THE TIME PROBLEM IN PRIMORDIAL PERTURBATIONS*

PhD seminar

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* "The time problem in primordial perturbations"– **A.B.**, P. Peter, P. Małkiewicz, soon on ArXiv

Time problem in physics

Time problem in physics

Clock
in physics

Time problem in physics

Clock
in physics

Reduction
formalism

Time problem in physics

Clock
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Canonical model of
primordial
spacetime

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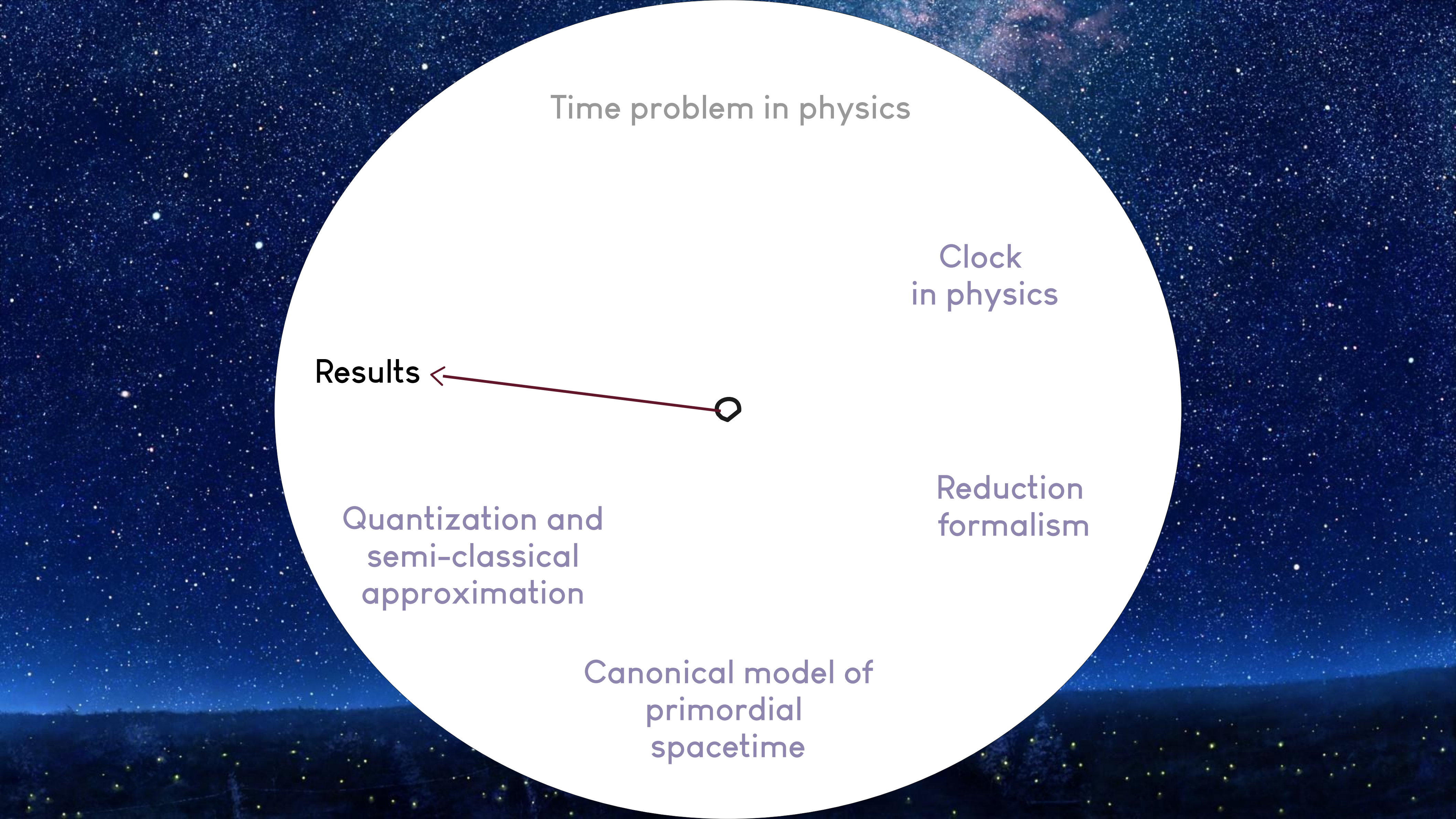
Quantization and
semi-classical
approximation


Time problem in physics
Clock in physics
Results
Reduction formalism
Quantization and semi-classical approximation
Canonical model of primordial spacetime

Time problem in physics

Clock in physics

Reduction formalism

Canonical model of primordial spacetime

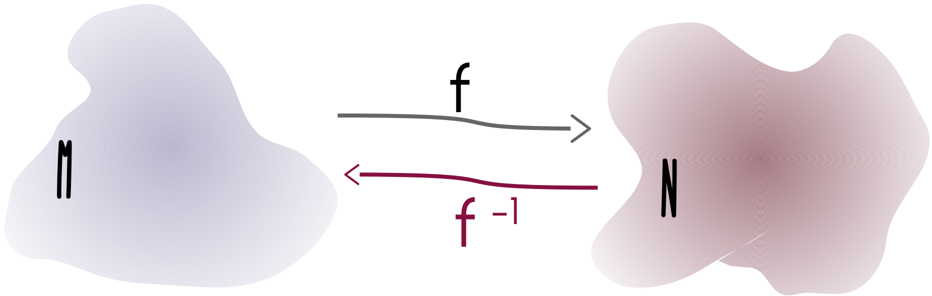
Quantization and semi-classical approximation

Results

Conclusions

TIME PROBLEM IN PHYSICS

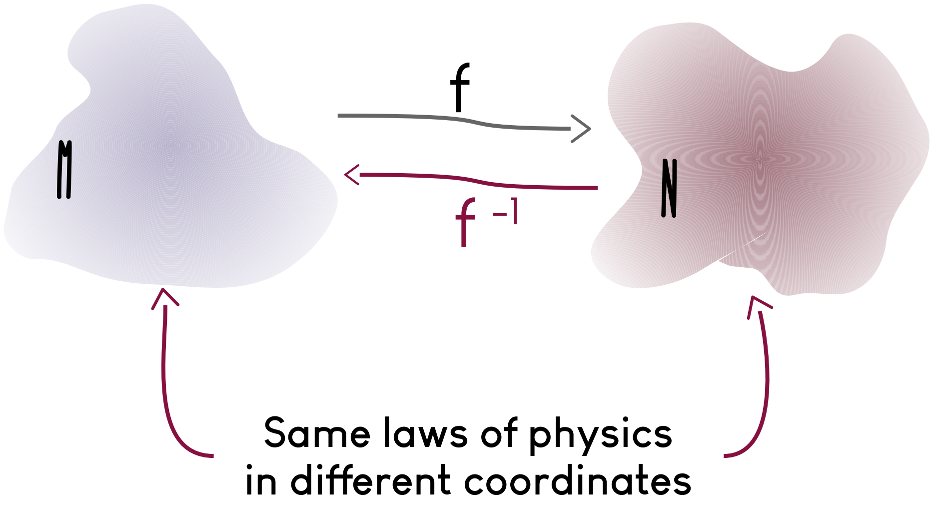
GR is diffeomorphism **invariant**, thus the free choice of internal time variable has no physical consequence.



$$\boxed{f, f^{-1} \in C^{\infty}}$$

TIME PROBLEM IN PHYSICS

GR is diffeomorphism invariant, thus the free choice of internal time variable has no physical consequence.



TIME PROBLEM IN PHYSICS

GR is **diffeomorphism invariant,** thus the free choice of internal time variable has no physical consequence.

Upon passing to **quantum theory**, however, different choices of internal time variables are known to produce unitarily inequivalent quantum models.

The problem of finding the correct interpretation of these non-equivalent models is commonly known as **the time problem**.

TIME PROBLEM IN PHYSICS

“The prime source of the problem of time in quantum gravity is the invariance of classical general relativity under the group Diff(M) of diffeomorphisms of the spacetime 4-manifold M.
This stands against the simple Newtonian picture of a fixed time parameter, and tends to produce quantisation schemes that apparently lack any fundamental notion of time at all.

From this perspective, the heart of the problem is contained in the following questions”:

Canonical Quantum Gravity and the Problem of Time - Chris J. Isham 10.48550/ARXIV.GR-QC/9210011

TIME PROBLEM IN PHYSICS

1. How should the notion of time be re-introduced into the quantum theory of gravity?

Canonical Quantum Gravity and the Problem of Time - Chris J. Isham 10.48550/ARXIV.GR-QC/9210011

TIME PROBLEM IN PHYSICS

1. **How should the notion of time be re-introduced into the quantum theory of gravity?**

The absence of time in a quantum theory might not be a problem after all as any **internal variable** chosen in our theory will give the same prediction in the classical limit.

Canonical Quantum Gravity and the Problem of Time - Chris J. Isham 10.48550/ARXIV.GR-QC/9210011

TIME PROBLEM IN PHYSICS

2. Should attempts to identify time be made at the classical level, i.e., before quantisation, or should the theory be quantised first?

Canonical Quantum Gravity and the Problem of Time - Chris J. Isham 10.48550/ARXIV.GR-QC/9210011

TIME PROBLEM IN PHYSICS

2. **Should attempts to identify time be made at the classical level, i.e., before quantisation, or should the theory be quantised first?**

In our method we choose an **internal time variable** at the **classical level, prior to quantisation,** this is called **reduced phase space quantisation.** The problem of time would not change if we were to use the **Dirac quantisation**.

Canonical Quantum Gravity and the Problem of Time - Chris J. Isham 10.48550/ARXIV.GR-QC/9210011

TIME PROBLEM IN PHYSICS

Open question

3. Can 'time' still be regarded as a fundamental concept in a quantum theory of gravity, or is its status purely phenomenological?

Canonical Quantum Gravity and the Problem of Time - Chris J. Isham 10.48550/ARXIV.GR-QC/9210011

TIME PROBLEM IN PHYSICS

Open question

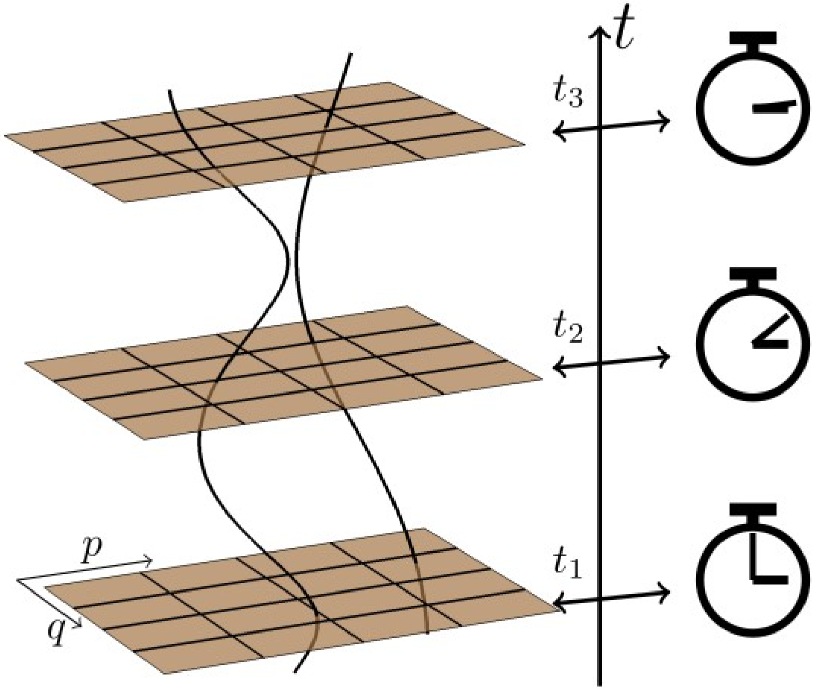
4. If 'time' is only an approximate concept, how reliable is the rest of the quantum-mechanical formalism in those regimes where the normal notion of time is not applicable?

In particular, how closely tied to the concept of time is the idea of probability?

This is especially relevant in those approaches to quantum gravity in which the notion of time emerges only after the theory has been quantised.

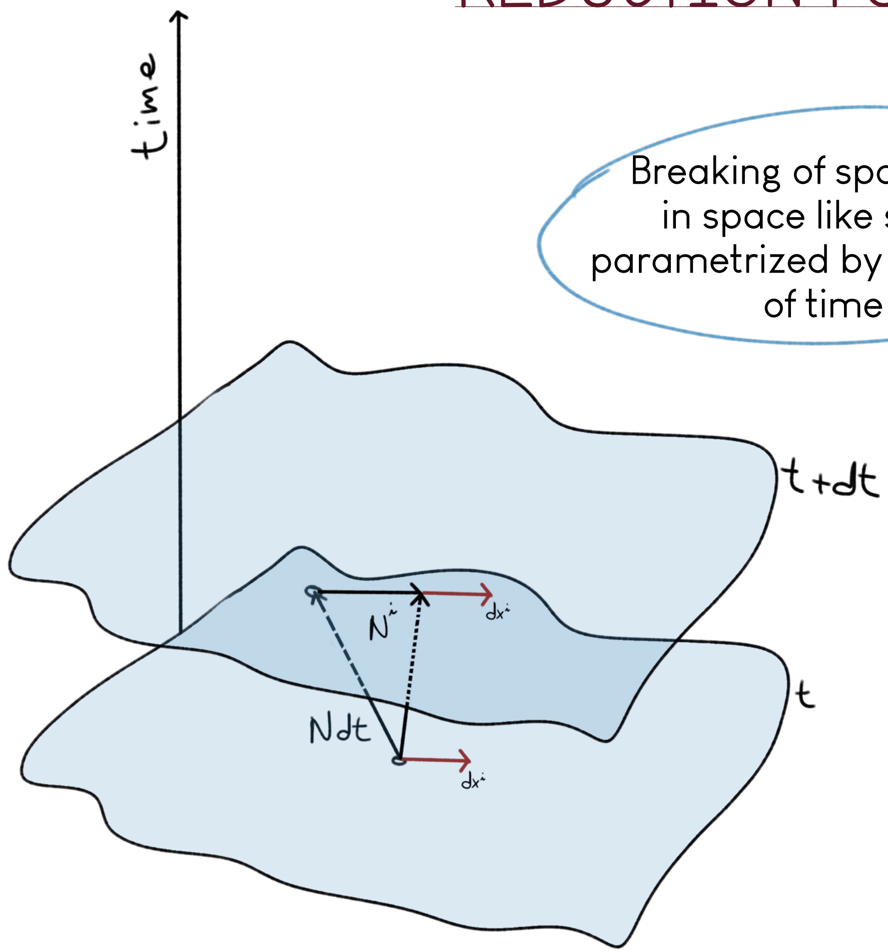
Canonical Quantum Gravity and the Problem of Time - Chris J. Isham 10.48550/ARXIV.GR-QC/9210011

CLOCK IN PHYSICS



Internal clock formulation of quantum mechanics- P. Małkiewicz, A. Miroszewski 10.1103/physrevd.96.046003

REDUCTION FORMALISM



Breaking of space time in space like sheets parametrized by a function of time

Lapse: Measures the velocity with which the proper time varies from one sheet to the other

Shift vector: Measures shift of local coordinates

Lapse function N

Shift vector N^i

NOT DYNAMICAL

The evolution from one sheet to the other extremizes the action

[R. Wald, general relativity]

REDUCTION FORMALISM

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

Spatial metric of a hypersurface

Einstein-Hilbert action $$\boxed{S_{gravity} = \frac{1}{2\kappa}\int R\sqrt{-g}\,d^4x}$$

$$\delta S_{tot} = \frac{1}{16}\int \left(\pi^{ij}\frac{\delta g_{ij}}{\delta t} - N\mathcal{H}(g_{ij},\pi^{ij}) - N^i\mathcal{H}_i(g_{ij},\pi^{ij})\right)d^4x + \int \delta\mathcal{L}_{field}\,d^4x$$

Lagrange multipliers

[R. Wald, general relativity]

REDUCTION FORMALISM

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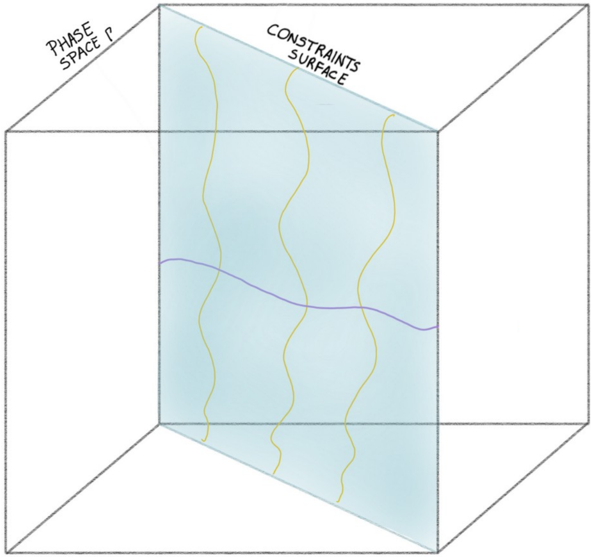
Constraints

[R. Wald, general relativity]

REDUCTION FORMALISM

Hamiltonian constraint

$$C(q_I, p^J) = 0$$



Assume q_0 varies monotonically with the evolution generated by the constraint

$$\{q_0, C\}_{\mathrm{PB}} \neq 0$$

we can assign to it the role of the **internal clock** in which the evolution of the remaining variables occurs

The dynamics reads

$$\boxed{\frac{\mathrm{d}q_I}{\mathrm{d}q_0} = \frac{\partial H}{\partial p_I}} \qquad \boxed{\frac{\mathrm{d}p^I}{\mathrm{d}q_0} = -\frac{\partial H}{\partial q^I}}$$

Let's call $q_0 = t$

REDUCTION FORMALISM

We can define a new clock as function of the old clock and the old canonical variables

$$\tilde{t} = \tilde{t}(q_I, p^I, t)$$

There must exist an invertible map between the **old** and the **new** variables.
We find a complete set of **canonical constants of motion**, denoted by

$$D_I \; = \; D_I(q_J, p^J, t)$$

KEY FORMULA

$$\boxed{\tilde{t} = \tilde{t}(q_I, p^I, t), \quad D_I(q_J, p^J, t) = D_I(\tilde{q}_J, \tilde{p}^J, \tilde{t})}$$

CANONICAL MODEL OF PRIMORDIAL SPACETIME

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)(\delta_{ij} + h_{ij}(x,t))\mathrm{d}x^i\mathrm{d}x^j$$

Flat Friedmann universe filled with **radiation** (w=1/3) and perturbed by gravitational waves

$$H = H^{(0)} + \sum_k H_k^{(2)}$$

$$H^{(0)} = \frac{1}{2}p_a^2$$

Hamiltonian in reduced phase space

$$H_k^{(2)} = -\frac{1}{2}|\pi_\pm(\vec{k})|^2 - \frac{1}{2}\left(k^2 - \frac{\ddot{a}}{a}\right)|\mu_\pm(\vec{k})|^2$$

Expansion of the gravitational constraint up to **second order**

CANONICAL MODEL OF PRIMORDIAL SPACETIME

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$$H = H^{(0)} + \sum_k H_k^{(2)}$$

$$H^{(0)} = \frac{1}{2}p_a^2$$

Derivatives with respect to conformal time

$$H_k^{(2)} = -\frac{1}{2}|\pi_\pm(\vec{k})|^2 - \frac{1}{2}\left(k^2 - \frac{\ddot{a}}{a}\right)|\mu_\pm(\vec{k})|^2$$

$$\mu_\pm = a h_\pm$$

Expansion of the gravitational constraint up to second order

CANONICAL MODEL OF PRIMORDIAL SPACETIME

We find the constants of motion that form canonical pairs

$$\frac{\mathrm{d}D}{\mathrm{d}t} = \frac{\partial D}{\partial t} + \{D, H^{(0)} + H^{(2)}\} = 0$$

Zeroth order

$$\boxed{D_1 = a - pt, \quad D_2 = p}$$

First order

$$\boxed{\begin{aligned} \delta D_{1,\pm} &= \mu_\pm \sqrt{k}\sin(\omega) - \pi_\pm \frac{1}{\sqrt{k}}\cos(\omega) \\ \delta D_{2,\pm} &= \mu_\pm \sqrt{k}\cos(\omega) + \pi_\pm \frac{1}{\sqrt{k}}\sin(\omega) \end{aligned}}$$

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$$\{.,.\} = \frac{\partial}{\partial a}\frac{\partial}{\partial p_a} - \frac{\partial}{\partial p_a}\frac{\partial}{\partial a} + \frac{\partial}{\partial \mu}\frac{\partial}{\partial \pi_\mu} - \frac{\partial}{\partial \pi_\mu}\frac{\partial}{\partial \mu}$$

CANONICAL MODEL OF PRIMORDIAL SPACETIME

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$$\delta D_{1,\pm} = \mu_\pm \sqrt{k} \sin(\omega) - \pi_\pm \frac{1}{\sqrt{k}} \cos(\omega)$$

$$\delta D_{2,\pm} = \mu_\pm \sqrt{k} \cos(\omega) + \pi_\pm \frac{1}{\sqrt{k}} \sin(\omega)$$

$$\omega = k(t + a/p)$$

CANONICAL MODEL OF PRIMORDIAL SPACETIME

We find the constants of motion that form canonical pairs

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$$\omega = k(t + a/p)$$

They form canonical pairs

$$\{D_1, D_2\} = 1, \quad \{\delta D_{1,\pm}, \delta D_{2,\pm}\} = \delta_{\pm,\pm}$$

CANONICAL MODEL OF PRIMORDIAL SPACETIME

Clock transformation $\longrightarrow t \to \tilde{t} = t + \Delta(q, p, t)$

Delay function

1) The clock must move forward

$$\frac{\partial(\tilde{q}, \tilde{p})}{\partial(q, p)} = \begin{vmatrix} \frac{\partial \tilde{q}}{\partial q} & \frac{\partial \tilde{q}}{\partial p} \\ \frac{\partial \tilde{p}}{\partial q} & \frac{\partial \tilde{p}}{\partial p} \end{vmatrix} > 0$$

2) Preserves the ranges of the background variables

$$\lim_{p \to \pm\infty} \tilde{p}(q, p, t) = \pm\infty$$

$$\tilde{q}(q, p, t)\big|_{q=0} = 0$$

CANONICAL MODEL OF PRIMORDIAL SPACETIME

The Dirac observables are invariant under the clock transformation

The dynamical varaibles are not invariant, i.e.

$$(\tilde{a}, \tilde{p}) \in \mathbb{R}_+ \times \mathbb{R}$$

$$\boxed{\begin{gathered} \tilde{a} = a + p\Delta, \quad \tilde{p} = p, \\ \begin{bmatrix} \tilde{\mu}_k \\ k^{-1}\tilde{\pi}_k \end{bmatrix} = \begin{bmatrix} \cos k\Delta & -\sin k\Delta \\ \sin k\Delta & \cos k\Delta \end{bmatrix} \begin{bmatrix} \mu_k \\ k^{-1}\pi_k \end{bmatrix} \end{gathered}}$$

The two classical frameworks generate the same physical dynamics of the system

QUANTIZATION AND SEMI-CLASSICAL APPROXIMATION

Semi-classical background Hamiltonian

$$H_{\text{sem}} = \frac{1}{2}\left(p^2 + \frac{\hbar^2 \mathfrak{K}}{a^2}\right)$$

$\mathfrak{K} > 0$

Physical perturbation Hamiltonian

$$H_k^{(2)} = -\tfrac{1}{2}|\pi(\vec{k})|^2 - \tfrac{1}{2}\left(k^2 - \tfrac{\ddot{a}}{a}\right)|\mu(\vec{k})|^2$$

$$\dot{a} = \frac{\partial H_{\text{sem}}}{\partial p}$$

$$\dot{p} = -\frac{\partial H_{\text{sem}}}{\partial a}$$

Hamilton equations

QUANTIZATION AND SEMI-CLASSICAL APPROXIMATION

Quantization of the perturbation

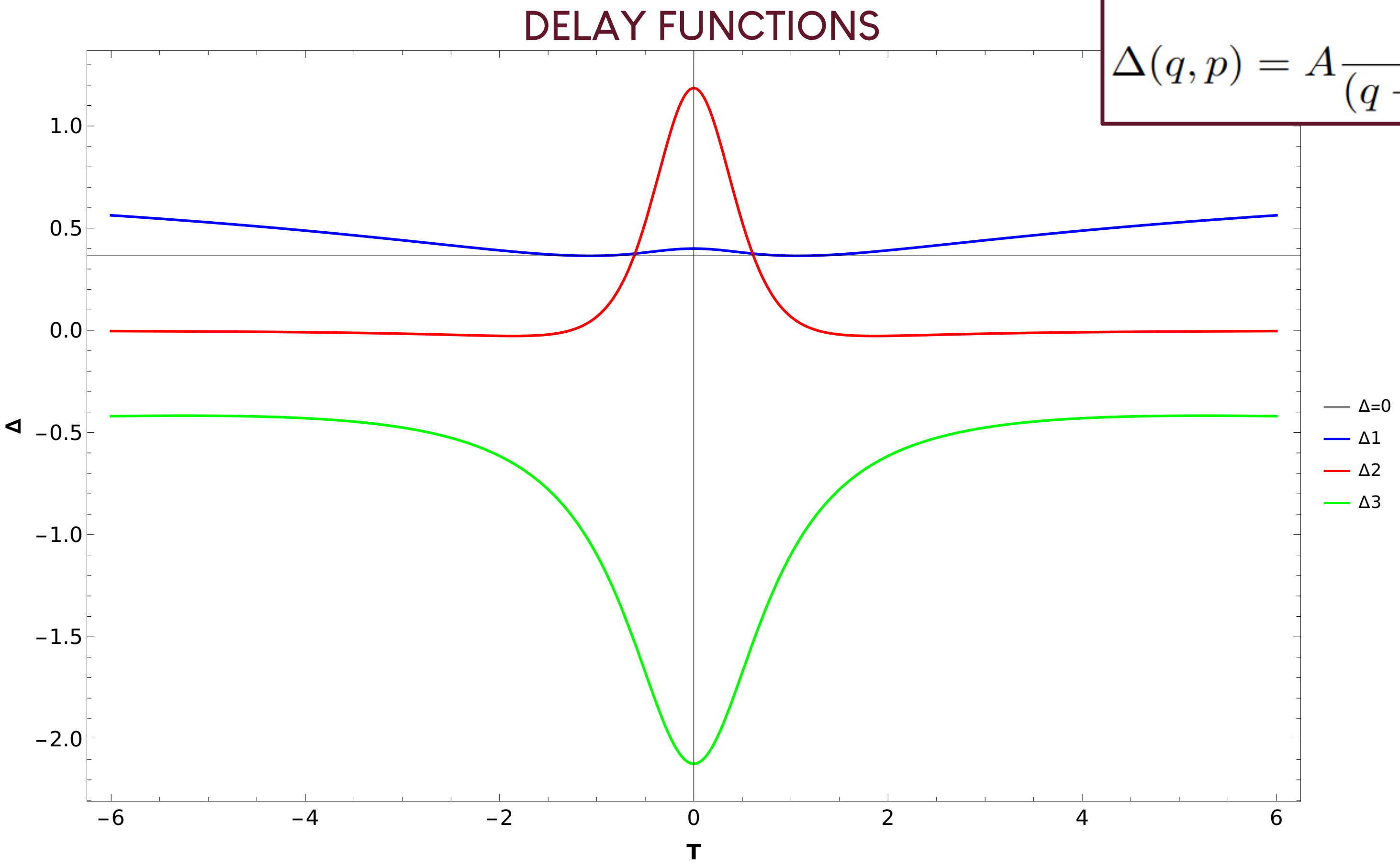
$$\mu_{\boldsymbol{k}} \mapsto \widehat{\mu}_{\boldsymbol{k}} = \sqrt{\frac{\hbar}{2}}\left[a_{\boldsymbol{k}}\bar{\mu}_k(\tau) + a^\dagger_{-\boldsymbol{k}}\mu_k(\tau)\right] \quad \pi_{\boldsymbol{k}} \mapsto \widehat{\pi}_{\boldsymbol{k}} = \sqrt{\frac{\hbar}{2}}\left[a_{\boldsymbol{k}}\dot{\bar{\mu}}_k(\tau) + a^\dagger_{-\boldsymbol{k}}\dot{\mu}_k(\tau)\right]$$

Equation of motion in Heisenberg picture

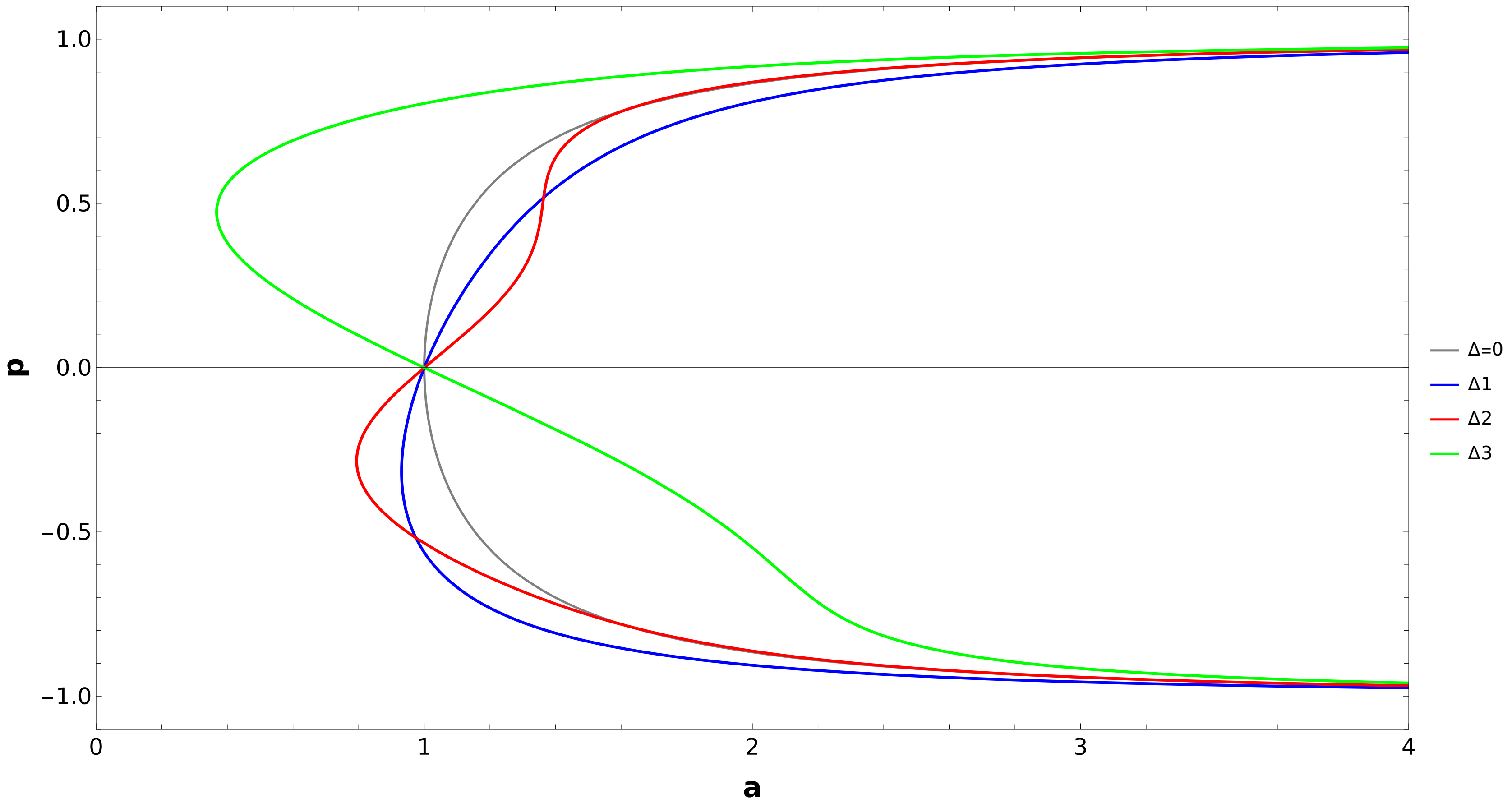
$$\boxed{\frac{d^2\mu_k}{d\tau^2} + \left(k^2 + \frac{\hbar^2\mathfrak{K}}{a^4}\right)\mu_k = 0}$$

RESULTS

$$\Delta(q,p) = A\frac{q^B}{(q+C)^D}\frac{\sin(Ep)}{p}$$



RESULTS



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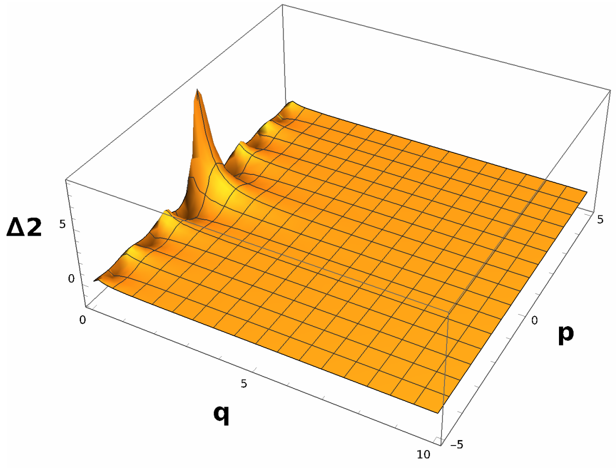
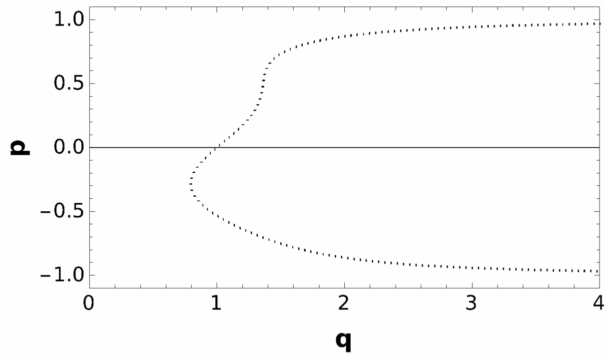
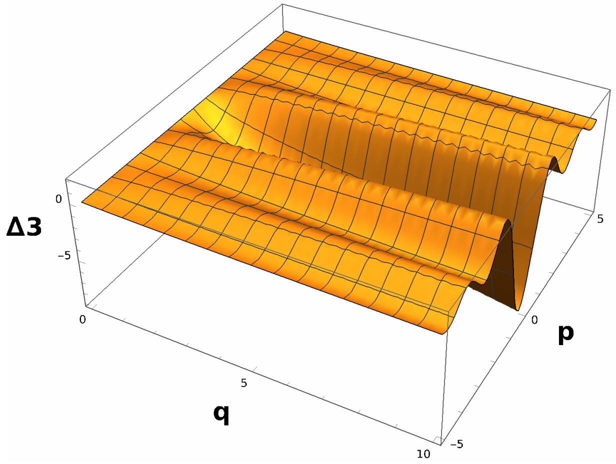


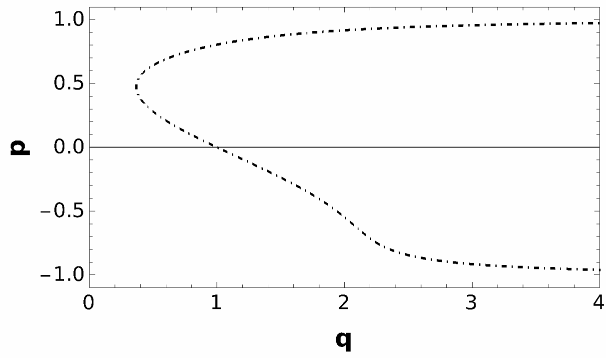
Δ2
5
0
0
5
10
q
5
0
-5
p



1.0
0.5
0.0
−0.5
−1.0
p
0
1
2
3
4
q

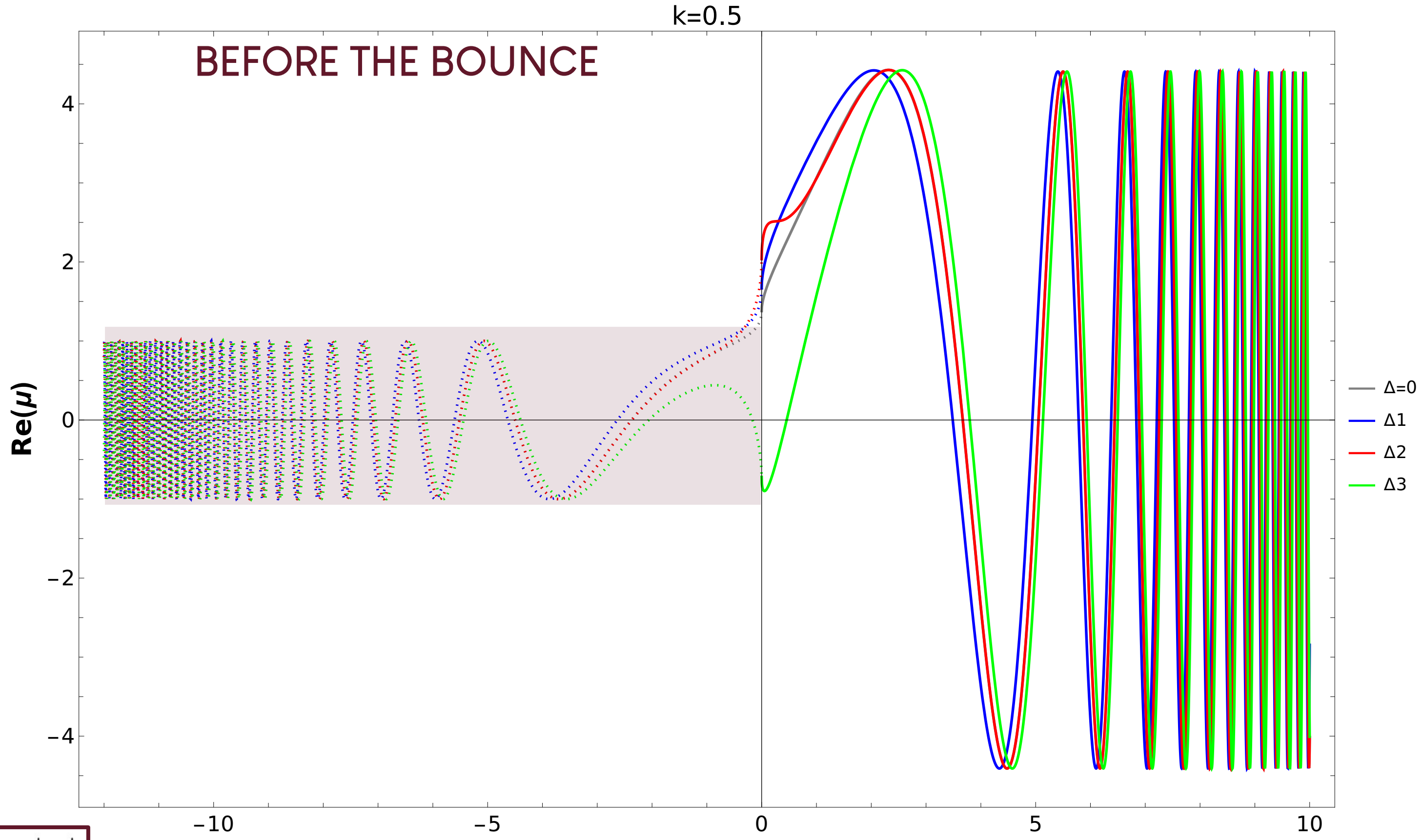


Δ3
0
-5
0
5
10
q
5
0
-5
p



1.0
0.5
0.0
−0.5
−1.0
p
0
1
2
3
4
q

RESULTS

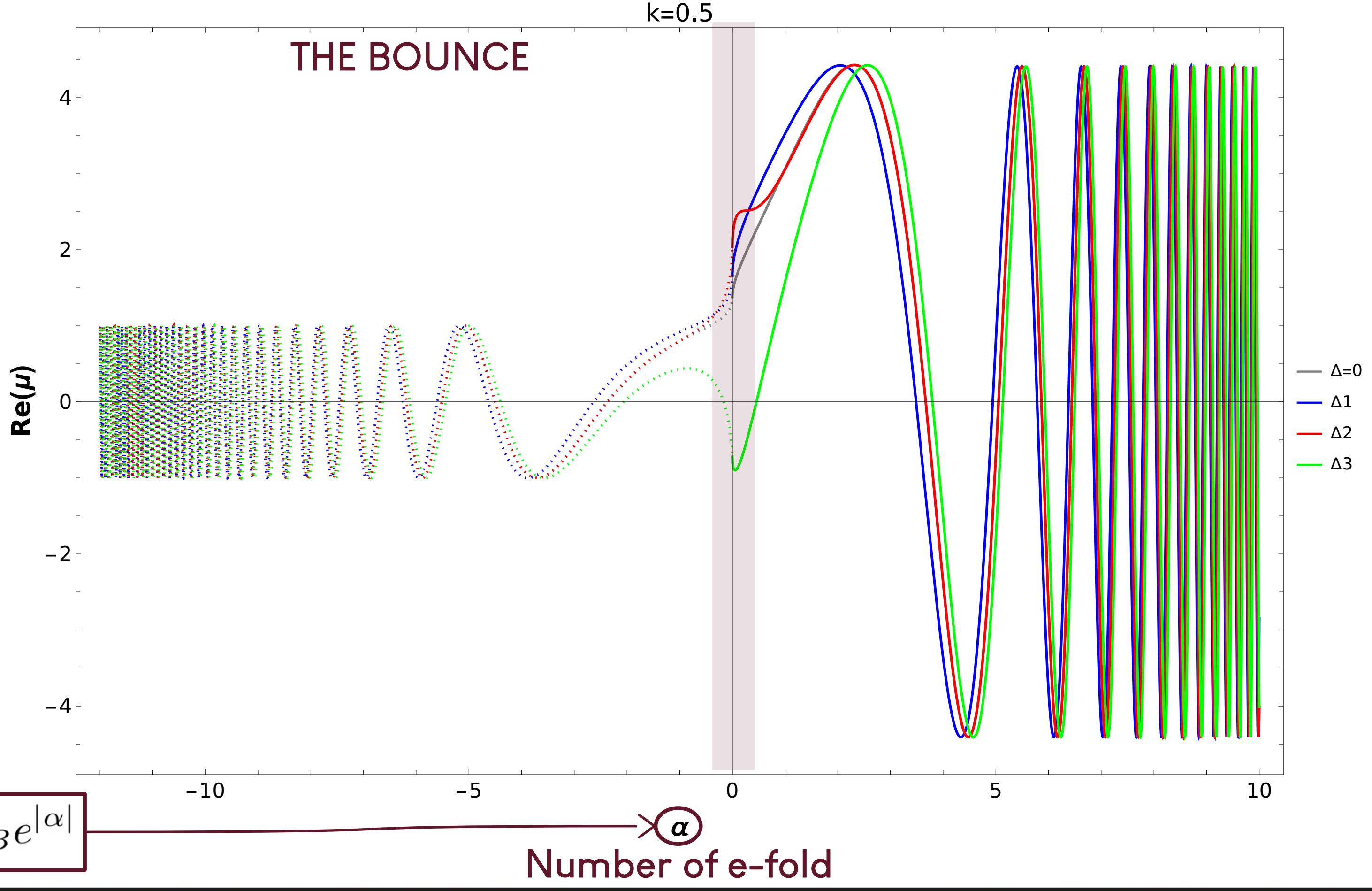


$$a = a_B e^{|\alpha|}$$

α

Number of e-fold

RESULTS



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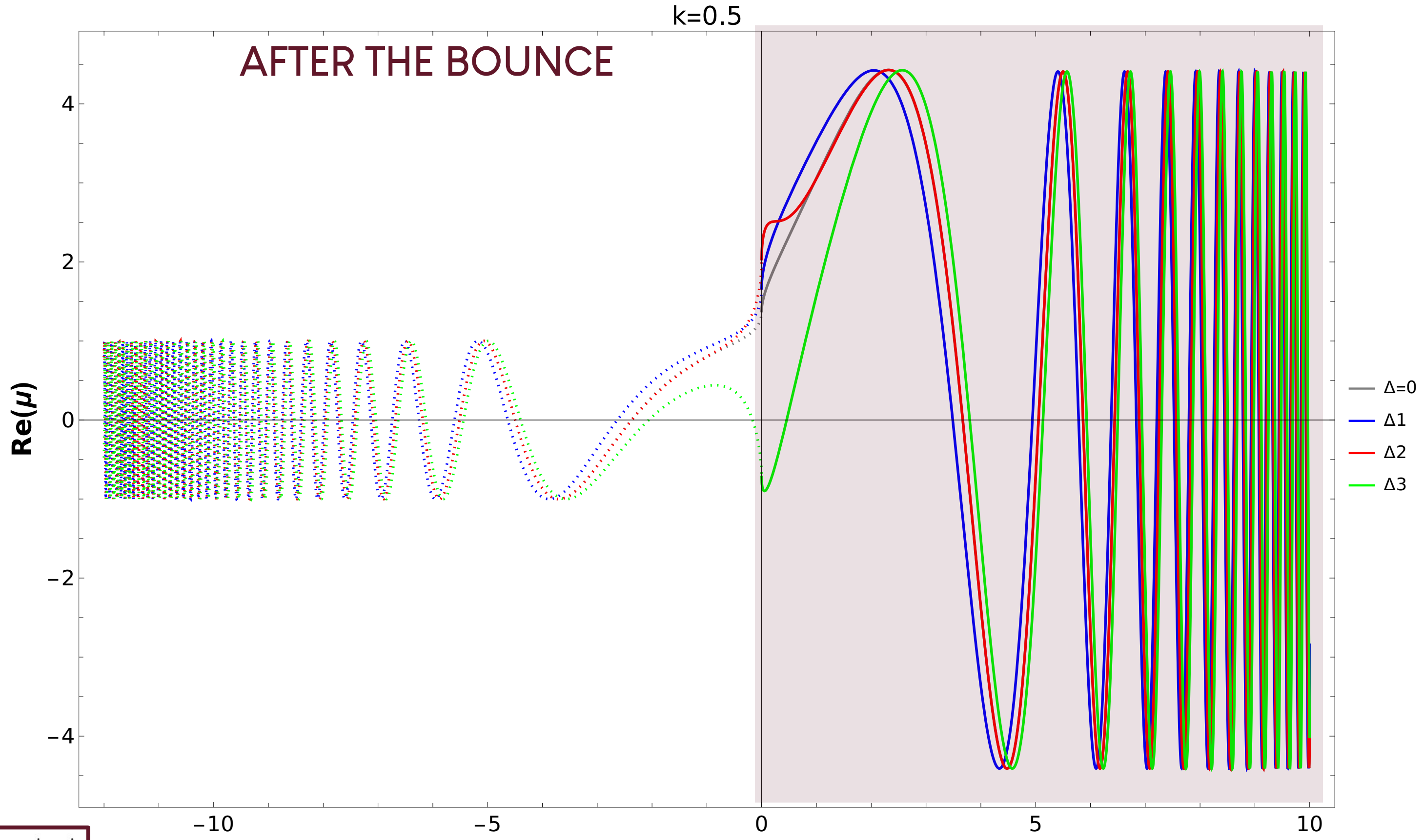

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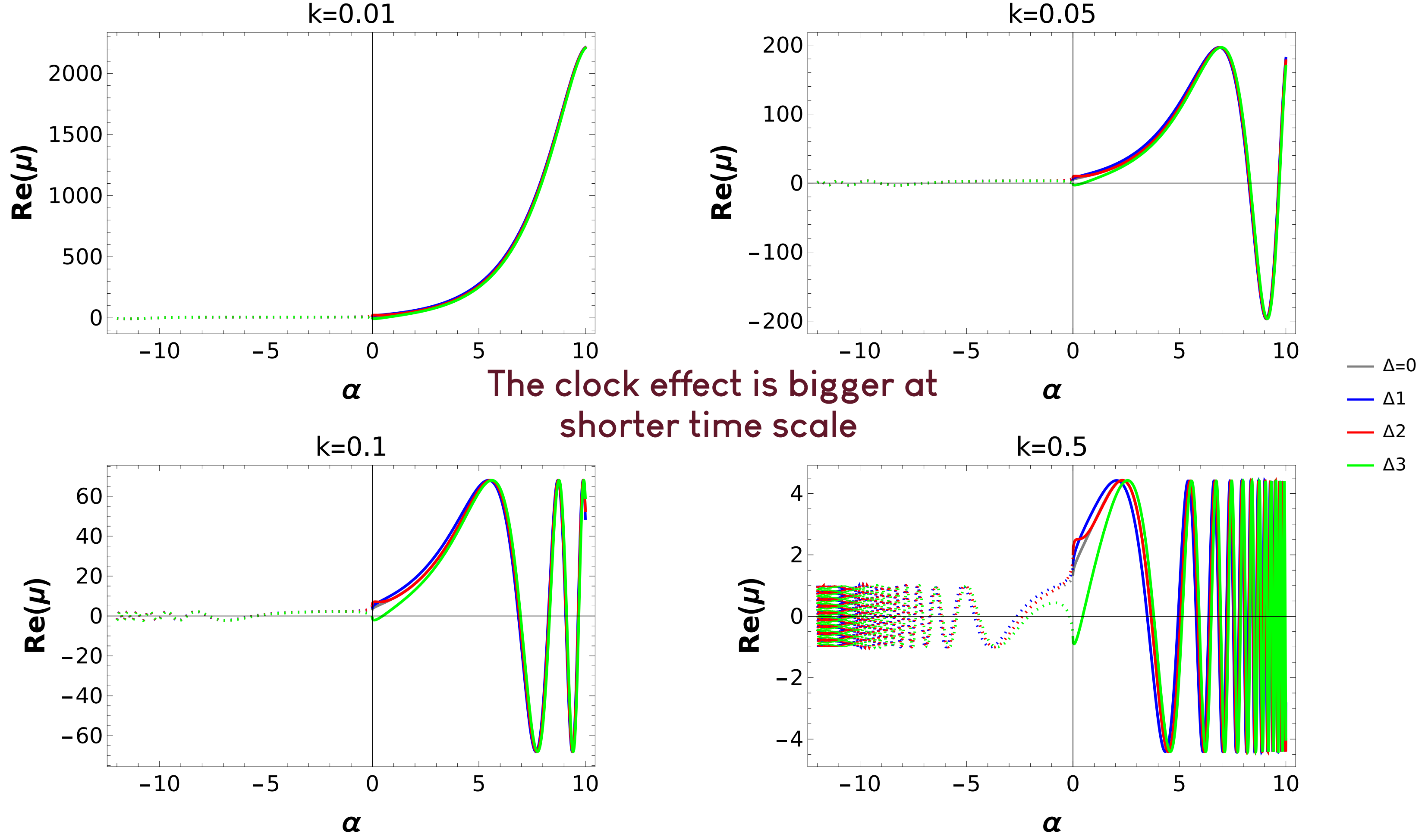
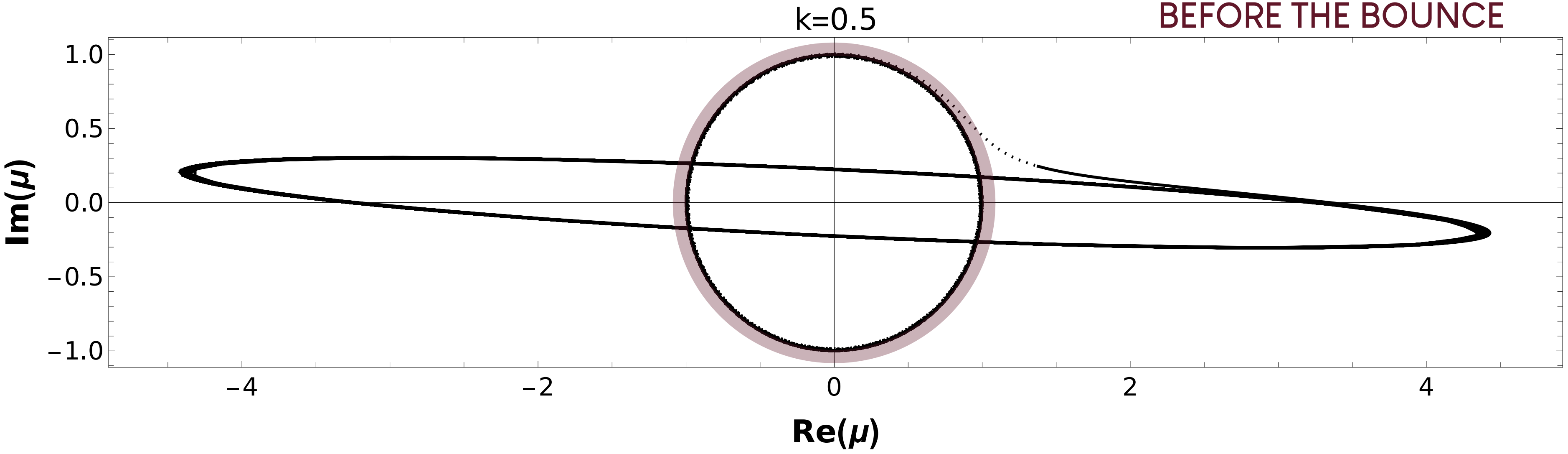


The clock effect is bigger at shorter time scale

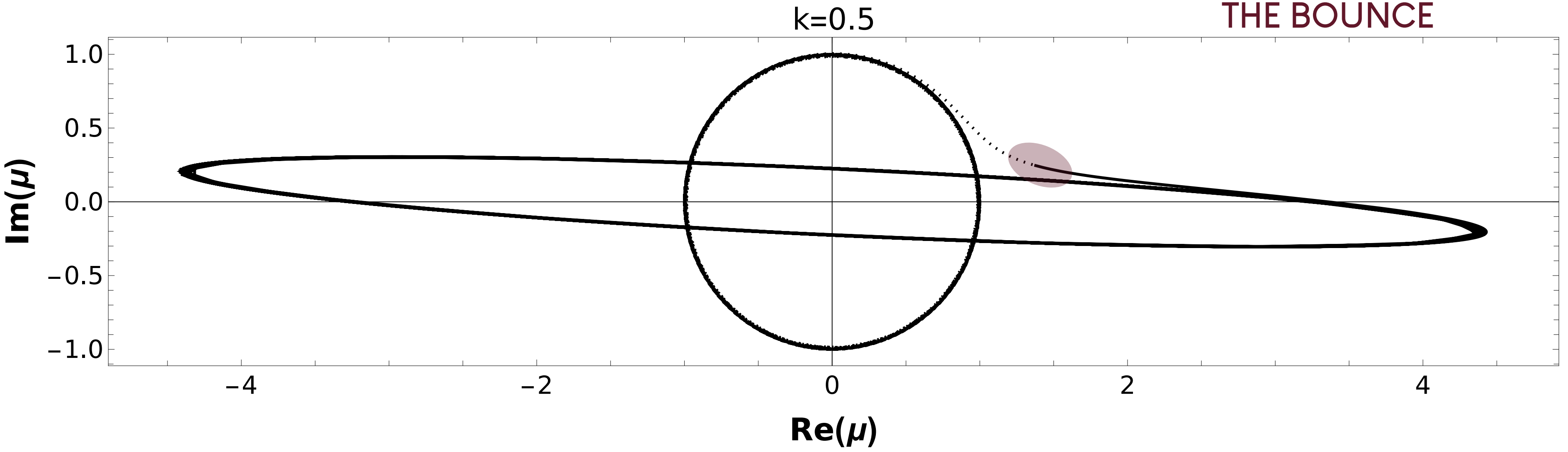
RESULTS

BEFORE THE BOUNCE



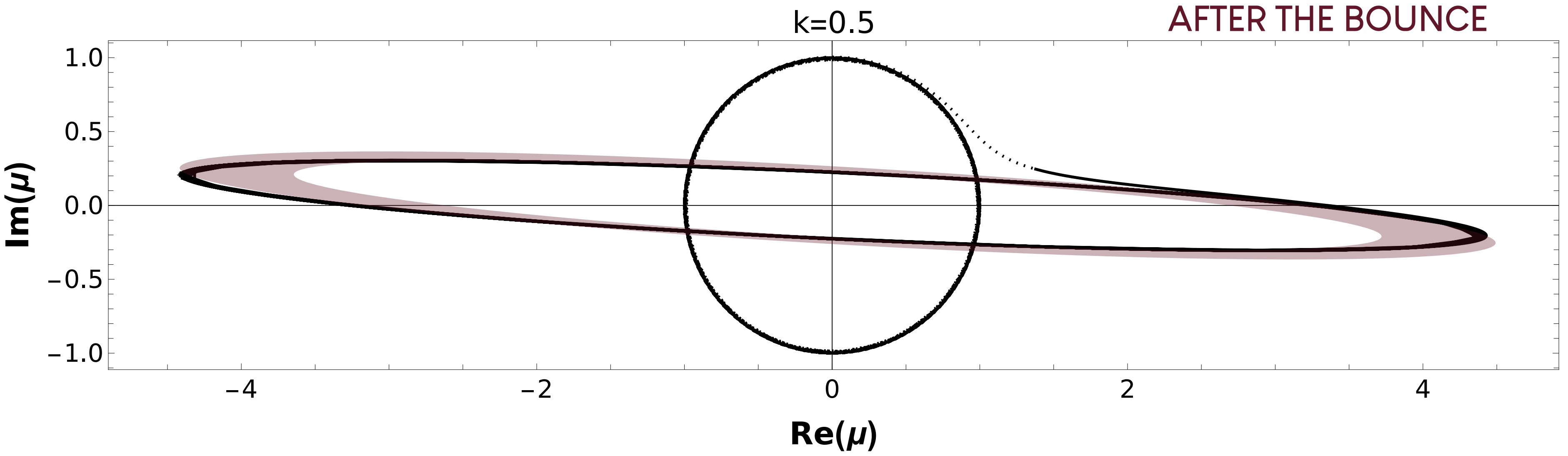
RESULTS

THE BOUNCE



RESULTS

AFTER THE BOUNCE



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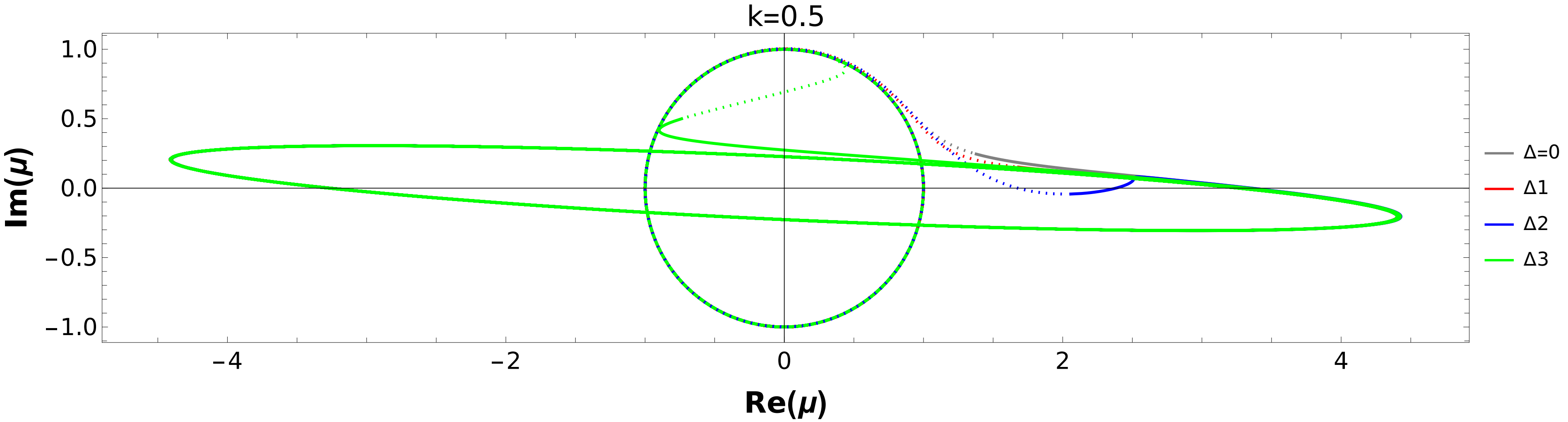

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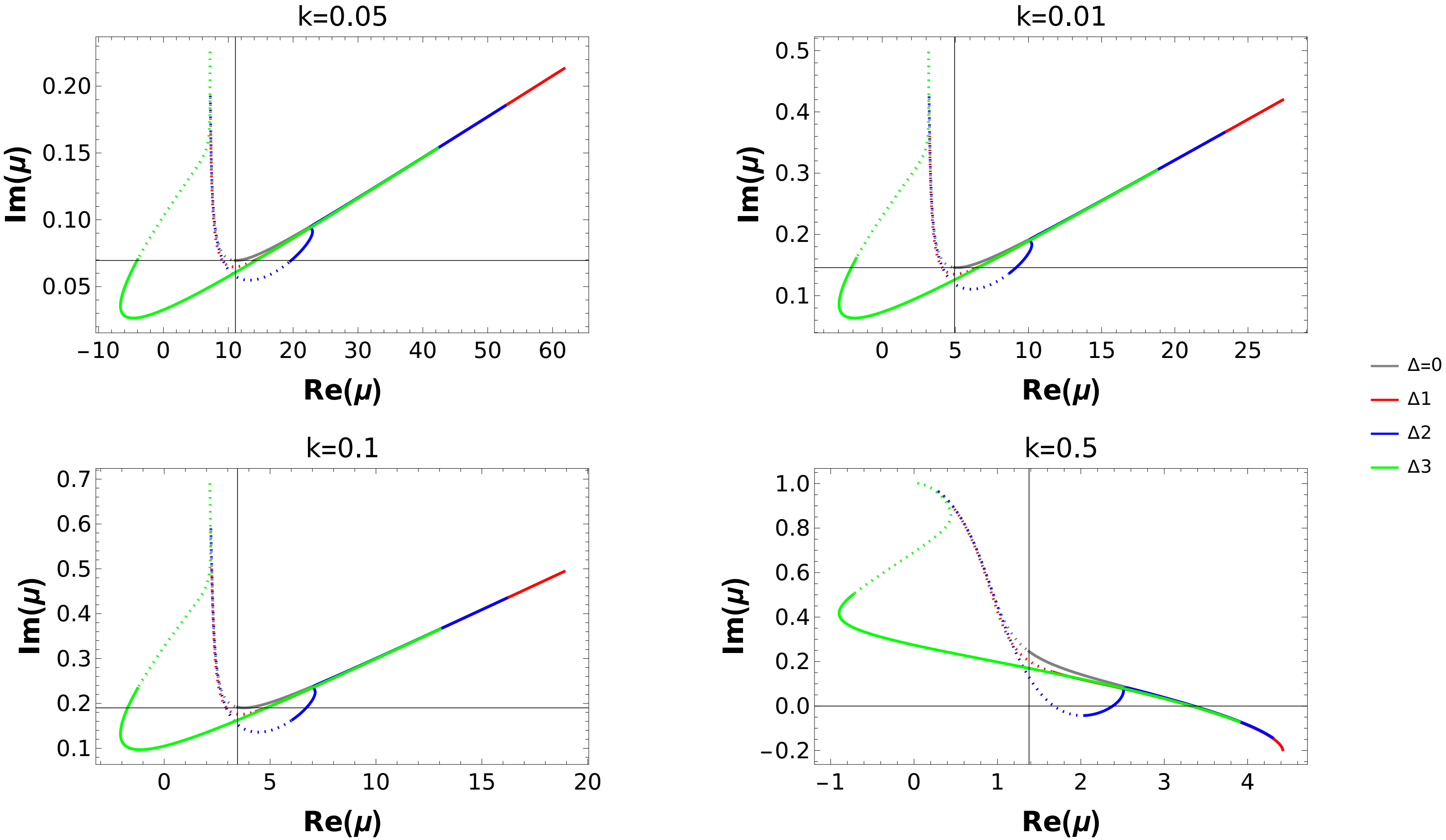

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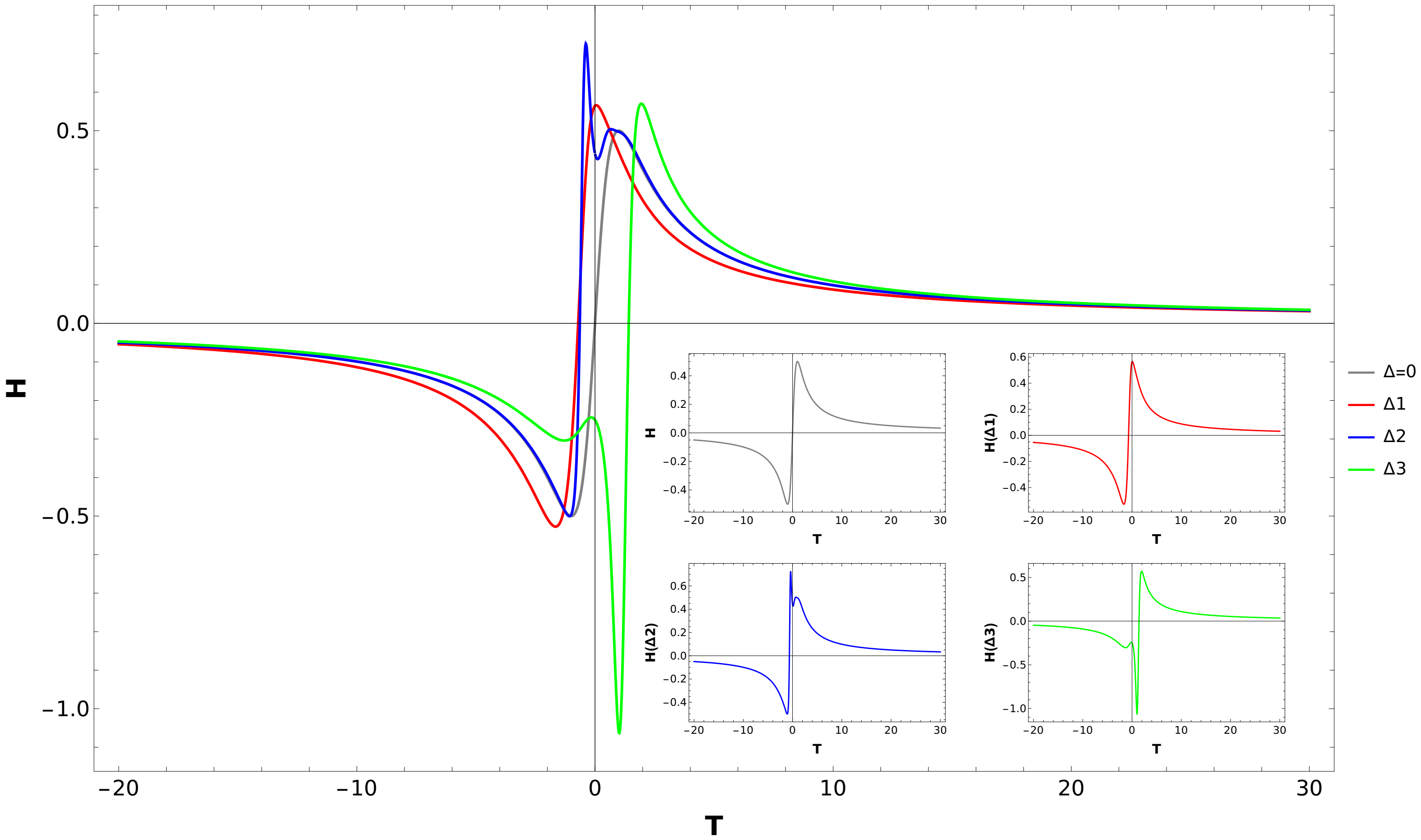
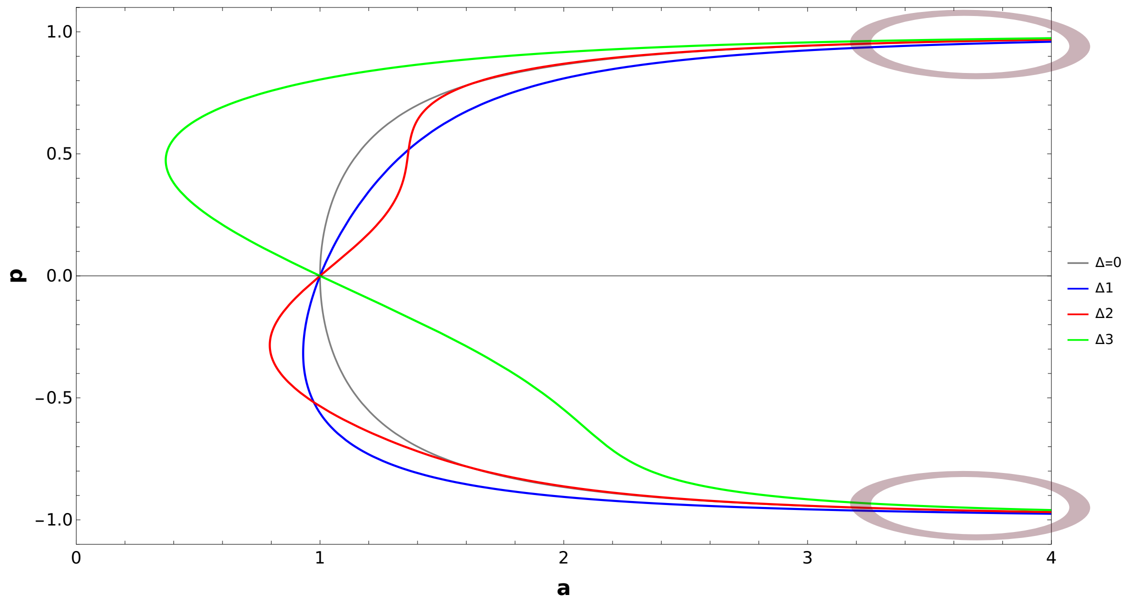


CONCLUSIONS

The expectation value of the dynamical variables is invariant under clock transformations away from the bounce where the behavior of the expectation values becomes classical.



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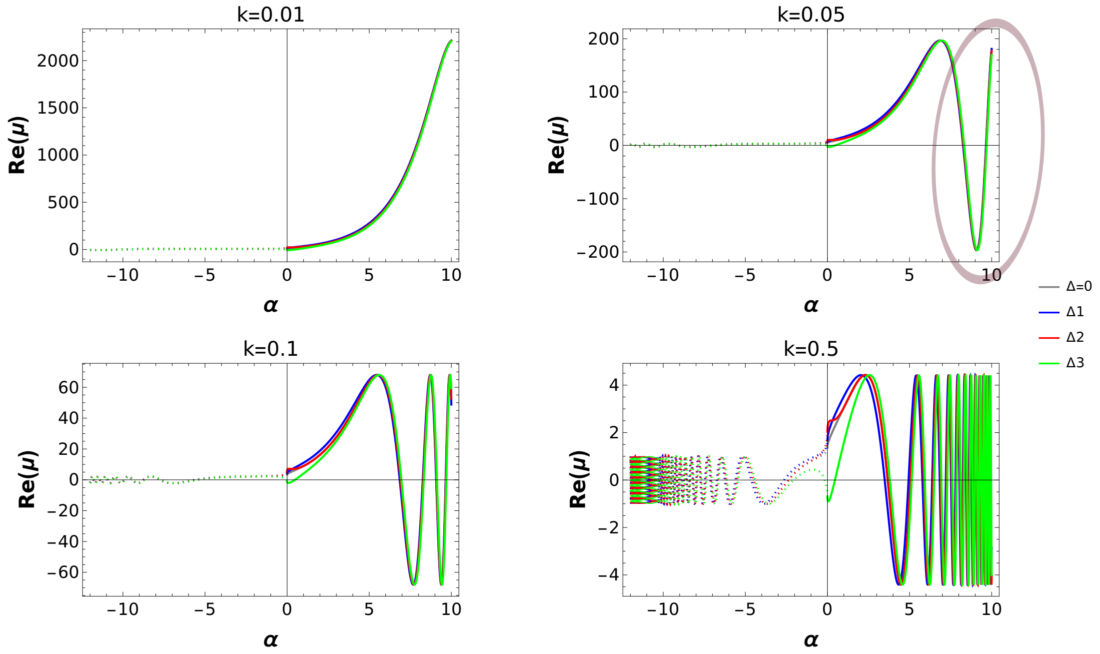

For asymptotically large universes the dynamical predictions of quantum gravity do not depend on the clock

THANK YOU

CLOCK TRANSFORMATIONS

Hamiltonian constraint

$$C(q_I, p^J) = 0$$

Assume q_0 varies monotonically with the evolution generated by the constraint

$$\{q_0, C\}_{\mathrm{PB}} \neq 0$$

we can assign to it the role of the internal clock in which the evolution of the remaining variables occurs

Hamiltonian constraint $\quad C \propto p_0 + H$

Reduced Hamiltonian formalism

$$\begin{aligned}\Omega\big|_{C=0} &= \left(\mathrm{d}q_I \wedge \mathrm{d}p^I + \mathrm{d}t \wedge \mathrm{d}p^0\right)\big|_{C=0} \\ &= \mathrm{d}q_I \wedge \mathrm{d}p^I - \mathrm{d}t \wedge \mathrm{d}H\end{aligned}$$

The dynamics reads

$$\boxed{\frac{\mathrm{d}q_I}{\mathrm{d}q_0} = \frac{\partial H}{\partial p_I}} \qquad \boxed{\frac{\mathrm{d}p^I}{\mathrm{d}q_0} = -\frac{\partial H}{\partial q^I}}$$