Can a quantum mixmaster universe undergo an spontaneous inflationary phase?

Jaime de Cabo Martín

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Work in collaboration with:
Przemysław Małkiewicz (NCBJ, Warsaw), Hervé Bergeron (ISMO, Orsay) and Jean-Pierre Gazeau (Université Paris Cité, Paris)

Talk based on: [arXiv:2303.07873]

Outline:

1) Introduction about Mixmaster Universe + Motivation

2) Mathematical background.

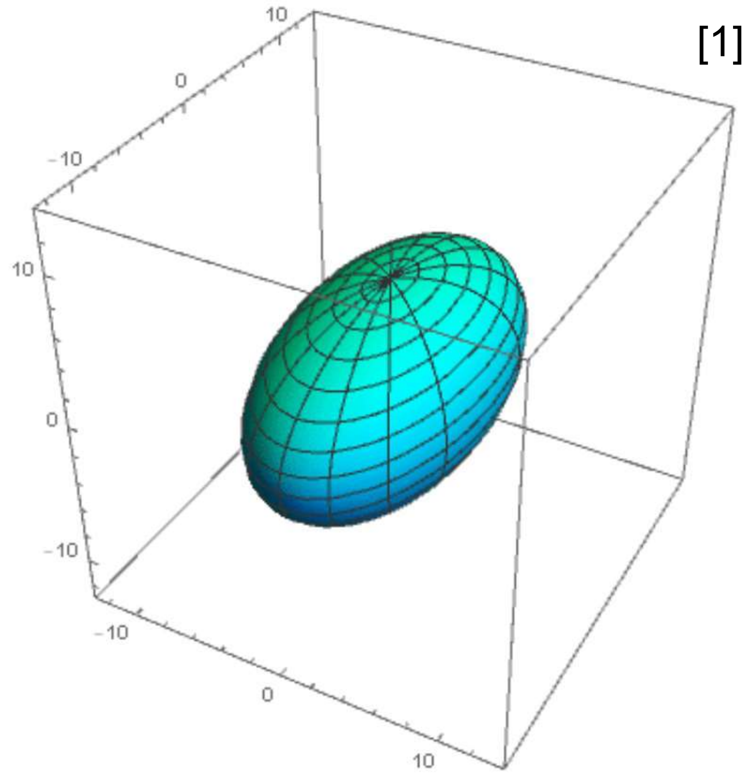
3) Investigation on the dynamics of the system. Comological interpretation.

4) Analysis of the (possible?) inflationary-expansion behaviour.

5) Conclusions and prospects for the future.

Introduction: Mixmaster Universe

- Homogeneous model of Early Universe.
  Studied by Belinski, Khalatnikov and Lifshitz (BKL) and independently by Misner.
- Oscillatory and Chaotic behaviour close to the initial singularity (Big Bang).
- Random and repeated squeezing and blowing up of spatial directions.
  3D Mixing→ Anisotropy.



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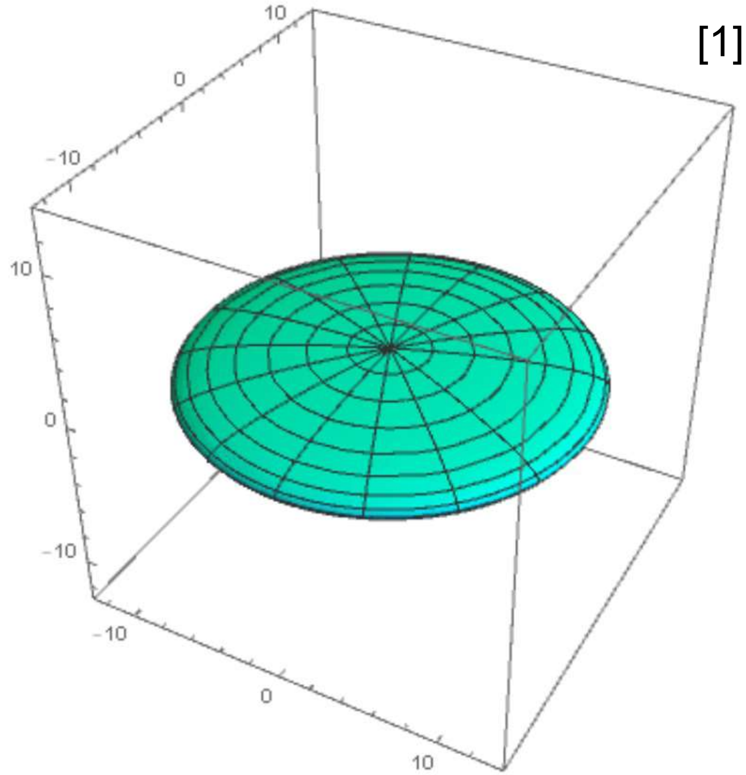

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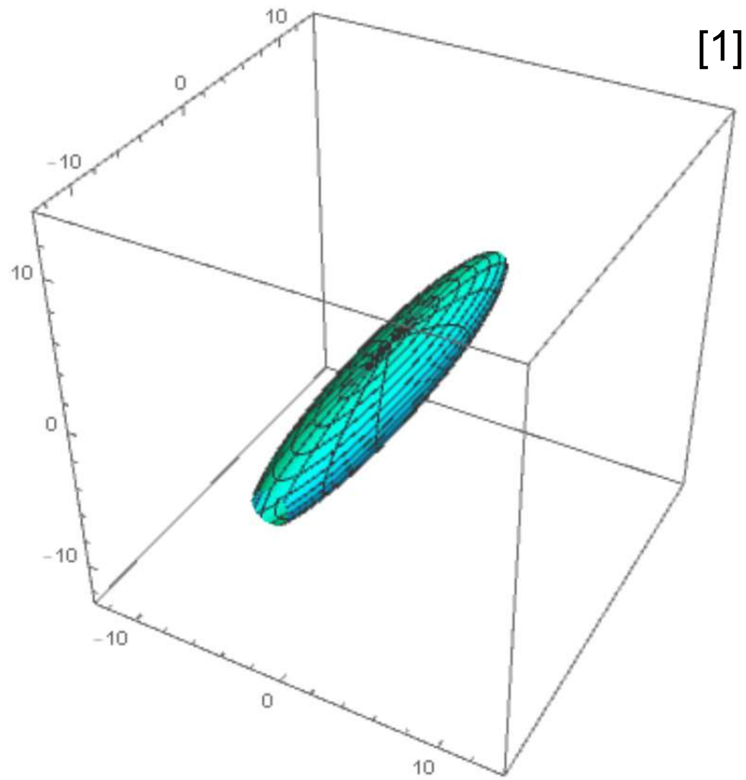

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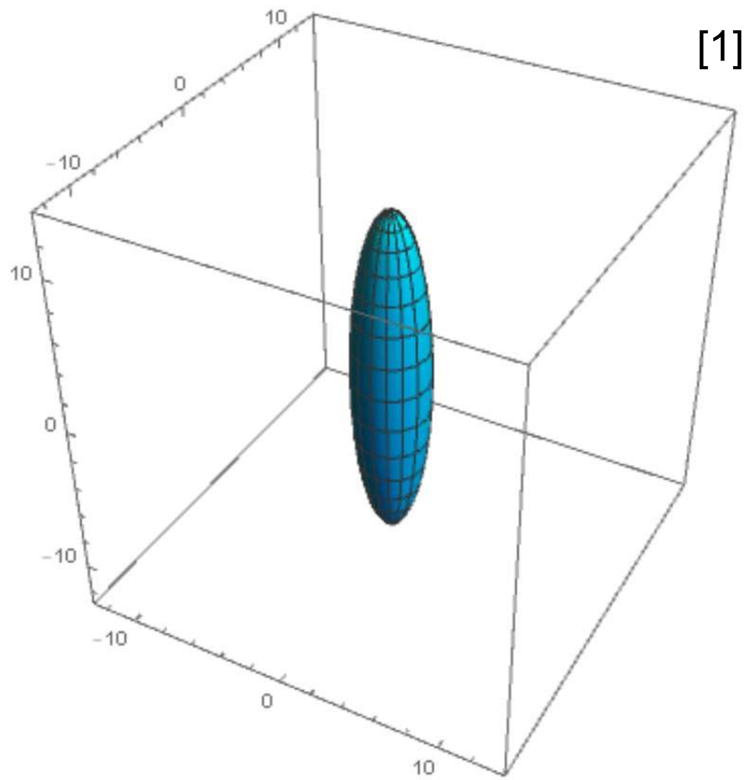

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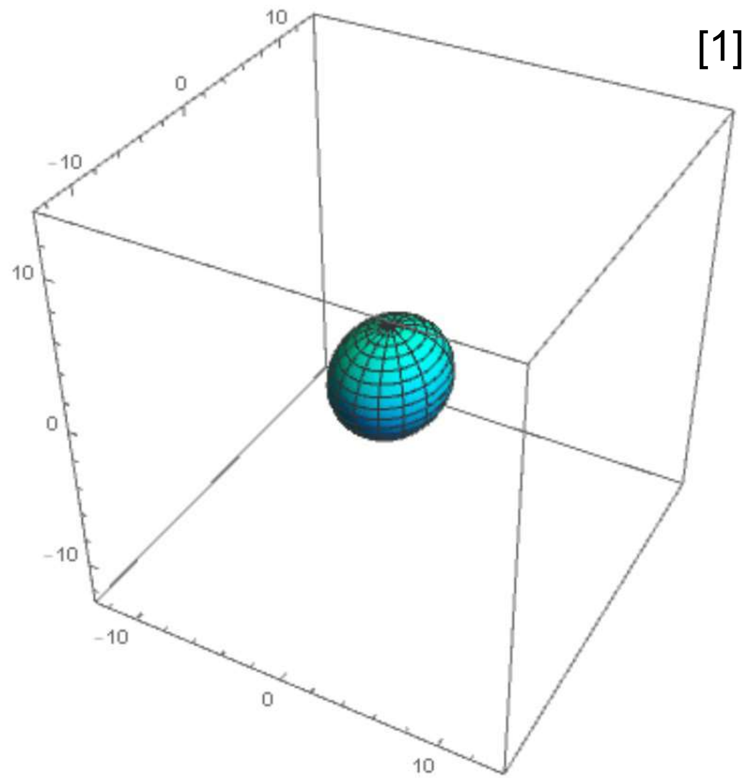

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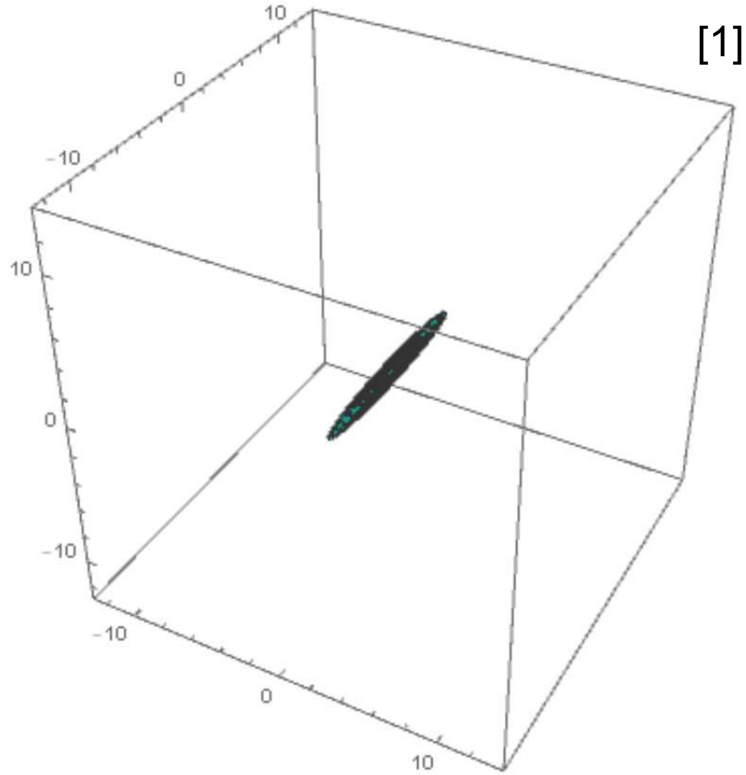

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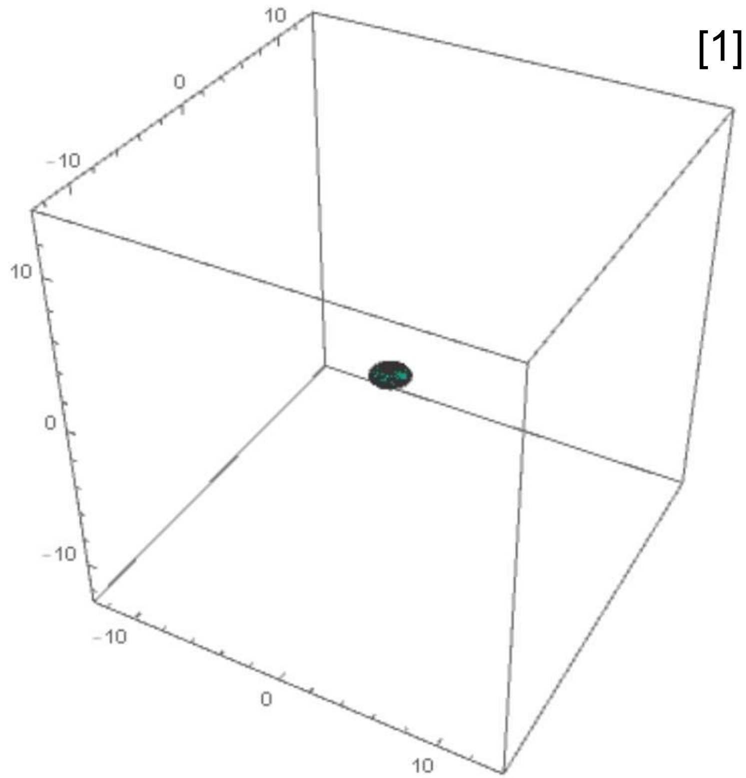

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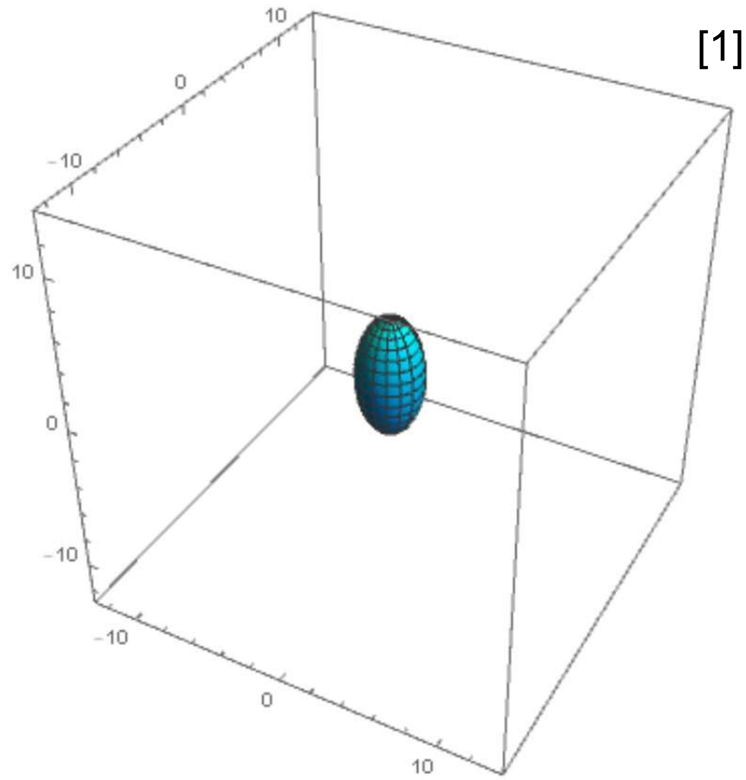

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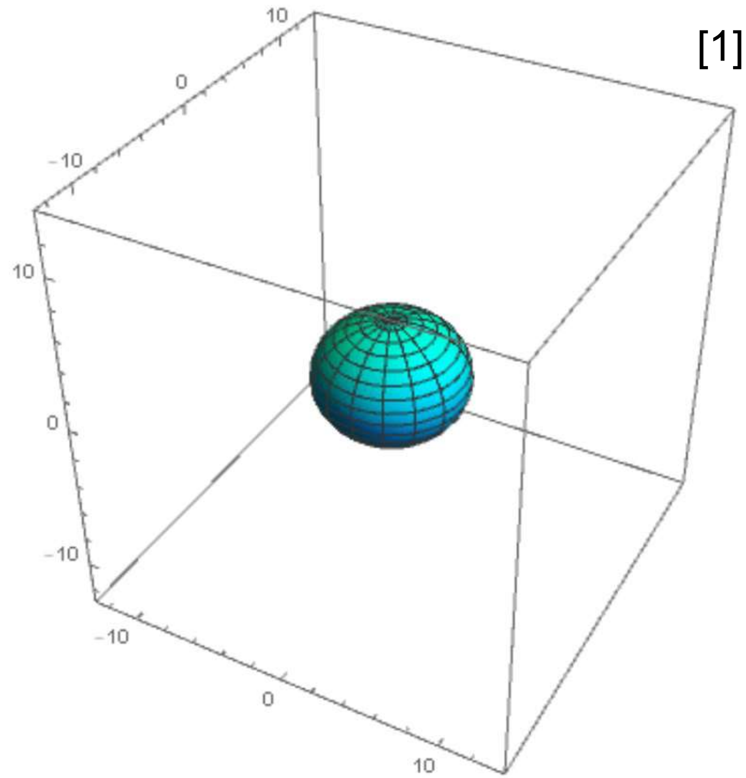

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Motivation: Why Anisotropy?

- Anisotropic very early universe → Expansion flattens the effect → Isotropy
- More generic model, we need less primordial symmetries.
- Isotropic bouncing models → OK, but slightly blue-tilted spectrum. Can anisotropy improve that?

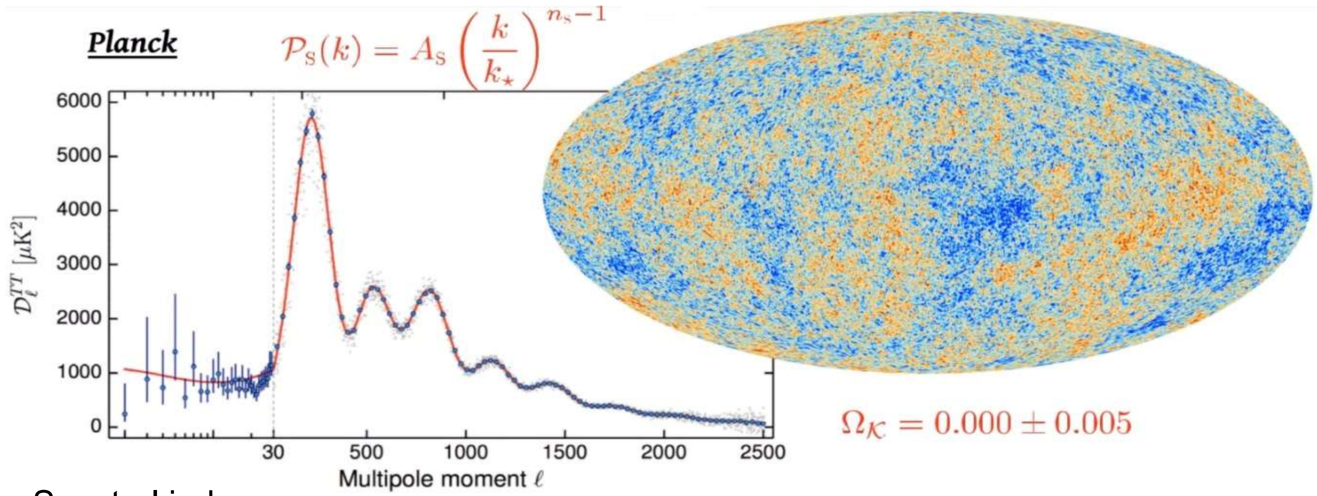
For large scales (small modes)

$$n_s = \frac{6w}{3w+1} + 1$$

$$n_S = \frac{3w+3}{3w+1} + 1$$

Blue tilted means: $n_s > 1$

But CMB says:



Spectral index

$n_s = 0.9639 \pm 0.0047$ almost scale invariant → (slightly red-tilted n_s<1 For large scales (infrared))

Power spectrum: Measures deviation from homogeneity

Correlation function:

$$\xi(r) = \langle \delta(\mathbf{x}) \delta(\mathbf{x}') \rangle = \frac{1}{V} \int d^3\mathbf{x}\, \delta(\mathbf{x}) \delta(\mathbf{x} - \mathbf{r}),$$

$$\xi(r) = \int \frac{d^3 k}{(2\pi)^3} P(k) e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}$$

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- Anisotropic very early universe → Expansion flattens the effect → Isotropy
- More generic model, we need less primordial symmetries.
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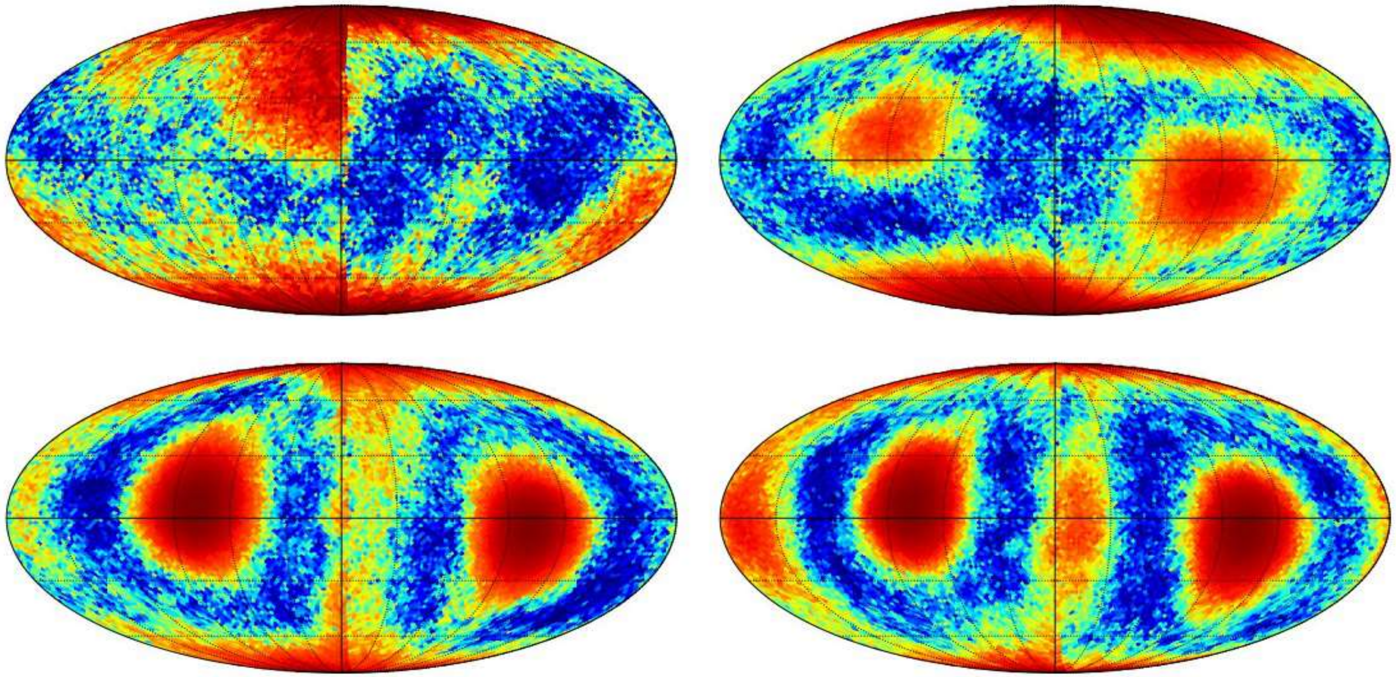
- Can anisotropy account for the origin of “depart from explanation” of primordial universe behaviour? Some observational data [2] suggest anomalies at large scales.



[2] A Durakovica, *et al*. (2018) *Reconstruction of a direction-dependent primordial power spectrum from Planck CMB data,* JCAP 1802

Quantum Primordial Universe

- End goal: Full description of primordial Universe → Quantum description
- Quantization: Replace singularity by quantum bounce.
- Interplay between isotropic and anisotropic variables → Complex quantum dynamics.

Mathematical description: Hamiltonian formulation

· **Bianchi IX metric**:

$$\mathrm{d}s^2 = -\mathcal{N}^2 \mathrm{d}\tau^2 + \sum_i a_i^2 \underbrace{(\omega^i)^2}$$

$\mathcal{N}$: Lapse function

$(\omega^i)^2$: Spatial hypersurface: S^3 topology, Closed universe

with: $\xi_i \cdot \omega^j = \delta_i^j$

↳ Killing vector: $SO(3, \mathbb{R})$ isometry group generator

$a_i(t)$: 3 different principal direction scale factors (anisotropy)

Mathematical description: Hamiltonian formulation

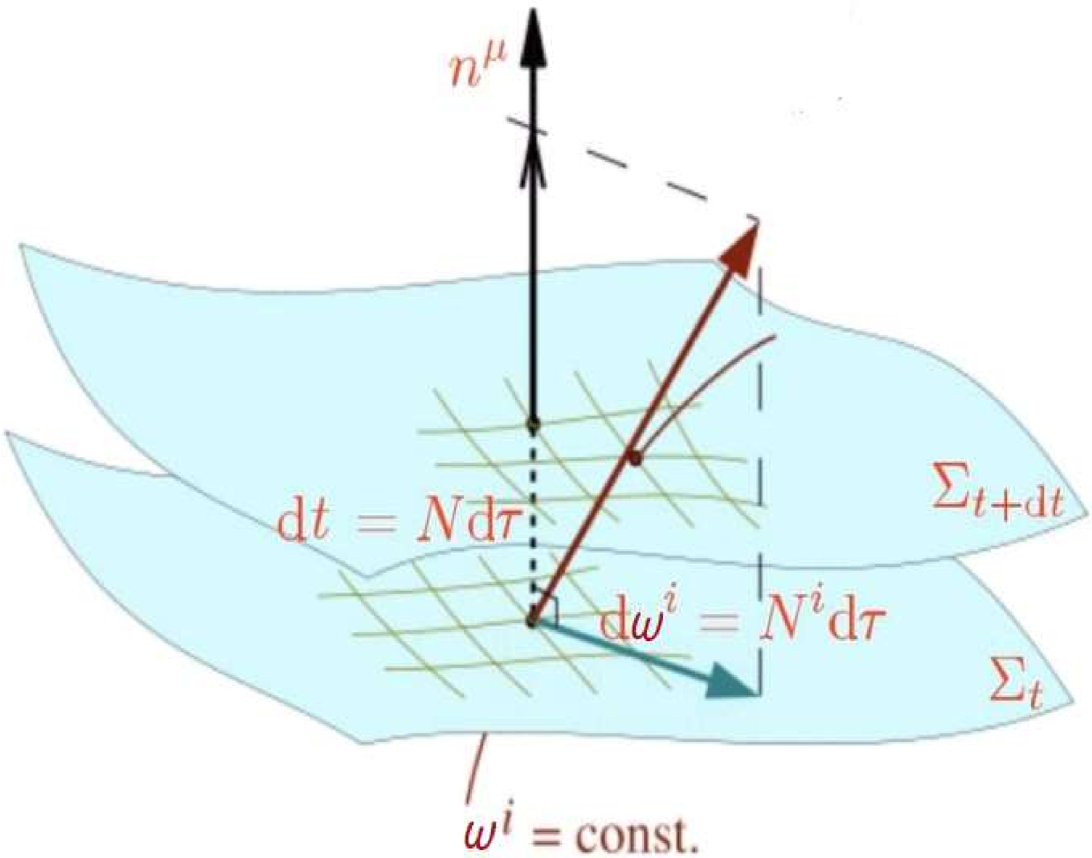
Extrinsic curvature:

$$K^{ij} = \frac{1}{2\mathcal{N}}(2\nabla^{(i}N^{j)} - \dot{\gamma}^{ij})$$

3+1 ADM formalism: $\mathrm{d}s^2 = -\mathcal{N}^2\mathrm{d}\tau^2 + \sum_i \gamma_{ij}(\omega^i + N^i\mathrm{d}\tau)(\omega^j + N^j\mathrm{d}\tau)$

Phase space: Bianchi IX 3-metric: a_i + 3-momemtum: $\pi^{ij} = \sqrt{\gamma(K^{ij} - K\gamma^{ij})}$

Hamiltonian constraint: $H = \mathcal{N}\sqrt{\gamma}\left((-^{(3)}R + \gamma^{-1}(\pi_{ij}\pi^{ij} - \frac{1}{2}\pi^2)\right)$



Mathematical description: Hamiltonian formulation

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Using **Misner Variables**: $\Omega = \frac{1}{3}\ln a_1a_2a_3,\ \beta_+ = \frac{1}{6}\ln\frac{a_1a_2}{a_3^2},\ \beta_- = \frac{1}{2\sqrt{3}}\ln\frac{a_1}{a_2}$

\+ Conjugate momenums:

$$\begin{pmatrix} a_1p_1 \\ a_2p_2 \\ a_3p_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{6} & \frac{1}{12} & \frac{\sqrt{3}}{12} \\ \frac{1}{6} & \frac{1}{12} & -\frac{\sqrt{3}}{12} \\ \frac{1}{6} & -\frac{1}{6} & 0 \end{pmatrix} \begin{pmatrix} p_\Omega \\ p_+ \\ p_- \end{pmatrix}$$

Isotropic geometry (Ω, p_Ω)

Anisotropic variables (β_+, β_-, p_+, p_-)

Canonical transformation for isotropic variables: $q = e^{\frac{3}{2}\Omega},\ p = \frac{2}{3}e^{-\frac{3}{2}\Omega}p_\Omega$

($q > 0$ always)

Mathematical description: Hamiltonian formulation

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Hamiltonian Constraint in Misner variables:

$$C = -\frac{9}{4}p^2 - 36q^{\frac{2}{3}} + \frac{\mathbf{p}_\pm^2}{q^2} + 36q^{\frac{2}{3}}V(\boldsymbol{\beta}_\pm)$$

→ Particle in 3D Minkowski s-t. with time dependent potential

Where: $V(\boldsymbol{\beta}) = \frac{e^{4\beta_+}}{3}\left[\left(2\cosh(2\sqrt{3}\beta_-) - e^{-6\beta_+}\right)^2 - 4\right] + 1$

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Isotropic geometry

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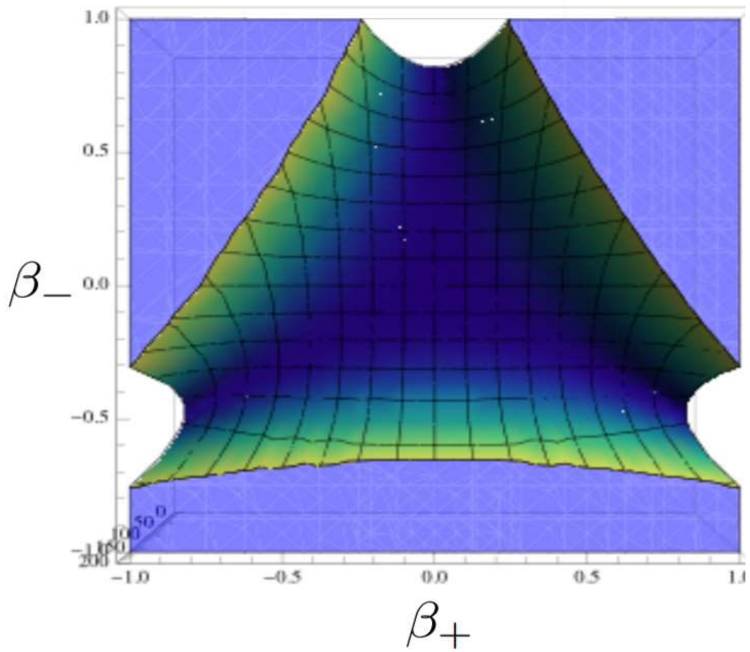

Quantization and semicassical portrait

- Isotropic canonical variables: $(q\,,p)\in\mathbb{R}_+^*\times\mathbb{R}$ → Positive half-plane ⚠

↓

Affine group structure :

Covariant affine group quantization, [3,4]
and semiclassical portrait
using affine coherent states

Bounce

p^2 → $\widecheck{(p^2)} = p^2 + \frac{K(\nu)}{q^2}\,,$ q^α → $\widecheck{(q^\alpha)} = Q_\alpha(\nu) q^\alpha\,,$

(quantization/semiclassical coefficients) $K(\nu) = \frac{K_1(\nu)^2\left(1+\nu\frac{K_0(\nu)}{K_1(\nu)}\right)}{4K_0(\nu)K_2(\nu)}\,,\quad Q_\alpha(\nu) = \frac{K_\alpha(\nu)K_{\alpha+1}(\nu)}{K_0(\nu)K_1(\nu)}.$

[3] H. Bergeron, *et al,* (2020), *Quantum Mixmaster as a Model of the Primordial Universe,* Universe **6, 7**.
[4] J.-P. Gazeau, *et al.* (2016) *Covariant affine integral quantization(s)*, J. Math. Phys.**57,** 052102

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- Anisotropic variables: $(\beta_\pm, p_\pm) \in \mathbb{R}^2$ → Full plane

↓

Covariant Weyl-Heisenberg integral quantization

Weyl-Heisenberg group:

Semiclassical portrait:

$$p_\pm^2 \longrightarrow \widecheck{(p_\pm^2)} = p_\pm^2 + \frac{8}{\sigma_\pm^2}, \qquad V(\beta_\pm) \longrightarrow \check{V}(\beta_\pm) = \frac{1}{3}\left(D(4\sqrt{3},4)e^{4\sqrt{3}\beta_- + 4\beta_+} + D(4\sqrt{3},4)e^{-4\sqrt{3}\beta_- + 4\beta_+} + D(0,8)e^{-8\beta_+}\right)$$
$$- \frac{2}{3}\left(D(2\sqrt{3},2)e^{-2\sqrt{3}\beta_- - 2\beta_+} + D(2\sqrt{3},2)e^{2\sqrt{3}\beta_- - 2\beta_+} + D(0,4)e^{4\beta_+}\right) + 1$$

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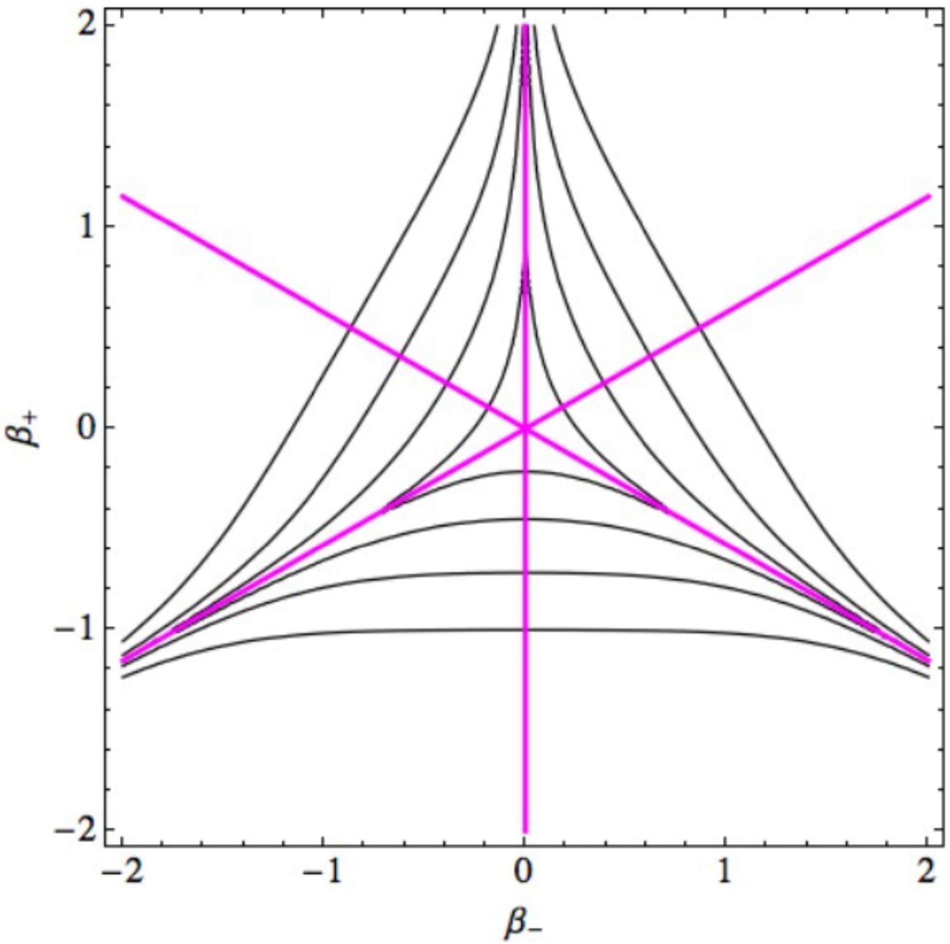

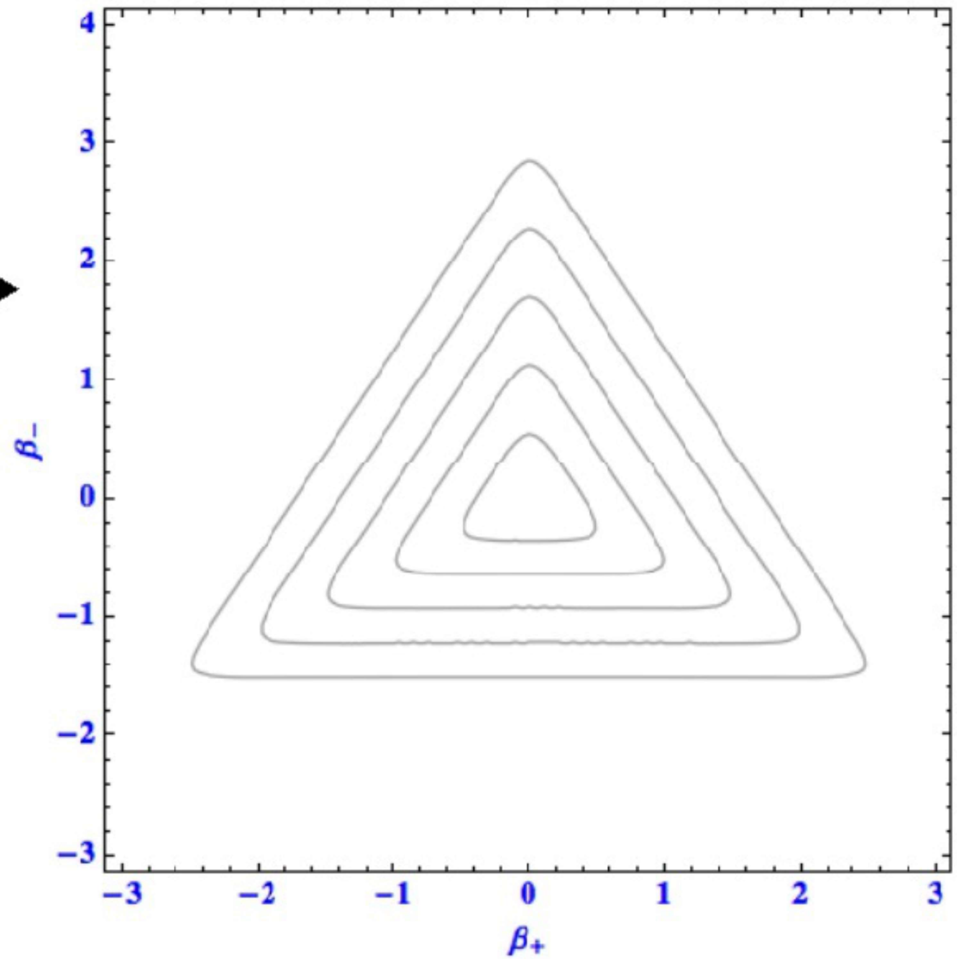

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Semiclassical portrait of full Hamiltonian constraint

$$\check{\mathrm{C}} = \frac{9}{4}\left(p^2 + \frac{K(\nu)}{q^2}\right) - Q_{-2}(\nu)\frac{p_\pm^2 + \frac{8}{\sigma_\pm^2}}{q^2} - 36Q_{\frac{2}{3}}(\nu)q^{\frac{2}{3}}[\check{V}(\boldsymbol{\beta}) - 1] - \frac{R}{q^{2/3}}$$

$\xi, \nu, \sigma_\pm, \omega_\pm$ → 6 quantization + semiclassical parameters

Hamilton equations:

$$\dot{q} = \frac{9}{2}p\,,$$

$$\dot{p} = \frac{9}{2}\frac{K}{q^3} - 2Q_{-2}\frac{p_\pm^2 + \frac{8}{\sigma_\pm^2}}{q^3} + 24Q_{\frac{2}{3}}q^{-\frac{1}{3}}[\check{V}(\boldsymbol{\beta}) - 1] - \frac{2}{3}Rq^{-\frac{5}{3}}\,,$$

$$\dot{\beta}_\pm = -2Q_{-2}\frac{p_\pm}{q^2}\,,$$

$$\dot{p}_\pm = 36Q_{\frac{2}{3}}q^{\frac{2}{3}}\partial_\pm\check{V}(\boldsymbol{\beta})\,,$$

(we added Radiation)

- Very rich model:
 Let's investigate, numerically, the effects of interplay between anisotropy and quantum bounce.

Solution for Isotropic case: $\beta_\pm = 0 = p_\pm$

$$\check{C}_{isotropic} = \frac{9}{4}\left(p^2 + \frac{K(\nu) - \frac{4}{9}Q_{-2}(\nu)\frac{8}{\sigma_\pm^2}}{q^2}\right) + 36Q_{\frac{2}{3}}(\nu)q^{\frac{2}{3}} - \frac{R}{q^{2/3}}$$

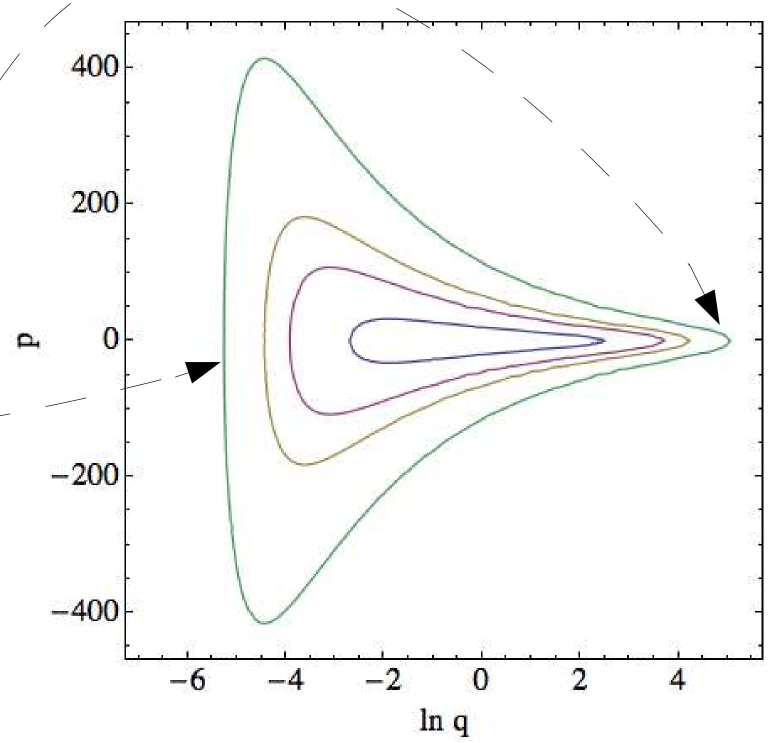
Let's call for simplicity: $L := 36Q_{\frac{2}{3}}$ (intrinsic isotropic curvature) $K_{eff} := K - \frac{4}{9}Q_{-2}\frac{8}{\sigma_\pm^2}$ (isotropic repulsive strength)

Analytical aproximations for quantum bounce and classical recolapse:

$$q_{min} = \left(\frac{9K_{iso}}{4R}\right)^{\frac{3}{4}}, \qquad q_{max} = \left(\frac{R}{L}\right)^{\frac{3}{4}}$$

Phase-space solutions:

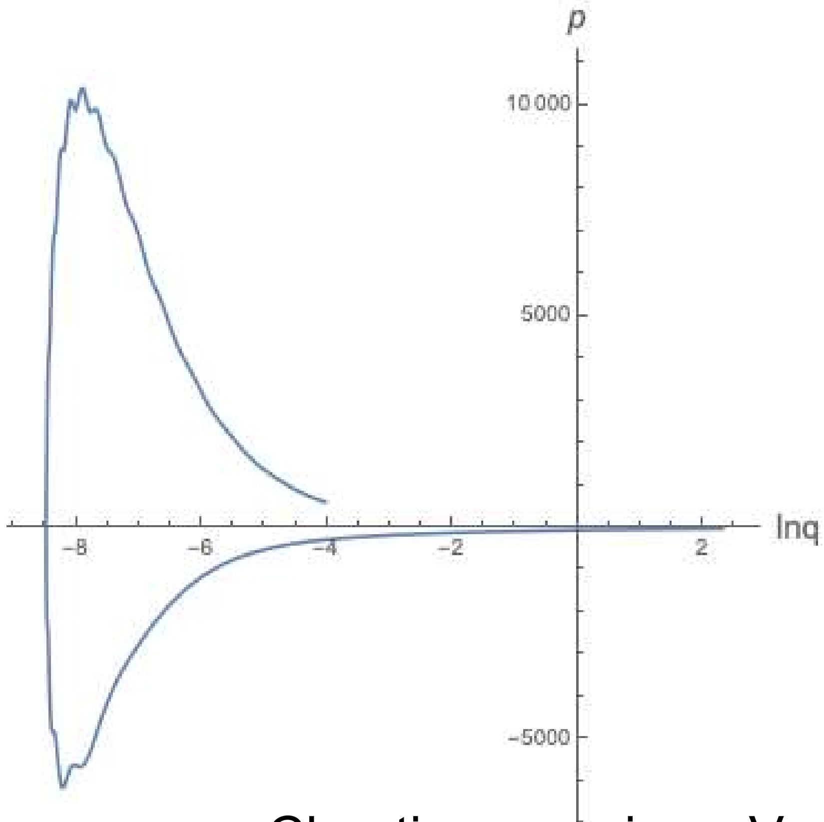
(for different R radiation contents)

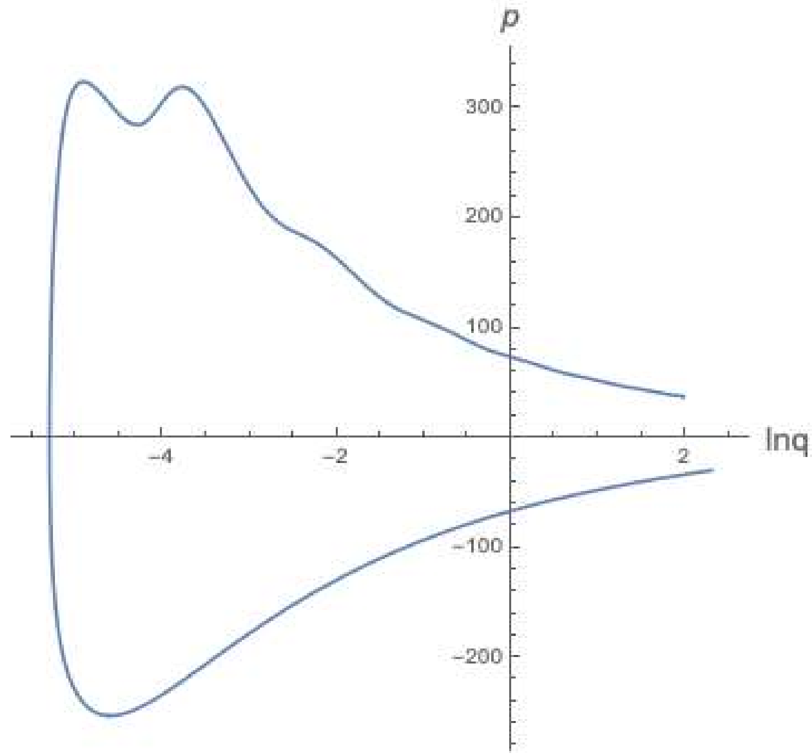


Full anisotropic solution:

$$\mathrm{C} = \frac{9}{4}\left(p^2 + \frac{K_{eff}}{q^2}\right) - M\frac{p_{\pm}^2}{q^2} - Lq^{\frac{2}{3}}[V-1] - \frac{R}{q^{\frac{2}{3}}}$$

Phase space of isotropic variables: <u>Asymmetric bounce</u> of the universe, due to increase of the role of anisotropy energy.
→ <u>Extra boost to the post-bounce expansion</u>





Chaotic scenario→ Very sensitive to initial conditions

(+ we have 6 quantization/semiclassical parameters)

A realistic scenario:

Generalised Friedmann equation:

$\check{\rho}_{ani}$ = Anisotropic Kinetic term (σ^2) + Anisotropic Potential term (R_{ani})

$$H^2 = \frac{1}{6}\rho_r - \frac{1}{6}\check{R}_{iso} + \frac{1}{3}\check{\sigma}^2 - \frac{1}{6}\check{R}_{ani} - \frac{1}{6}\check{R}_Q$$

(radiation)

Quantum Curvature (R_Q)

$$\check{R}_{iso} = \frac{3Q_{\frac{2}{3}}}{2q^{\frac{4}{3}}}, \quad \check{R}_{ani} = -\frac{3Q_{\frac{2}{3}}\check{V}(\boldsymbol{\beta})}{2q^{\frac{4}{3}}}, \quad \check{\sigma}^2 = \frac{Q_{-2}\mathbf{p}^2}{48q^4}, \quad \check{R}_Q = \frac{3K_{eff}}{32q^4},$$

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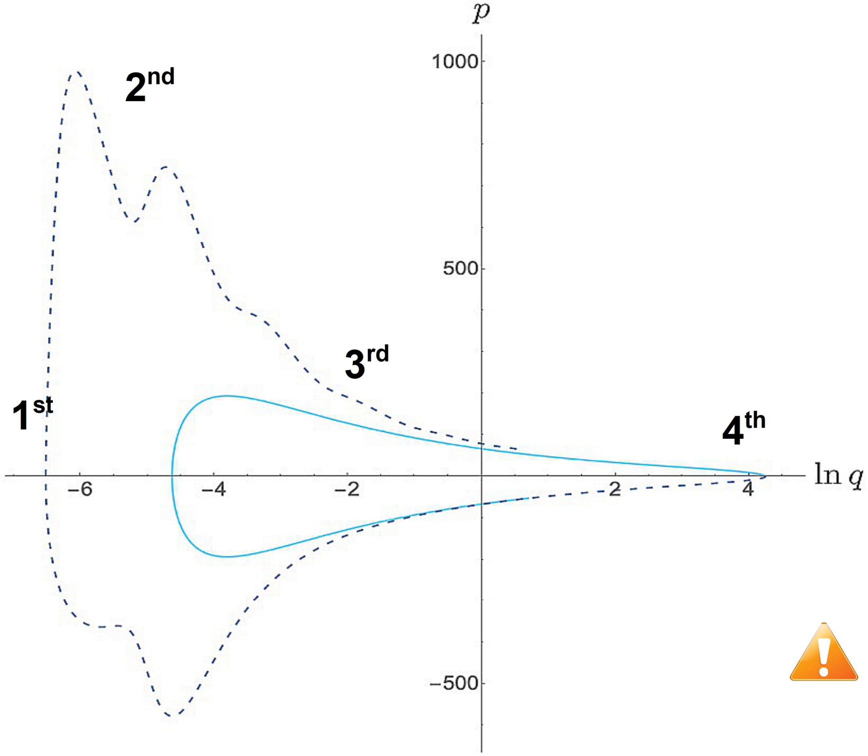
$$H^2 = \frac{1}{6}\rho_r - \frac{1}{6}\check{R}_{iso} + \frac{1}{3}\check{\sigma}^2 - \frac{1}{6}\check{R}_{ani} - \frac{1}{6}\check{R}_Q$$

(radiation)

Quantum Curvature (R_Q)

$$\check{R}_{iso} = \frac{3Q_{\frac{2}{3}}}{2q^{\frac{4}{3}}}, \quad \check{R}_{ani} = -\frac{3Q_{\frac{2}{3}}\check{V}(\beta)}{2q^{\frac{4}{3}}}, \quad \check{\sigma}^2 = \frac{Q_{-2}\mathbf{p}^2}{48q^4}, \quad \check{R}_Q = \frac{3K_{eff}}{32q^4},$$

Initially (close to the bounce) we want: $\check{R}_{Quantum} > \check{\rho}_{ani} > \rho_r \gg \check{R}_{iso}$



1st: Quantum/Semiclassical Repulsion dominates → Expansion

2nd: Anisotropy plays a significant role → non-trivial Asymmetry

3rd: Transition to radiation, matter dominates dynamics.

4th: Classical recolapse.

⚠ But still different initial values of variables and parameters can reproduce this scenario → Different $\beta\pm$ trajectories inside the potential = Different shapes

Inflationary-expansion behaviour?

The Big Question: Can anisotropy make the phase of accelerated expansion to last long enough?

For inflationary scenario: $\ddot{a} > 0 \rightarrow \dot{\mathcal{H}} > 0$

Increasing # modes leaving the horizon (super-Hubble)

Friedmann equation during inflation: $H^2 = \underbrace{\frac{1}{6}\check{\rho}_{ani} - \frac{1}{6}\check{R}_Q}_{} + ... > 0$ where $\frac{1}{6}\check{\rho}_{ani} = \frac{E_{ani}}{a^{2+n_{ani}}}$

Driving terms during inflation (quantum + anisotropy in our model)

Power law approximation of the scale factor for the anisotropy term

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e-folds $\Delta N = \ln\left(\frac{a_{fin}}{a_{ini}}\right)$

Above conditions translates into: $n_{ani} = 4e^{-4\Delta N}$ → If during inflation n_{ani} is mantained at that value, the phase will last for ΔN e-folds

In the standard inflationary scenario (from observations): $\Delta N \approx$ 20 e-folds → Very small n_{ani} → We want: $\check{\rho}_{ani} \propto \frac{1}{a^2}$

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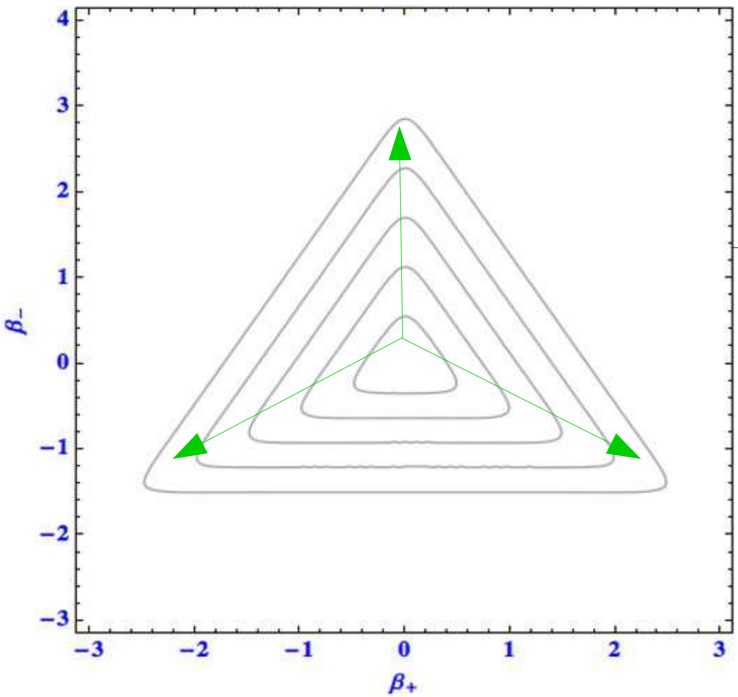
Our Friedmann equation: $\mathrm{H}^2 = -\frac{1}{64}\frac{K_{eff}}{a^6} + \frac{M}{144}\frac{p_{\pm}^2}{a^6} + \frac{L}{144}\frac{[V-1]}{a^2} + \frac{1}{144}\frac{R}{a^4}$

$E_{ANI} \cdot a^{2+n_{ani}}$

We have to make this potential V to drive the dynamics just after $\Omega_{Quantum}$ domination, to have $\sim a^{-2}$ extended expansion

Specific situation for the system:
Extremal case for inflationary behaviour

How to make potential increase?



$V(\beta_\pm) \longrightarrow$ Very steep triangular walls, with

<u>3 flatter canyons</u> and very flat central part.

Length and flatness of the (closed) canyons in the vertex modulated by semiclassical parameter $\boldsymbol{\omega}_\pm$

We throw the particle in the exact direction of the canyons
to make the value of the potential big and ~constant at the same time
for the longest time possible, while momentum becomes small.
Afterwards it will roll down in the opposite direction.

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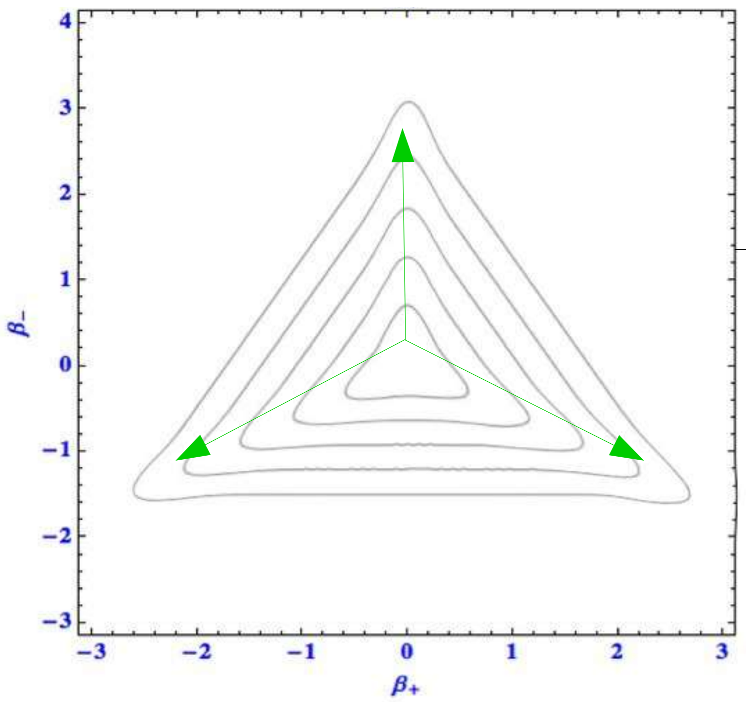

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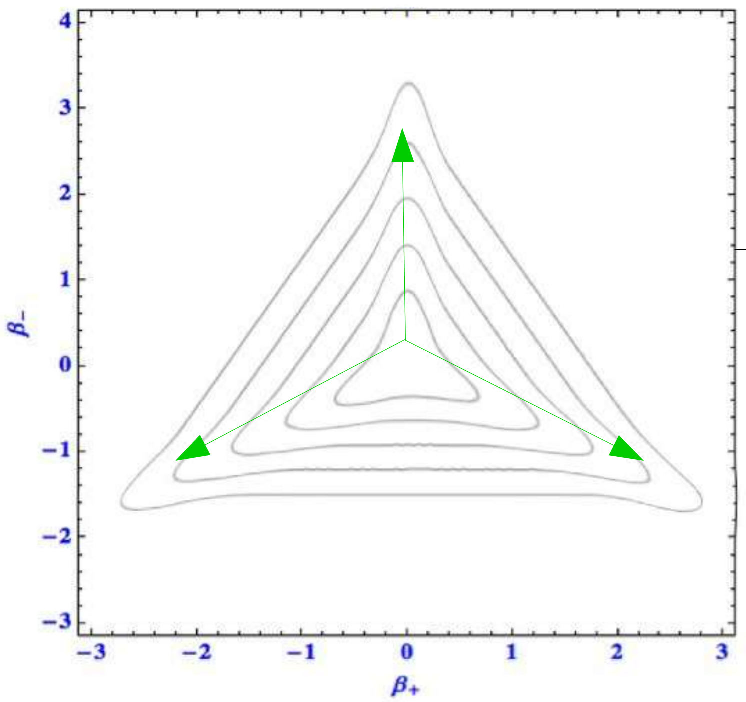

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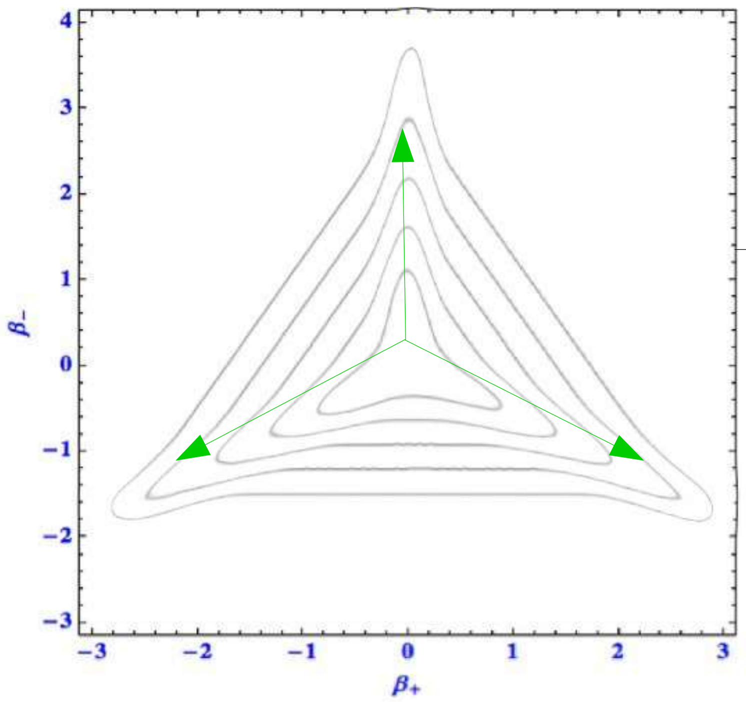

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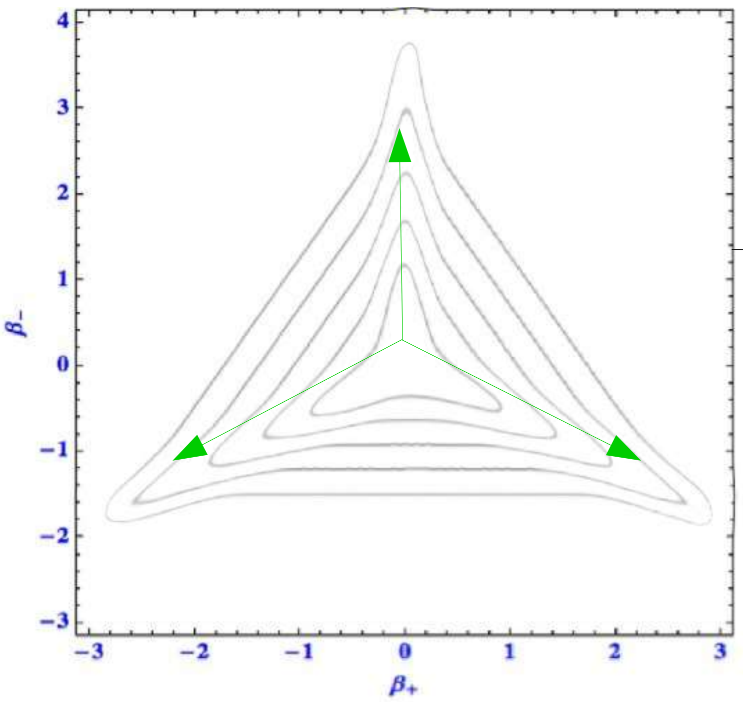

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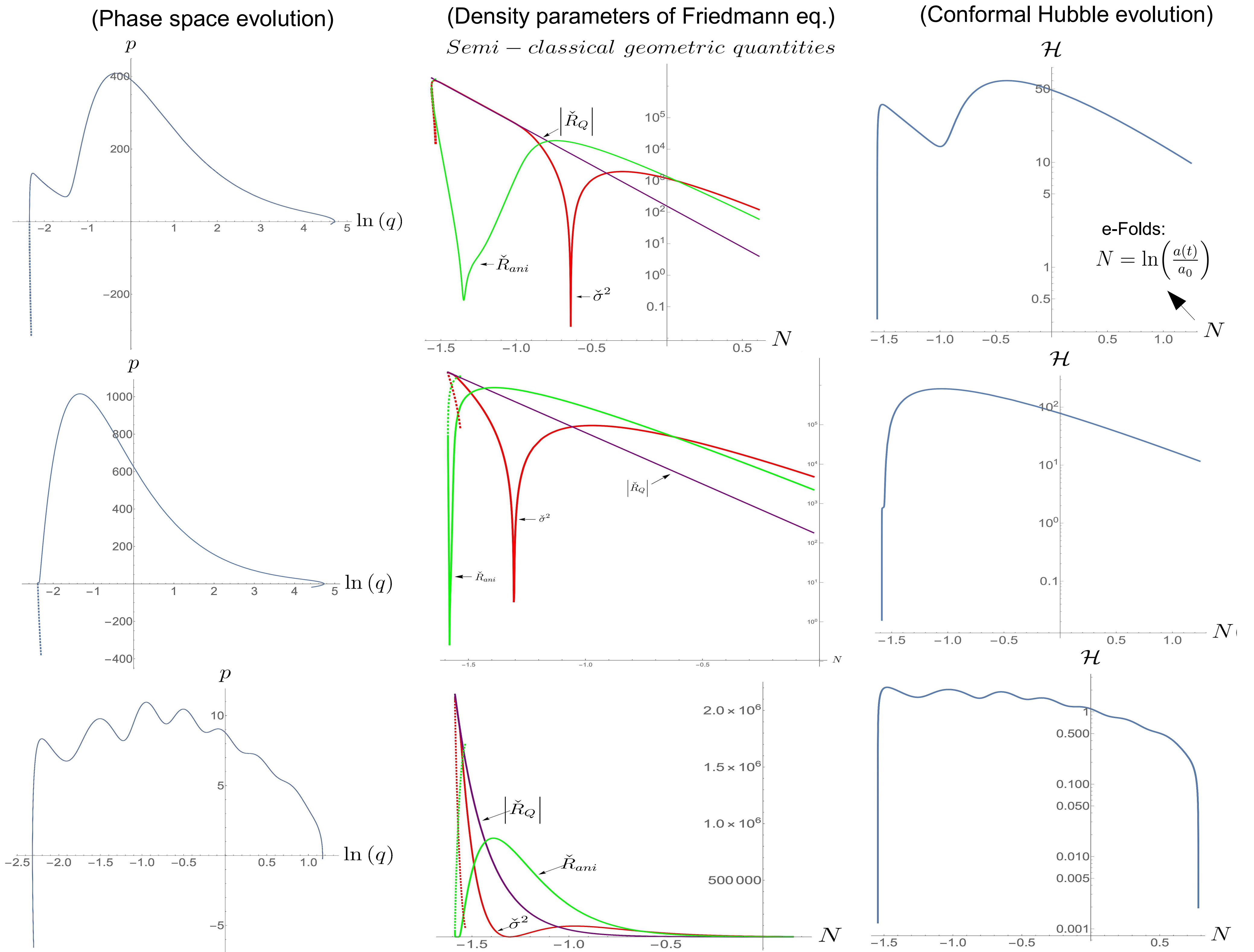
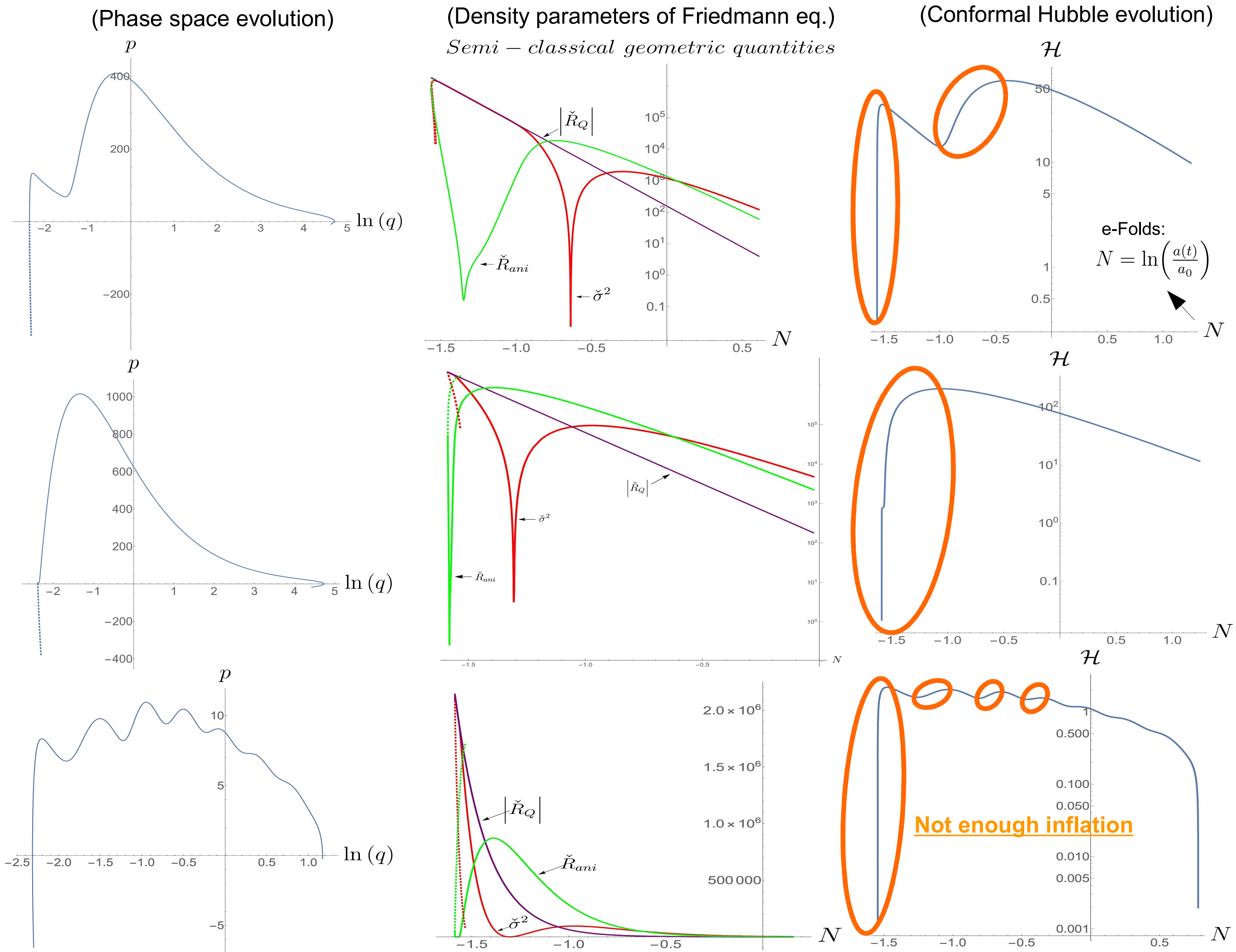


(Phase space evolution)
(Density parameters of Friedmann eq.)
Semi − classical geometric quantities
(Conformal Hubble evolution)
p
ln (q)
$|\check{R}_Q|$
$\check{R}_{ani}$
$\check{\sigma}^2$
N
$\mathcal{H}$
e-Folds:
$N = \ln\left(\frac{a(t)}{a_0}\right)$



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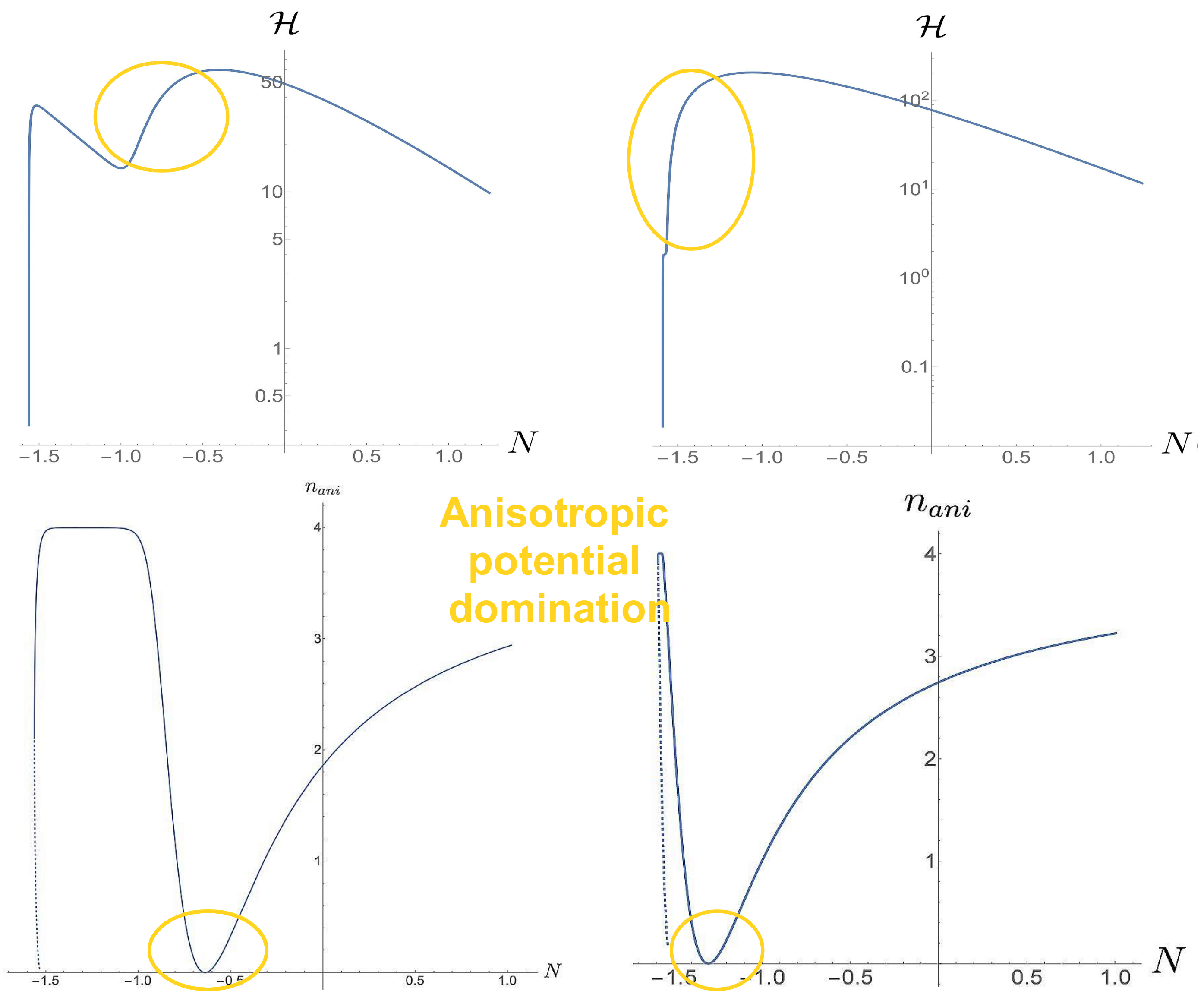
Not enough inflation

Why not enough?

(Conformal Hubble evolution)

(n_{ani} evolution $\in (0, 4)$)



Another explanation (from equations of motion of anisotropy variables):

Imposing relative change in the potential to be small during ΔN e-folds gives:

$$\frac{\check{V}_{,\pm}}{\check{V}} \ll \frac{\sqrt{Q_{\frac{2}{3}} Q_{-2}}}{\Delta N}.$$

→ RHS is very small

However in our potential we have:

$$2 < \frac{|\check{V}_{,\pm}|}{|\check{V}|} < 8$$

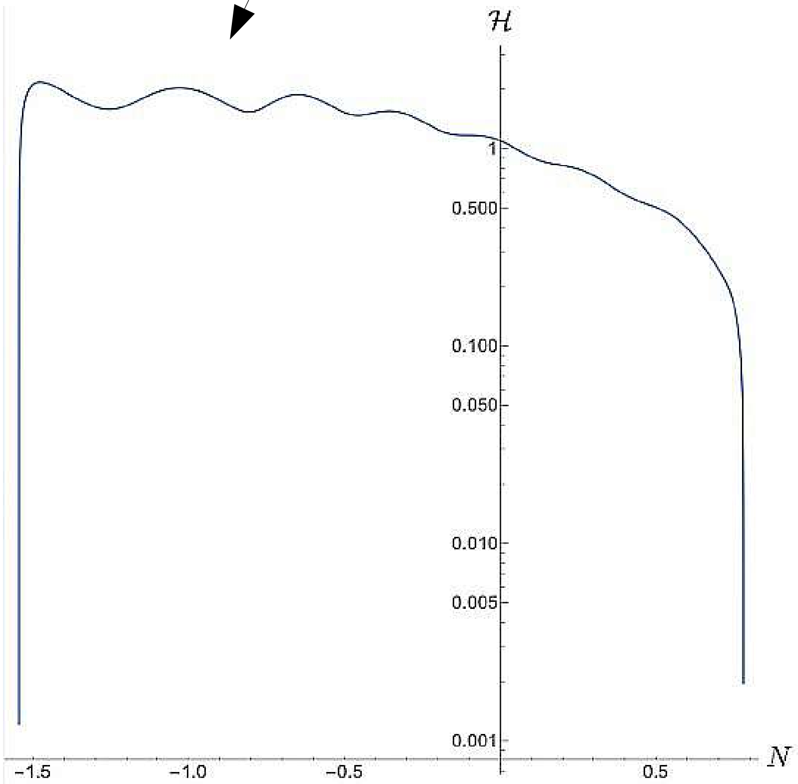
(Except close to the origin $\beta_{\pm}=0$ where the potential is very flat but small)

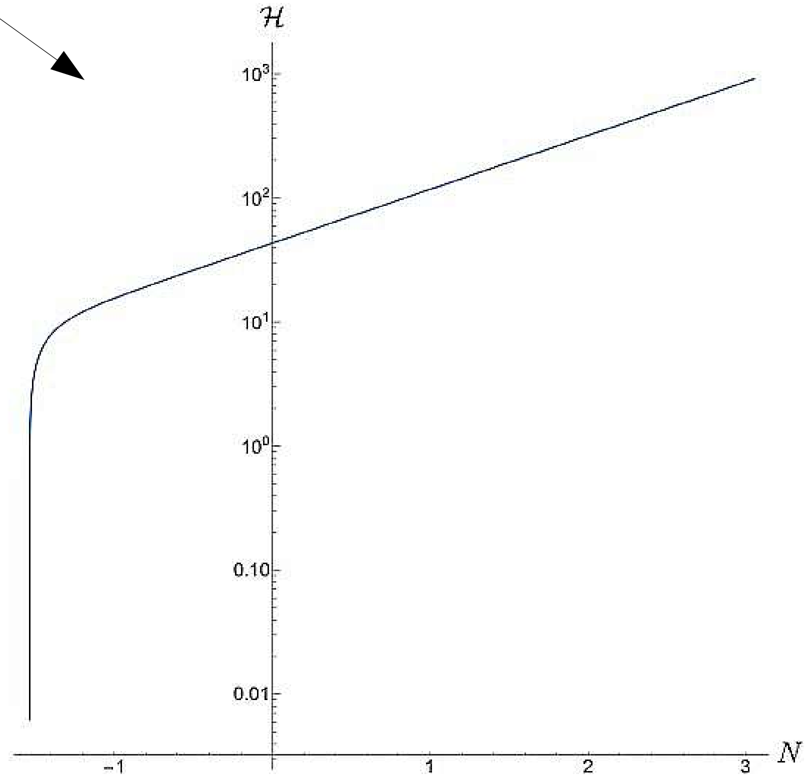
Alternatives where there is enough inflation:

- Same model, harmonic approximation of potential → $\frac{|V_{,\pm}|}{V} = \frac{2}{|\beta|}$

Found in previous work: H.Bergeron, E.Czuchry, J.-P.Gazeau, and P. Malkiewicz,
Nonadiabatic bounce and an inflationary phase in the quantum mixmaster universe,
Phys.Rev. D93(2016)124053.

- Replace anisotropy by standard inflaton with quadratic potential





Conclusions and Future Investigations:

- Very simple model → Rich dynamics, many possibilities.

- Solve singularity problem → Quantum Bounce

- Anisotropy + bounce by themselves do not generate sufficient inflationary dynamics.

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- Anisotropy + bounce by themselves do not generate sufficient inflationary dynamics.

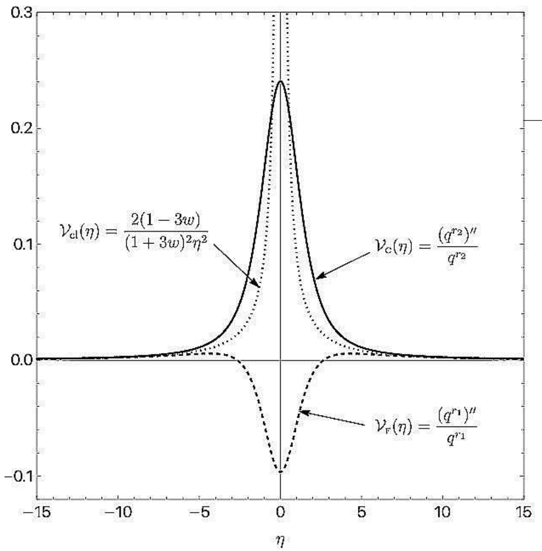
(Does not mean we cannot generate structures)

→ **BUT**: Might be the seed for future investigations:

- Generation of different gravitational potential?
- Interplay with primordial perturbations?

→ **Another approach:** Full quantum model → Maybe semiclassicality erase some features close to the bounce. Full quantum is more complicated.

Gravitational potentials $\frac{a''}{a}$ for other previously studied *isotropic* models:



→ Isotropic bouncing models + perturbations give this kind of gravitational potential → Generation of cosmological structures.

The primordial spectrum is nearly scale invariant but slightly blue-tilted for $w > 0$ → Can anisotropy improve this?

(It can produce an effective cosmological fluid parameter $w_{\rm ani}$ for the equation of state)

[5] J. de Cabo Martin, P. Małkiewicz, and P. Peter, (2021) [arXiv:2111.02963]

Thank you for your attention!