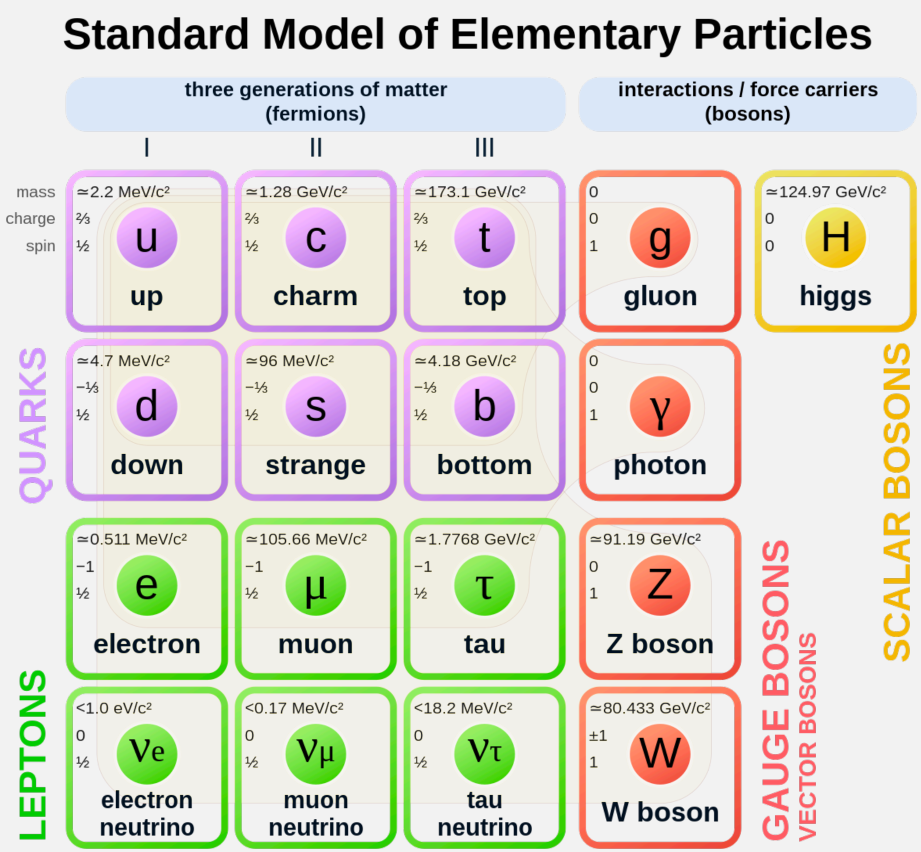
Quark-gluon plasma in magnetic fields

Patrycja Słoń

National Centre for Nuclear Research, Warsaw, Poland

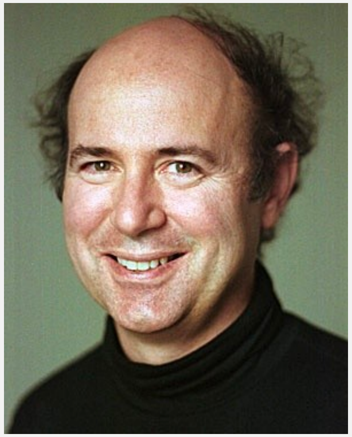
Supervised by **Stanisław Mrówczyński**

Quarks and gluons

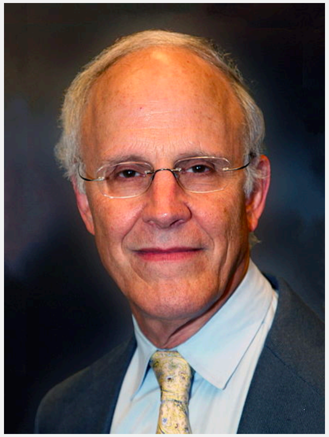


Quantum chromodynamics (QCD):

- color confinement
- asymptotic freedom (2004 Nobel Prize in Physics)



Frank Wilczek

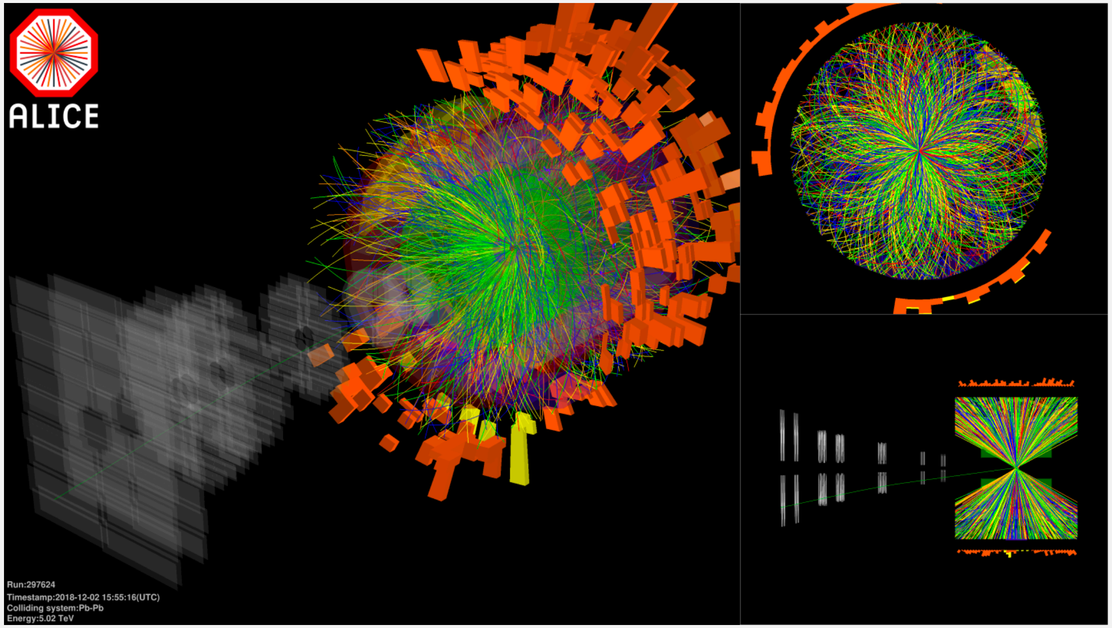


David Jonathan Gross

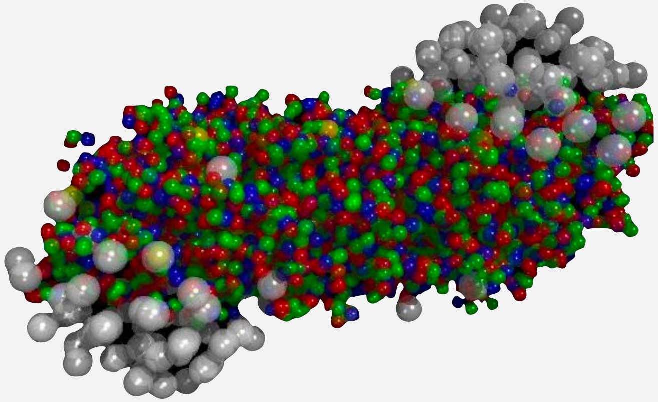
Hugh David Politzer

Quark-gluon plasma

QGP formation



Particle trajectories and energy deposition in the ALICE detector during the last lead–lead collisions in the second LHC run



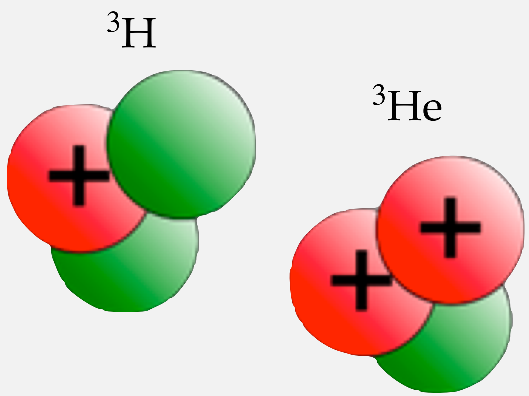
Temperature: 300 MeV
Energy density: 12-14 GeV/fm^3

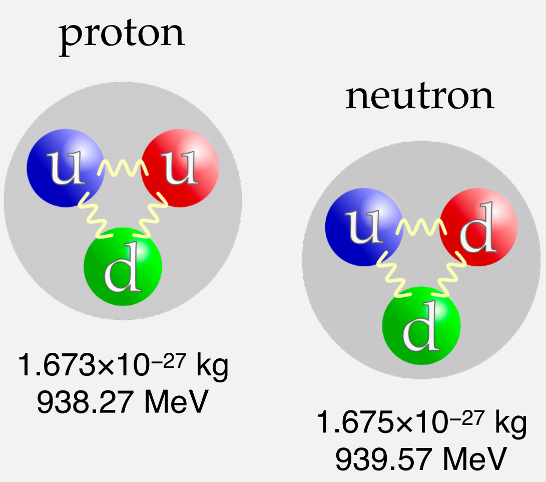
Why even bother with the electromagnetic field in the presence of the strong interaction?

Interaction	Relative strength
Strong	10^{38}
Electromagnetic	10^{36}
Weak	10^{33}
Gravitation	1

Why even bother with the electromagnetic field in the presence of the strong interaction?

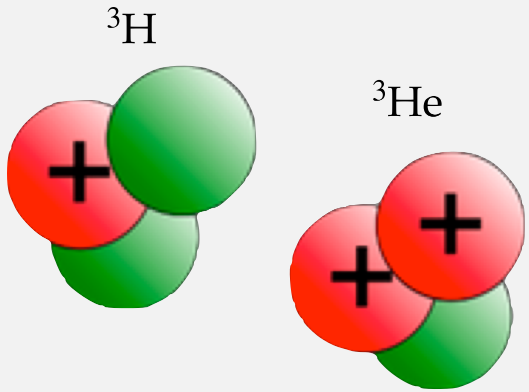
Interaction	Relative strength
Strong	10^{38}
Electromagnetic	10^{36}
Weak	10^{33}
Gravitation	1





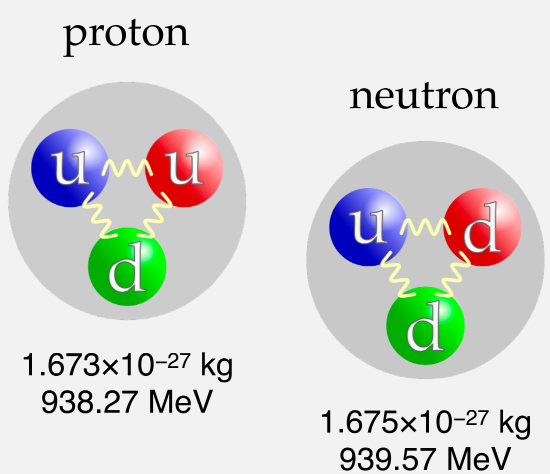
Why even bother with the electromagnetic field in the presence of the strong interaction?

Interaction	Relative strength
Strong	10^{38}
Electromagnetic	10^{36}
Weak	10^{33}
Gravitation	1



Magnetic field of a size characteristic for strong interactions

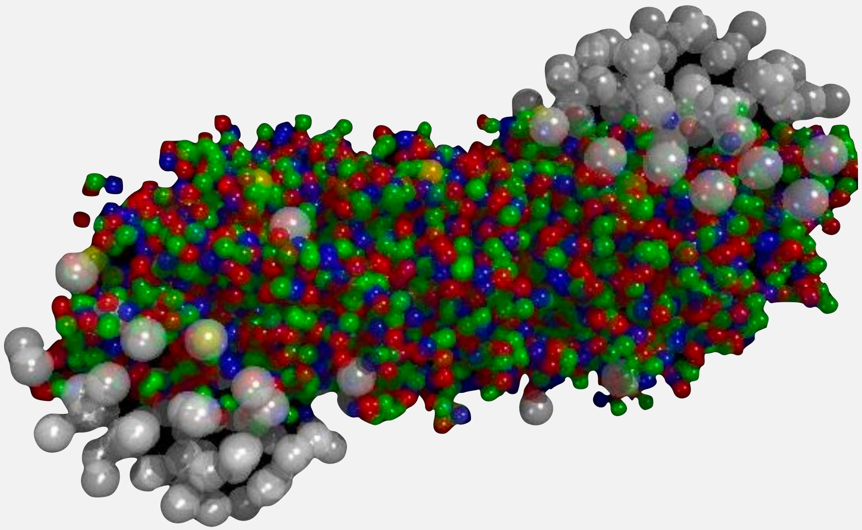
$$eB \sim m_\pi^2$$



$m_\pi = 140\,\mathrm{MeV}$ $\qquad$ $m_\pi^2 = 19.6 \cdot 10^3\,\mathrm{MeV}^2$

$eB \sim m_\pi^2$

- Electromagnetic field is important in the presence of the strong interaction
- Not accessible experimentally
- Magnetic field of a great amplitude appears very briefly during the relativistic heavy ion collisions
- The field is sustained according to the Faraday's law through electric currents induced by created **quark-gluon plasma**



$m_\pi = 140\,\mathrm{MeV}$ $\qquad m_\pi^2 = 19.6 \cdot 10^3\,\mathrm{MeV}^2$

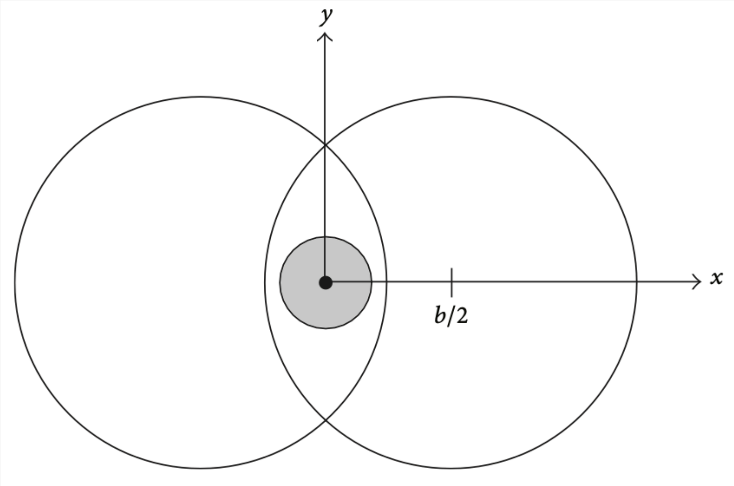
What happened with the magnetic field?

- There are theoretical predictions of various phenomena influenced by the magnetic field

- Experimentally EM field's effects are not seen

The problem is described in

K. Tuchin, *Particle production in strong electromagnetic fields in relativistic heavy-ion collisions*, Adv. High Energy Phys. 2013, 490495



Heavy-ion collision geometry as seen along the collision axis z

What happened with the magnetic field?

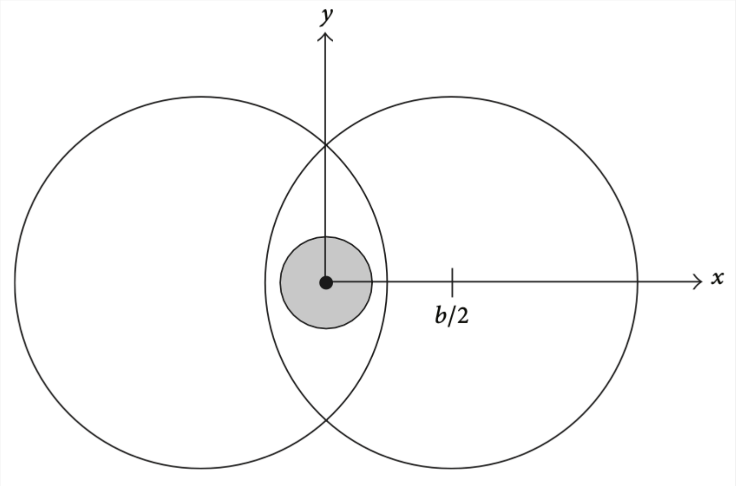
- There are theoretical predictions of various phenomena influenced by the magnetic field

- Experimentally EM field's effects are not seen

The problem is described in

K. Tuchin, *Particle production in strong electromagnetic fields in relativistic heavy-ion collisions*, Adv. High Energy Phys. 2013, 490495

What's the idea to improve the agreement between theory and experiment?



Heavy-ion collision geometry as seen along the collision axis z

What happened with the magnetic field?

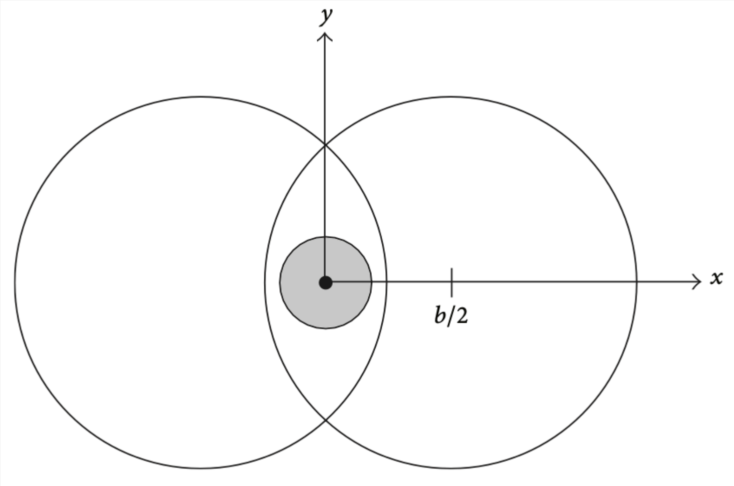
- There are theoretical predictions of various phenomena influenced by the magnetic field
- Experimentally EM field's effects are not seen

The problem is described in

K. Tuchin, *Particle production in strong electromagnetic fields in relativistic heavy-ion collisions*, Adv. High Energy Phys. 2013, 490495

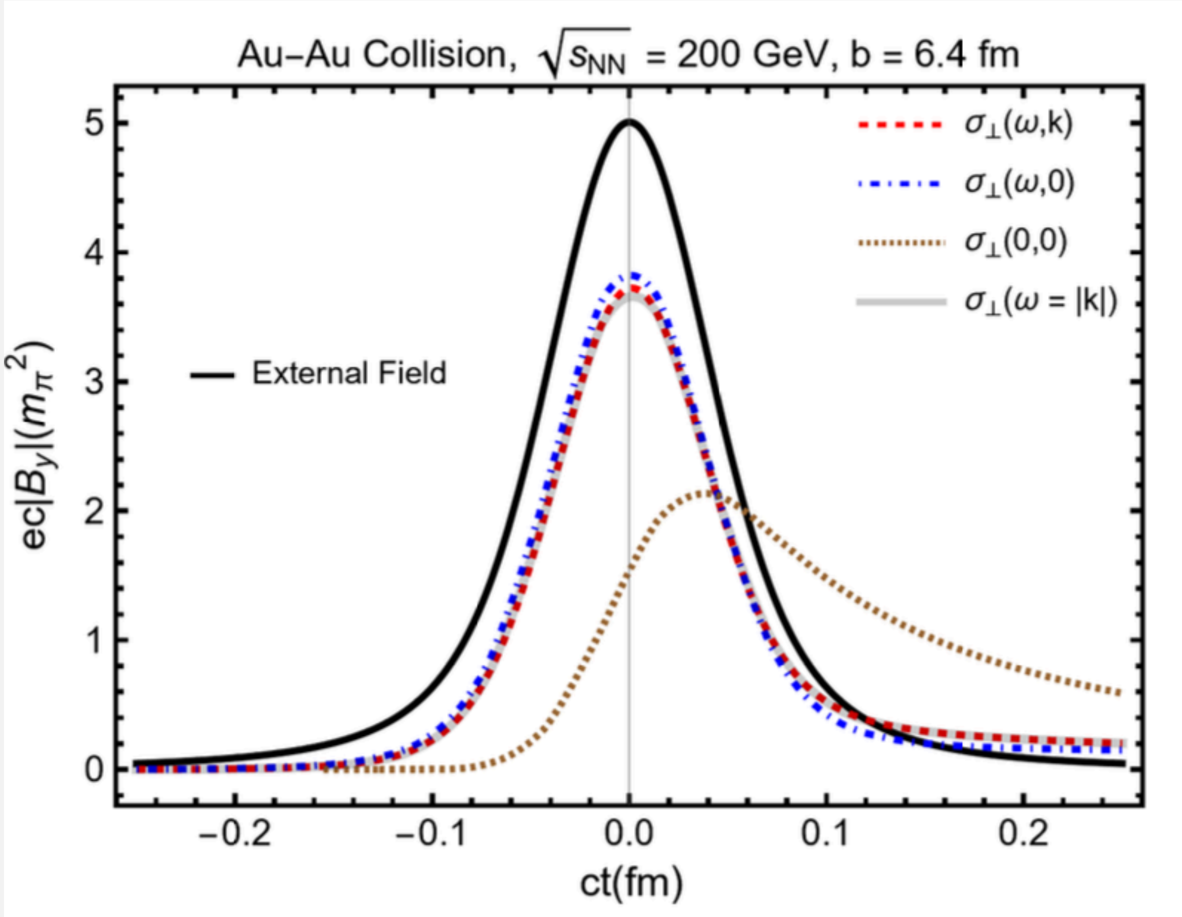
What's the idea to improve the agreement between theory and experiment?

Let's make the calculations involve some other details



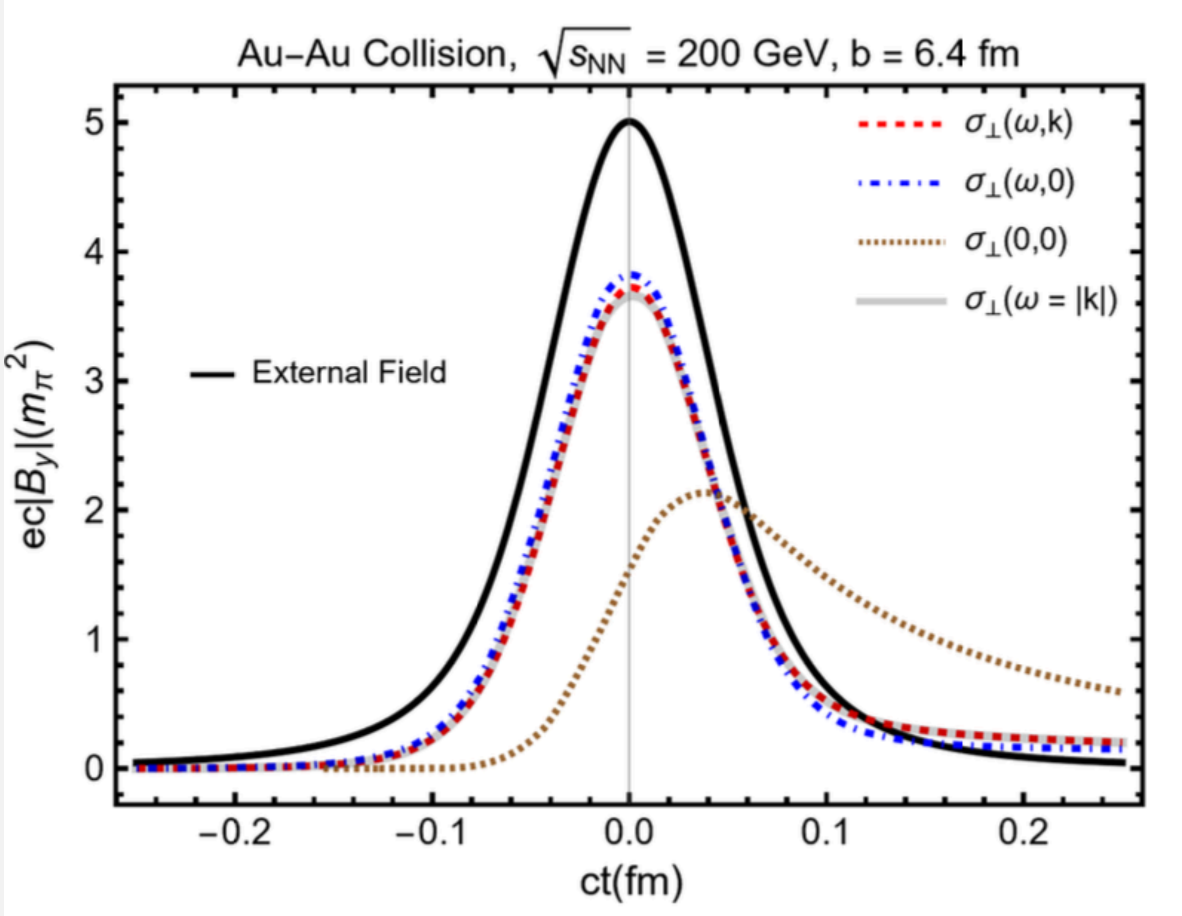
Heavy-ion collision geometry as seen along the collision axis z

Example of an improvement



C. Grayson, M. Formanek, J. Rafelski, B. Mueller, *Dynamic magnetic response of the quark-gluon plasma to electromagnetic fields*, Phys. Rev. D **106** (2022) no.1, 014011

Example of an improvement



How do we get such plots?

C. Grayson, M. Formanek, J. Rafelski, B. Mueller, *Dynamic magnetic response of the quark-gluon plasma to electromagnetic fields*, Phys. Rev. D **106** (2022) no.1, 014011

Electromagnetic field in a vacuum

In the research project we want to analyze the influence of the magnetic field on various phenomena produced by particles in medium, when there's plasma. But first we need a point of reference.

Vector potential

$$\mathbf{A}(t,\mathbf{r}) = \frac{e\mathbf{v}}{\left(\mathbf{R}^2 - (\mathbf{R}\times\mathbf{v})^2\right)^{1/2}}$$

Magnetic field

$$\mathbf{B}(t,\mathbf{r}) = \frac{e(1-\mathbf{v}^2)\mathbf{v}\times\mathbf{R}}{\left(\mathbf{R}^2 - (\mathbf{R}\times\mathbf{v})^2\right)^{1/2}}$$

We define:

$$\mathbf{R} \equiv \mathbf{r} - \mathbf{r}_0 - \mathbf{v}t$$

Available in literature

Specific example of our reference plot

Collision of two nuclei in a vacuum

Let's set the parameters

$$\begin{cases} \mathbf{r}_{01} = (b/2,0,0), \\ \mathbf{v}_1 = (0,0,v), \\ \mathbf{R}_1 = (x - b/2, y, z - vt), \end{cases} \qquad \begin{cases} \mathbf{r}_{02} = (-b/2,0,0), \\ \mathbf{v}_2 = (0,0,-v), \\ \mathbf{R}_2 = (x + b/2, y, z + vt), \end{cases}$$

Specific example of our reference plot

Collision of two nuclei in a vacuum

Let's set the parameters

$$\begin{cases} \mathbf{r}_{01} = (b/2, 0, 0), \\ \mathbf{v}_1 = (0, 0, v), \\ \mathbf{R}_1 = (x - b/2, y, z - vt), \end{cases} \qquad \begin{cases} \mathbf{r}_{02} = (-b/2, 0, 0), \\ \mathbf{v}_2 = (0, 0, -v), \\ \mathbf{R}_2 = (x + b/2, y, z + vt), \end{cases}$$

And input them in out reference equation from before

$$\mathbf{B}(t, \mathbf{r}) = \frac{q(1 - \mathbf{v}_1^2)\mathbf{v}_1 \times \mathbf{R}_1}{\left(\mathbf{R}_1^2 - (\mathbf{R}_1 \times \mathbf{v}_1)^2\right)^{3/2}} + \frac{q(1 - \mathbf{v}_2^2)\mathbf{v}_2 \times \mathbf{R}_2}{\left(\mathbf{R}_2^2 - (\mathbf{R}_2 \times \mathbf{v}_2)^2\right)^{3/2}}$$

Specific example of our reference plot

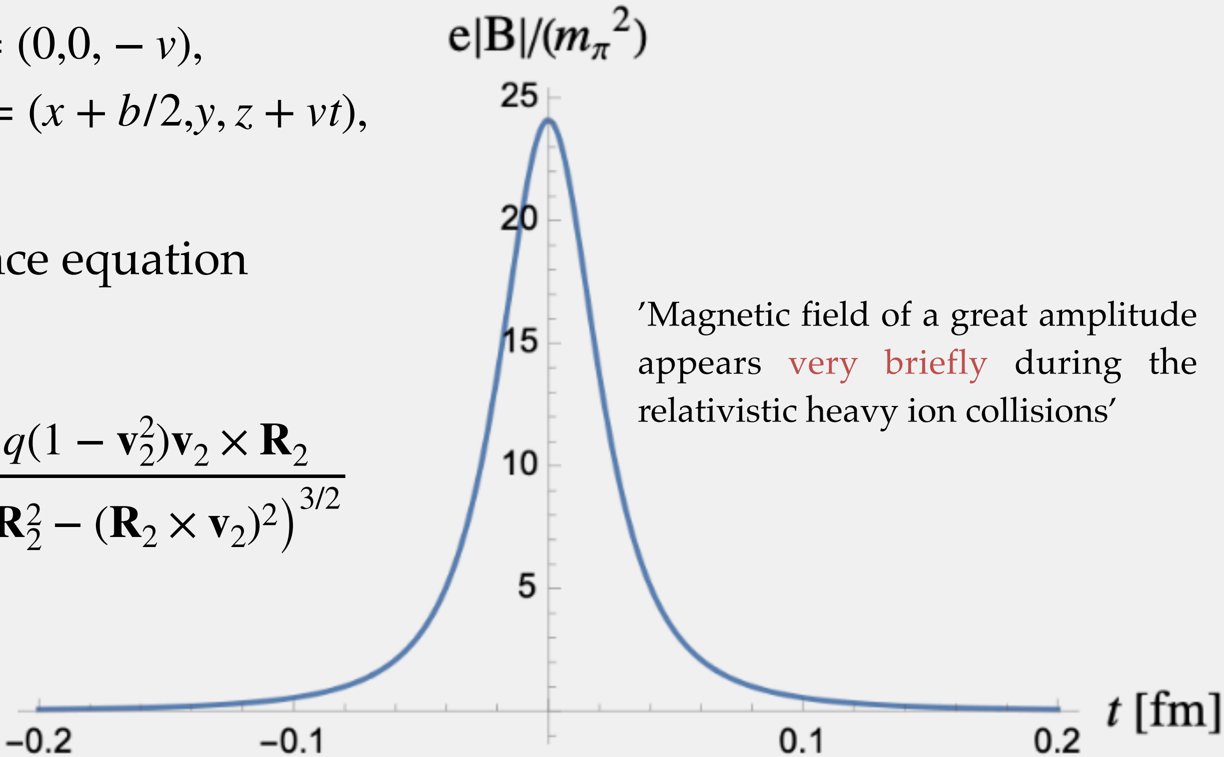
Collision of two nuclei in a vacuum

Let's set the parameters

$$\begin{cases} \mathbf{r}_{01} = (b/2,0,0), \\ \mathbf{v}_1 = (0,0,v), \\ \mathbf{R}_1 = (x - b/2, y, z - vt), \end{cases} \qquad \begin{cases} \mathbf{r}_{02} = (-b/2,0,0), \\ \mathbf{v}_2 = (0,0,-v), \\ \mathbf{R}_2 = (x + b/2, y, z + vt), \end{cases}$$

And input them in out reference equation from before

$$\mathbf{B}(t,\mathbf{r}) = \frac{q(1-\mathbf{v}_1^2)\mathbf{v}_1 \times \mathbf{R}_1}{\left(\mathbf{R}_1^2 - (\mathbf{R}_1 \times \mathbf{v}_1)^2\right)^{3/2}} + \frac{q(1-\mathbf{v}_2^2)\mathbf{v}_2 \times \mathbf{R}_2}{\left(\mathbf{R}_2^2 - (\mathbf{R}_2 \times \mathbf{v}_2)^2\right)^{3/2}}$$



'Magnetic field of a great amplitude appears very briefly during the relativistic heavy ion collisions'

Magnetic field as a function of time for Au-Au collision

Specific example of our reference plot

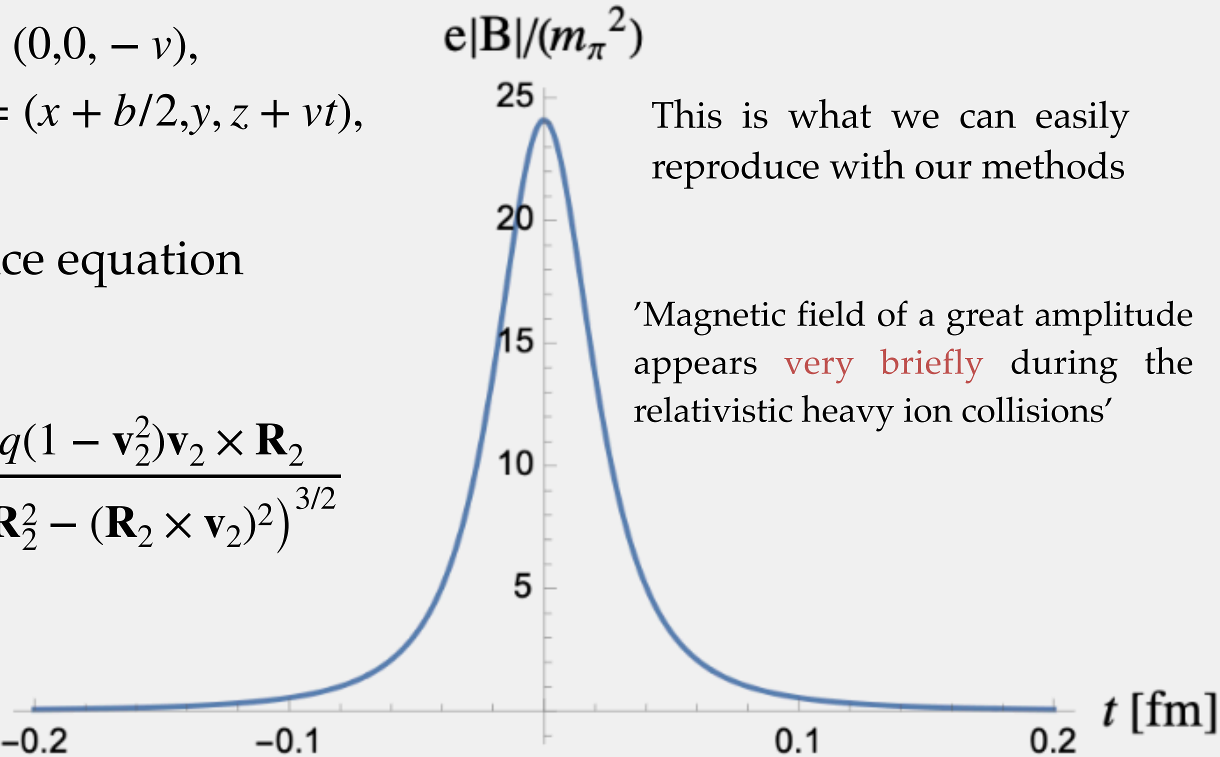
Collision of two nuclei in a vacuum

Let's set the parameters

$$\begin{cases} \mathbf{r}_{01} = (b/2, 0, 0), \\ \mathbf{v}_1 = (0, 0, v), \\ \mathbf{R}_1 = (x - b/2, y, z - vt), \end{cases} \qquad \begin{cases} \mathbf{r}_{02} = (-b/2, 0, 0), \\ \mathbf{v}_2 = (0, 0, -v), \\ \mathbf{R}_2 = (x + b/2, y, z + vt), \end{cases}$$

And input them in out reference equation from before

$$\mathbf{B}(t, \mathbf{r}) = \frac{q(1 - \mathbf{v}_1^2)\mathbf{v}_1 \times \mathbf{R}_1}{\left(\mathbf{R}_1^2 - (\mathbf{R}_1 \times \mathbf{v}_1)^2\right)^{3/2}} + \frac{q(1 - \mathbf{v}_2^2)\mathbf{v}_2 \times \mathbf{R}_2}{\left(\mathbf{R}_2^2 - (\mathbf{R}_2 \times \mathbf{v}_2)^2\right)^{3/2}}$$



This is what we can easily reproduce with our methods

'Magnetic field of a great amplitude appears very briefly during the relativistic heavy ion collisions'

Magnetic field as a function of time for Au-Au collision

What are the possible methods to acquire the equation for magnetic field and its components?

- We can start from the Maxwell's equations in medium

$$
\begin{cases}
\mathbf{k} \cdot \mathbf{D}(\omega, \mathbf{k}) = 4\pi\rho_{ext}(\omega, \mathbf{k}), & \text{Gauss's law} \\
\mathbf{k} \cdot \mathbf{B}(\omega, \mathbf{k}) = 0, & \text{Gauss's law for magnetism} \\
\mathbf{k} \times \mathbf{E}(\omega, \mathbf{k}) = \omega\mathbf{B}(\omega, \mathbf{k}), & \text{Faraday's law} \\
\mathbf{k} \times \mathbf{B}(\omega, \mathbf{k}) = -\omega\mathbf{D}(\omega, \mathbf{k}) - 4\pi i\mathbf{j}_{ext}(\omega, \mathbf{k})\,. & \text{Modified Ampere's law}
\end{cases}
$$

What are the possible methods to acquire the equation for magnetic field and its components?

- We can start from the Maxwell's equations in medium

$$\begin{cases} \mathbf{k} \cdot \mathbf{D}(\omega, \mathbf{k}) = 4\pi\rho_{ext}(\omega, \mathbf{k}), \\ \mathbf{k} \cdot \mathbf{B}(\omega, \mathbf{k}) = 0, \\ \mathbf{k} \times \mathbf{E}(\omega, \mathbf{k}) = \omega\mathbf{B}(\omega, \mathbf{k}), \\ \mathbf{k} \times \mathbf{B}(\omega, \mathbf{k}) = -\omega\mathbf{D}(\omega, \mathbf{k}) - 4\pi i\mathbf{j}_{ext}(\omega, \mathbf{k})\,. \end{cases}$$

Gauss's law
Gauss's law for magnetism
Faraday's law
Modified Ampere's law

- The displacement field **D**

$$D^i(\omega, \mathbf{k}) = \varepsilon^{ij}(\omega, \mathbf{k})E^j(\omega, \mathbf{k})$$

What are the possible methods to acquire the equation for magnetic field and its components?

- We can start from the Maxwell's equations in medium

$$\begin{cases} \mathbf{k}\cdot\mathbf{D}(\omega,\mathbf{k}) = 4\pi\rho_{ext}(\omega,\mathbf{k}), & \text{Gauss's law} \\ \mathbf{k}\cdot\mathbf{B}(\omega,\mathbf{k}) = 0, & \text{Gauss's law for magnetism} \\ \mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}) = \omega\mathbf{B}(\omega,\mathbf{k}), & \text{(1) Faraday's law} \\ \mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) = -\omega\mathbf{D}(\omega,\mathbf{k}) - 4\pi i\mathbf{j}_{ext}(\omega,\mathbf{k})\,. & \text{(2) Modified Ampere's law} \end{cases}$$

- The displacement field $\mathbf{D}$

3

$$D^i(\omega,\mathbf{k}) = \varepsilon^{ij}(\omega,\mathbf{k})E^j(\omega,\mathbf{k})$$

What are the possible methods to acquire the equation for magnetic field and its components?

- We can start from the Maxwell's equations in medium

$$\begin{cases} \mathbf{k} \cdot \mathbf{D}(\omega, \mathbf{k}) = 4\pi\rho_{ext}(\omega, \mathbf{k}), \\ \mathbf{k} \cdot \mathbf{B}(\omega, \mathbf{k}) = 0, \\ \mathbf{k} \times \mathbf{E}(\omega, \mathbf{k}) = \omega \mathbf{B}(\omega, \mathbf{k}), \\ \mathbf{k} \times \mathbf{B}(\omega, \mathbf{k}) = -\omega \mathbf{D}(\omega, \mathbf{k}) - 4\pi i \mathbf{j}_{ext}(\omega, \mathbf{k}) \,. \end{cases}$$

Gauss's law

Gauss's law for magnetism

1 Faraday's law

2 Modified Ampere's law

- The displacement field $\mathbf{D}$

3

$$D^i(\omega, \mathbf{k}) = \varepsilon^{ij}(\omega, \mathbf{k}) E^j(\omega, \mathbf{k})$$

1 + 3 $\rightarrow$ 2

What are the possible methods to acquire the equation for magnetic field and its components?

- We can start from the Maxwell's equations in medium

$$\begin{cases} \mathbf{k}\cdot\mathbf{D}(\omega,\mathbf{k}) = 4\pi\rho_{ext}(\omega,\mathbf{k}), & \text{Gauss's law} \\ \mathbf{k}\cdot\mathbf{B}(\omega,\mathbf{k}) = 0, & \text{Gauss's law for magnetism} \\ \mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}) = \omega\mathbf{B}(\omega,\mathbf{k}), & \text{1 Faraday's law} \\ \mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) = -\,\omega\mathbf{D}(\omega,\mathbf{k}) - 4\pi i\mathbf{j}_{ext}(\omega,\mathbf{k})\,. & \text{2 Modified Ampere's law} \end{cases}$$

- The displacement field **D**

$$D^i(\omega,\mathbf{k}) = \varepsilon^{ij}(\omega,\mathbf{k})E^j(\omega,\mathbf{k}) \qquad 3$$

1 + 3 → 2

$$\boxed{j^i_{ext}(\omega,\mathbf{k}) = \frac{i}{4\pi}\left[\frac{1}{\omega}(k^ik^j - \mathbf{k}^2\delta^{ij}) + \omega\varepsilon^{ij}(\omega,\mathbf{k})\right]E^j(\omega,\mathbf{k})}$$

The dielectric tensor $\varepsilon^{ij}(\omega, \mathbf{k})$

Provides a lot of informations about the characteristics of the magnetic field

Very useful for the analysis

- Because we are dealing with an isotropic plasma:

$$\varepsilon^{ij}(\omega, \mathbf{k}) = \varepsilon_L(\omega, \mathbf{k})\frac{k^i k^j}{\mathbf{k}^2} + \varepsilon_T(\omega, \mathbf{k})\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

The dielectric tensor $\varepsilon^{ij}(\omega, \mathbf{k})$

Provides a lot of informations about the characteristics of the magnetic field

Very useful for the analysis

- Because we are dealing with an isotropic plasma:

$$\varepsilon^{ij}(\omega, \mathbf{k}) = \varepsilon_L(\omega, \mathbf{k})\frac{k^i k^j}{\mathbf{k}^2} + \varepsilon_T(\omega, \mathbf{k})\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

$$\varepsilon_L(\omega, \mathbf{k}) = \frac{k^i k^j}{\mathbf{k}^2}\varepsilon^{ij}(\omega, \mathbf{k}) \qquad \varepsilon_T(\omega, \mathbf{k}) = \frac{1}{2}(\varepsilon^{ii}(\omega, \mathbf{k}) - \varepsilon_L(\omega, \mathbf{k}))$$

What information can those components provide?

What information can the dielectric tensor give?

Let's consider two boundary conditions

$k \to 0$

$\omega \to 0$

What information can the dielectric tensor give?

Let's consider two boundary conditions

$k \to 0$

$\omega \to 0$

$$\varepsilon_L(\omega) \overset{k\to 0}{=} 1 - \frac{\omega_p^2}{\omega^2}$$

Plasma frequency

$$\boxed{\omega_p^2 \sim e^2 T^2}$$

Indicates a normal mode of plasma oscillations

What information can the dielectric tensor give?

Let's consider two boundary conditions

$k \to 0$

$$\varepsilon_L(\omega) \overset{k\to 0}{=} 1 - \frac{\omega_p^2}{\omega^2}$$

Plasma frequency

$$\boxed{\omega_p^2 \sim e^2 T^2}$$

Indicates a normal mode of plasma oscillations

$\omega \to 0$

$$\varepsilon_L(\mathbf{k}) \overset{\omega\to 0}{=} 1 + \frac{m_D^2}{\mathbf{k}^2}$$

Debye mass

$$\boxed{m_D \sim e^2 T^2 / 3}$$

Connected to the screening length of the fields

$\lambda = 1/m_D$

What's the other method of analyzing the behaviour of plasma in the electromagnetic field?

Properties of the plasma can be described by
the kinetic theory

- Boltzmann transport equation

$$\left[\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_p\right] f(t, \mathbf{r}, \mathbf{p}) = C(t, \mathbf{r}, \mathbf{p})$$

$f(t, \mathbf{r}, \mathbf{p})$ - distribution function
$C(t, \mathbf{r}, \mathbf{p})$ - collision term - describes the rate of change of the distribution function f as a result of collisions.

What's the other method of analyzing the behaviour of plasma in the electromagnetic field?

Properties of the plasma can be described by
the kinetic theory

- Boltzmann transport equation

$$\left[\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_p\right] f(t, \mathbf{r}, \mathbf{p}) = C(t, \mathbf{r}, \mathbf{p})$$

- Vlasov equation

$$\left[\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_p\right] f(t, \mathbf{r}, \mathbf{p}) = 0$$

$$(p_\mu \partial^\mu - e p_\mu F^{\mu\nu} \partial_\nu^p) f(x, p) = 0$$

$f(t, \mathbf{r}, \mathbf{p})$ - distribution function
$C(t, \mathbf{r}, \mathbf{p})$ - collision term - describes the rate of change of the distribution function f as a result of collisions. Disappears, when:

- we study the properties of the plasma on a short time scale compared to a collision time
- the system reaches the state of local equilibrium

$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ - field-strength tensor

How can we acquire the properties of plasma in this method?

By linearizing the Vlasov equation

- How can we find the four-current?
  From the Maxwell equations $\partial_\mu F^{\mu\nu} = j^\nu$:

$$j^\nu(x) = 2e \int \frac{d^3p}{(2\pi)^3} \frac{p^\nu}{E_p} [f(x,p) - \bar{f}(x,p)]$$

How can we acquire the properties of plasma in this method?

By linearizing the Vlasov equation

- How can we find the four-current?
  From the Maxwell equations $\partial_\mu F^{\mu\nu} = j^\nu$:

$$j^\nu(x) = 2e \int \frac{d^3p}{(2\pi)^3} \frac{p^\nu}{E_p} [f(x,p) - \bar{f}(x,p)]$$

- Distribution function as their value in equilibrium plus a small perturbation

$$f(x,p) = f(p) + \delta f(x,p)$$
$$\bar{f}(x,p) = \bar{f}(p) + \delta\bar{f}(x,p)$$

How can we acquire the properties of plasma in this method?

By linearizing the Vlasov equation

- How can we find the four-current?
  From the Maxwell equations $\partial_\mu F^{\mu\nu} = j^\nu$:

$$j^\nu(x) = 2e \int \frac{d^3p}{(2\pi)^3} \frac{p^\nu}{E_p} [f(x,p) - \bar{f}(x,p)]$$

- Distribution function as their value in equilibrium plus a small perturbation

$$f(x,p) = f(p) + \delta f(x,p)$$
$$\bar{f}(x,p) = \bar{f}(p) + \delta \bar{f}(x,p)$$

Assumptions:

$$f(p) \gg |\delta f(x,p)|$$
$$\bar{f}(p) \gg |\delta \bar{f}(x,p)|$$

$$|\partial_p^\nu f(p)| \gg |\partial_p^\nu \delta f(x,p)|$$
$$|\partial_p^\nu \bar{f}(p)| \gg |\partial_p^\nu \delta \bar{f}(x,p)|$$

How can we acquire the properties of plasma in this method?

By linearizing the Vlasov equation

- How can we find the four-current?
  From the Maxwell equations $\partial_\mu F^{\mu\nu} = j^\nu$:

$$j^\nu(x) = 2e \int \frac{d^3p}{(2\pi)^3} \frac{p^\nu}{E_p} [f(x,p) - \bar{f}(x,p)]$$

- Distribution function as their value in equilibrium plus a small perturbation

$$f(x,p) = f(p) + \delta f(x,p)$$
$$\bar{f}(x,p) = \bar{f}(p) + \delta \bar{f}(x,p)$$

Assumptions:

$$f(p) \gg |\delta f(x,p)|$$
$$\bar{f}(p) \gg |\delta \bar{f}(x,p)|$$

$$|\partial_p^\nu f(p)| \gg |\partial_p^\nu \delta f(x,p)|$$
$$|\partial_p^\nu \bar{f}(p)| \gg |\partial_p^\nu \delta \bar{f}(x,p)|$$

- The linearized Vlasov equation

$$p_\mu \partial^\mu \delta f(x,p) = e p_\mu F^{\mu\nu} \partial_\nu^p f(p)$$
$$p_\mu \partial^\mu \delta \bar{f}(x,p) = -e p_\mu F^{\mu\nu} \partial_\nu^p \bar{f}(p)$$

The linearized Vlasov equation and the four-current

- The linearized Vlasov equation after the Fourier transformation

$$-ip_\mu k^\mu \delta f(p,k) = -iep_\mu [k^\mu A^\nu(k) - k^\nu A^\mu(k)] \partial_\nu^p f(p)$$
$$-ip_\mu k^\mu \delta \bar{f}(p,k) = iep_\mu [k^\mu A^\nu(k) - k^\nu A^\mu(k)] \partial_\nu^p \bar{f}(p)$$

The linearized Vlasov equation and the four-current

- The linearized Vlasov equation after the Fourier transformation

$$-ip_\mu k^\mu \delta f(p,k) = -iep_\mu[k^\mu A^\nu(k) - k^\nu A^\mu(k)]\delta^p_\nu f(p)$$
$$-ip_\mu k^\mu \delta \bar{f}(p,k) = iep_\mu[k^\mu A^\nu(k) - k^\nu A^\mu(k)]\delta^p_\nu \bar{f}(p)$$

- We solve for $\delta f(p,k)$ and $\delta \bar{f}(p,k)$

$$\delta f(p,k) = \frac{e}{p_\rho k\rho} p_\mu(k^\mu A^\nu - k^\nu A^\mu)\partial^p_\nu f(p)$$
$$\delta \bar{f}(p,k) = -\frac{e}{p_\rho k\rho} p_\mu(k^\mu A^\nu - k^\nu A^\mu)\partial^p_\nu \bar{f}(p)$$

The linearized Vlasov equation and the four-current

- The linearized Vlasov equation after the Fourier transformation

$$-ip_\mu k^\mu \delta f(p,k) = -\, iep_\mu[k^\mu A^\nu(k) - k^\nu A^\mu(k)]\delta^p_\nu f(p)$$
$$-ip_\mu k^\mu \delta \bar{f}(p,k) = iep_\mu[k^\mu A^\nu(k) - k^\nu A^\mu(k)]\delta^p_\nu \bar{f}(p)$$

- We solve for $\delta f(p,k)$ and $\delta \bar{f}(p,k)$

$$\delta f(p,k) = \frac{e}{p_\rho k \rho} p_\mu (k^\mu A^\nu - k^\nu A^\mu)\partial^p_\nu f(p)$$
$$\delta \bar{f}(p,k) = -\frac{e}{p_\rho k \rho} p_\mu (k^\mu A^\nu - k^\nu A^\mu)\partial^p_\nu \bar{f}(p)$$

- The four-current

$$j^\sigma(k) = 4e^2 \int \frac{d^3p}{(2\pi)^3} \frac{p^\sigma}{E_p} \frac{p_\mu (k^\mu g^{\nu\lambda} - k^\nu g^{\mu\lambda})}{p_\rho k^\rho} A_\lambda \partial^p_\nu f(p)$$

What the four-current leads us to?

To other quantities useful in the field analysis

The current is connected to the potential by the polarization tensor…

$$j^{\sigma}(k) = -\,\Pi^{\sigma\lambda}(k)A_{\lambda}(k)$$

What the four-current leads us to?

To other quantities useful in the field analysis

The current is connected to the potential by the polarization tensor…

$$j^{\sigma}(k) = -\,\Pi^{\sigma\lambda}(k)A_{\lambda}(k)$$

…which is connected to the dielectric tensor

$$\boxed{\varepsilon^{ij}(\omega,\mathbf{k}) = \delta^{ij} + \frac{1}{\omega^2}\Pi^{ij}(\omega,\mathbf{k})}$$

We see there are many ways of finding quantities necessary to check how the quark-gluon plasma behaves in a magnetic field.

And what form the distribution can have?

Let's think about what constitutes the $e^2 f(p)$

We have two types of quarks, up and down, with the charges equal $\dfrac{2}{3}e$ and $-\dfrac{1}{3}e$

$$\left(\frac{2}{3}e\right)^2 f_u(p) + \left(-\frac{1}{3}e\right)^2 f_d(p) = \frac{5}{9}e^2 f_q(p) = \frac{5}{9}e^2 \frac{8}{e^{\beta E_p}+1}$$

$$\beta = \frac{1}{T}$$

And what form the distribution can have?

Let's think about what constitutes the $e^2 f(p)$

We have two types of quarks, up and down, with the charges equal $\frac{2}{3}e$ and $-\frac{1}{3}e$

$$\left(\frac{2}{3}e\right)^2 f_u(p) + \left(-\frac{1}{3}e\right)^2 f_d(p) = \frac{5}{9}e^2 f_q(p) = \frac{5}{9}e^2 \frac{8}{e^{\beta E_p}+1}$$

$$\beta = \frac{1}{T}$$

And why the 8?

And what form the distribution can have?

Let's think about what constitutes the $e^2 f(p)$

We have two types of quarks, up and down, with the charges equal $\frac{2}{3}e$ and $-\frac{1}{3}e$

$$\left(\frac{2}{3}e\right)^2 f_u(p) + \left(-\frac{1}{3}e\right)^2 f_d(p) = \frac{5}{9}e^2 f_q(p) = \frac{5}{9}e^2 \frac{8}{e^{\beta E_p}+1}$$

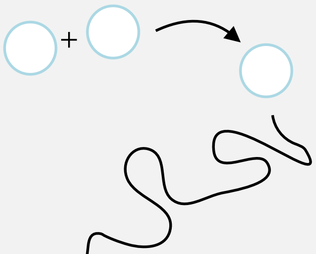
$$\beta = \frac{1}{T}$$

$$f_u(p) = f_d(p) = f_q(p) = \frac{2 \cdot 2 \cdot 2}{e^{\beta E_p}+1}$$

And why the 8?

- 2 types of particles
- 2 possibilities: particle or antiparticle
- 2 possible spin configurations

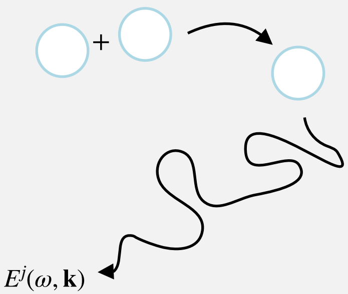
All this talk about the fields, let's see them!



Do you remember how we manipulated the Maxwell's equations to acquire the current density?

$$j^i_{ext}(\omega, \mathbf{k}) = \frac{i}{4\pi}\left[\frac{1}{\omega}(k^i k^j - \mathbf{k}^2\delta^{ij}) + \omega\varepsilon^{ij}(\omega, \mathbf{k})\right]E^j(\omega, \mathbf{k})$$

All this talk about the fields, let's see them!



Do you remember how we manipulated the Maxwell's equations to acquire the current density?

$$j^i_{ext}(\omega, \mathbf{k}) = \frac{i}{4\pi}\left[\frac{1}{\omega}(k^i k^j - \mathbf{k}^2\delta^{ij}) + \omega\varepsilon^{ij}(\omega, \mathbf{k})\right]E^j(\omega, \mathbf{k})$$

We can use it to get the electric field..

$$E^j(\omega, \mathbf{k}) = -\frac{4\pi i}{\omega} j^i_{ext}(\omega, \mathbf{k})\left[\frac{1}{\varepsilon_L(\omega, \mathbf{k})}\frac{k^i k^j}{\mathbf{k}^2} + \frac{1}{\varepsilon_T(\omega, \mathbf{k}) - \frac{\mathbf{k}^2}{\omega^2}}\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)\right]$$

..and the magnetic field!

$$\mathbf{B}(\omega, \mathbf{k}) = \frac{1}{\omega}\mathbf{k} \times \mathbf{E}(\omega, \mathbf{k}) = -\frac{4\pi i}{\omega^2}\frac{1}{\varepsilon_T(\omega, \mathbf{k}) - \frac{\mathbf{k}^2}{\omega^2}}[\mathbf{k} \times \mathbf{j}_{ext}(\omega, \mathbf{k})]$$

I mentioned comparing our equation for the magnetic field with the reference point

Our equation: $\mathbf{B}(\omega, \mathbf{k}) = -4\pi i \dfrac{\mathbf{k} \times \mathbf{j}_{ext}(\omega, \mathbf{k})}{\omega^2 - \mathbf{k}^2}$

Reference point: $\mathbf{B}(t, \mathbf{r}) = \dfrac{e(1 - \mathbf{v}^2)\mathbf{v} \times \mathbf{R}}{\left(\mathbf{R}^2 - (\mathbf{R} \times \mathbf{v})^2\right)^{1/2}}$

I mentioned comparing our equation for the magnetic field with the reference point

$$\text{Our equation: } \mathbf{B}(\omega, \mathbf{k}) = -4\pi i \frac{\mathbf{k} \times \mathbf{j}_{ext}(\omega, \mathbf{k})}{\omega^2 - \mathbf{k}^2}$$

$$\text{Reference point: } \mathbf{B}(t, \mathbf{r}) = \frac{e(1 - \mathbf{v}^2)\mathbf{v} \times \mathbf{R}}{\left(\mathbf{R}^2 - (\mathbf{R} \times \mathbf{v})^2\right)^{1/2}}$$

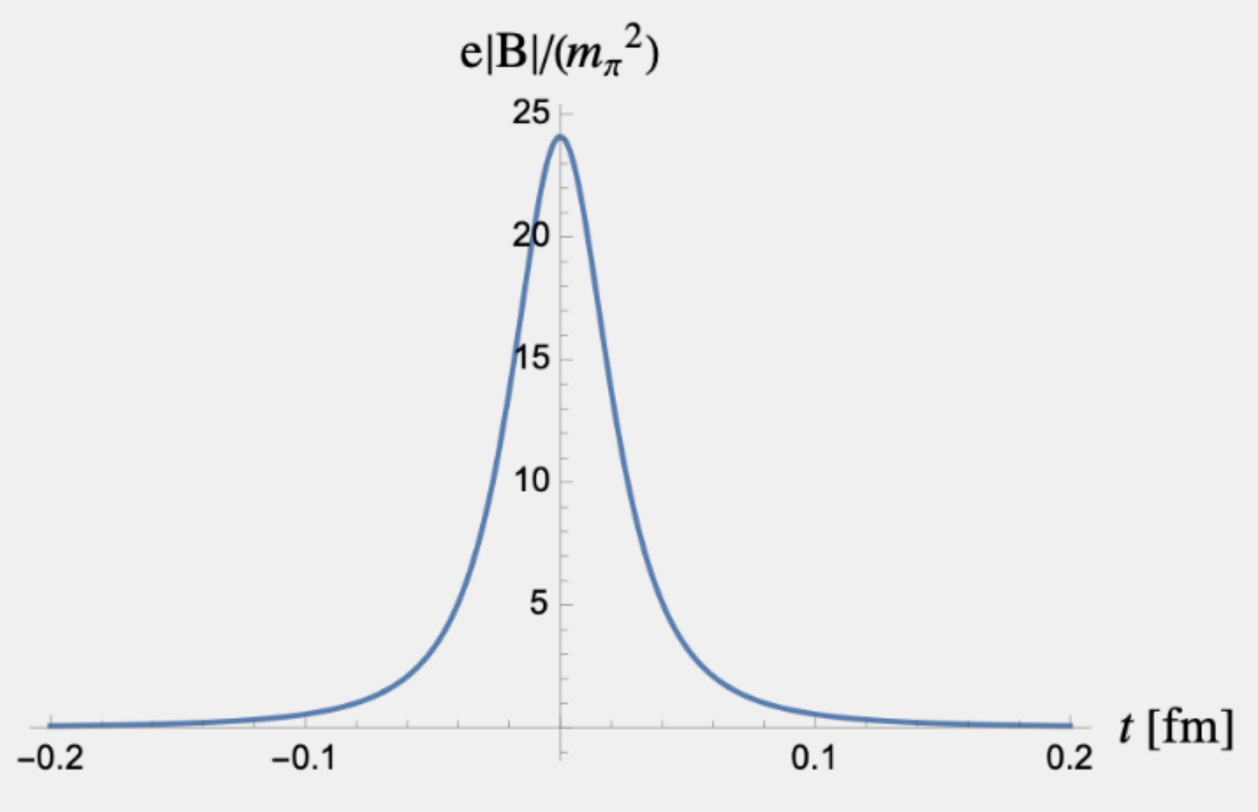
What do we need to do?

- Set $\varepsilon_T = 1$ - for the vacuum
- Set the current density for $\mathbf{j}_{ext}(t, \mathbf{r}) = Ze\mathbf{v}\delta(\mathbf{r} - \mathbf{r}_0 - \mathbf{v}t)$
- Use the Fourier transformation on the magnetic field

$$\boxed{\mathbf{B}(t, \mathbf{k}) = -4\pi i Ze \frac{\mathbf{k} \times \mathbf{v}}{(\mathbf{k} \cdot \mathbf{v})^2 - \mathbf{k}^2} e^{-i(\mathbf{k}\cdot\mathbf{v}t + \mathbf{k}\cdot\mathbf{r}_0)}}$$

Did it work?

Our calculations for the magnetic field in vacuum agree with the reference point!



What about some improvements to the idea?

What do we know up to this point?

- We can study the influence of the magnetic field on various processes during the relativistic heavy ion collisions with production of the quark gluon plasma
- We need to focus on the magnetic field in the medium and choose a certain model for $\varepsilon_T(\omega, \mathbf{k})$

$$\mathbf{B}(\omega, \mathbf{k}) = -\frac{4\pi i}{\omega^2}\frac{1}{\varepsilon_T(\omega, \mathbf{k}) - \frac{\mathbf{k}^2}{\omega^2}}[\mathbf{k} \times \mathbf{j}_{ext}(\omega, \mathbf{k})]$$

What about some improvements to the idea?

What do we know up to this point?

- We can study the influence of the magnetic field on various processes during the relativistic heavy ion collisions with production of the quark gluon plasma
- We need to focus on the magnetic field in the medium and choose a certain model for $\varepsilon_T(\omega, \mathbf{k})$

$$\mathbf{B}(\omega, \mathbf{k}) = -\frac{4\pi i}{\omega^2} \frac{1}{\varepsilon_T(\omega, \mathbf{k}) - \frac{\mathbf{k}^2}{\omega^2}} [\mathbf{k} \times \mathbf{j}_{ext}(\omega, \mathbf{k})]$$

What is something that we can improve?

- This model of the magnetic field due to the heavy ion collisions is unrealistic!
- The magnetic field should be sensitive to the actual run of events (time)
- We worked in $(\omega, \mathbf{k})$ instead $(t, \mathbf{r})$ to be able to use relations like $D^i(\omega, \mathbf{k}) = \varepsilon^{ij}(\omega, \mathbf{k}) E^j(\omega, \mathbf{k})$

Why is it unrealistic?

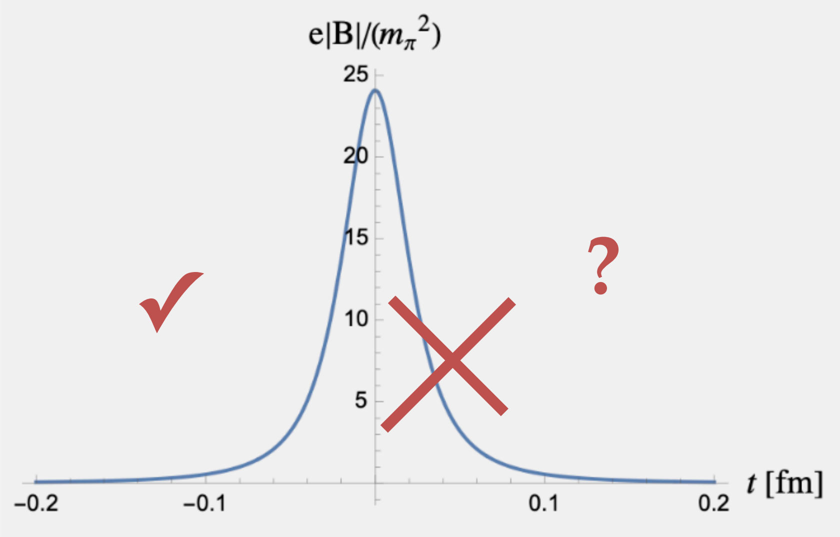
What are the original model assumptions?

- The quark-gluon plasma exists and has electric charges since the beginning of the impact, from $t = -\infty$ to $t = \infty$.

Why is it unrealistic?

What are the original model assumptions?

- The quark-gluon plasma exists and has electric charges since the beginning of the impact, from $t = -\infty$ to $t = \infty$.



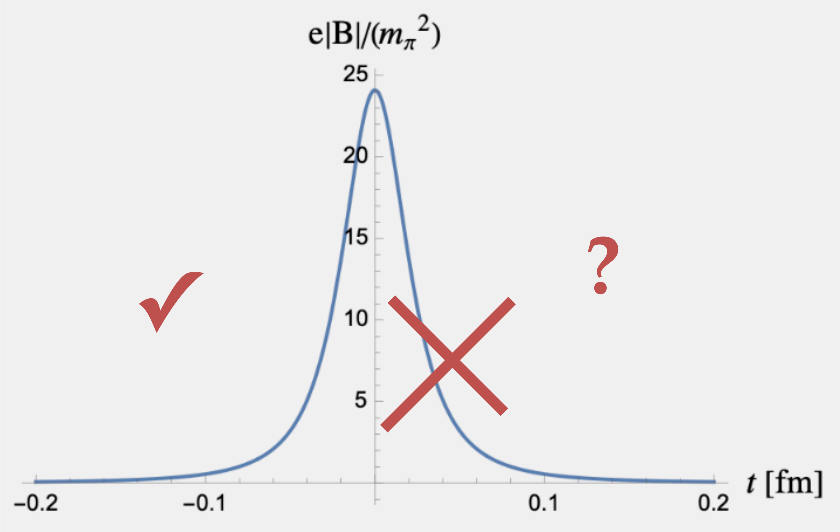
How is it in reality?

- In the first moments of the impact ($t < 0$) there is no plasma. Only the approaching nuclei generate the magnetic field.
- The additional fields generated due to the plasma appear at $t = 0$
- After $t = 0$ we see the effects of electromagnetic field on the quark-gluon plasma.

Why is it unrealistic?

What are the original model assumptions?

- The quark-gluon plasma exists and has electric charges since the beginning of the impact, from $t = -\infty$ to $t = \infty$.



How is it in reality?

- In the first moments of the impact ($t < 0$) there is no plasma. Only the approaching nuclei generate the magnetic field.
- The additional fields generated due to the plasma appear at $t = 0$
- After $t = 0$ we see the effects of electromagnetic field on the quark-gluon plasma.

What should we do about it?

- Take into the consideration the initial field values.

The initial value problem

- We will, again, start with the Maxwell's equations…

$$\begin{cases} \nabla \cdot \mathbf{D}(t, \mathbf{r}) = 4\pi\rho_{ext}(t, \mathbf{r}), \\ \nabla \cdot \mathbf{B}(t, \mathbf{r}) = 0, \\ \nabla \times \mathbf{E}(t, \mathbf{r}) = -\dfrac{\partial \mathbf{B}(t, \mathbf{r})}{\partial t}, \\ \nabla \times \mathbf{B}(t, \mathbf{r}) = 4\pi\mathbf{j}_{ext}(t, \mathbf{r}) + \dfrac{\partial \mathbf{D}(t, \mathbf{r})}{\partial t}. \end{cases}$$

…and manipulate them in an analogous way as before.

- But this time!…

The initial value problem

- …We will use the one-sided Fourier transform
- It will result in the appearance of the initial fields

The one-sided Fourier transform

$$f(\omega, \mathbf{k}) \equiv \int_0^\infty dt \int d^3 r e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})} f(t, \mathbf{r})$$

$$f(t, \mathbf{r}) = \int_{-\infty + i\sigma}^{\infty + i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3 k}{(2\pi)^3} e^{-(\omega t - \mathbf{k}\cdot\mathbf{r})} f(\omega, \mathbf{k})$$

The real parameter $\sigma > 0$ is chosen is such a way that the integral over ω is taken along a straight line in the complex ω-plane, parallel to the real axis, above all singularities of $f(\omega, \mathbf{k})$.

The initial value problem

- How do the Maxwell's equations change?

$$
\begin{cases}
ik^i \varepsilon^{ij}(\omega, \mathbf{k}) E^j(\omega, \mathbf{k}) = 4\pi \rho_{ext}(\omega, \mathbf{k}), \\
ik^i B^i(\omega, \mathbf{k}) = 0, \\
i\epsilon^{ijk} k^j E^k(\omega, \mathbf{k}) = i\omega B^i(\omega, \mathbf{k}) + B_0^i(\mathbf{k}), \\
i\epsilon^{ijk} k^j B^k(\omega, \mathbf{k}) = 4\pi j_{ext}^i(\omega, \mathbf{k}) - i\omega D^i(\omega, \mathbf{k}) - D_0^i(\mathbf{k}) \,.
\end{cases}
$$

The initial fields have appeared!

The initial value problem

- The electromagnetic field has changed as well

$$E^i(\omega, \mathbf{k}) = -\, i\left[\frac{1}{\omega^2\varepsilon_L(\omega, \mathbf{k})}\frac{k^ik^j}{\mathbf{k}^2} + \frac{1}{\omega^2\varepsilon_T(\omega, \mathbf{k}) - \mathbf{k}^2}\left(\delta^{ij} - \frac{k^ik^j}{\mathbf{k}^2}\right)\right] \times$$

$$\times\left[4\pi\omega j^j_{ext}(\omega, \mathbf{k}) + \epsilon^{ijk}k^j B^k_0(\mathbf{k}) - \omega D^j_0(\mathbf{k})\right]$$

$$B^i(\omega, \mathbf{k}) = -\frac{i}{\omega^2\varepsilon_T(\omega, \mathbf{k}) - \mathbf{k}^2}\left[\epsilon^{ijk}k^j\left(4\pi j^k_{ext}(\omega, \mathbf{k}) - D^k_0(\mathbf{k})\right) - \frac{\mathbf{k}^2}{\omega}B^i_0(\mathbf{k})\right] + \frac{i}{\omega}B^i_0(\mathbf{k})$$

The initial value problem

$\boxed{\text{How do we find the initial fields?}}$

We only need to remember our conditions:

- At $t < 0$ there is no plasma $\Longrightarrow \mathbf{D}_0(\mathbf{k}) = \mathbf{E}_0(\mathbf{k})$
- The initial fields are generated at $t = 0$
- We consider a collision in a vacuum $\Longrightarrow \varepsilon^{ij}(\omega, \mathbf{k}) = \delta^{ij}$, $\varepsilon_T = \varepsilon_L = 1$

The initial value problem

How do we find the initial fields?

We only need to remember our conditions:

- At $t < 0$ there is no plasma $\Longrightarrow \mathbf{D}_0(\mathbf{k}) = \mathbf{E}_0(\mathbf{k})$
- The initial fields are generated at $t = 0$
- We consider a collision in a vacuum $\Longrightarrow \varepsilon^{ij}(\omega, \mathbf{k}) = \delta^{ij}$, $\varepsilon_T = \varepsilon_L = 1$

This gives us:

$$\mathbf{E}_0(\mathbf{k}) = -4\pi i \int \frac{d\omega}{2\pi}\left[\frac{\mathbf{k}\left(\mathbf{k}\cdot\mathbf{j}_{ext}(\omega,\mathbf{k})\right)}{\omega(\omega^2 - \mathbf{k}^2)} + \frac{\omega\mathbf{j}_{ext}(\omega,\mathbf{k})}{\omega^2 - \mathbf{k}^2}\right]$$

$$\mathbf{B}_0(\mathbf{k}) = -4\pi i \int \frac{d\omega}{2\pi}\frac{\mathbf{k}\times\mathbf{j}_{ext}(\omega,\mathbf{k})}{\omega^2 - \mathbf{k}^2}$$

The initial value problem

How do we check if the calculated initial fields are correct?

The initial value problem

How do we check if the calculated initial fields are correct?

- We put the found initial conditions into the equation for the magnetic field acquired through the one-sided Fourier transform
- While remembering to transform the current accordingly $\mathbf{j}_{ext}(\omega, \mathbf{k}) = i \frac{Ze\mathbf{v}e^{-i\mathbf{k}\cdot\mathbf{r}_0}}{\omega - \mathbf{k}\cdot\mathbf{v} + i0^+}$
- It will give us the magnetic field at the beginning of events - without the plasma

The initial value problem

How do we check if the calculated initial fields are correct?

- We put the found initial conditions into the equation for the magnetic field acquired through the one-sided Fourier transform.
- While remembering to transform the current accordingly $\mathbf{j}_{ext}(\omega, \mathbf{k}) = i \dfrac{Ze\mathbf{v}e^{-i\mathbf{k}\cdot\mathbf{r}_0}}{\omega - \mathbf{k}\cdot\mathbf{v} + i0^+}$
- It will give us the magnetic field at the beginning of events - without the plasma

$$\mathbf{B}(t, \mathbf{k}) = -4\pi i Ze \frac{\mathbf{k}\times\mathbf{v}}{(\mathbf{k}\cdot\mathbf{v})^2 - \mathbf{k}^2} e^{-i(\mathbf{k}\cdot\mathbf{v}t + \mathbf{k}\cdot\mathbf{r}_0)}$$

The calculations give identical result to the one acquired through the two-sided Fourier transform, as in our first case!

Our calculations must be going in a right direction.

Conclusions

- Electromagnetic field is important in the presence of the strong interaction.

- A way to study the electromagnetic field is to analyze it as it appears briefly during the relativistic heavy ion collisions and is sustained by the currents in the **quark-gluon plasma** in agreement with the Faraday's law.

- There are theoretical predictions of various phenomena influenced by the magnetic field, but experimentally its effects are not seen. Why?

- We suggest that the the theoretical calculations could be done more precisely taking into consideration a more realistic run of events during the heavy ion collisions. This could bring the theoretical and experimental results closer to an agreement.

- A way to include the more precise model is to acquire the magnetic field formula using the **one-sided Fourier transform**.