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Forward photon+jet production in pA collisions at next-to-eikonal accuracy

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Outline

- Introduction to Color Glass Condensate
- The Eikonal limit and corrections
- Photon + jet: S-matrix elements
- Cross-section

Light-Cone coordinates

Let us first define the Light-cone coordinates

$$x^+ = \frac{x^0 + x^3}{\sqrt{2}}$$
$$x^- = \frac{x^0 - x^3}{\sqrt{2}}$$

The product of two 4-vectors:

$$x^\mu y_\mu = x^+ y^- + x^- y^+ - \mathrm{x} \cdot \mathrm{y}.$$

The complete metric in this light-cone frame is

$$g_{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

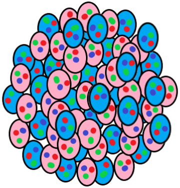
Deep Inelastic Scattering at High Energy (I)

What happens in a hadronic collision within QCD?

- Content of a hadron: quarks and gluons
- Nucleon: three quarks bounded by gluons
- They can fluctuate
- Need of a probe to see this

Low energy collisions:

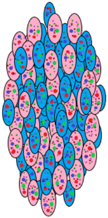
- Visible for the probe: quarks and very few fluctuations.
- Collisions very complicated because of the interactions of the constituents.



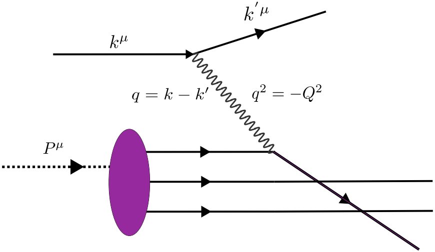
Deep Inelastic Scattering at High Energy (II)

High energy collisions:

- Lorentz contraction.
- Time scales multiplied by the Lorentz factor
- Time dilation as a result
- Constituents unlikely to interact during the time probed
- Lifetimes of fluctuations dilated as well
- Result: number of probed gluons increased



Deep Inelastic Scattering



The photon probes the proton's structure. Transverse size roughly 1/Q.
DIS parameters:

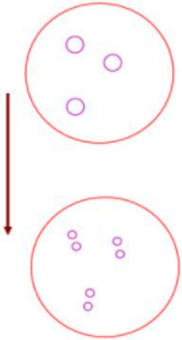
$$Q^2 = -q^2 \geq 0$$

$$x = \frac{Q^2}{2P \cdot q}$$

$$s \simeq 2P \cdot Q = Q^2/x$$

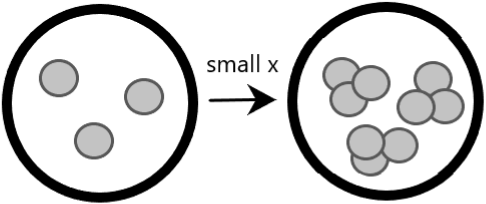
Deep Inelastic Scattering at High Energy (IV): Bjorken limit

The Bjorken limit consists on keeping a fixed Bjorken-x and increase the energy by increasing Q^2.
The resulting effect is an increase in the resolution scale and a final system that is more dilute than the initial one.



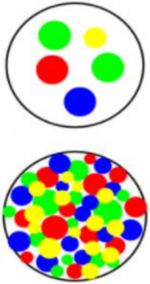
Deep Inelastic Scattering at High Energy (V): Regge-Gribov limit

The Regge-Gribov limit consists on keeping a fixed Q^2 and move towards smaller values of Bjorken-x.
The number of partons increases while keeping the transverse scale, therefore we obtain a final system that looks denser than the initial one.
Gluons will outnumber all other parton species.



Gluon saturation

We can pack more and more gluons if their size is smaller. The system becomes denser with increasing energy.
We reach a point where no more gluons can be packed inside, we reached saturation.
-New scale is introduced: Saturation scale Q_s that separates the 'homogeneous' from the 'grainy' regime.
For worse resolution than $1/Q_s$ gluons strongly overlap.



Color Glass Condensate (I)

Effective theory in the saturation regime
Dynamics is slowed down by Lorentz time dilation.
We use effective dregrees of freedom according to the MV model: Current for the fast partons and gauge fields for the slow partons.

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$$J_a^\mu(x) = \delta^{\mu +}\delta(x^-)\rho_a(\mathrm{x})$$

$$\mathcal{A}_a^\mu(x)$$

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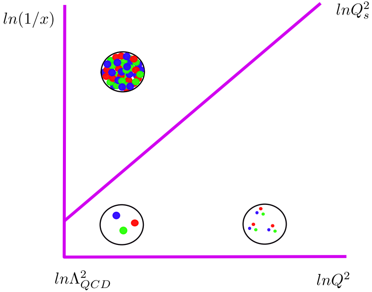

Color Glass Condensate (II)

CGC formalism used for dilute-dense high energy scattering so we apply Semi-classical approximation

- Dense target: classical background field $A_a^\mu(x)$
- Dilute projectile: virtual photon treated in perturbation theory

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We also adopt Eikonal approximation which amount to taking the high energy limit $s \to \infty$. Beyond eikonal limit give corrections of order $1/s$. We can obtain this limit boosting the target in following way:

$$A^{\mu}_{a}(x) \to \begin{cases} \gamma_t A^{-}_{a}\left(\gamma_t x^{+}, \frac{1}{\gamma_t}x^{-}, x_{\perp}\right) \\ \frac{1}{\gamma_t} A^{+}_{a}\left(\gamma_t x^{+}, \frac{1}{\gamma_t}x^{-}, x_{\perp}\right) \\ A^{i}_{a}\left(\gamma_t x^{+}, \frac{1}{\gamma_t}x^{-}, x_{\perp}\right) \end{cases}$$

Eikonal approximation

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- Target with finite width: transverse motion of the parton within the medium

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- Interactions with the perpendicular component of the field
  T. Altinoluk, G. Beuf. A.Czajka, A.Tymowska (2021) [arXiv:2012.03886] see also G.A.Chirilli [arXiv:1807.11435], [arxiv:2101.12744]

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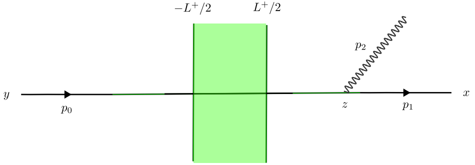
- Taking into account x^- -dependence
  T. Altinoluk, G. Beuf (2021) [arXiv:2109.01620] , T. Altinoluk, G. Beuf. A.Czajka, A.Tymowska (2022) [arXiv:2212.10484] and T. Altinoluk, N.Armesto, G. Beuf. and A.Tymowska (2023) [ongoing collaboration]

Motivation for studying forward Photon-Jet production

We want to calculate the following Photon-jet production at next-to eikonal order, we will have 3 possible diagrams.



-Lower energies at RHIC compared to LHC $\rightarrow$ NEik corrections

-Establishing an important formalism for future EIC.

- Wilson line: $\mathcal{U}_R(x) = \mathcal{P}_+ exp\big[ig \int dx^+ T^a_R A^-_a(x^+, x_\perp)\big]$

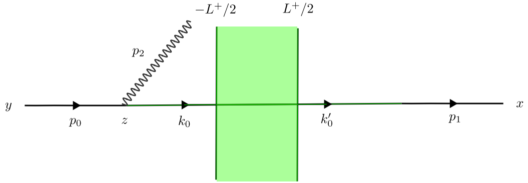
-The LSZ reduction formula for this process:

$$S^{\rm LO}_{q_1\,\gamma_2 \leftarrow q_0} = \lim_{y^+ \to -\infty} \int d^2 y \int dy^- \, e^{-iy\cdot \check{p}_0} \lim_{x^+ \to +\infty} \int d^2 x \int dx^- \, e^{+ix\cdot \check{p}_1}$$

$$\times \; (-i) e \, e_f \int d^4 z \; e^{+iz\cdot \check{p}_2} \;\; \bar{u}(\underline{p_1}, h_1)\gamma^+ \left[S_F(x,z)\right]_{\alpha_1 \beta} \; \not{\epsilon}_\lambda(\underline{p_2})^* \; \left[S_F(z,y)\right]_{\beta\alpha_0} \; \gamma^+ u(\underline{p_0}, h_0)$$

Photon emission before the medium

The first diagram contributing to photon-jet production at both eikonal and NEik order is:



Our final amplitude for this case is:

$$S'^{\text{LO-bef}}_{q_1\,\gamma_2\leftarrow q_0} = (-i)e\,e_f(1)_{\beta\alpha_0}\theta(p_0^+)\theta(p_1^+)\int d^2\mathbf{v}\,e^{-i\mathbf{v}\cdot(\mathbf{p_1}-\mathbf{p_0}+\mathbf{p_2})}\bar{u}(1)(-i\bar{\Delta}')\Bigg\{$$

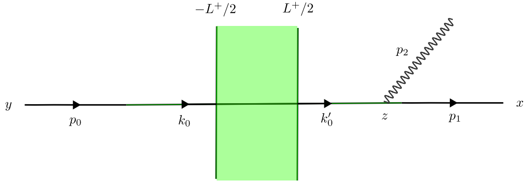
$$\times\int dv^-\,e^{iv^-(p_1^+-p_0^++p_2^+)}\mathcal{U}_F\left(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{v},v^-\right)+\frac{2\pi\delta(p_1^+-p_0^++p_2^+)}{2p_1^+}$$

$$\times\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}}dv^+\,\mathcal{U}_F\left(\frac{L^+}{2},v^+;\mathbf{v}\right)\left[\frac{[\gamma^i,\gamma^j]}{4}gt\cdot\mathcal{F}_{ij}(v)-\frac{(\mathbf{p}_1^j+\mathbf{p}_0^j-\mathbf{p}_2^j)}{2}\overleftrightarrow{\mathcal{D}_{\mathbf{v}^j}}-i\overleftarrow{\mathcal{D}_{\mathbf{v}^j}}\overrightarrow{\mathcal{D}_{\mathbf{v}^j}}\right]\mathcal{U}_F\left(v^+,-\frac{L^+}{2};\mathbf{v}\right)\Bigg\}$$

$$\times\,\epsilon_\lambda^k(p_2)\gamma^+\left(\left(p_2^l-\frac{p_2^+}{p_0^+}p_0^l\right)\left(2\delta^{kl}-\frac{p_2^+}{p_0^+}\gamma^k\gamma^l\right)+\left(\frac{p_2^+}{p_0^+}\right)^2m\gamma^k\right)u(0)$$

Photon emission after the medium

The second diagram contributing to photon-jet production at both eikonal and NEik order is:



with the amplitude :

$$S'^{\text{LO-aft}}_{q_1\,\gamma_2\leftarrow q_0} = (-i)e\,e_f\,\epsilon^k_\lambda(p_2)(1)_{\alpha_1\beta}\theta(p_1^+)\theta(p_0^+)\int d^2\mathbf{v}\,e^{-i\mathbf{v}\cdot(\mathbf{p_1}+\mathbf{p_2}-\mathbf{p_0})}\bar{u}(1)i\bar{\Delta}\gamma^+$$

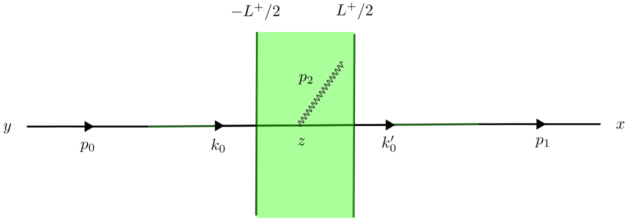
$$\times\left(\left(p_2^l-\frac{p_2^+}{p_1^+}p_1^l\right)\left(2\delta^{kl}-\frac{p_2^+}{p_1^+}\gamma^l\gamma^k\right)-\left(\frac{p_2^+}{p_1^+}\right)^2 m\gamma^k\right)\left\{\int dv^-\,e^{iv^-(p_1^++p_2^+-p_0^+)}\mathcal{U}_F\left(\frac{L^+}{2},-\frac{L^+}{2};\mathbf{v},v^-\right)\right.$$

$$+\frac{2\pi\delta(p_1^++p_2^+-p_0^+)}{(2p_0^+)}$$

$$\left.\times\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}}dv^+\,\mathcal{U}_F\left(\frac{L^+}{2},v^+;\mathbf{v}\right)\left[\frac{[\gamma^i,\gamma^j]}{4}gt\cdot\mathcal{F}_{ij}(v)-\frac{(\mathbf{p_0^j}+\mathbf{p_1^j}+\mathbf{p_2^j})}{2}\overleftrightarrow{\mathcal{D}_{\mathbf{v}^j}}-i\overleftarrow{\mathcal{D}_{\mathbf{v}^j}}\overrightarrow{\mathcal{D}_{\mathbf{v}^j}}\right]\mathcal{U}_F\left(v^+,-\frac{L^+}{2};\mathbf{v}\right)\right\}_{\beta\alpha_0}u(0)$$

Photon emission inside the medium

The third and last diagram contributing to photon-jet production contributing only at NEik order is:



with the amplitude :

$$S'^{\text{LO-in}}_{q_1\,\gamma_2\leftarrow q_0} = (-i)e\, e_f \epsilon_{\lambda k}(\underline{p_2})^* \int d^2\mathbf{z}\; 2\pi\delta(p_1^+ + p_2^+ - p_0^+)\theta(p_0^+)\theta(p_1^+)e^{-i\mathbf{z}\cdot(\mathbf{p_1}+\mathbf{p_2}-\mathbf{p_0})}\bar{u}(\underline{p_1},h_1)$$
$$\times \int_{-L^+/2}^{L^+/2} dz^+ \mathcal{U}_F\Big(\frac{L^+}{2},z^+;\mathbf{z}\Big)\left[\frac{i\gamma^+\gamma^i\gamma^k}{2p_1^+} + \frac{i\gamma^k\gamma^i\gamma^+}{2p_0^+}\right]\frac{1}{2}\overleftrightarrow{\mathcal{D}_{\mathbf{z}^i}}\mathcal{U}_F\Big(z^+,\frac{-L^+}{2};\mathbf{z}\Big)u(\underline{p_0},h_0)$$

Cross-section

We have the following definitions for the Wilson lines appearing in our cross-section

$$\mathcal{U}_{F;j}^{(1)}(\mathrm{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+ \, \mathcal{U}_F\left(\frac{L^+}{2}, v^+; \mathrm{v}\right) \overleftrightarrow{\mathcal{D}_{\mathrm{v}^j}} \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \mathrm{v}\right)$$

$$\mathcal{U}_{F;j}^{(2)}(\mathrm{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+ \, \mathcal{U}_F\left(\frac{L^+}{2}, v^+; \mathrm{v}\right) \overleftarrow{\mathcal{D}_{\mathrm{v}^j}} \, \overrightarrow{\mathcal{D}_{\mathrm{v}^j}} \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \mathrm{v}\right)$$

$$\mathcal{U}_{F;ij}^{(3)}(\mathrm{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+ \, \mathcal{U}_F\left(\frac{L^+}{2}, v^+; \mathrm{v}\right) gt \cdot \mathcal{F}_{ij}(\underline{v}) \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \mathrm{v}\right)$$

$$\mathcal{U}_F(\mathrm{v}) = \mathcal{U}_F\left(\frac{L^+}{2}, -\frac{L^+}{2}; \mathrm{v}\right)$$

All factors will be expressed in terms of:

$$\mathrm{k}_\perp = \mathrm{p}_1 + \mathrm{p}_2 - \mathrm{p}_0$$

$$\mathrm{P}_\perp = \frac{p_2^+ \mathrm{p}_1 - p_1^+ \mathrm{p}_2}{p_1^+ + p_2^+}$$

$$\mathrm{p}_0 = 0$$

We also have that:

$$\overrightarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\partial_{z^\mu}} + igt \cdot \mathcal{A}_\mu(z)$$

$$\overleftarrow{\mathcal{D}_{z^\mu}} \equiv \overleftarrow{\partial_{z^\mu}} - igt \cdot \mathcal{A}_\mu(z)$$

$$\overleftrightarrow{\mathcal{D}_{z^\mu}} \equiv \overrightarrow{\mathcal{D}_{z^\mu}} - \overleftarrow{\mathcal{D}_{z^\mu}} = \overleftrightarrow{\partial_{z^\mu}} + 2igt \cdot \mathcal{A}_\mu(z)$$

$$\mathcal{F}_{\mu\nu}^a(z) \equiv \partial_{\mathrm{z}^\mu} \mathcal{A}_\nu^a(z) - \partial_{\mathrm{z}^\nu} \mathcal{A}_\mu^a(z) - gf^{abc} \mathcal{A}_\mu^b(z) \mathcal{A}_\nu^c(z)$$

Cross-section (I)-Generalized eikonal contribution

$$\Bigg[S^{\text{LO-aft}}_{q_1\,\gamma_2\leftarrow q_0}\left(S^{\text{LO-aft}}_{q_1\,\gamma_2\leftarrow q_0}\right)^\dagger + S^{\text{LO-bef}}_{q_1\,\gamma_2\leftarrow q_0}\left(S^{\text{LO-bef}}_{q_1\,\gamma_2\leftarrow q_0}\right)^\dagger$$

$$+ S^{\text{LO-aft}}_{q_1\,\gamma_2\leftarrow q_0}\left(S^{\text{LO-bef}}_{q_1\,\gamma_2\leftarrow q_0}\right)^\dagger + S^{\text{LO-bef}}_{q_1\,\gamma_2\leftarrow q_0}\left(S^{\text{LO-aft}}_{q_1\,\gamma_2\leftarrow q_0}\right)^\dagger\Bigg]_{\text{G-Eik}} =$$

$$(e\,e_f)^2\int d^2\mathrm{v}\int d^2\mathrm{w}\int dv^-\int dw^-\, e^{-i(\mathrm{v}-\mathrm{w})\mathrm{k}} e^{i(v^- - w^-)(p_1^+ + p_2^+ - p_0^+)}$$

$$\times\Bigg[(\bar{\Delta})^2\mathrm{F}_a[\mathrm{P},p_0^+,p_1^+,p_2^+] + \theta(p_0^+ - p_2^+)\,(\bar{\Delta}')^2\mathrm{F}_b[\mathrm{P},\mathrm{k},p_0^+,p_1^+,p_2^+]$$

$$+ 2\theta(p_0^+ - p_2^+)\bar{\Delta}'\bar{\Delta}\mathrm{F}_{a-b}[\mathrm{P},\mathrm{k},p_0^+,p_1^+,p_2^+]\Bigg]\frac{1}{N_c}tr\left[\mathcal{U}_F\left(\mathrm{v},v^-\right)\mathcal{U}_F^\dagger\left(\mathrm{w},w^-\right)\right]$$

with the following dependence:

$$\bar{\Delta} = \bar{\Delta}(\mathrm{P},p_1^+,p_2^+) = \left(\left(-\frac{(p_1^+ + p_2^+)}{p_1^+}\mathrm{P}\right)^2 + m^2\left(\frac{p_2^+}{p_1^+}\right)^2\right)^{-1}$$

$$\bar{\Delta}' = \bar{\Delta}'(\mathrm{P},\mathrm{k},p_0^+,p_1^+,p_2^+) = \left(\left(-\frac{(p_1^+ + p_2^+)}{p_0^+}\mathrm{P} + \frac{p_2^+}{p_0^+}\mathrm{k}\right)^2 + m^2\left(\frac{p_2^+}{p_0^+}\right)^2\right)^{-1}$$

Cross-section (II) Next-to-eikonal corrections

The first contribution at NEik accuracy.

$$\left[S^{\text{LO-all}}_{q_1\,\gamma_2\leftarrow q_0}\left(S^{\text{LO-all}}_{q_1\,\gamma_2\leftarrow q_0}\right)^\dagger\right]_{\text{NEik}-\mathcal{U}^{(1)}_{F;j}(\mathrm{v})} = (e\,e_f)^2\int d^2\mathrm{v}\int d^2\mathrm{w}\,e^{-i(\mathrm{v}-\mathrm{w})\cdot\mathrm{k}}(2\pi)^2\delta(p_1^+ + p_2^+ - p_0^+)$$

$$\times\Bigg\{(-1)(\bar{\Delta})^2\frac{\mathrm{k}^j}{2}\frac{\mathrm{F}_a[\mathrm{P},p_0^+,p_1^+,p_2^+]}{2p_0^+} - (\bar{\Delta}')^2\frac{\left(\frac{(p_1^+-p_2^+)}{p_0^+}\mathrm{k}^j + 2\mathrm{P}^j\right)}{2}\frac{\mathrm{F}_b[\mathrm{P},\mathrm{k},p_0^+,p_1^+,p_2^+]}{2p_1^+}$$

$$-(\bar{\Delta}\bar{\Delta}')\left(\frac{(2p_1^+-p_2^+)}{4p_0^+p_1^+}\mathrm{k}^j + \frac{\mathrm{P}^j}{2p_1^+}\right)\mathrm{F}_{a-b}[\mathrm{P},\mathrm{k},p_0^+,p_1^+,p_2^+]$$

$$+\frac{1}{2}\left(\left(-\mathrm{P}^j + \frac{p_2^+}{p_0^+}\mathrm{k}^j\right)\bar{\Delta}'\left(\frac{p_1^+}{p_0^+}(p_0^+ + p_2^+) + p_0^+\right) - \left(\frac{p_0^+}{p_1^+}\mathrm{P}^j\right)\bar{\Delta}\left(\frac{p_0^+}{p_1^+}(p_1^+ + p_2^+) + p_1^+\right)\right)\Bigg\}$$

$$\times\frac{1}{N_c}tr\left[\mathcal{U}^{(1)}_{F;j}(\mathrm{v})\mathcal{U}^\dagger_F(\mathrm{w})\right]$$

Cross-section (III) Next-to-eikonal corrections

The second contribution at NEik accuracy:

$$\left[S^{\text{LO-all}}_{q_1\,\gamma_2\leftarrow q_0} \left(S^{\text{LO-all}}_{q_1\,\gamma_2\leftarrow q_0}\right)^\dagger \right]_{\text{NEik}-\mathcal{U}^{(2)}_{F;j}(\mathrm{v})} = (e\, e_f)^2 \int d^2\mathrm{v} \int d^2\mathrm{w}\, e^{-i(\mathrm{v}-\mathrm{w})\cdot(\mathrm{p_1}+\mathrm{p_2}-\mathrm{p_0})}$$
$$\times (2\pi)^2 \delta(p_1^+ + p_2^+ - p_0^+) \Bigg\{ (-1)(\bar{\Delta})^2 \frac{1}{2p_0^+} \mathrm{F}_a[\mathrm{P}, p_0^+, p_1^+, p_2^+] - (\bar{\Delta}')^2 \frac{1}{2p_1^+} \mathrm{F}_b[\mathrm{P}, \mathrm{k}, p_0^+, p_1^+, p_2^+]$$
$$+ (\bar{\Delta}\bar{\Delta}') \left(\frac{1}{2p_0^+} + \frac{1}{2p_1^+} \right) \mathrm{F}_{a-b}[\mathrm{P}, \mathrm{k}, p_0^+, p_1^+, p_2^+] \Bigg\} i \frac{1}{N_c} tr \left[\mathcal{U}^{(2)}_{F;j}(\mathrm{v}) \mathcal{U}^\dagger_F(\mathrm{w}) \right]$$

Cross-section (IV) Next-to-eikonal corrections

The third and last contribution at NEik accuracy:

$$\left[S^{\text{LO-aft}}_{q_1\,\gamma_2\leftarrow q_0}\left(S^{\text{LO-bef}}_{q_1\,\gamma_2\leftarrow q_0}\right)^{\dagger}+S^{\text{LO-bef}}_{q_1\,\gamma_2\leftarrow q_0}\left(S^{\text{LO-aft}}_{q_1\,\gamma_2\leftarrow q_0}\right)^{\dagger}\right]_{\text{NEik-Fij}}=(e\,e_f)^2$$
$$\times\int d^2\mathrm{v}\,d^2\mathrm{w}\,e^{-i(\mathrm{v}-\mathrm{w})\cdot(\mathrm{p}_1+\mathrm{p}_2-\mathrm{p}_0)}(2\pi)^2\delta(p_1^+ + p_2^+ - p_0^+)$$
$$\times\,(8\bar{\Delta}\bar{\Delta}')\frac{(p_2^+)^3}{p_1^+}\left(-\mathrm{P}^i+\frac{p_2^+}{p_0^+}\mathrm{k}^i\right)\left(\frac{1}{p_1^+}\mathrm{P}^j\right)\frac{1}{N_c}tr\left[\mathcal{U}^{(3)}_{F;ij}(\mathrm{v})\mathcal{U}^{\dagger}_{F}(\mathrm{w})\right]$$

We would need to take also into account the conjugate of all these contributions.

Outlook

- We computed the cross section for the case of forward photon-jet production at full NEik order
- Next-to eikonal corrections include:
  - Relaxing the shockwave approximation $\rightarrow$ transverse motion through the target
  - Including interactions with transverse component of the background field
  - Taking into account z^--dependence
- We plan on taking the back to back limit for this process.

Thank you for your attention