASA/ESA/CSA/STScI/j. Roger

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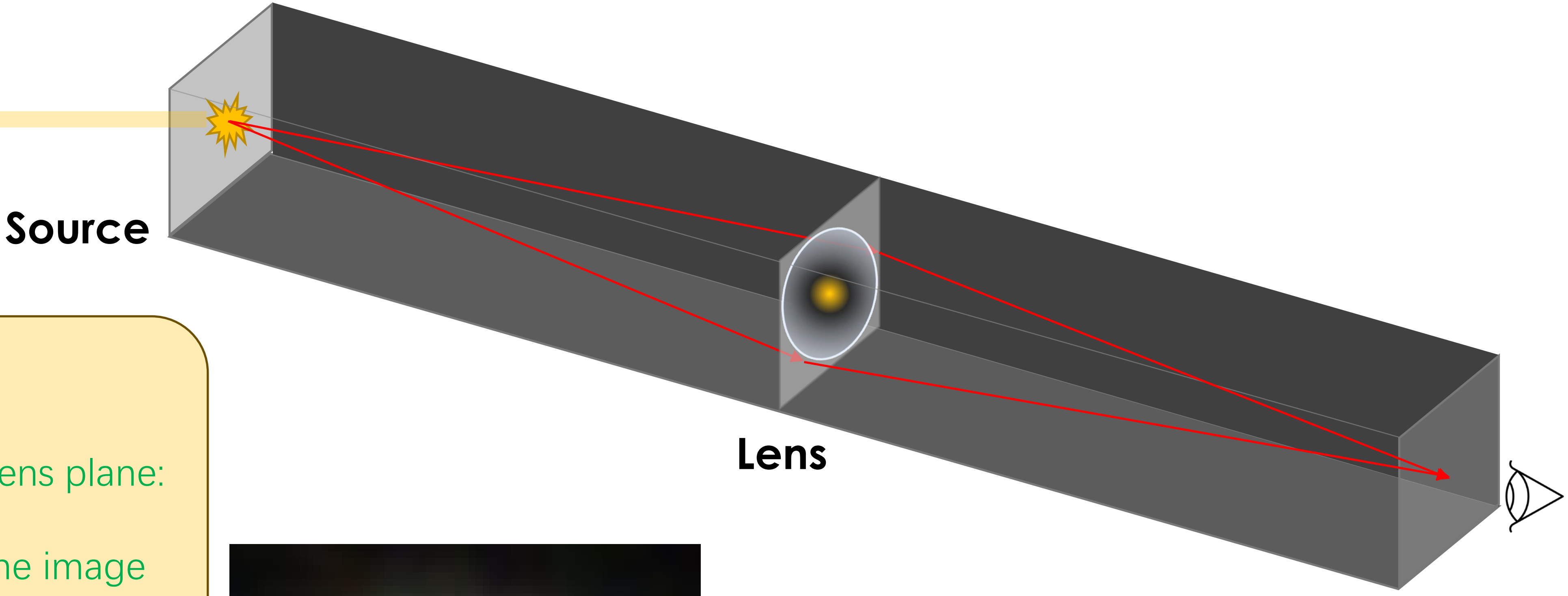
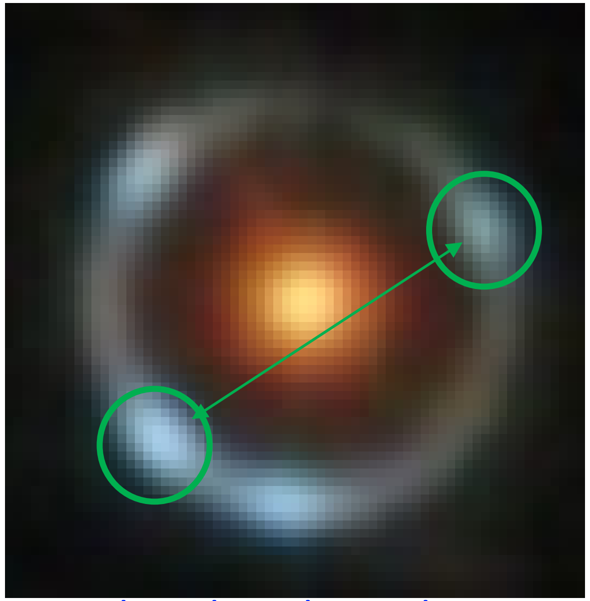


Source:

- redshift Zs

Source Images on the lens plane:

- Images' separations
- Flux & flux ratio of the image
- Time-delays



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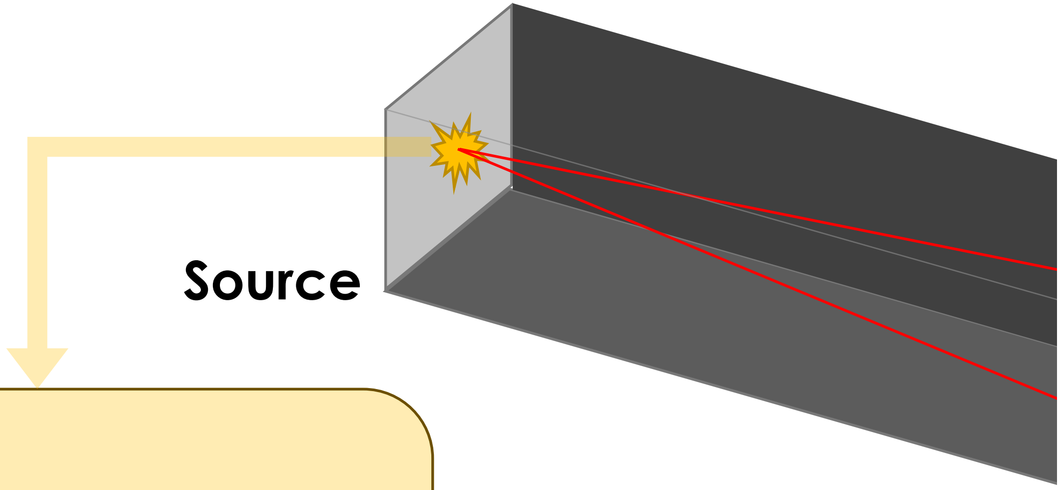

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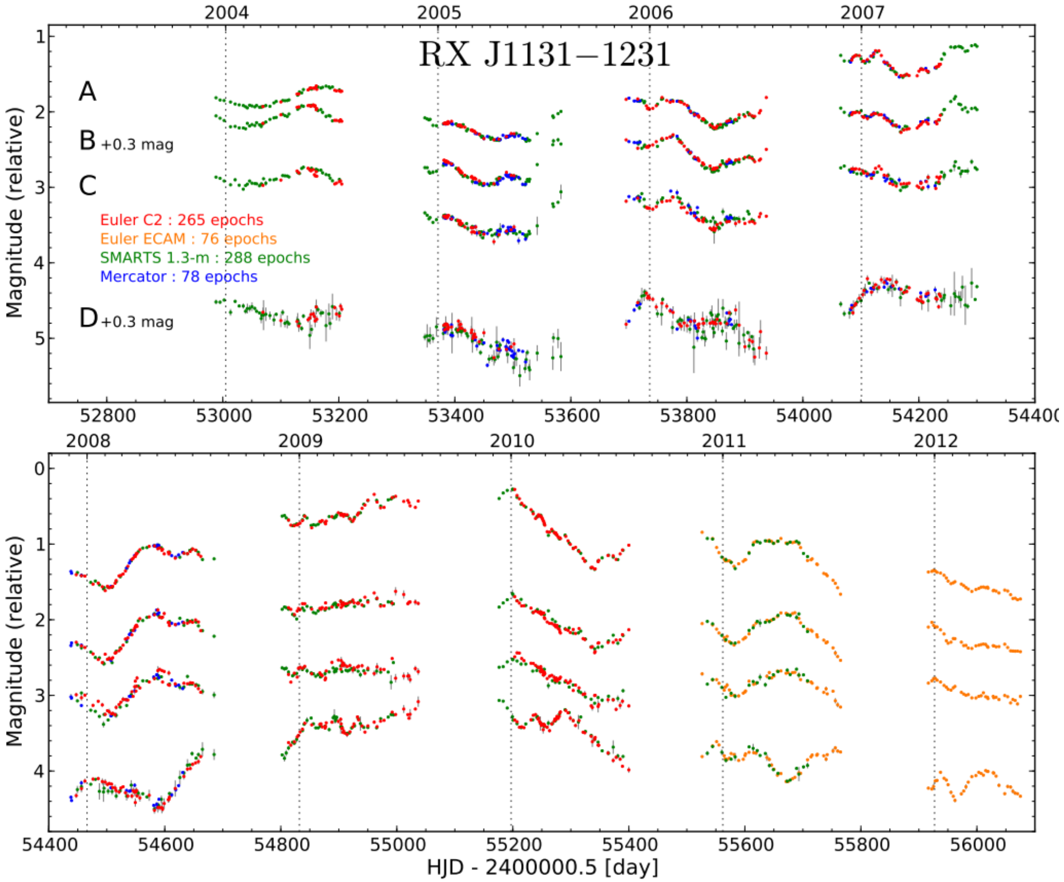
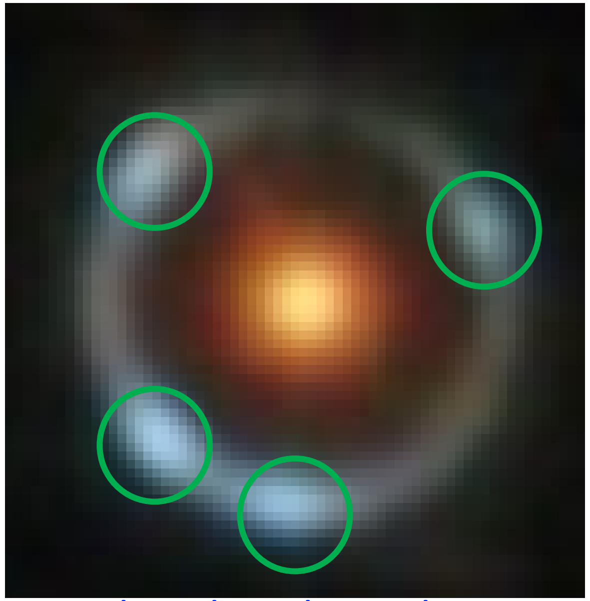


Tewes et al. 2021



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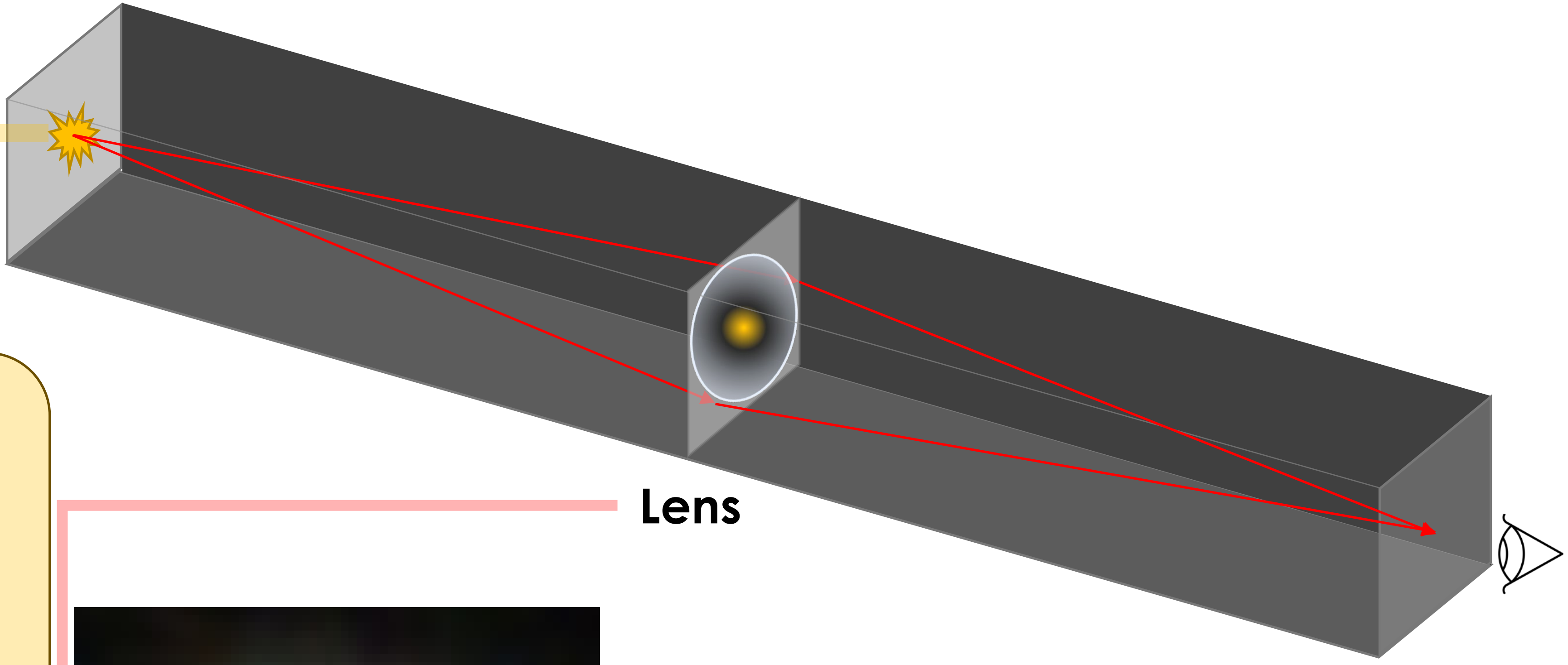

Source:
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Source Images on the lens place:
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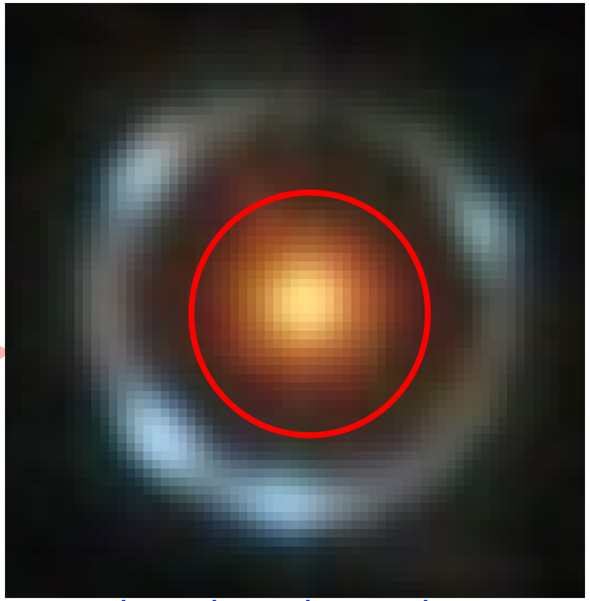
- Time-delays

Lens plane:
- Lens redshift Zl
- Velocity dispersion



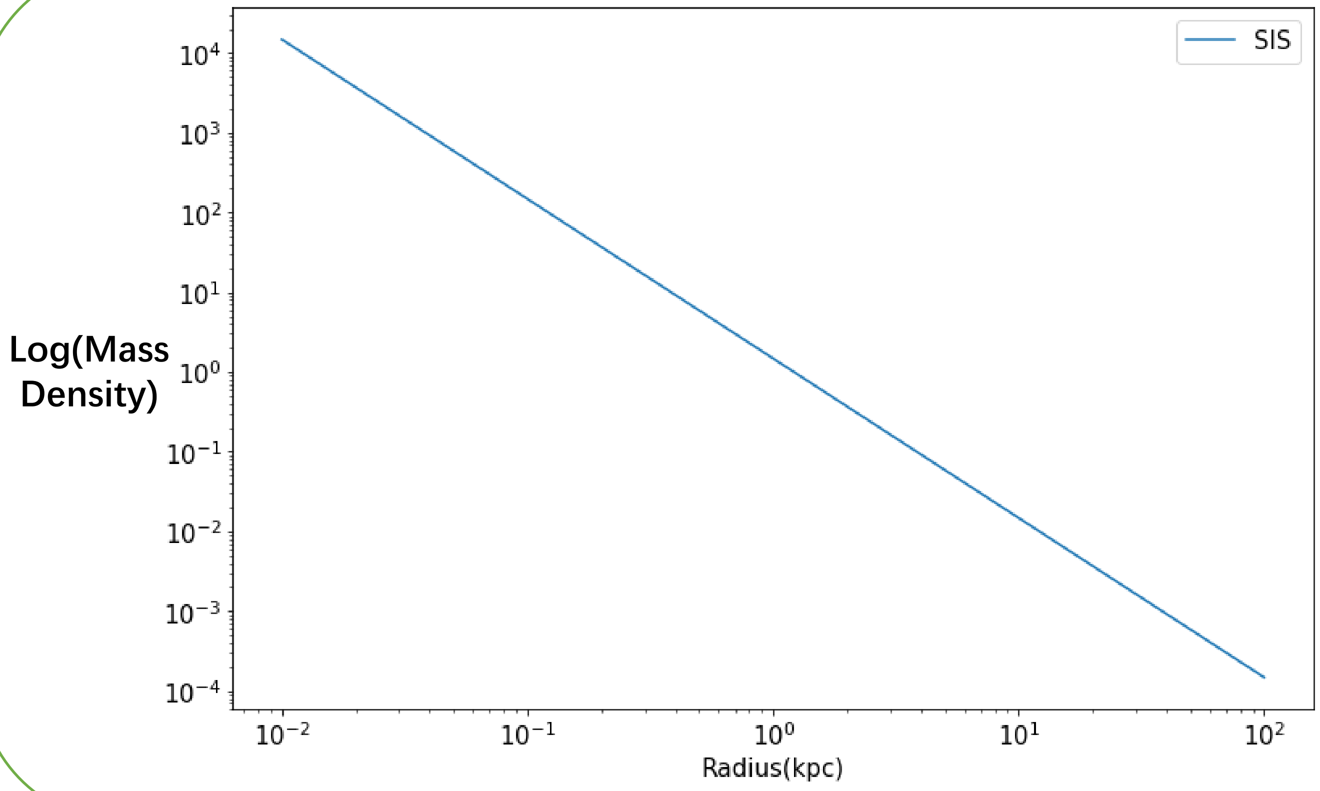
NASA/ESA/CSA/STScI/j. Roger

Lens model

Mass density profile

- Point mass
- Singular Isothermal Sphere (SIS)
- Power-law
- Dark Matter (NFW ···)
- Dark Matter + Luminous Matter

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$$



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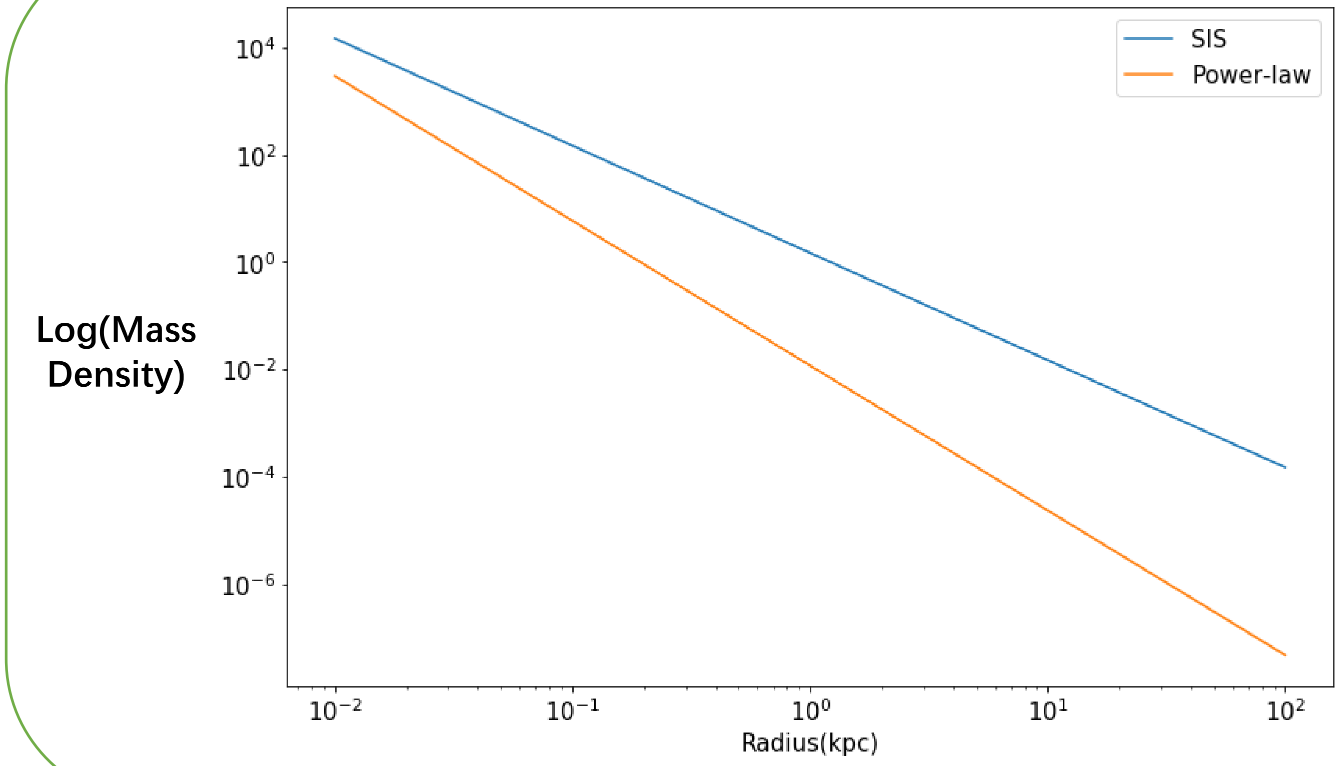
$$\rho(r) \propto r^{-\gamma}$$



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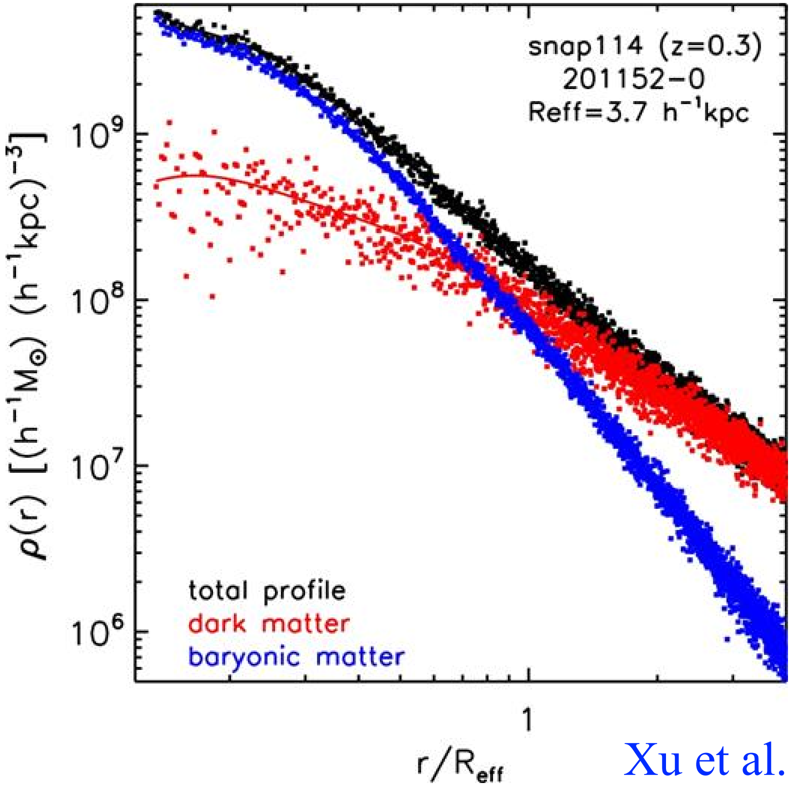
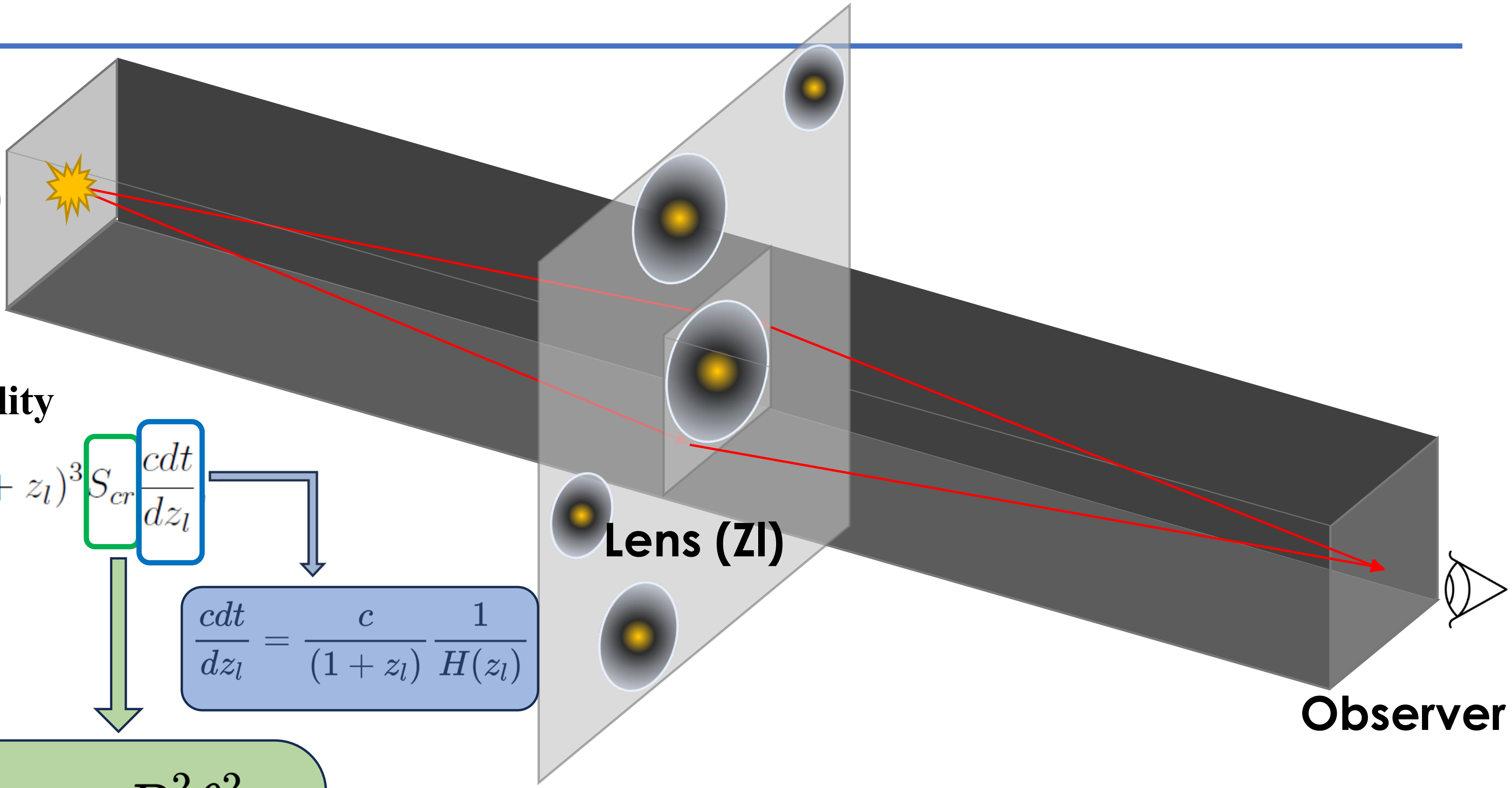


Xu et al. 2017

Optical depth



Lensing Probability

$$\frac{d\tau}{dz_l} = n(\theta_E, z_l)(1+z_l)^3 S_{cr} \frac{cdt}{dz_l}$$

$$\frac{cdt}{dz_l} = \frac{c}{(1+z_l)} \frac{1}{H(z_l)}$$

$$\int_{\Delta\theta-\delta\theta}^{\Delta\theta+\delta\theta} \frac{dn}{d\sigma} \cdot \frac{d\sigma}{d\Delta\theta} d\Delta\theta$$

Velocity dispersion function (VDF):

Modified Schechter Function

$$\frac{dn}{d\sigma} = n_*(\frac{\sigma}{\sigma_*})^\alpha \exp[-(\frac{\sigma}{\sigma_*})^\beta] \frac{\beta}{\Gamma(\alpha/\beta)} \frac{1}{\sigma}$$

$$S_{cr} = \pi D_l^2 \theta_E^2$$

Power-law Model

$$\theta_E = 4\pi\left(\frac{\sigma_{ap}}{c}\right)^2 \frac{D_{ls}}{D_s}\left(\frac{\theta_E}{\theta_{ap}}\right)^{2-\gamma} f(\gamma)$$

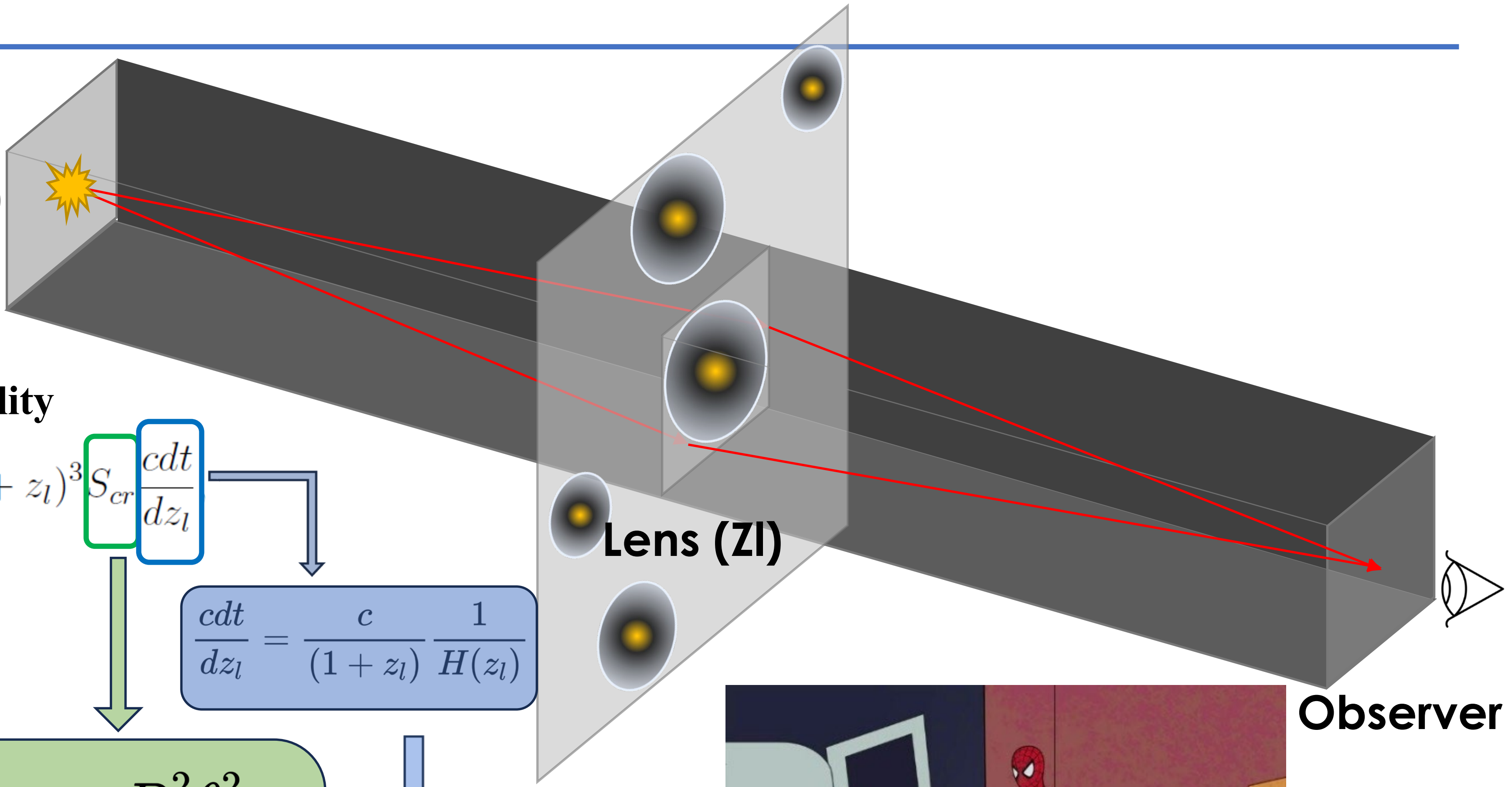
$\Delta\theta$: image separation (Einstein Radius)

σ: velocity dispersion (mass)

$f(\gamma)$: a function of γ (power-law index)

H(z): Hubble parameter

Optical depth



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Motivations

Cosmology

Assuming VDF and Lens model:
Alternative method to study cosmology
- Different systematics
- Wide redshift range

Galaxy Evolution

Assuming Lens model and Get distance:
Constrain the evolution of the galaxy population
- H0 independent
- Supplement to the survey results

Cosmology

Cosmology

FLRW metric

$$ds^2 = c^2dt^2 - a(t)^2[\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2sin^2\theta d\phi^2]$$

Coordinate distance

$$D_{cd}(r_1) = \int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}}$$

$$= \frac{c}{a_0H_0}\int_0^{z_1}\frac{dz}{E(z)}$$

Angular Diameter Distance

$$d_A = a(t_1)r_1 d\theta$$

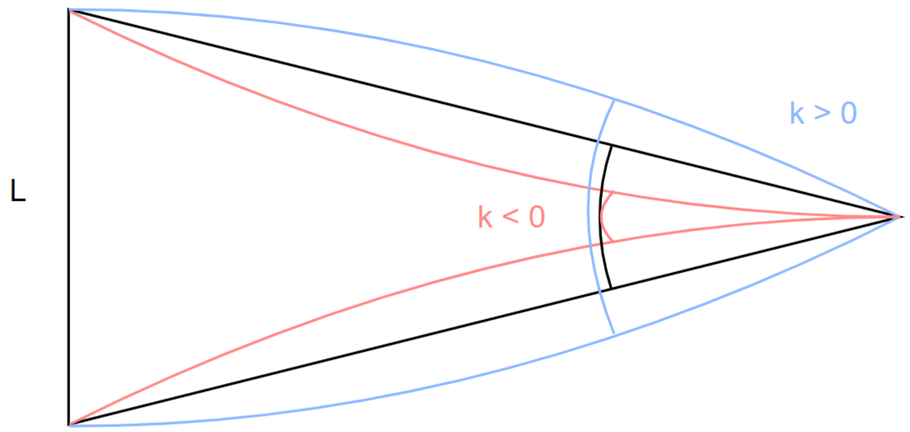
$$D_A = a(t_1)r_1 = \frac{a_0 D_{cd}}{1+z_1}$$

$$= \frac{c}{1+z_1}\int_0^{z_1}\frac{dz}{H_0\cdot E(z)}$$

Cosmology parameters

$$H^2 = H_0^2\left[\Omega_r\frac{a_0^4}{a^4} + \Omega_m\frac{a_0^3}{a^3} + \Omega_k\frac{a_0^2}{a^2} + \Omega_\Lambda\right]$$

$E(z)$



Cosmology

$$\frac{cdt}{dz_l} = \frac{c}{(1+z_l)} \frac{1}{H(z_l)}$$

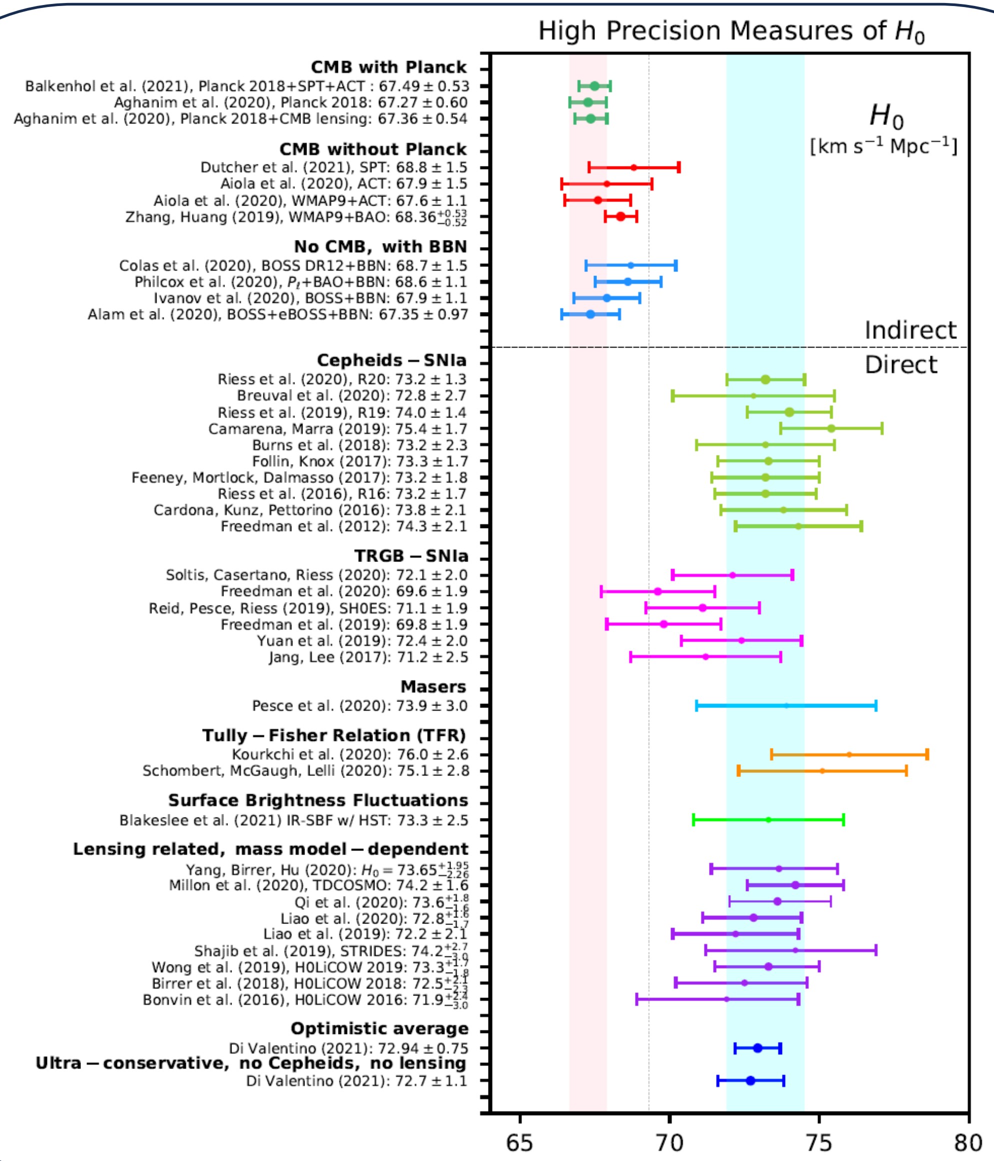
$$\left.\begin{matrix} D_l \\ D_s \\ D_{ls} \end{matrix}\right\} D_A = \frac{c}{1+z_1} \int_0^{z_1} \frac{dz}{H_0 \cdot E(z)}$$

$$\frac{D_{ls}}{D_s} = \frac{\theta_E}{4\pi} \frac{c^2}{\sigma_{ap}^2} \left(\frac{\theta_E}{\theta_{ap}}\right)^{\gamma-2} f(\gamma)^{-1}$$

$$\Delta t = \frac{1+z_l}{c} \frac{D_{ol} D_{os}}{D_{ls}} \left[\frac{(\vec{\theta} - \vec{\beta})^2}{2} - \phi(\vec{\theta}) \right]$$

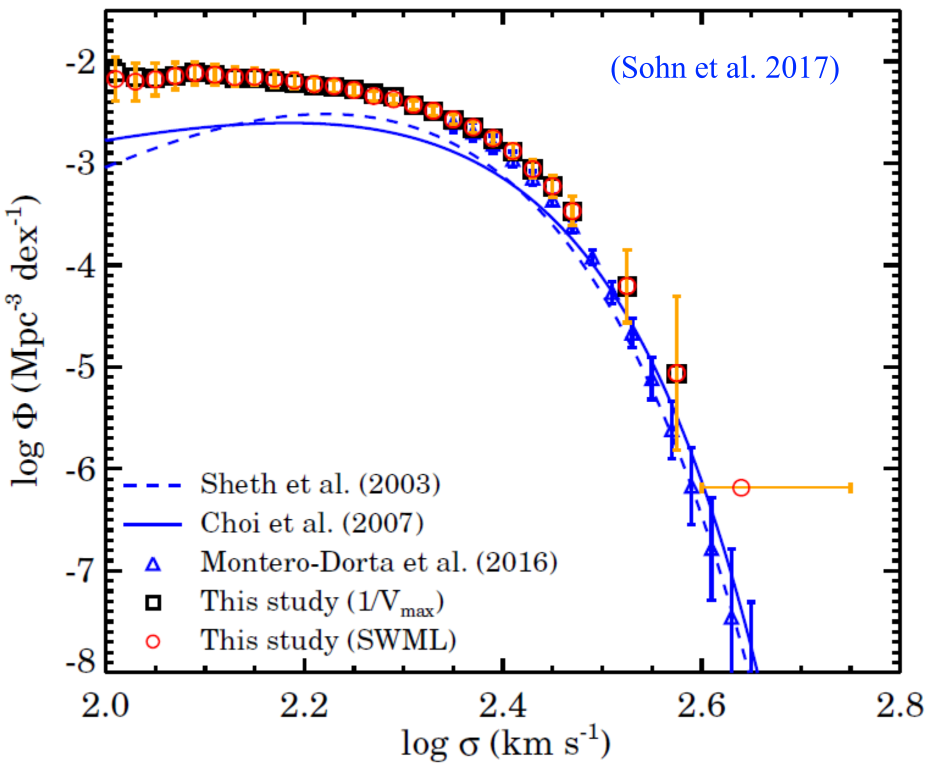
… …

Di Valentino et al. 2021



Galaxies

Galaxies



Velocity dispersion function:

Modified Schechter Function (Sheth et al. 2003)

$$\frac{dn}{d\sigma} = n_*\left(\frac{\sigma}{\sigma_*}\right)^{\alpha}\exp\left[-\left(\frac{\sigma}{\sigma_*}\right)^{\beta}\right]\frac{\beta}{\Gamma(\alpha/\beta)}\frac{1}{\sigma}$$

Characterisitc parameters

$\boldsymbol{n_*}$: characteristic number density

$\boldsymbol{\sigma_*}$: characteristic velocity dispersion

Shape parameters

α: low-velocity power-law index

β: high-velocity cut-off index

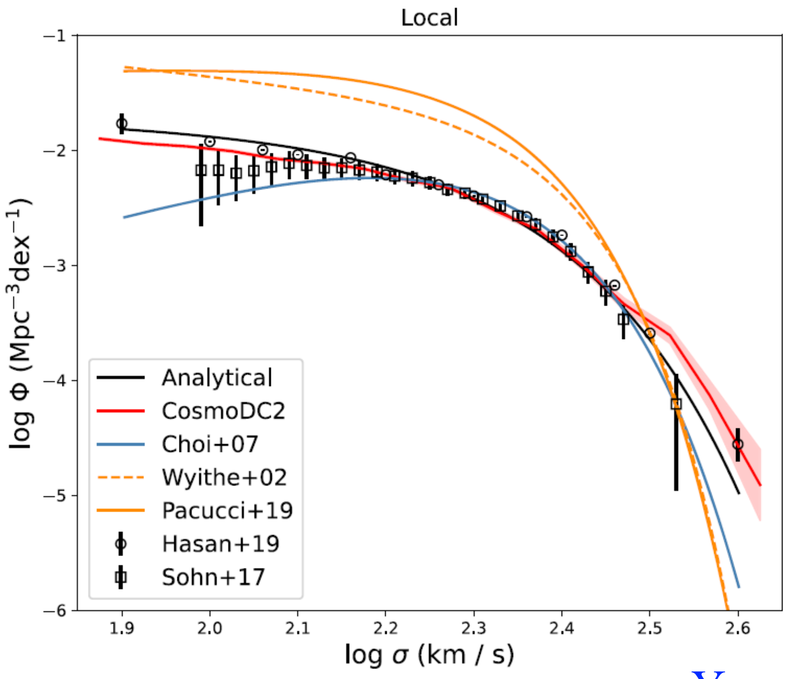
Galaxies

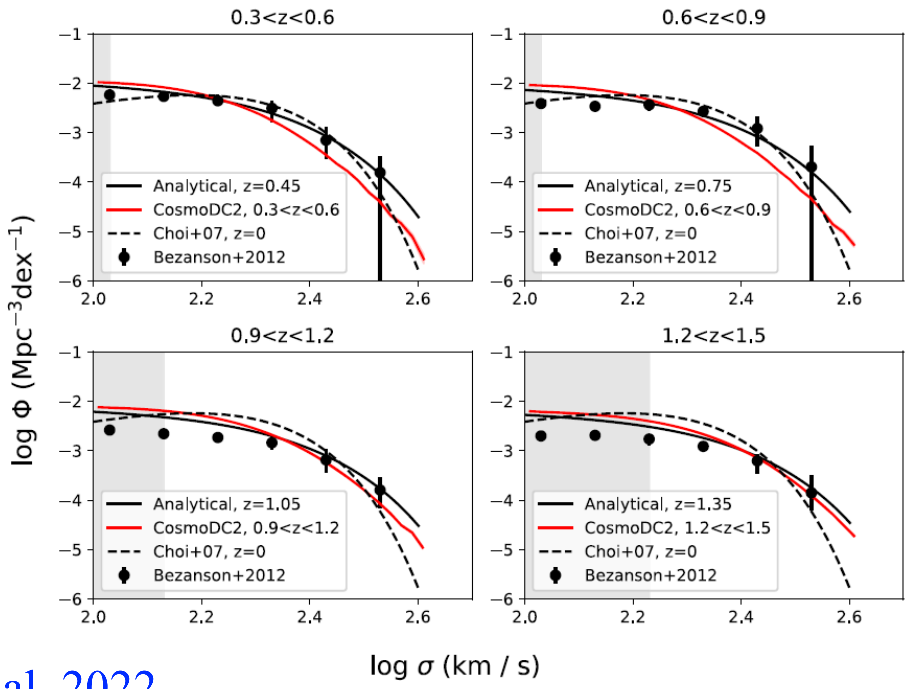
(Chae & Mao 2003)

$$n_*(z_l) = n_*(1+z_l)^{\nu_n}$$
$$\sigma_*(z_l) = \sigma_*(1+z_l)^{\nu_v}$$

(Ofek et al. 2003)

$$n_*(z_l) \rightarrow n_* 10^{P z_l}$$
$$\sigma_*(z_l) \rightarrow \sigma_* 10^{Q z_l}$$





Yue et al. 2022

Redshift test

Lensing Probability

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$$\theta_E = 4\pi \left(\frac{\sigma_{ap}}{c}\right)^2 \frac{D_{ls}}{D_s} \left(\frac{\theta_E}{\theta_{ap}}\right)^{2-\gamma} f(\gamma)$$

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$$\theta_{E*} = \left(4\pi(\frac{\sigma_*}{c})^2 \frac{D_{ls}}{D_s} (\frac{\theta_{eff}}{2\theta_{ap}})^{0.08} {\theta_{ap}}^{\gamma-2} f(\gamma)\right)^{\frac{1}{\gamma-1}}$$

$$\left(\frac{\theta_E}{\theta_{E*}}\right) = \left(\frac{\sigma}{\sigma_*}\right)^{\frac{2}{\gamma-1}}$$

$$n(\theta_E, z_l)$$

$$\delta p = \frac{d\tau}{dz_l} / \tau = \frac{d\tau}{dz_l} / \int_0^{z_s} \frac{d\tau}{dz_l} dz_l$$

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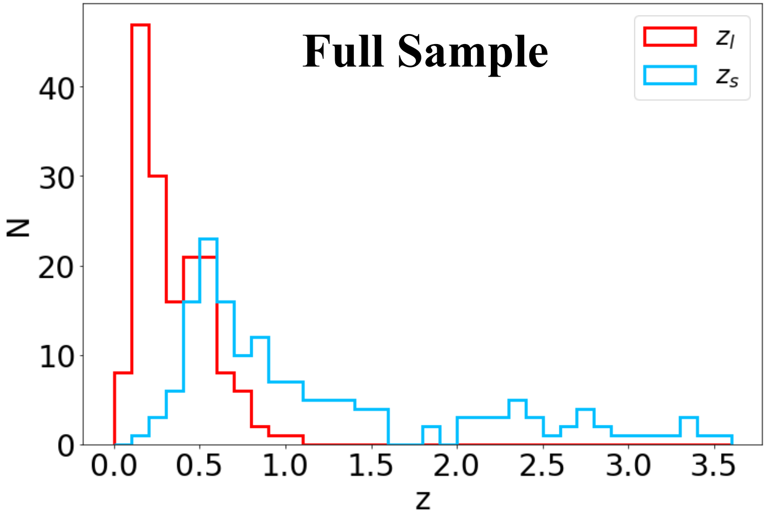
Galaxy global properties and their redshift evolution

Dataset



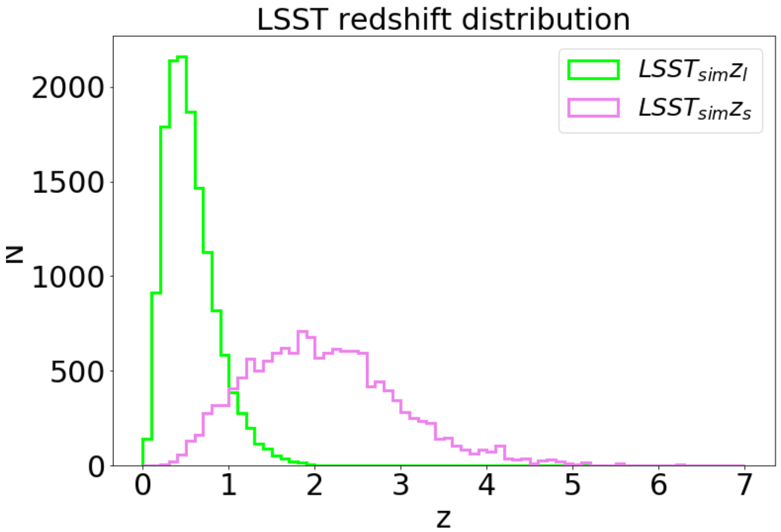
A Full Sample

97 from SLACS and S4TM, 25 from BELLS and BELLS GALLERY, 31 from SL2S, and 5 from LSD

B Subsample

SLACS and S4TM + BELLS and BELLS GALLERY (126)

(Chen et al. 2019)



S Simulation data

LSST

The Legacy Survey of Space and Time

Cosmology

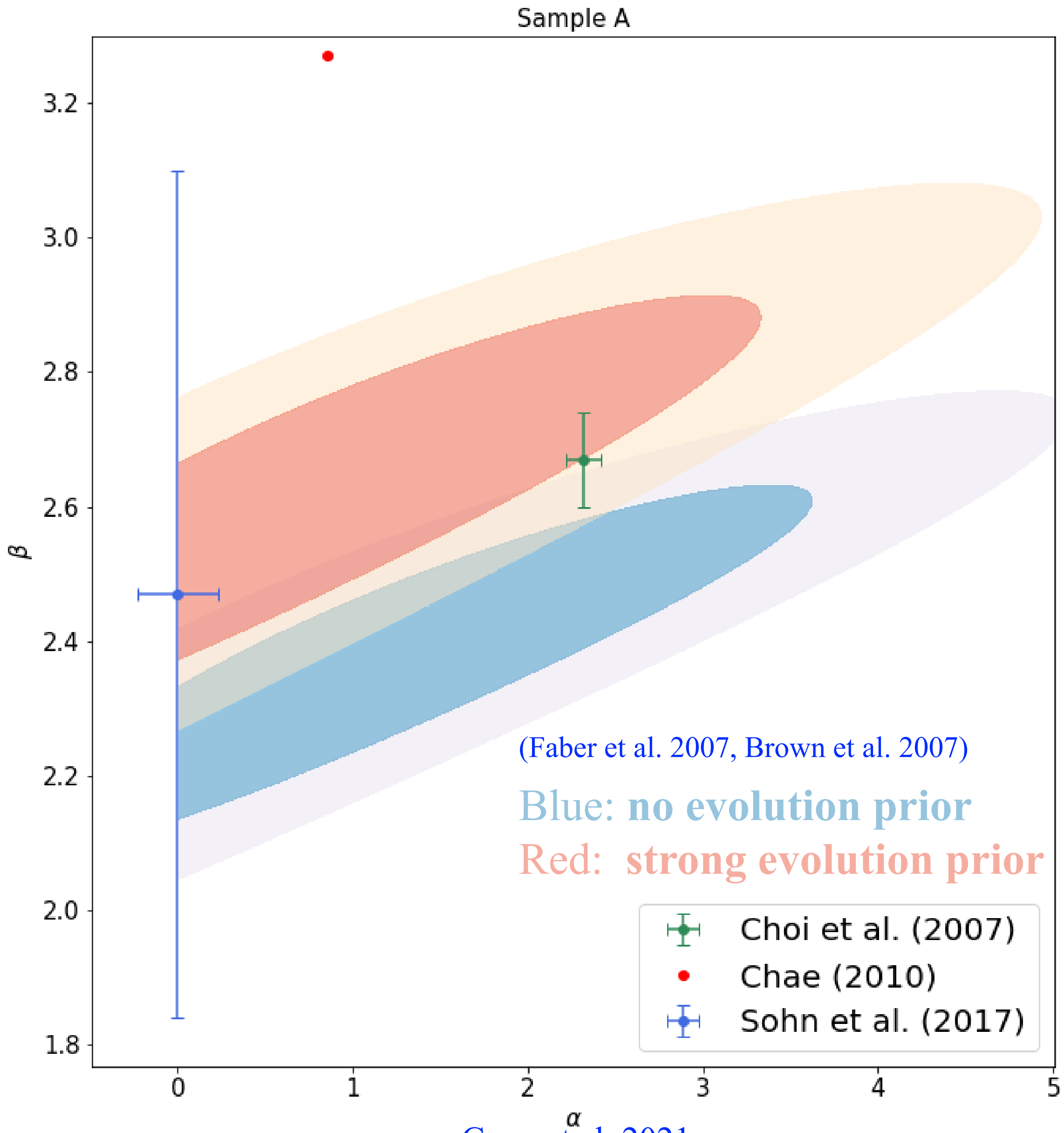
Dark Matter

Dark Energy

Huge amount of data (15 TB per night)

(Collett 2015)

Results



Geng et al. 2021

arXiv: 2102.12140 / DOI: 10.1093/mnras/stab519

Green cross

$(\sigma_*, \ \alpha, \ \beta)_{\rm DR5} = [161 \pm 5 \ \ {\rm km/s},$

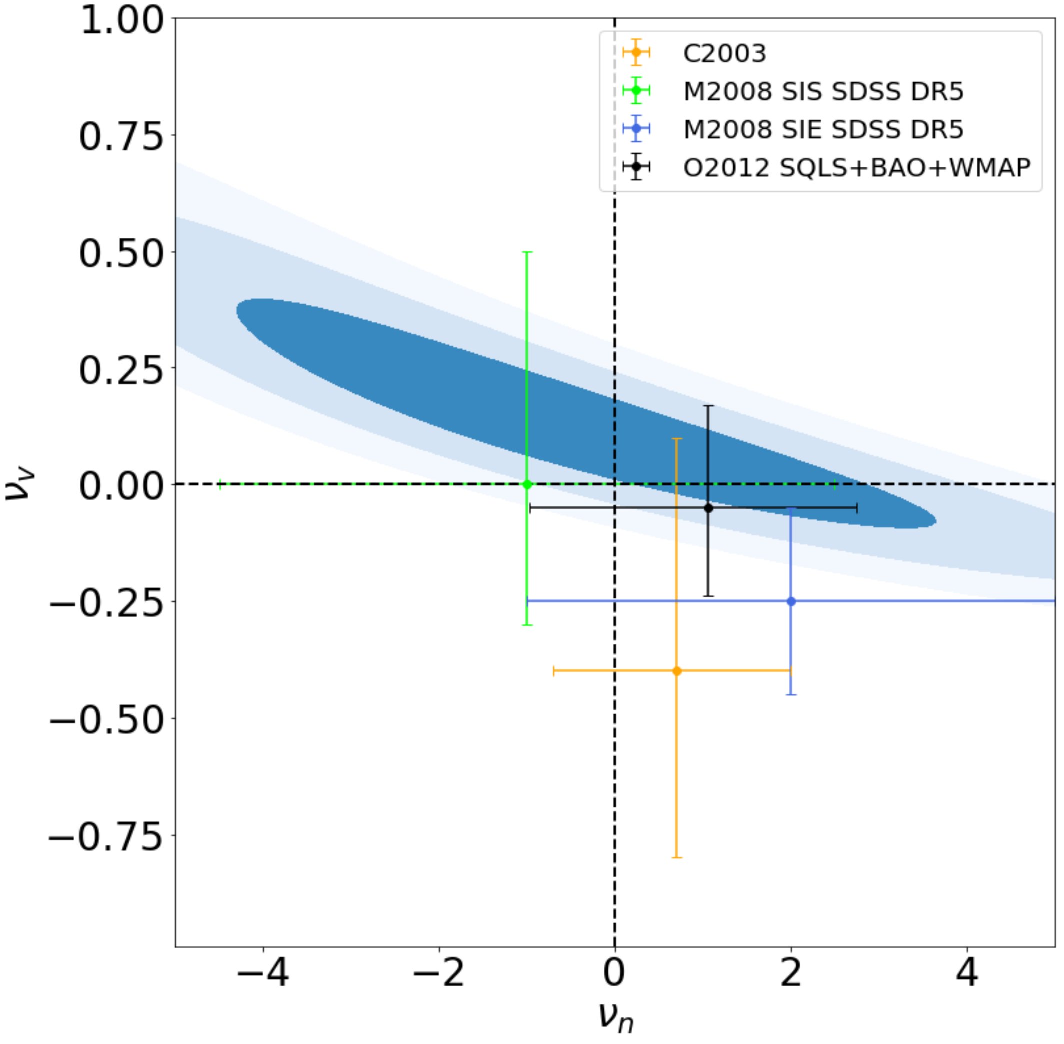
(Choi et al. 2007) $2.32 \pm 0.10, \ \ 2.67 \pm 0.07]$

Results:

Prior	α	β
$\nu_n = \nu_v = 0:$	$0.66^{+2.13}_{-0.66}$	$2.28^{+0.24}_{-0.18}$
$\nu_n = -1.0, \nu_v = 0.25:$	$0.45^{+2.38}_{-0.45}$	$2.55^{+0.28}_{-0.20}$

Taking the strong evolving prior, the constraints on the shape parameters of the VDF are in good agreement with the results obtained from the survey (SDSS DR5)

Results



Geng et al. 2021
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$$n_*(z_l) = n_*(1+z_l)^{\nu_n}, \qquad \nu_n = -1.18$$

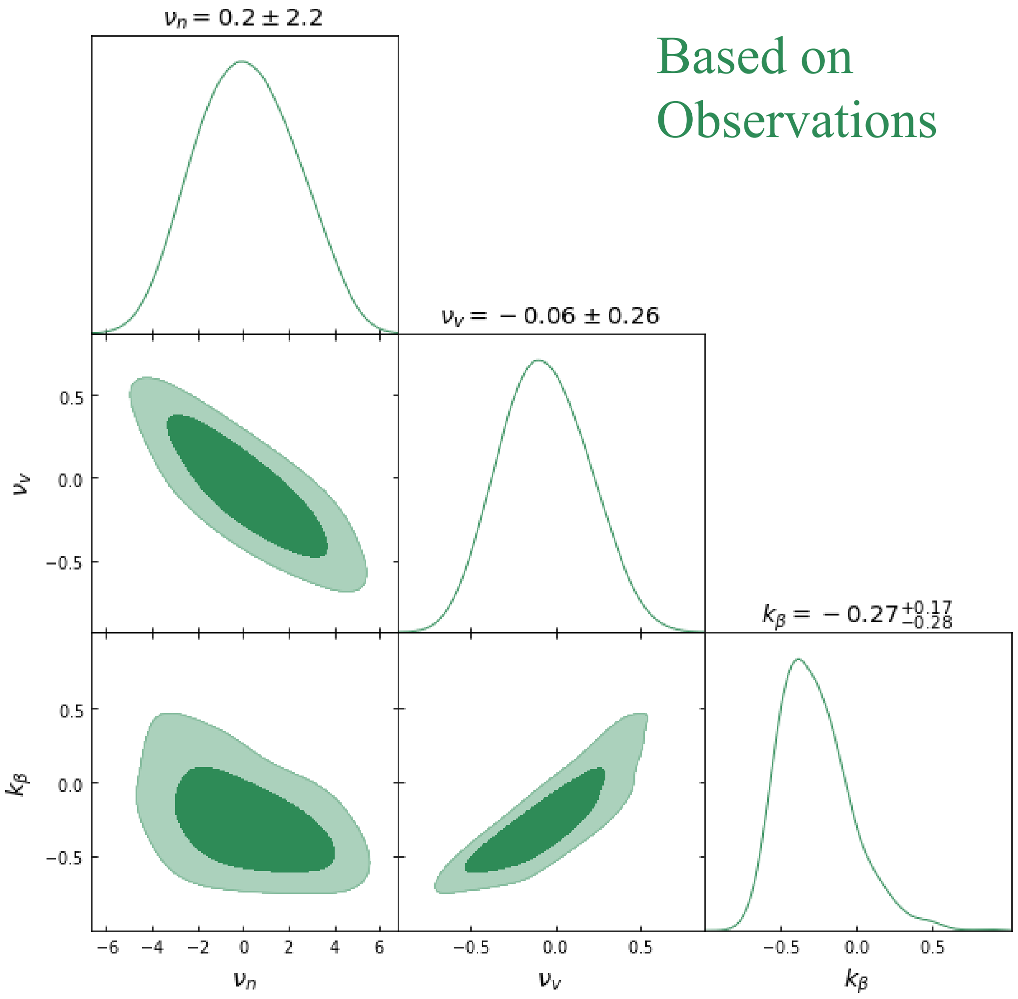
$$\sigma_*(z_l) = \sigma_*(1+z_l)^{\nu_v}, \qquad \nu_v = 0.18$$

(Chae & Mao 2003)

Under different semi-analytic models, the observational data combined with statistical methods of strong gravitational lensing redshift distributions tend towards a strong redshift evolution of the number density (n_*) of early-type galaxies.

The number density of early-type galaxies would double from z~1 to the local universe, according our constraints.

Results



Based on Observations

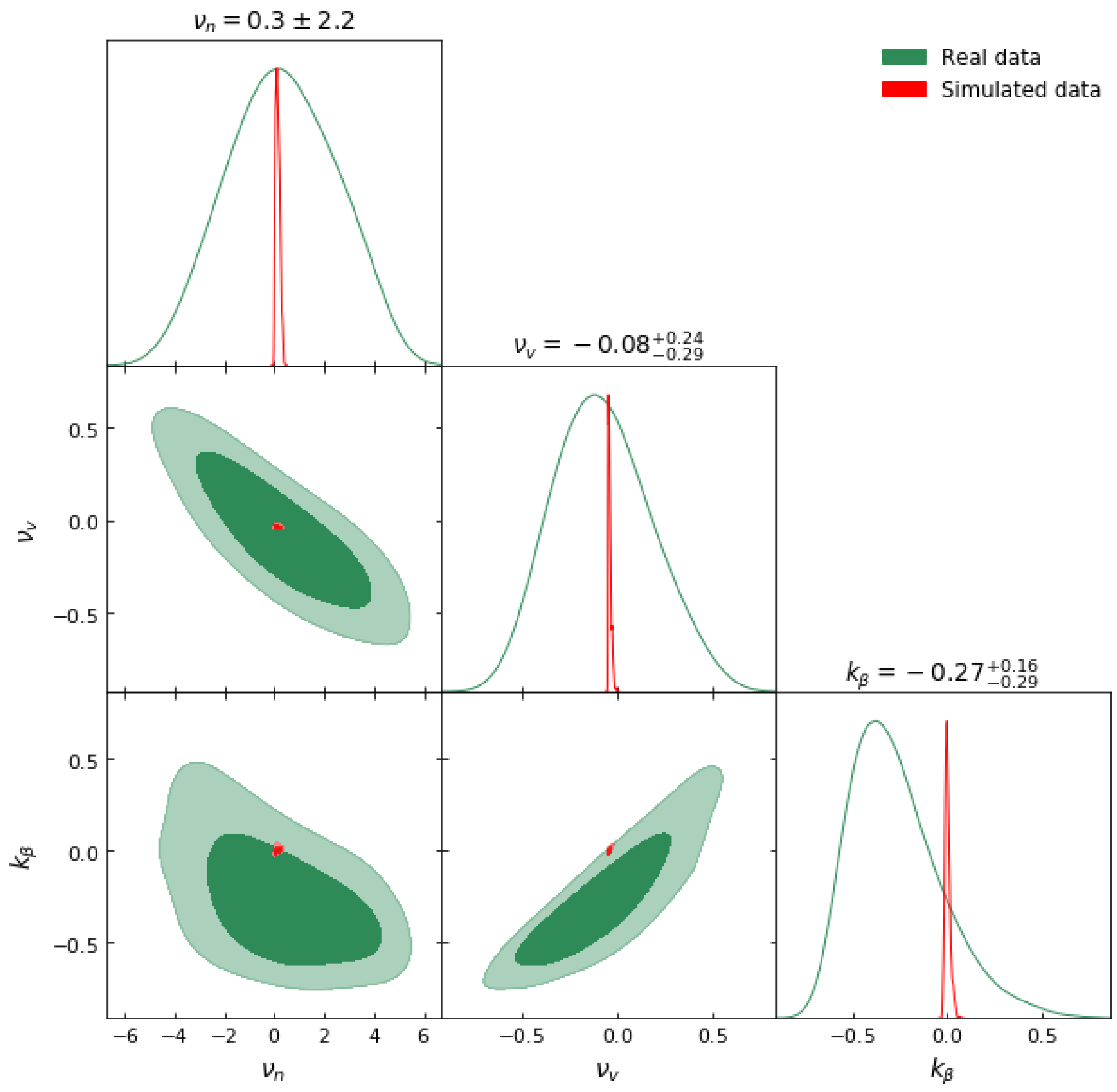
Geng et al. 2021
arXiv: 2102.12140 / DOI: 10.1093/mnras/stab519

$$\alpha \rightarrow \alpha \left(1 + k_\beta \frac{z_l}{1+z_l}\right), \ \beta \rightarrow \beta \left(1 + k_\beta \frac{z_l}{1+z_l}\right).$$

Including the redshift evolution of shape parameters, we found:

- The non-evolution case was not excluded
- There are strong degeneracies between redshift evolution parameters.

Results



Including the redshift evolution of shape parameters, we found:

- Non-evolution cases were not excluded
- There are strong degeneracies between redshift evolution parameters.
- Next generation of wide and deep sky surveys (such as LSST) may improve the precision by an order of magnitude.

Geng et al. 2021
arXiv: 2102.12140 / DOI: 10.1093/mnras/stab519

Cosmology constraints – Ωm

Flat Λ Cold Dark Matter (ΛCDM) model

$$D_A^{\Lambda CDM}(z_1, z_2) = \frac{c}{H_0(1+z_2)} \int_{z_1}^{z_1} \frac{dz}{\sqrt{\Omega_m(1+z)^3 + 1 - \Omega_m}}$$

Flat Dvali–Gabadadze–Porrati (DGP) model

$$D_A^{DGP}(z_1, z_2) = \frac{c}{H_0(1+z_2)} \int_{z_1}^{z_1} \left[\sqrt{\Omega_m(1+z)^3 + \Omega_{r_e}} + \sqrt{\Omega_{r_e}}\right]^{-1} dz.$$

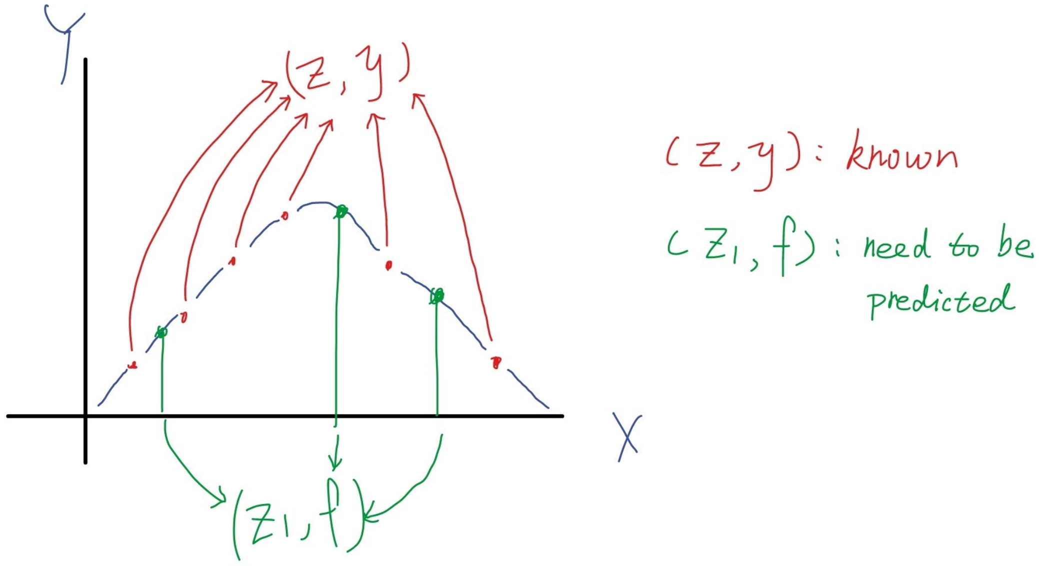
Theoretical predictions

$$\frac{D_{ls}}{D_s} = \frac{\theta_E}{4\pi} \frac{c^2}{\sigma_{ap}^2} \left(\frac{\theta_E}{\theta_{ap}}\right)^{\gamma-2} f(\gamma)^{-1}$$

From observations

Gaussian process regression

Data		Points' positions	
y	given data	Z	data y measured at a set of points Z
f	data we want to predict	Z1	points position corresponding to f



$$\begin{bmatrix} \mathbf{y} \\ \mathbf{f} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mathbf{m}(\mathbf{Z}) \\ \mathbf{m}(\mathbf{Z}_1) \end{bmatrix}, \begin{bmatrix} K_y(Z, Z) & K(Z, Z_1) \\ K(Z_1, Z) & K(Z_1, Z_1) \end{bmatrix} \right)$$

Gaussian process regression

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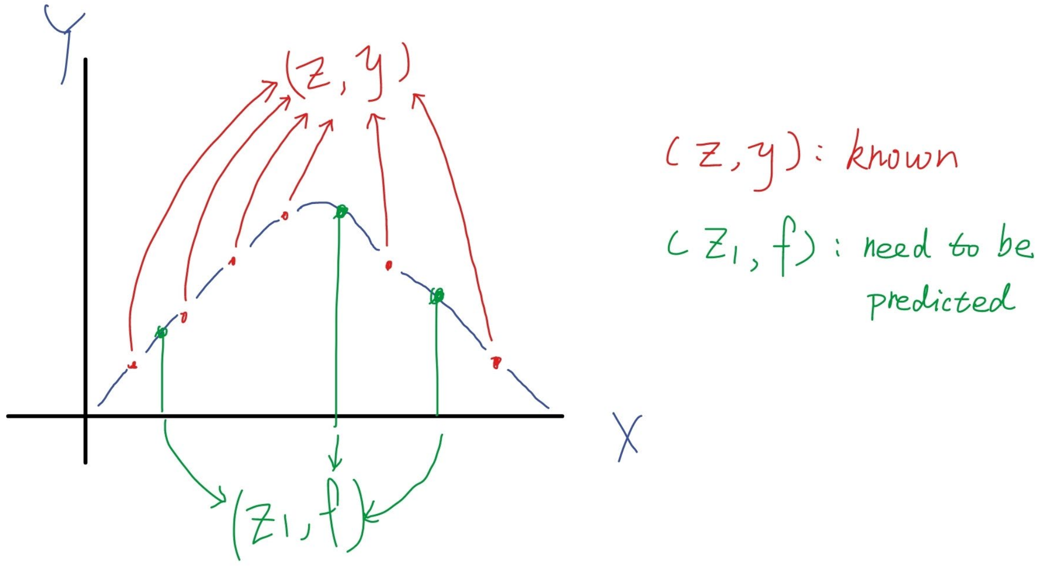

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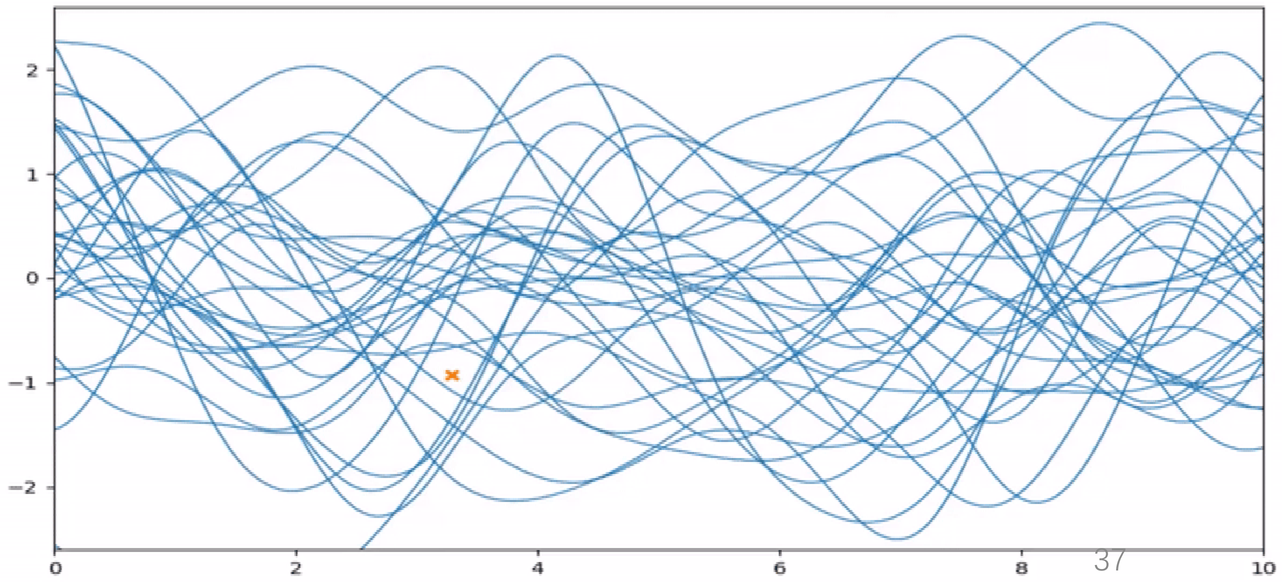

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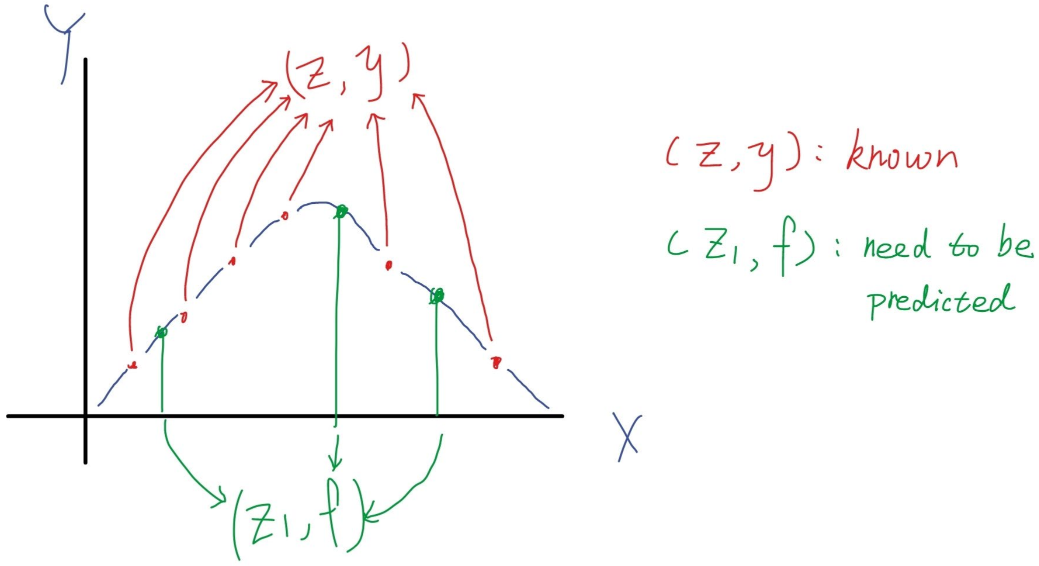

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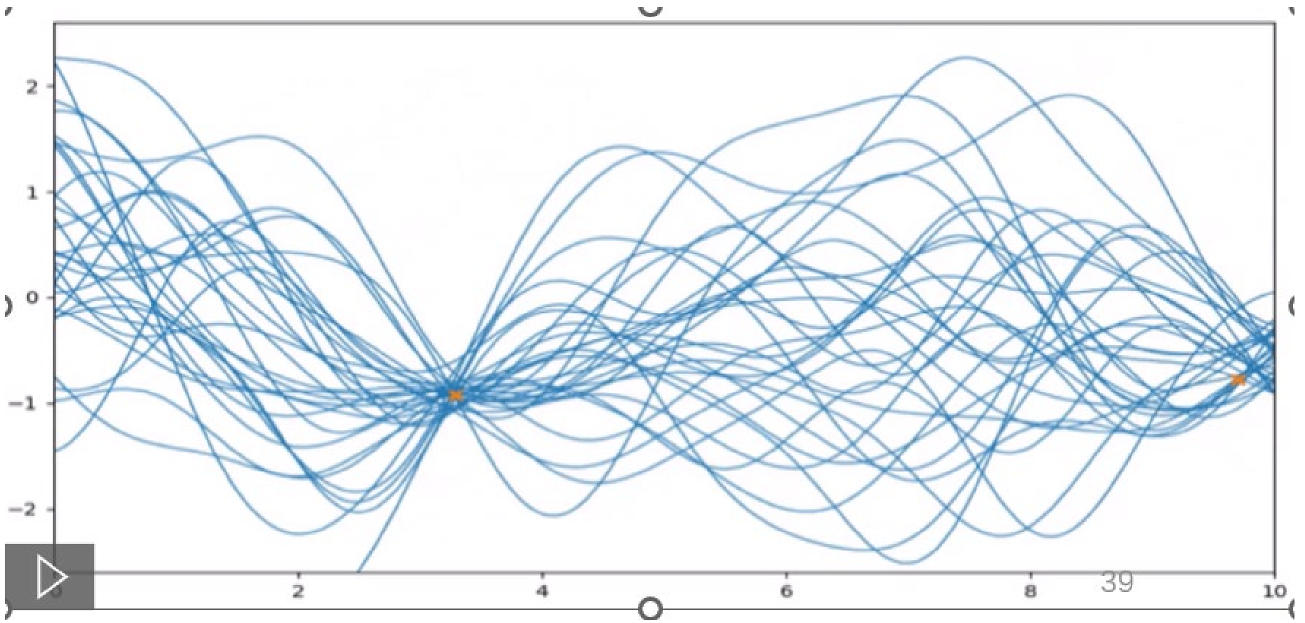

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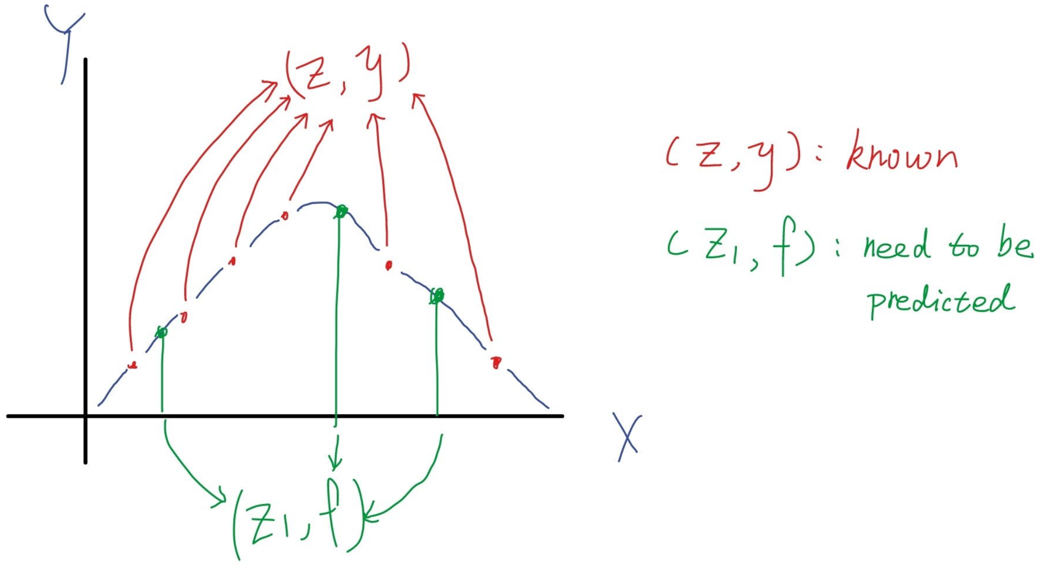

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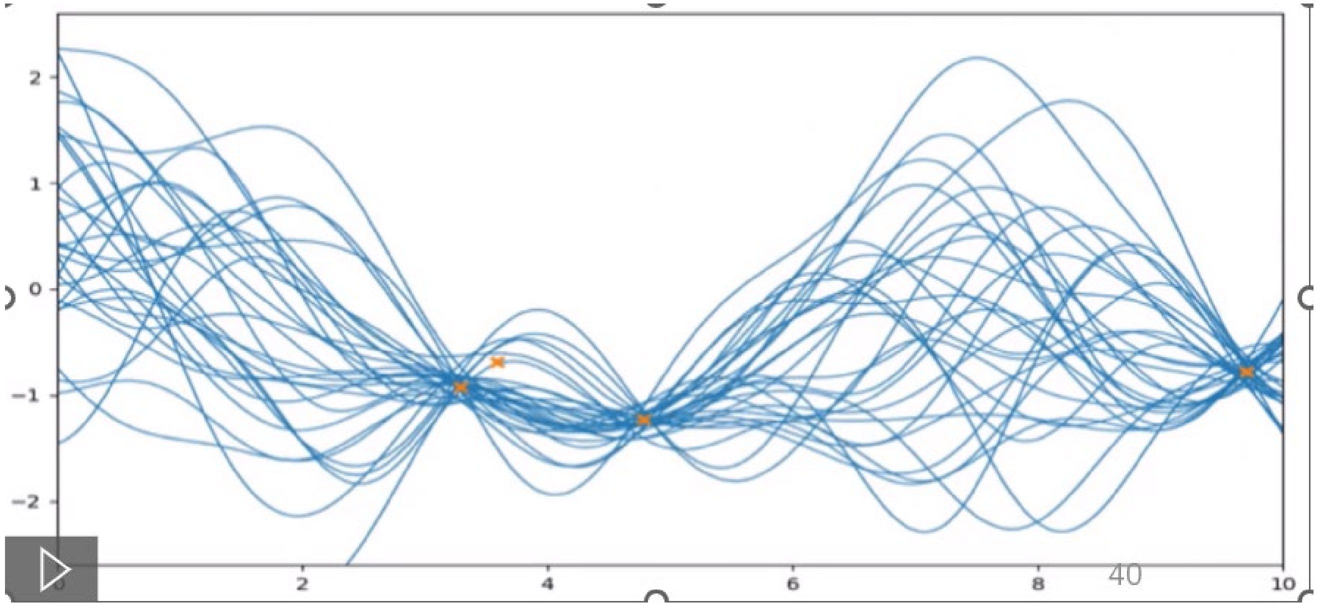

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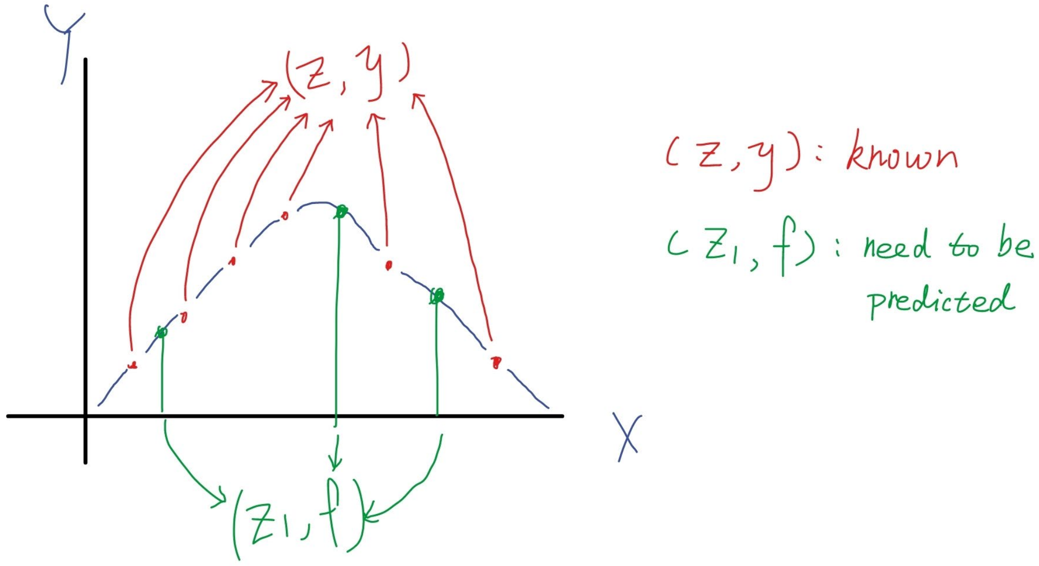

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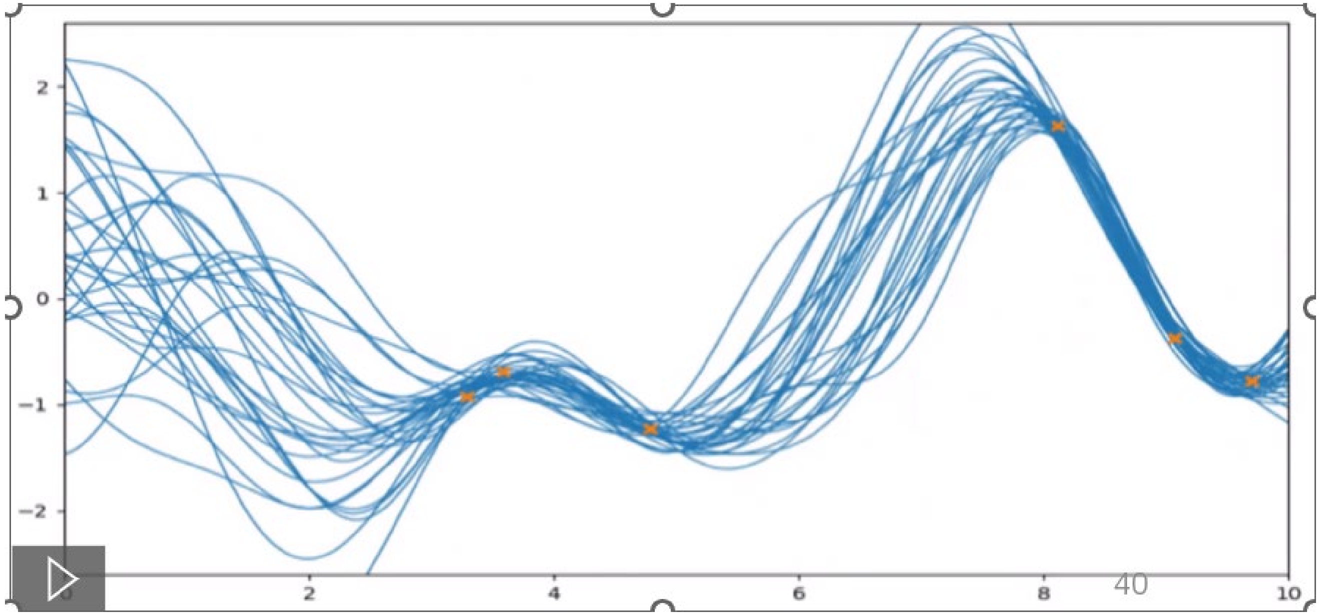

Gaussian process regression

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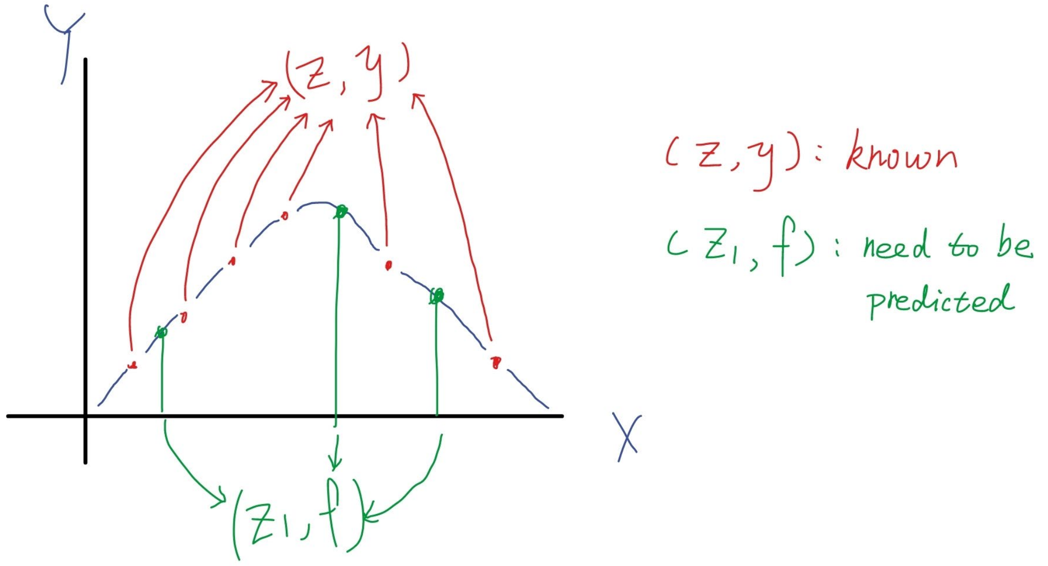

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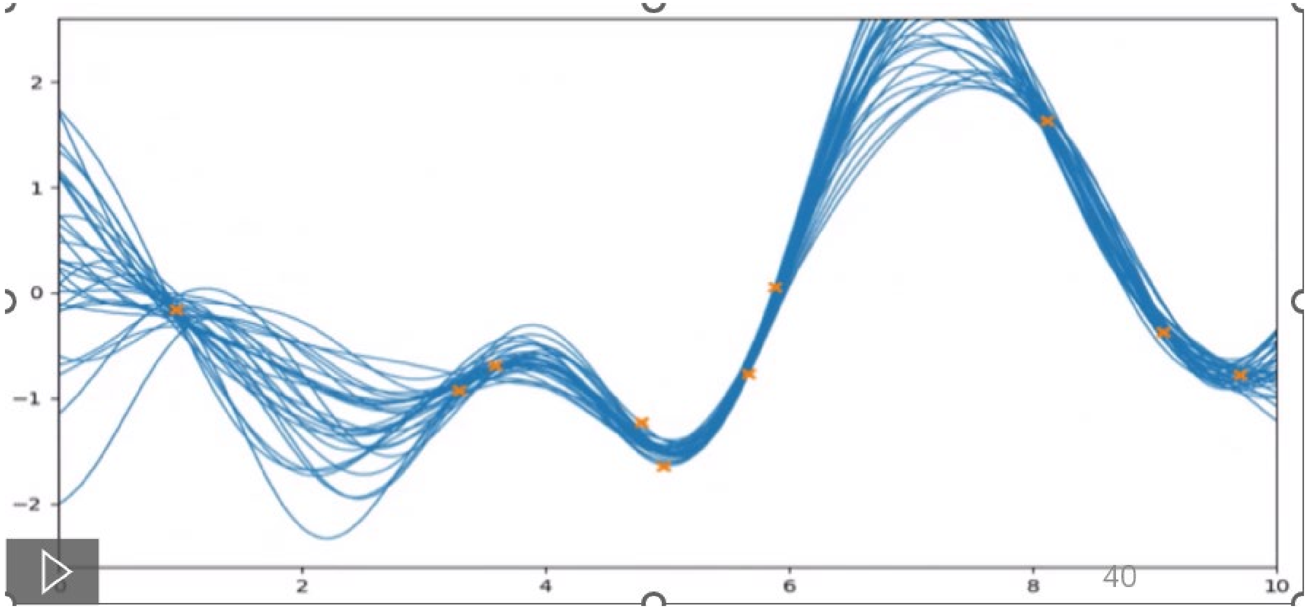
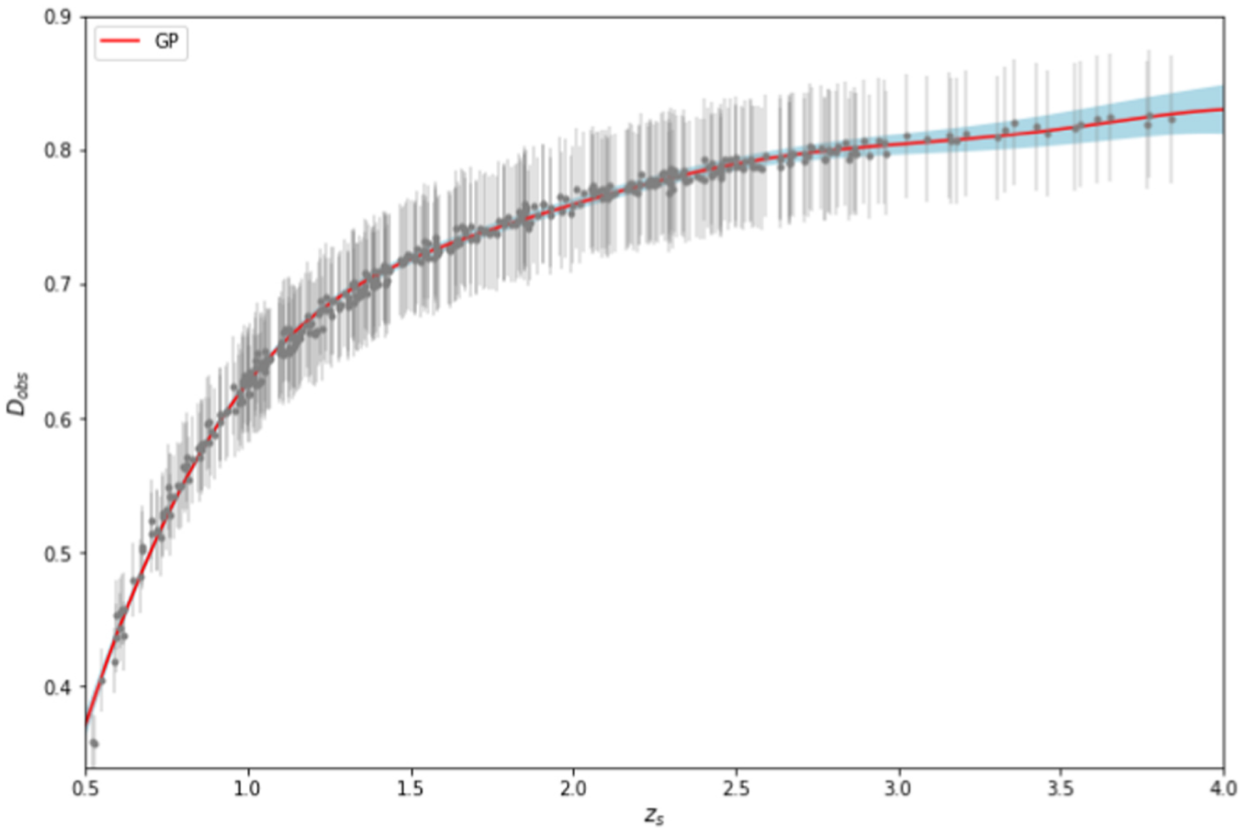


Cosmology constraints – Ωm

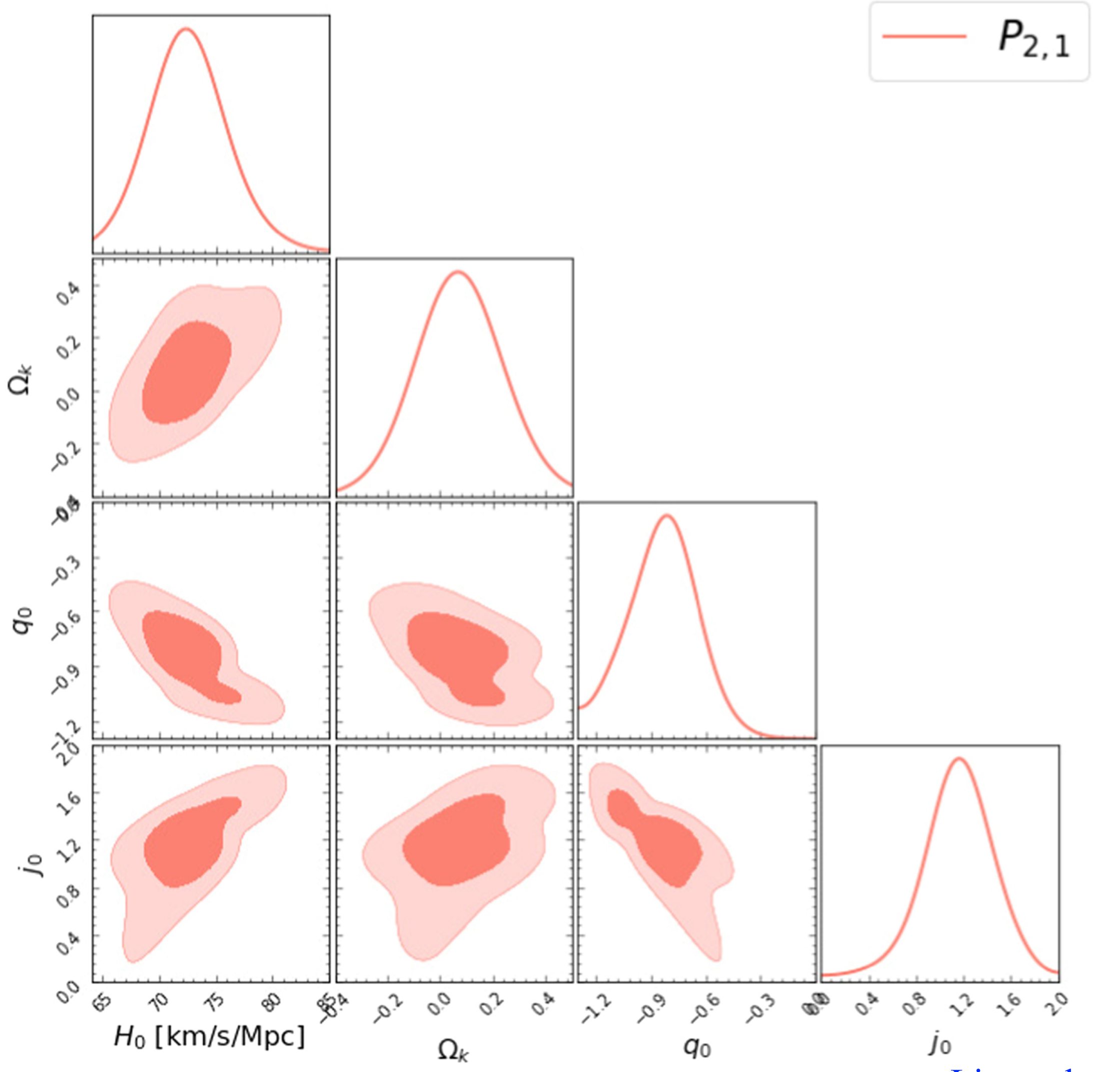
GP

- Simulated strong lensing systems from forthcoming Rubin Observatory's Legacy Survey of Space and Time (LSST)
- Precision of $\Delta\Omega_m$~0.015 in the concordance ΛCDM model (comparable constraint on Ω_m with Planck 2015)



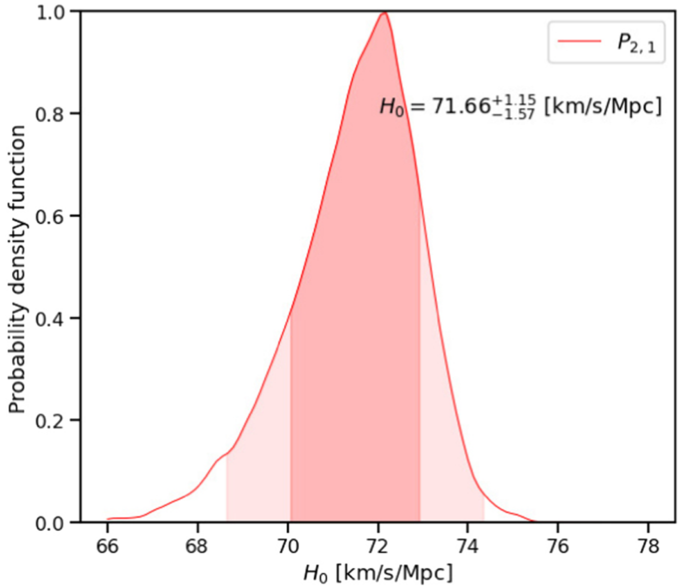
Liu et al. 2022 ApJ 939 37
DOI: 10.3847/1538-4357/ac93f3

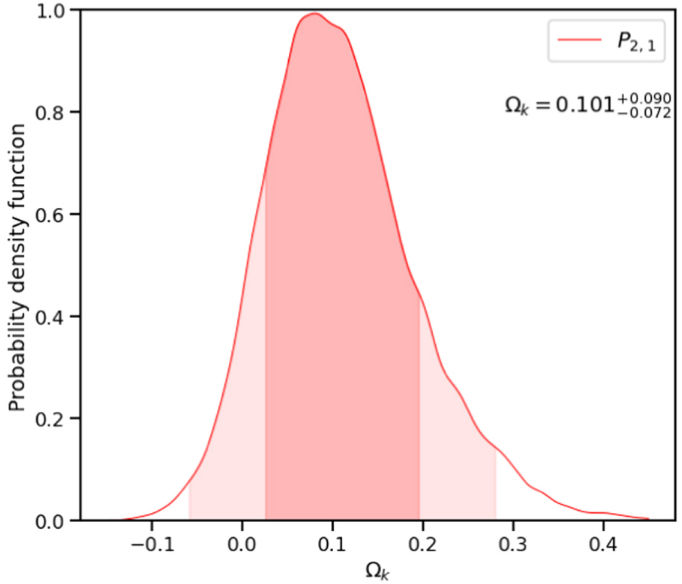
Cosmology constraints – H0, Ωk



Time-delay distance

$$\Delta t = \frac{1+z_l}{c}\frac{D_{ol}D_{os}}{D_{ls}}\left[\frac{(\vec{\theta}-\vec{\beta})^2}{2} - \phi(\vec{\theta})\right]$$



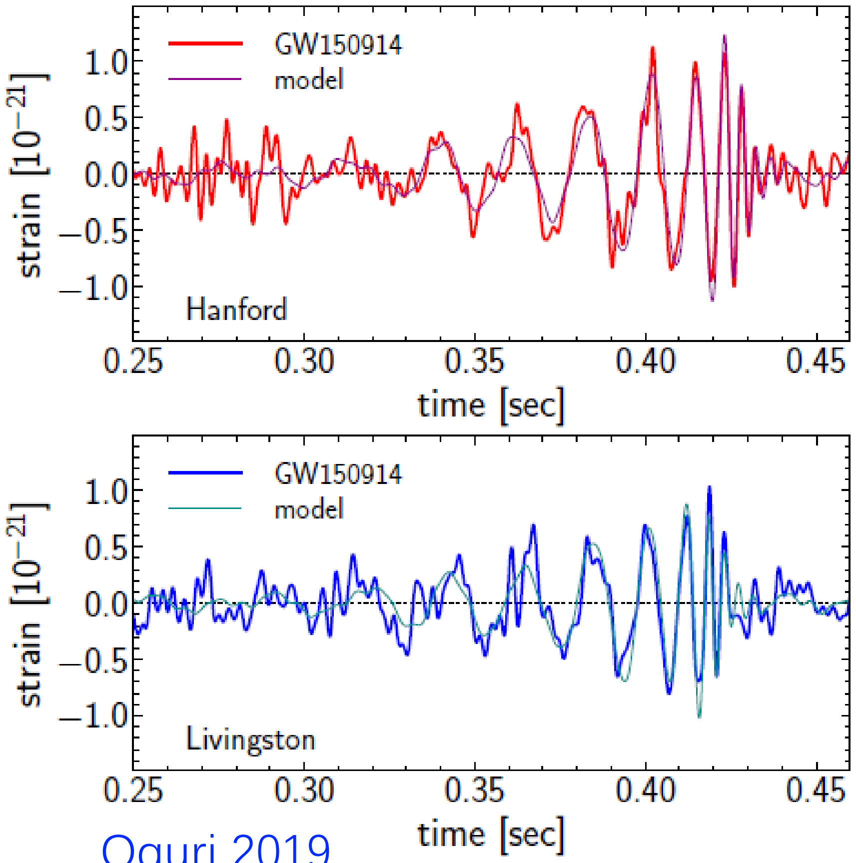


Liu et al. 2022 ApJ 939 37

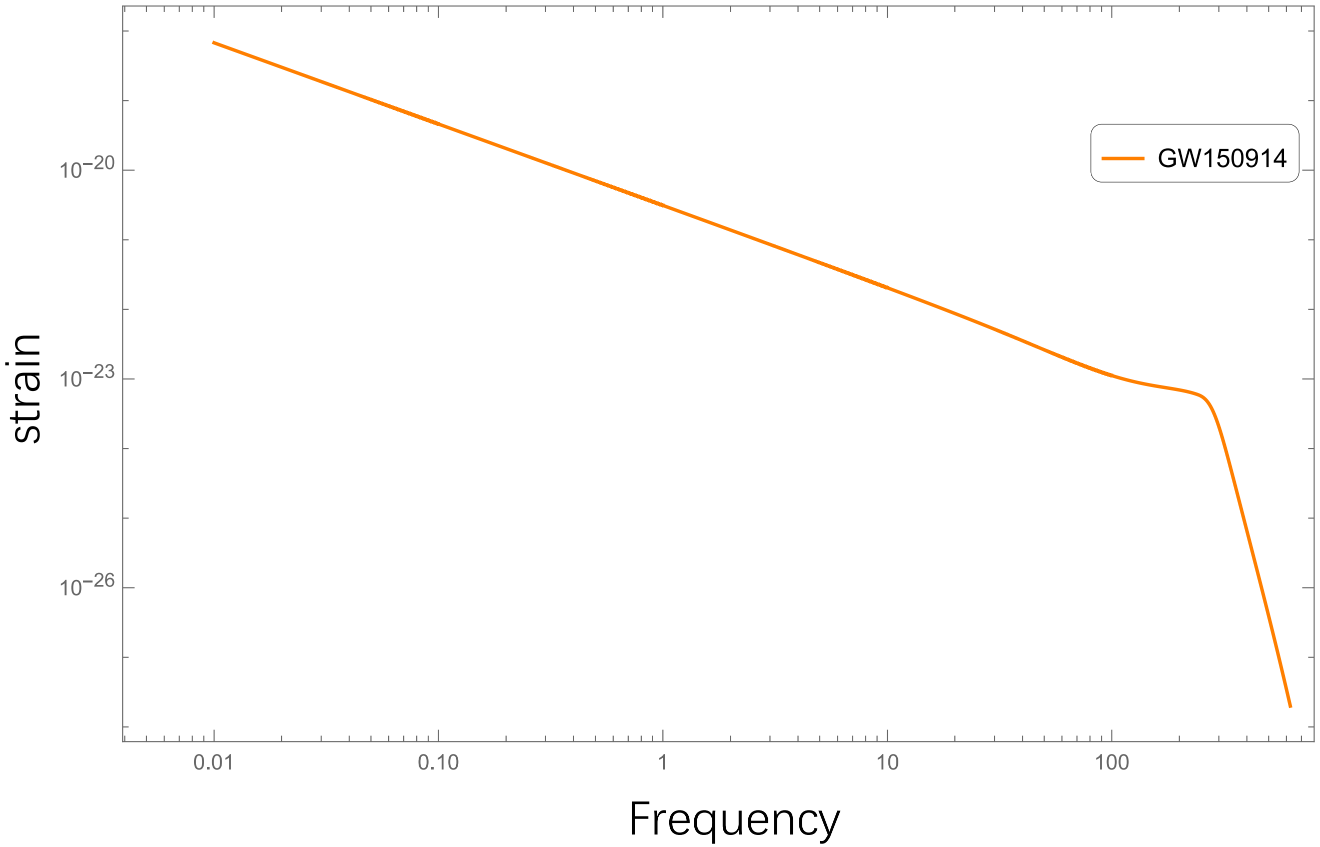
DOI: 10.3847/1538-4357/ac93f3

Future perspective

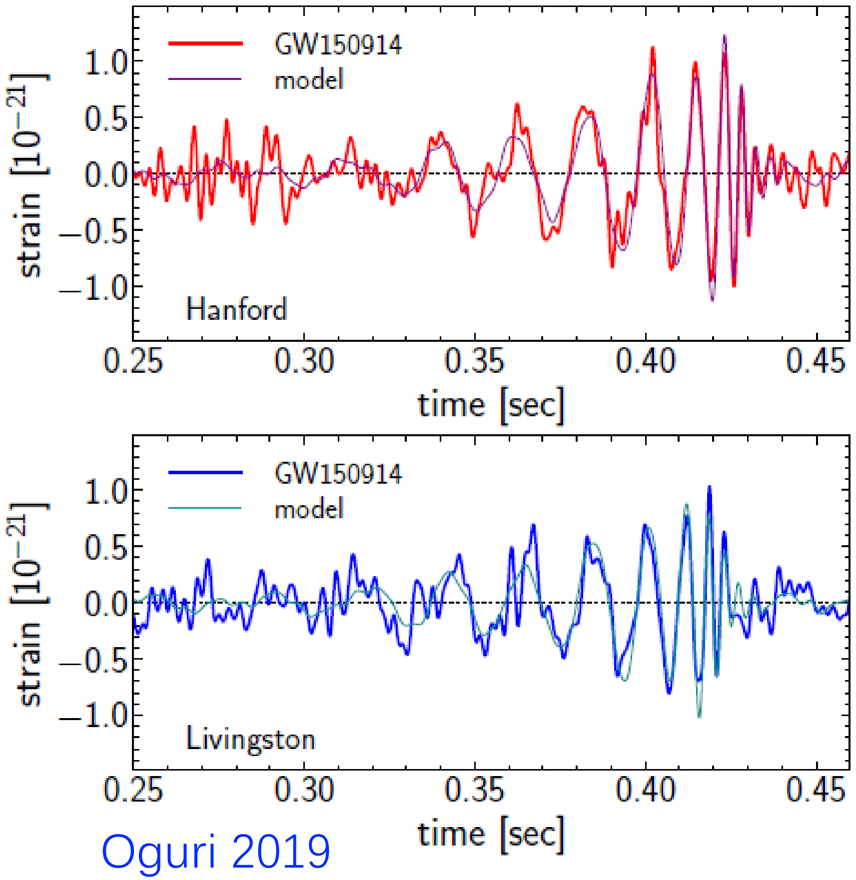
Gravitational Waves (GW)



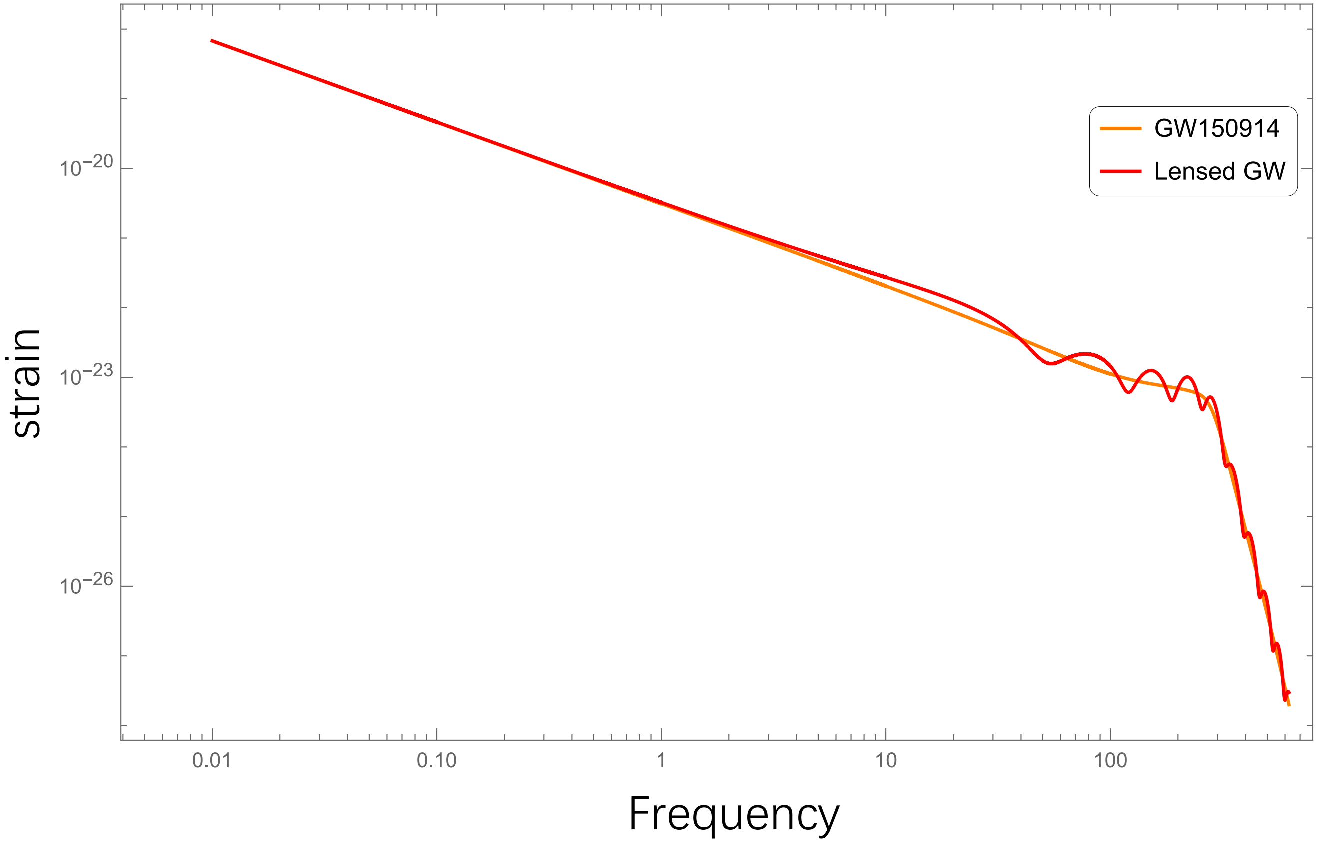
Oguri 2019



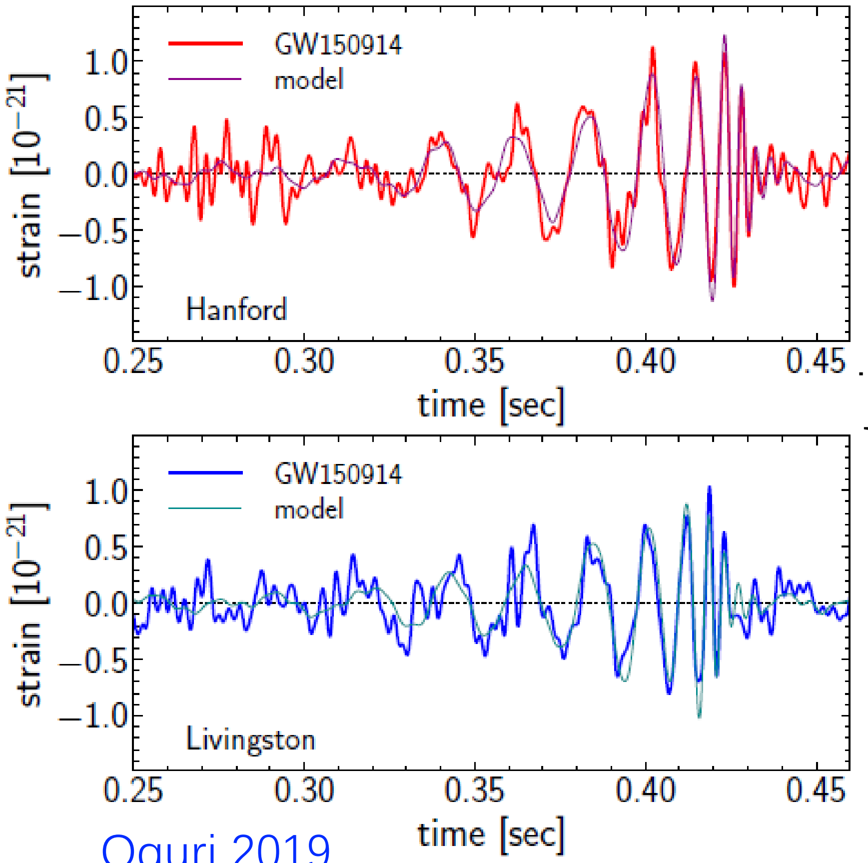
Lensed GW



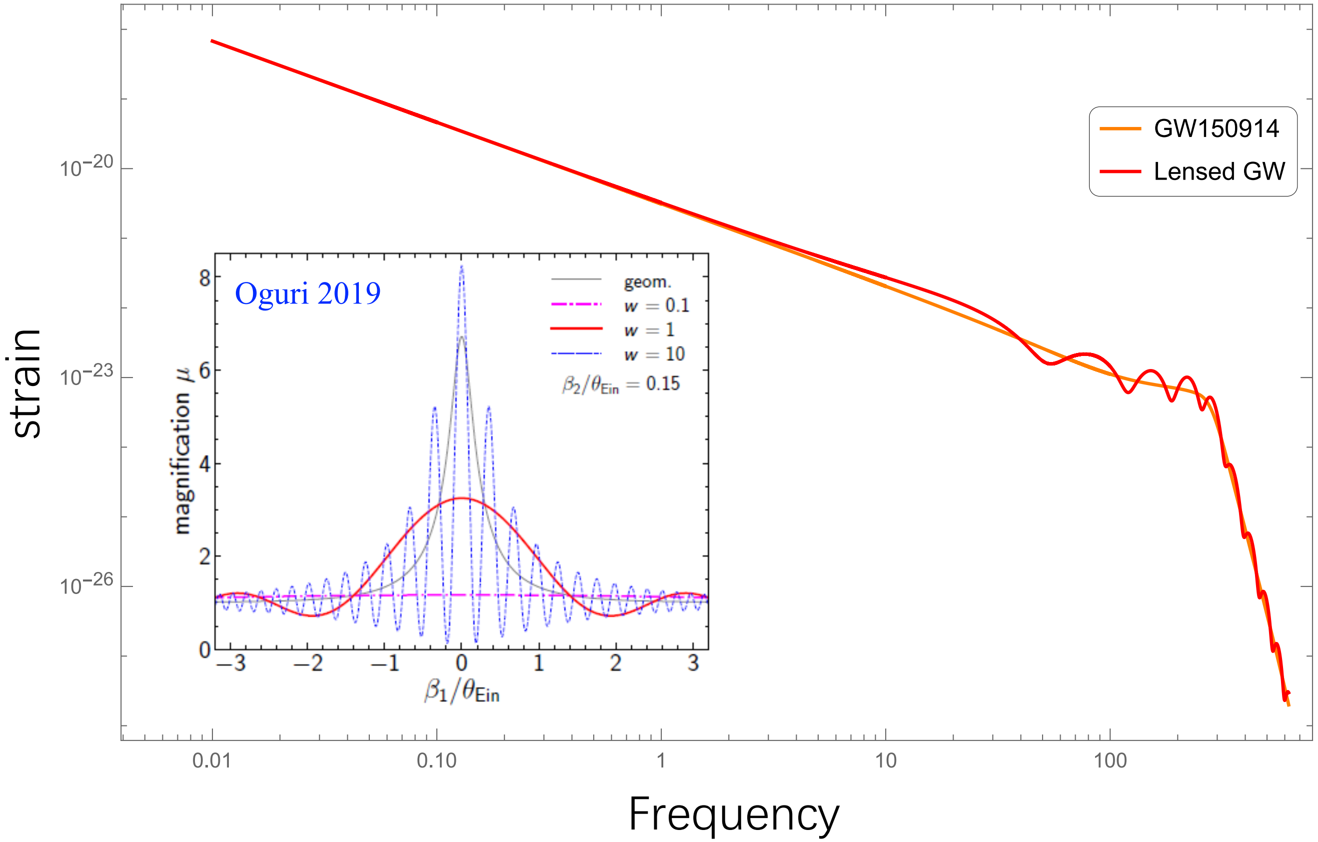
Oguri 2019



Lensed GW

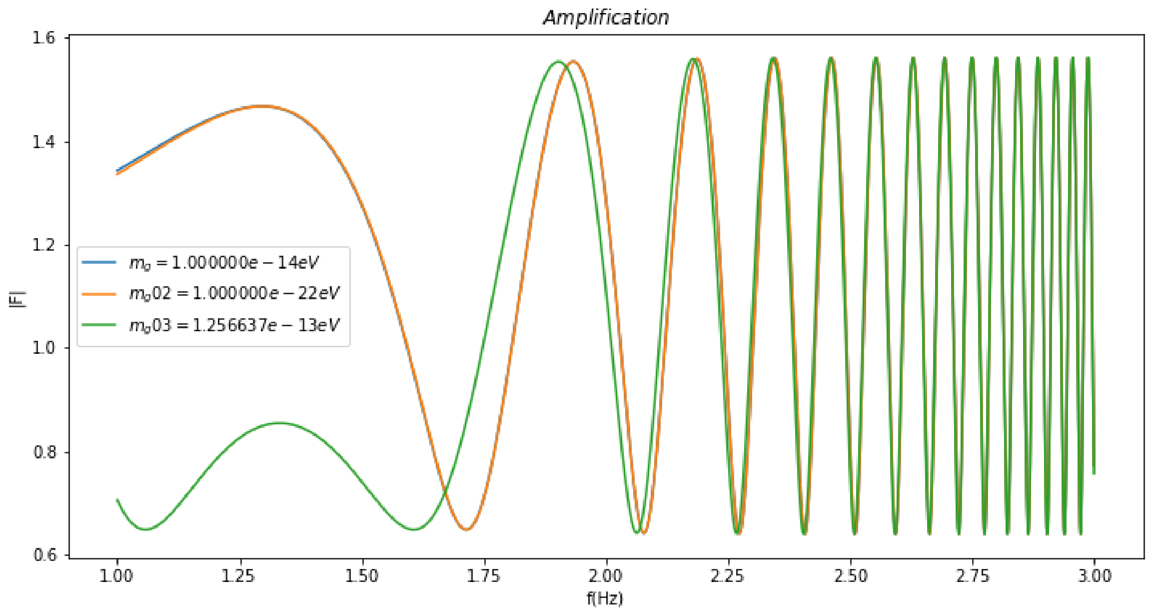


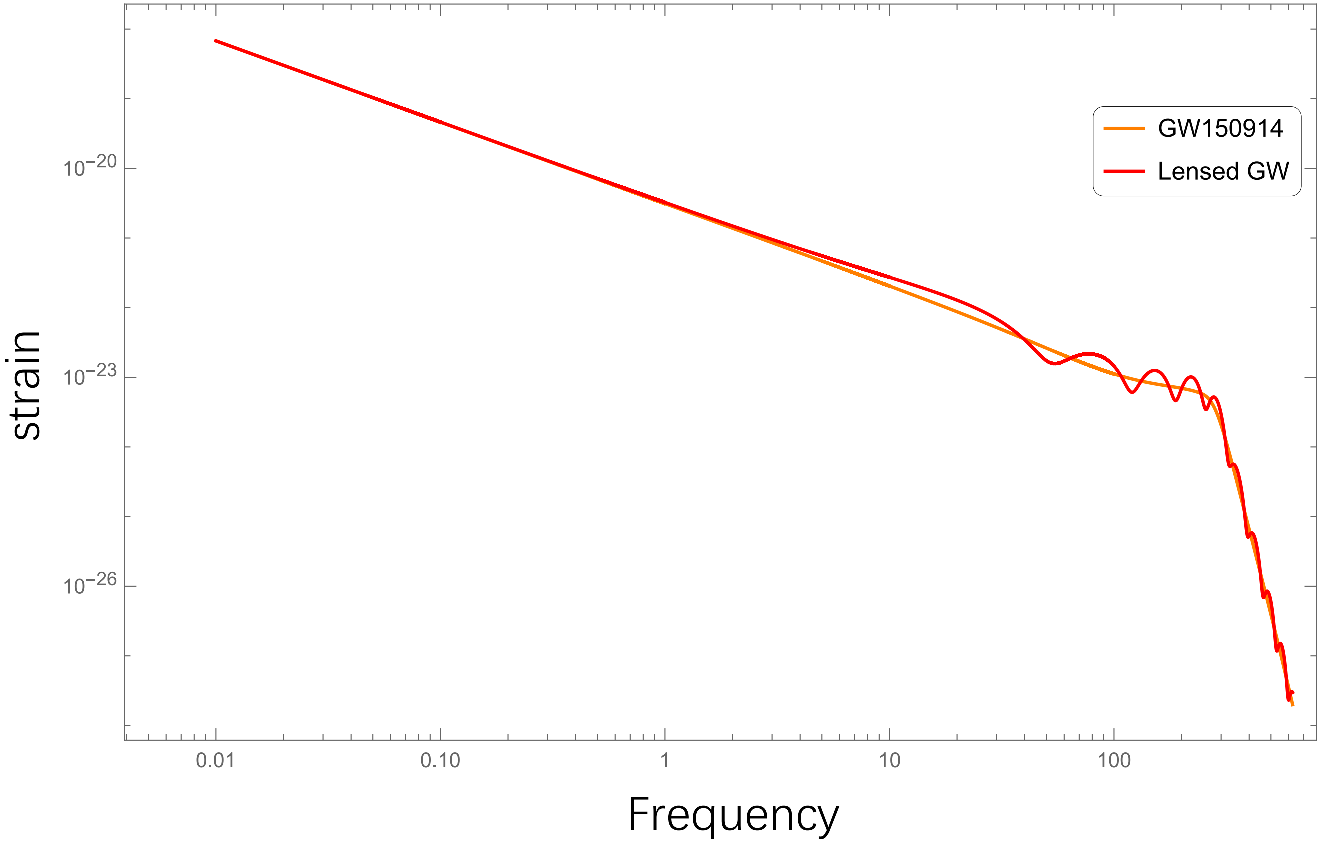
Oguri 2019



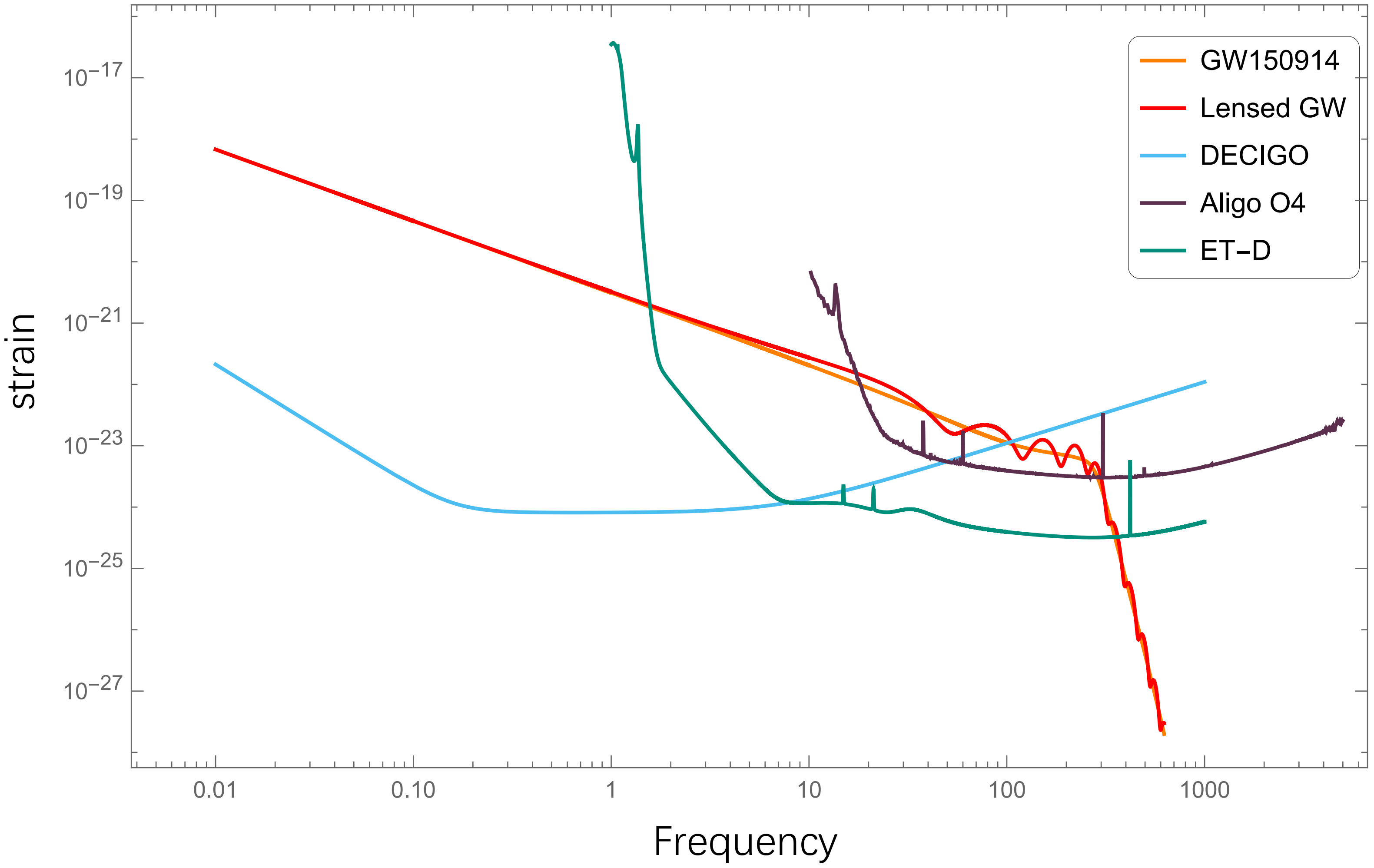
Lensed GW – mass of the graviton

$$\omega^2 = k^2 + m_g^2$$





Lensed GW – mass of the graviton



Virgo O5 designed
Unlensed SNR: 139.923
Lensed SNR: 165.592
Lensed/Unlensed: 1.183

DECIGO
Unlensed SNR: **24688.592**
Lensed SNR: **25058.167**
Lensed/Unlensed: 1.015

ET-D
Unlensed SNR: 1573.476
Lensed SNR: 2008.419
Lensed/Unlensed: **1.276**

Summary

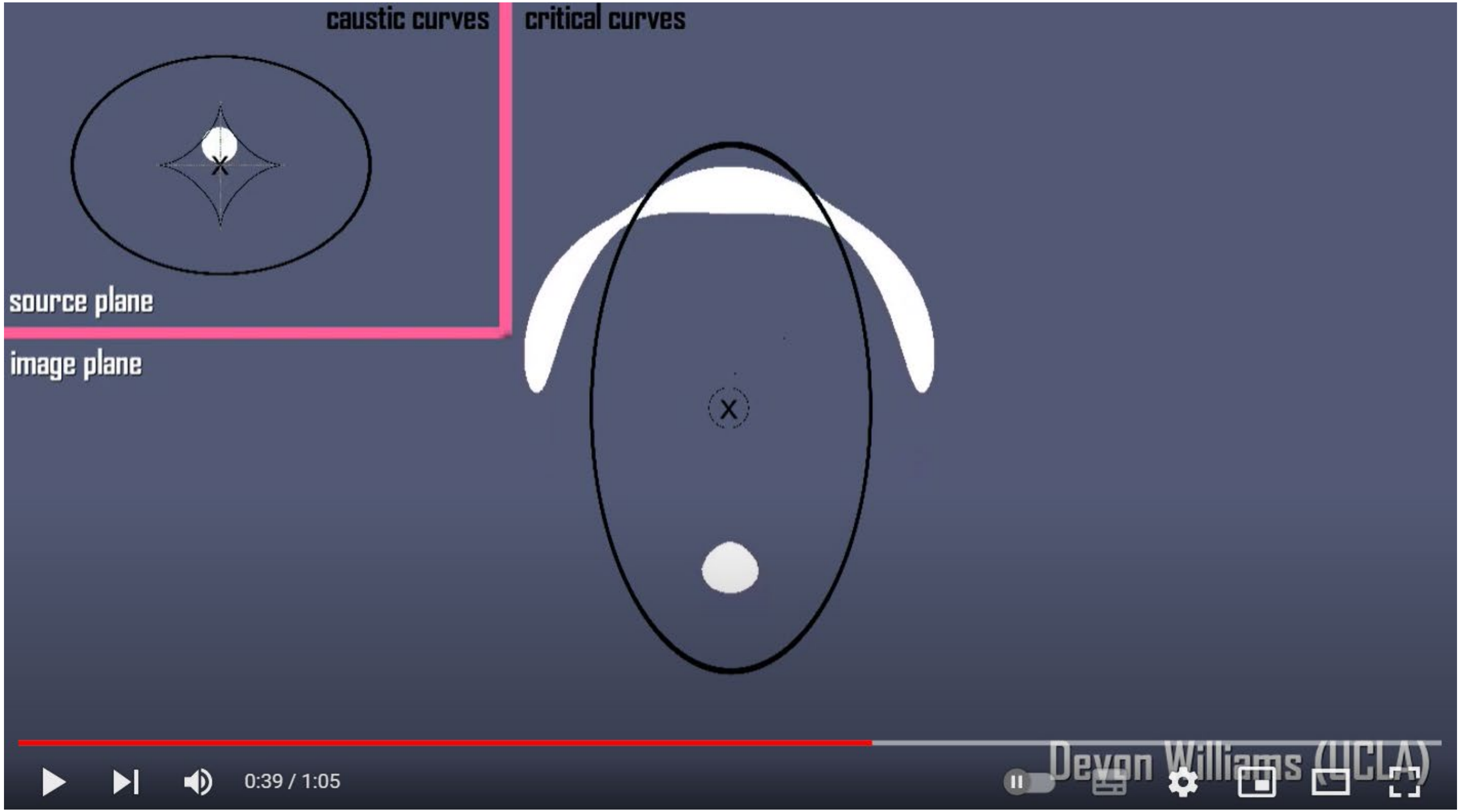
- Strong lensing systems serve as an invaluable tool for examining the characteristics and redshift evolution of galaxy populations.
- Strong lensing systems provide a robust complementary method for constraining cosmological parameters.
- Strong lensed transients will open up new applications, including new tests on the fundamental physics.

Thank you for your listening!

Something more …

Interesting link

- https://www.youtube.com/watch?v=i33kfYNMBnI&ab_channel=DevonWilliams



Lens model

Mass density profile

Point mass

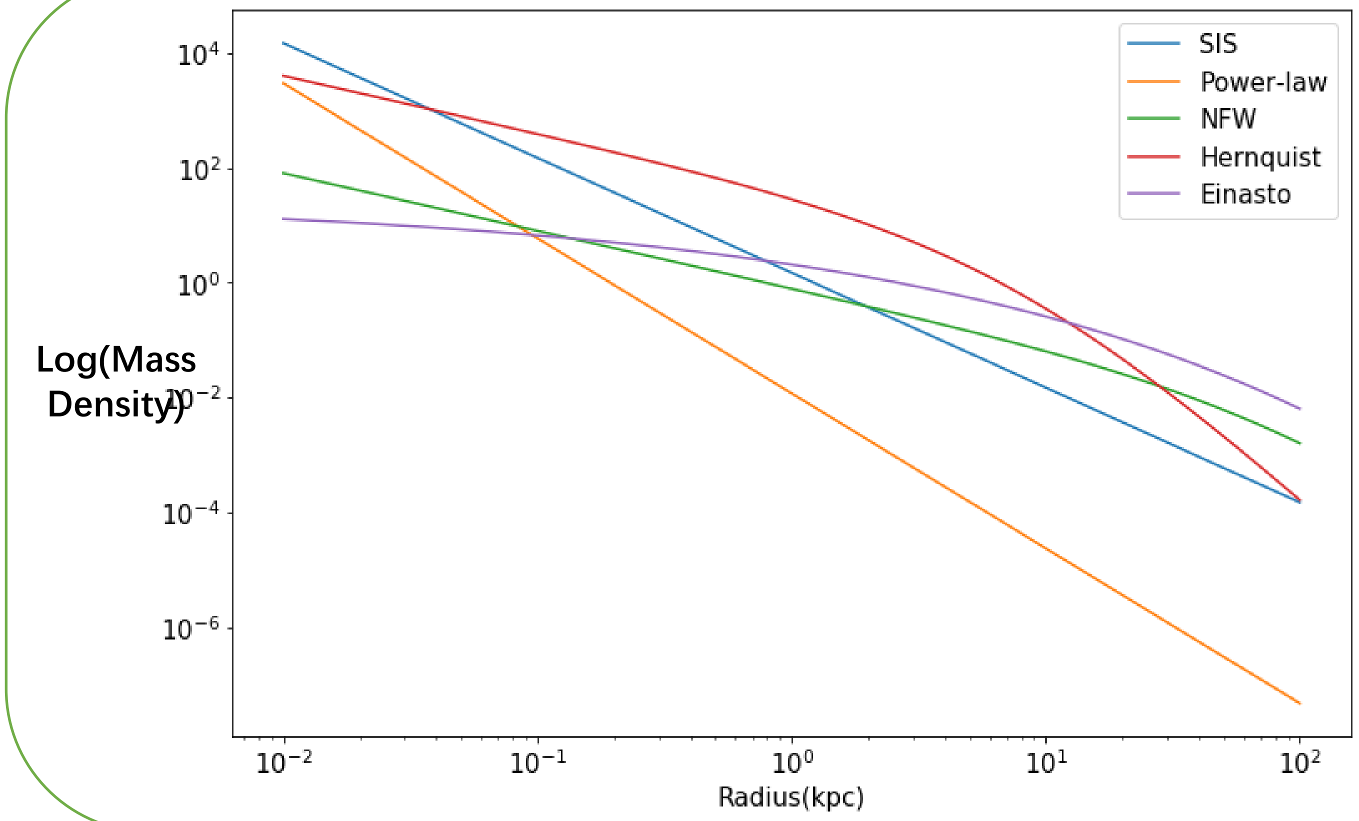
Singular Isothermal Sphere (SIS)

Power-law

Dark Matter (NFW ···)

Dark Matter + Luminous Matter

$$\rho(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right)\left(1 + \frac{r}{r_s}\right)^2}$$



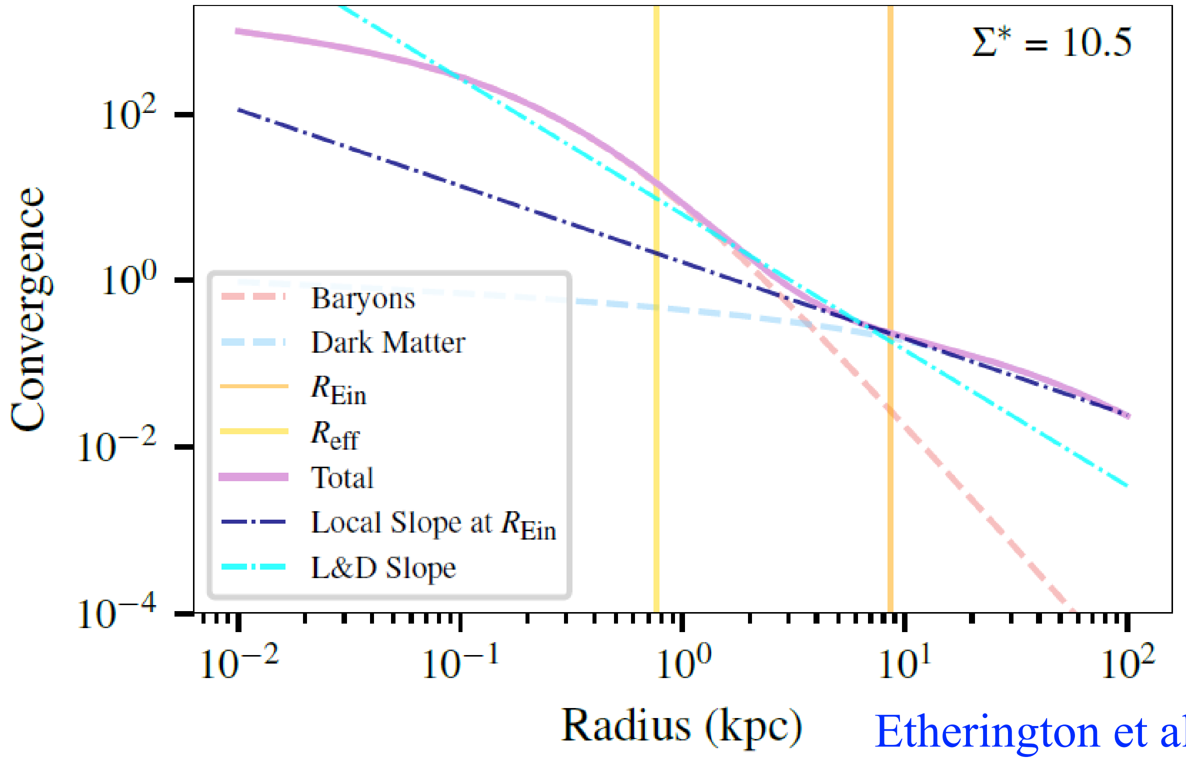
Lens model

Mass density profile

- Point mass
- Singular Isothermal Sphere (SIS)
- Power-law
- Dark Matter (NFW ⋯)
- Dark Matter + Luminous Matter

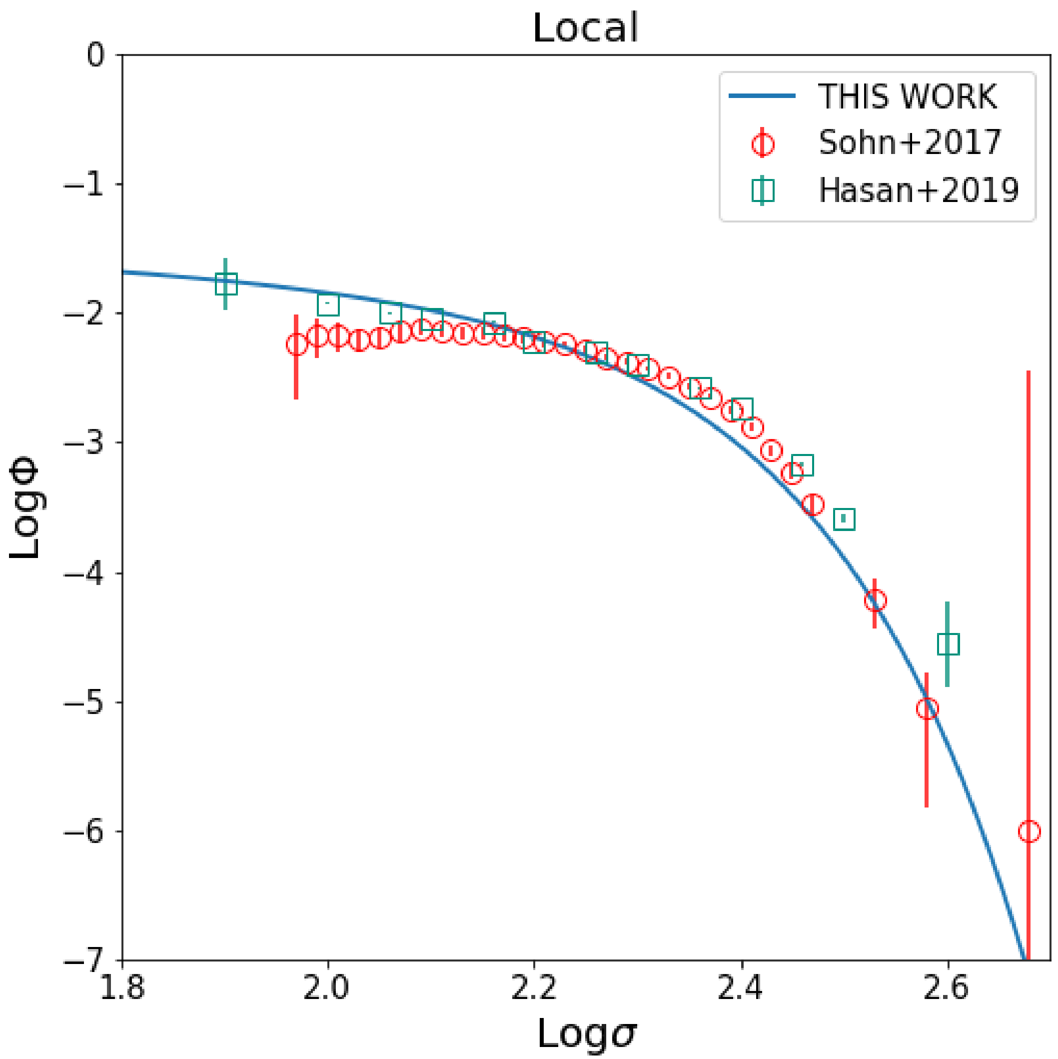
Hernquist (stellar) + NFW (dark matter)

$$\rho(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^3} \qquad \rho(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^2}$$



Etherington et al. 2022

VDF



Local
THIS WORK
Sohn+2017
Hasan+2019
LogΦ
Logσ
0
−1
−2
−3
−4
−5
−6
−7
1.8
2.0
2.2
2.4
2.6

Cosmology constraints – H0, Ωk

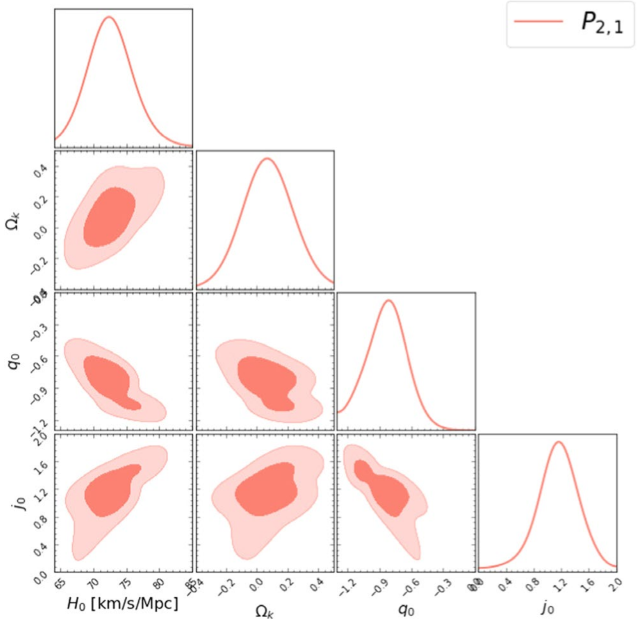
$$H \equiv \frac{d}{a}, q \equiv \frac{-a}{aH^2}, j \equiv \frac{a^{(3)}}{aH^3}, s \equiv \frac{a^{(0)}}{aH^4}$$

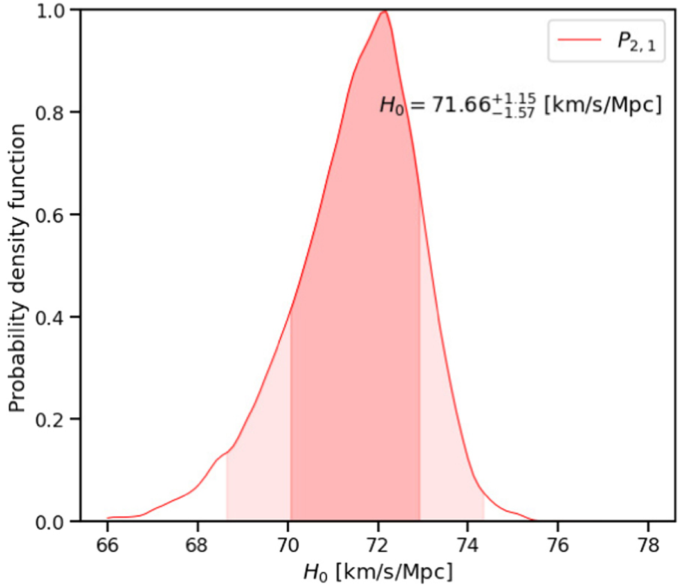
$$H(z) = H_0 + \frac{dH}{dz}\bigg|_{z=0} z + \frac{1}{2!}\frac{d^2H}{dz^2}\bigg|_{z=0} z^2 + \frac{1}{3!}\frac{d^3H}{dz^3} l_{z=0} z^3 + \dots$$
$$= H_0 \sum_{i=0} \mathcal{H}_z^i z^i$$

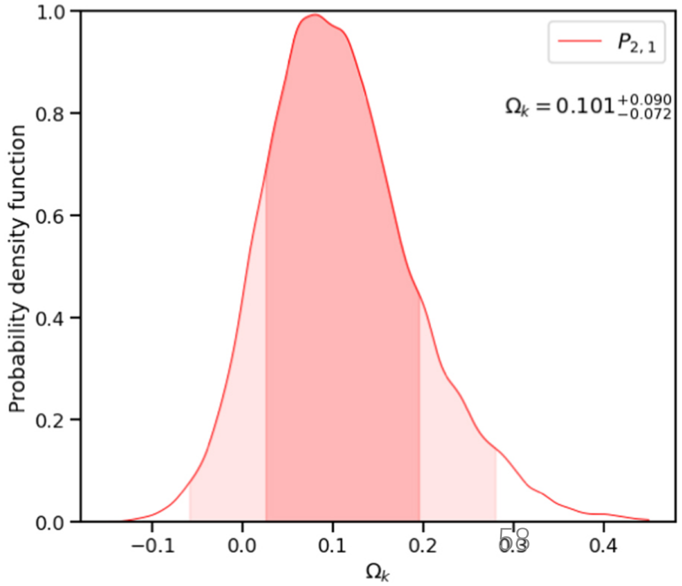
$$\mathcal{H}_z^0 = 1,$$
$$\mathcal{H}_z^1 = 1 + q_0,$$
$$\mathcal{H}_z^2 = \frac{1}{2}\left(j_0 - q_0^2\right),$$
$$\mathcal{H}_2^3 = \frac{1}{6}\left(-3j_0 - 4j_0q_0 + 3q_0^2 + 3q_0^3 - s_0\right)$$

Time-delay distance

$$\Delta t = \frac{1+z_l}{c}\frac{D_{ol}D_{os}}{D_{ls}}\left[\frac{(\vec{\theta}-\vec{\beta})^2}{2} - \phi(\vec{\theta})\right]$$







Cosmology constraints – Ωm

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$$H^2(z) = H_0^2\left[\Omega_m(1+z)^3 + (1-\Omega_m)\right]$$

$$D_A^{\Lambda CDM}(z_1, z_2) = \frac{c}{H_0(1+z_2)}\int_{z_1}^{z_1}\frac{dz}{\sqrt{\Omega_m(1+z)^3+1-\Omega_m}}$$

Flat Dvali–Gabadadze–Porrati model

$$H^2 - \frac{H}{r_c} = \frac{8\pi G}{3}\rho_m \quad H^2 = H_0^2\left(\sqrt{\Omega_m(1+z)^3+\Omega_{r_c}} + \sqrt{\Omega_{r_c}}\right)^2$$

$$\Omega_{r_c} = 1/(4r_c^2 H_0^2) = \frac{1}{4}(1-\Omega_m)^2$$

$$D_A^{DGP}(z_1, z_2) = \frac{c}{H_0(1+z_2)}\int_{z_1}^{z_1}\left[\sqrt{\Omega_m(1+z)^3+\Omega_{r_e}} + \sqrt{\Omega_{r_e}}\right]^{-1} dz.$$

Theoretical predictions

$$\frac{D_{ls}}{D_s} = \frac{\theta_E}{4\pi}\frac{c^2}{\sigma_{ap}^2}\left(\frac{\theta_E}{\theta_{ap}}\right)^{\gamma-2} f(\gamma)^{-1}$$

From observations

Massive graviton

$$E^2 = p^2c^2 + m^2c^4$$
$$m^2c^4 = E^2 - p^2c^2$$
$$= \hbar^2\omega^2 - \hbar^2k^2c^2$$

$\hbar = c = 1$

$$m^2 = \omega^2 - k^2$$
$$\omega = (m^2 + k^2)^{1/2}$$

speed $v = \frac{\partial\omega}{\partial k}$

$$v = (m^2 + k^2)^{-1/2}k$$
$$= [(\frac{m}{k})^2 + 1]^{-1/2}$$
$$\simeq 1 - \frac{1}{2}(\frac{m}{k})^2$$
$$k = 2\pi f$$
$$v \simeq 1 - \frac{1}{8\pi^2}\frac{m^2}{f^2}$$

Upper bounds of m_g	Methods and data	References
10^-19 eV/c^2	From the lag measured between the GW and EM signals of GW170817	Clifford 2018 1805.10523
10^-22 eV/c^2	In bimetric theories	Baker et al. 2017 1710.06394
7.7^-23 eV/c^2	GW150914+GW151226+GW170104	LIGO Virgo Collaboration 1706.01812
10^-24 eV/c^2	Solar system's test	Clifford 2018 1805.10523
10^-28 eV/c^2	From 14 Binary Pulsars	Shao et al. 2020 2007.04531